What can we measure

with CBC parameter estimation?

John Veitch

Rattle and Shine

KITP, August 2012

NIKHEF

What is there to measure?

What parameters are there are to measure?

- The simplest model of a compact binary in a circular orbit has 9 parameters

2 masses, time, sky position, distance, 3 orientation angles

- Add spins for another six

2 magnitudes, 4 orientation angles

- and neutron stars have equation of state parameters

Tidal deformability of each star λ_1, λ_2

- Then there are possible deviations from the GR model...

The observations

Advanced detectors will provide huge amounts of data

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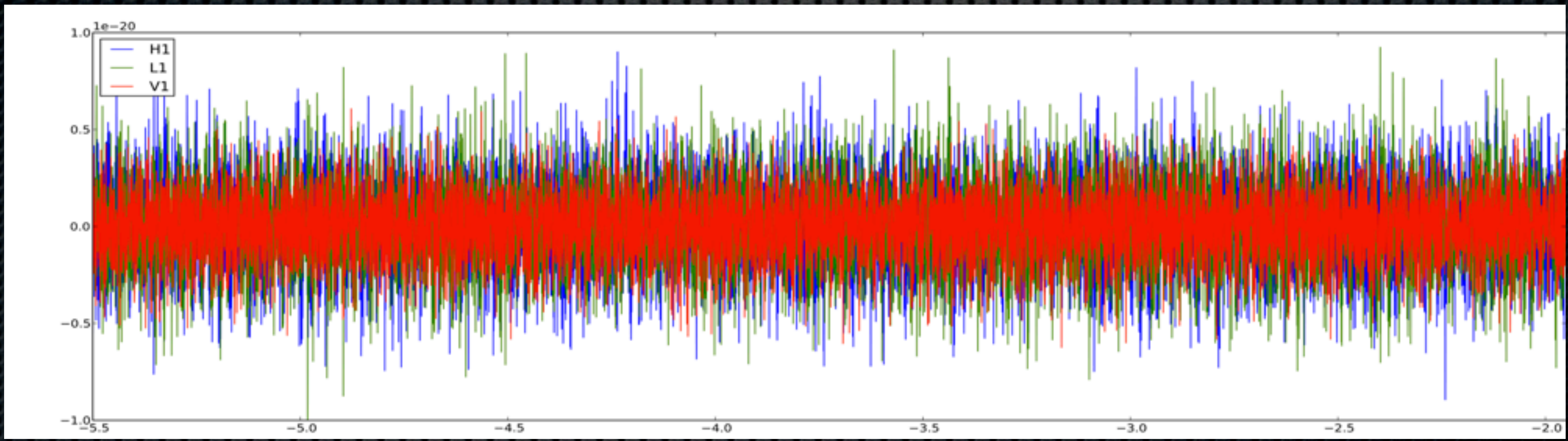
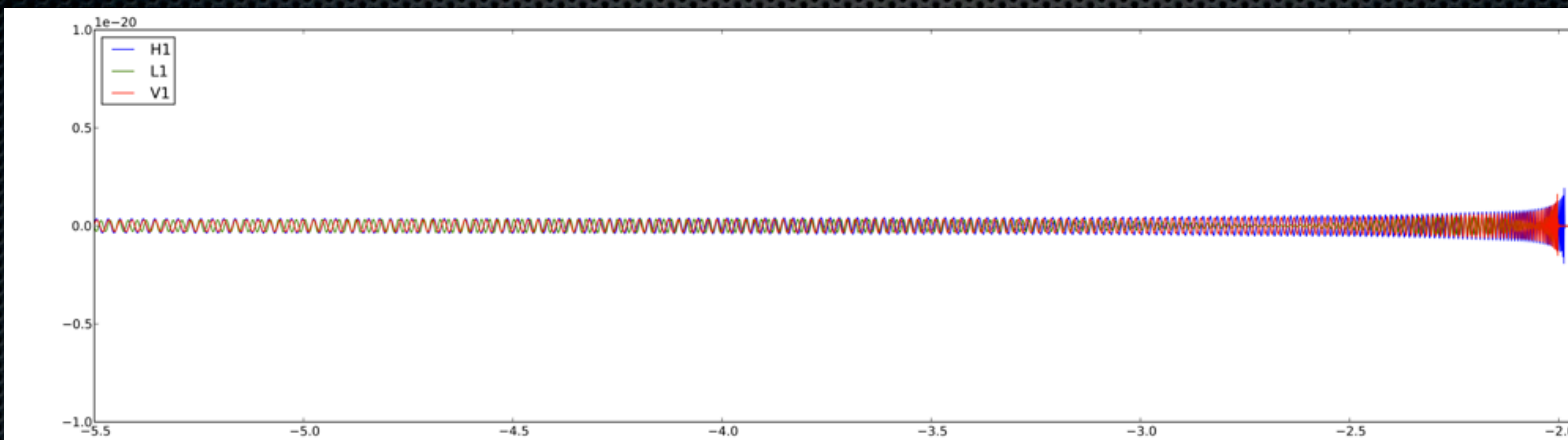


(on a good day!)

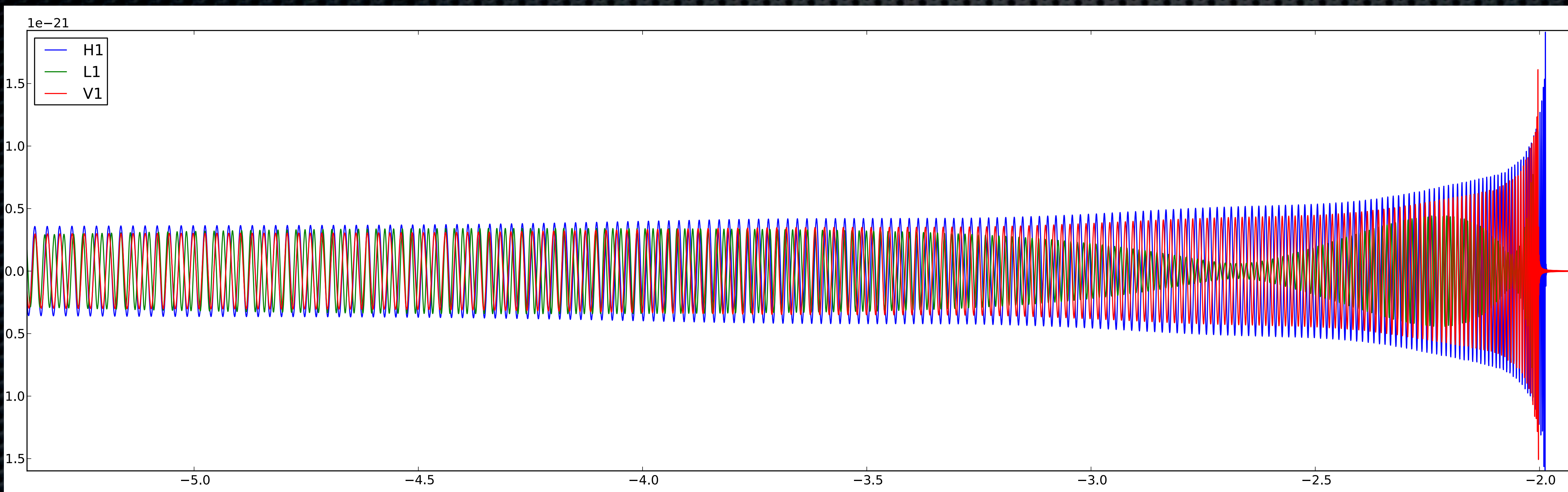
The observations

Advanced detectors will provide huge amounts of data



(on a good day!)

GW signal



Fortunately the observed inspiral is well described by post-Newtonian theory and understanding of the detectors. (i.e. we have a mapping from parameters ⇒ waveform)

The gravitational wave signal contains a lot of information encoded by the amplitude and phase evolution. (30-40 bits at signal-to-noise of 10)

“Just” need to convert the observed waveform ⇒ parameters

Parameter Estimation

Our signal model *H* has some free parameters θ, represent the knowledge of those parameters with a probability distribution

$$\int_{\Theta} p(\vec{\theta}|H,I)\,d\vec{\theta} = 1$$

where "*p*" represents probability *density*.

After observing some data *d*, calculate the posterior probability density function (PDF)

$$p(\theta|d,H) = \frac{p(d|\theta,H)p(\theta|H)}{p(d|H)}$$

Using likelihood, prior and evidence,

$$p(d|H) = \int_{\Theta} p(\theta|H)p(d|\theta,H)\,d\theta$$

Model Selection

When we observe real signals will not know the “true” waveform.

- Use Bayesian evidence to compare models in light of the data
- Model selection tells us which is a better fit
- Automatically takes into account Occam’s razor

Compute Odds Ratio between A and B:

$$\frac{P(A|d,I)}{P(B|d,I)} = \frac{P(A|I)}{P(B|I)} \frac{\int d\theta_A p(\theta_A|A,I) p(d|\theta_A,A,I)}{\int d\theta_B p(\theta_B|B,I) p(d|\theta_B,B,I)}$$

A and B can have different numbers of parameters.

Sampling the probability distribution

For a linear model we can find the best fit values easily, but if not we need to search the parameter space.

Search codes: maximise over parameters if possible. Parameter estimation: compute full probability density function (PDF)

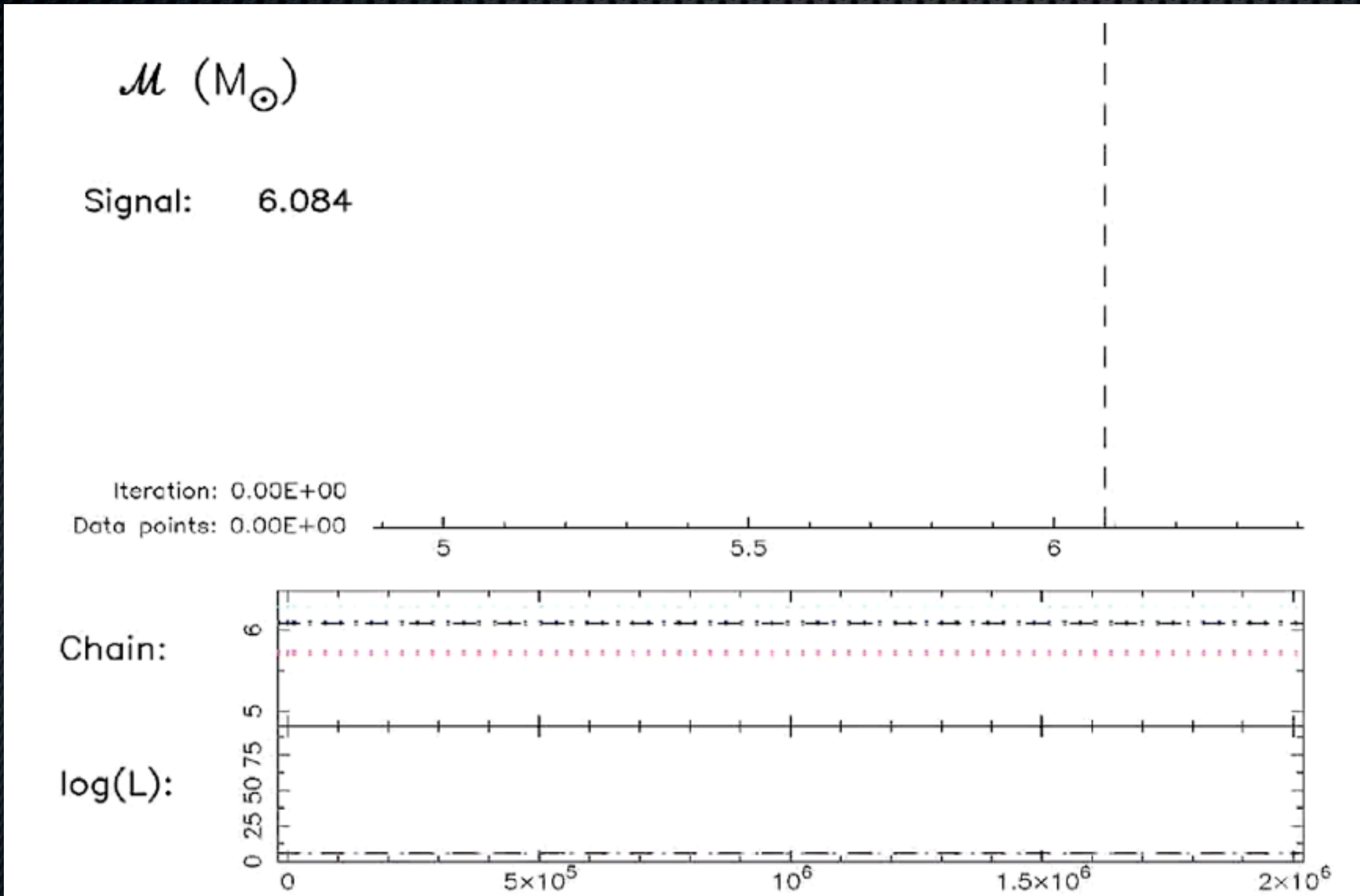
Generating samples directly from posterior PDF is hard.

- Need to know the shape of the distribution, which is the point of doing the analysis.

Instead, use probabilistic algorithms (MCMC or Nested Sampling)

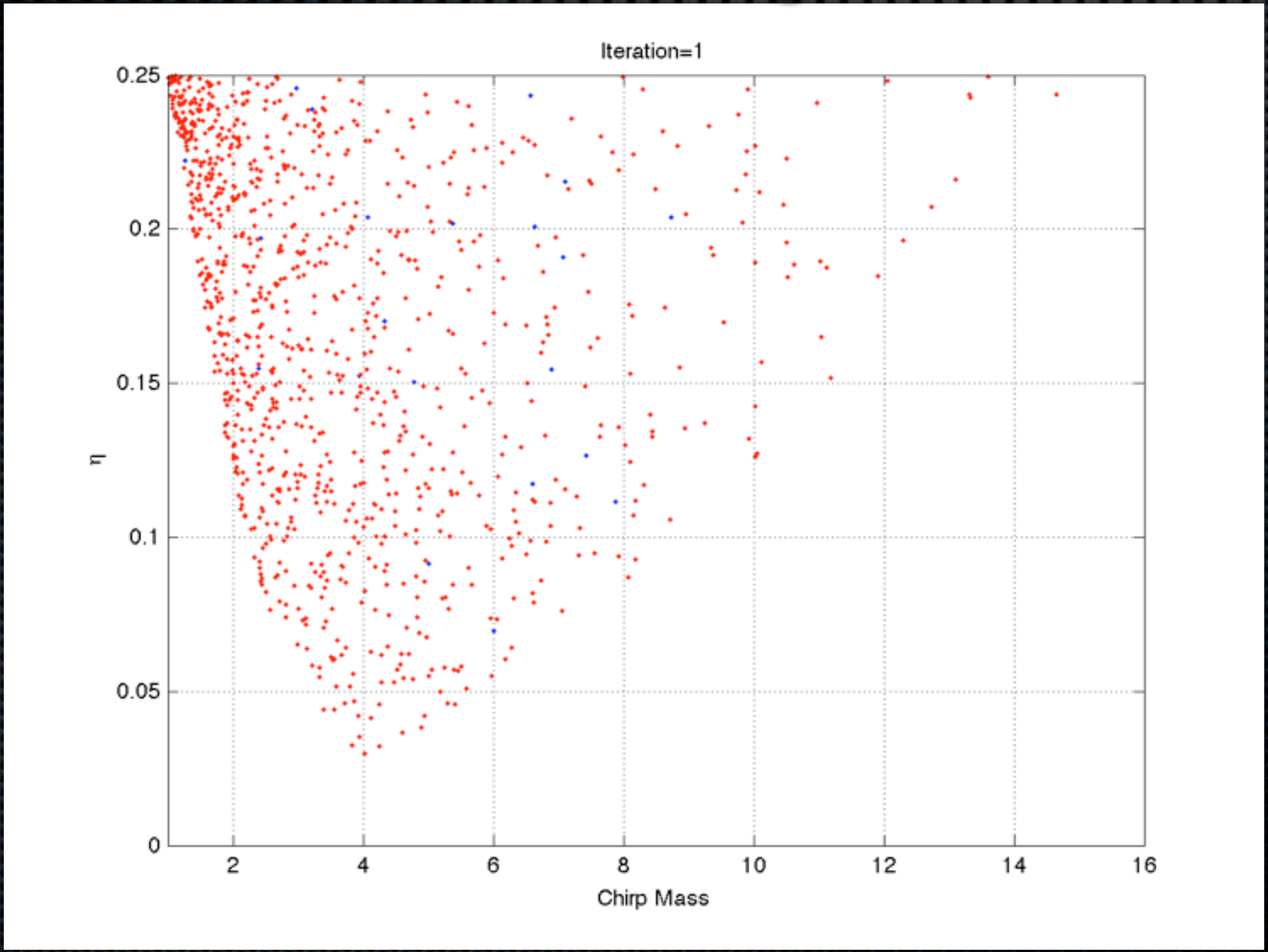
- Generate random points from the parameter space
- Evolve on a random walk
- Keep samples dependent on the value of the posterior PDF.

MCMC



$\mathcal{M}$ ($M_\odot$)
Signal: 6.084
Iteration: 0.00E+00
Data points: 0.00E+00
5
5.5
6
Chain:
5
6
log(L):
0 25 50 75
0
5×10^5
10^6
1.5×10^6
2×10^6

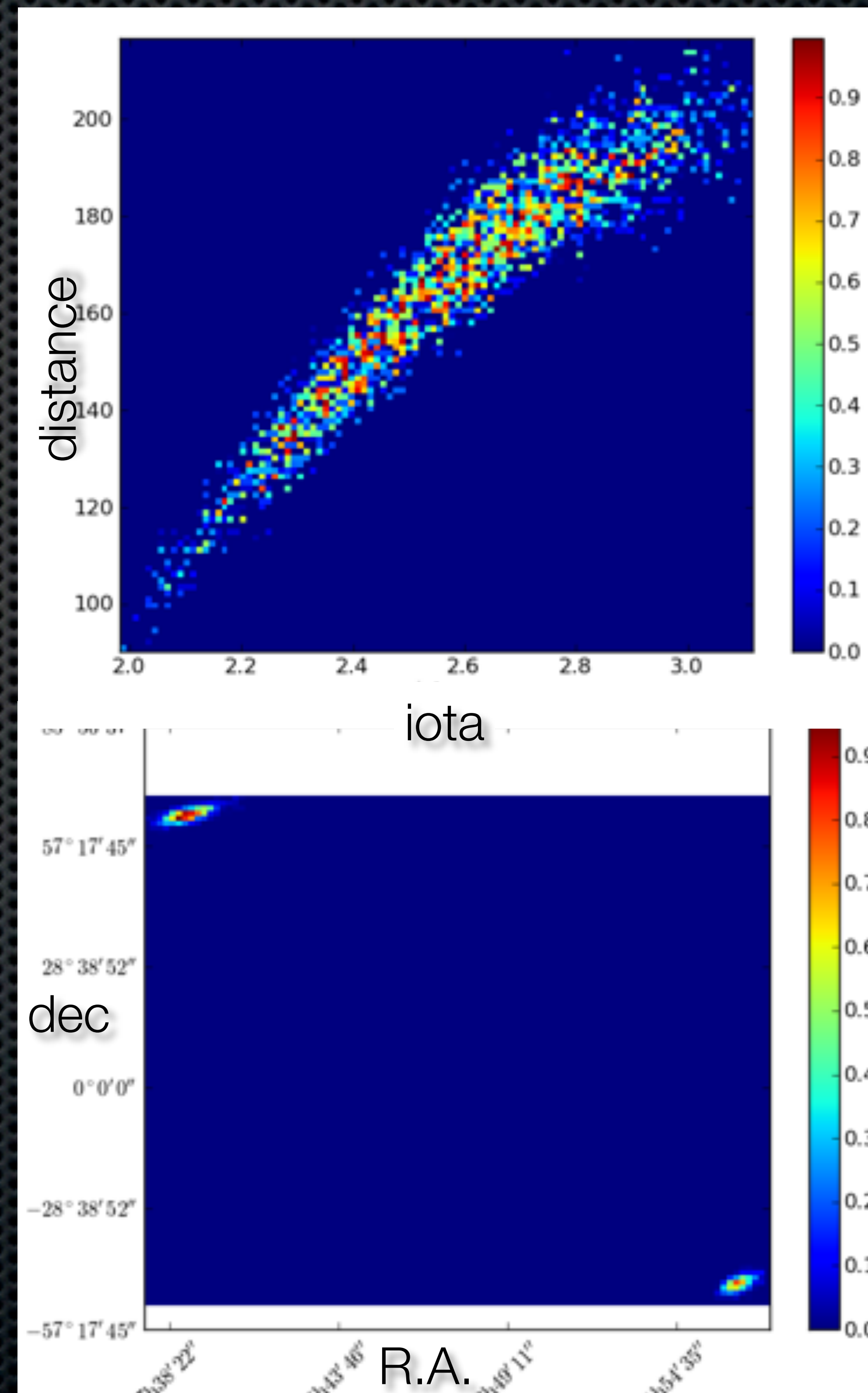
Nested Sampling



Degeneracies and Correlations

Correlations and (partial) degeneracies exist between certain parameters:

- Distance / inclination
- Sky location
- Polarisation / phase
- Masses / spins
- Spin magnitude / tilt

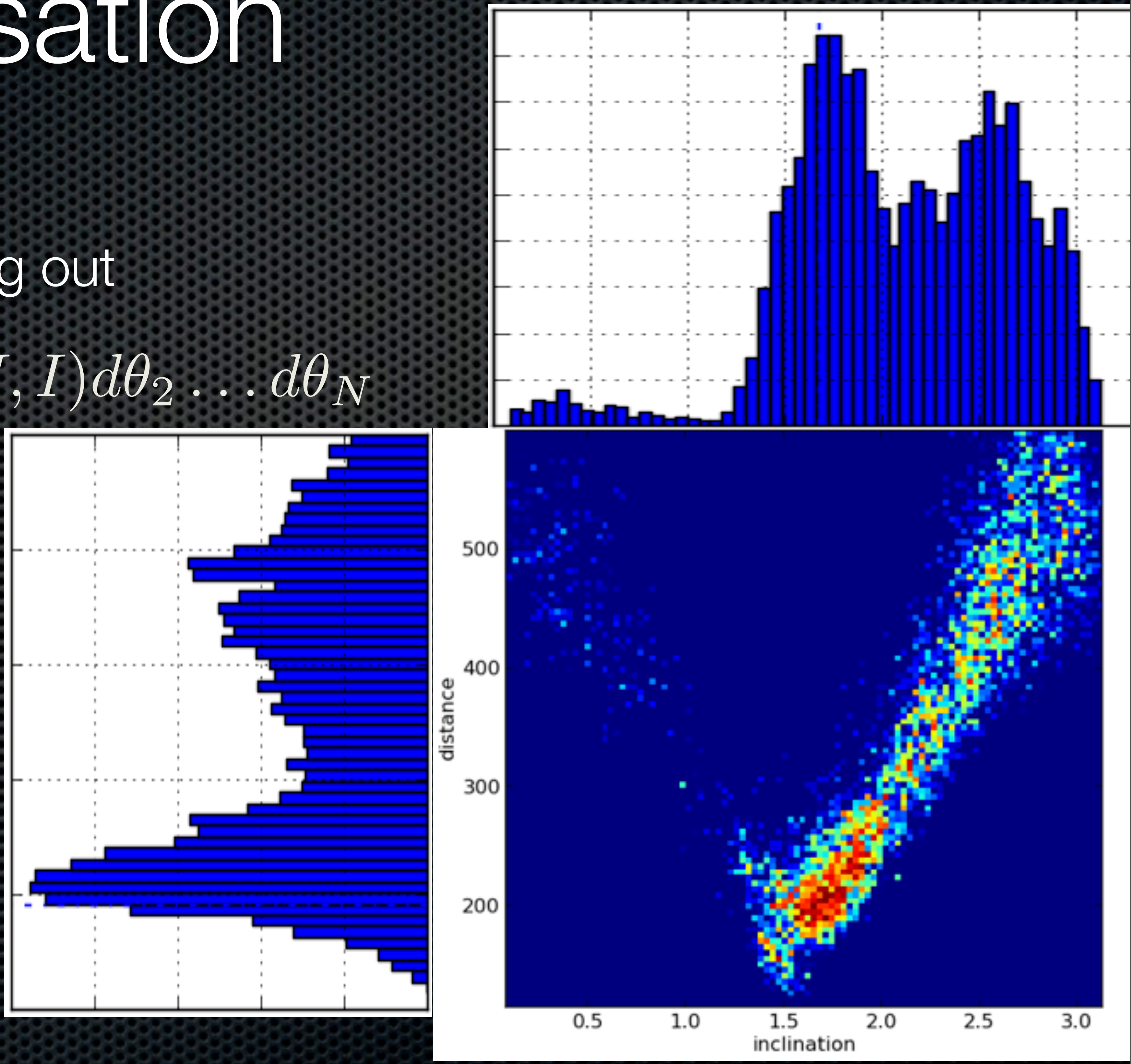


Marginalisation

Eliminate nuisance parameters by integrating out

$$p(\theta_1|H,I) = \int p(\vec{\theta}|H,I)d\theta_2 \ldots d\theta_N$$

Can be done easily with collection of posterior samples by producing histograms for the parameters of interest. Can use 2D histograms to see correlations, etc



Implementation

Implementing these algorithms efficiently and correctly is a tricky task in itself.

MCMC & nested sampling algorithms have been developed over several years by the LSC/Virgo CBC group.

- MCMC: Röver, *et al* CQG 23 (2006) [4 params]; PRD 75 (2007) [9 params] ... van der Sluys *et al* ApJL 688 L61 (2008) [15 params] .... Raymond *et al* CQG 26 (2009) ... Littenberg & Cornish PRD 80 (2009)
- Nested Sampling: Veitch & Vecchio PRD 78 (2008) [4 params] ... Feroz *et al* CQG 26 (2009) ... Veitch & Vecchio PRD 81 (2010) [9 params]

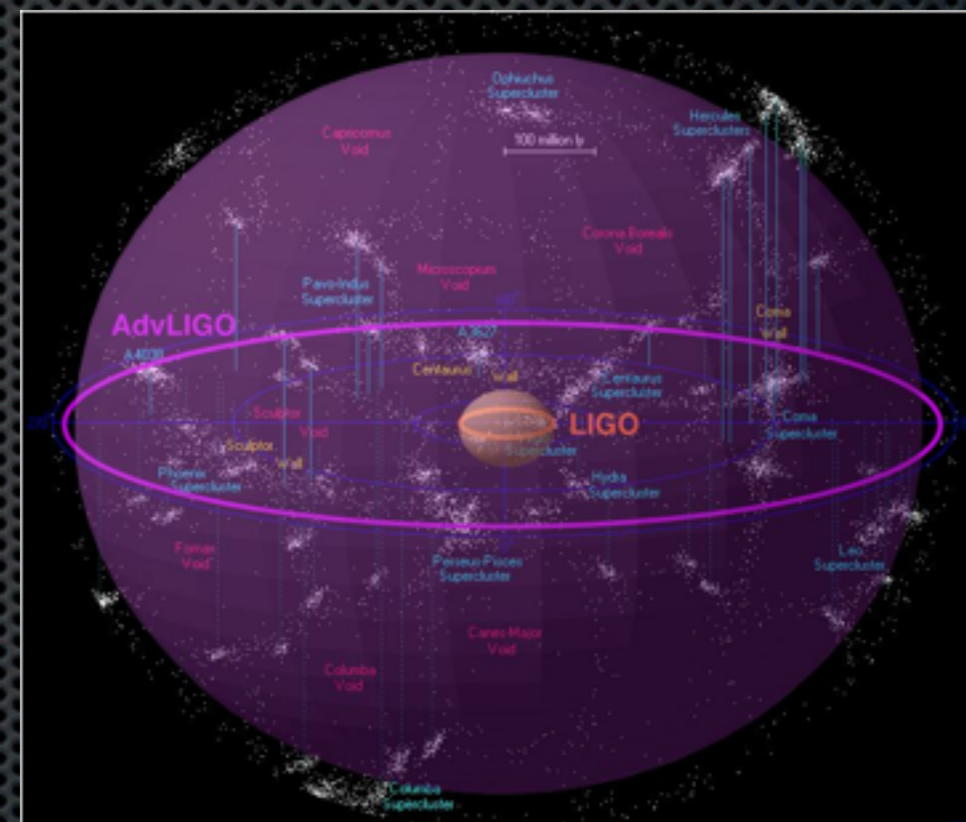
Multiple approaches vital for cross-checking results (and healthy competition)

Combined forces to produce LALInference: open source library for GW inference. (part of LALSuite)

Working with the LSC Waveforms group has reduced run times from O(month) to O(day) for spinning, precessing waveforms. Further improvements still needed.

Advanced Detector Network

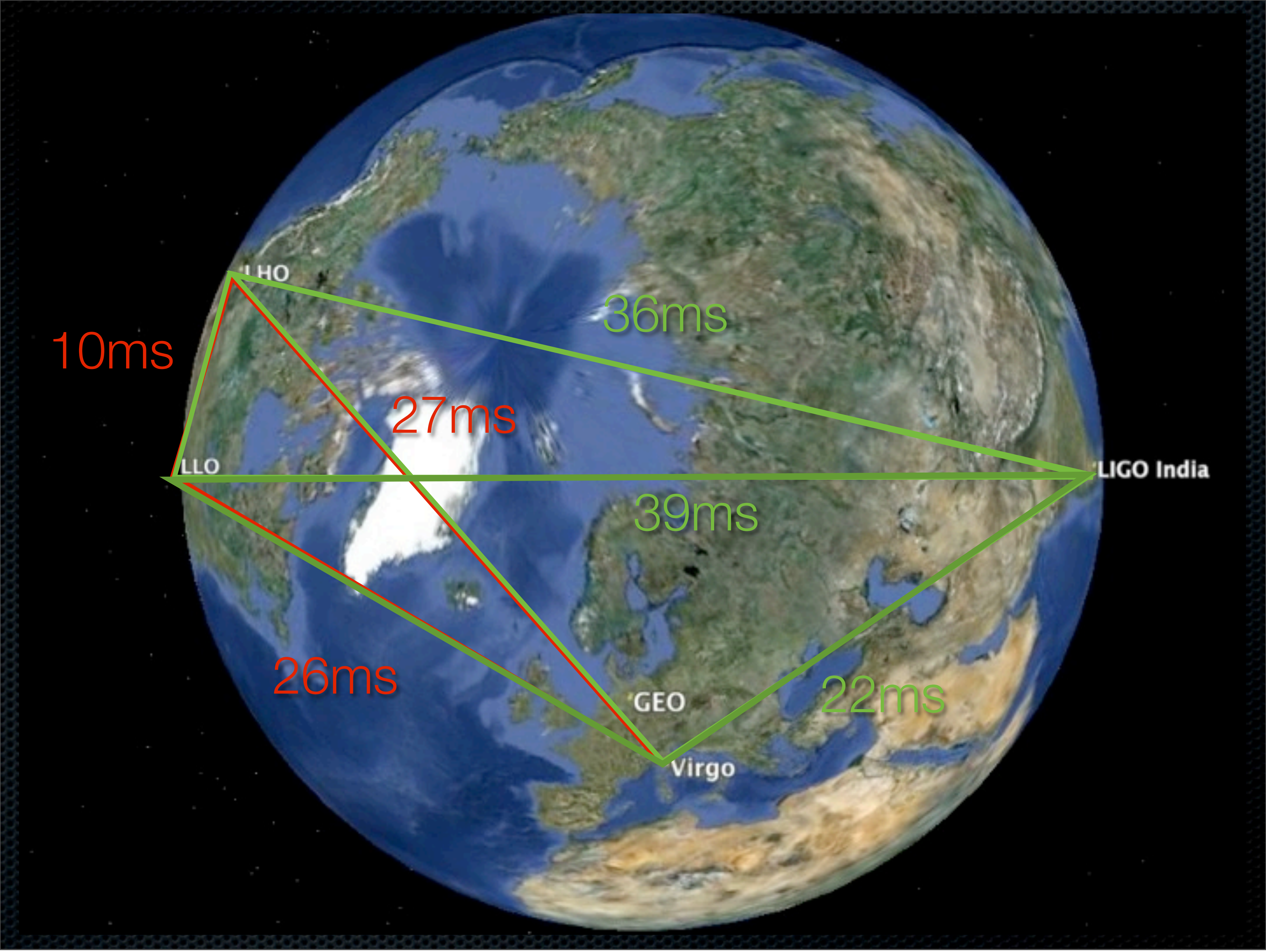
- From ~2015, Advanced detectors will begin to come online.
- When at design sensitivity, are expected to detect per year:
    - 0.4 - 400 BNS
    - 0.2 - 300 NSBH
    - 0.4 - 1000 BBH



How well will we do with these signals?

[Abadie et al, C.Q.G. 27 (1) 2010]


LHO
10ms
27ms
LLO
LIGO India
26ms
GEO
Virgo

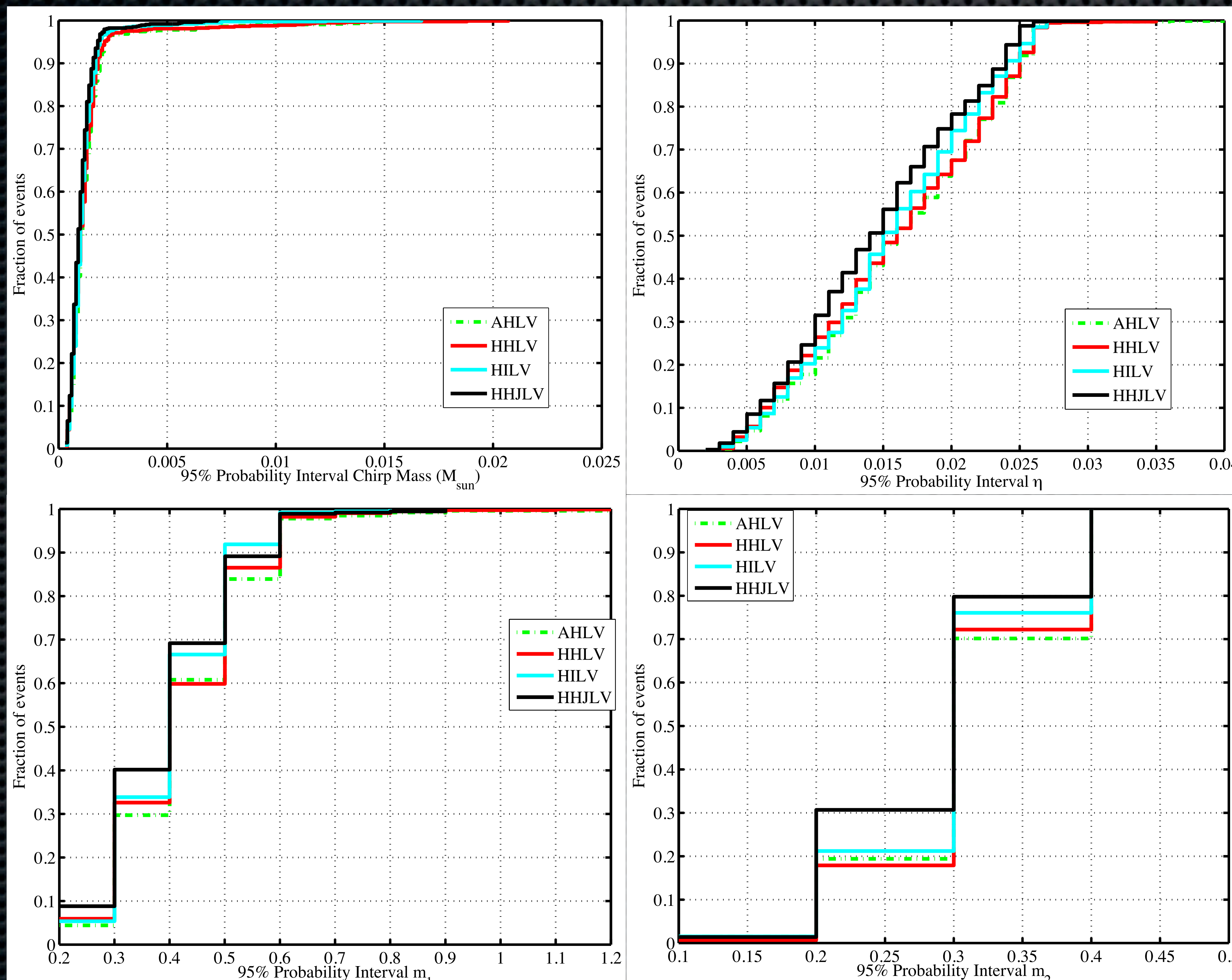

LHO
10ms
36ms
27ms
LLO
LIGO India
39ms
26ms
GEO
22ms
Virgo

What will we measure?

Analysed 750 BNS (1.4-1.4) sources distributed throughout parameter space to build up statistics about resolvability of first 9 parameters.

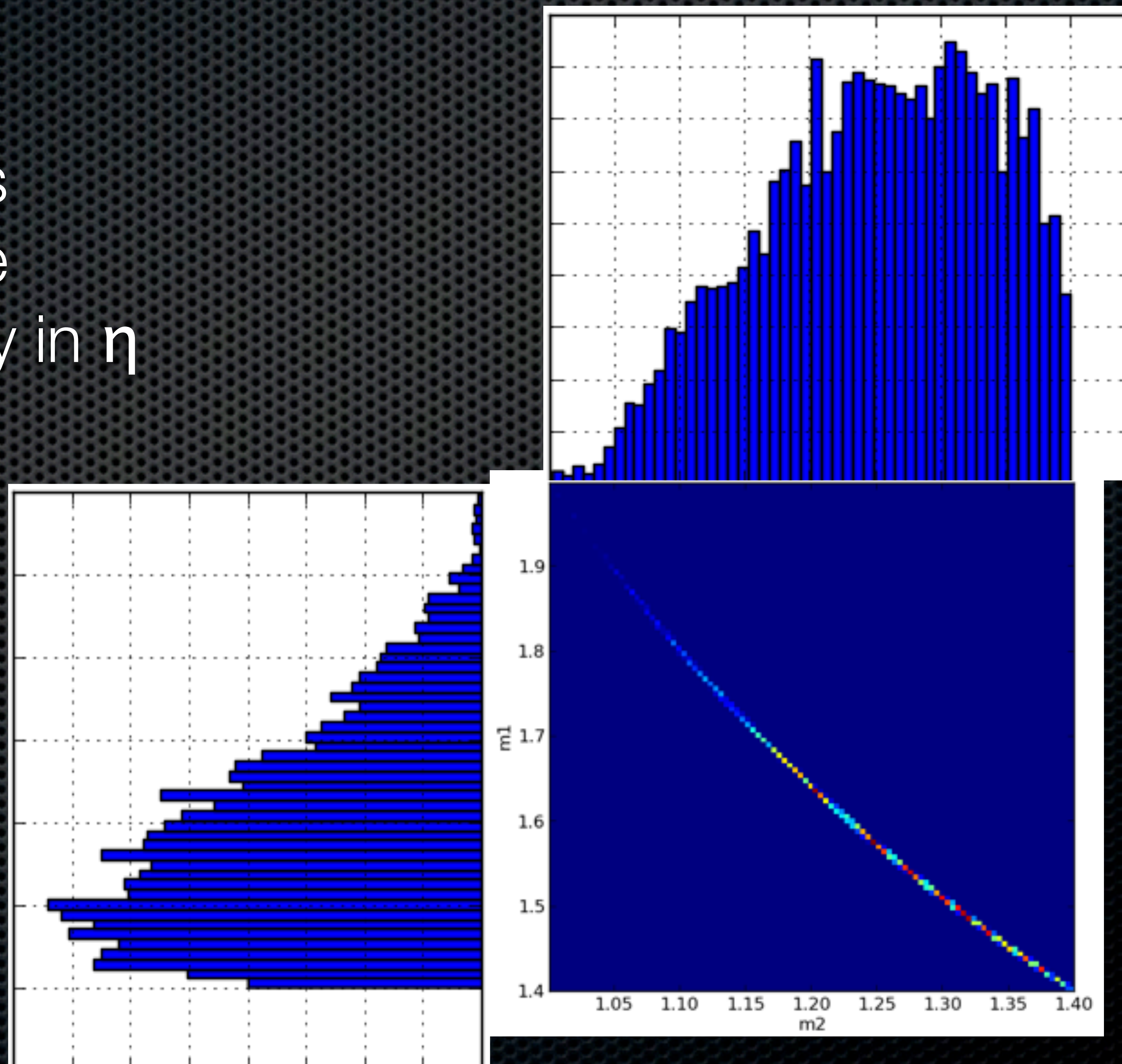
Looked at potential network configurations to compare performance. [Veitch *et al* PRD 85 2012]

Masses


AHLV
HHLV
HILV
HHJLV
Fraction of events
95% Probability Interval Chirp Mass (M_{sun})
95% Probability Interval η
95% Probability Interval m_1
95% Probability Interval m_2

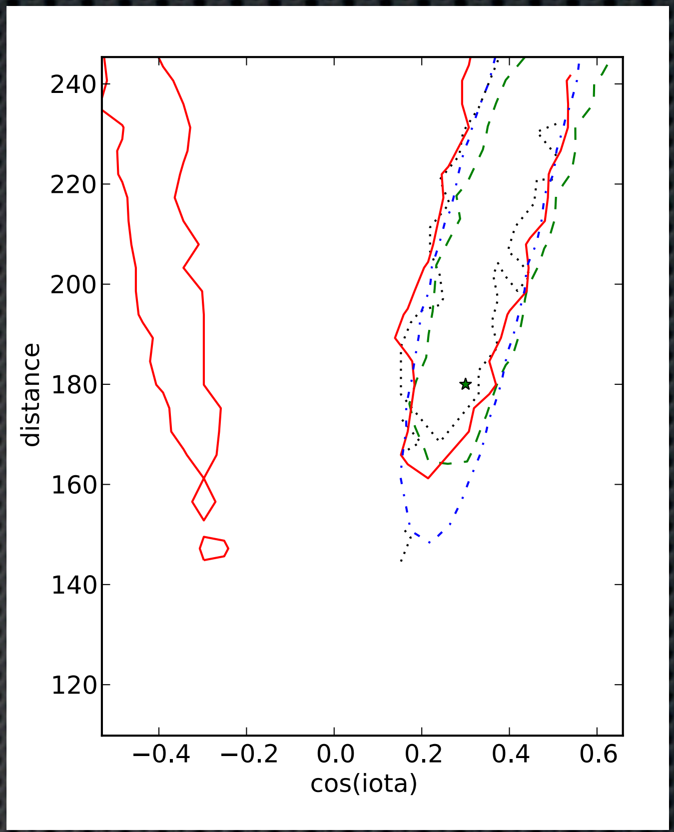
Masses - components

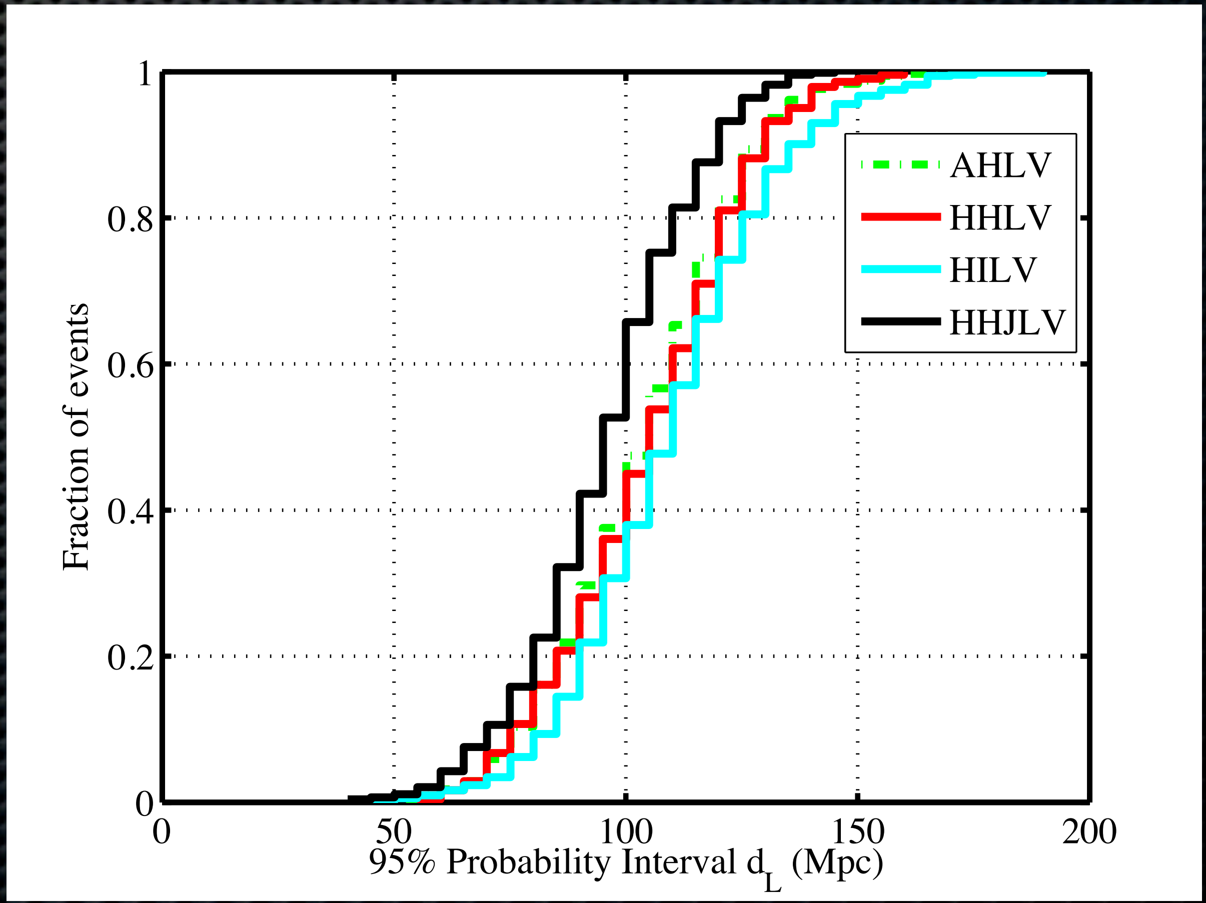
Although chirp mass measured well, large statistical uncertainty in η translates to high correlation between recovered masses.



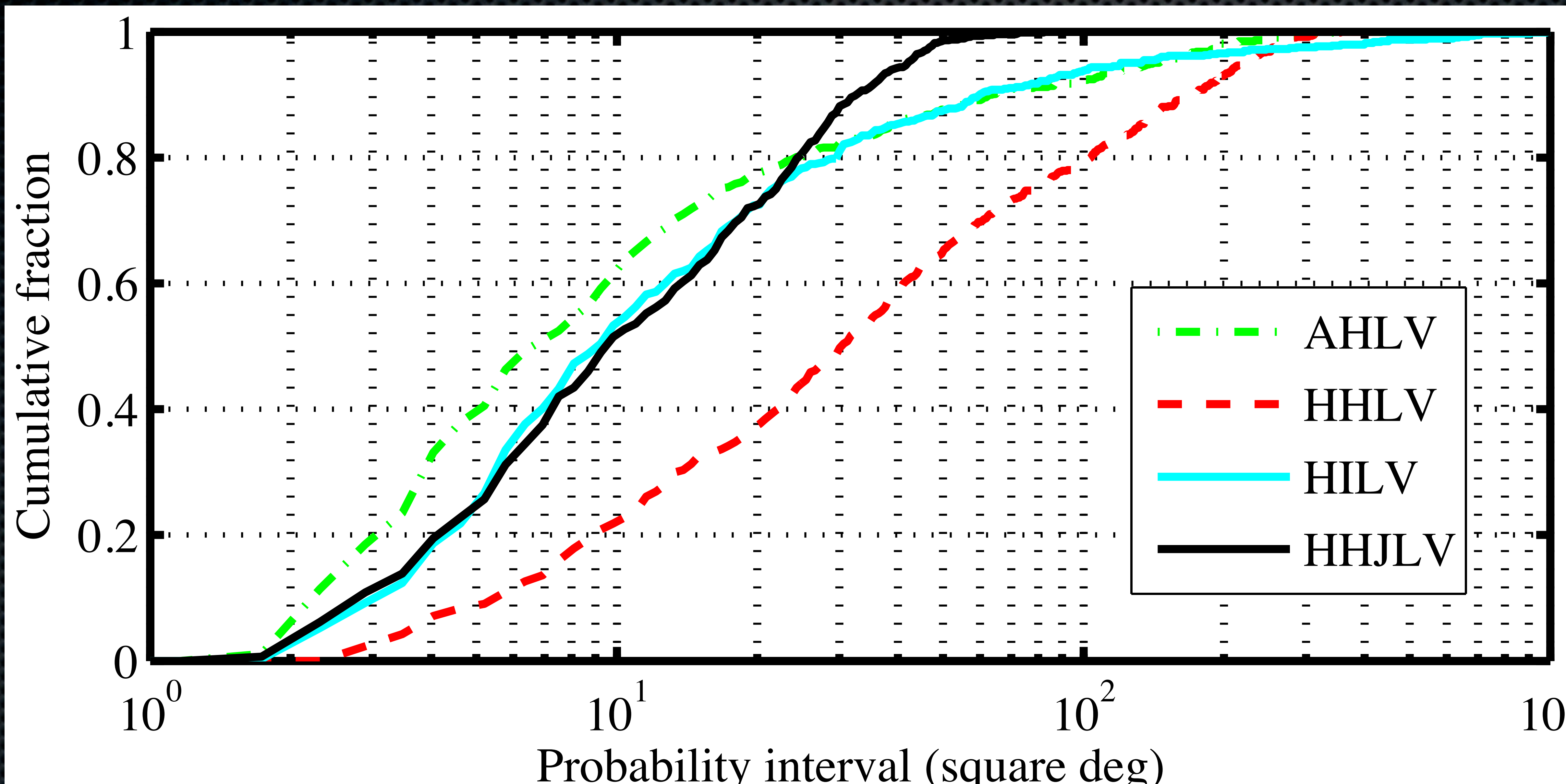
Distance

- Distance is highly correlated with inclination angle
- Can be improved with:-
  - EM counterpart (GRB)
  - Precessing orbital plane from non-aligned spin effects
  - Using higher amplitude modes of the signal (esp. for BBH) which have different emission patterns (See Ben Farr's poster)



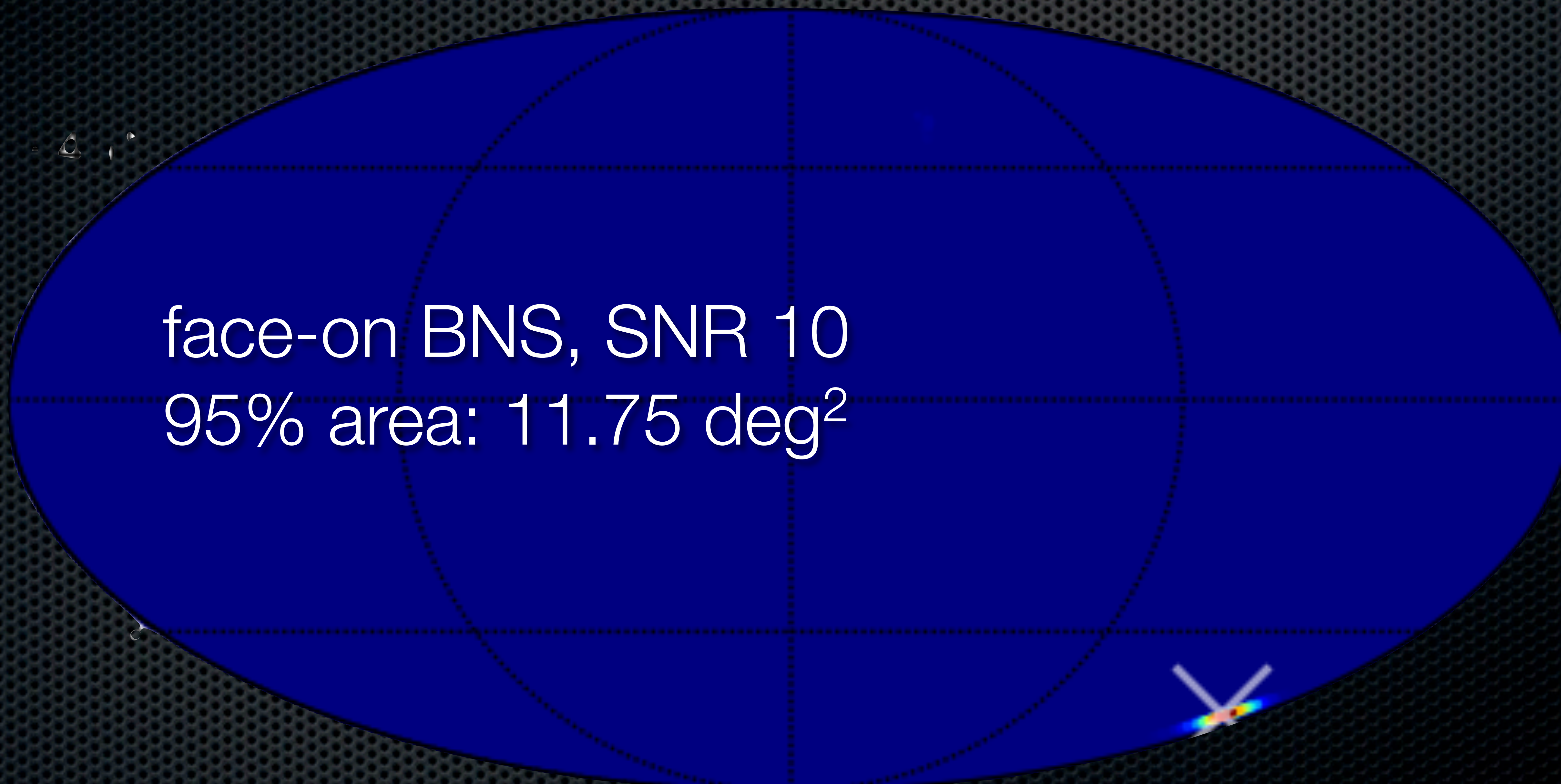


Sky localisation

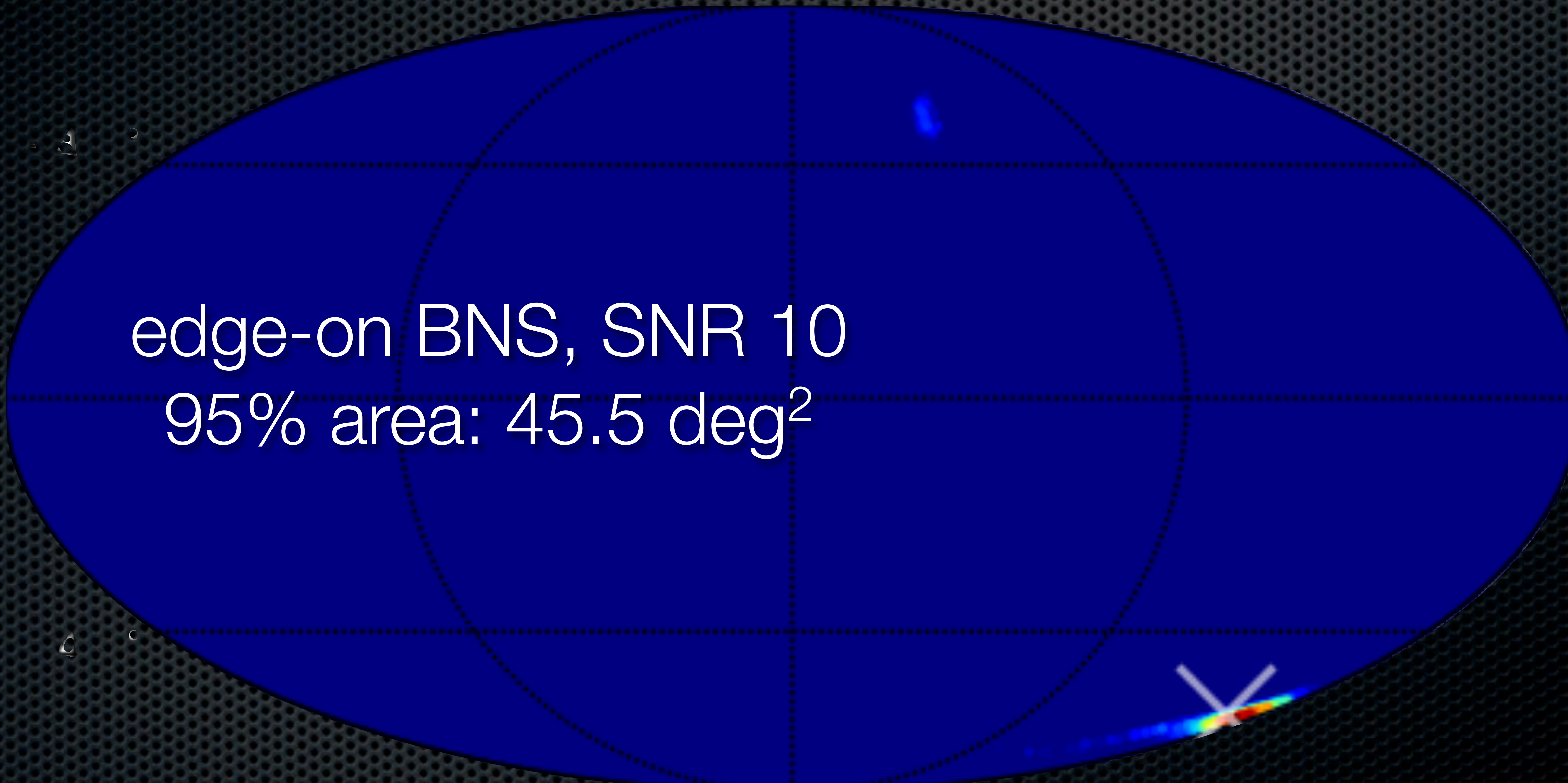


See also Steve Fairhursts's talk, Vivien Raymond's talk; Fairhurst CQG 28 (10) (2011); Nissanke *et al* ApJ 739:99 (2011); Klimenko *et al* PRD 83 (2011)

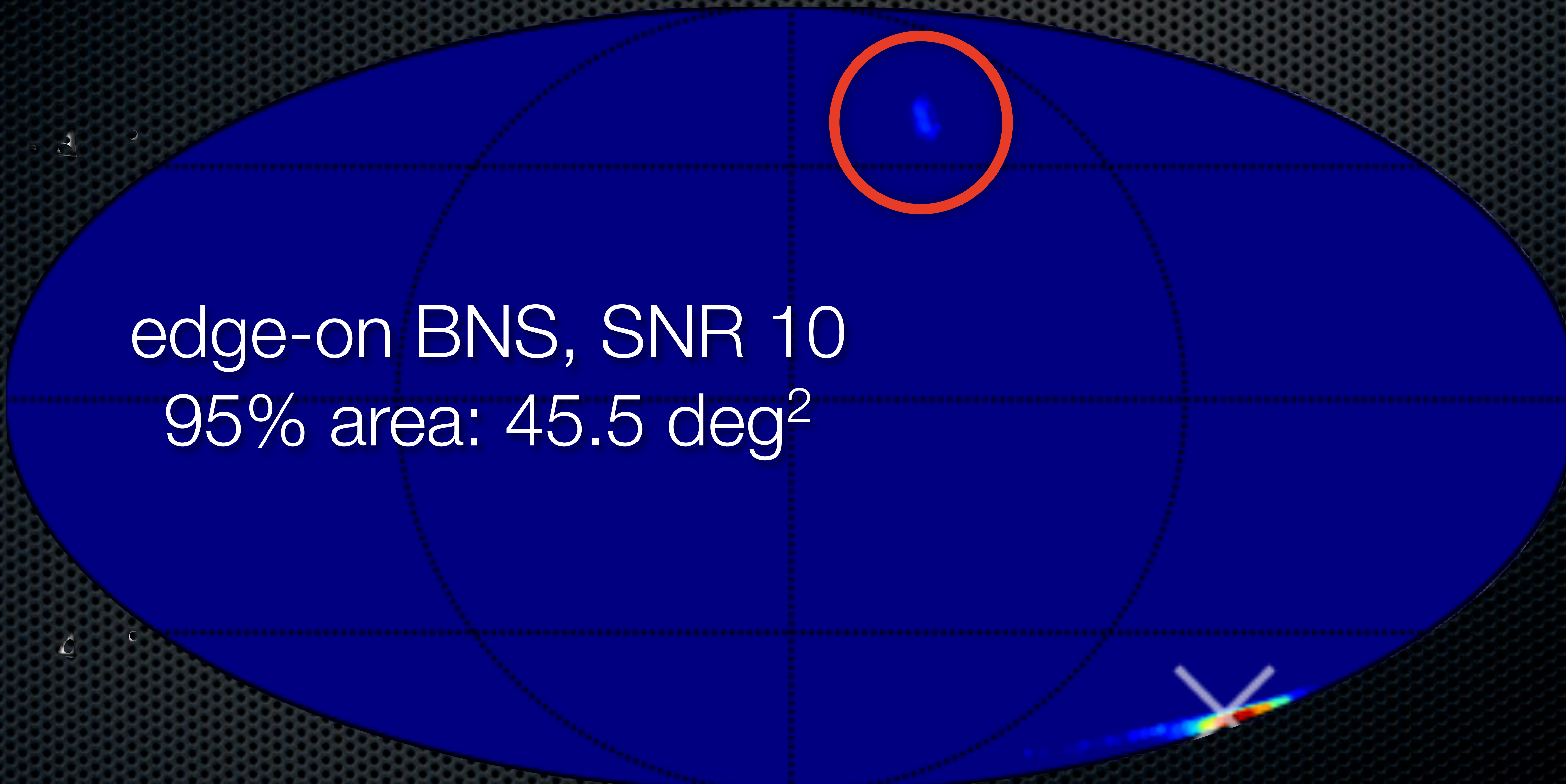
Breaking sky position degeneracy



Breaking sky position degeneracy



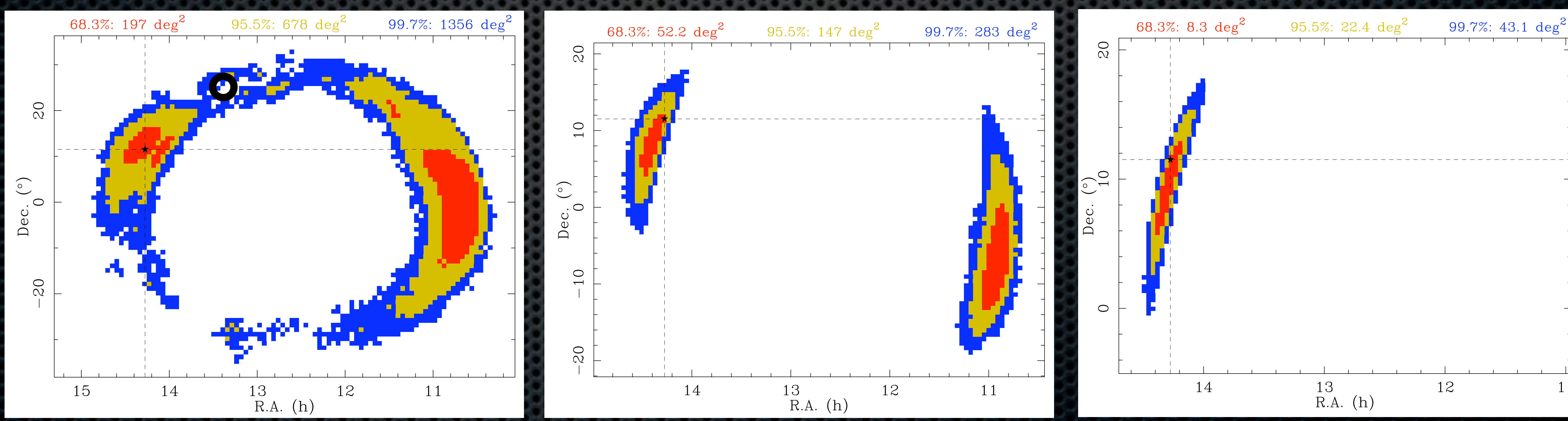
Breaking sky position degeneracy



Break sky position degeneracy - spin

Precessing spin also breaks degeneracy in sky position, including for only 2 detectors. (see talk by Vivien Raymond)

$spin_1$: 0 $spin_1$ 0.5, θ_1 =20° $spin_1$: 0.5, θ_1=55°



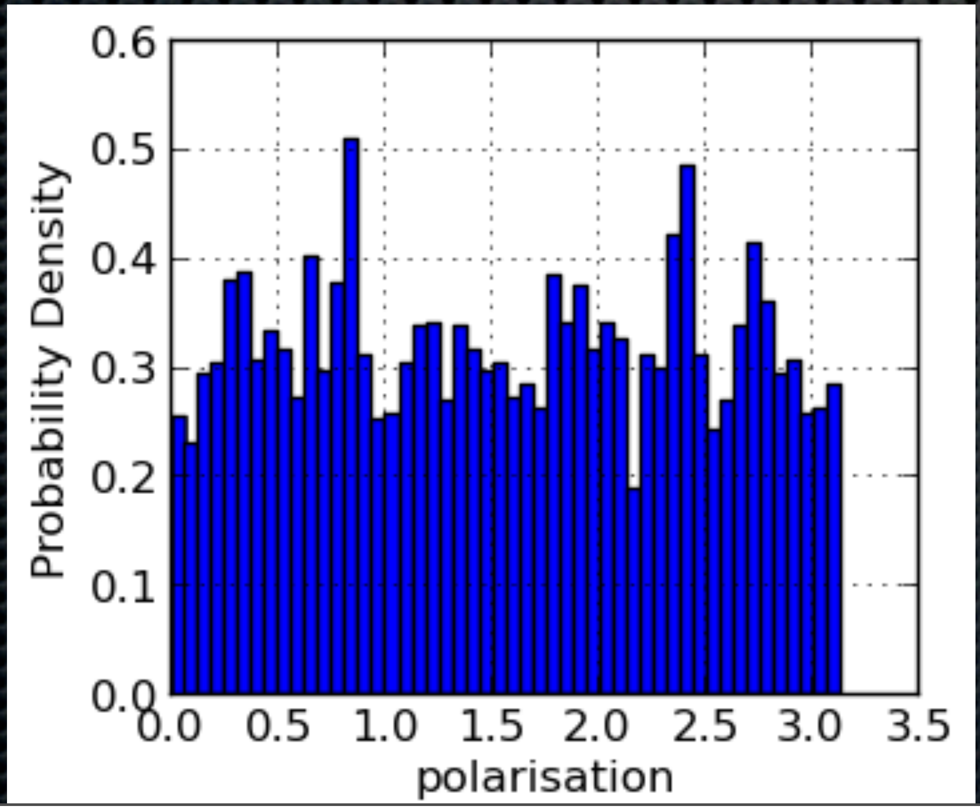
Raymond *et al* Class.Quant.Grav **26** 114007

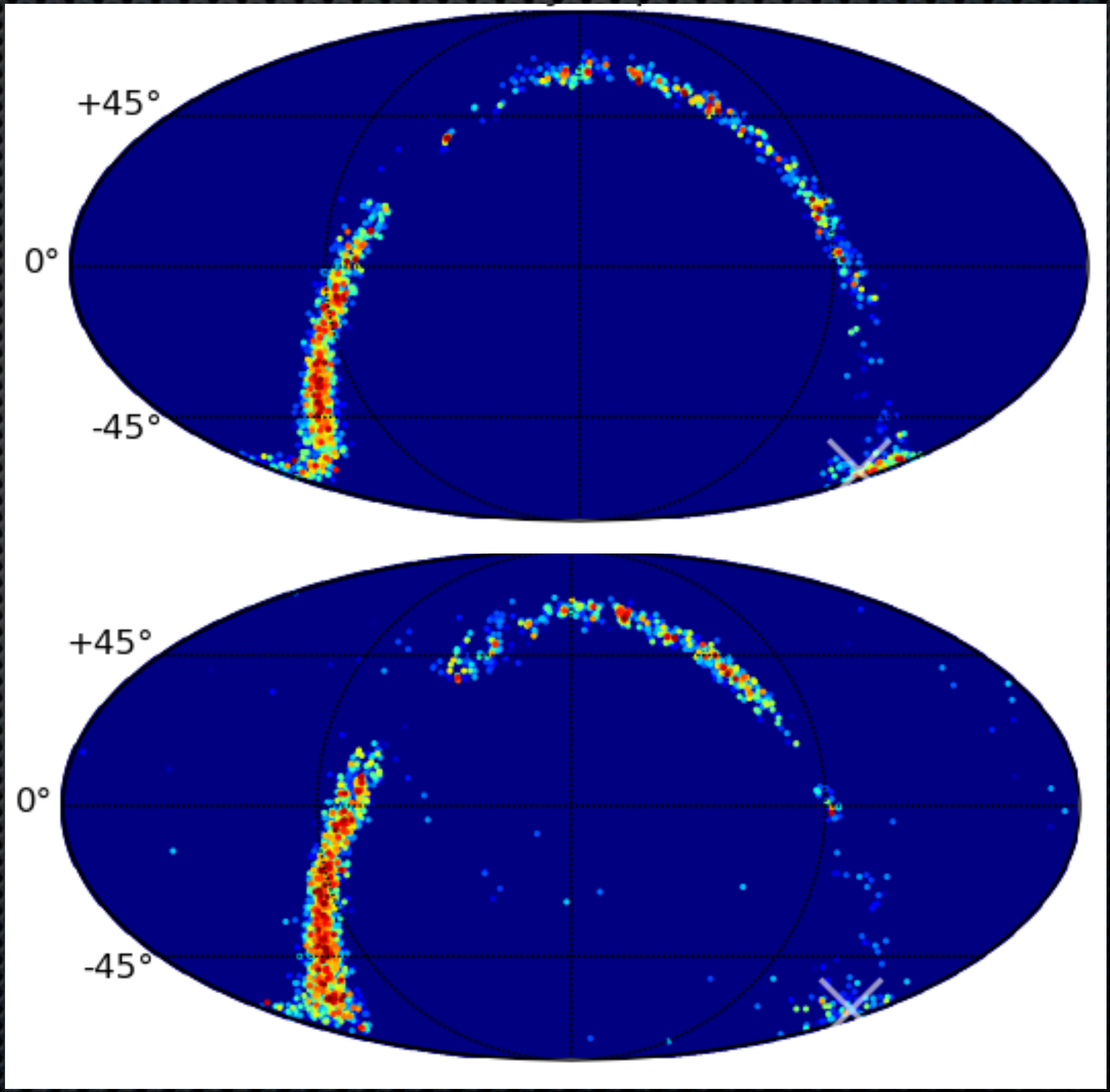
Sub-optimal cases

2 Detector network:

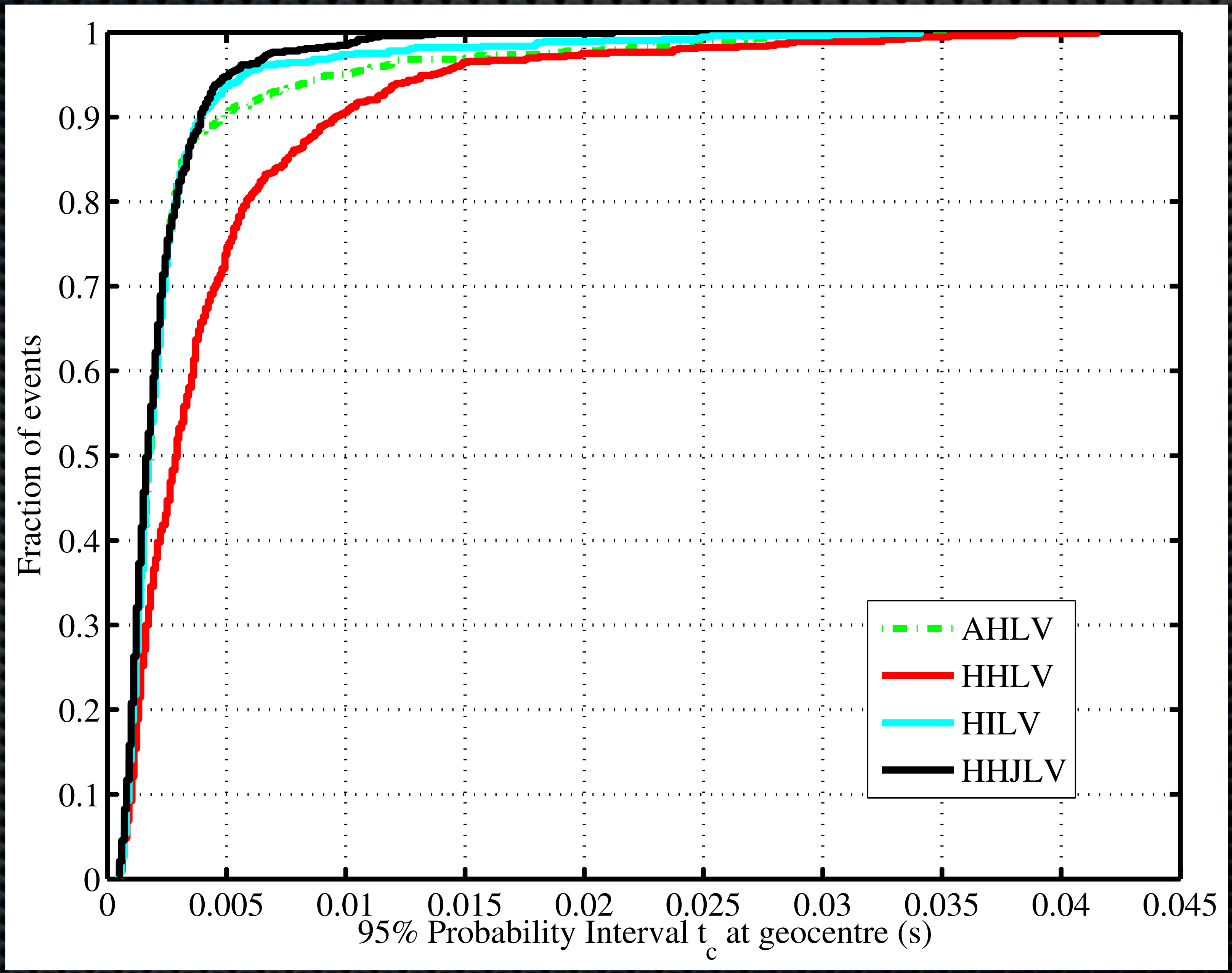
Sky localisation reduced to ring

Extrinsic parameters become degenerate

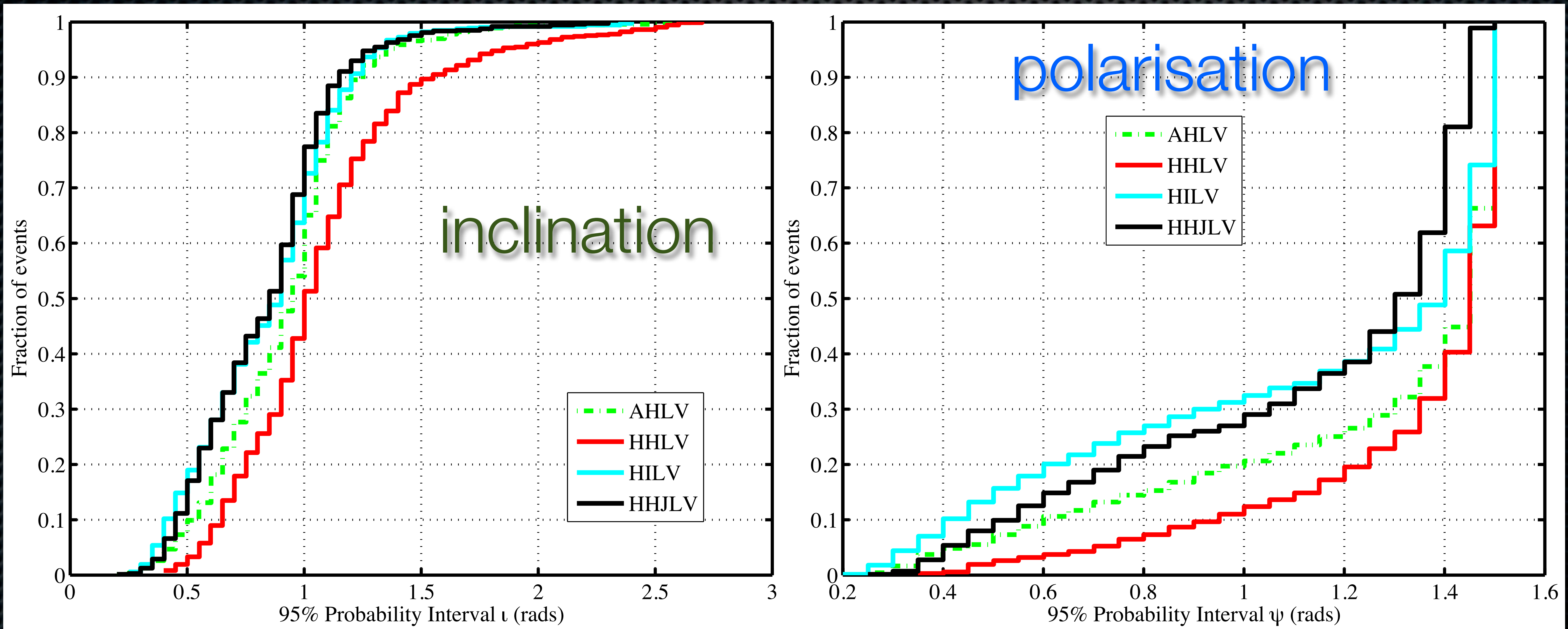




Time of coalescence



Extrinsic parameters

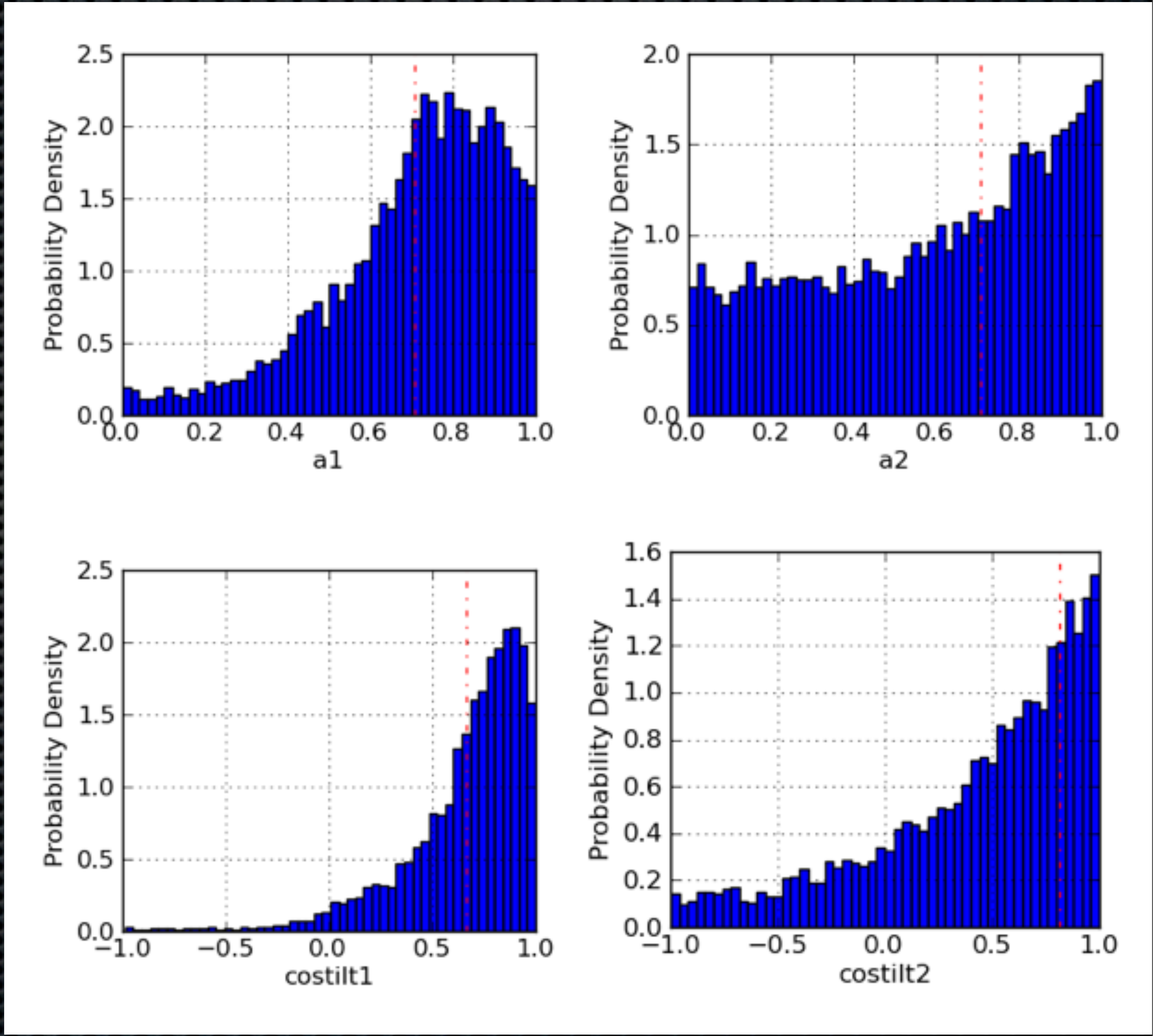


Spins

No large-scale Monte Carlo results (until recently waveform generation too slow)

Ability to measure spin magnitude and angles is dependent on the orientation of the system

- Prefer *edge-on* systems, as precession becomes visible

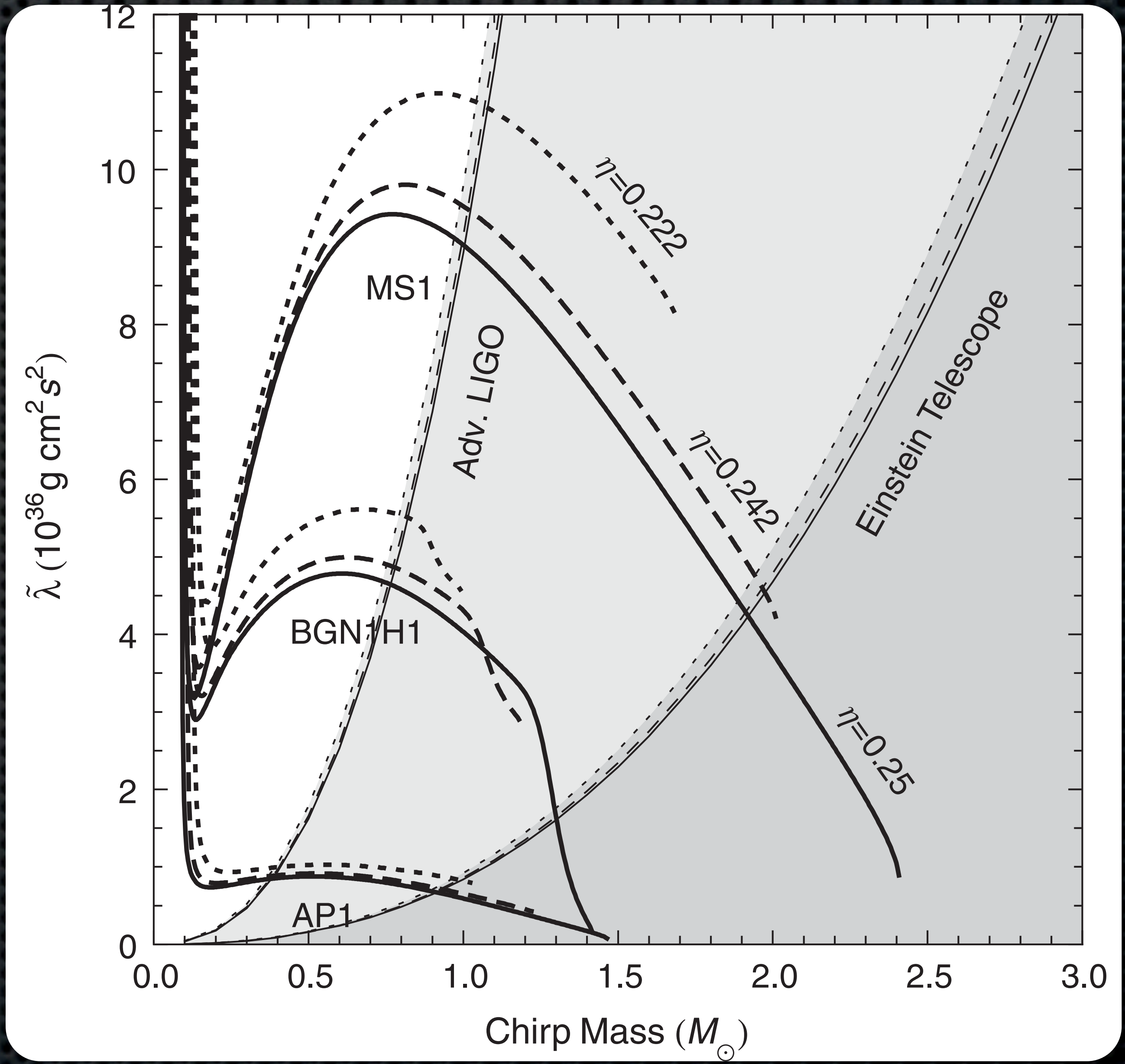


Beyond point masses

Beyond the point particle approximation, the effects of matter on the inspiral phase can be parameterised by tidal deformability λ

- Marginal effect in adv. detectors, but in competition with systematic errors (see poster by Favata)
- Combination of multiple sources can boost detectability (Markakis *et al* WSPC proceedings (2010))

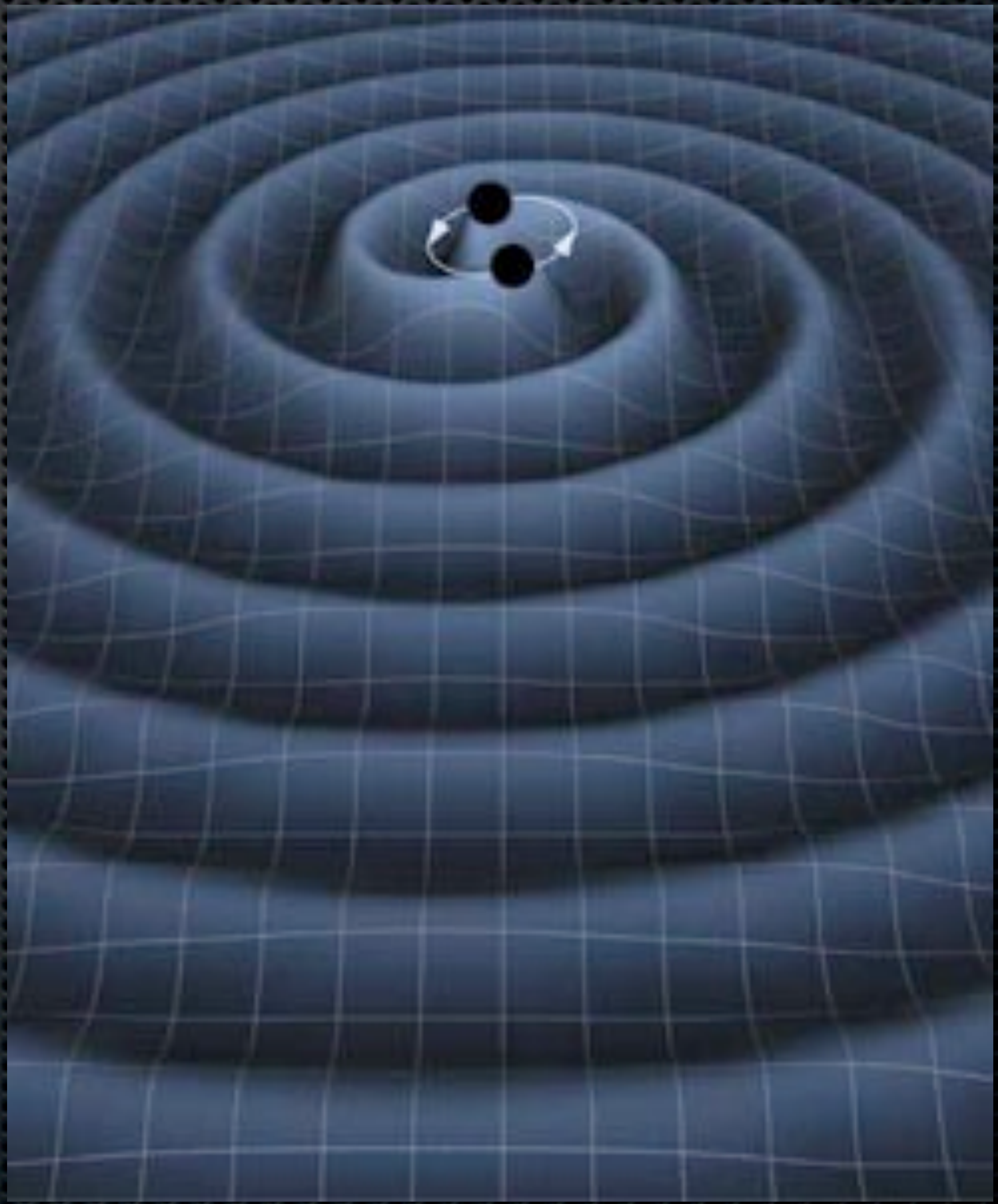
See Jocelyn Read's talk



Hinderer, T., Lackey, B. D., Lang, R. N., & Read, J. S. (2010). Phys Rev D, 81(1)

Tests of Relativity

- Binary pulsars provide best strong-field laboratory so far
- J0737-3039 relativistic binary
    - $(v/c) \sim 10^{-3}$
    - $GM/(Rc^2) \sim 10^{-6}$
- Contrast with binary black hole or neutron star just before merger
    - $(v/c) \sim 0.4$
    - $GM/(Rc^2) \sim 0.2$

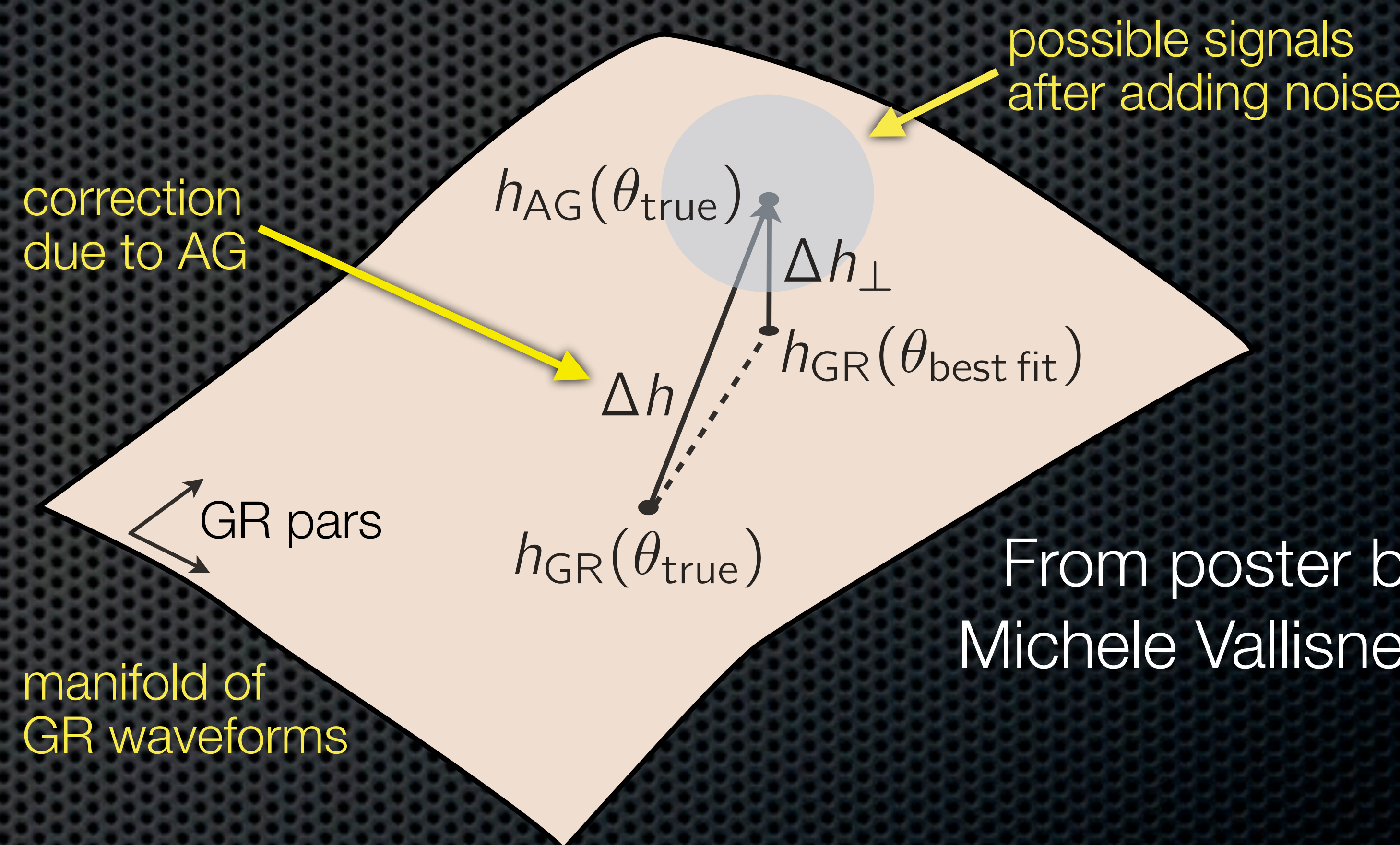


Testing GR with Bayesian Methods

Parameters beyond known GR(+EOS) physics form a larger model which embeds GR.

Projection of real (AG) template onto GR parameter space can introduce bias (even when GR appears to be favoured).

Detecting AG amounts to performing model selection between the larger and smaller models.



From poster by Michele Vallisneri

$$FF = \frac{(h_{AG}(\theta_{true}), h_{GR}(\theta_{bestfit}))}{|h_{AG}(\theta_{true})| \cdot |h_{GR}(\theta_{bestfit})|}$$

$$O_{AG,GR} \approx \exp\left[\frac{x^2}{2} + \sqrt{2(1-FF)}\mathrm{SNR} + (1-FF)\mathrm{SNR}^2\right]$$

TIGER - testing PN expansion

$$\Phi(v(t)) = \left(\frac{v}{c}\right)^{-5} \sum_{n=0}^{7} \left[\psi_n + \psi_n^{(l)} \ln\left(\frac{v}{c}\right)\right] \left(\frac{v}{c}\right)^{n}$$

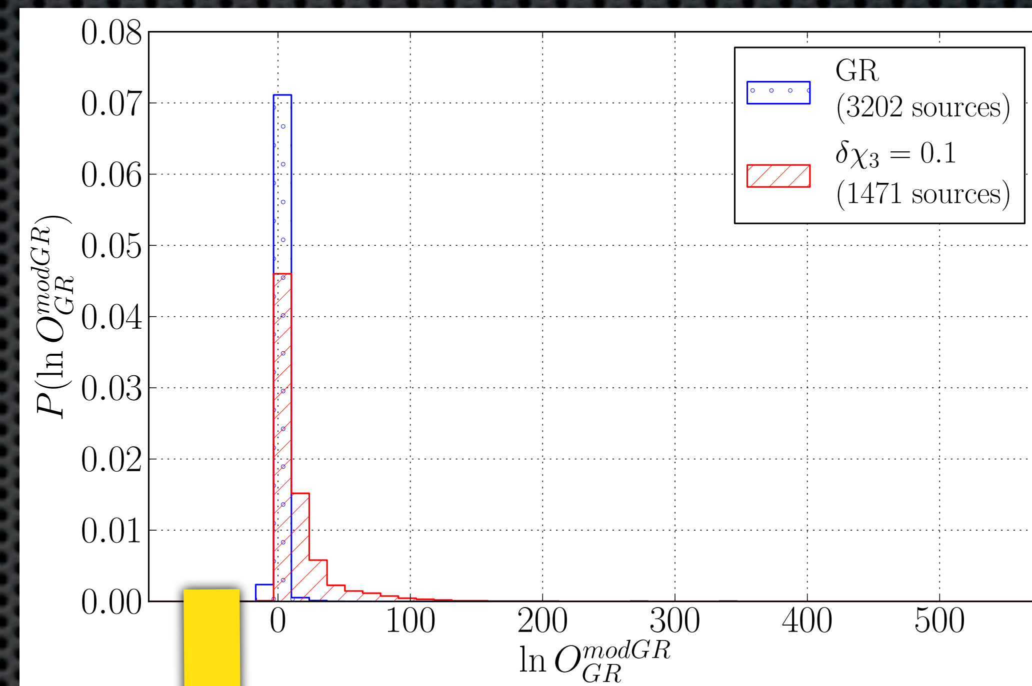
- Parameterise deviations away from GR as changes in the PN coefficients

$$\psi_n{}' = \psi_n[1 + \Delta\psi_n]$$

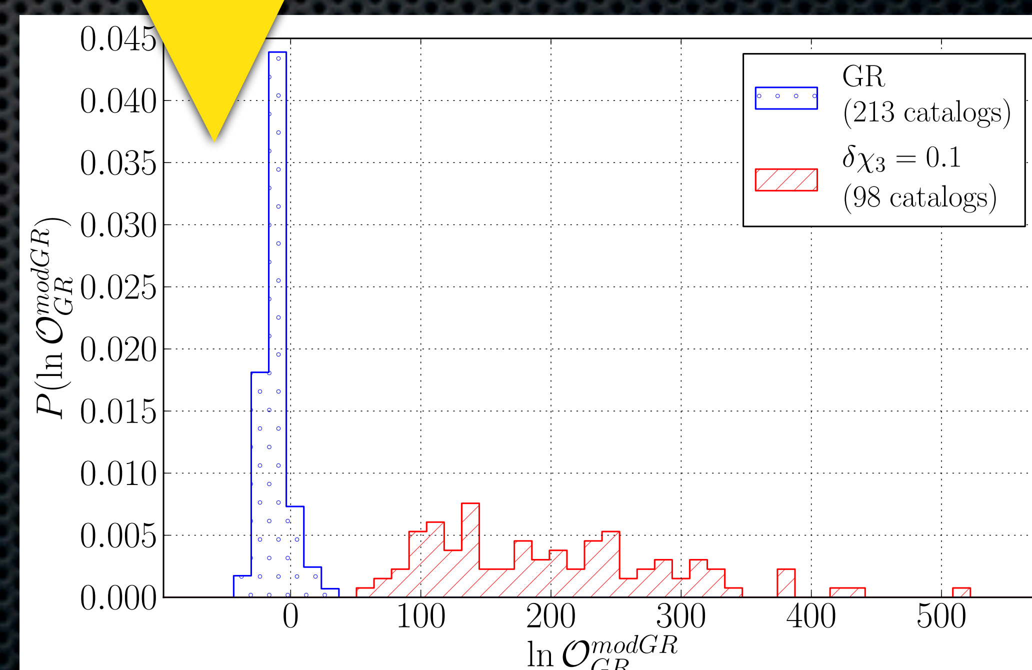
- Perform parameter estimation of the $\Delta\psi_n$

- Perform model selection with GR model ($\Delta\psi_n = 0$)

(See poster by Tjonnie Li)



Combine 15 sources



Li *et al* PRD 85 (8) 2012; JPCS 363 (1) 2012

Tests of Relativity - PPE

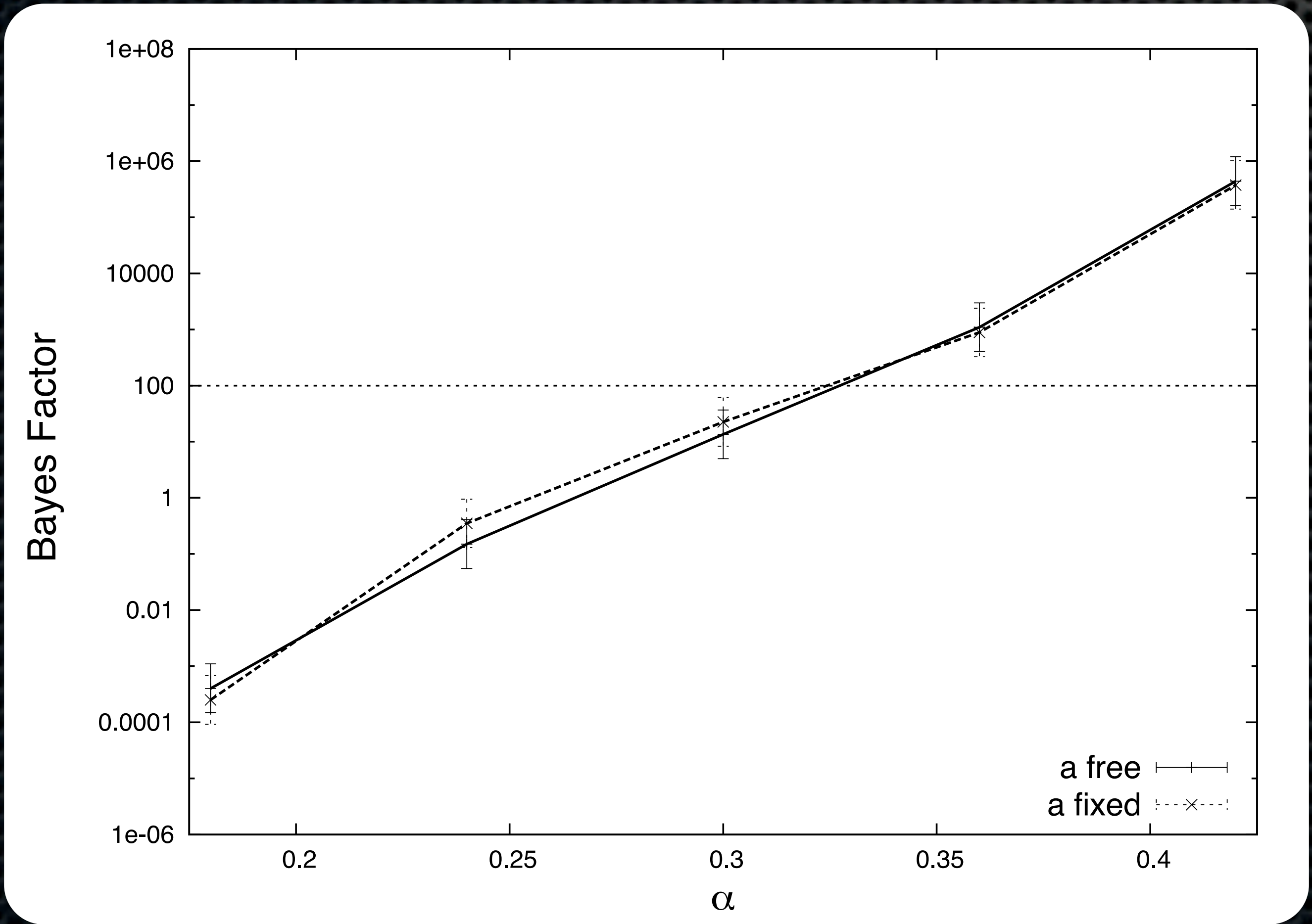
Can also parameterise beyond the PN expansion

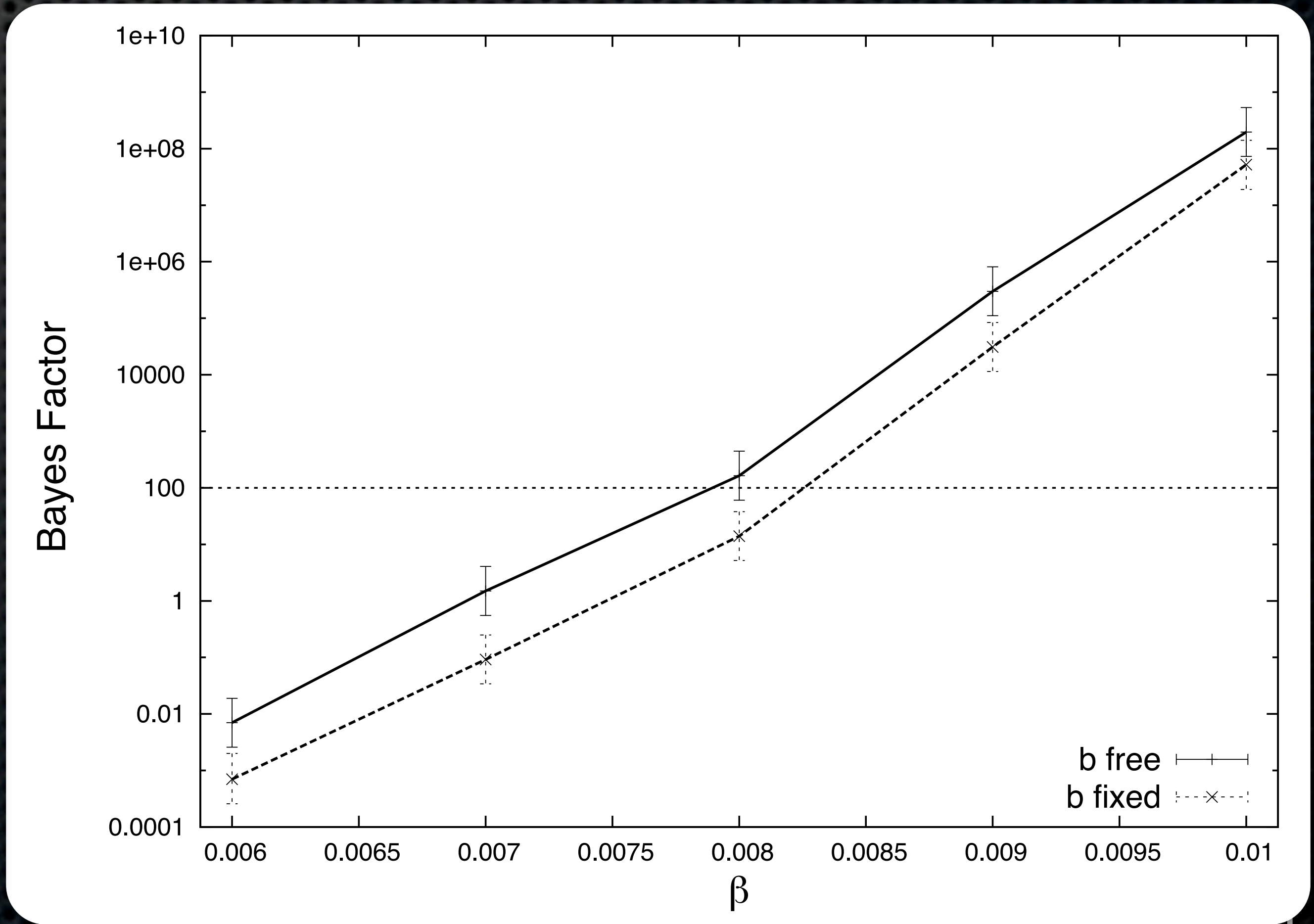
$$\tilde{h}(f) = \tilde{h}_{GR}(f)\left[1 + \alpha\left(\frac{v}{c}\right)^a\right] e^{i\beta\left(\frac{v}{c}\right)^b}$$

and perform PE/model selection with the α,a and β,b

(See talk by Nico Yunes)

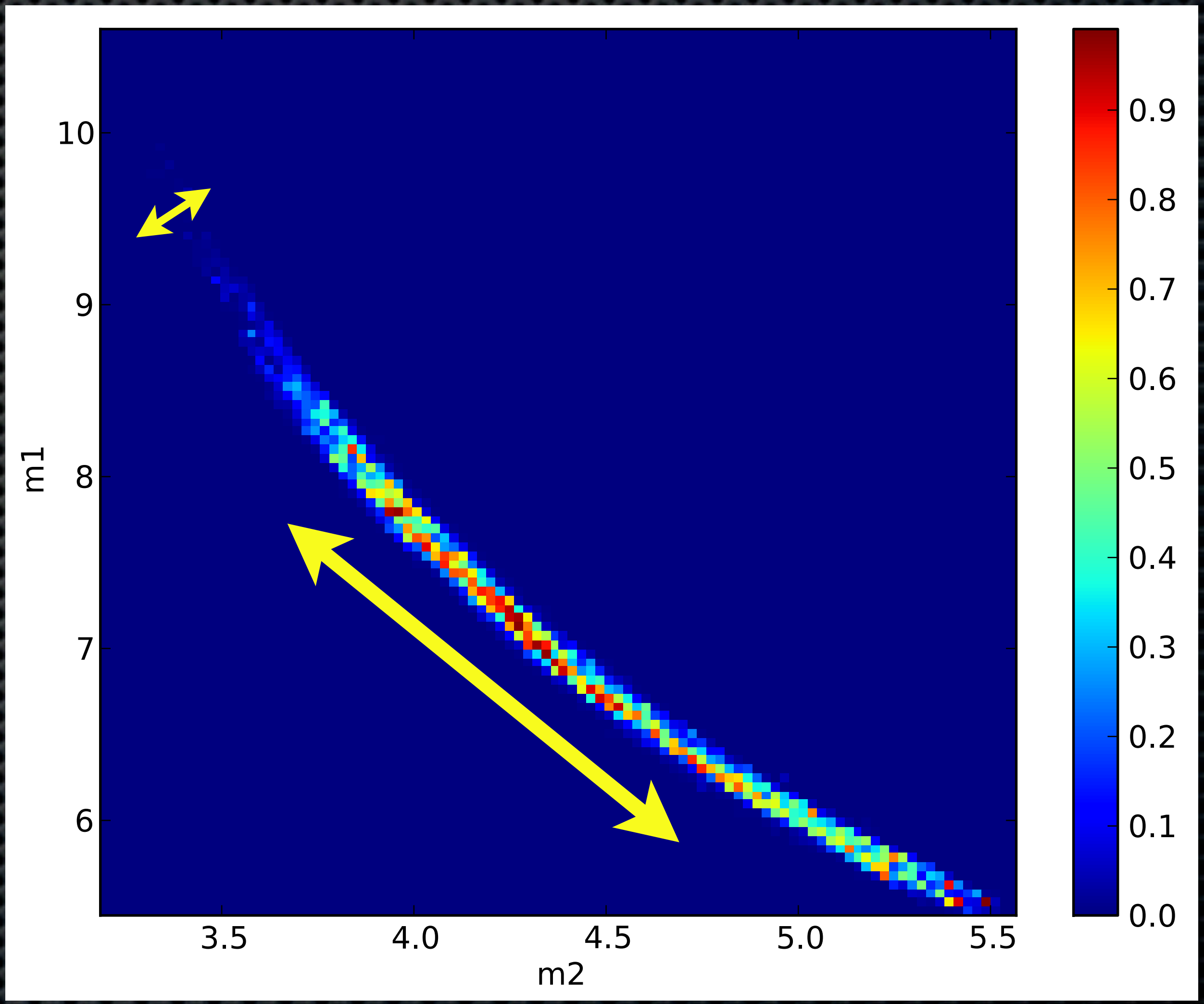
Cornish *et al* PRD 84 (6) 2011





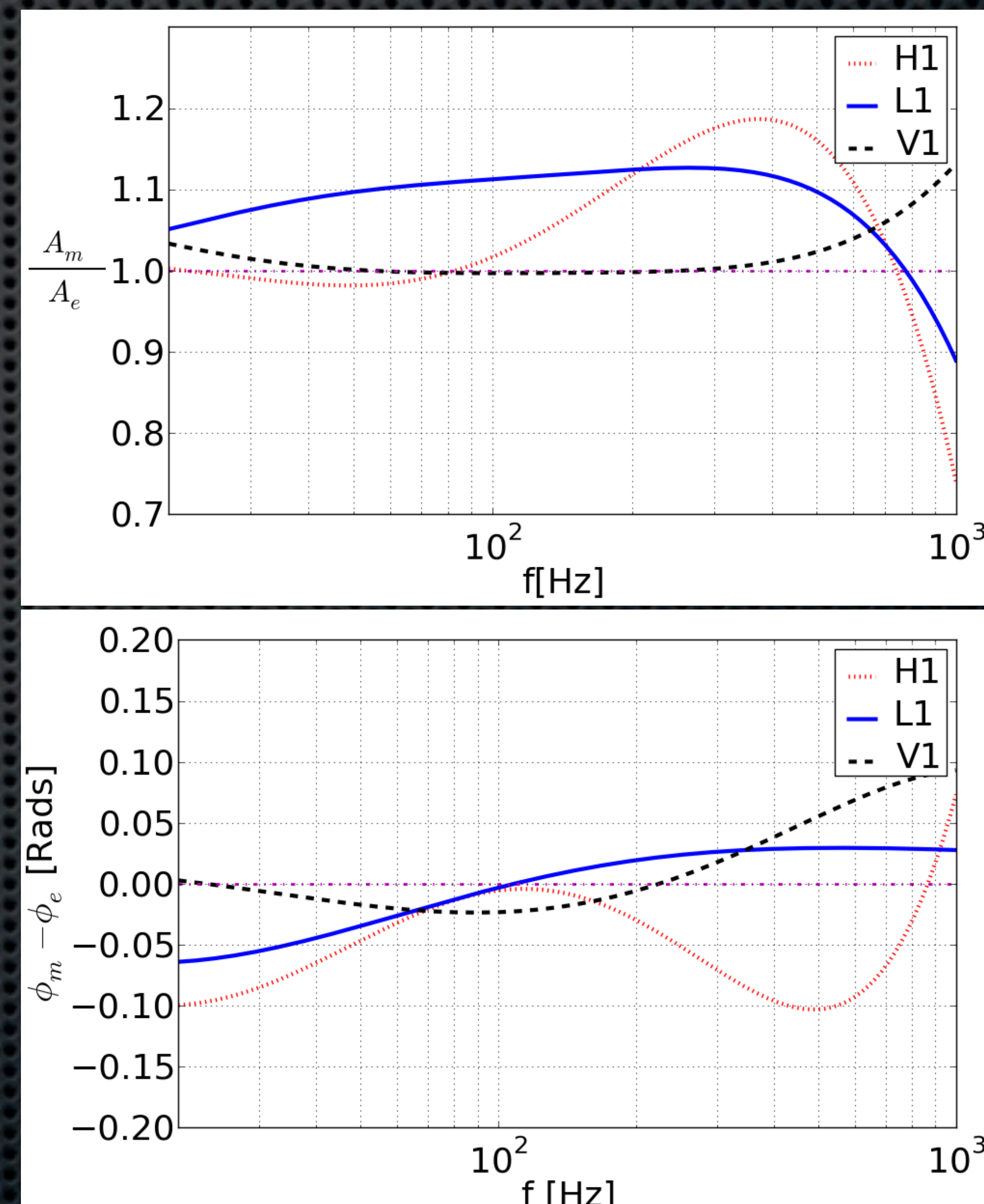
Systematic Errors

- Waveform errors primarily affect recovery of the mass and intrinsic parameters.
- So precision tests rely on low systematic error in the waveform approximants
- Expect large systematic as well as statistical errors in mass ratio. (See poster by Marc Favata)
- Can we account for this uncertainty? - "ignorance is preferable to error"



Calibration Errors

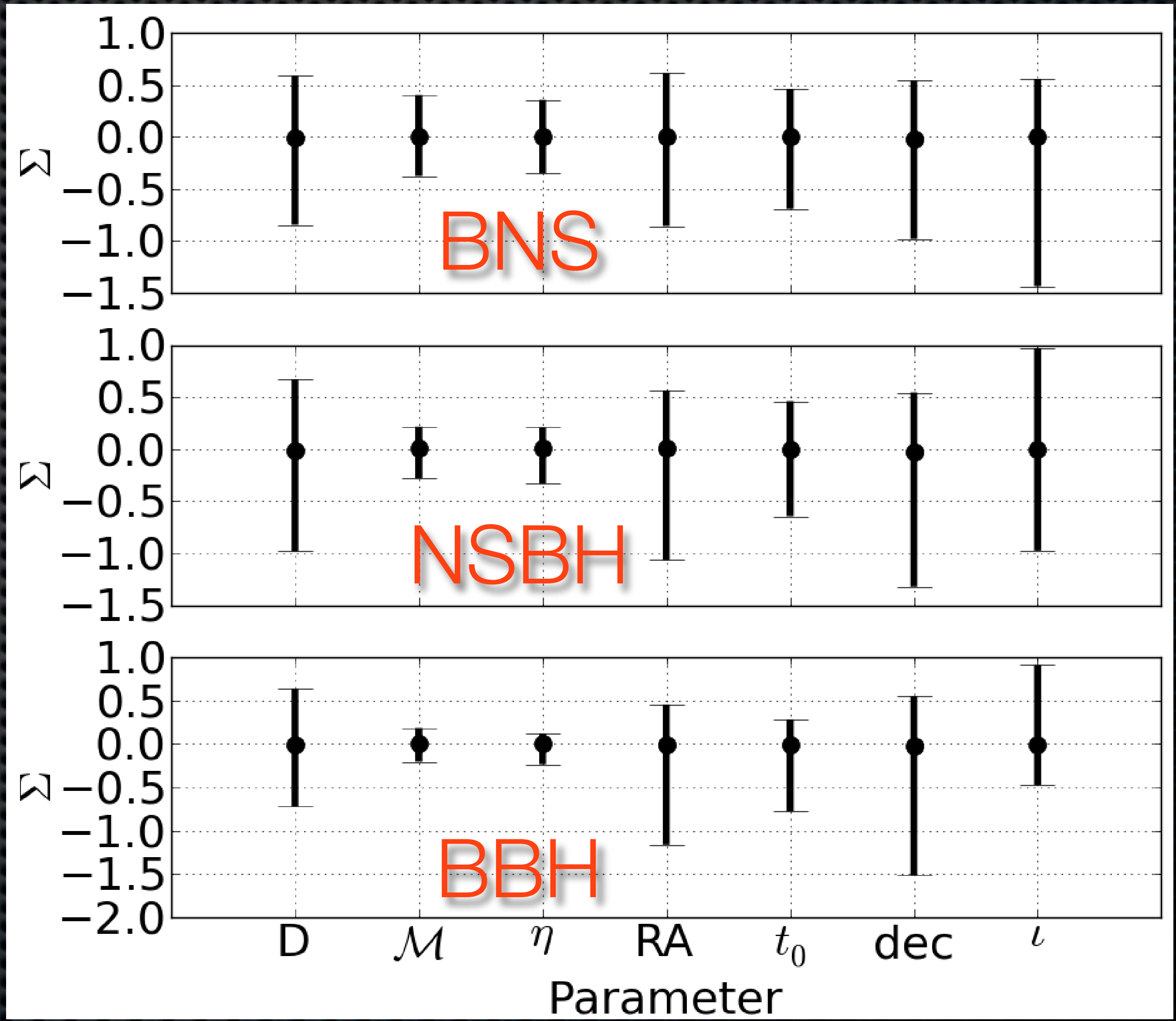
- Calibration of detectors is imperfect in phase and amplitude.
- Can these errors in the observation bias the parameter estimation?
- Vitale *et al* performed Monte Carlo parameter estimation with a range of realistic calibration error curves



Vitale *et al* P.R.D. 85 6 (2011)

Calibration Errors

- Σ = ratio of systematic error to statistical std. dev.
- Extrinsic parameters are affected much more severely by the calibration errors
- Results indicate that statistical errors will dominate for population of initial detections.



Vitale *et al* P.R.D. 85 6 (2011)

Glitches

Likelihood function used in parameter estimation is based on the assumption that the noise follows a stationary Gaussian distribution with known power spectrum

$$p(\{d\}|\vec{\theta}) \propto \exp\left[-\sum_k \frac{\left|d_k - h(f_k;\vec{\theta})\right|^2}{2\sigma_k^2}\right]$$

$$p(\{d\}|\vec{\theta}) \propto \exp\frac{1}{2}\left[-\langle d|d\rangle - \langle h|h\rangle + 2\langle d|h\rangle\right]$$

Glitches are instances of noise that deviate from the background Gaussian distribution, likelihood is no longer a good description of the data.

Glitches

For detection, already use signal-based vetoes and background estimation through time slides

Bayesian P.E. approaches to glitch robustness

- Include a glitch hypothesis in the noise model
  - Incoherent signal model [Veitch et al 2010]
  - Glitch fitting (wavelets) [Littenburg et al 2010]
- Relax Gaussian likelihood function [Röver et al 2011]

Challenges Ahead

Advanced detectors and first detections will require parameter estimation to be ready!

We have come a long way to get a reliable, reviewed code base ready, but further goals include:

- Truly rapid Bayesian sky localisation
- Long templates (starting @10 Hz)
  - glitches occurring during the templates
- Handling systematic uncertainties better
- Keep making things faster!!

Questions

?