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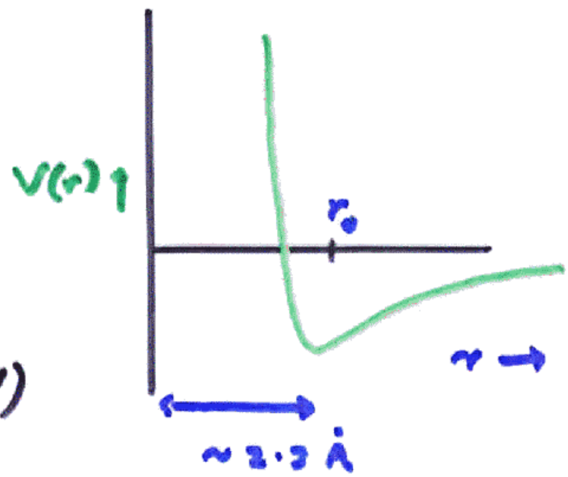
LESSONS FROM ^{3}HE

A.J. Leggett
UIUC

Normal state of ^{3}He: Fermi liquid, $k_F \sim 1\,\text{Å}^{-1}$

Interatomic potential:



$k_F r_0 \sim 2.5$

$\Rightarrow$ s-wave pairing suppressed by hard core.

$$V_{kk'} \sim f(k,k') \sum_l V_l P_l(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')$$

slowly varying, can approx. by $k = k' = k_F$

$V_0 > 0$, V_1 (and V_2 ?) < 0

$\Rightarrow$ pairs form in $l = 1$ state (well verified in expt., e.g. ultrasound abs?)

What does this mean?

Yang: $\rho_2(r_1 r_2 \sigma_1 \sigma_2, r_1' r_2' \sigma_1' \sigma_2') = \sum_i n_i \chi_i(r_1 r_2 \sigma_1 \sigma_2)\chi_i^*(r_1' r_2' \sigma_1' \sigma_2')$

if $N_0 \sim N$, $\chi_0(r_1 r_2 \sigma_1 \sigma_2) = \chi_0(\mathbf{R}, \mathbf{r} : \text{spins})$

"pair wf" $\sim Y_{1m}(\hat{\mathbf{r}})$

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ABM phase (^3He-A):

$$\chi_0(\mathbf{R}, \mathbf{r} : \sigma_1 \sigma_2) = (\text{spin f.}^n \times \text{COM f.}^n \times f(|r|) \times)$$

rel.ve coord.

$\sin\theta \cdot e^{i\varphi}$

total a.m.

The \$64K question: what is $\langle \hat{L} \rangle$?

A: not "obvious" that it is $N\hbar/2$!

[cf: ^3He-A_1: all pairs form in $\uparrow\uparrow$ state, but $\langle \mathbf{S} \rangle \neq N$!]

"Pair wf" describes those particles which are condensed, not liquid as a whole!

In literature, find at least

(a) $\langle L \rangle = N\hbar/2$

(b) $\langle L \rangle \sim N\hbar/2 \times (\Delta/\epsilon_F)$

(c) $\langle L \rangle \sim N\hbar/2 \times (\Delta/\epsilon_F)^2$

Complication: boundary cond.^{ns} on $\mathbf{l}$!

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"Naive" calcuⁿ (ignore boundary conditions):

(a) BCS:

$$\Psi_{BCS} = \prod_{\underline{k}} (u_{\underline{k}} + v_{\underline{k}} a^{+}_{k\uparrow} a^{+}_{-k\uparrow})|vac\rangle$$

$$u_k = |u_k|, \quad v_k = |v_k| \exp i\phi_k$$ ← $\angle$ of $\hat{\underline{k}}$ around $\hat{\underline{l}}$

$\langle N\rangle = \sum_k |v_k|^2 \Rightarrow$ intuitively, create N/2 pairs w. ang. momⁿ $\hbar$ each $\Rightarrow \langle L\rangle = N\hbar/2$

Formally, take N-particle projⁿ of Ψ_{BCS}:

$$\Psi_N = (\text{const.})\, \hat{\Omega}^{N/2} |vac\rangle$$

$$\hat{\Omega} \equiv \sum_k c_k a^{+}_{k\uparrow} a^{+}_{-k\uparrow}, \quad c_k = v_k/u_k$$

$$= |c_k| \exp i\varphi_k$$

$$[\hat{L}_z, \hat{\Omega}] = -i\hbar \sum_k \left(\frac{\partial c_k}{\partial \phi_k}\right) a^{+}_{k\uparrow} a^{+}_{-k\uparrow} = \hbar\Omega$$

$$\Rightarrow \quad \hat{L}_z \Psi_N = \frac{N\hbar}{2} \Psi_N$$

Note: result is indepᵗ of Δ!

→ macroscopic discontinuity in $\langle L\rangle$ at $\Delta = 0$. (? limits $\Delta \to 0$, $R \to \infty$ do not commute?)

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(b) alternative approach: consider (apart from norm^n)

$$\Psi'_N = (\hat{\Omega}^+)^{N_+} (\hat{\Omega}^-)^{N_-} |FS\rangle$$

normal-state Fermi sea

$$\hat{\Omega}^+ \equiv \sum_{k>k_F} c_k a^+_k a^+_{-k}$$

$$\hat{\Omega}^- \equiv \sum_{k<k_F} d_k a_{-k} a_k$$

with appropriate spin indices

$$N_+ = \sum_{k>k_F} \frac{|c_k|^2}{1+|c_k|^2}$$

$$c_k \equiv v_k/u_k, \quad d_k \equiv u_k/v_k$$

$$N_- = \sum_{k<k_F} \frac{|d_k|^2}{1+|d_k|^2}$$

This gives same value of K.E. as Ψ_N

It does not as it stands give same value of P.E.

bec. it does not allow scattering of a pair of particles into hole states and v.v. To remedy this, set for the moment $N_+ = N_- \equiv p$ and consider

$$\Psi''_N = \sum_p f_p (\hat{\Omega}^+)^p (\hat{\Omega}^-)^p |FS\rangle \qquad \sum_p |f_p|^2 = 1$$

with f_p slowly varying. Irrespective of form of v_k, this gives same value of both KE and PE as Ψ_{BCS}.

In fact, in s-wave case it is simply a trivial rewriting of Ψ_N. ($\sim \Psi_{BCS}$). But...

NMTS

WHAT IS VALUE OF $\langle L \rangle$ FOR "ALTERNATIVE" W.F. IN $\ell \neq 0$ CASE ?

Ans.: $\langle L \rangle = \frac{\hbar}{2}\left(\langle N_+ \rangle - \langle N_- \rangle\right)$

and so in limit $\langle N_+ \rangle = \langle N_- \rangle$ $(\Delta \to 0)$

$$\langle L \rangle = 0$$

no discontinuity at $\Delta = 0$!

Conjecture: for noninfinitesimal Δ,

$$N_+ - N_- = N_{\epsilon_F} - N_\mu$$

($N_\mu \equiv$ no. of states within Fermi sea with a k_F given by $\frac{\hbar^2 k_F^2}{2m} = \mu$). If so, then for small Δ

$$\langle L \rangle \sim \frac{N\hbar}{2}(\Delta / \epsilon_F)^2 \qquad \text{(since } \mu(\Delta) - \epsilon_F \sim (\Delta/\epsilon_F)^2 \text{)}$$

but for $\mu < 0$ ("BEC" side of crossover)

$$\langle L \rangle = N\hbar/2.$$

as expected.