Frames for precessing binaries

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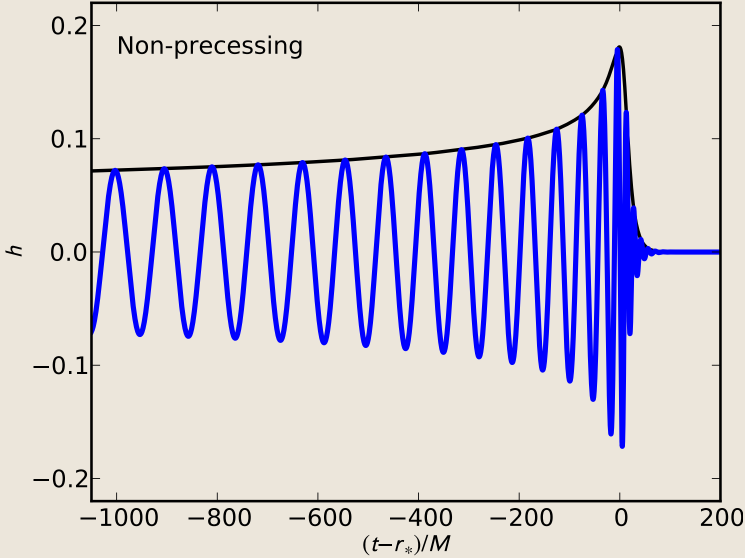
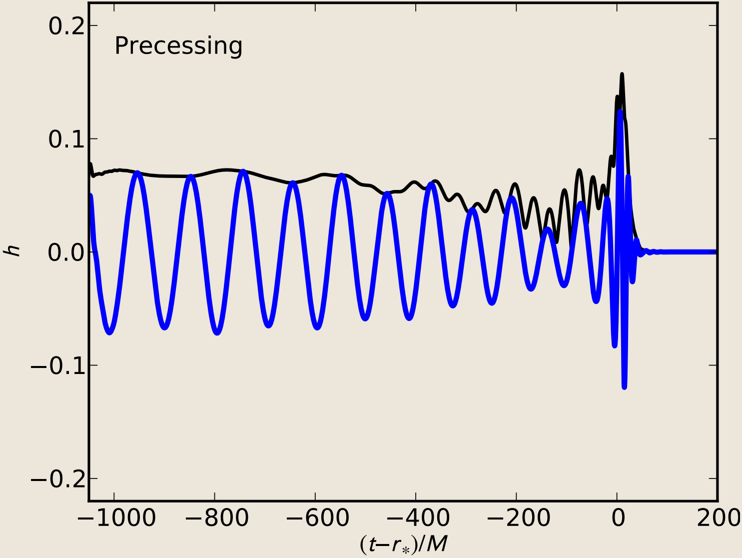
Outline

- The problem
  - Waveforms look ugly
  - Modeling and hybridization are hard
- The solution(s)
  - Schmidt frame
  - O'Shaughnessy frame
  - Minimal-rotation frames
  - Corotating frame
- Quaternions $\gg$ Euler angles

Waveforms



Waveforms



O'Shaughnessy frame

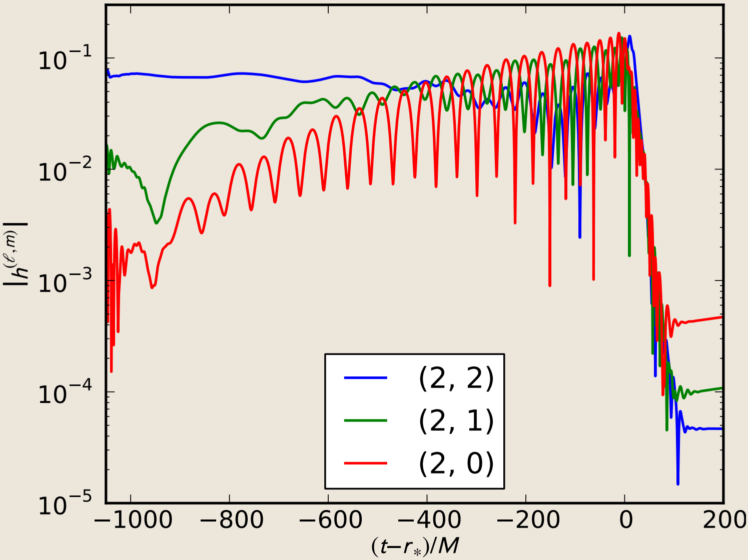
$$A_{ab} = \sum_{\ell,m,m'} \bar{h}_{\ell,m'} \, \langle \ell, m' | L_a \, L_b | \ell, m \rangle \, h_{\ell,m}$$

Principal eigenvector is radiation axis

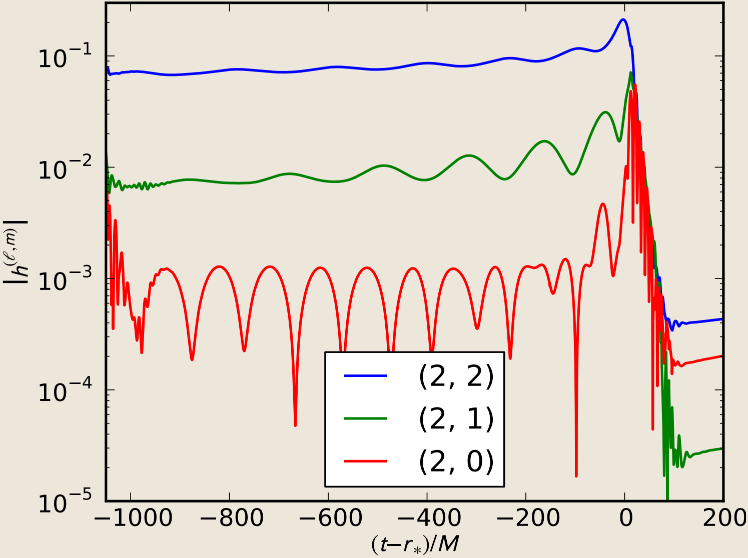
Schmidt frame

$$\max_{\mathcal{R}} \sum_{m=\pm 2} \left| \mathcal{R}\, h_{2,m} \right|^2$$

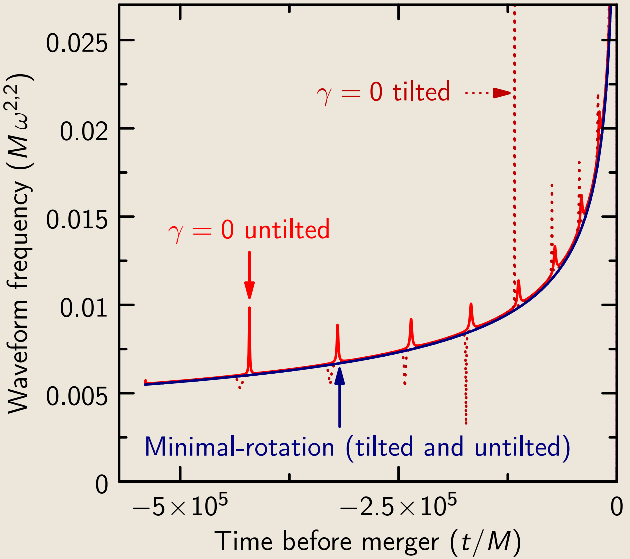
Results



Results



Minimal-rotation condition



Minimal-rotation condition

$$(\mathcal{R}^{-1}\,\dot{\mathcal{R}})\cdot Z = 0$$

$$\mathcal{R} \to \mathcal{R}\,\mathrm{e}^{\gamma Z}$$

$$\dot{\gamma} = -(\mathcal{R}^{-1}\,\dot{\mathcal{R}})\cdot Z$$

Corotating frame

$$\min \int \frac{d}{dt} \left| e^{-i\,\vec{\theta}(t)\cdot\vec{L}}\, h(t) \right|^2 \, dS^2$$

$$\dot{\vec{\theta}} = \vec{B}^{\mathsf{T}} A^{-1}$$

$$A_{ab} = \sum_{\ell,m,m'} \bar{h}_{\ell,m'} \,\langle \ell, m' | L_a L_b | \ell, m \rangle\, h_{\ell,m}$$

$$B_a = \sum_{\ell,m,m'} \Im \left[\dot{\bar{h}}_{\ell,m'} \,\langle \ell, m' | L_a | \ell, m \rangle\, h_{\ell,m} \right]$$

Comparing methods

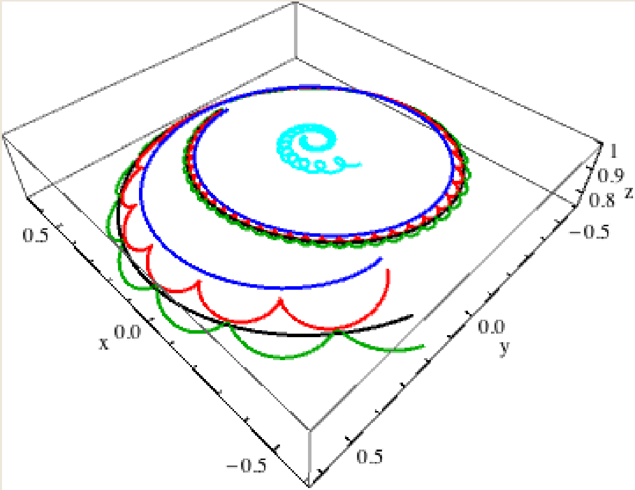


Image credit: Ochsner & O'Shaughnessy, arXiv:1205.2287

Quaternions $\gg$ Euler angles

- Clear geometric interpretation
- No coordinate singularities (unlike Euler angles)
- Operators (like rotation matrices)
- Trivially invertible
- Simple logarithm and exponent
- Linear and spline interpolation

Summary

Waveforms are now described by:

1. The inertial frame
2. The rotating frame
3. The waveform modes in the rotating frame