Spiral wave chaos: triggers, drivers, rotors, and wavelets

Roman Grigoriev

Georgia Institute of Technology

Outline

- Interaction between spiral waves
- Topological analysis of fibrillation
- Can ablation make things worse?

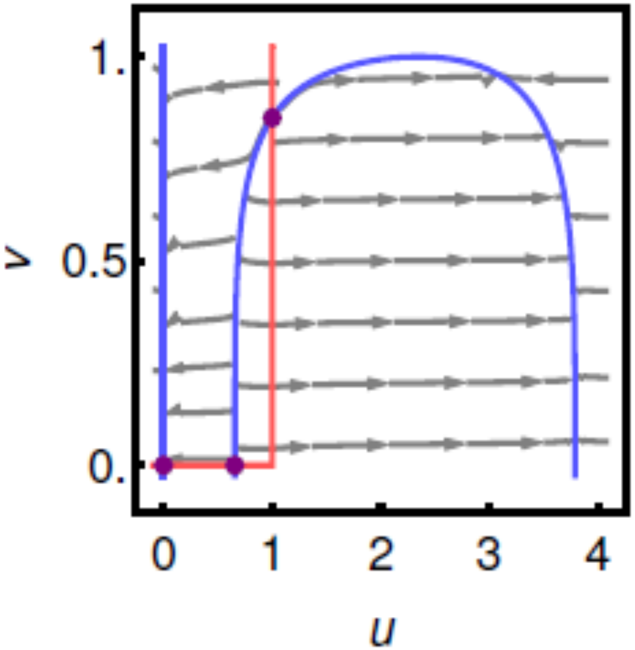
The Karma model

$$\partial_t u = D_u \nabla^2 u + \left(u_0 - v^4\right)\left[1 - \tanh(u-3)\right]\frac{u^2}{2} - v$$

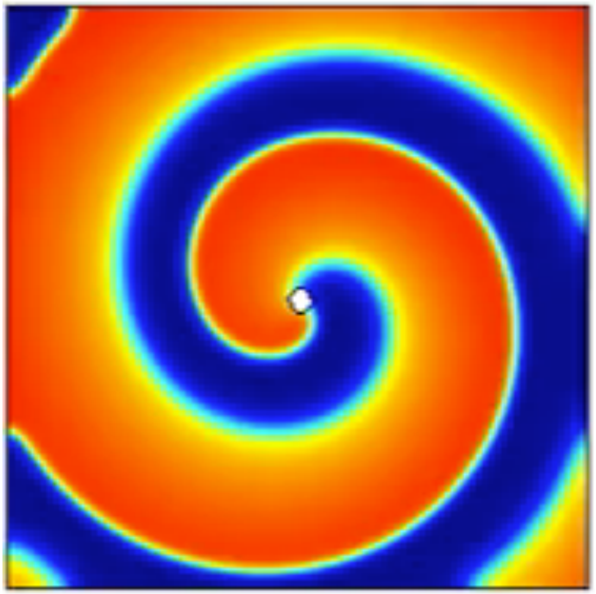
$$\partial_t v = \varepsilon\left[\beta\Theta_s(u-1) - v\right]$$

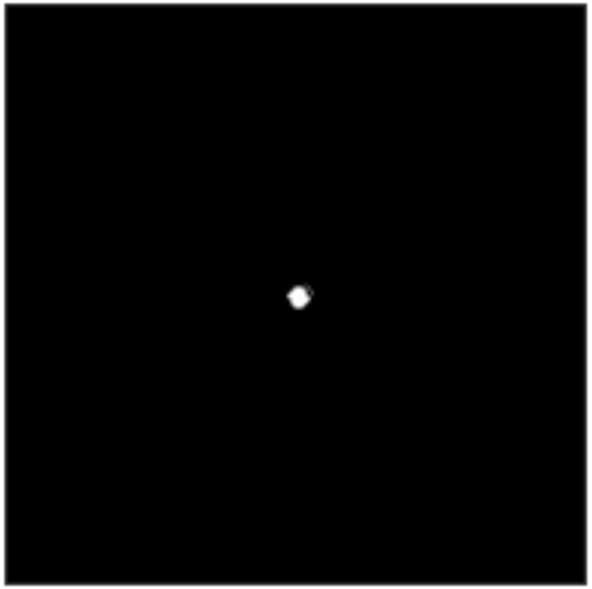
Restitution parameter:

$$\beta = \frac{1}{1 - e^{-R}}$$



Breakups and mergers in the Karma model



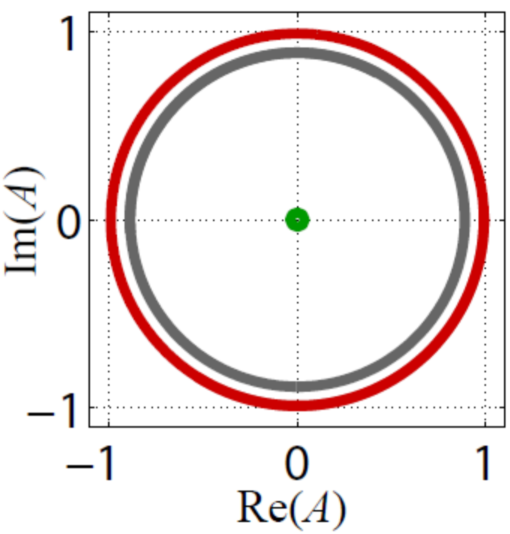


Plane and spiral waves unstable to alternans ($R > 1$)

Tiling

Complex Ginzburg-Landau Equation:

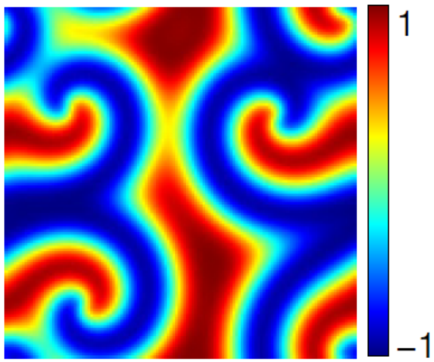
$$\partial_t A = A + (1+i\alpha)\nabla^2 A - (1+i\beta)|A|^2 A$$



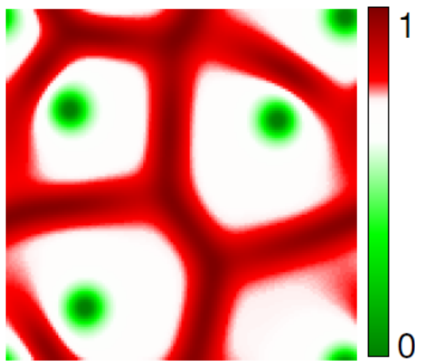
Amplitude/phase representation:

$$A = \rho e^{i\varphi}$$

Bohr, Huber, Ott (1996)



Re A

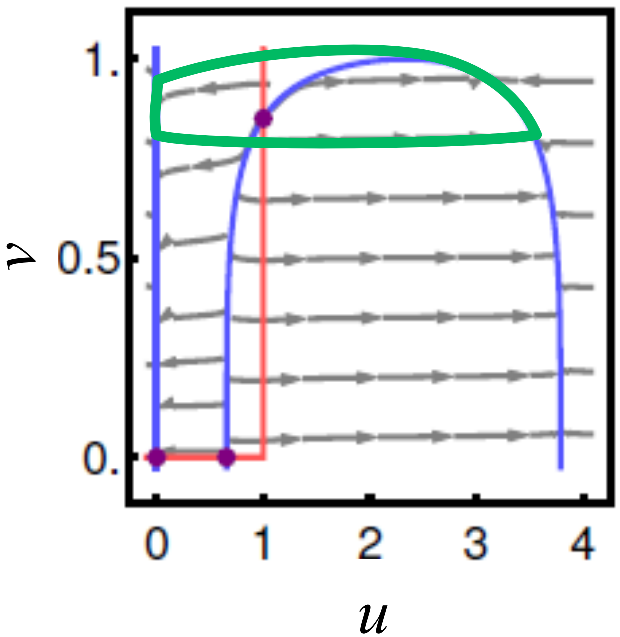


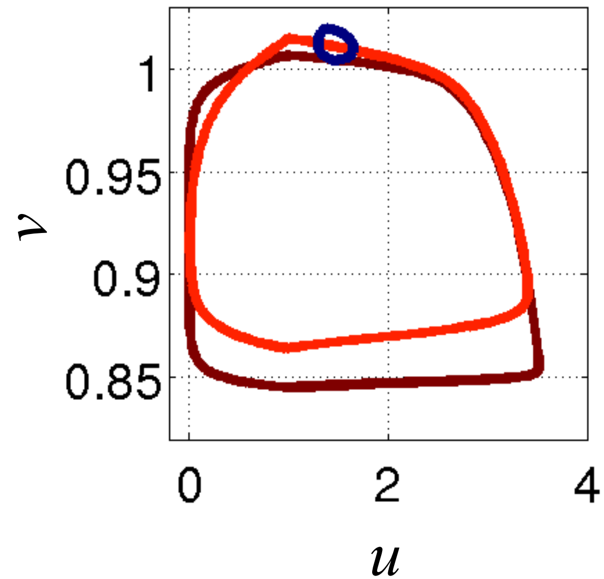
$|A|$

Amplitude Equation:

$$\partial_t \rho = [\nabla^2 - (\nabla\phi)^2]\rho - \alpha(2\nabla\phi\cdot\nabla\rho + \rho\nabla^2\phi) + (1-\rho^2)\rho$$

Strongly nonlinear waves



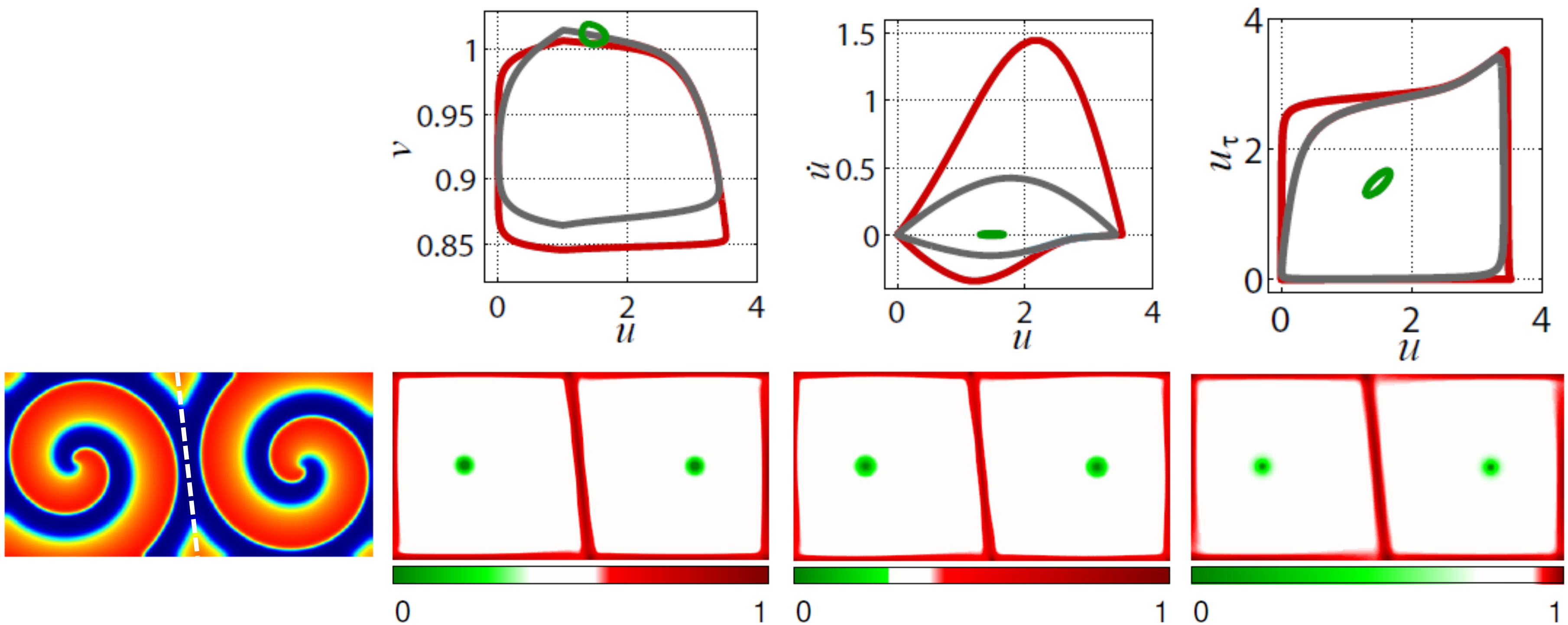


Use cycle area instead of amplitude: $I = \oint v\,du = \int_0^T v\dot{u}\,dt$

and elapsed time from crossing a Poincare section instead of phase: $\theta = \int \omega dt, \quad \omega = \frac{2\pi}{T}$

Alternative representations

$$I = \int v\, du \qquad I = \int \dot{u}\, du \qquad I = \int u_\tau\, du$$



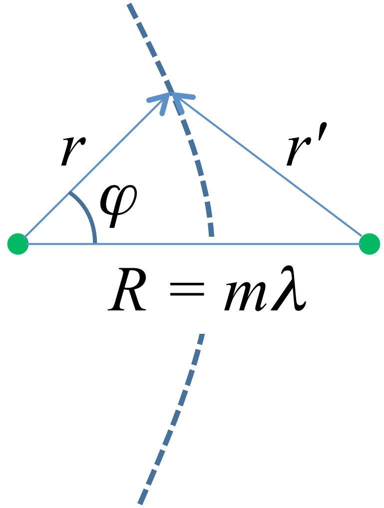
Byrne, Marcotte, Grigoriev (2014)

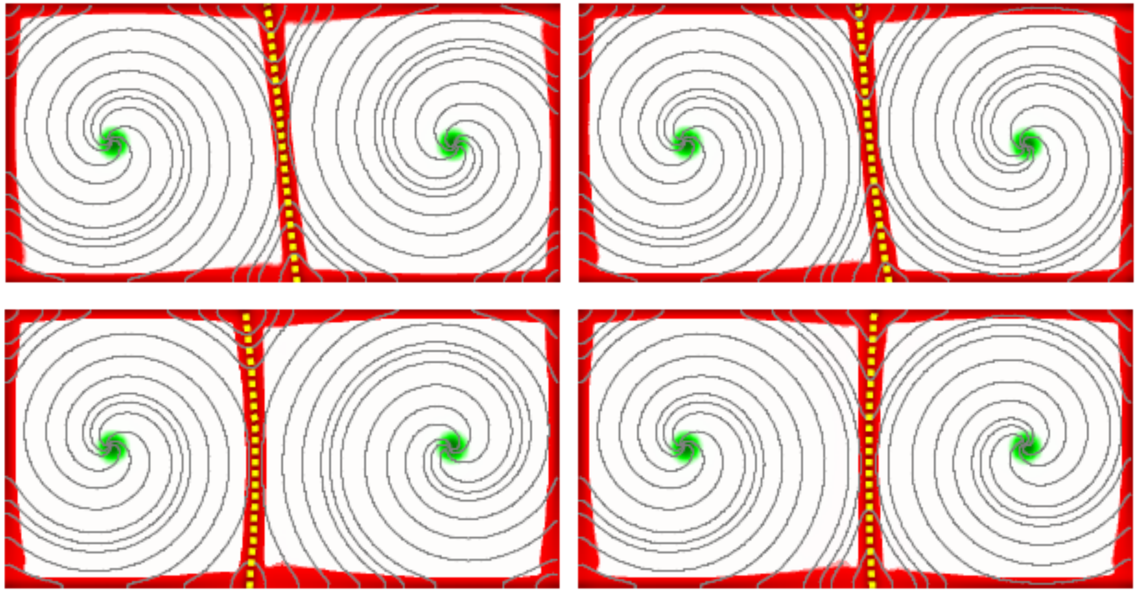
Tile boundaries

Can describe tile boundaries analytically:

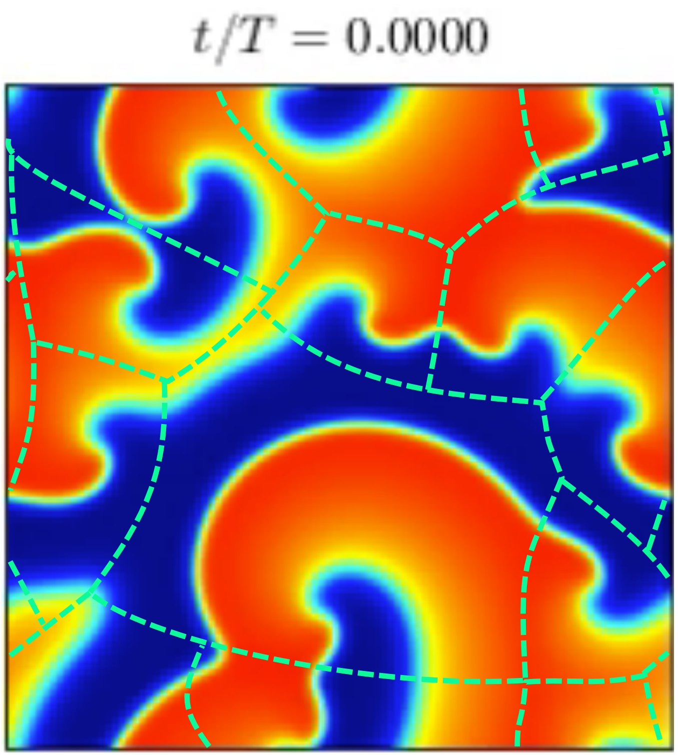
$$\frac{dr}{d\varphi} = \frac{-\sigma r'^2 - \sigma' r(R\cos\varphi - r) + 2\pi m r' r \sin\varphi}{\sigma' R^2 \sin\varphi + m\left(r'^2 - r'(r - R\cos\varphi)\right)}$$

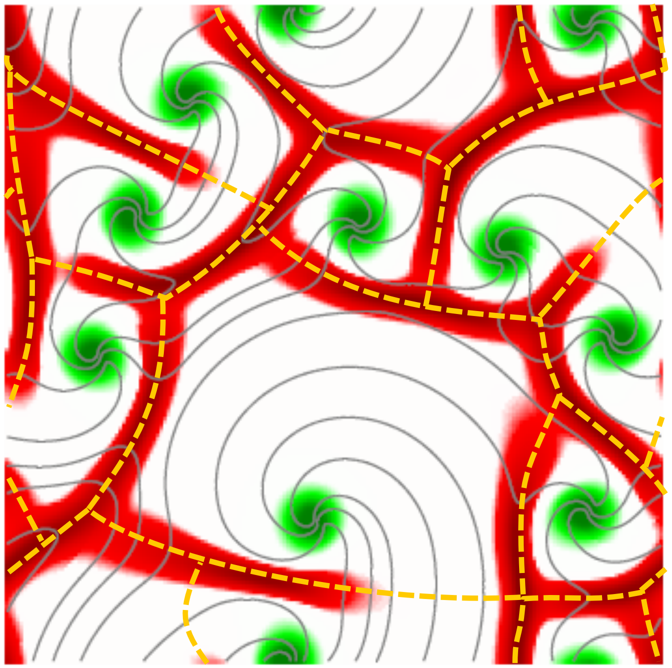
Luo, Zhang, Zhan (2009)





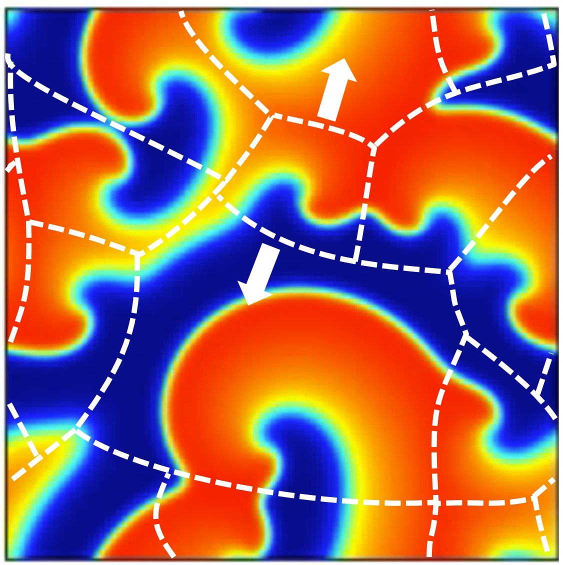
Tiling spiral wave chaos

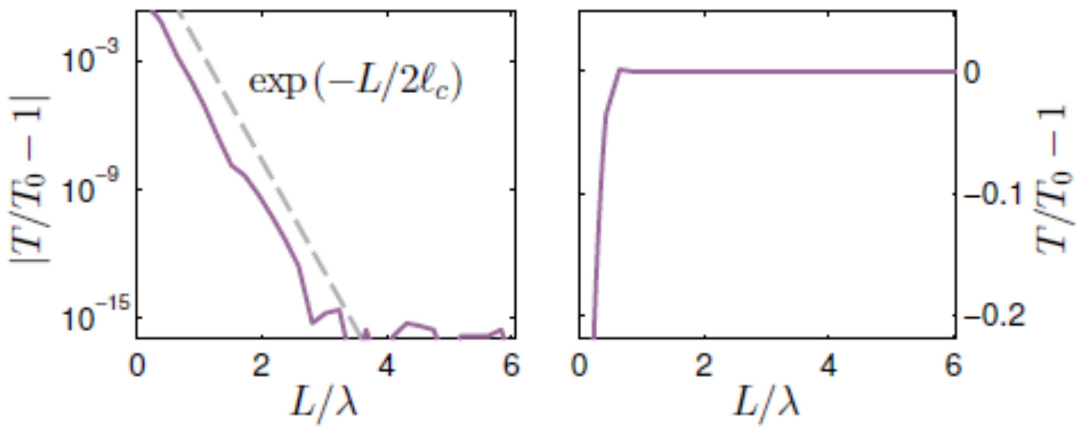




- Tiles are noncircular
- Neumann boundary conditions

Dynamics of the tiles



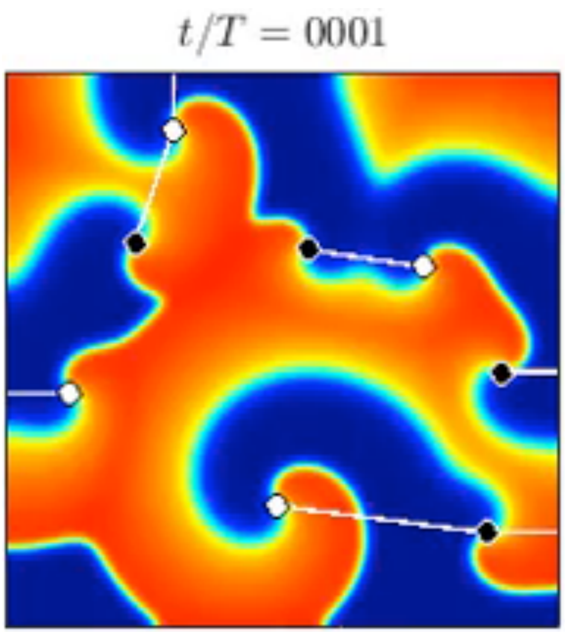


Motion of boundaries:

$$\mathbf{c} = (\omega_1 - \omega_2)\frac{\mathbf{k}_1 - \mathbf{k}_2}{|\mathbf{k}_1 - \mathbf{k}_2|^2}$$

Howard, Kopell (1977)

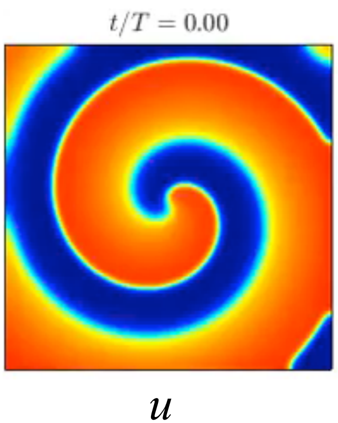
Dynamics of the spirals

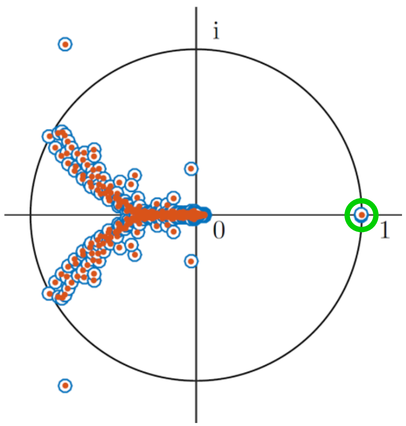


Stroboscopic map

- Why are some cores moving (and others are not)?
- Why is their motion so slow?
- What sets the distance between cores?

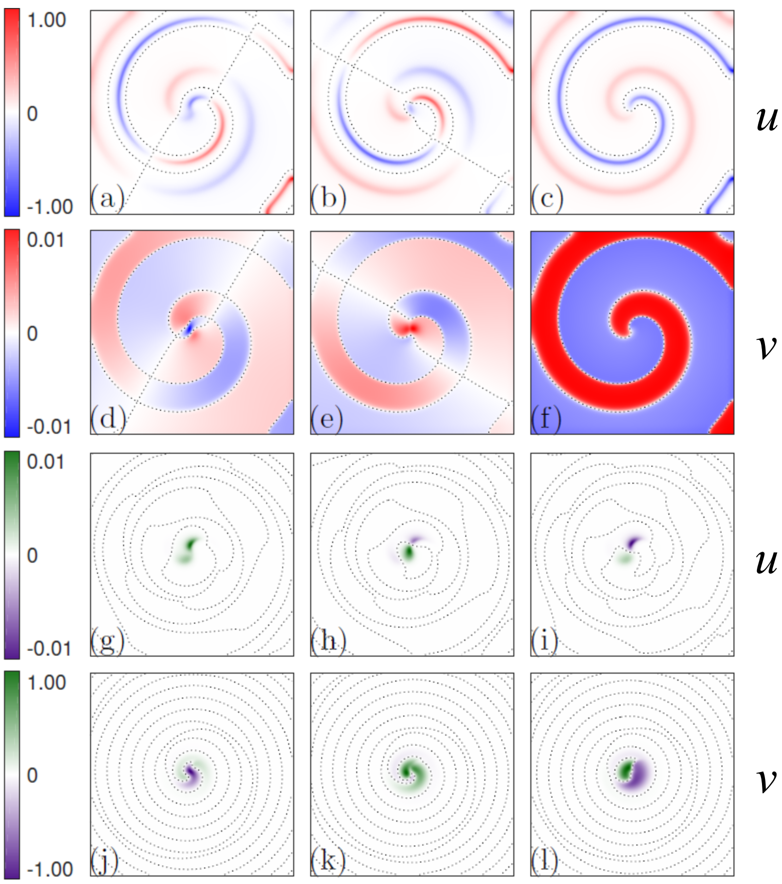
Local Euclidean symmetries





Floquet multipliers

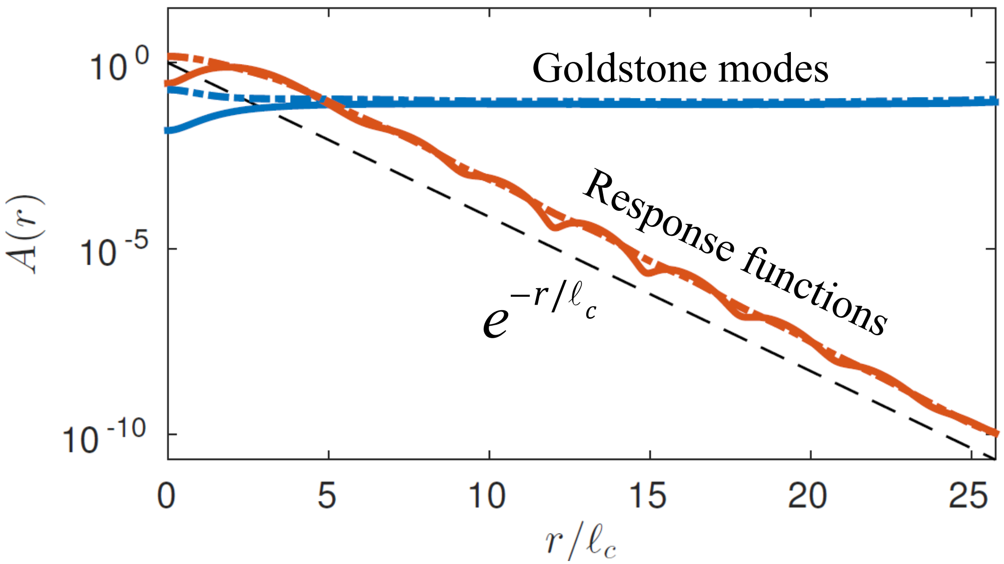
Goldstone modes

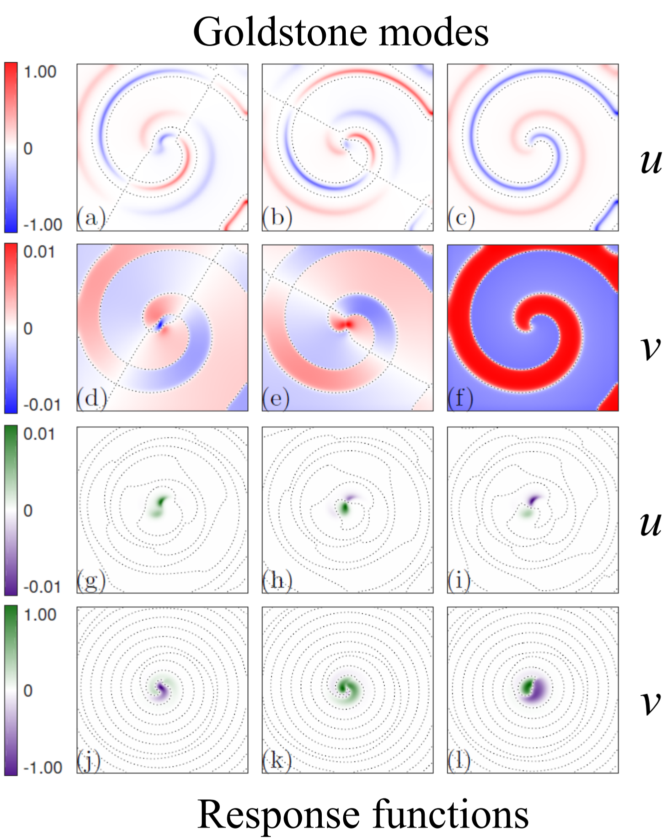


Response functions

Marcotte, Grigoriev (2016)

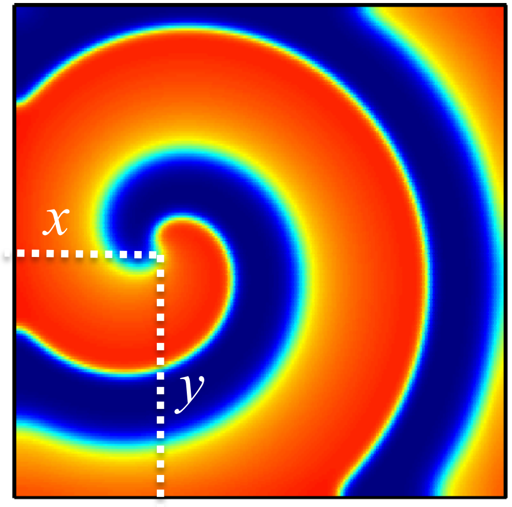
Local Euclidean symmetries

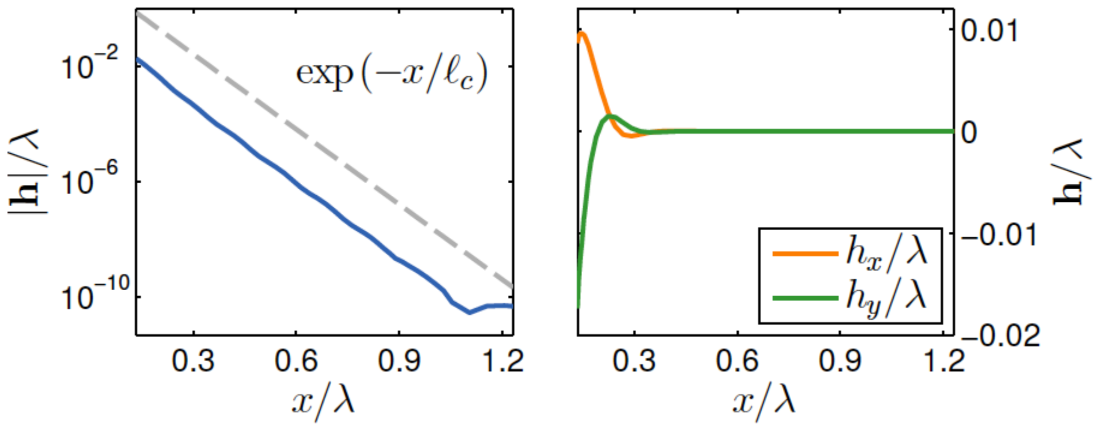




Marcotte, Grigoriev (2016)

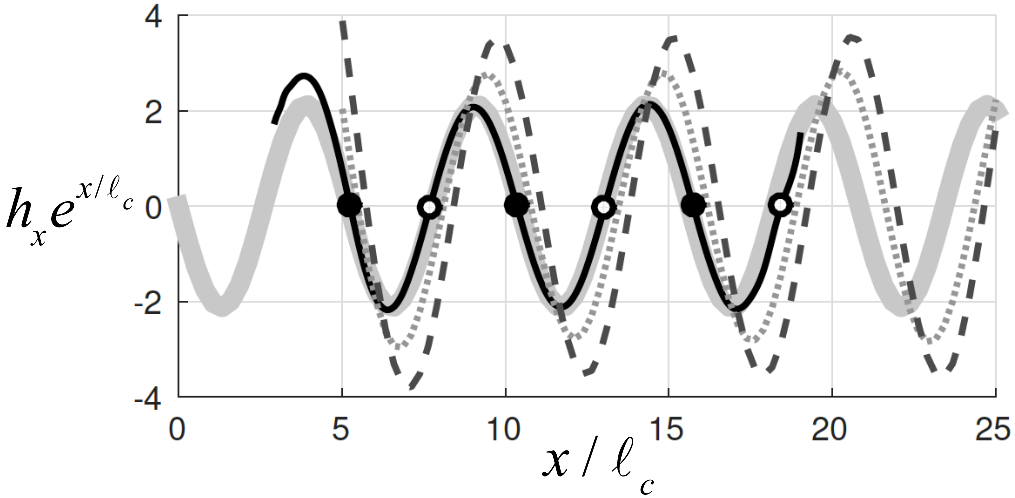
Interaction of cores with boundaries





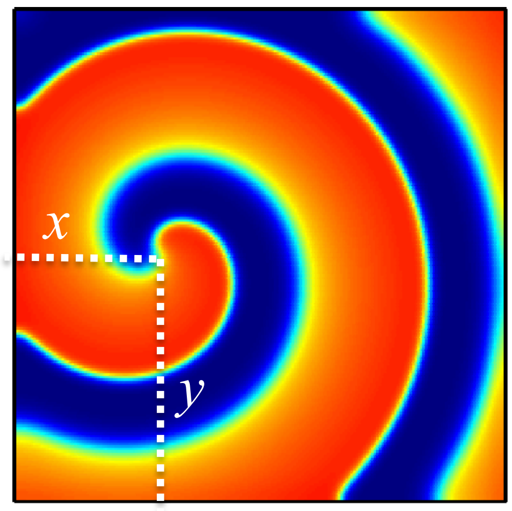
$$\mathbf{x}^{n+1} = \mathbf{x}^n + \mathbf{h}(\mathbf{x}^n),$$

$$\mathbf{x}^n = \mathbf{x}(nT)$$



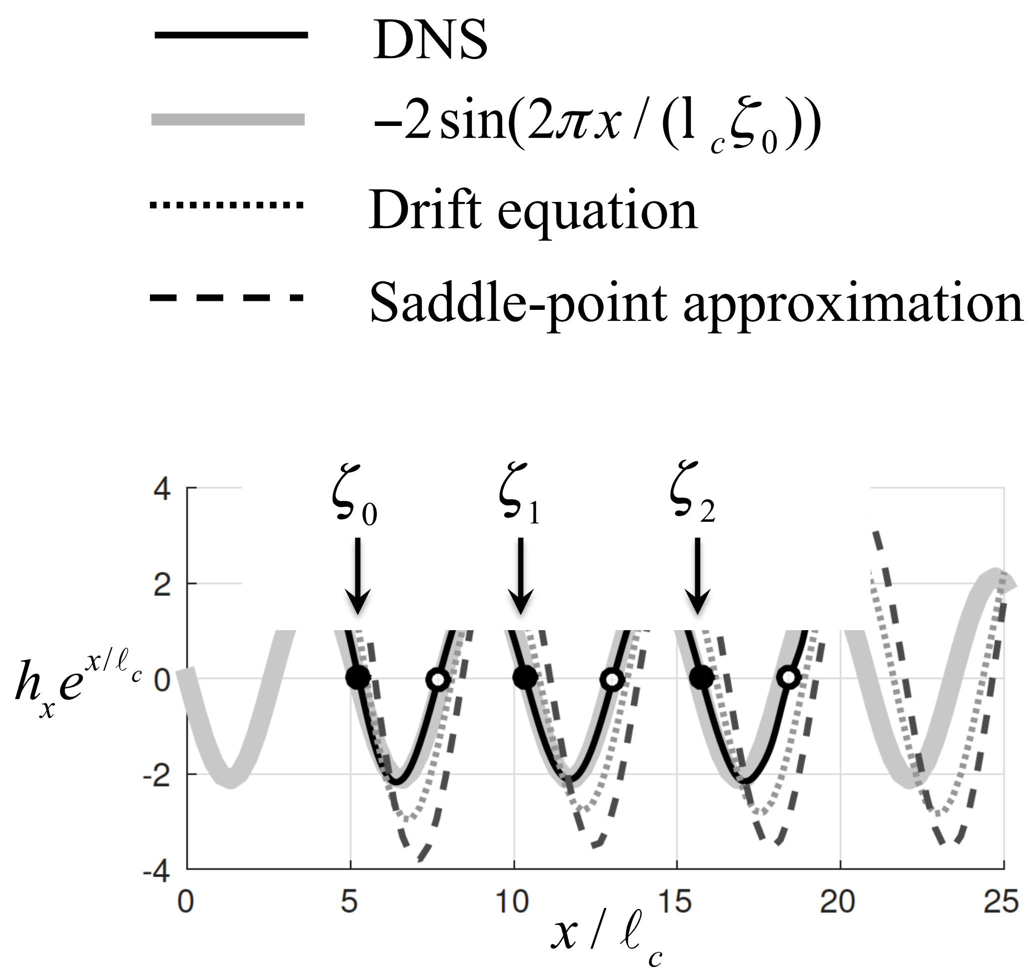
Marcotte, Grigoriev (2016)

Interaction of cores with boundaries

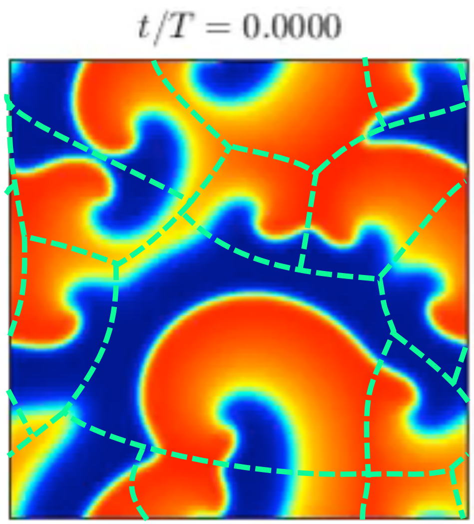


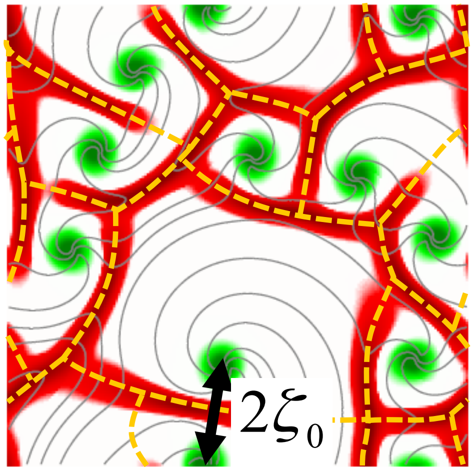
$$\mathbf{x}^{n+1} = \mathbf{x}^{n} + \mathbf{h}(\mathbf{x}^{n}),$$
$$\mathbf{x}^{n} = \mathbf{x}(nT)$$

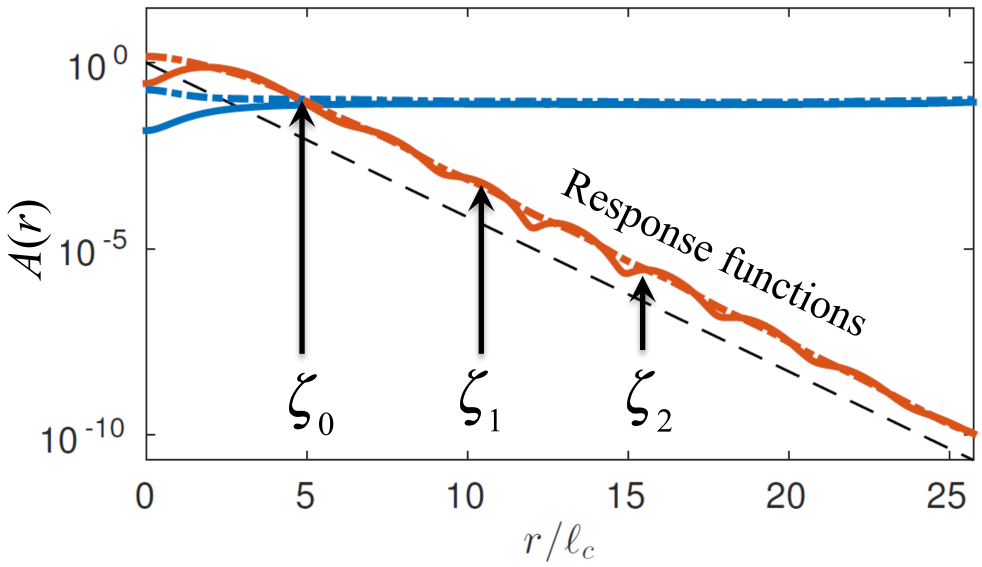
Marcotte, Grigoriev (2016)



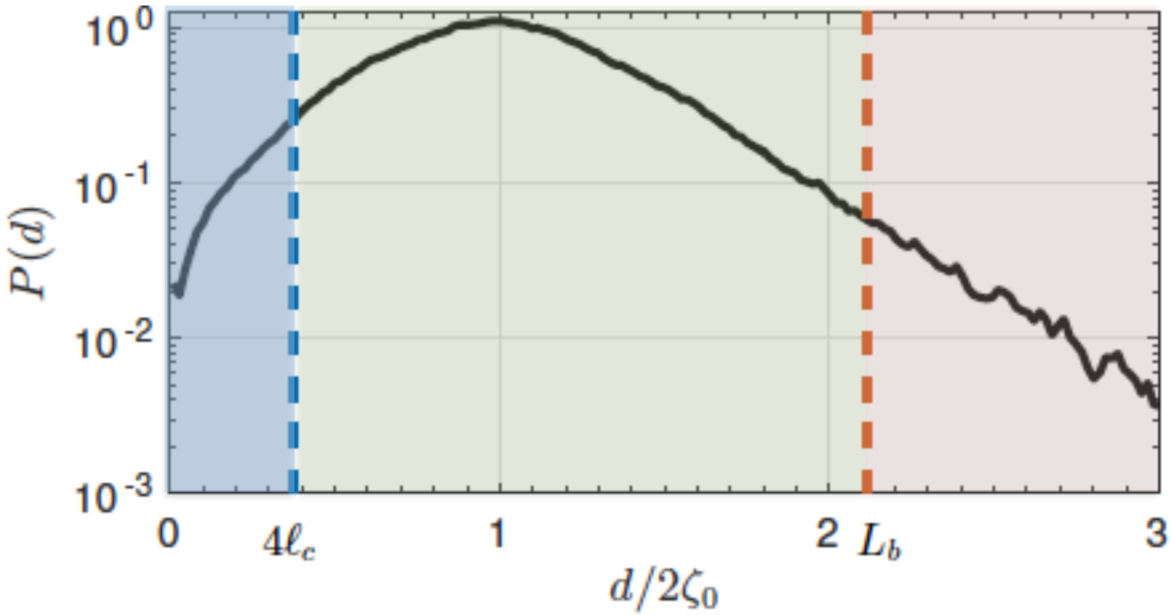
Core-core interaction







Core-core separation (& tile size)



No time-periodic solutions

Core drift/ meander

Solutions unstable (alternans)

The mechanism of spiral chaos

Tiles

- The tiles are of different size
- The frequencies of spirals differ
- The tiles boundaries drift (slowly)
- Small (fast) spirals grow at the expense of big (slow) ones

Spirals

- The dynamics of spirals is fast
- Small spirals ($L < 4l_c$) survive for less than one period
- Medium size spirals ($4l_c < L < \lambda$) merge as a result of meandering instability
- Large spirals ($L > \lambda$) break up due to alternans instability*

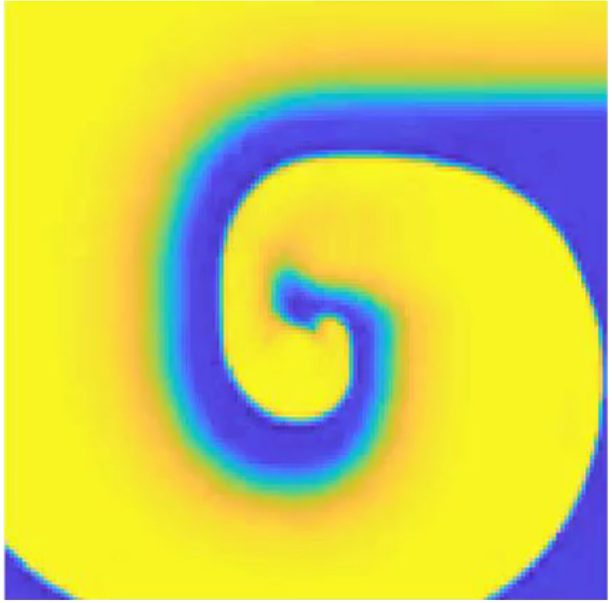
* for the Karma model

Outline

- Interaction between spiral waves
- Topological analysis of fibrillation
- Can ablation make things worse?

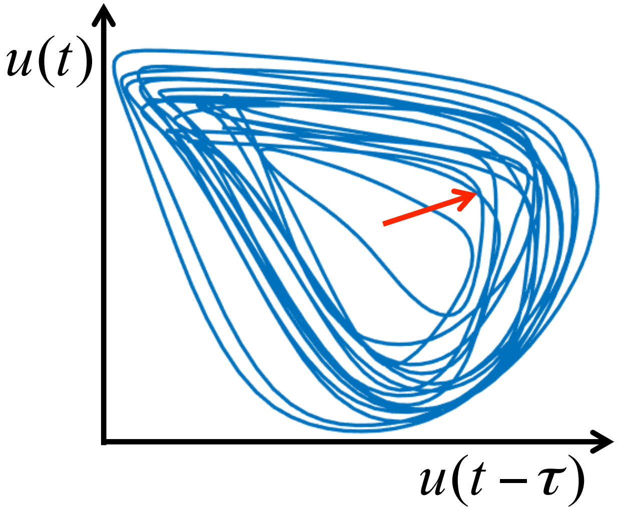
“Tachycardia” → “fibrillation” (2D)

Mitchell-Schaeffer model

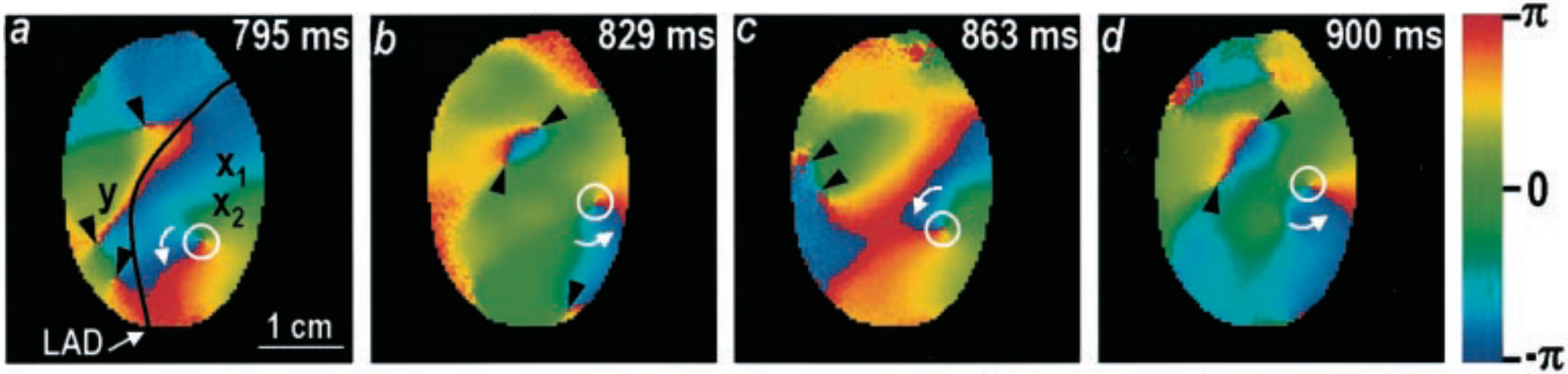


Stefanos Folias (2007)

Phase singularities



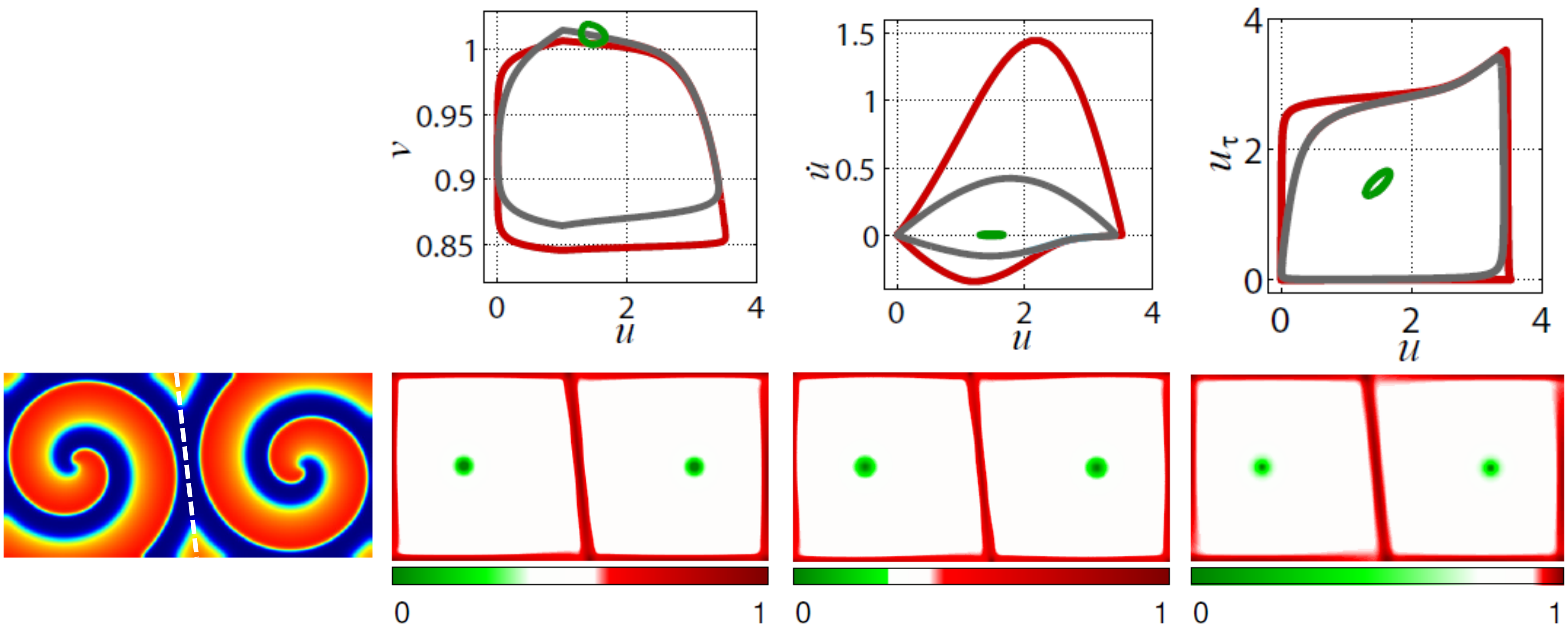
$$q = \frac{1}{2\pi}\oint d\varphi = \begin{cases} -1, & \text{PS (clockwise)} \\ 0, & \text{no PS inside} \\ +1, & \text{PS (counter-clockwise)} \end{cases}$$



Wu et al. (2004)

Amplitude description

$$I = \int v\,du \qquad I = \int \dot{u}\,du \qquad I = \int u_\tau\,du$$



Byrne, Marcotte, Grigoriev (2014)

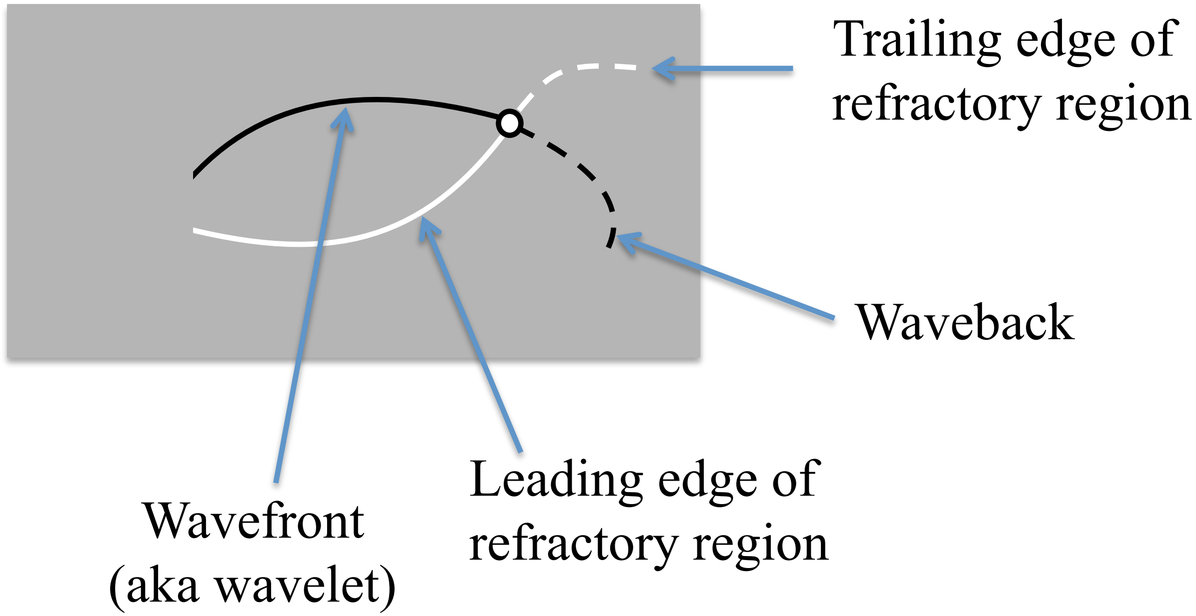
Topological description

- Complexity of the excitation pattern can be quantified by the number of phase singularities (PS)

Marcotte & Grigoriev (2017)

Topological description

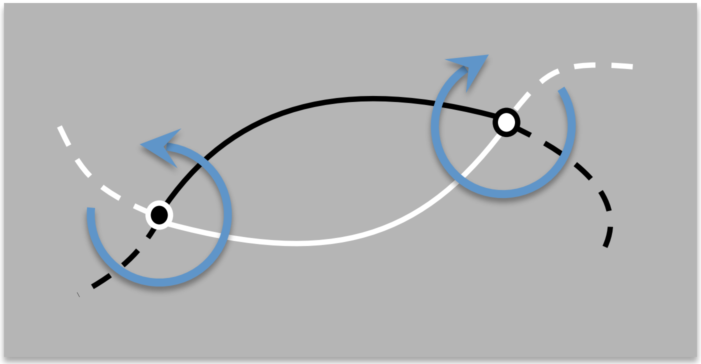
- Complexity of the excitation pattern can be quantified by the number of phase singularities (PS)
- Each PS lies at the intersection of two level sets (e.g., $\partial_t u = 0$ and $\partial_t v = 0$ or $\varphi = \varphi_1$ and $\varphi = \varphi_2$ or $\partial_t u = 0$ and $\partial_t^2 u = 0$).



Marcotte & Grigoriev (2017)

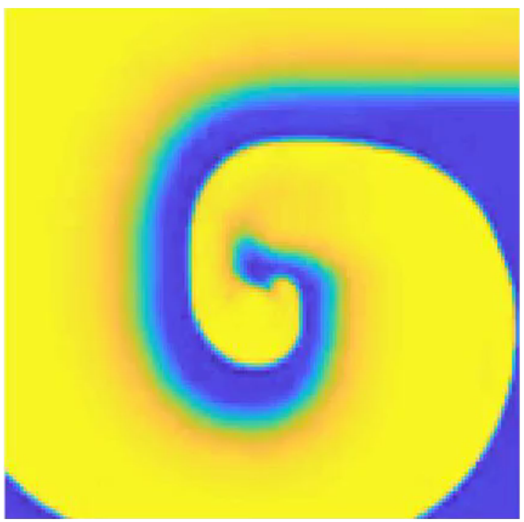
Topological description

- Complexity of the excitation pattern can be quantified by the number of phase singularities (PS)
- Each PS lies at the intersection of two level sets (e.g., $\partial_t u = 0$ and $\partial_t v = 0$ or $\partial_t u = 0$ and $\partial_t^2 u = 0$).

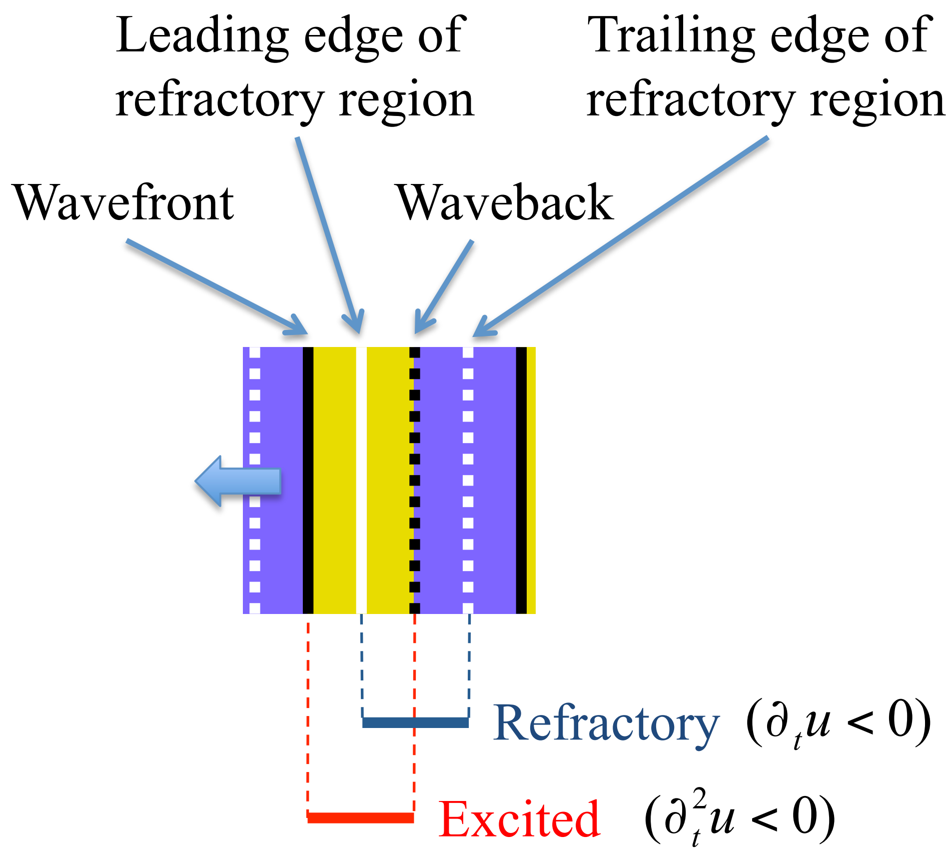


- Each PS has a topological charge: $q = \text{sign}(\hat{\mathbf{z}} \cdot \nabla d_1 \times \nabla d_2) = \pm 1$
- The net topological charge is conserved*: $\sum_i q_i = 0$
- Phase singularities can only be created/destroyed in pairs*

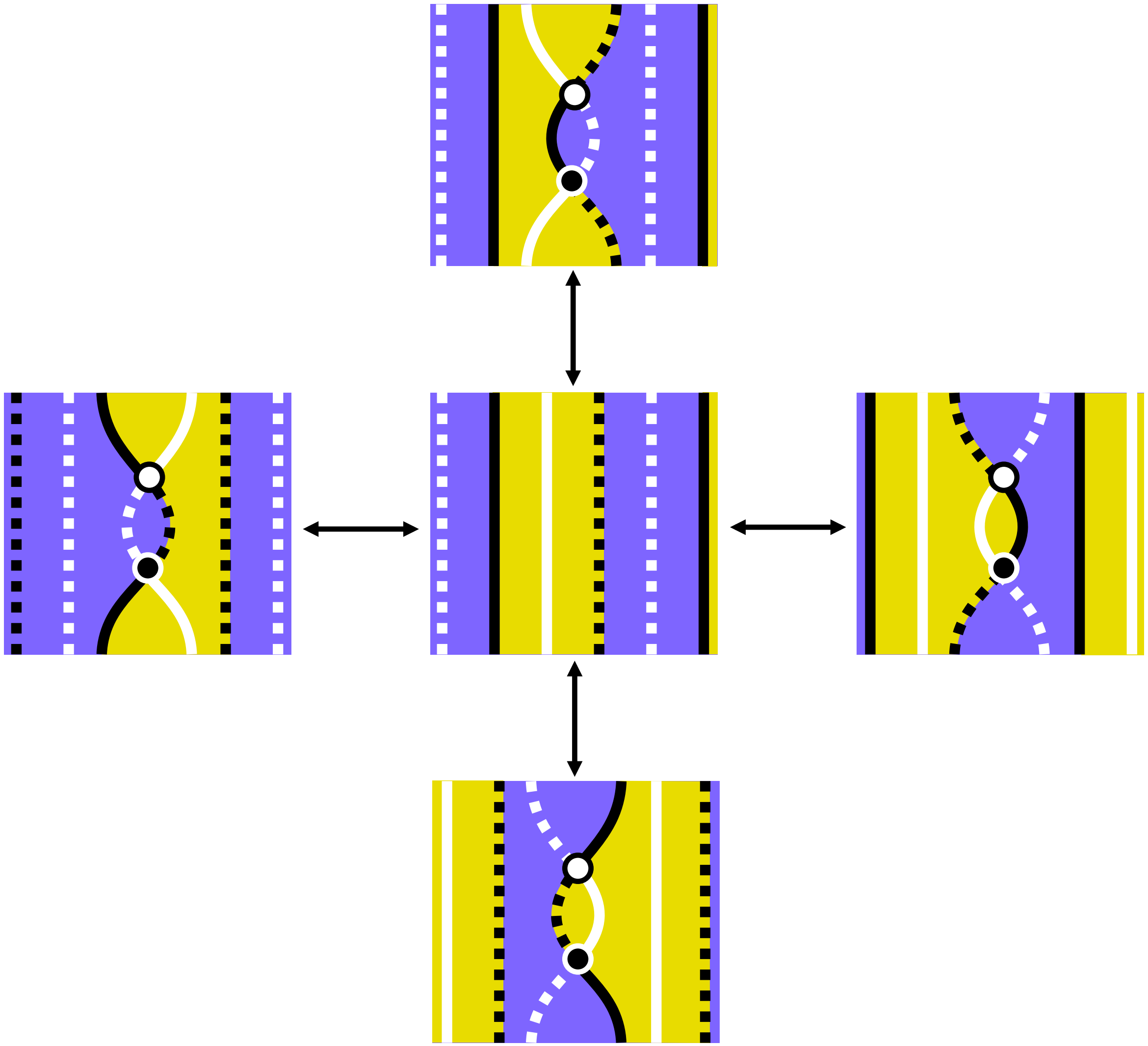
Topological transitions



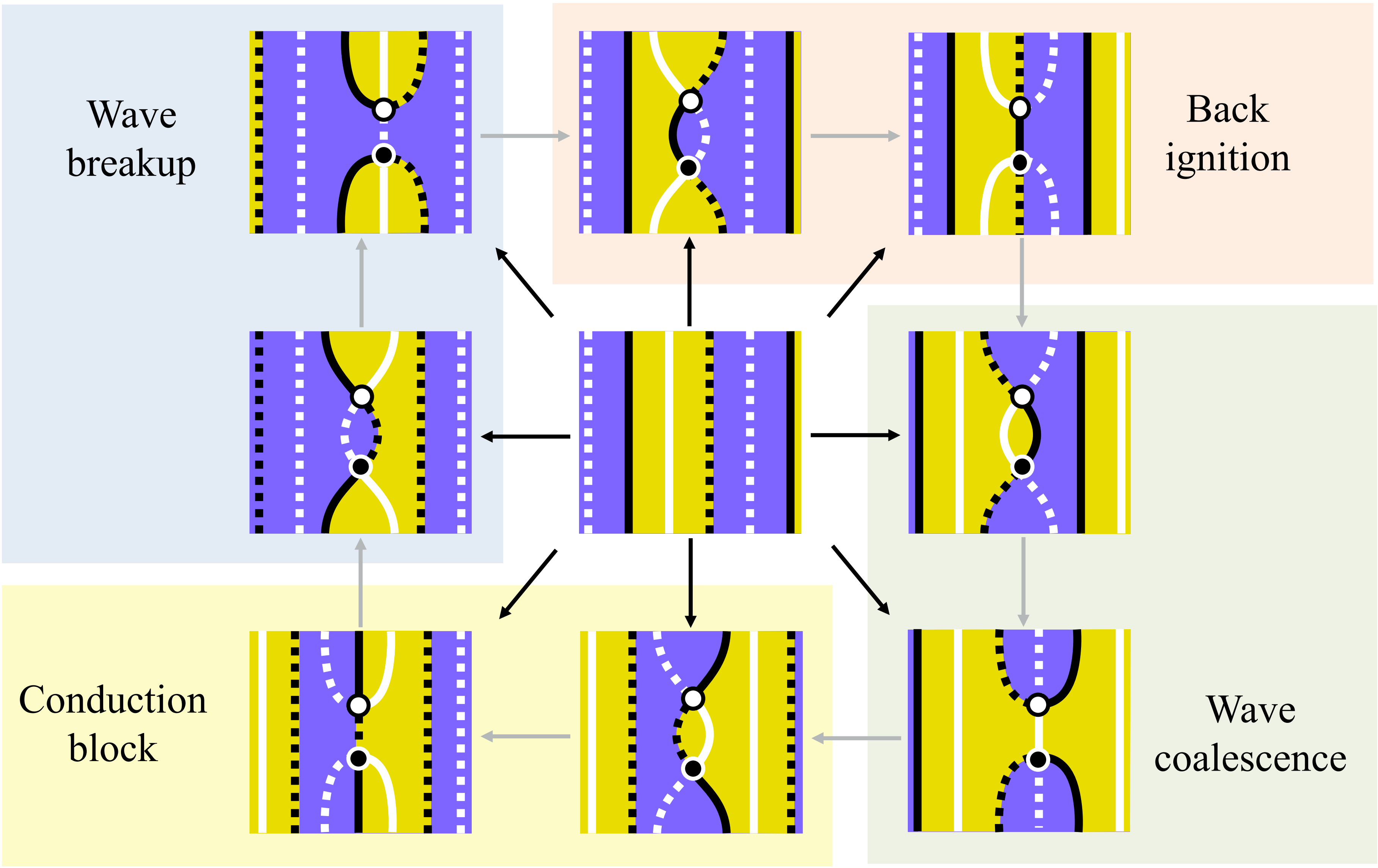
Mitchell-Schaeffer model



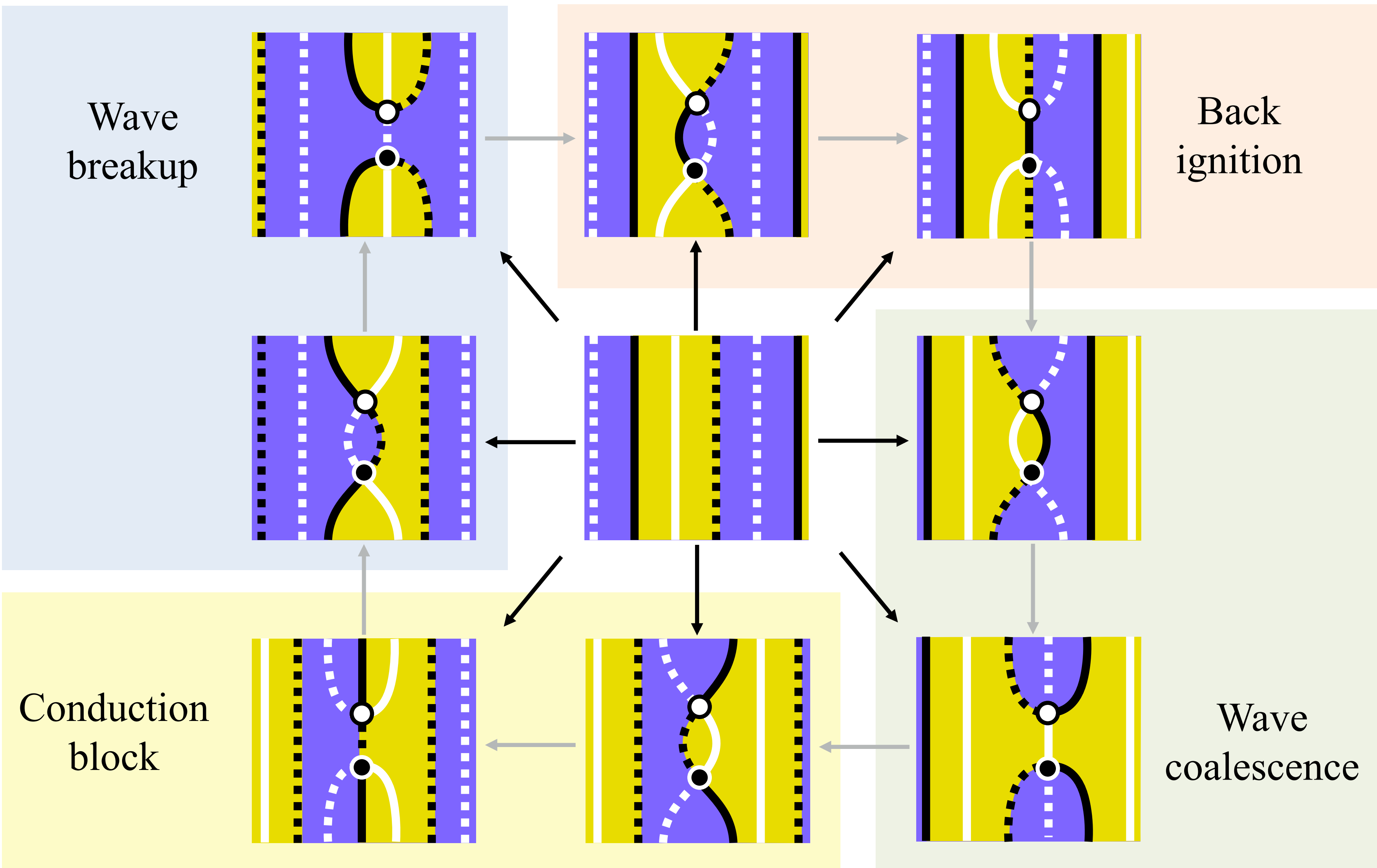
Pair creation/destruction



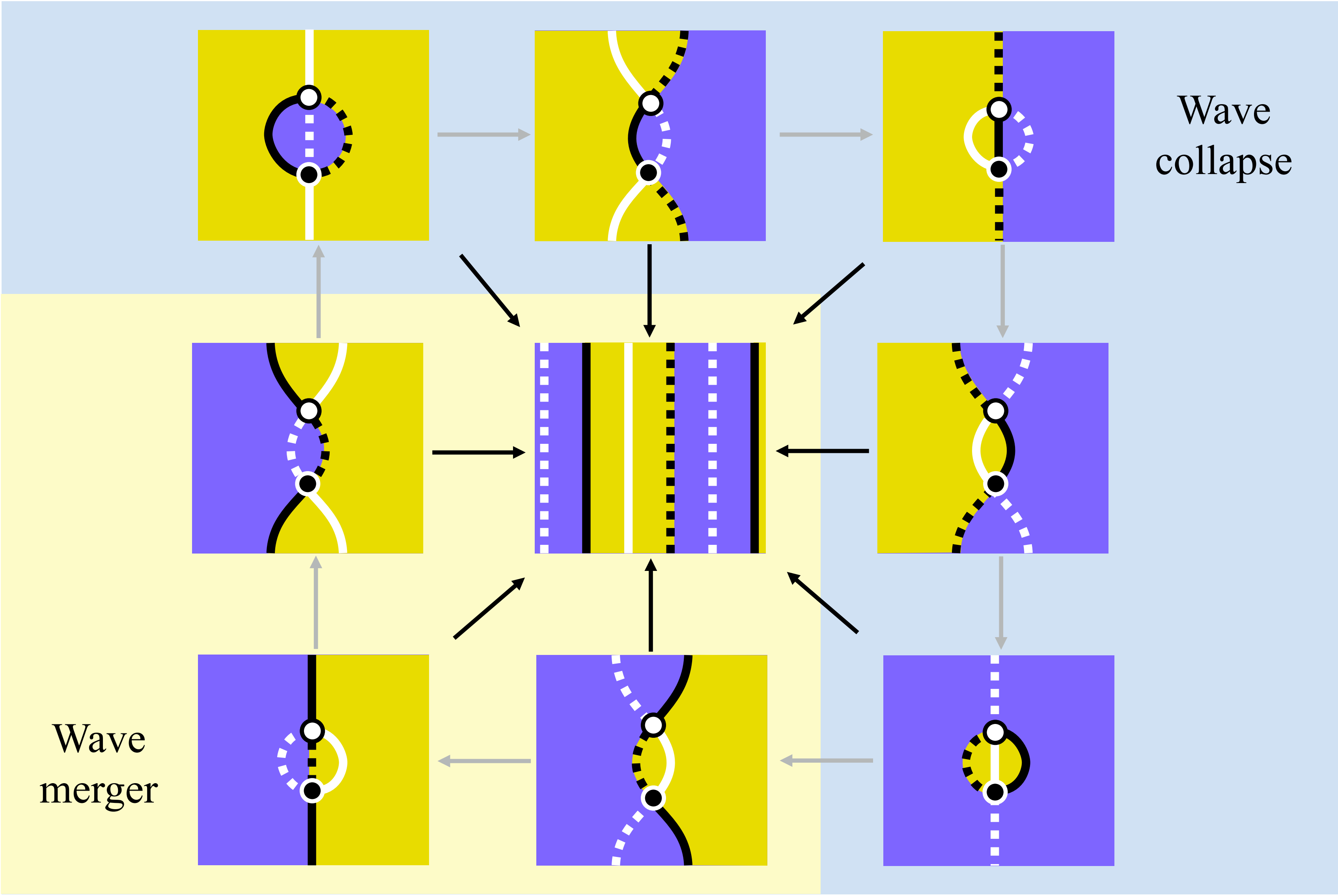
Pair creation



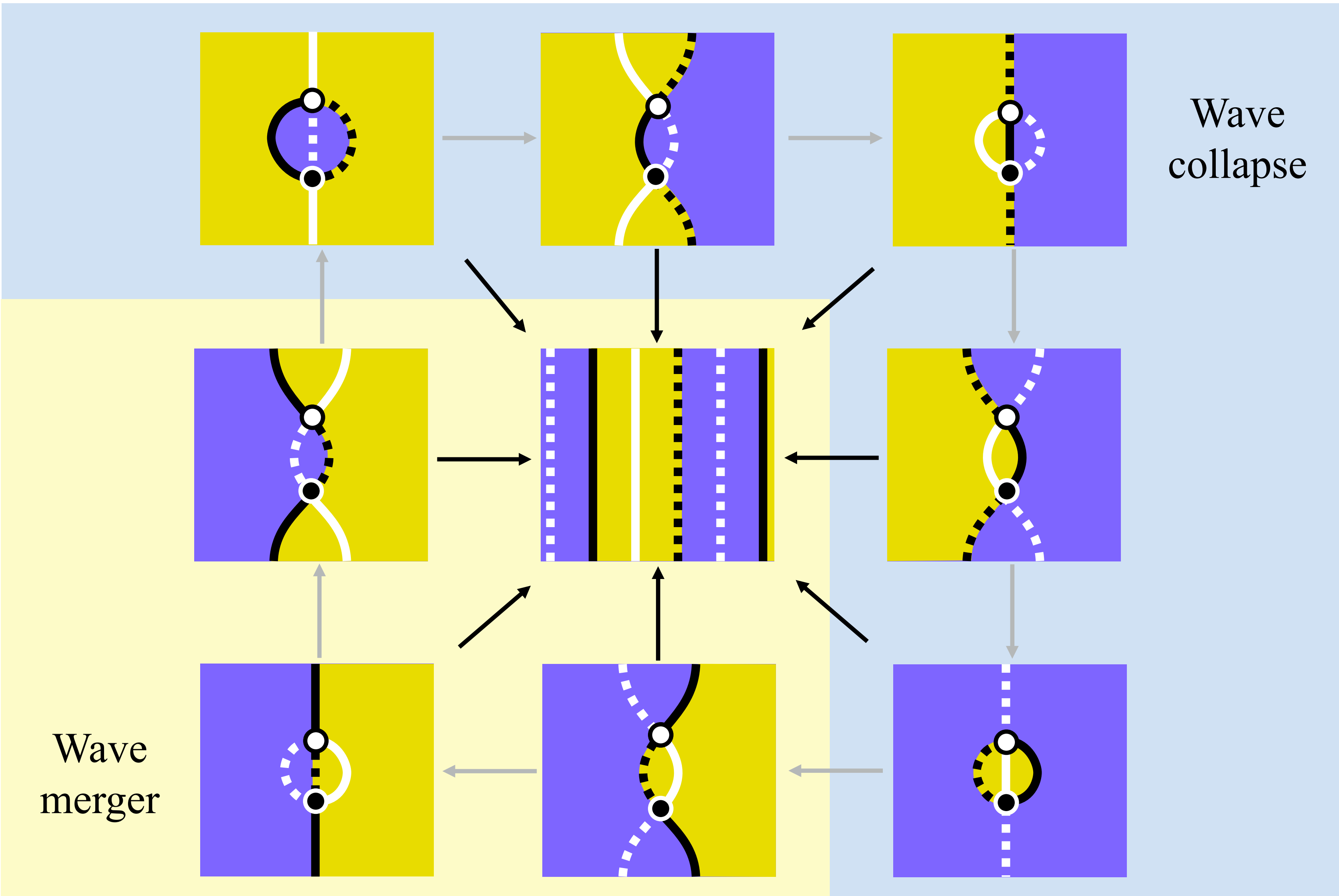
Pair creation



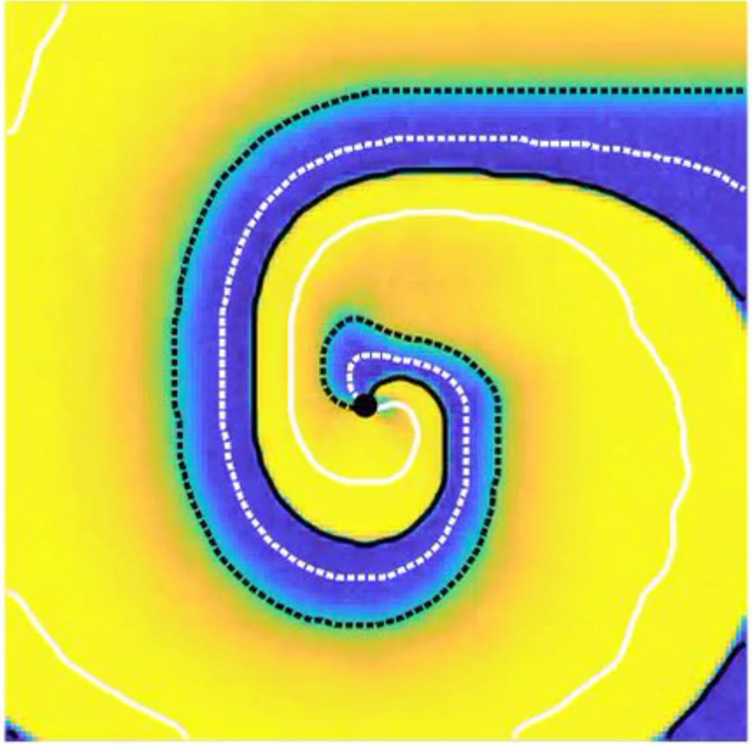
Pair destruction



Pair destruction

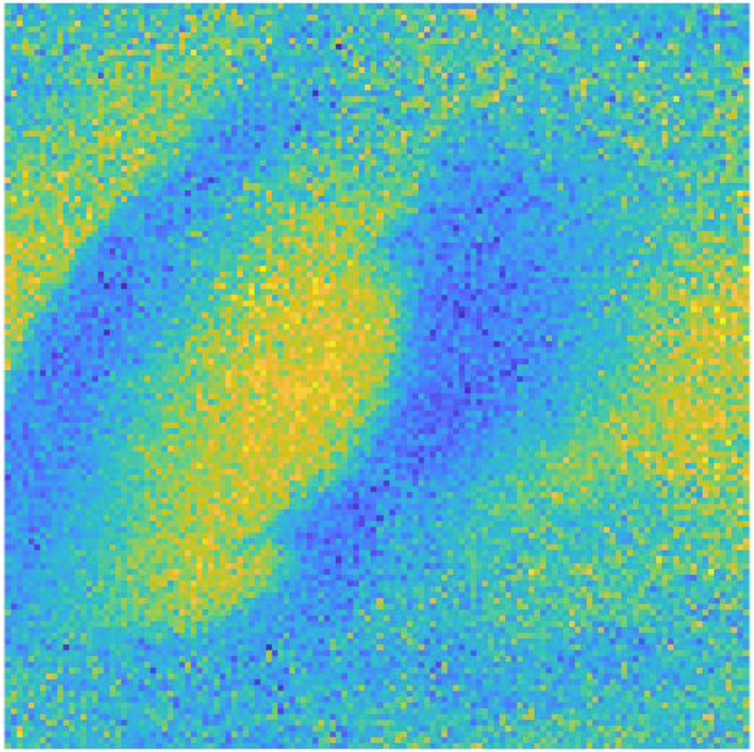


“Atrial fibrillation” (MS)



mpeg movie ➔ jpeg frames (with artifacts) ➔ mpeg movie

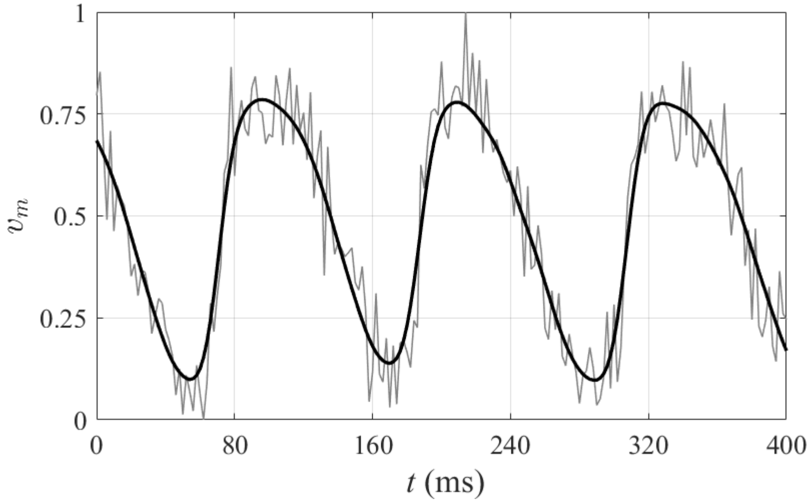
Ventricular fibrillation (pig)

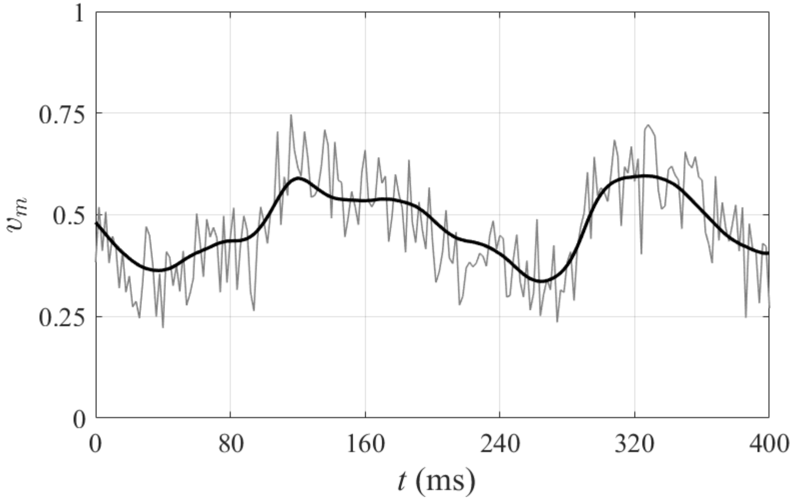


Herndon & Fenton

Optical mapping (voltage, epicardium)

Data quality

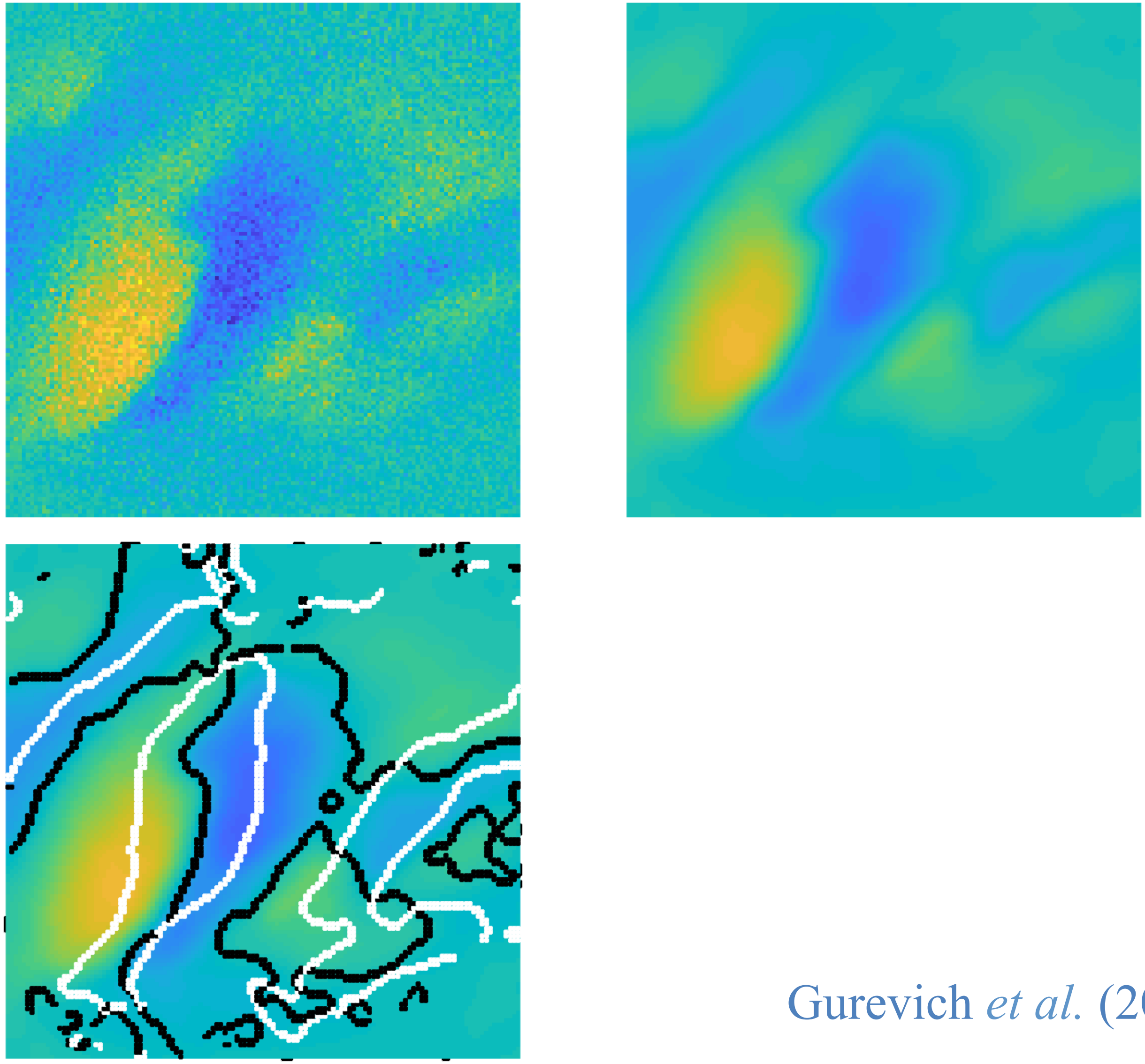




Far from PS

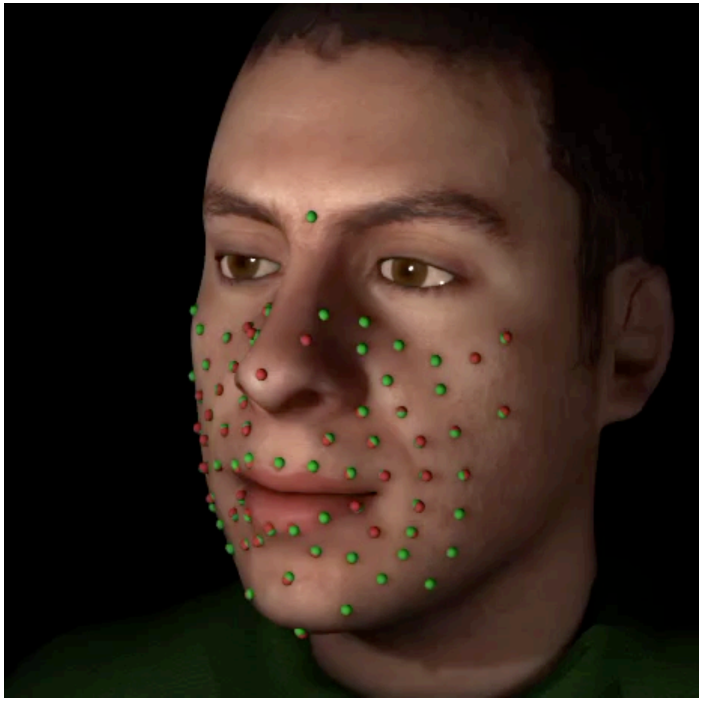
Near PS

Level-set based approach

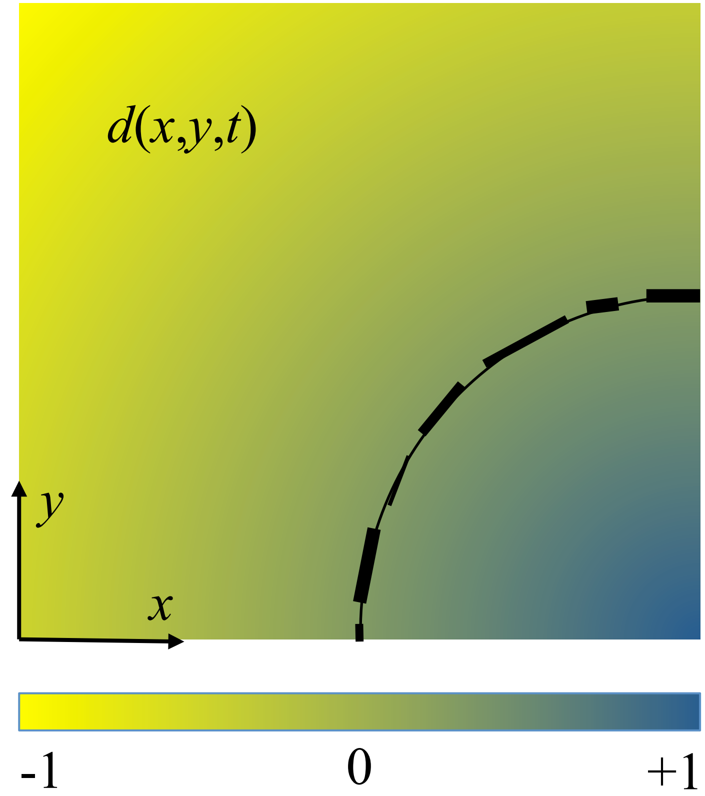


Gurevich *et al.* (2017)

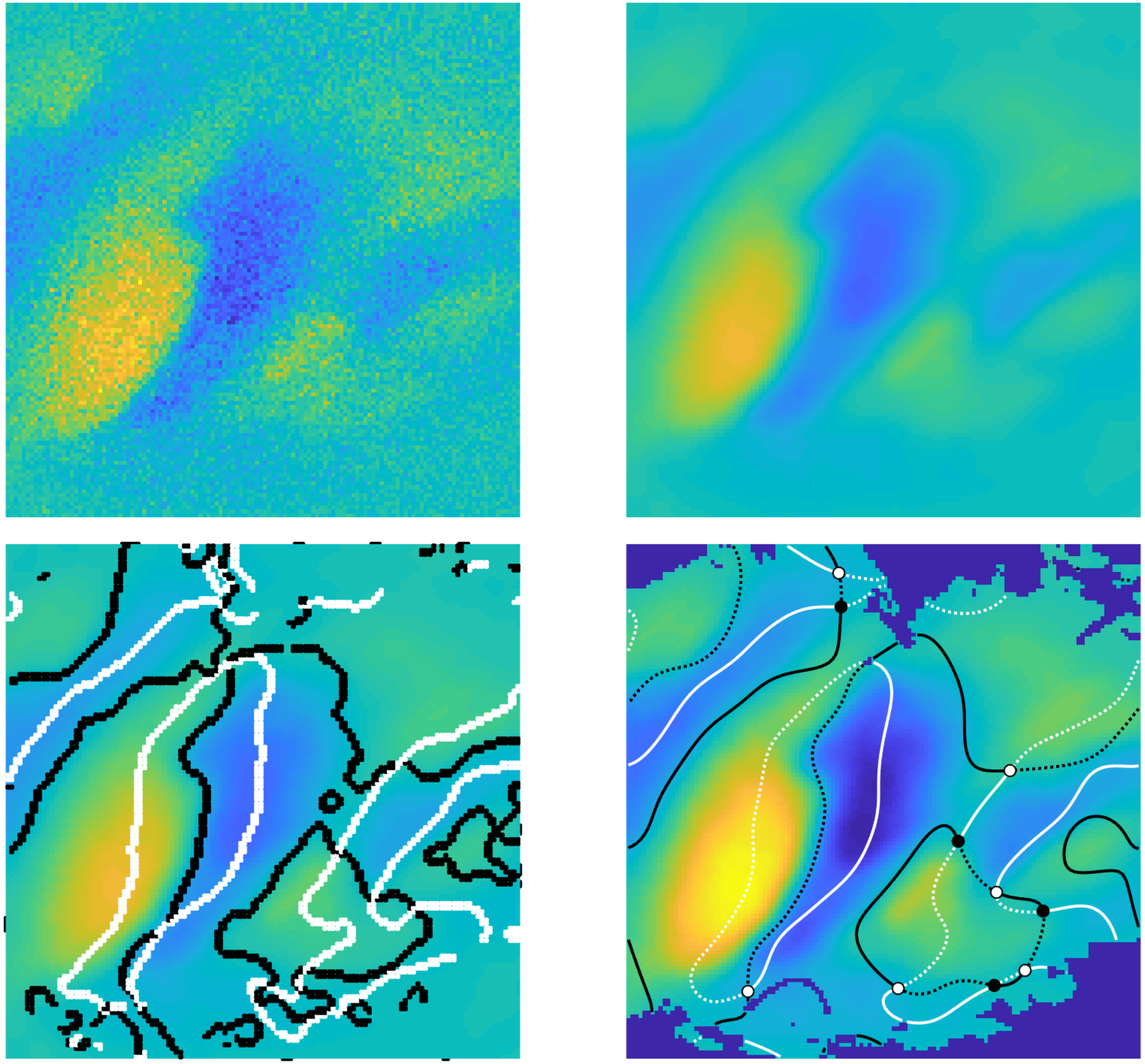
Signed distance function



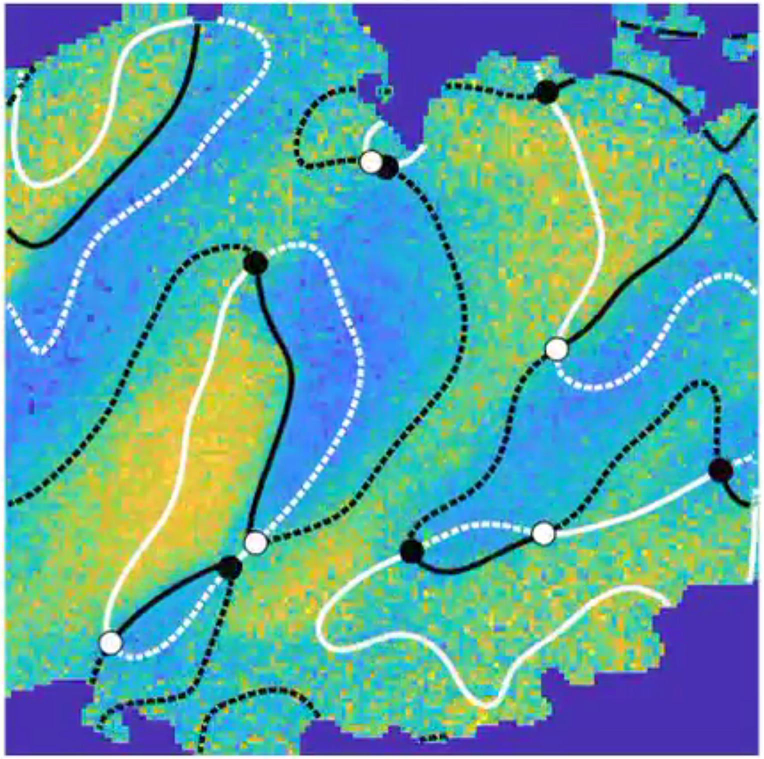
Ron Fedkiw



Level-set based approach



Ventricular fibrillation (3D)



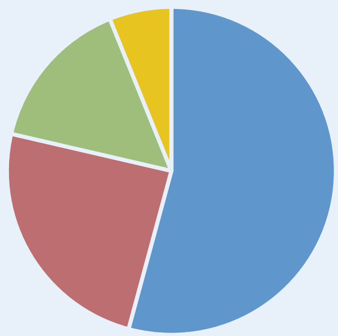
Gurevich *et al.* (2017)

Event statistics

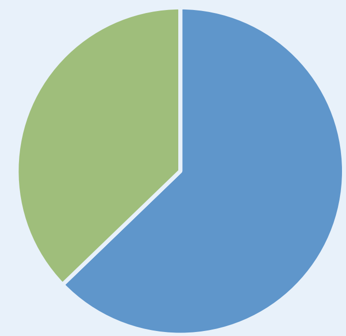
Pig VF (3D)

MS AF (2D)

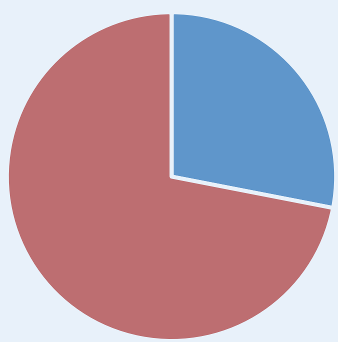
Creation



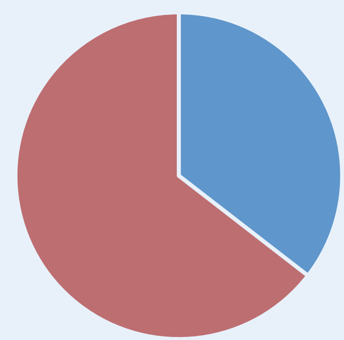
- wave coalescence
- wave breakup
- conduction block
- back ignition



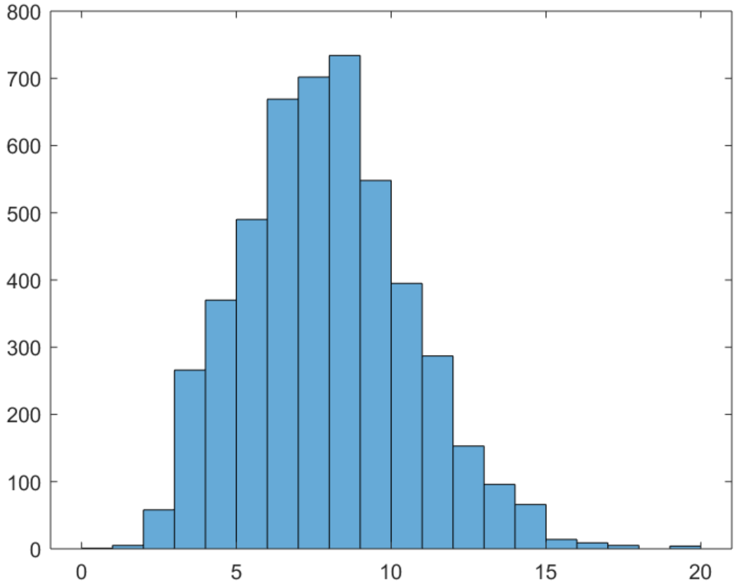
Destruction

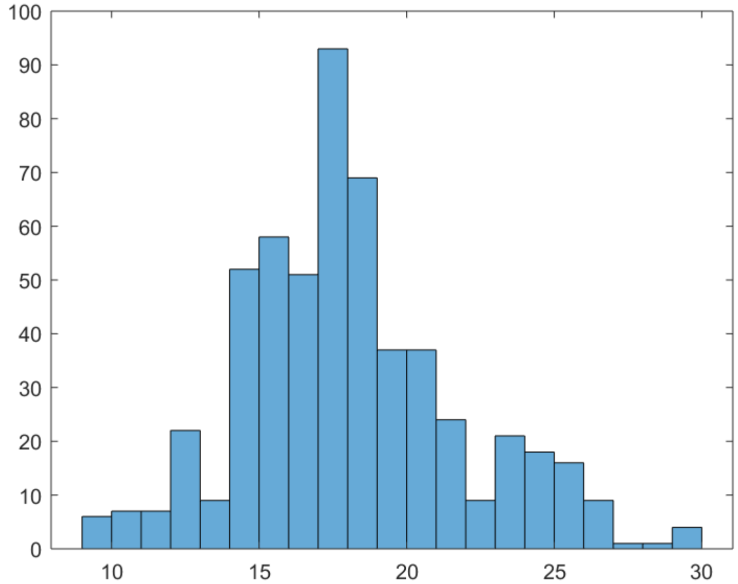


- wave collapse
- wave merger



Rotor number statistics

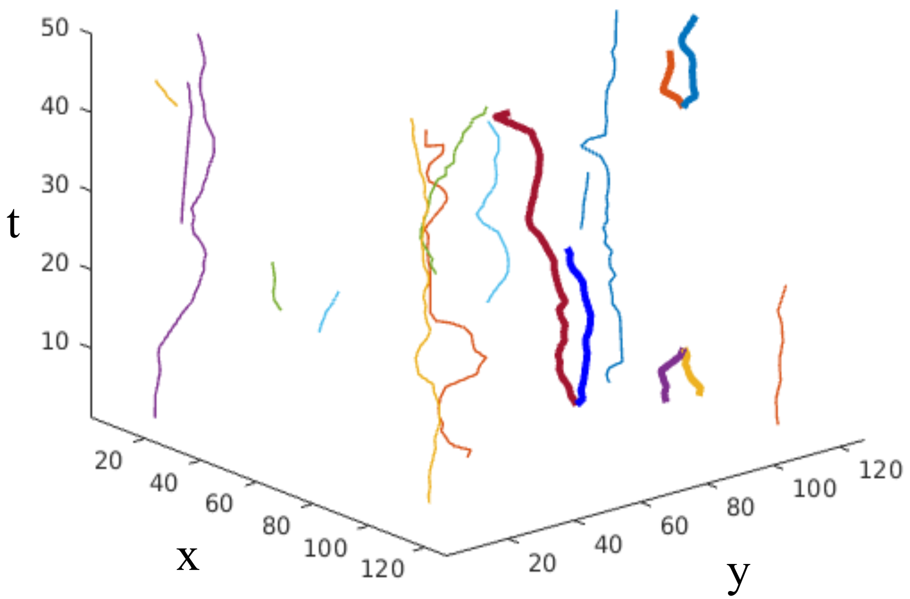


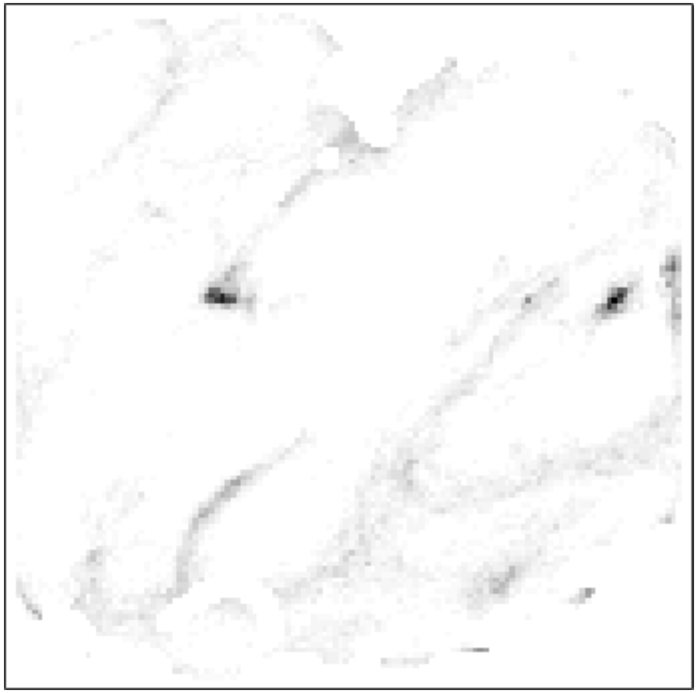


Pig VF

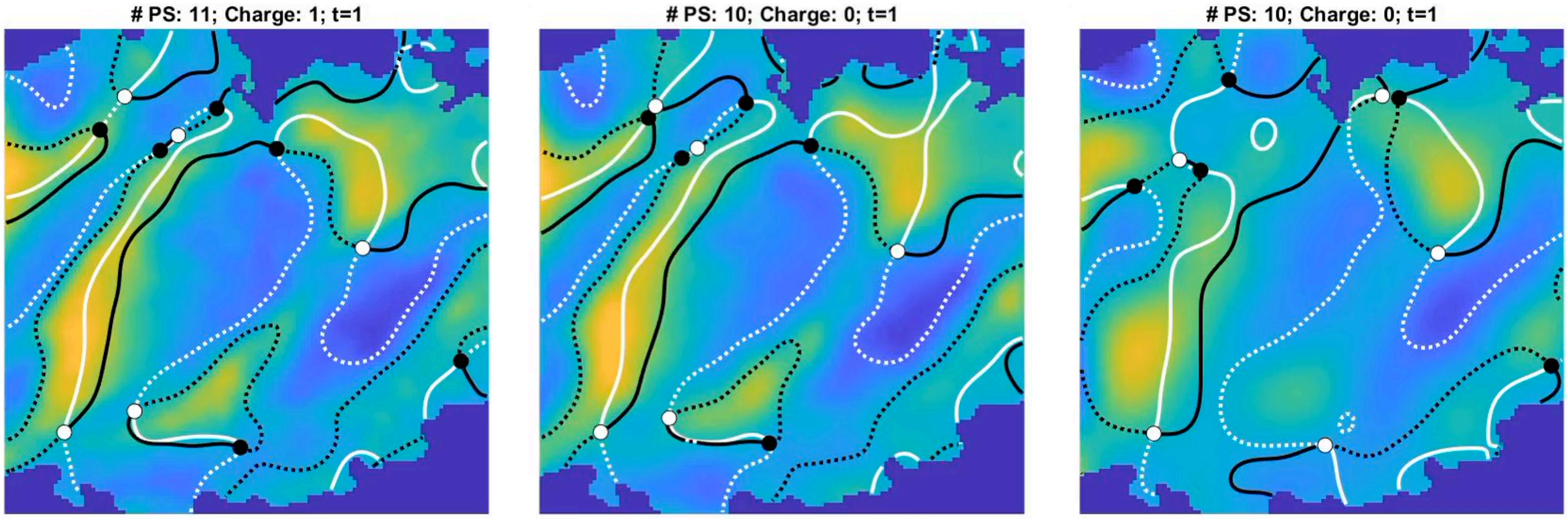
MS AF

Rotor tracking





Rotor mapping from sparse recordings



128 x 128

16 x 16

8 x 8

Topological perspective

- Topological analysis applies to both numerical simulations and experimental recordings
- It provided new insight into dynamical mechanisms underlying fibrillation.
- ***Initiation*** of fibrillation in 2D (AF), aka *the trigger*:
  - conduction block leading to wave breakup
- ***Sustained*** fibrillation, aka *the driver*, is a balance of:
  - wave coalescence
  - wave merger
- Similar results in 3D (VF)