Modeling Gravitational Waves from Compact Binary Systems

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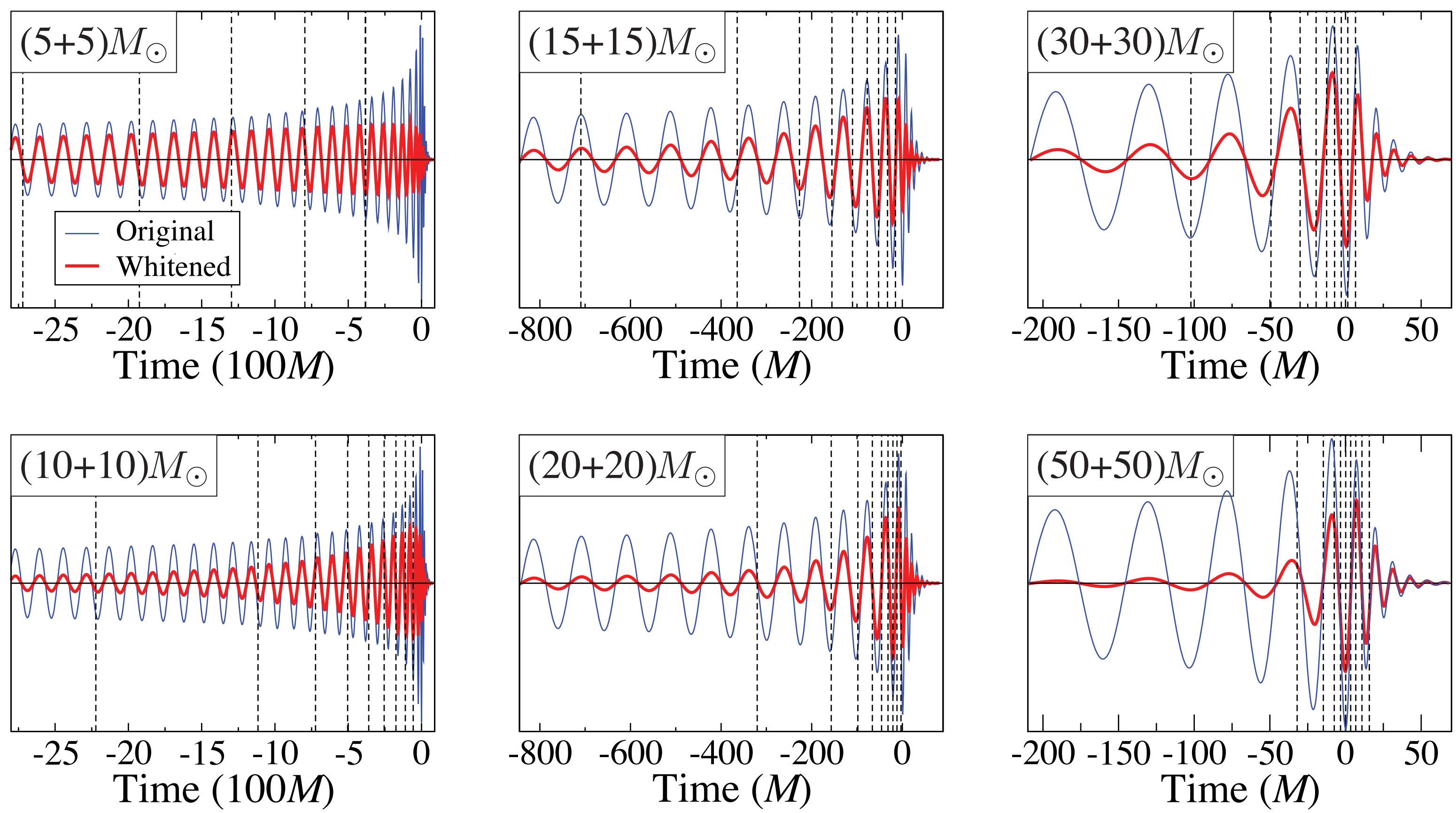
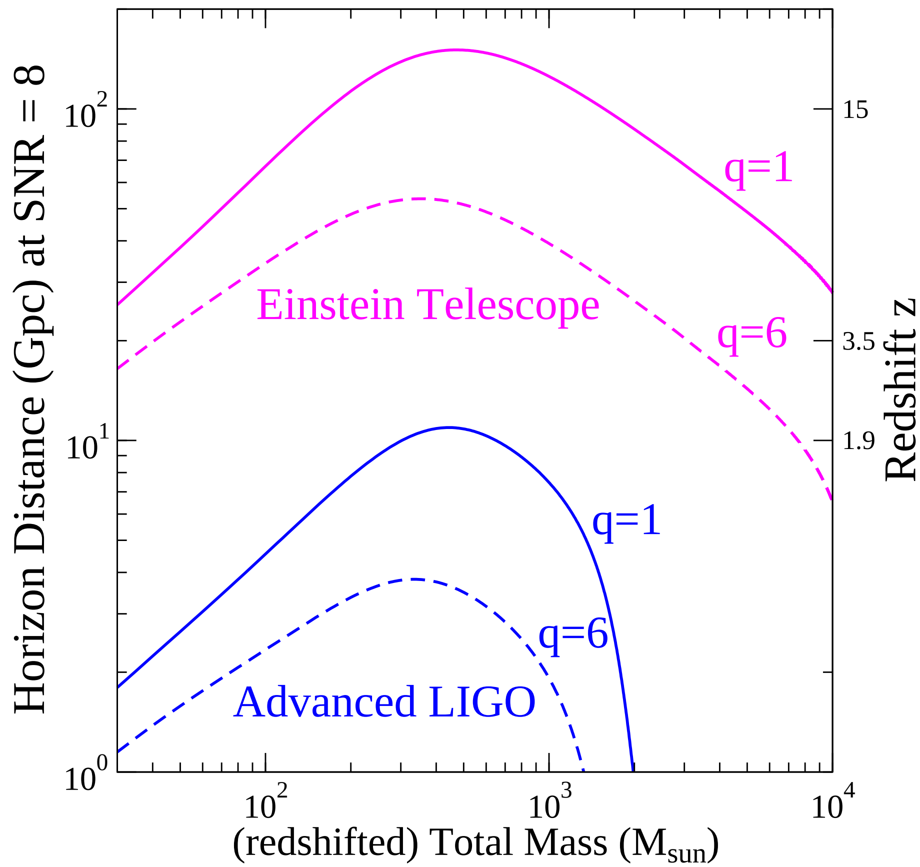
Accumulation of signal-to-noise ratio in binary signals

Initial LIGO! [Pan, AB, Pretorius & NASA-Goddard 07]




Horizon Distance (Gpc) at SNR = 8
(redshifted) Total Mass (M$_{sun}$)
Redshift z
15
3.5
1.9
10^0
10^1
10^2
10^3
10^4
q=1
q=6
Einstein Telescope
q=1
q=6
Advanced LIGO

Inspiral: number of GW cycles predicted by PN theory

$M = (1.4 + 1.4) M_\odot$

$f_{\rm in} = 40$ Hz; $f_{\rm fin} = 1570$ Hz

$\chi = |\mathbf{S}|/m^2$

	Number of cycles	Number of *useful* cycles:
Newtonian:	16034	247.8
1PN:	$+441$	$+24.0$
1.5PN	-211	-20.0
Spin-orbit:	$+65.7\chi_1 + 65.7\chi_2$	$6.2\chi_1 + 6.2\chi_2$
2PN	$+9.9$	$+1.5$
2.5PN	$-11.7 +9.2\chi_1 + 9.2\chi_2$	$-2.3 \; +0.8\chi_1 + 0.8\chi_2$
3PN:	$+2.6$	$+0.6$
3.5PN:	-0.9	-0.2

Inspiral: number of GW cycles predicted by PN theory

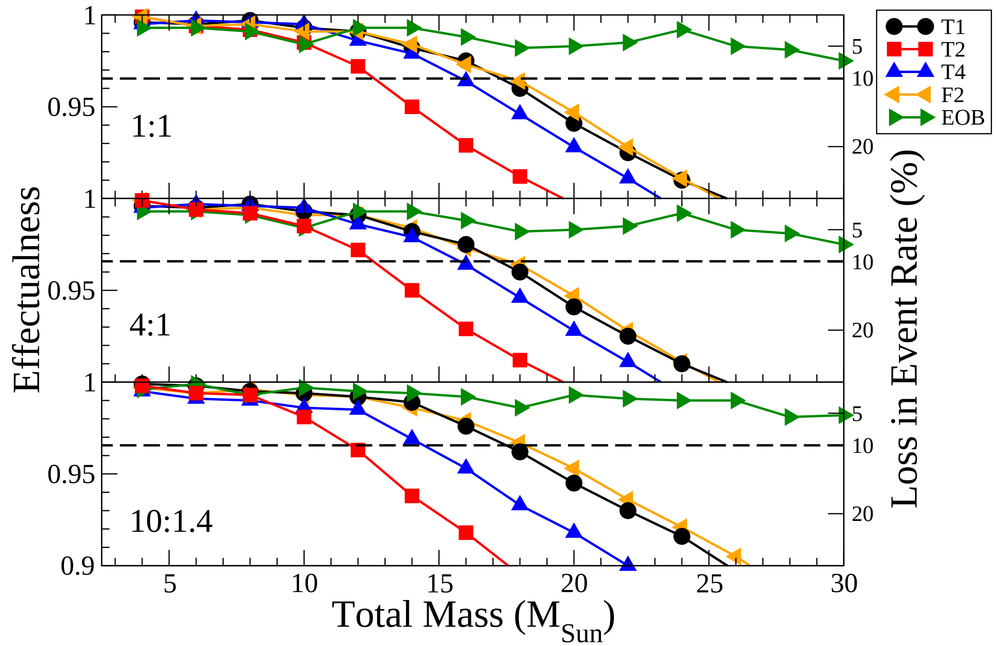
$M = (15 + 15) M_\odot$

$f_{\rm in} = 40$ Hz; $f_{\rm fin} = 147$ Hz

$\chi = |\mathbf{S}|/m^2$

	Number of cycles	Number of *useful* cycles:
Newtonian:	302	10.7
1PN:	$+39$	$+4.0$
1.5PN	-37	-6.2
Spin-orbit:	$+11.7\chi_1 + 11.7\chi_2$	$1.9\chi_1 + 1.9\chi_2$
2PN	$+3.3$	$+0.8$
Spin-spin:	$-1.7\chi_1\,\chi_2$	$-0.4\chi_1\,\chi_2$
2.5PN	$-6.2 + 3.6\chi_1 + 3.6\chi_2$	$-2.3 + 0.8\chi_1 + 0.8\chi_2$
3PN:	$+2$	$+1.2$
3.5PN:	-0.8	-0.5

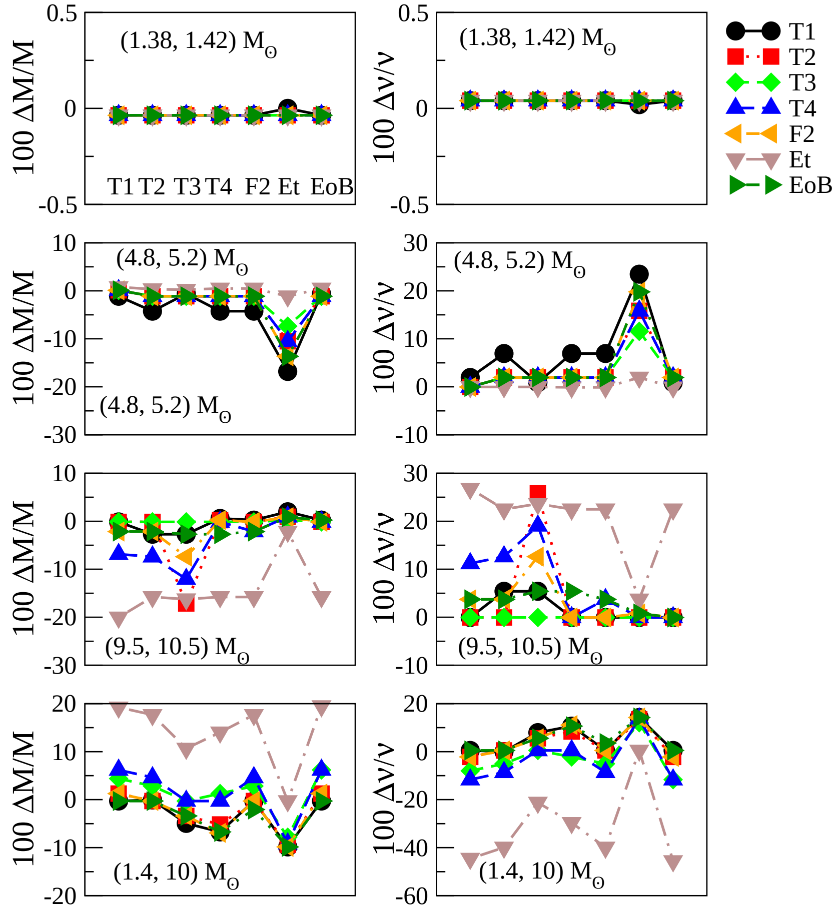
Closeness of non-spinning PN approximants



For detection, PN-approximants are "the same" for $M \lesssim 12 M_\odot$

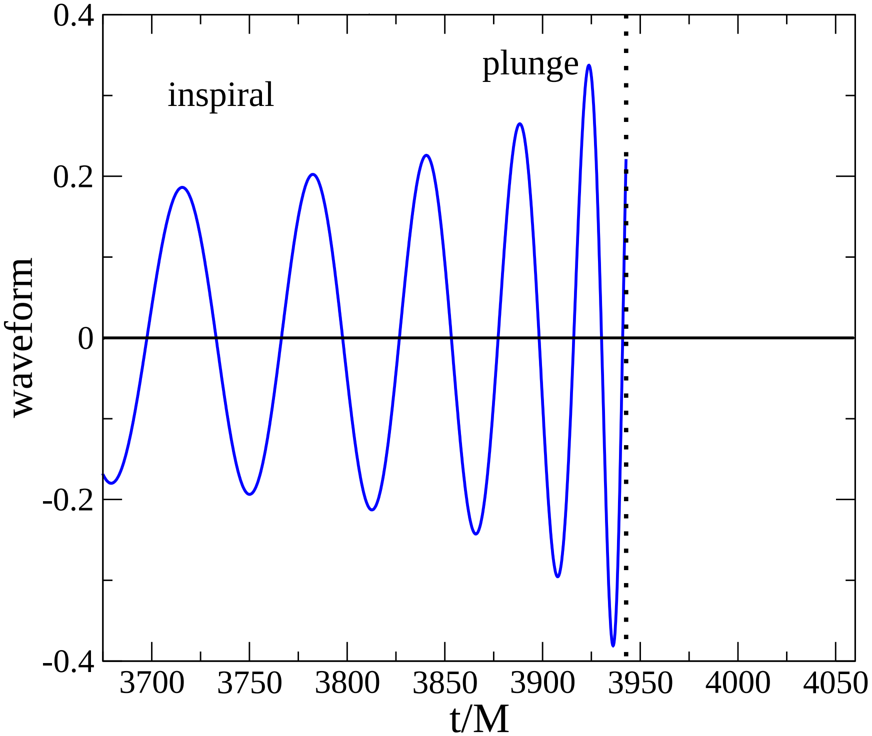
[AB, Iyer, Ochsner, Pan & Sathyaprakash 09]

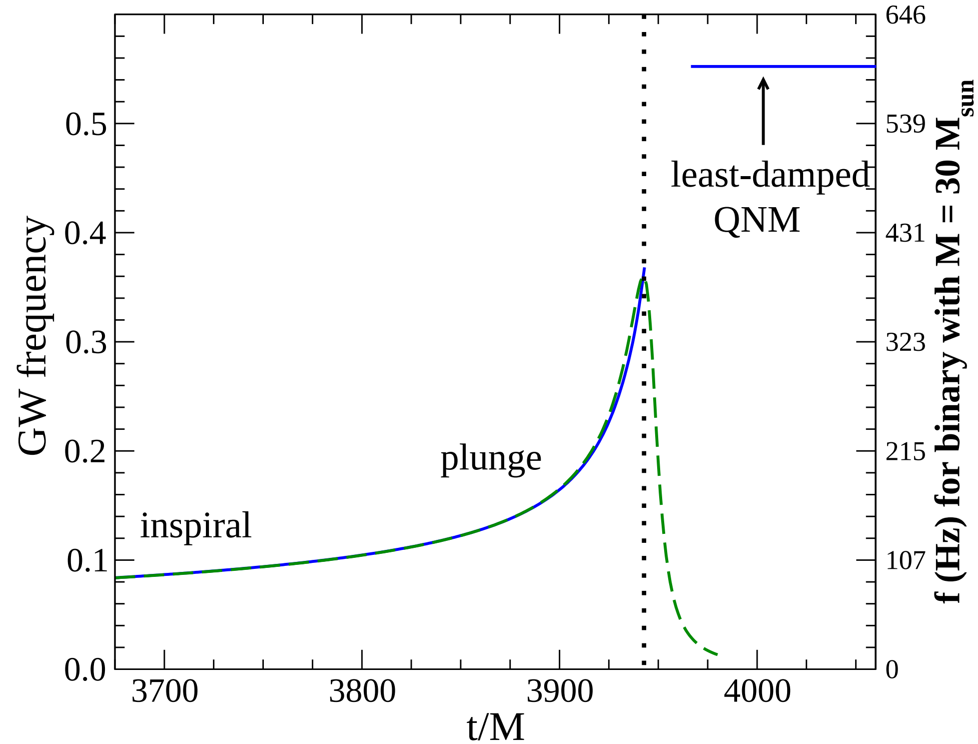
Biases in binary parameters



[AB, Iyer, Ochsner, Pan & Sathyaprakash 09]

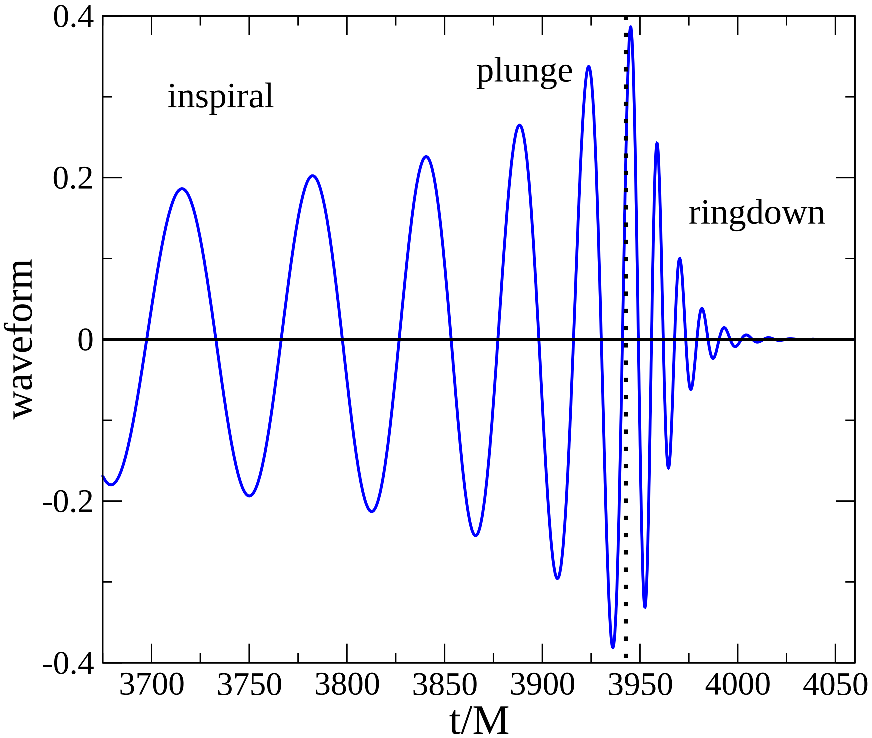
EOB inspiral-plunge waveform

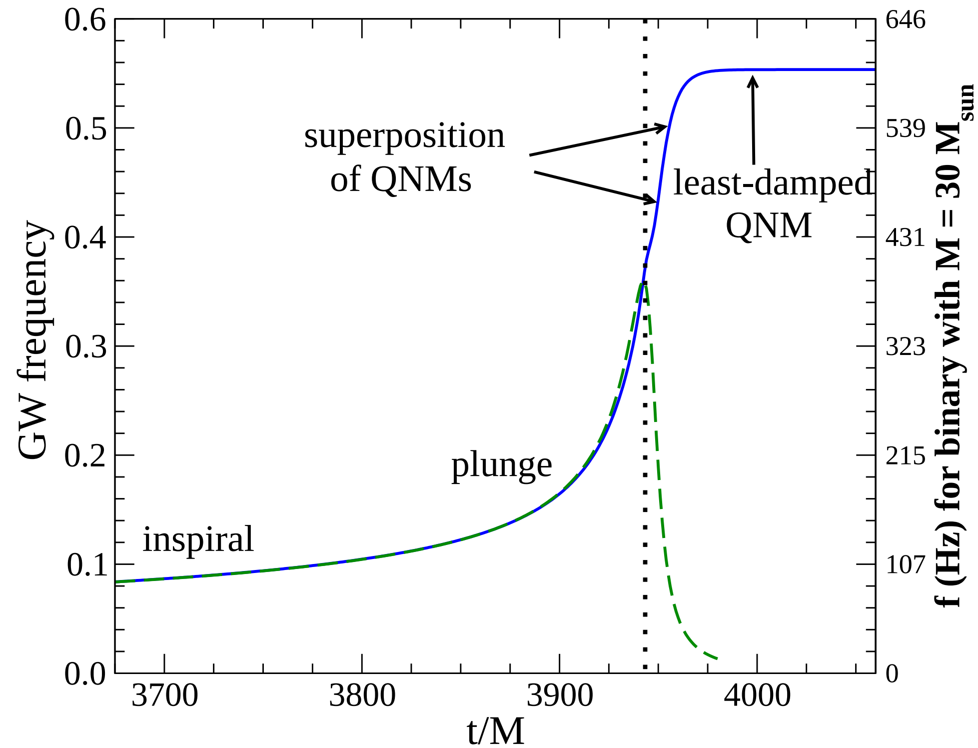




- **Plunge is adiabatic continuation of inspiral**
- **What could the "merger" signal be?**

EOB inspiral-merger-ringdown waveforms





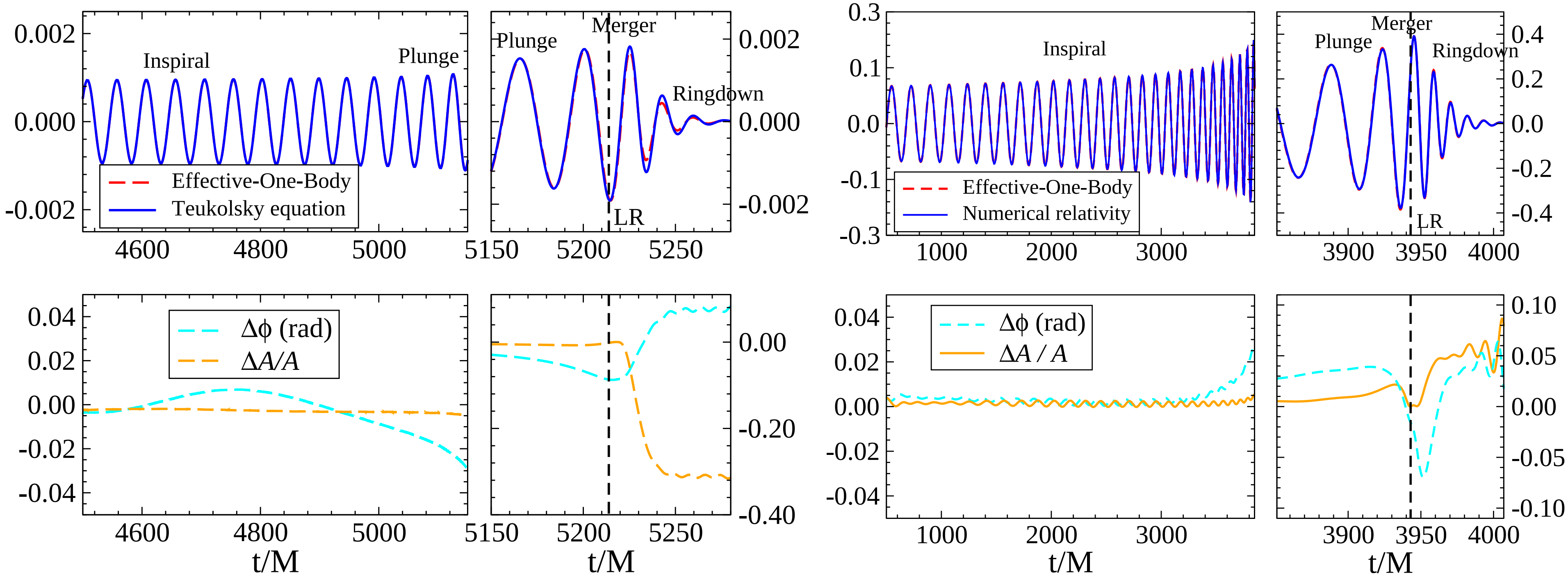
- **Very *short* transition merger–ringdown**
- **Energy quickly released during merger**
- $E_{\rm rad} \sim 2\%$–$12\%\ M\, c^2$

  $1 M_\odot\, c^2 \sim 10^{54}\,{\rm erg} \sim 10^{56}\,{\rm GeV!}$

Inspiral-merger-ringdown templates

Modeling analytically the inspiral, merger and ringdown of black-hole binary systems

large mass-ratio: $m_2/m_1 = 10^3$ **equal-mass:** $m_2/m_1 = 1$

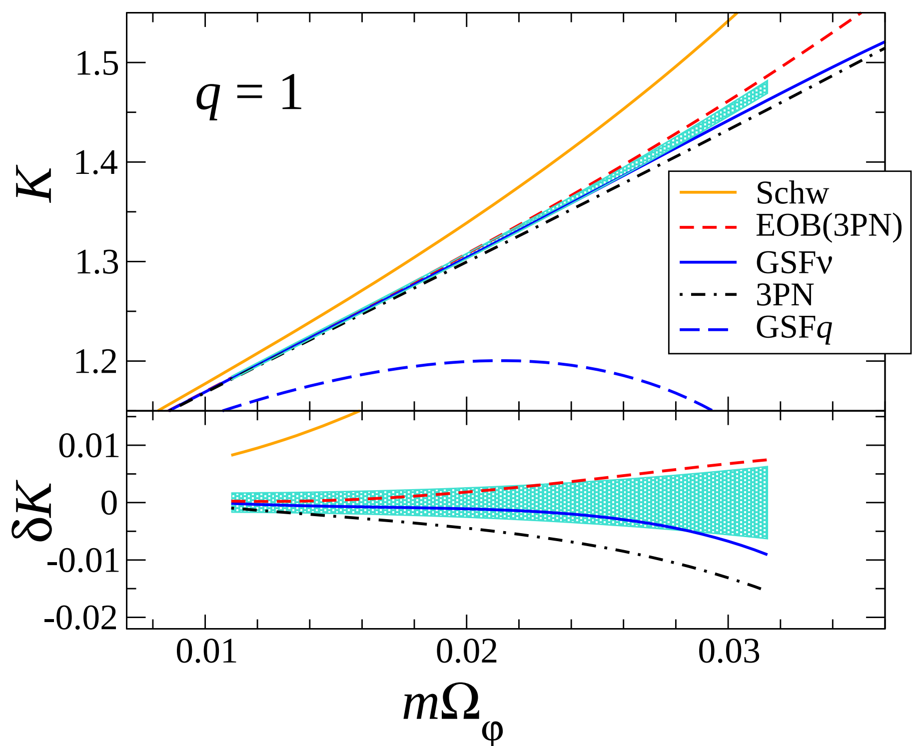


Simplicity and universality of merger signal over mass (and spin) range

Comparison between all techniques: periastron advance

[Le Tiec, Mroué, Barack, AB, Pfeiffer, Sago & Taracchini 11]

34 **orbits!**



- In 1915 Einstein derived the lowest order GR angular advance per orbit:

$$\Delta\Phi = \frac{6\pi G\, M_\odot}{c^2\, a\, (1-e^2)}$$

- Very accurate NR simulations
- Predictions from PN theory
- Predictions from (uncalibrated) EOB
- Predictions from gravitational self-force (GSF)

[Barack & Sago 09-11; Barack, Damour & Sago 10]

$$K = 1 + \frac{\Delta\Phi}{2\pi}, \qquad K = \frac{\Omega_\varphi}{\Omega_r}$$

Comparison between all techniques: binding energy

- **Predictions from PN theory**

- **Predictions from (uncalibrated) EOB**

- **Predictions from GSF**

  [Le Tiec, Barausse & AB 11]

  $$\hat{E} = E_{\rm Schw} + \nu\, E_{\rm GSF} + \mathcal{O}(\nu^2)$$

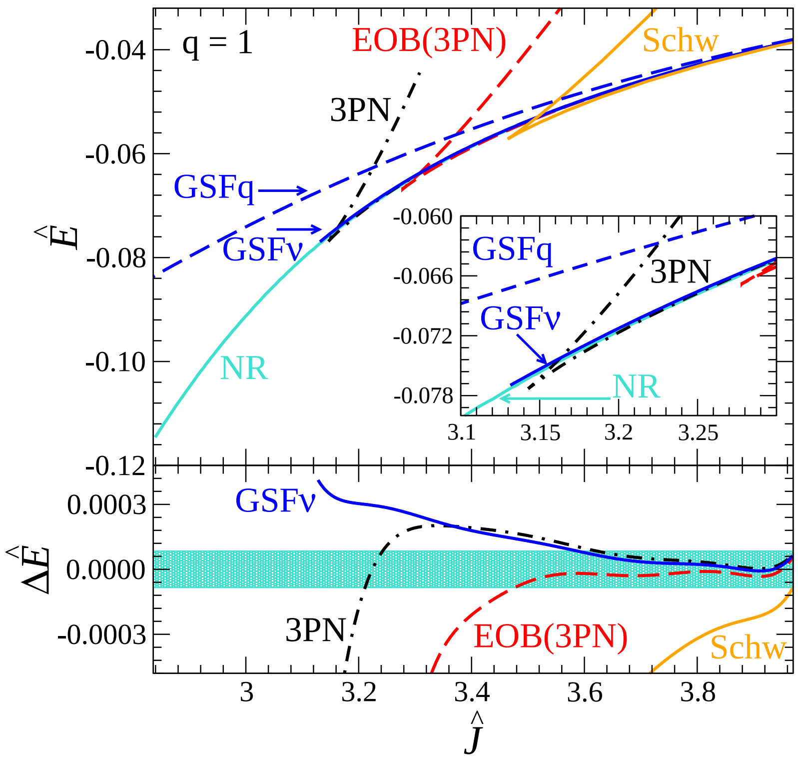
  $$\hat{J} = J_{\rm Schw} + \nu\, J_{\rm GSF} + \mathcal{O}(\nu^2)$$

  $\Rightarrow$ **complete EOB potentials A and B!**

  [Barausse, AB & Le Tiec 11]

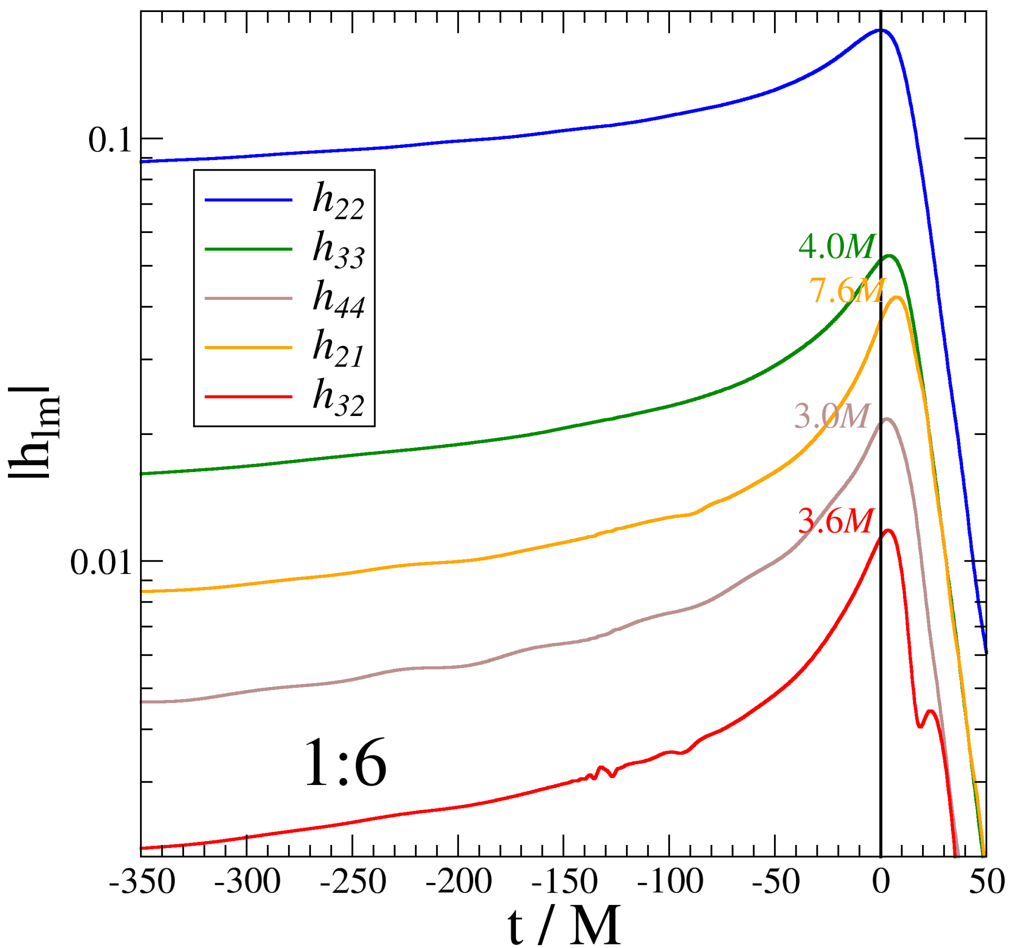
- **NR result**

  [Damour, Nagar, Pollney & Reisswig 11]



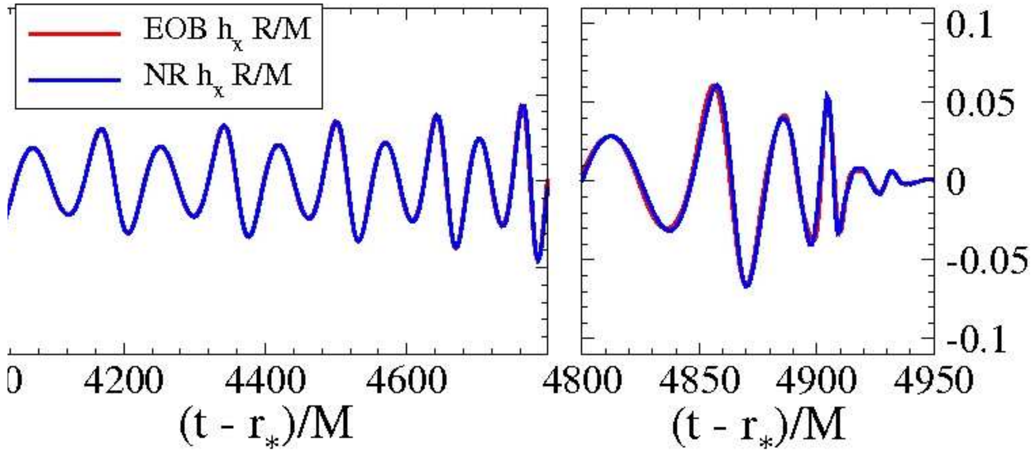
Similar results for $m_2/m_1 = 2, 3$

Nonspinning IMR Model

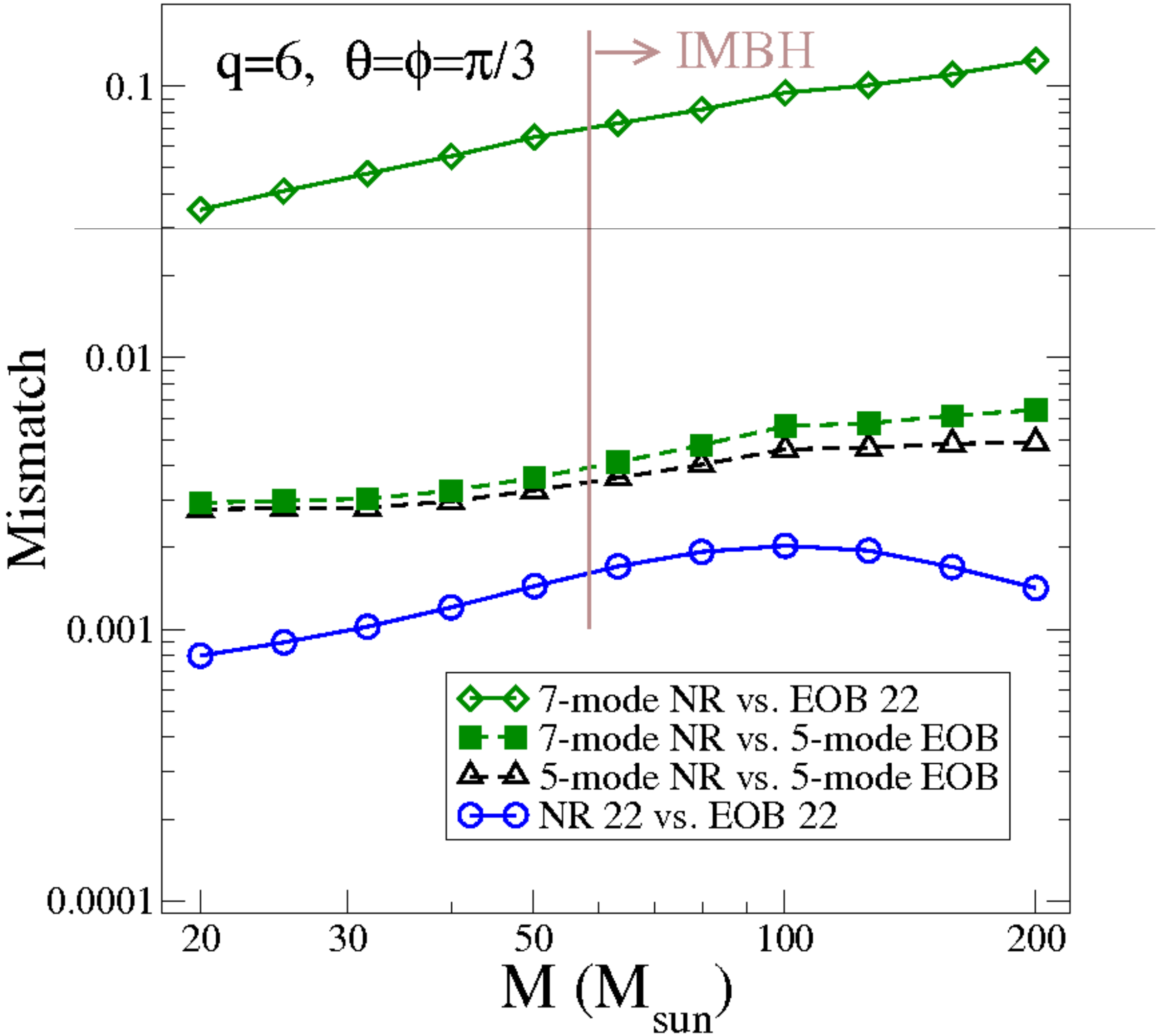


$$h_+ + ih_\times = \sum_{\ell=2}^{L} \sum_{m=-\ell}^{\ell} {}_{-2}Y_{\ell m} h_{\ell m}$$

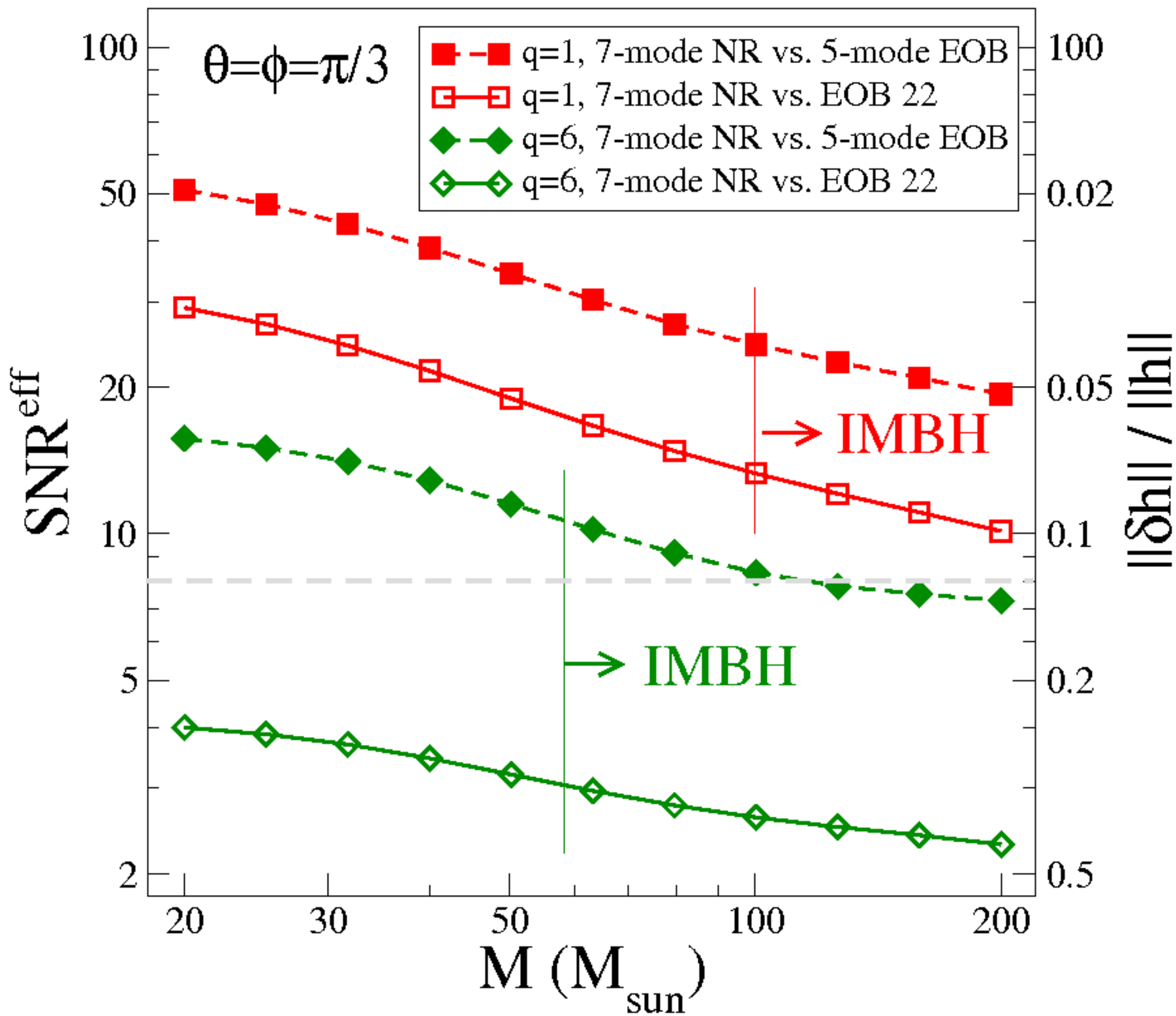
$q = 1/6, \theta = \phi = \pi/3$



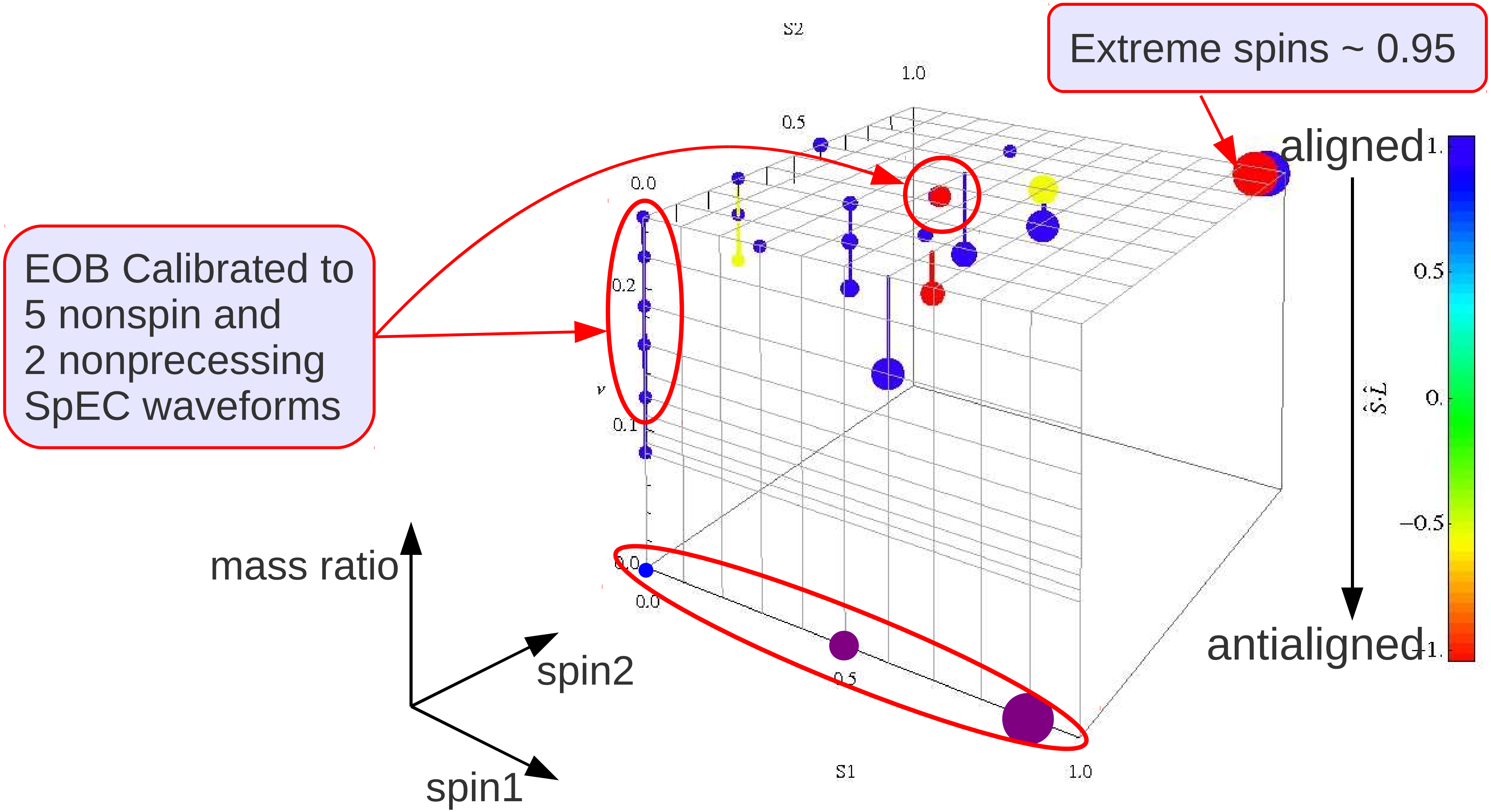
Effectualness



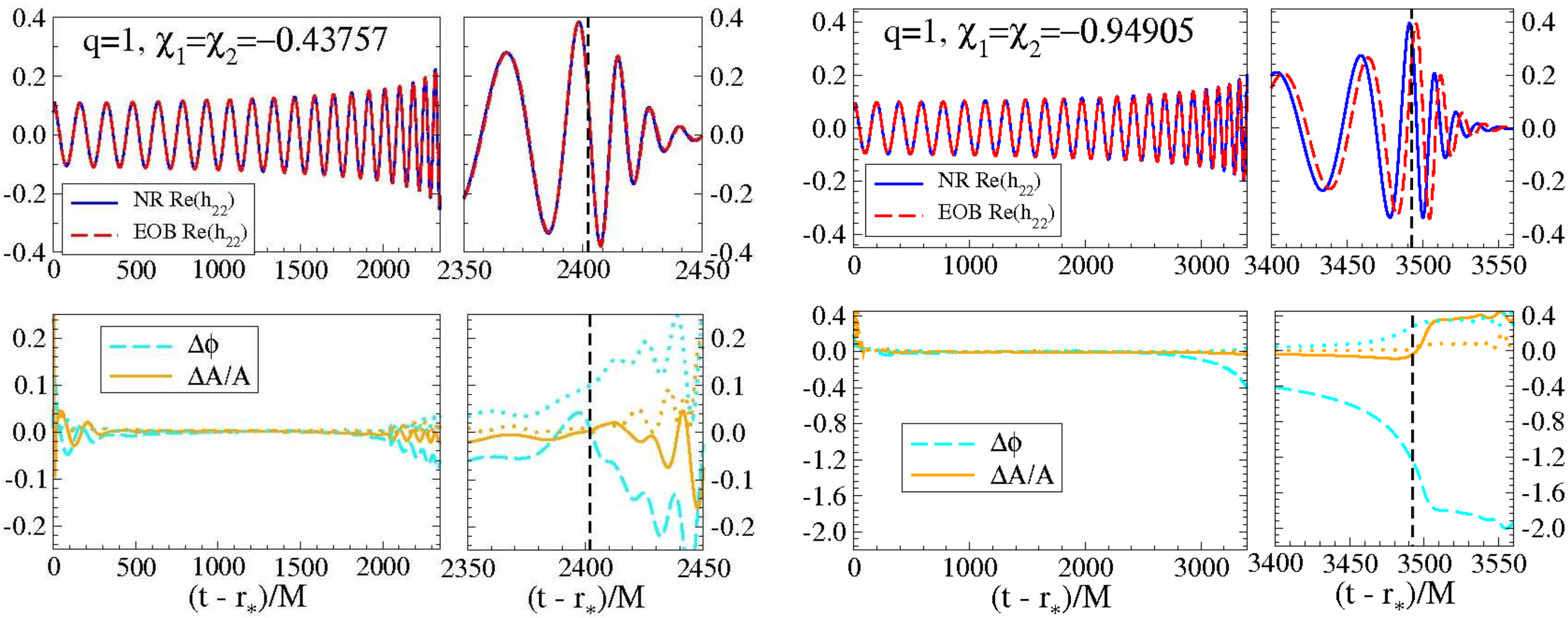
Faithfulness



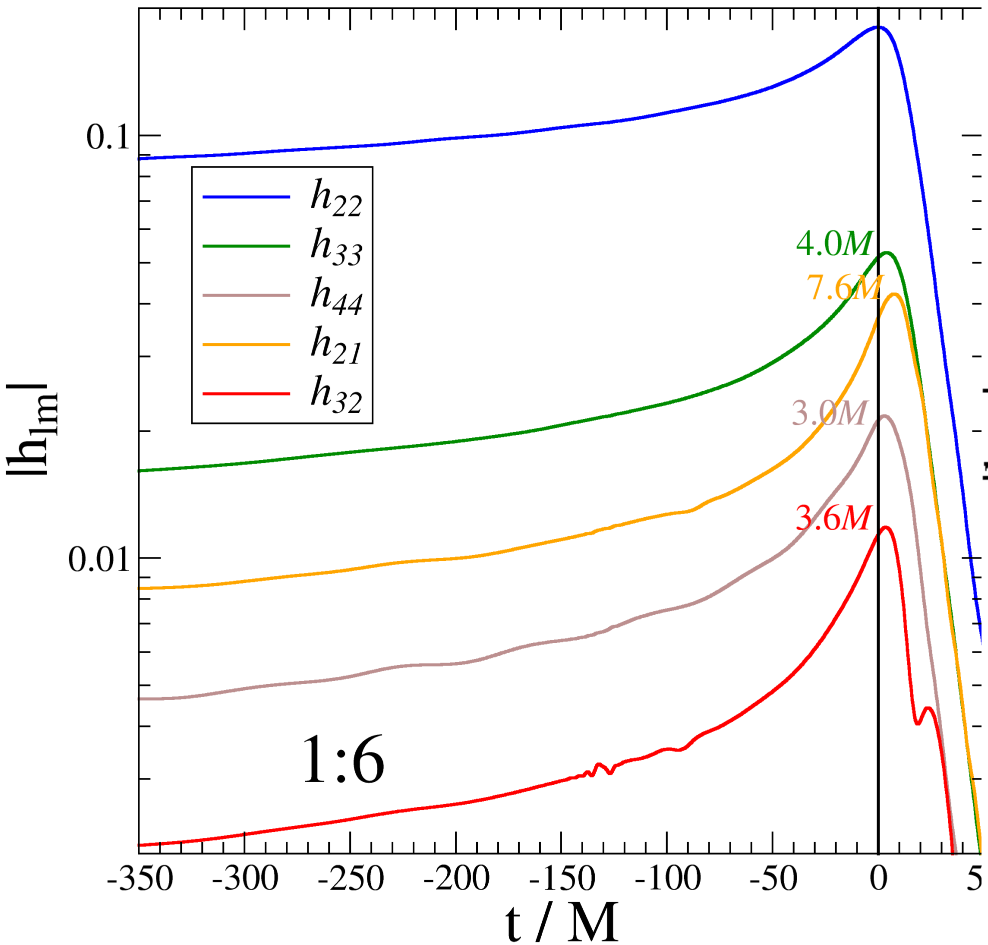
SEOBNRv1

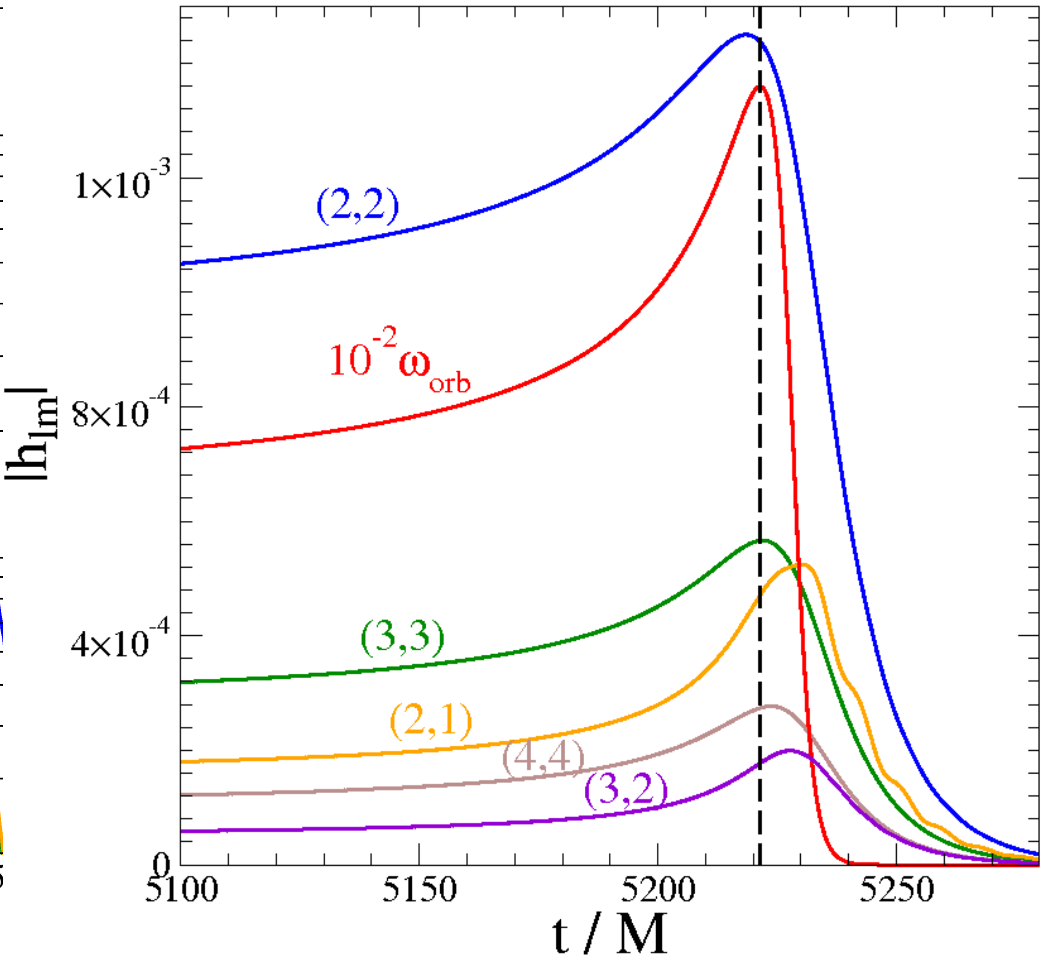

Extreme spins ~ 0.95
EOB Calibrated to
5 nonspin and
2 nonprecessing
SpEC waveforms
mass ratio
spin2
spin1
aligned
antialigned
S2
S1
1.0
0.5
0.0
0.2
0.1
0.0
0.0
0.5
1.0
ν
$\hat{S}\cdot\hat{L}$
1.
0.5
0.
−0.5
-1.

Extreme anti-aligned spins



Nonspinning IMR modes





QNM excitation