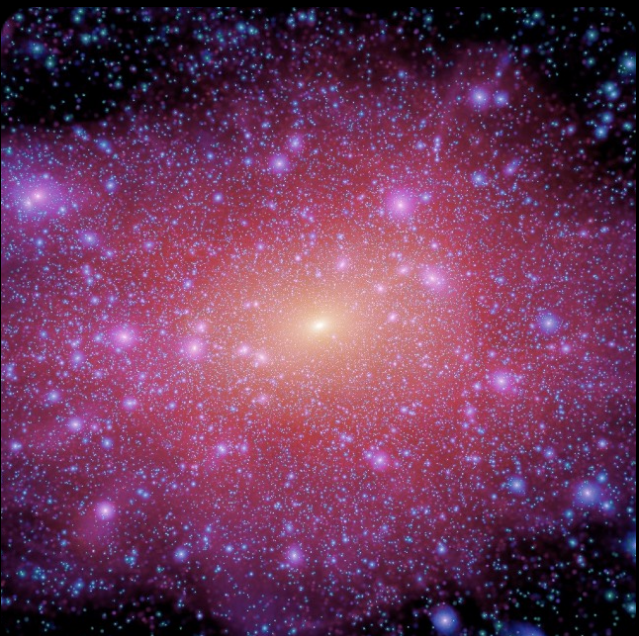
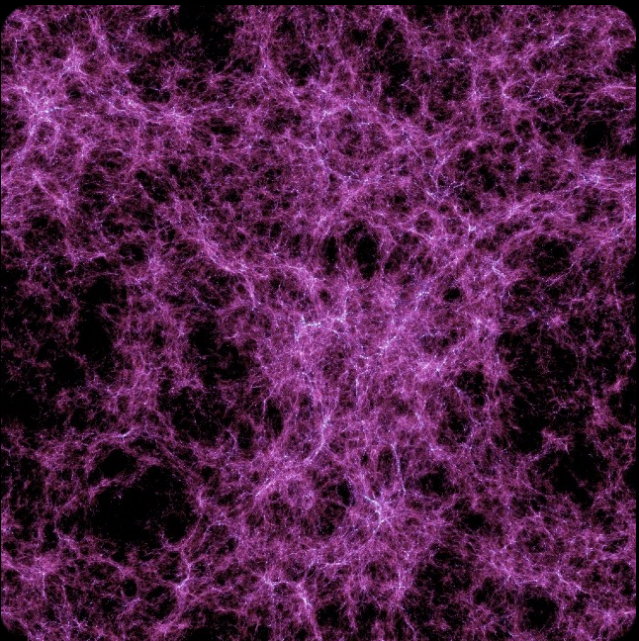


# Simulations Beyond CDM



Mike Boylan-Kolchin



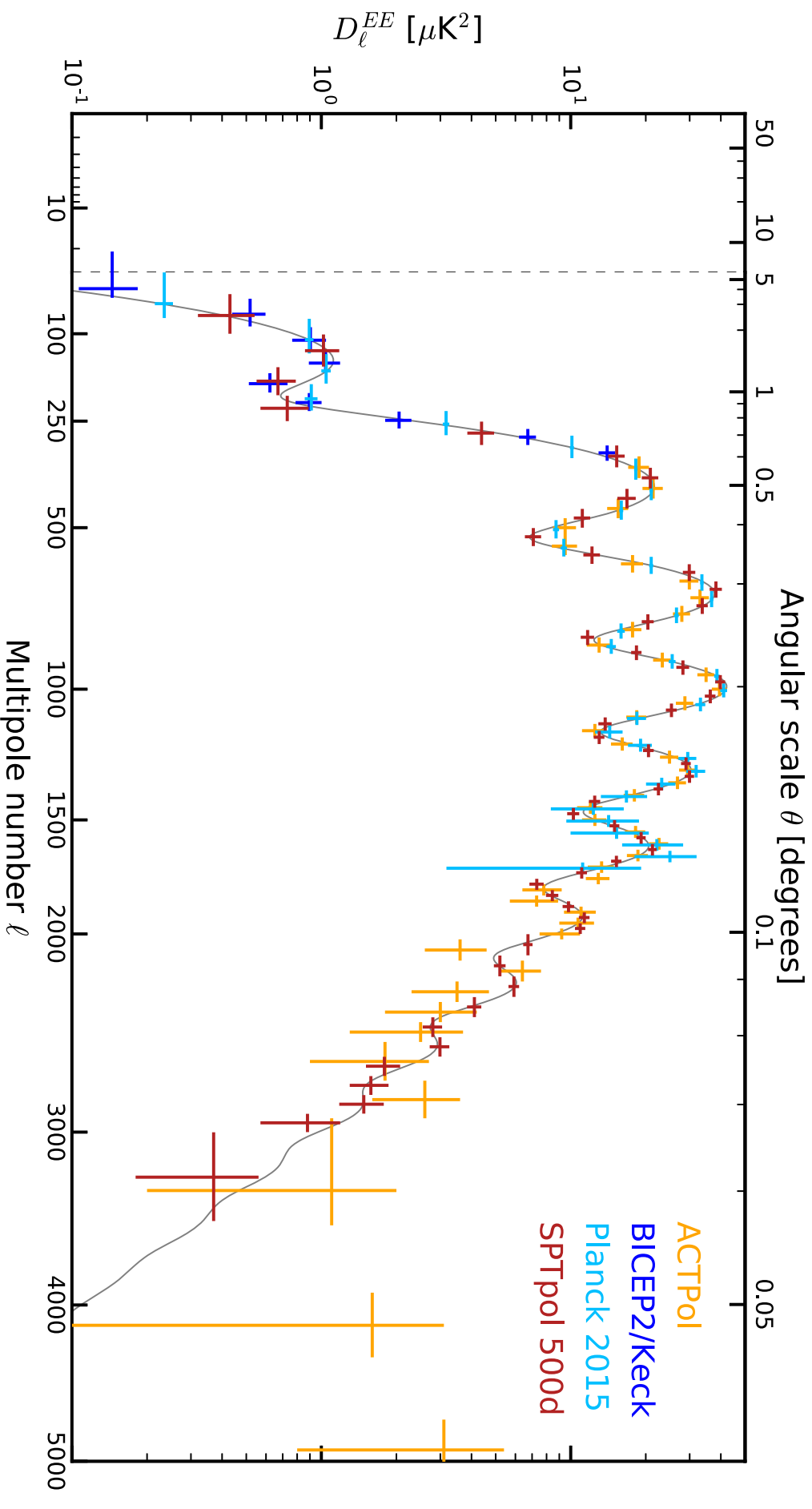
The University of Texas at Austin

KITP

21 June 2018

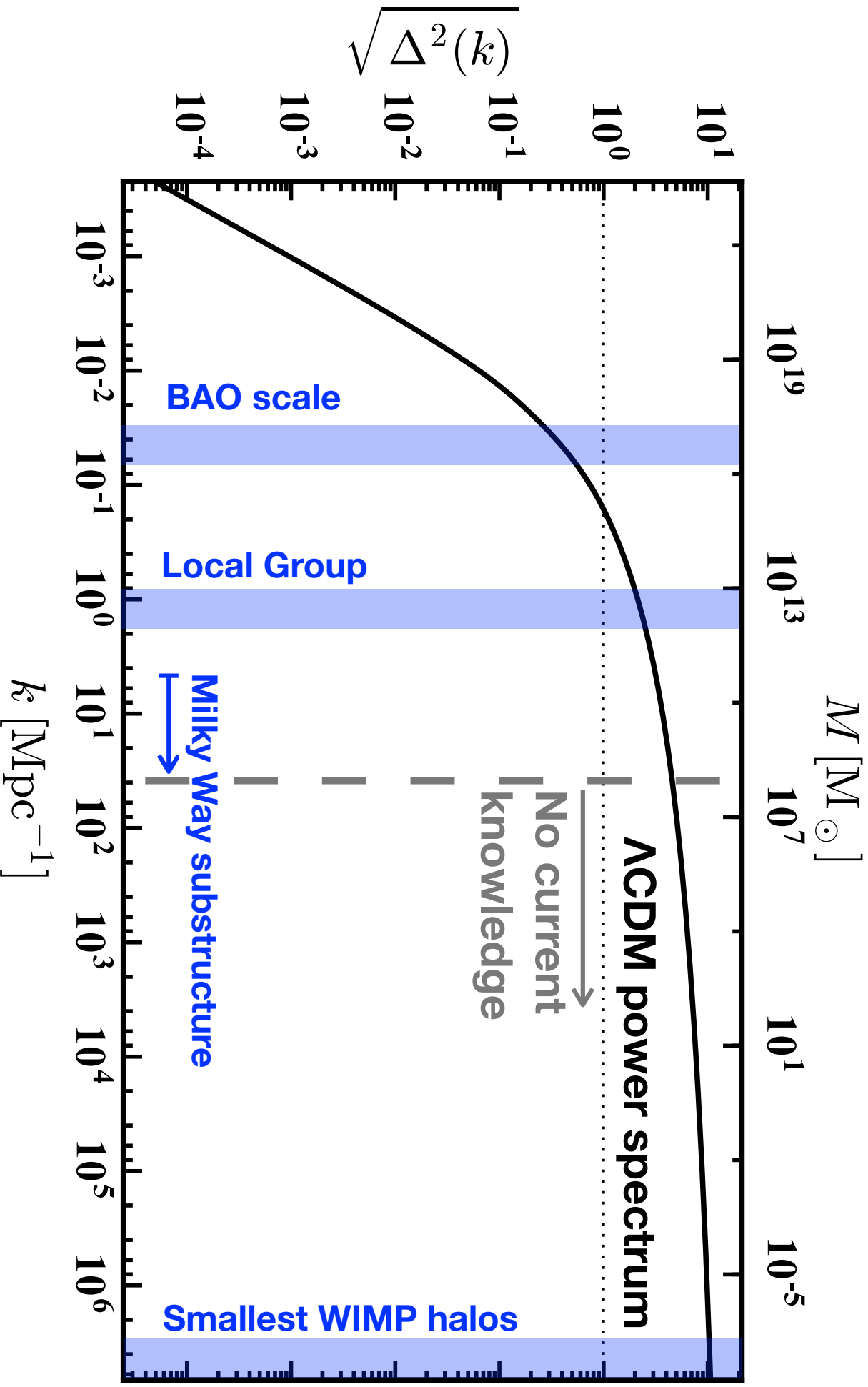
# Large Scales: Concordance $\Lambda$ CDM

---



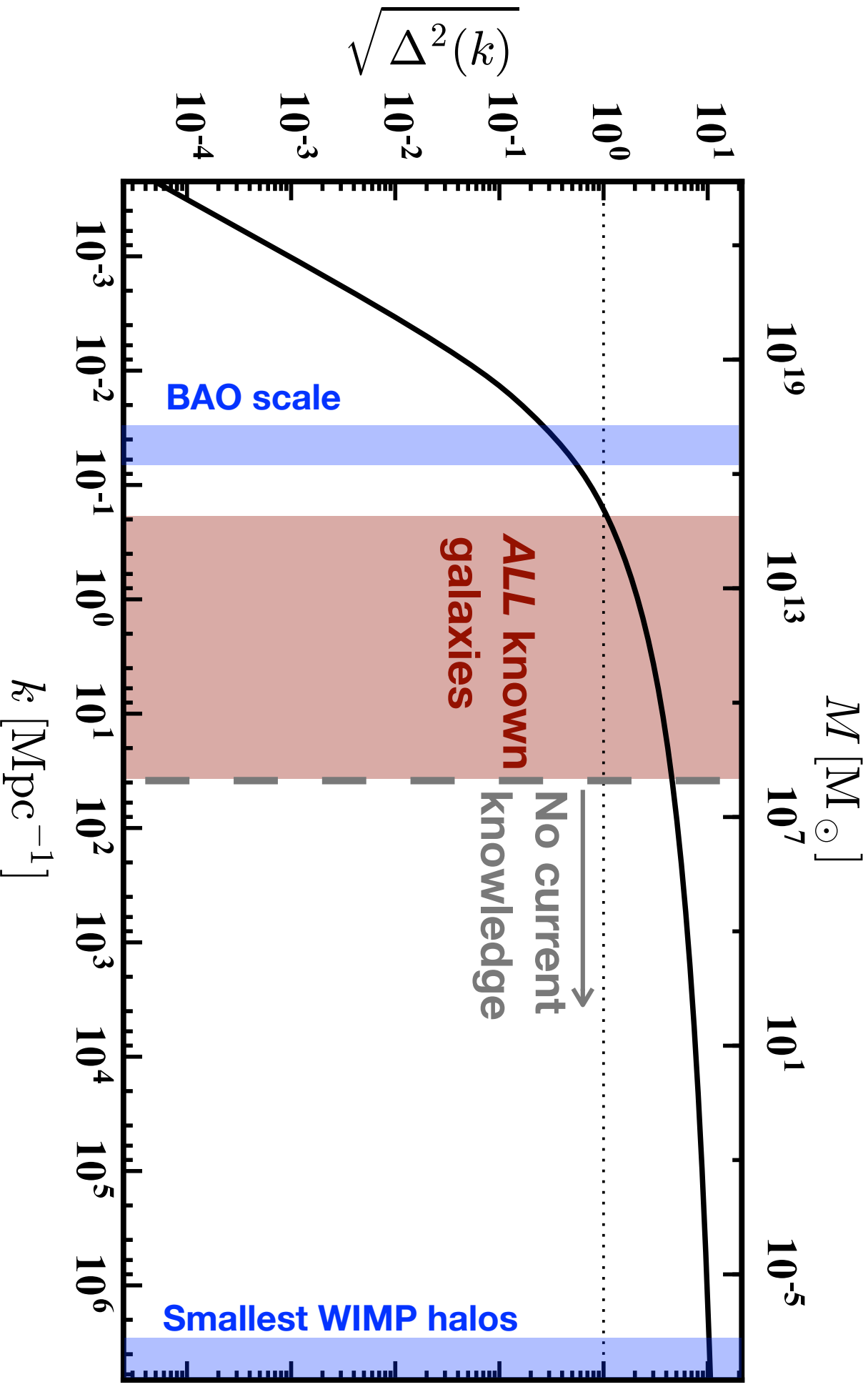
# The scales of $\Lambda$ CDM

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# The scales of $\Lambda$ CDM

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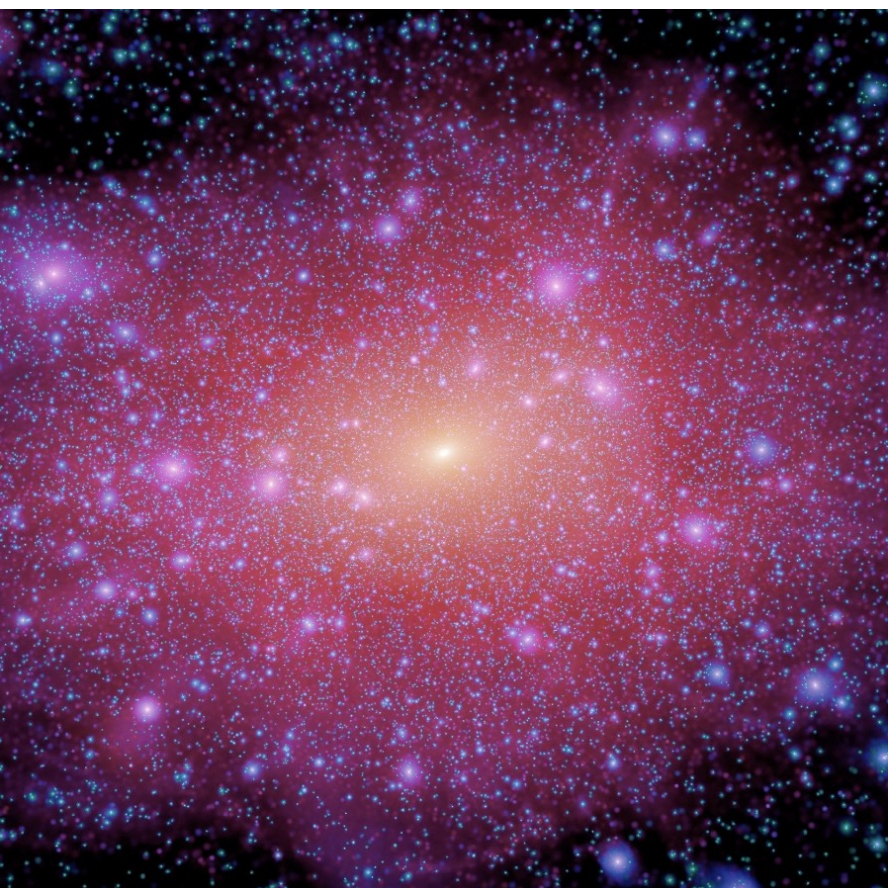
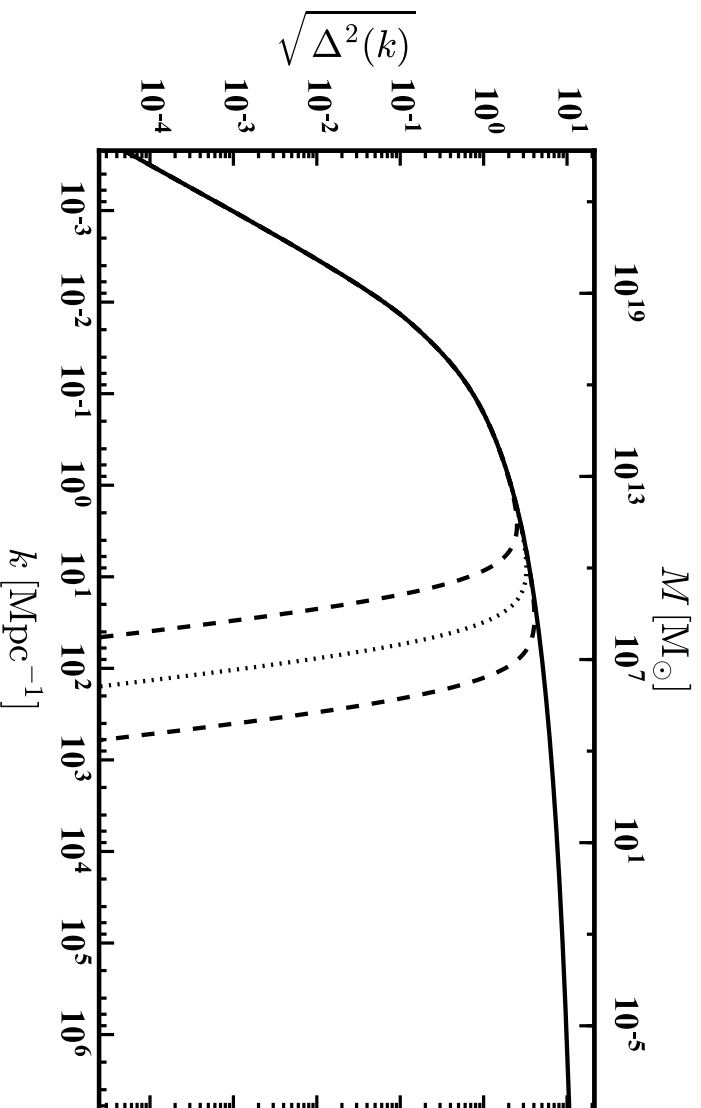
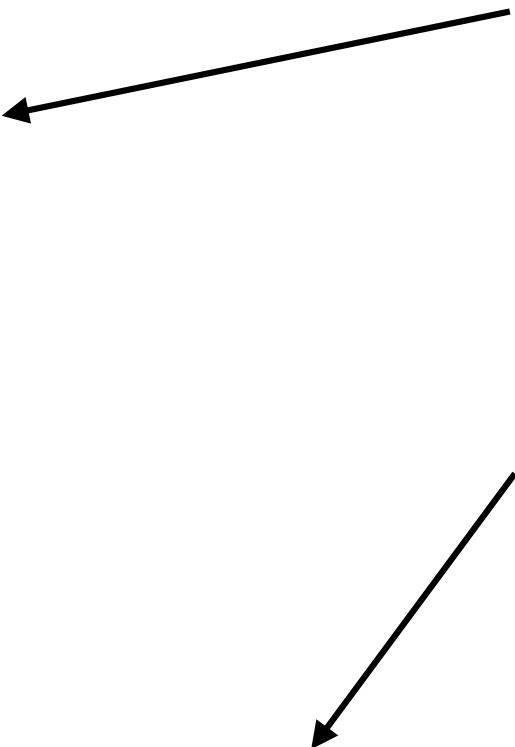




# Alternative Dark Matter Models

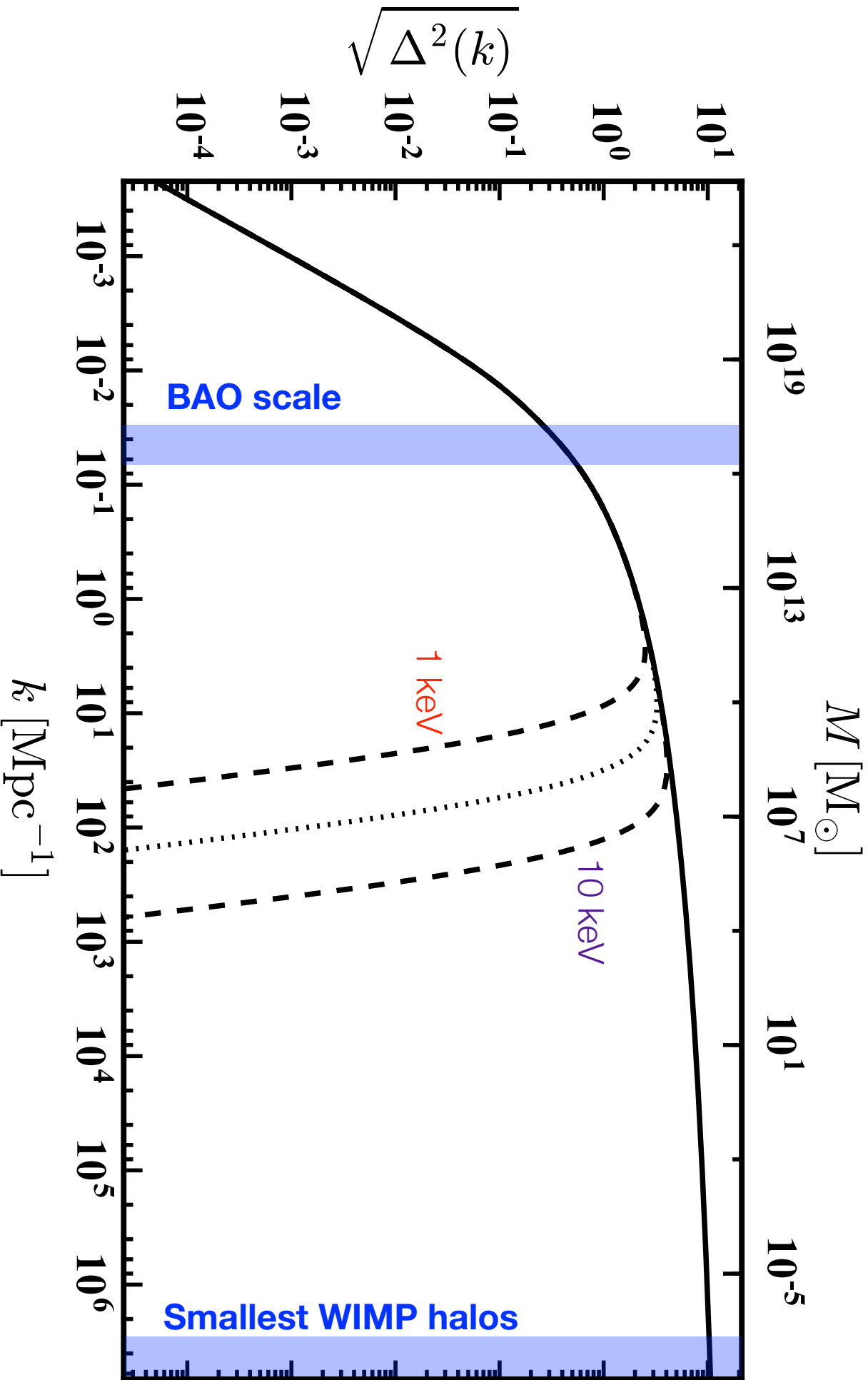
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Modify *linear* physics or *non-linear* physics or **both**



# Beyond CDM: power spectrum modifications

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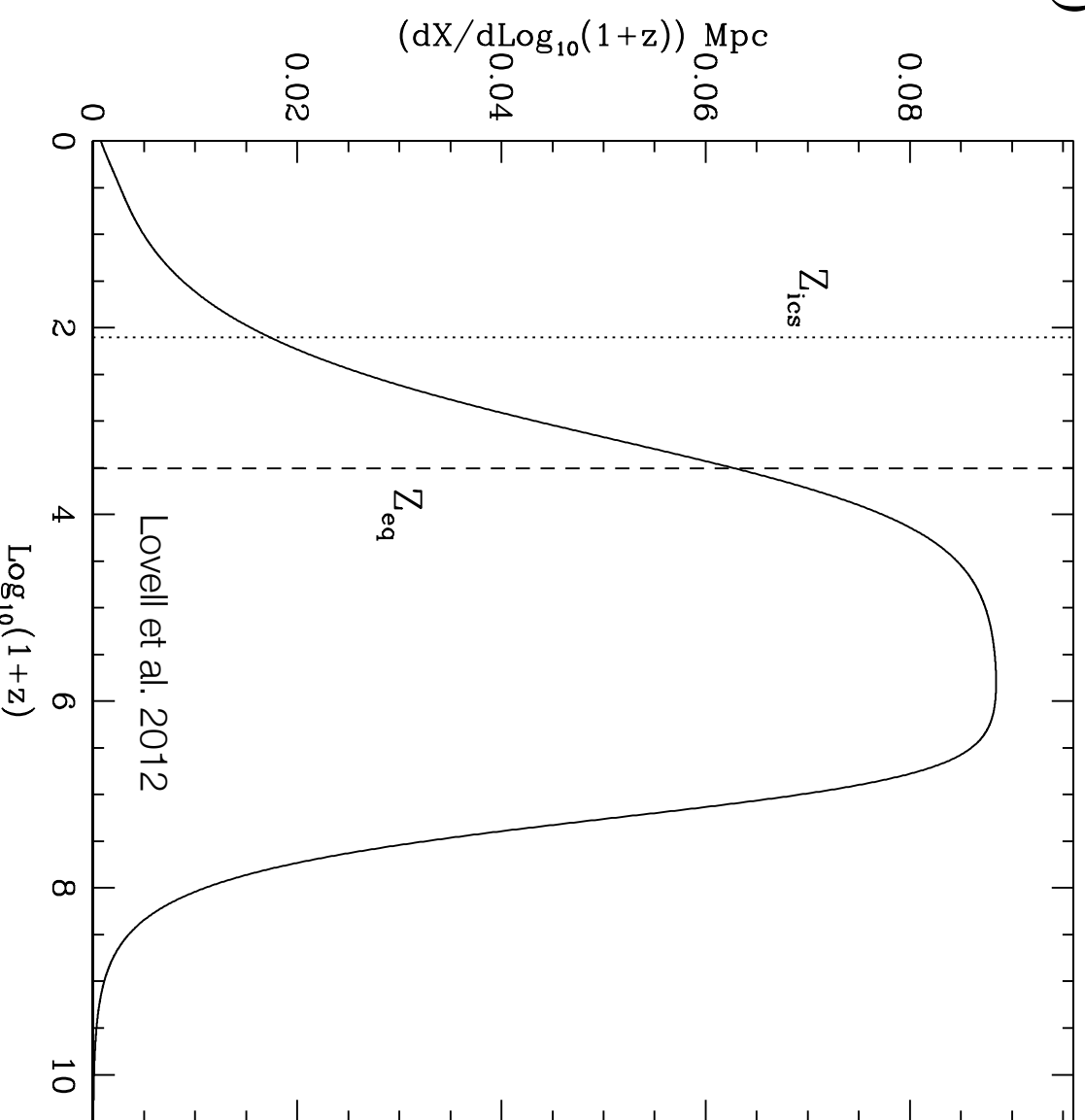
# Origin of cut-off: free-streaming

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$$T_{\text{CMB}}(z = 0) \sim 2.5 \times 10^{-4} \text{ eV}$$

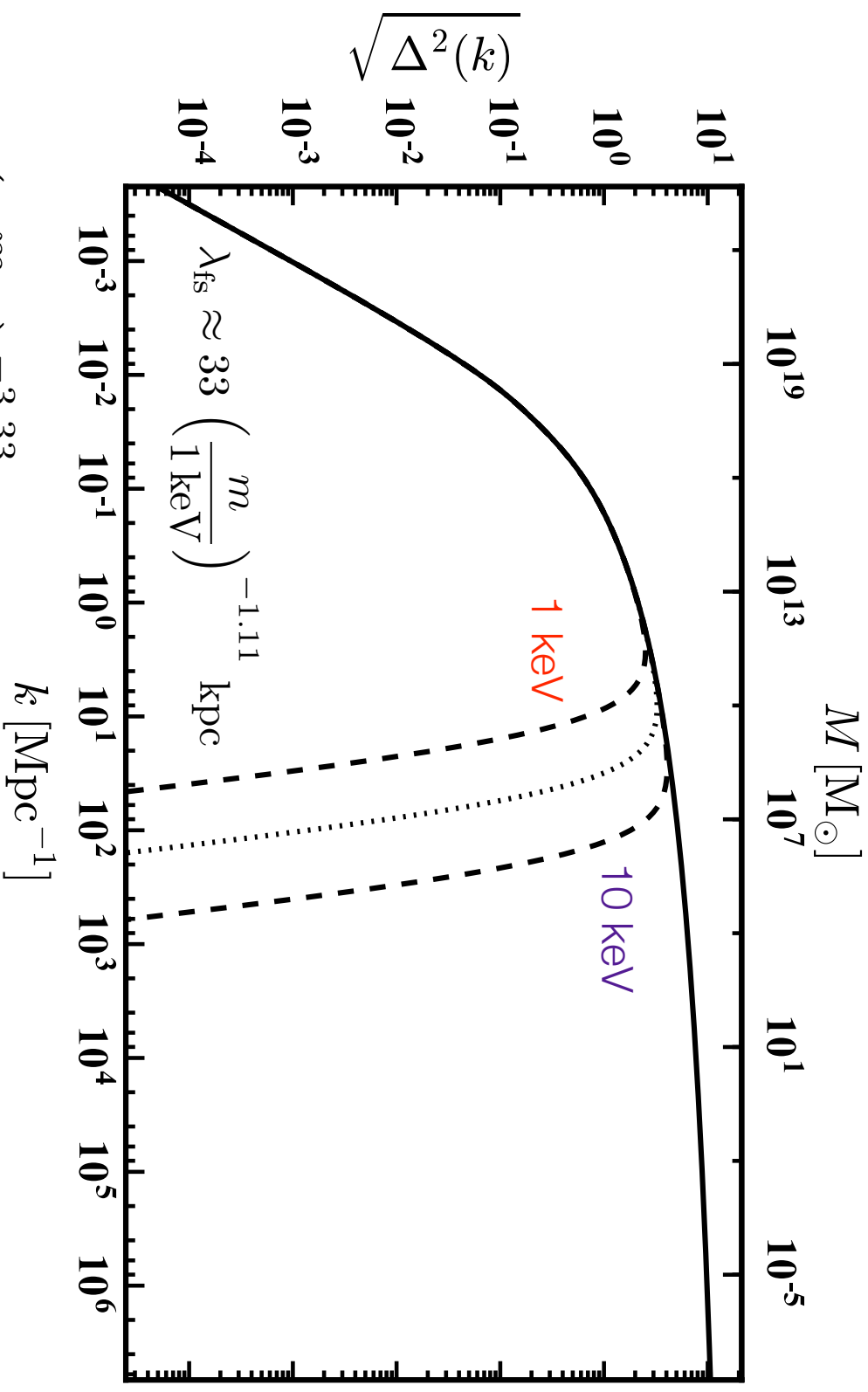
$$m \sim T \text{ at } z \sim 10^7 \text{ (for } m \sim 2 \text{ keV)}$$

$$v(z = 0) \sim 25 \text{ m s}^{-1} \text{ (for } m \sim 2 \text{ keV)}$$



# Origin of cut-off: free-streaming

---



$$M_{\text{fs}} \approx 2 \times 10^7 \left(\frac{m}{1 \text{ keV}}\right)^{-3.33} M_\odot$$

$$M_{\text{hm}} \approx 5.5 \times 10^{10} \left(\frac{m}{1 \text{ keV}}\right)^{-3.33} M_\odot$$

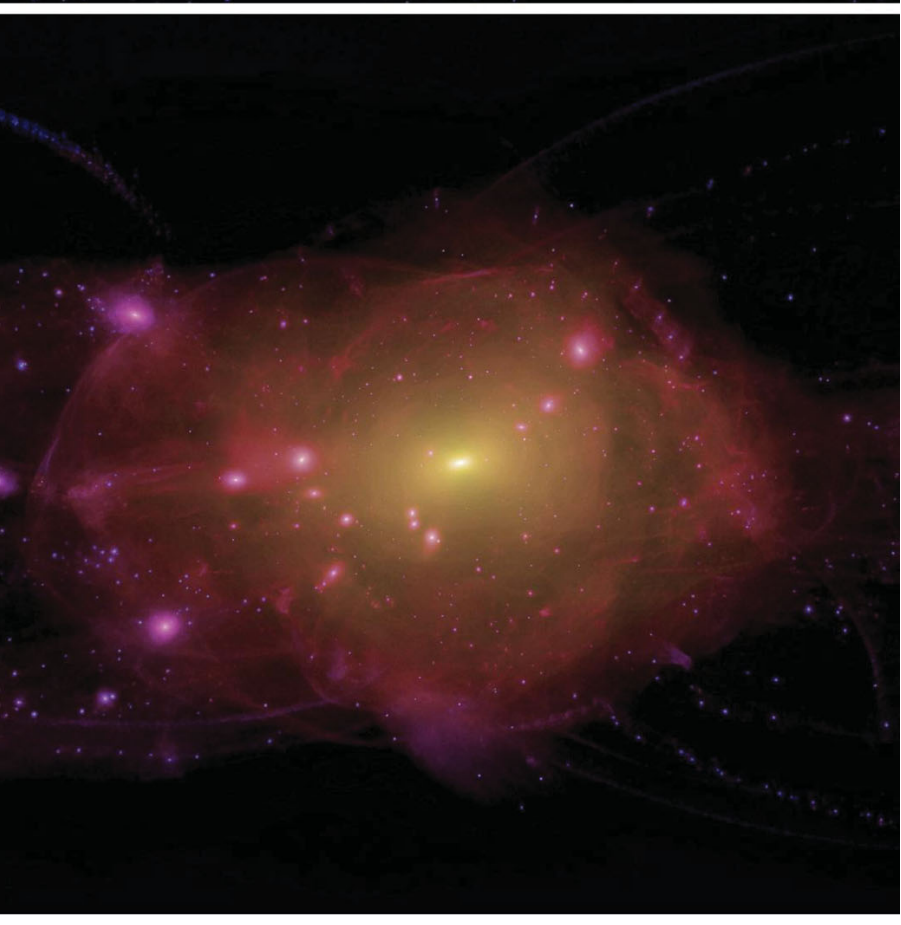
# Simulating Warm Dark Matter

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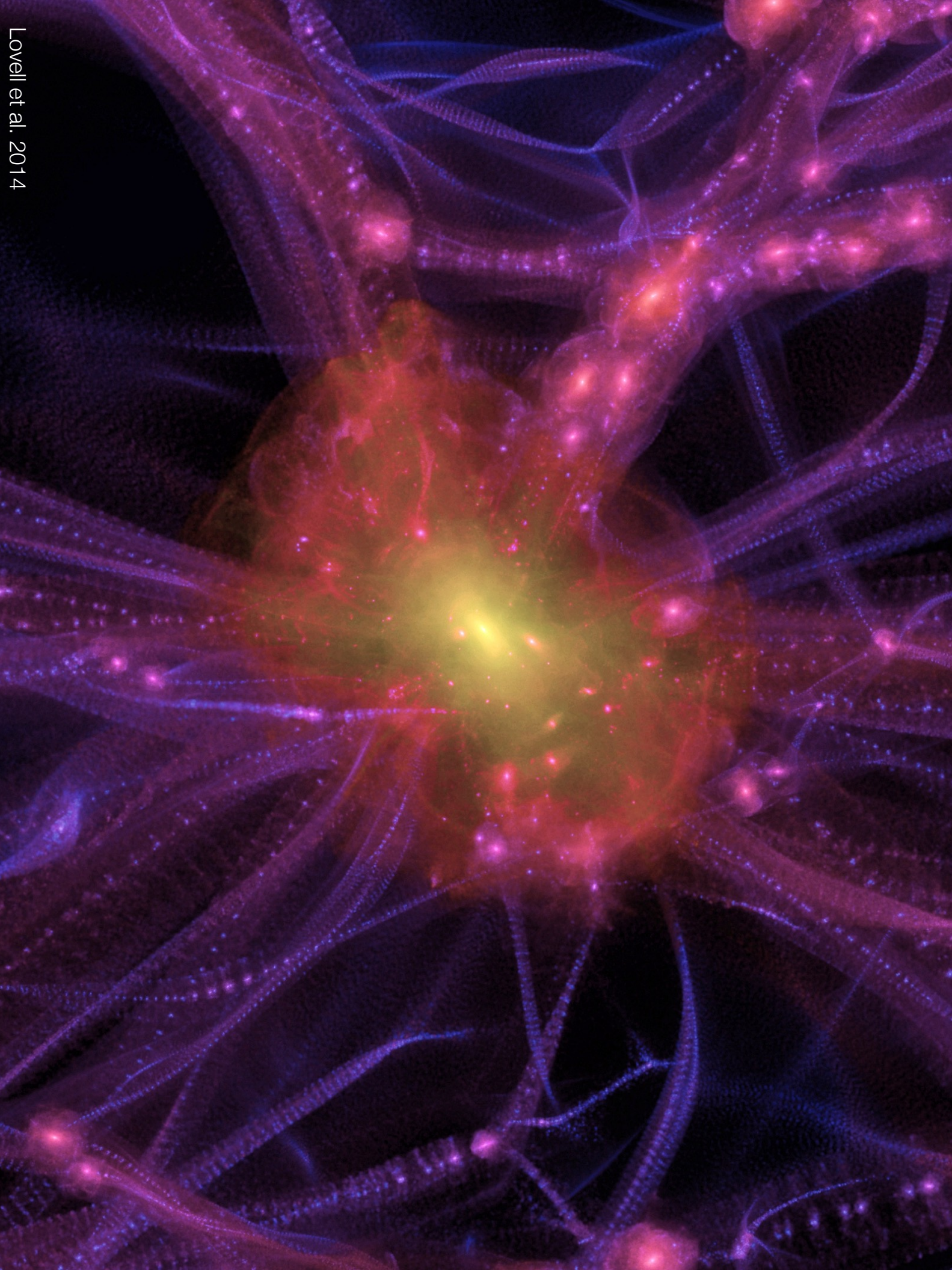
CDM



WDM (~2 keV)



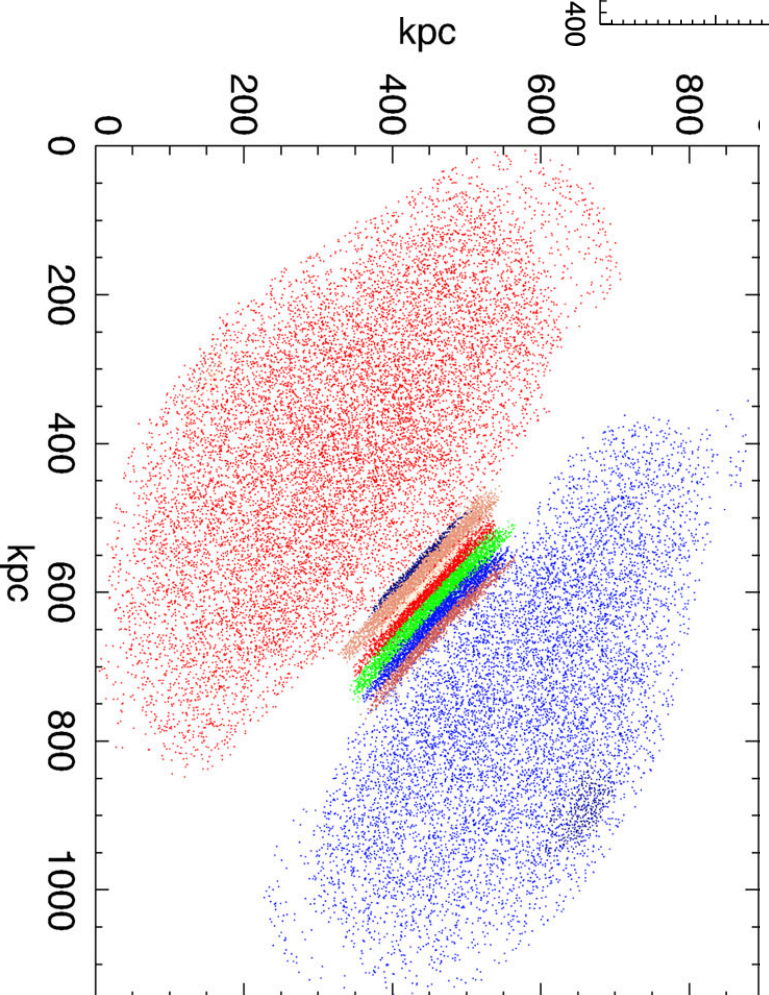
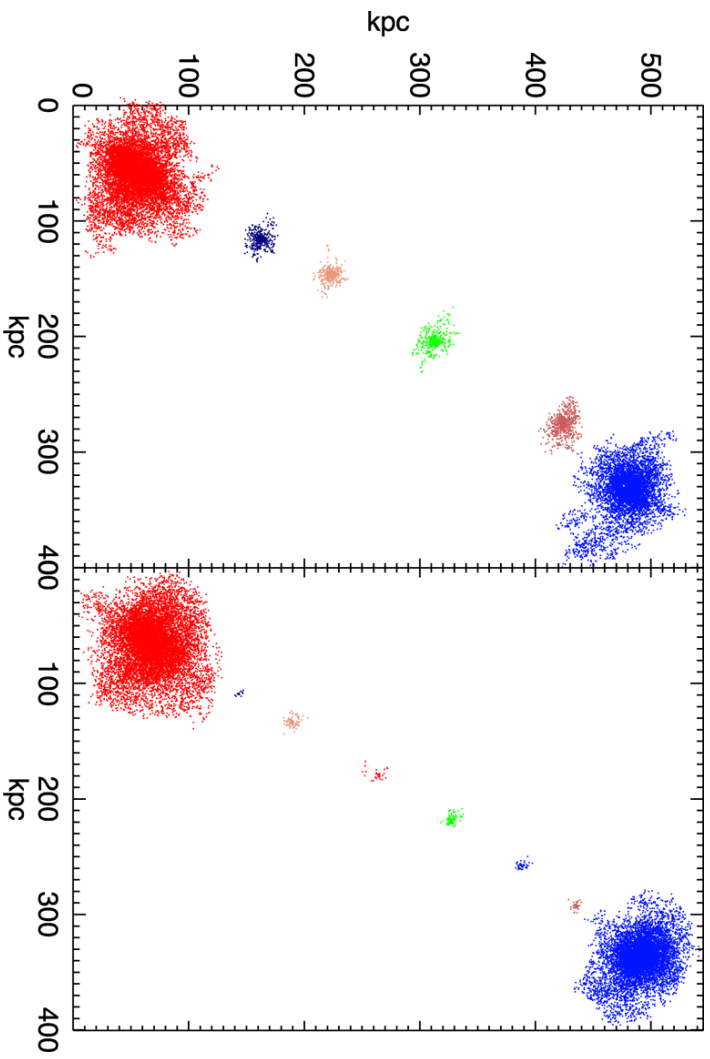






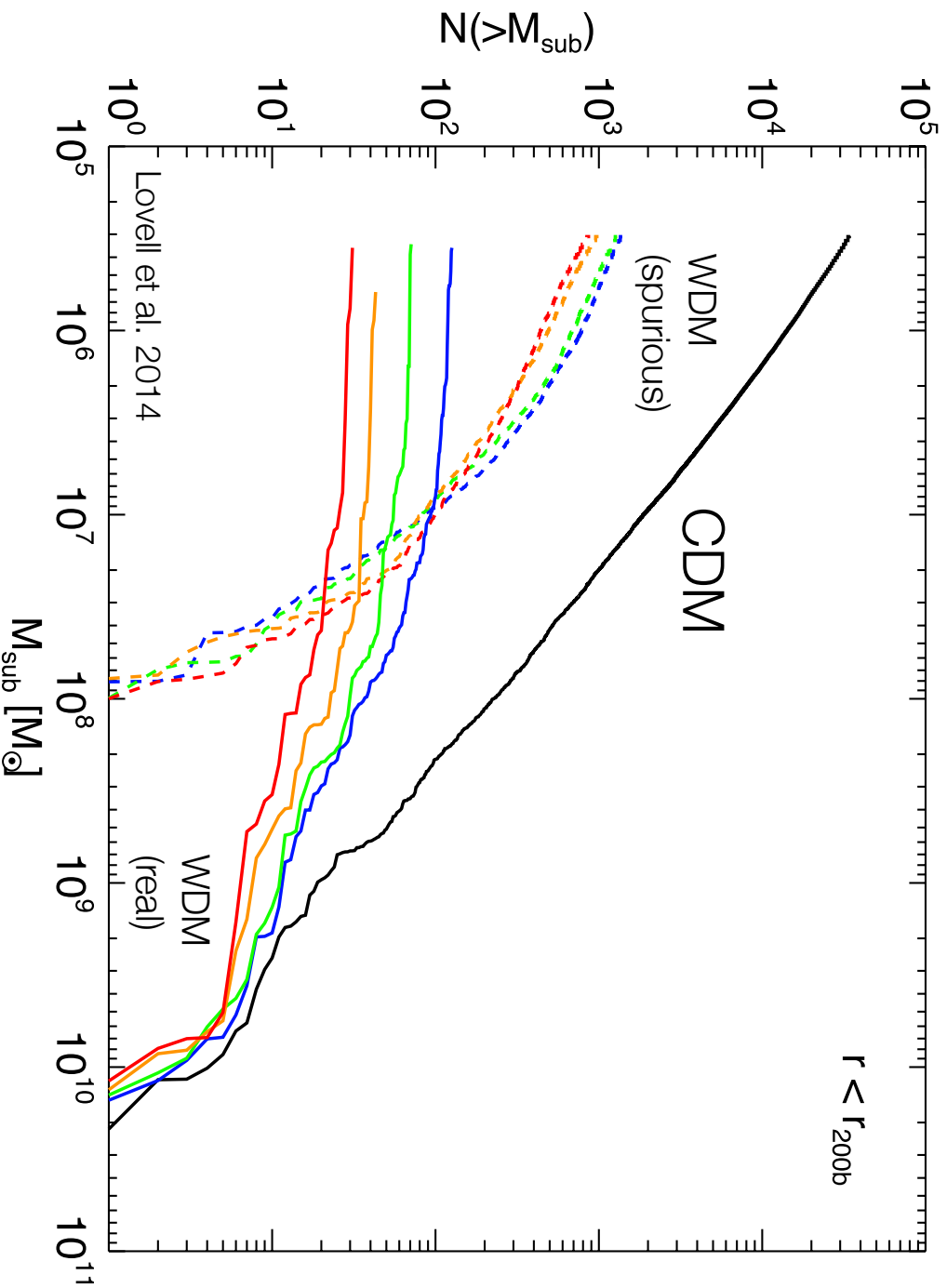
# Identifying spurious halos

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# Counts of spurious halos

---



Scale of spurious subhalos is related to the mean interparticle separation (scales with  $m_p^{1/3}$ )

# Simulating ultra-light axions

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- Basic idea: different regimes for dark matter

- if dark matter is made of particles with very low mass, get quantum behavior on astrophysical scales

$$\lambda = \frac{h}{mv} \approx 4 \text{ kpc} \left( \frac{m}{10^{-22} \text{ eV}} \right)^{-1} \left( \frac{v}{30 \text{ km s}^{-1}} \right)^{-1}$$

# Some math

---

$$i\hbar \left( \frac{\partial \psi}{\partial t} \right) = -\frac{\hbar^2}{2m} \nabla^2 \psi + mV\psi,$$

## Schrödinger-Poisson

$$\nabla^2 V = 4\pi G(\rho - \bar{\rho})$$

$$\psi = \sqrt{\rho} e^{iS/\hbar}$$

$$\mathbf{u} \equiv \nabla S/m$$



# Some more math

---

$$i\hbar \left( \frac{\partial \psi}{\partial t} \right) = -\frac{\hbar^2}{2m} \nabla^2 \psi + mV\psi,$$

$$\psi = \sqrt{\rho} e^{iS/\hbar}$$

$$\nabla^2 V = 4\pi G(\rho - \bar{\rho})$$

$$\mathbf{u} \equiv \nabla S/m$$

$$V_Q \equiv -\frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}.$$

$$\nabla V_Q = \frac{m}{\rho} \nabla \cdot \mathbb{P}_Q,$$

$$\mathbb{P}_Q \equiv -\left( \frac{\hbar}{2m} \right)^2 \rho \nabla \otimes \nabla \ln \rho.$$

# Correspondence

---

## Schrödinger-Poisson

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0,$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla V - \frac{1}{\rho} \nabla \cdot \mathbb{P}_Q$$

$$\mathbb{P}_Q \equiv - \left( \frac{\hbar}{2m} \right)^2 \rho \nabla \otimes \nabla \ln \rho.$$

## Jeans/Euler

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \langle \mathbf{v} \rangle) = 0$$

$$\frac{\partial \langle \mathbf{v} \rangle}{\partial t} + \langle \mathbf{v} \rangle \cdot \nabla \langle \mathbf{v} \rangle = -\nabla V - \frac{1}{\rho} \nabla \cdot \mathbb{P}_J$$

$$\mathbb{P}_J \equiv \rho \sigma_{ij}^2 = \rho (\langle v_i v_j \rangle - \langle v_i \rangle \langle v_j \rangle)$$

# Another look

---

## Madelung Equations

$$\dot{\rho} + 3H\rho + \frac{1}{R}\mathbf{V} \cdot (\rho\mathbf{v}) = 0,$$

$$\dot{\mathbf{v}} + H\mathbf{v} + \frac{1}{R}(\mathbf{v} \cdot \mathbf{V})\mathbf{v} = -\frac{1}{R}\mathbf{V}\Phi + \frac{\hbar^2}{2R^3 m^2}\mathbf{V}\left(\frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}\right)$$

# Phenomenology

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$$\lambda \lesssim R_{\text{vir}} = GM/V^2$$

Stationary solution, spherical symmetry:

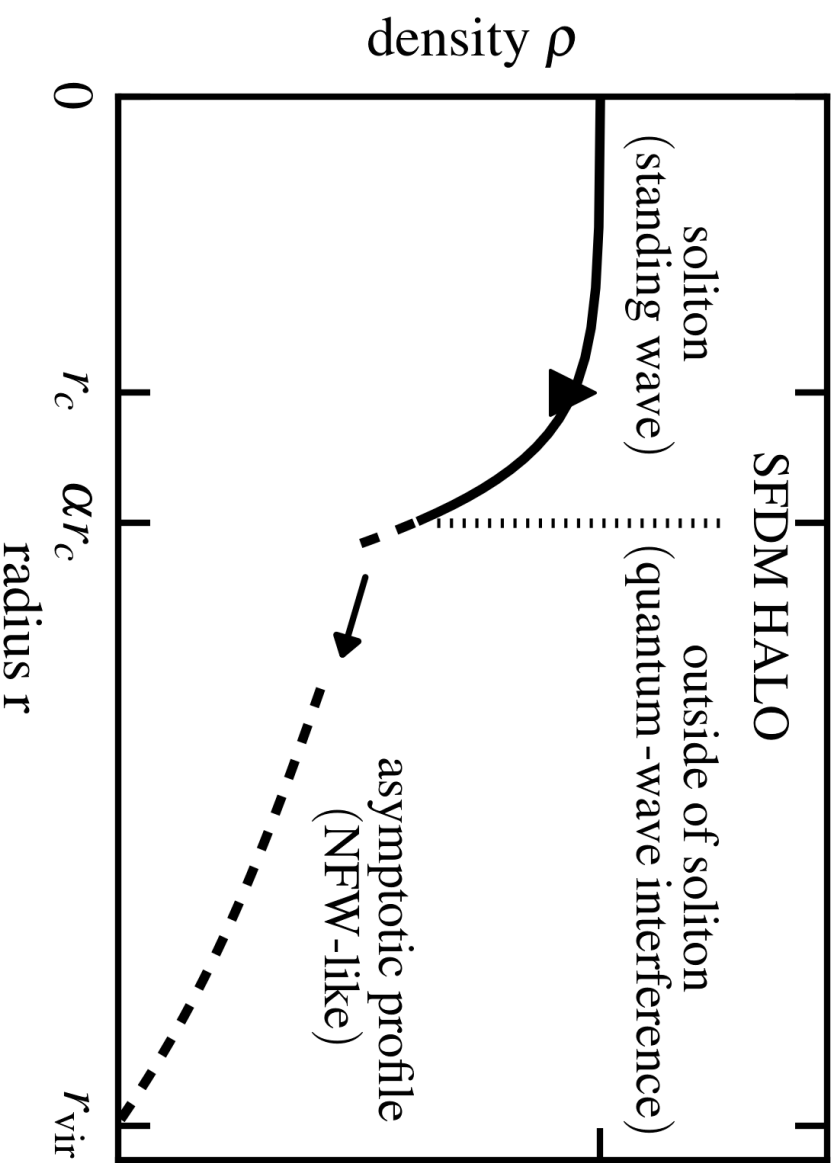
$$r_{1/2} \geq 3.925 \frac{\hbar^2}{GMm^2} = 0.335 \text{ kpc} \frac{10^9 M_{\odot}}{M} \left( \frac{10^{-22} \text{ eV}}{m} \right)^2$$

$$\begin{aligned} \rho_c &\leq 0.0044 \left( \frac{Gm^2}{\hbar^2} \right)^3 M^4 \\ &= 7.05 M_{\odot} \text{ pc}^{-3} \left( \frac{m}{10^{-22} \text{ eV}} \right)^6 \left( \frac{M}{10^9 M_{\odot}} \right)^4. \end{aligned}$$

Central “soliton” (standing wave)

# Schematic structure of halo

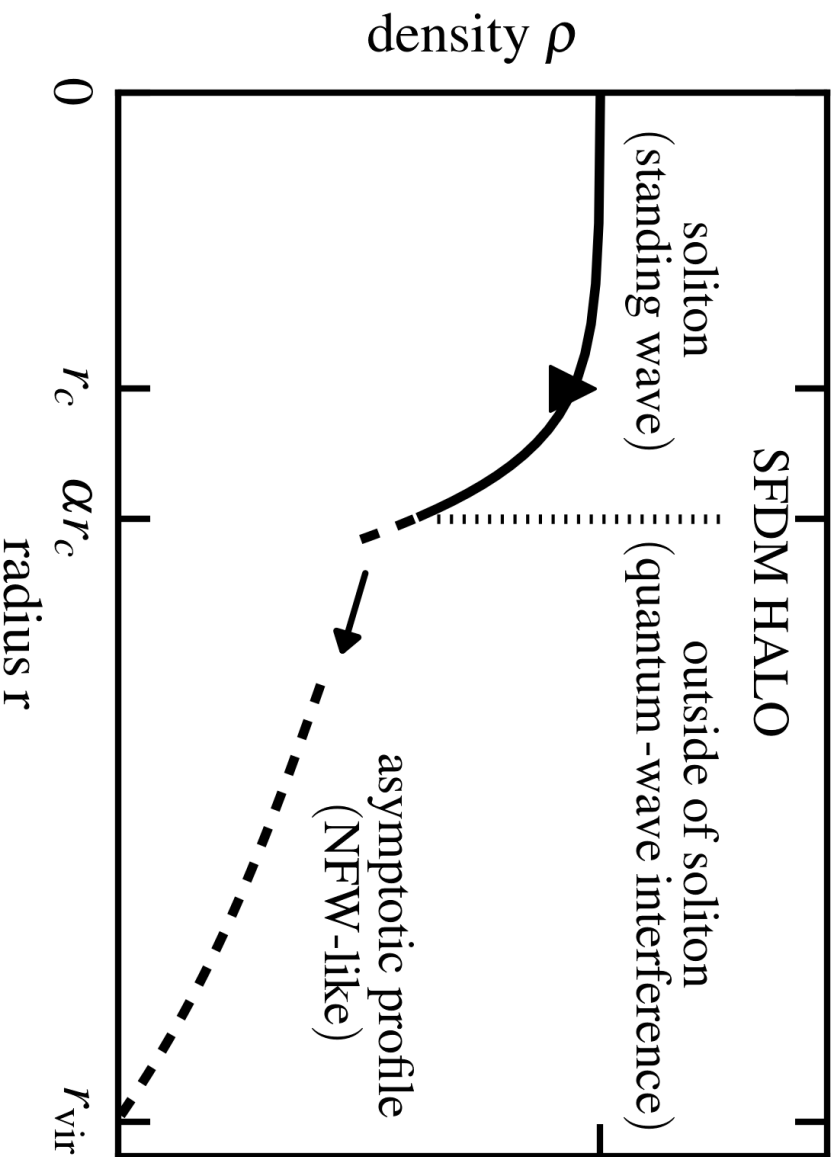
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# Schematic structure of halo

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Simple scaling:

$$r_{\text{core}} \propto \lambda \propto (m\nu)^{-1}$$

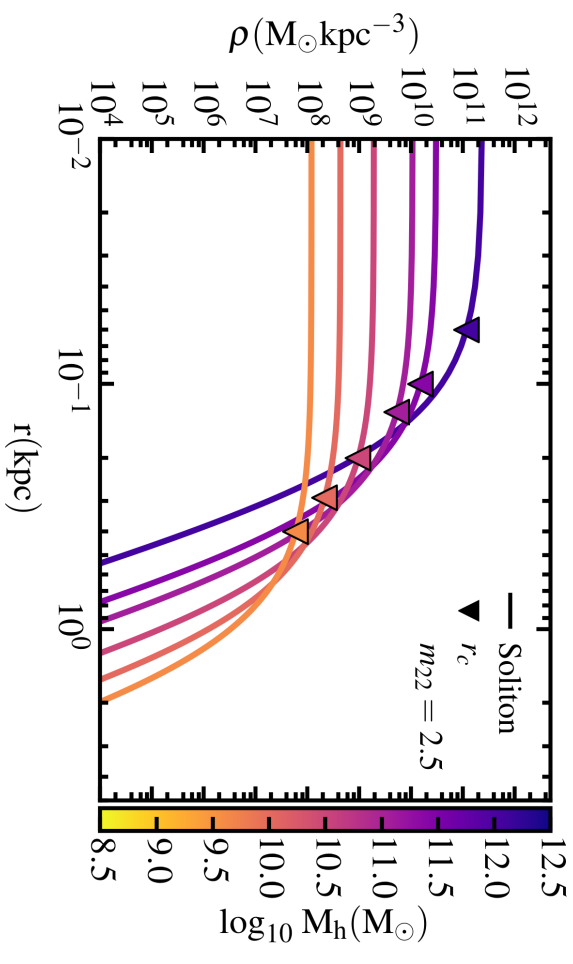
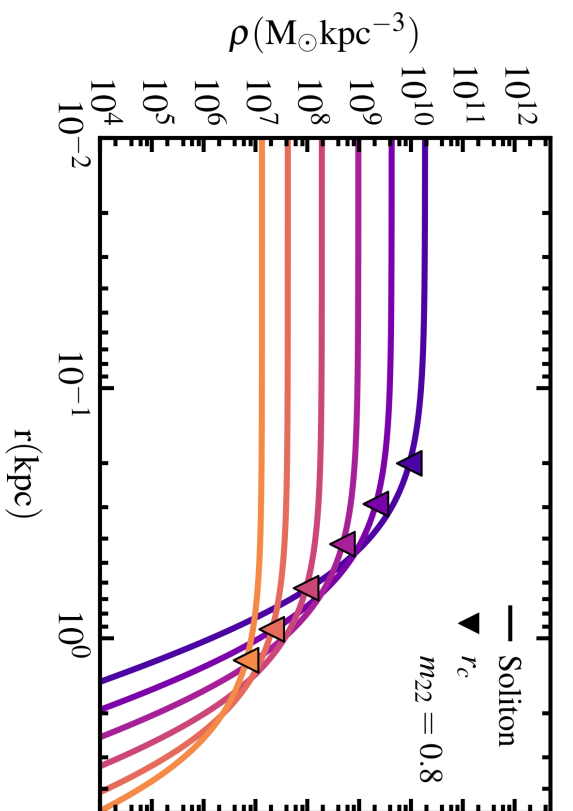
$$\nu \propto M_{\text{halo}}^{1/3}$$

$$r_{\text{core}} \propto 1/(m M_{\text{halo}}^{1/3})$$

$$\frac{r_{\text{core}}}{r_{\text{vir}}} \propto 1/(m M_{\text{halo}}^{2/3})$$

# Schematic structure of halo

---



lower-mass halo  $\rightarrow$  lower density, larger solitonic core

# Numerical difficulties

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$$\dot{\rho} + 3H\rho + \frac{1}{R} \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\dot{\mathbf{v}} + H\mathbf{v} + \frac{1}{R} (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{R} \nabla \Phi + \frac{\hbar^2}{2R^3 m^2} \nabla \left( \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \right)$$

Numerically challenging to handle third derivatives of density

Quantum pressure term is neither isotropic nor (necessarily) positive-definite

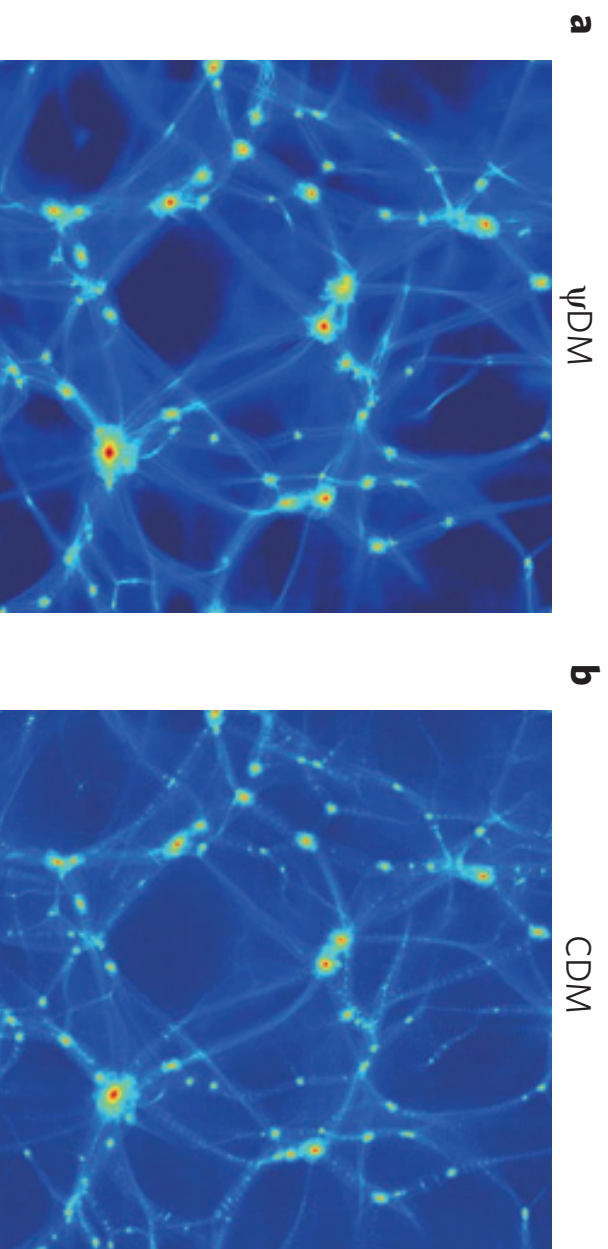
# Numerical frontiers

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- Quantum pressure term can be handled in a similar way to Braginskii conduction or cosmic-ray diffusion (implementations of these exist in current codes)
- Can we run fully cosmological simulations (with or without baryons) that resolve full non-linear structure of scalar field dark matter (including self-interference, quantum turbulence, caustics, ...)?
- How does addition of baryons change soliton structure (particularly if baryons dominate central potential at some point? Can we use analytic models to understand these situations (similar to Kaplinghat et al. for SIDM)?

# Cosmic structure as the quantum interference of a coherent dark wave

Hsi-Yu Schive<sup>1</sup>, Tzihong Chiueh<sup>1,2\*</sup> and Tom Broadhurst<sup>3,4</sup>





Mocz et al. 2017 (also Schwabe et al. 2016): idealized merger simulations

