

The Black Hole Interior in SYK

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[[hep-th/1810.02055](#)]

Outline

- SYK Black Hole Microstates. Atypical and Typical States.
- Necessity of State Dependence.
- AdS/CFT as a QEC.
- A Dictionary for the Interior from QEC.
- Comments on the Papadodimas-Raju proposal.

SYK BH Microstates

[Kourkoulou, Maldacena]

- N Majorana Fermions = N/2 Spins. $S_k = 2i\psi^{2k-1}\psi^{2k}$

SYK BH Microstates

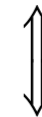
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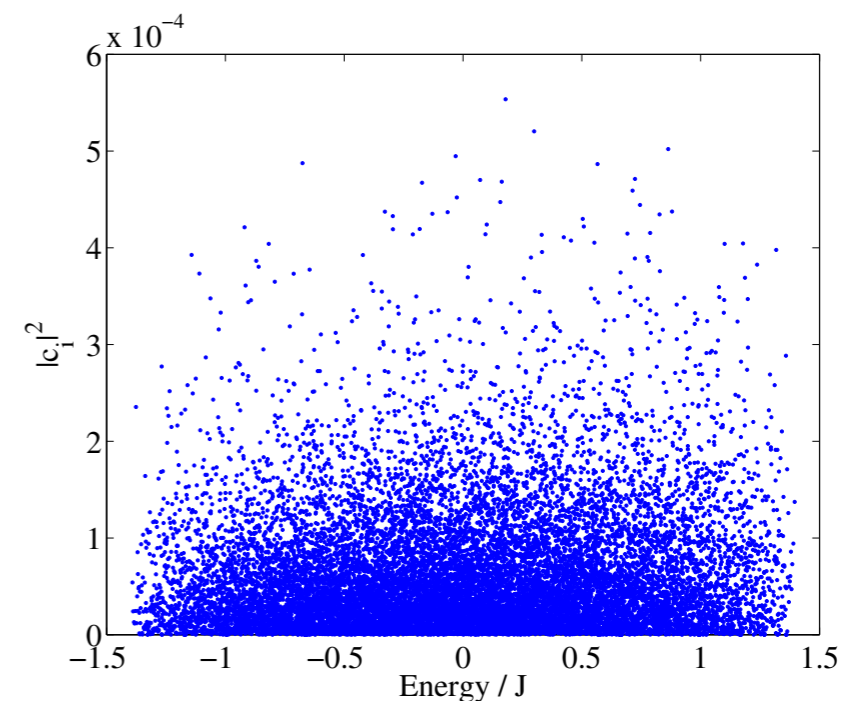
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- Uniformly distributed in Energy:

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$$|B_s\rangle = \sum_i c_i |E_i\rangle \quad |c_i|^2 \sim 2^{-N/2+1}$$



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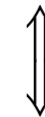
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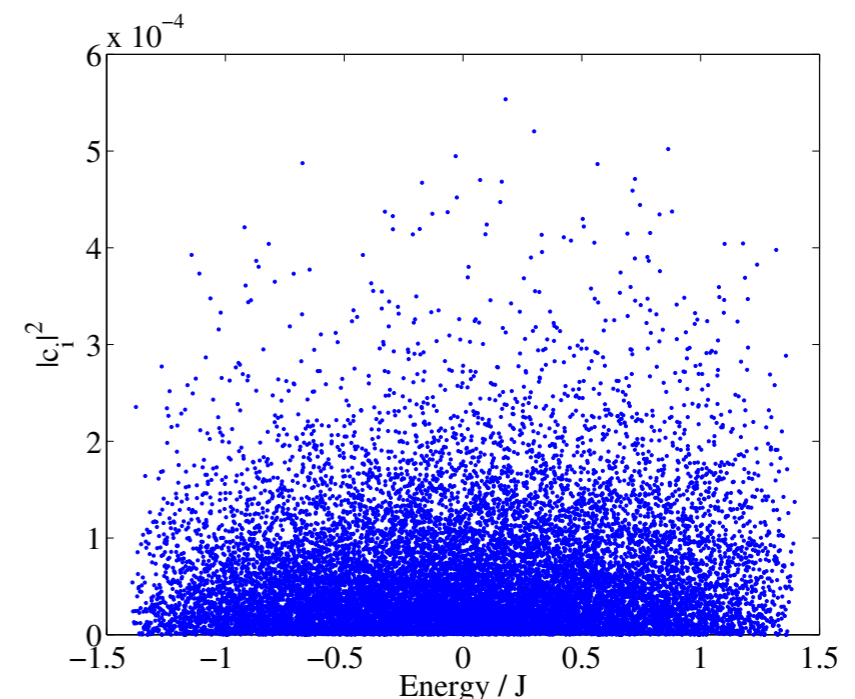
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- Lower Energy via Euclidean Ev.

$$|B_s^\beta\rangle = e^{-\frac{\beta}{2}H} |B_s\rangle$$



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[Kourkoulou, Maldacena]

- Obtain from TFD: $|\beta\rangle_{LR} = \sum_E e^{-\frac{\beta}{2}H_R} |E\rangle_L |E\rangle_R$

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- $\mathcal{O}(N)$ Symmetry, $\sim \mathcal{O}(1/N^{q-1})$. “Flip” subgroup: Flip sign of any even fermion. Relates any two states $|B_s\rangle$ and $|B_{s'}^\beta\rangle$

e.g. $\psi^2 \rightarrow -\psi^2 \implies |\uparrow\uparrow \dots \uparrow\rangle \rightarrow |\downarrow\uparrow \dots \uparrow\rangle$

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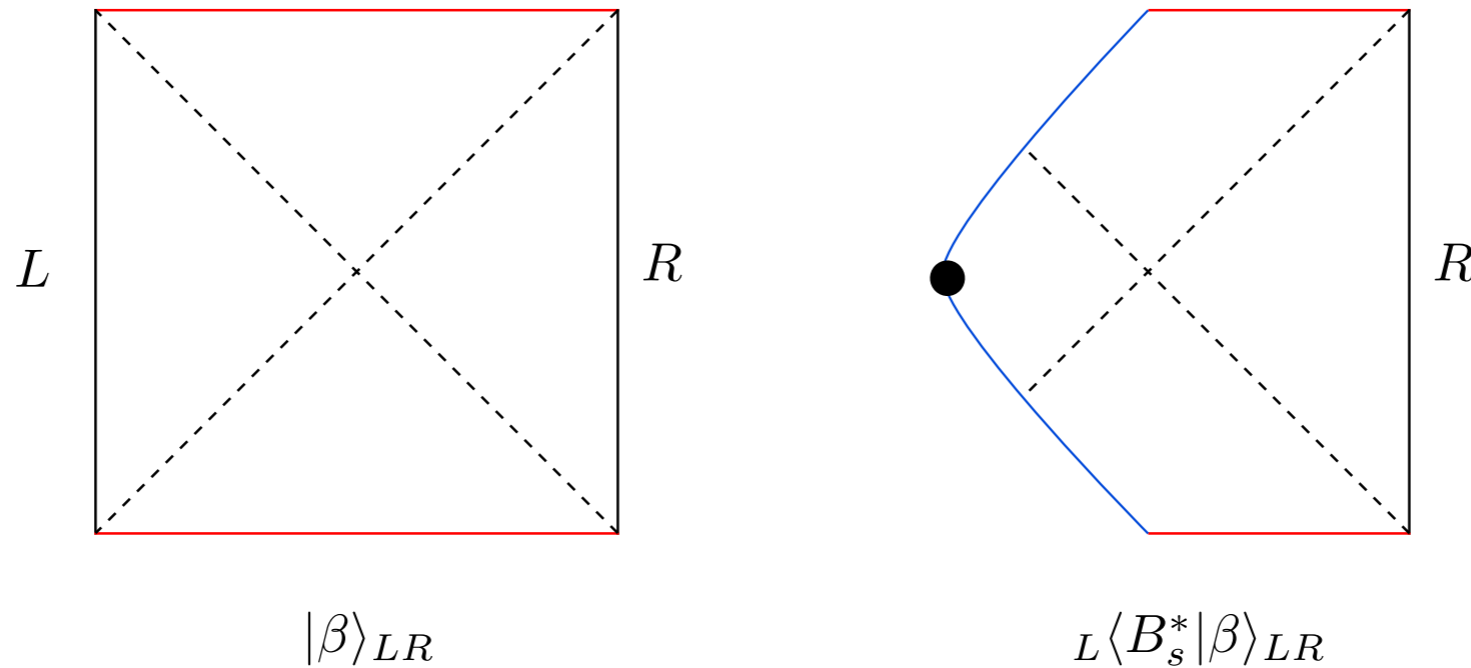
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Bulk Picture

[Kourkoulou, Maldacena]



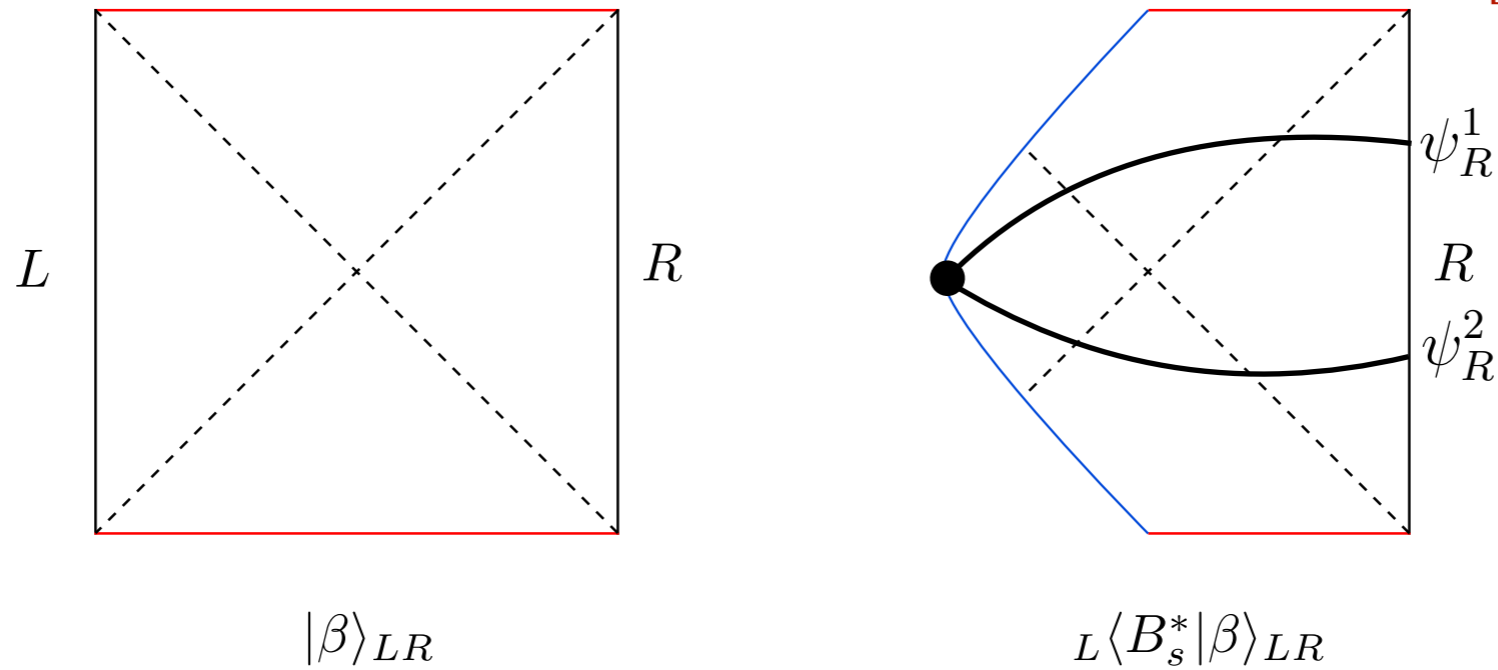
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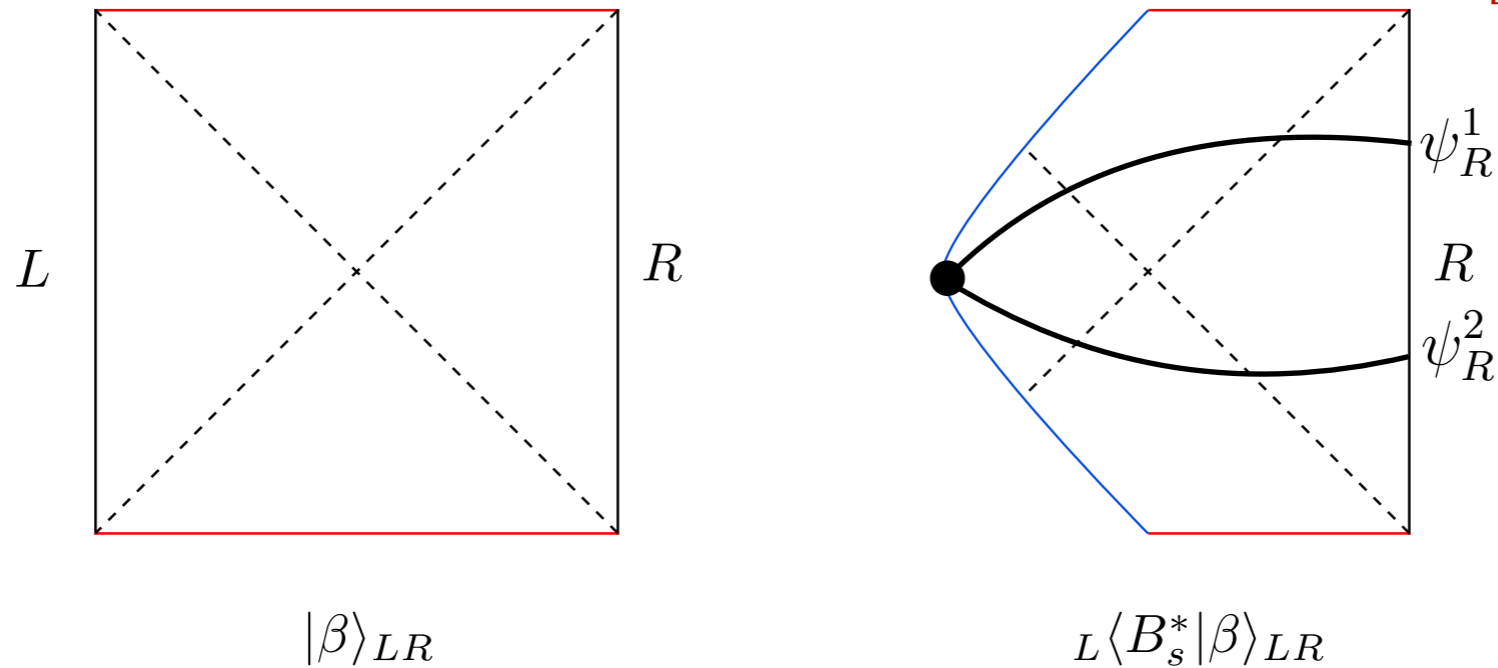
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$$R\langle B_s^\beta | \psi_R^1 \psi_R^2 | B_s^\beta \rangle_R \propto 2is_1 \times LR\langle \beta | \psi_L^1(0) \psi_R^1 | \beta \rangle_{LR} \times LR\langle \beta | \psi_L^2(0) \psi_R^2 | \beta \rangle_{LR}$$

- These are atypical BH microstates: Simple observables have not thermalized.

More Typical Microstates

- Begin with wormhole with OTO shockwaves: $|W\beta\rangle_{LR} \equiv W_L|\beta\rangle_{LR}$
- Then project: $|B_s^{W\beta}\rangle_R \equiv {}_L\langle B_s|W\beta\rangle_{LR}$

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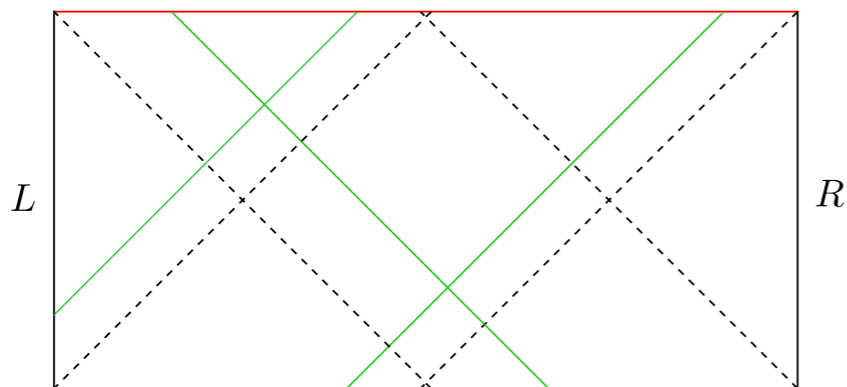
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 ${}_R\langle B_s^{W\beta}|\psi_R^1\psi_R^2|B_s^{W\beta}\rangle_R \propto 2is_1 \times {}_{LR}\langle W\beta|\left[\psi_L^1(0)\psi_L^2(0) \otimes \psi_R^1\psi_R^2\right]|W\beta\rangle_{LR}$
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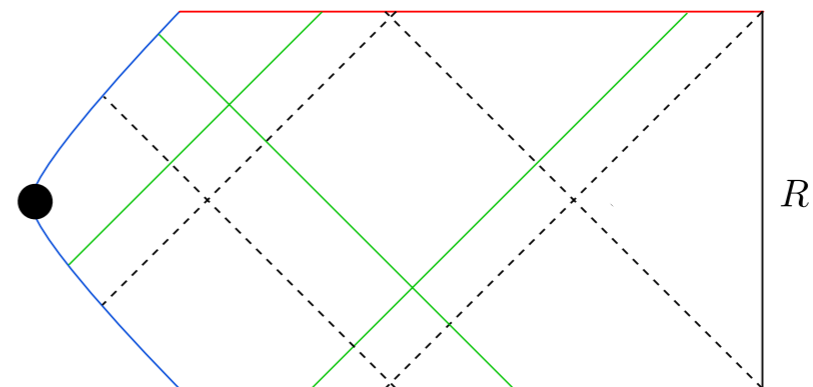
More Typical Microstates

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$W_L|\beta\rangle_{LR}$

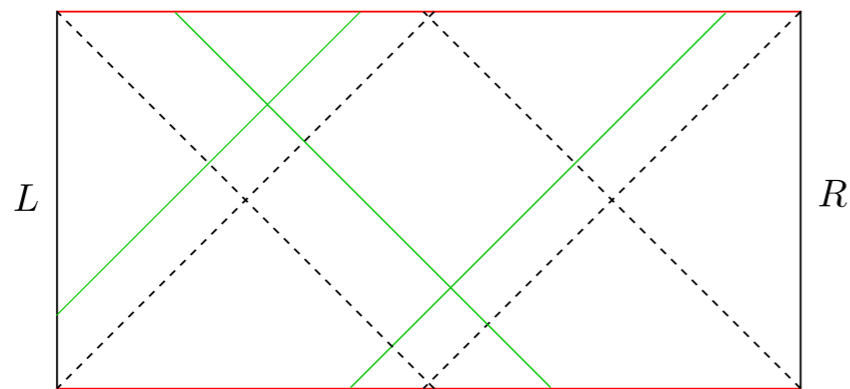


${}_L\langle B_s^*|W_L|\beta\rangle_{LR}$

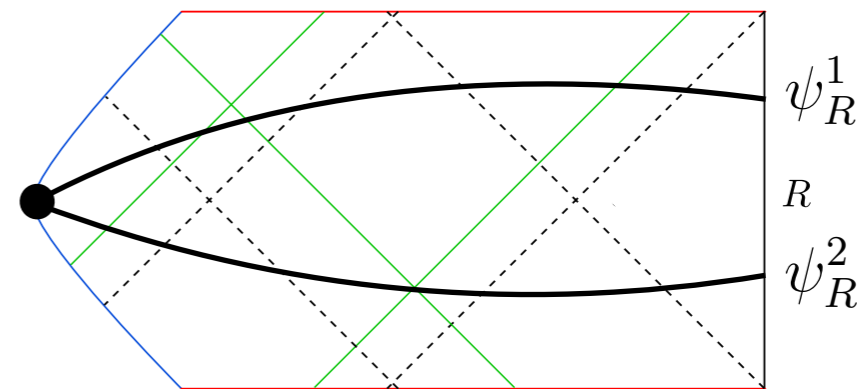
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Also: More Microstates from Modified SYK

[AA, Zhenbin Yang - WIP]

- SYK: $H_{SYK} \sim \sum_{i_1 \dots i_q}^N J_{i_1 \dots i_q} \psi^{i_1} \dots \psi^{i_q}$
- Raising and lowering operators: $\chi_k^\sigma = \psi^{2k-1} + i\sigma_k \psi^{2k}$
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 $\langle J_{a_1, \dots, a_q}^{\sigma_1, \dots, \sigma_q} J_{b_1, \dots, b_q}^{\tilde{\sigma}_1, \dots, \tilde{\sigma}_q} \rangle = \mathcal{J}^2 \delta_{a_1, b_1} \dots \delta_{a_q, b_q} \delta_{\sigma_1 + \tilde{\sigma}_1} \dots \delta_{\sigma_q + \tilde{\sigma}_q}$

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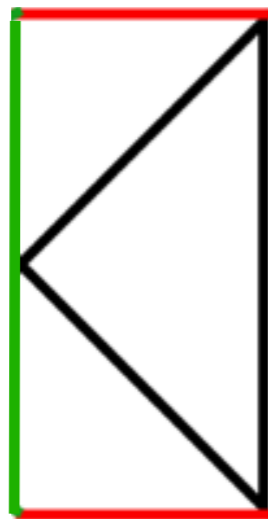
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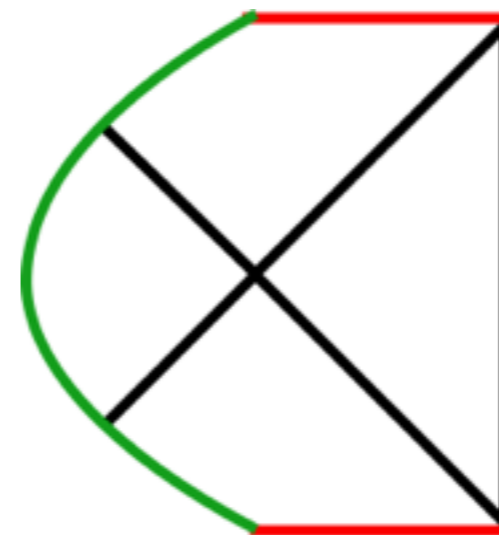
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$|\uparrow\uparrow \dots \uparrow\rangle + O(1)$ spin flips



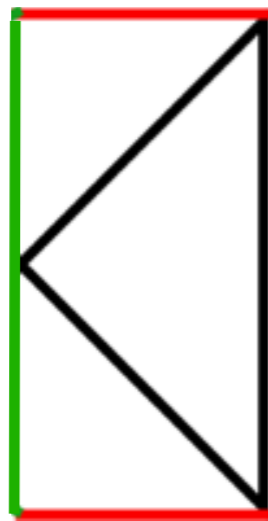
$|\uparrow\downarrow\uparrow \dots \downarrow\rangle, O(N)$ spin flips

Also: More Microstates from Modified SYK

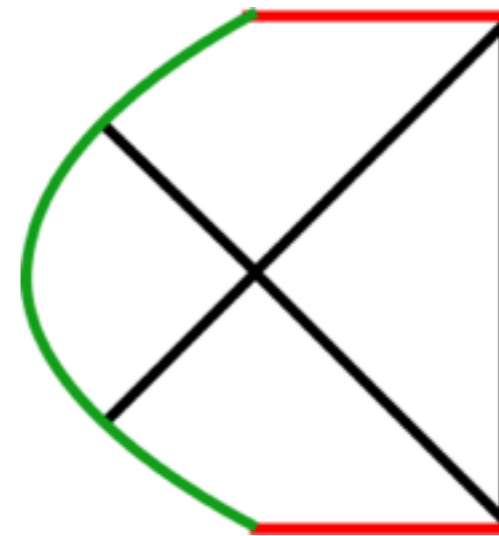
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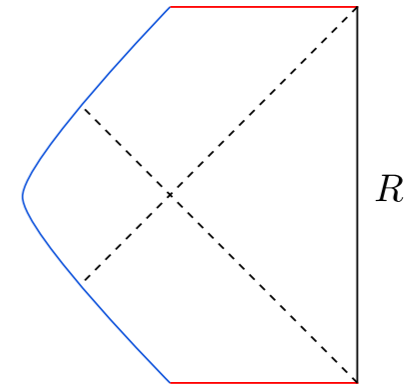
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Different Microstates

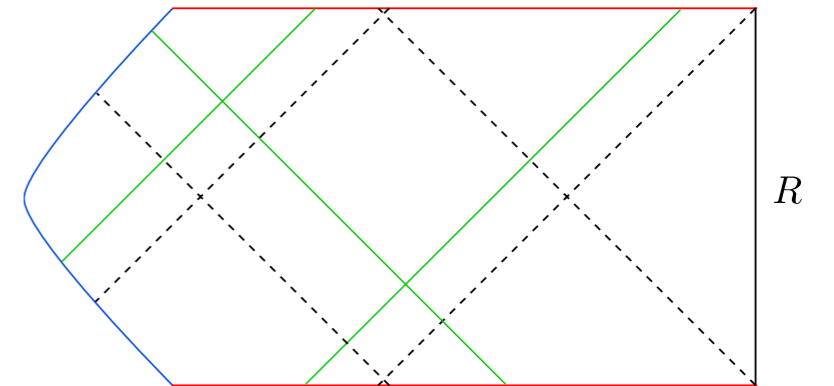
- Atypical Microstates

$$|B_s^\beta\rangle_R = {}_L\langle B_s|\beta\rangle_{LR}$$



- More Typical Microstates

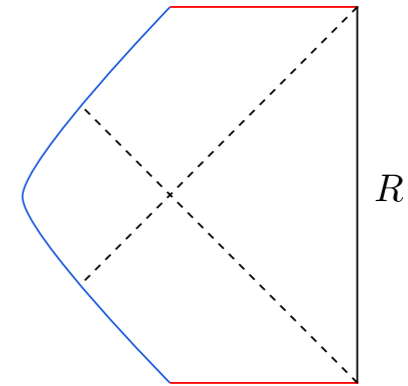
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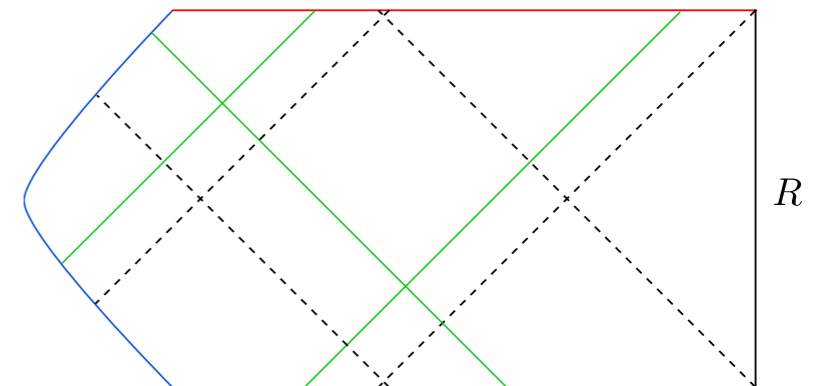
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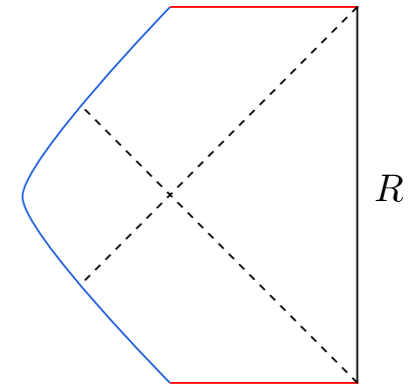


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Different Microstates

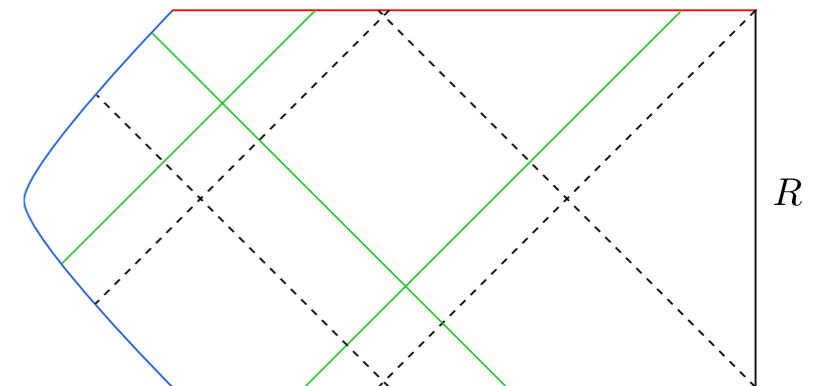
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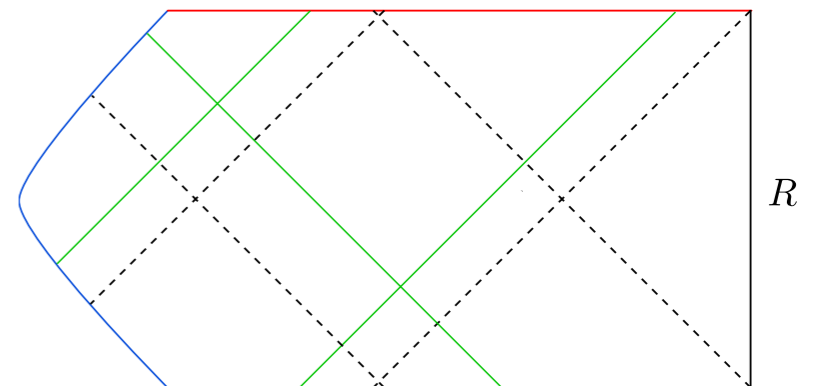
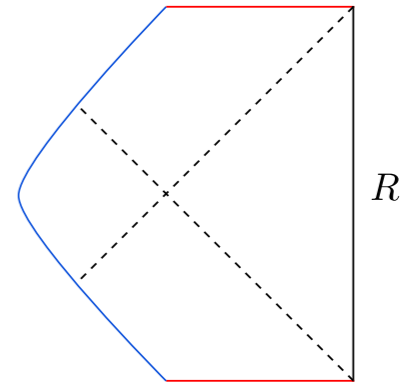
- Both are over-complete bases of BH microstates of temperature β
- Does this have consequences for interior reconstruction?

Necessity of State-Dependence

- Suppose the existence of a linear operator N_R^F that measures whether there is a shockwave behind the horizon.

$${}_R\langle B_s^\beta | N_R^F | B_s^\beta \rangle_R$$

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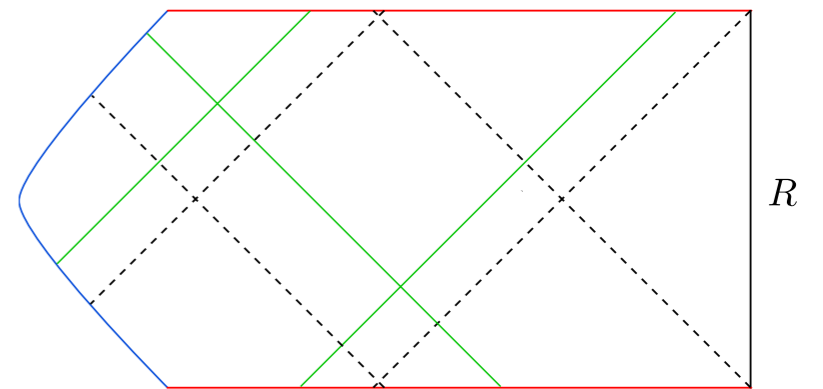
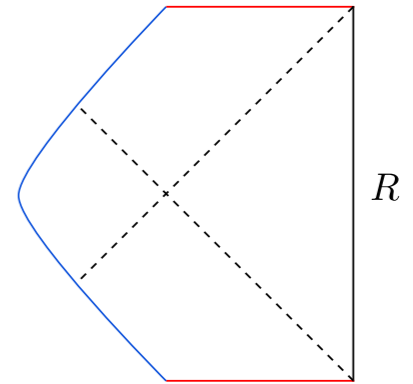


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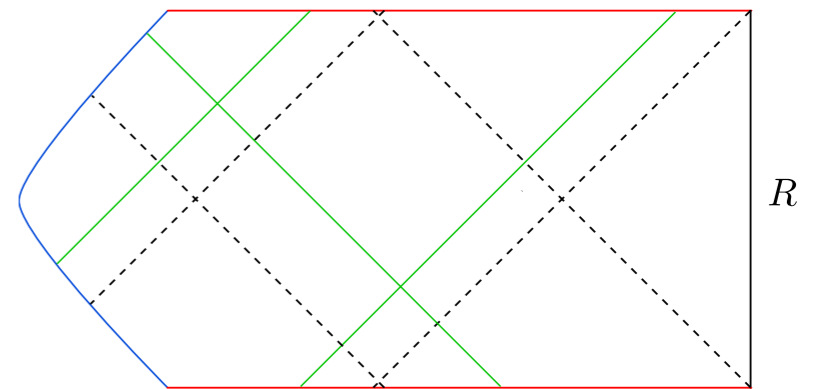
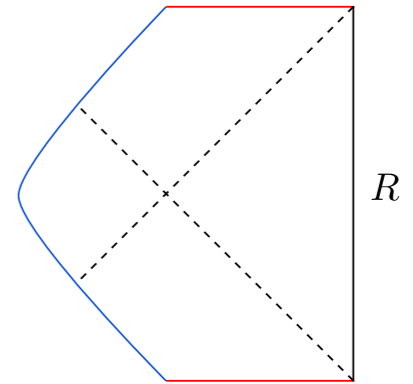


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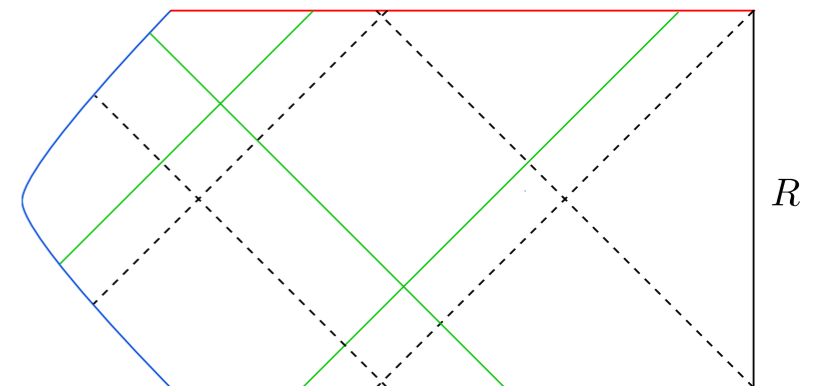
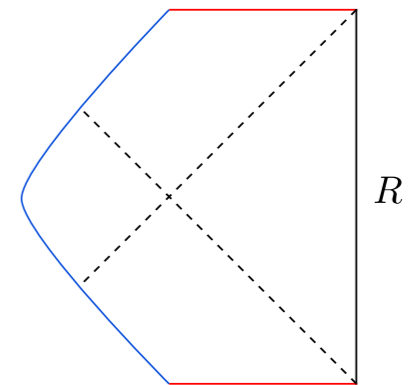


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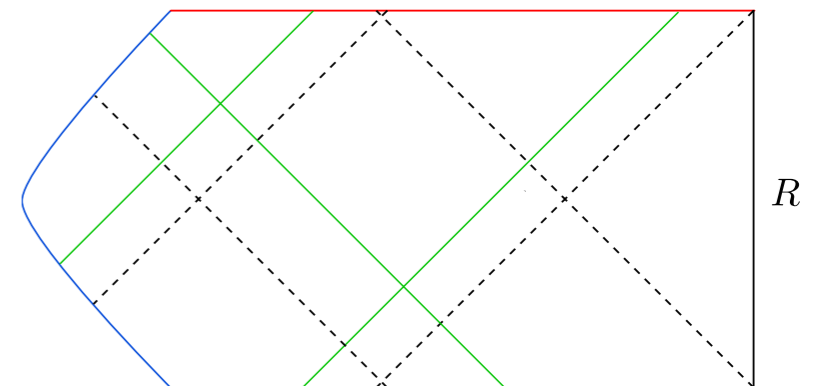
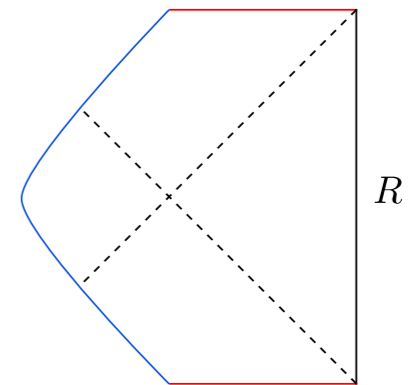


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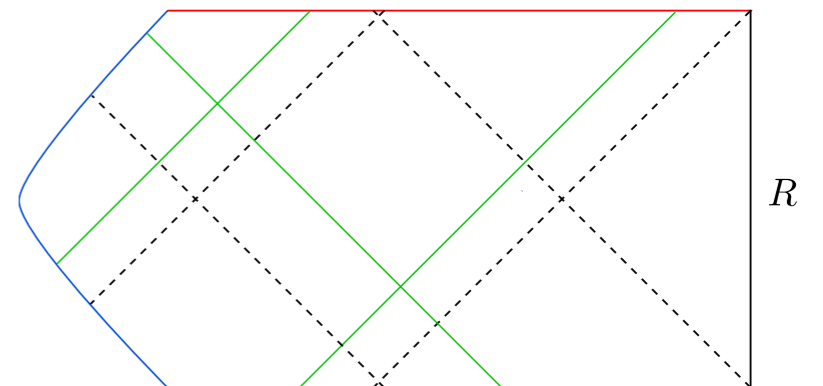
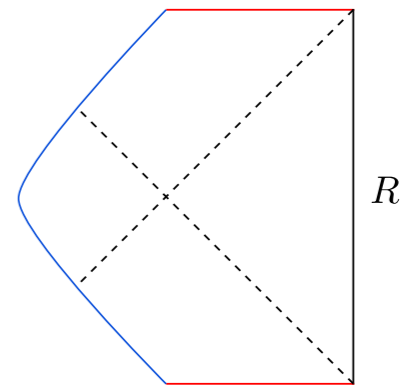


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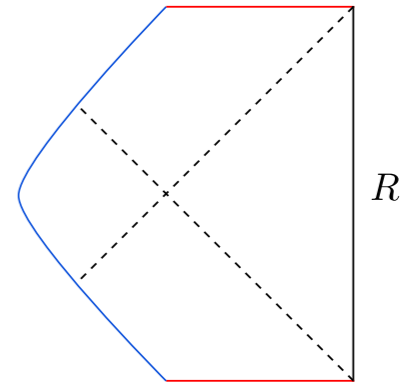
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Necessity of State-Dependence

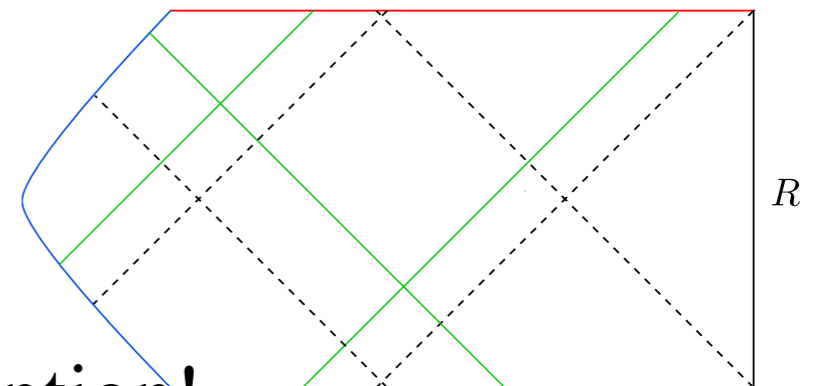
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$$= {}_{LR} \langle \beta | N_R^F | \beta \rangle_{LR}$$



- Contradiction! Can't be the same by assumption!

Take Away

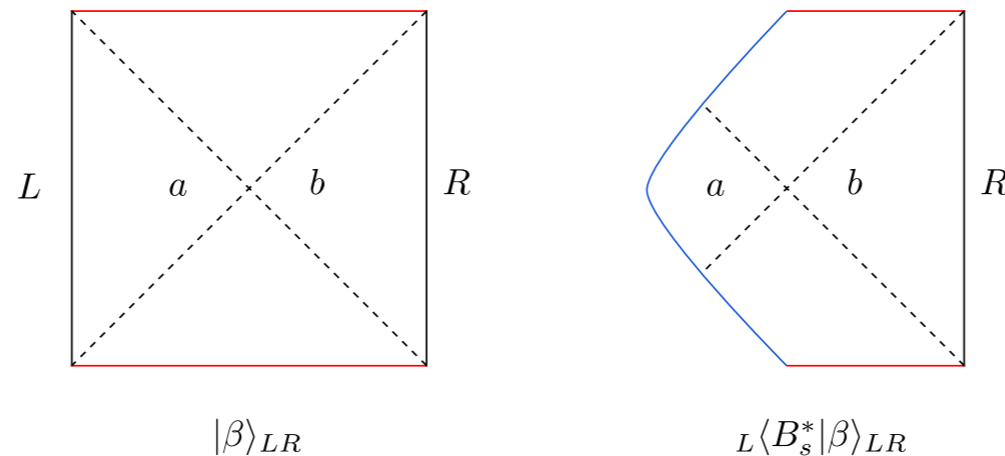
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- The dictionary for the interior must be state dependent.
- Can we find such a dictionary?
- Yes! By assuming the AdS/CFT dictionary to be a QEC code, one can generate such a dictionary for the interior!
- Key: the dictionary is fluid, responding to the projection in a way that maps the interior operators to the remaining boundary in a state dependent way.



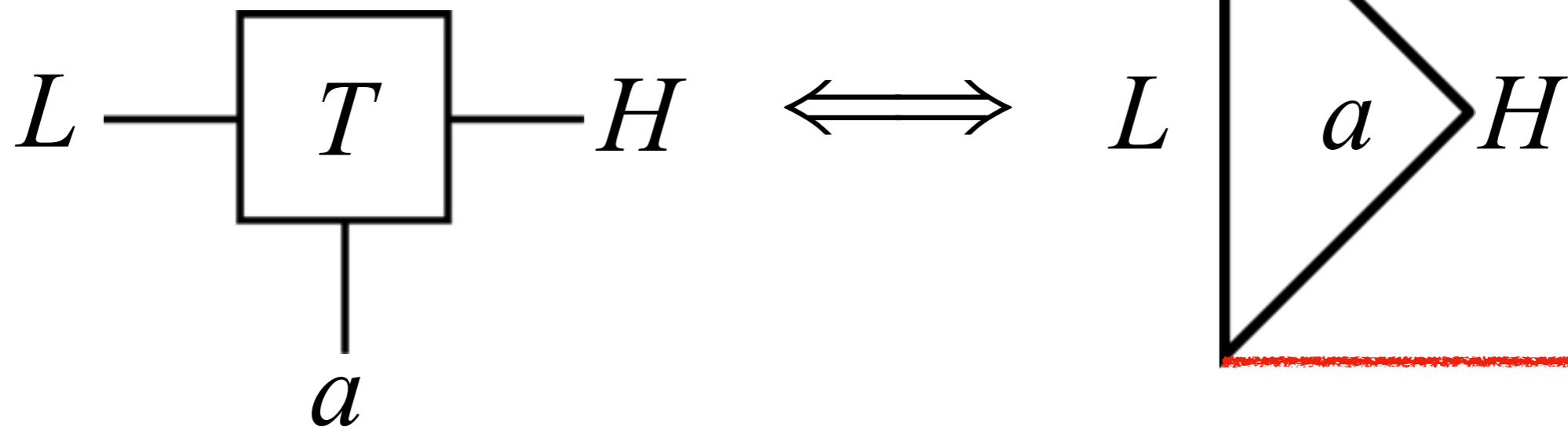
AdS/CFT as QEC

[AA, Dong, Harlow;...]

- I will describe the duality using a circuit diagram.
- Consider Isometry T from systems a & H into L .

$$\mathcal{D}_L \gg \mathcal{D}_a \times \mathcal{D}_H$$

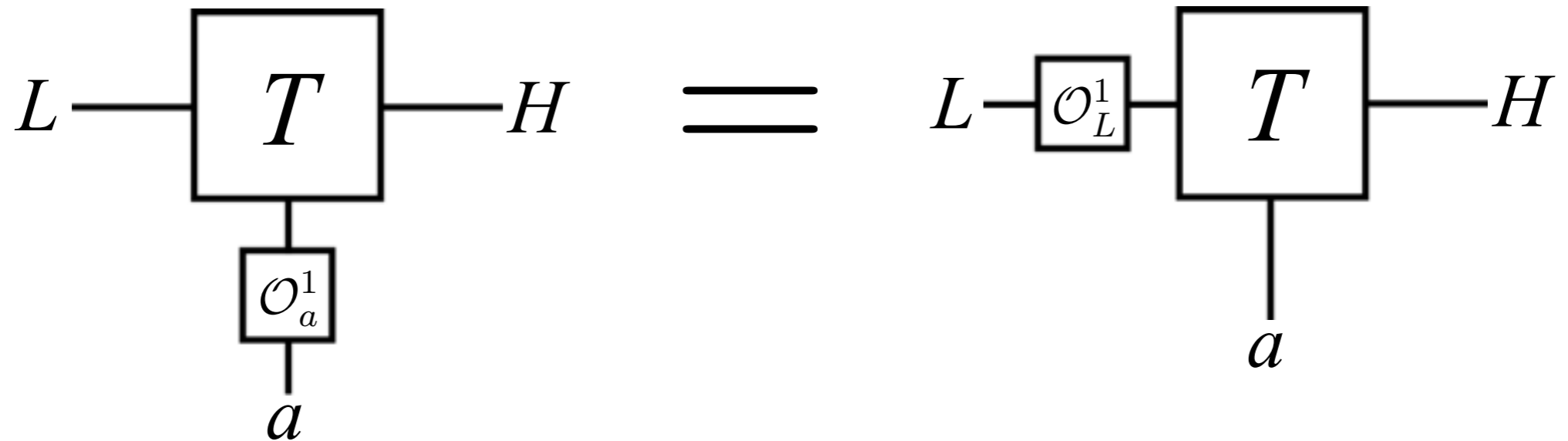
$$\mathcal{D}_H > \mathcal{D}_a$$



Operator Mapping

[HPPY, Hayden et al]

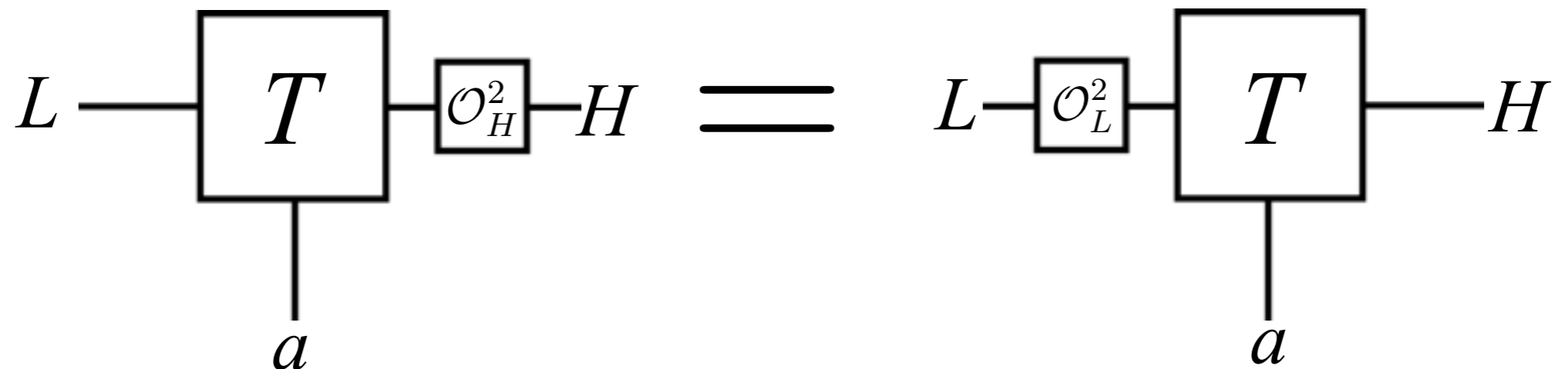
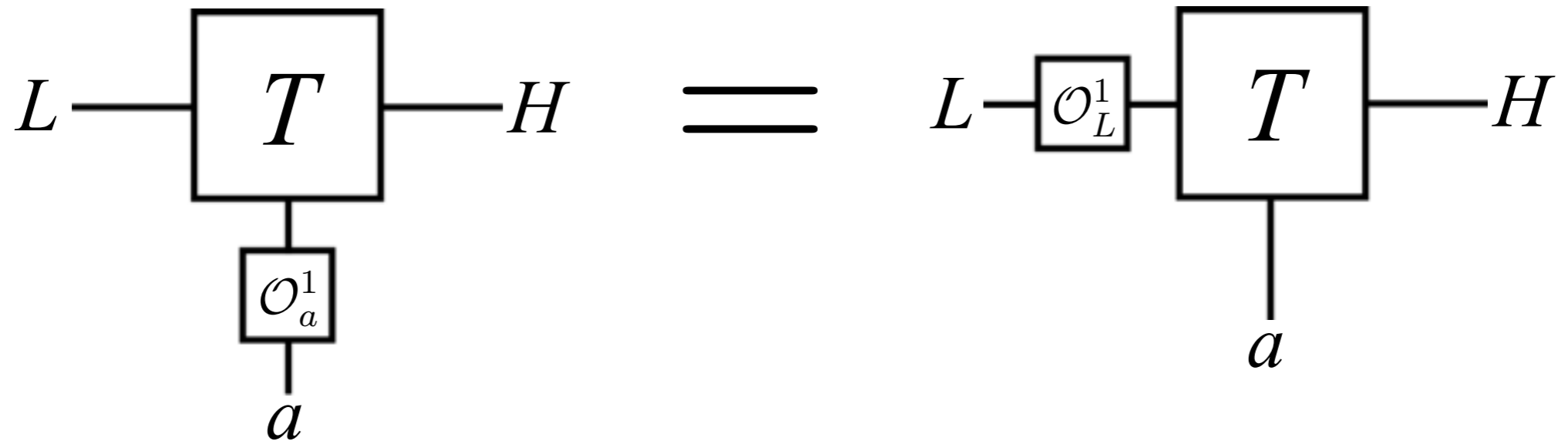
- Operators can be pushed through the tensors:



Operator Mapping

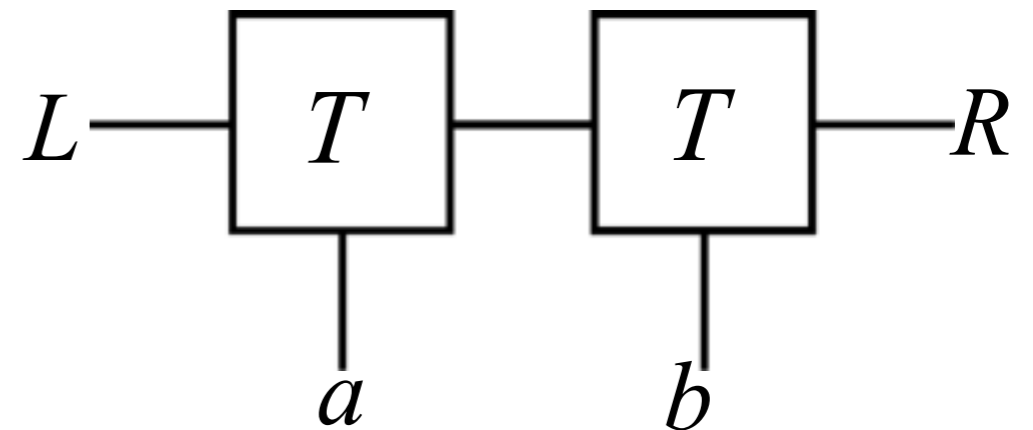
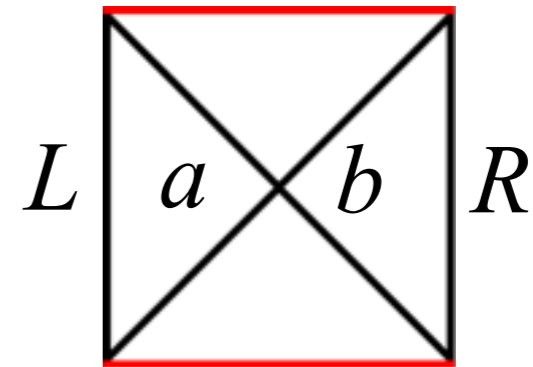
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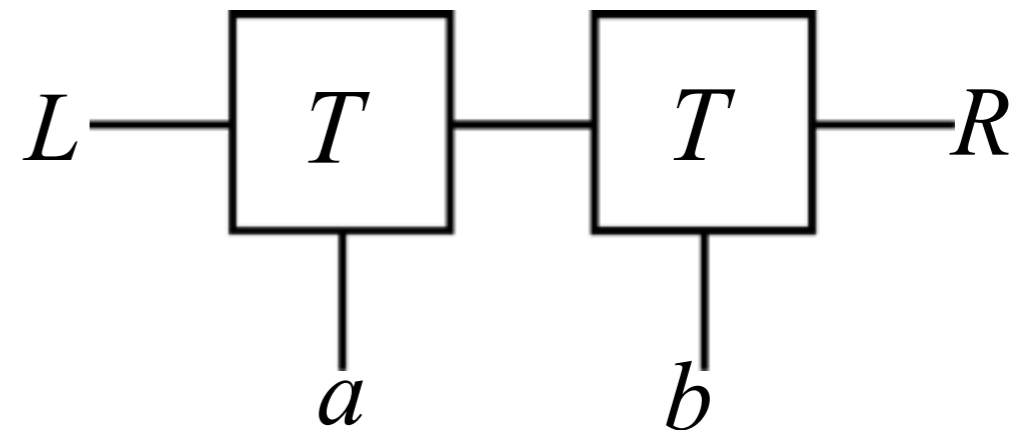
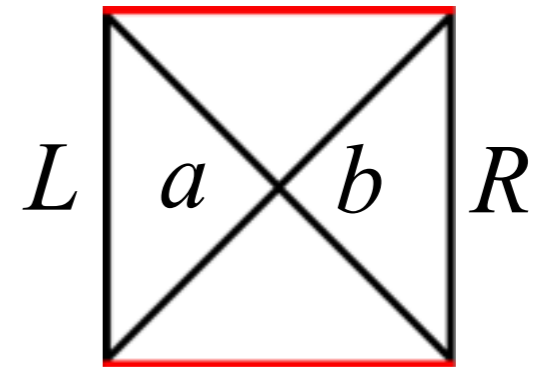
Eternal BH Code

- Eternal BH dictionary is given by:



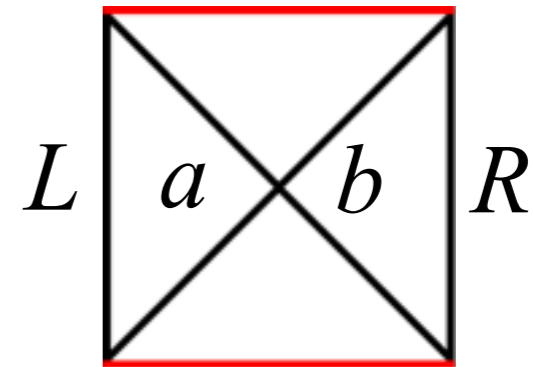
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- a and b represent bulk quantum fields.

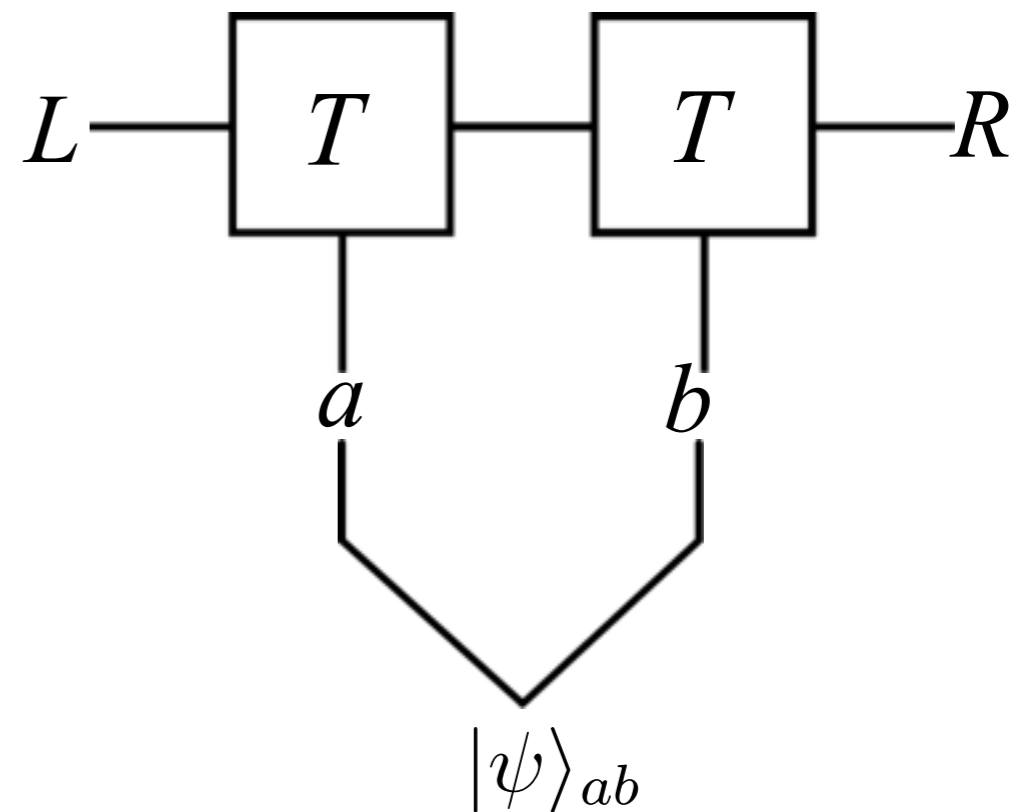


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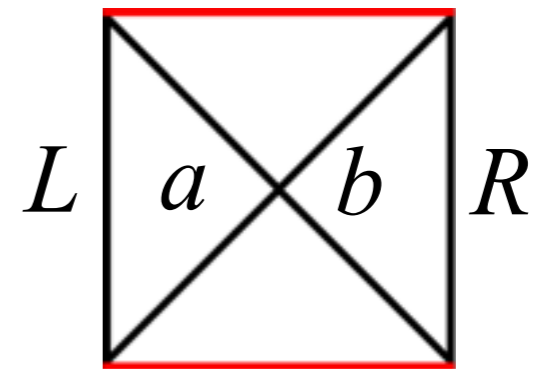


$$|\psi\rangle_{ab} \rightarrow |\tilde{\psi}\rangle_{LR} \in \mathcal{H}_{code} \subset \mathcal{H}_L \otimes \mathcal{H}_R$$



Eternal BH Code

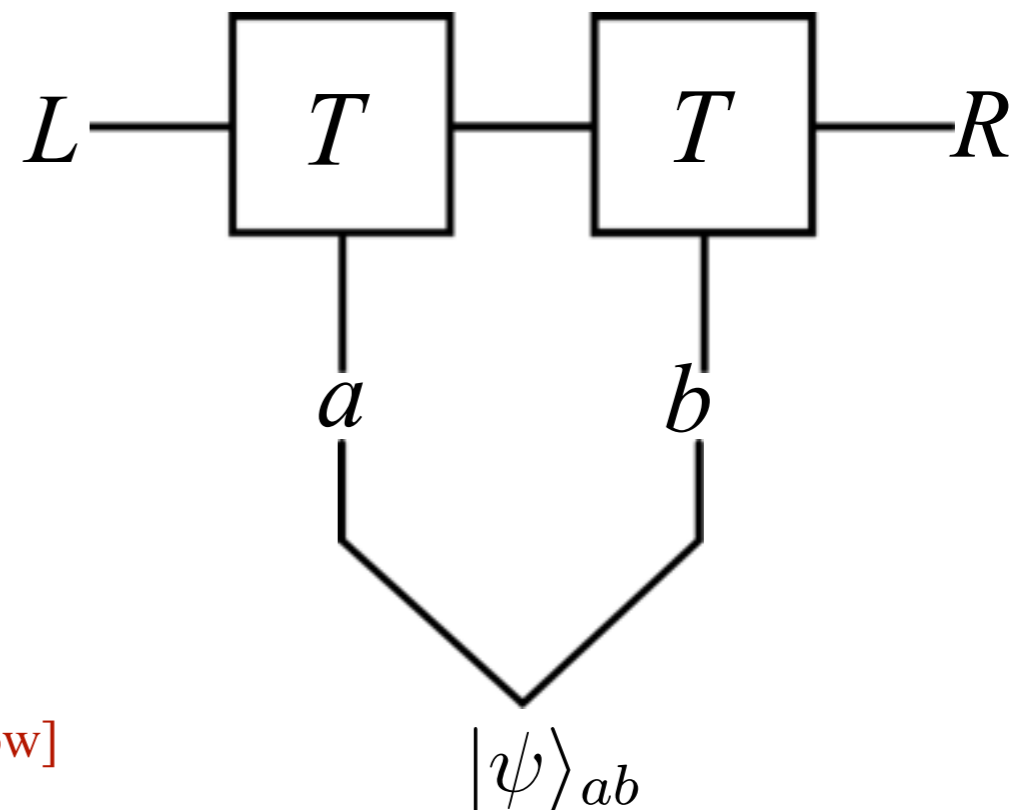
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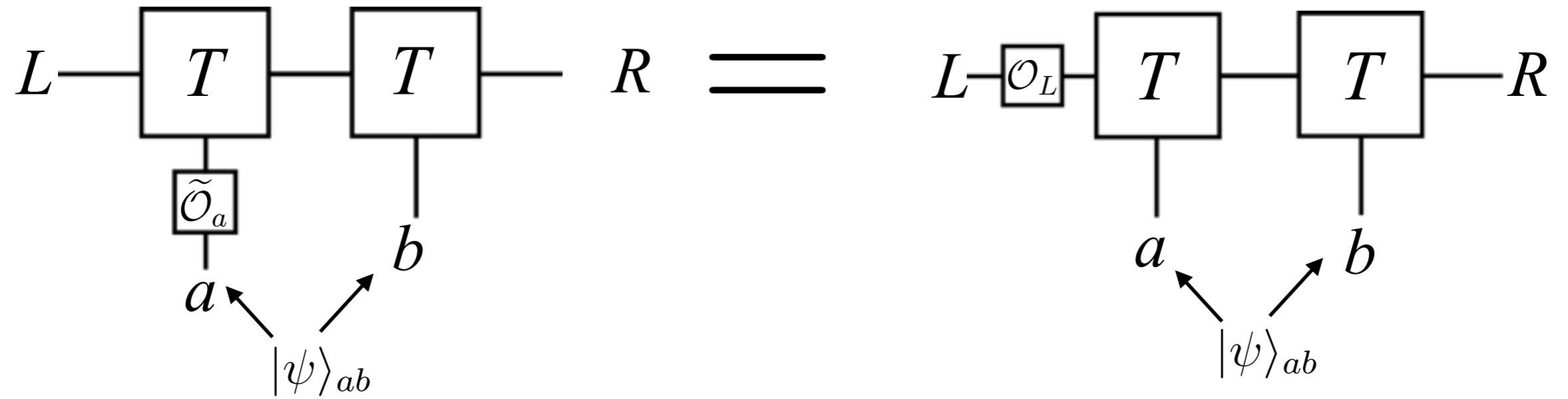
- Reproduces quantum corrected RT formula.

$$S(\rho_R^{\tilde{\psi}}) = \ln \mathcal{D}_H + S(\rho_b^{\psi})$$

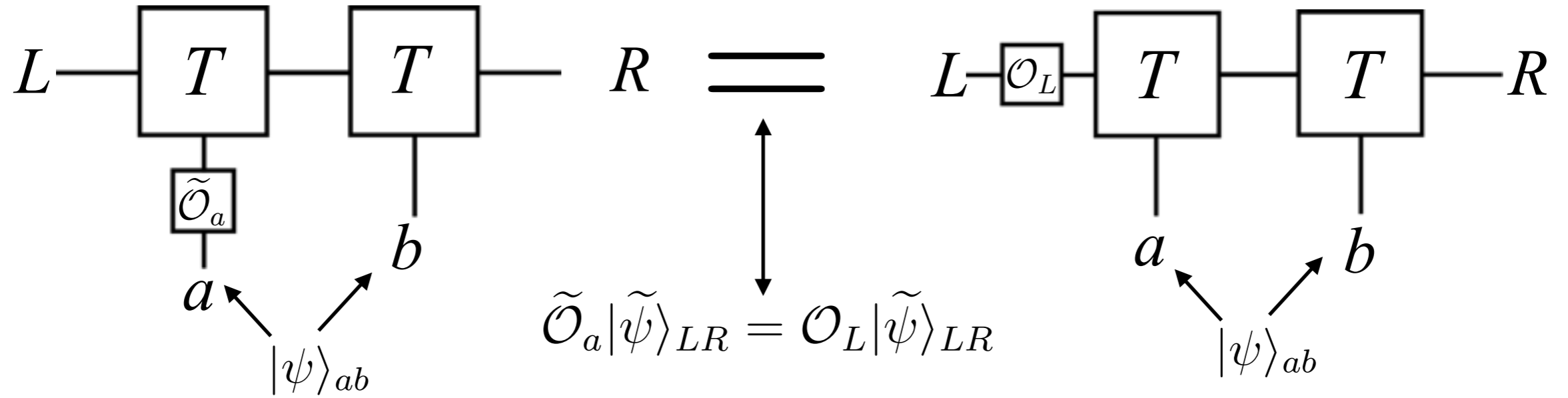


[Harlow]

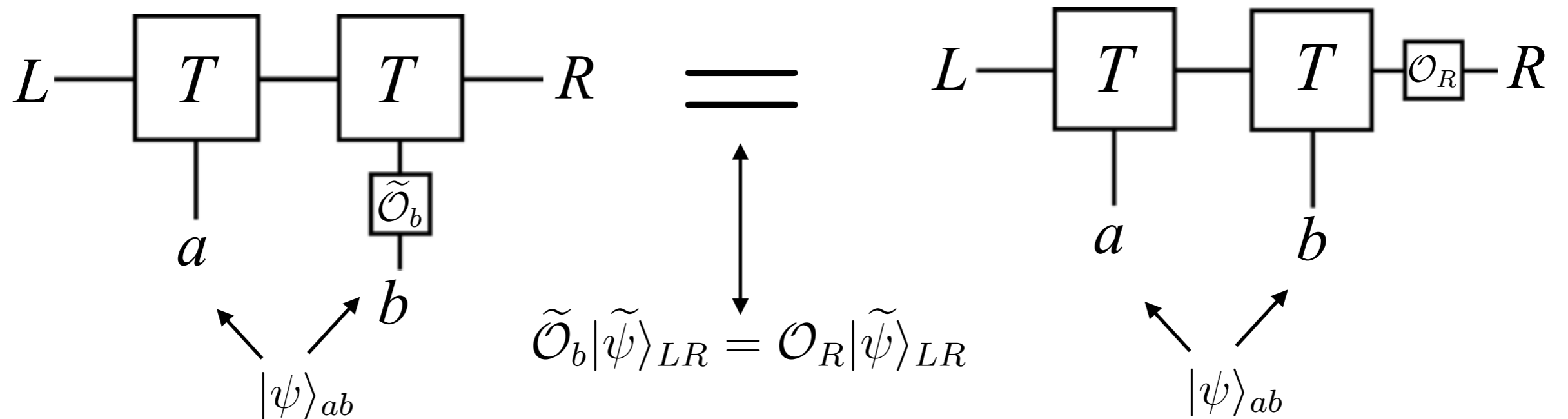
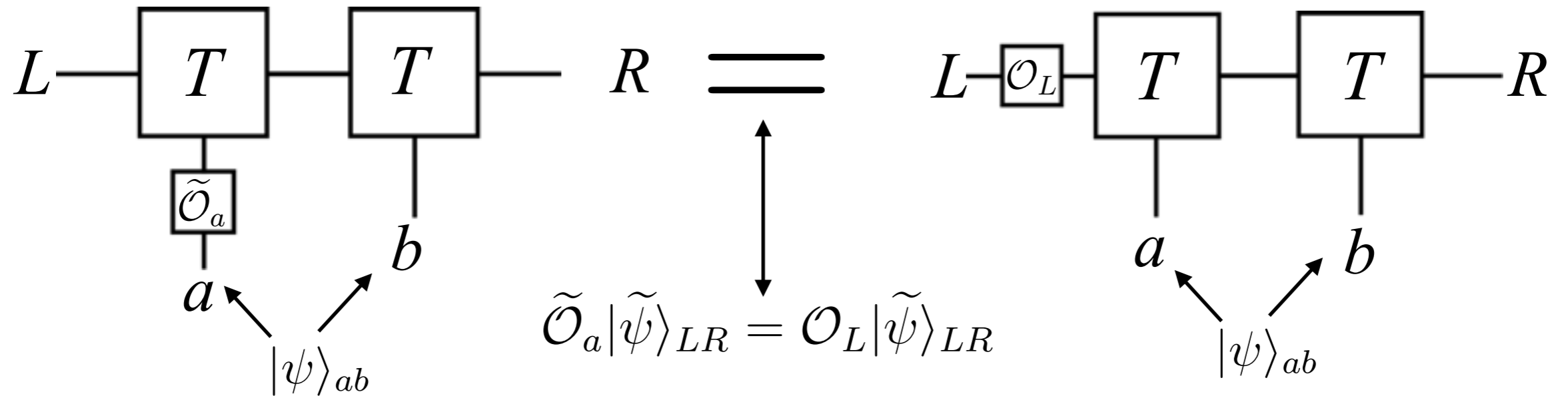
Subregion-Subregion Duality



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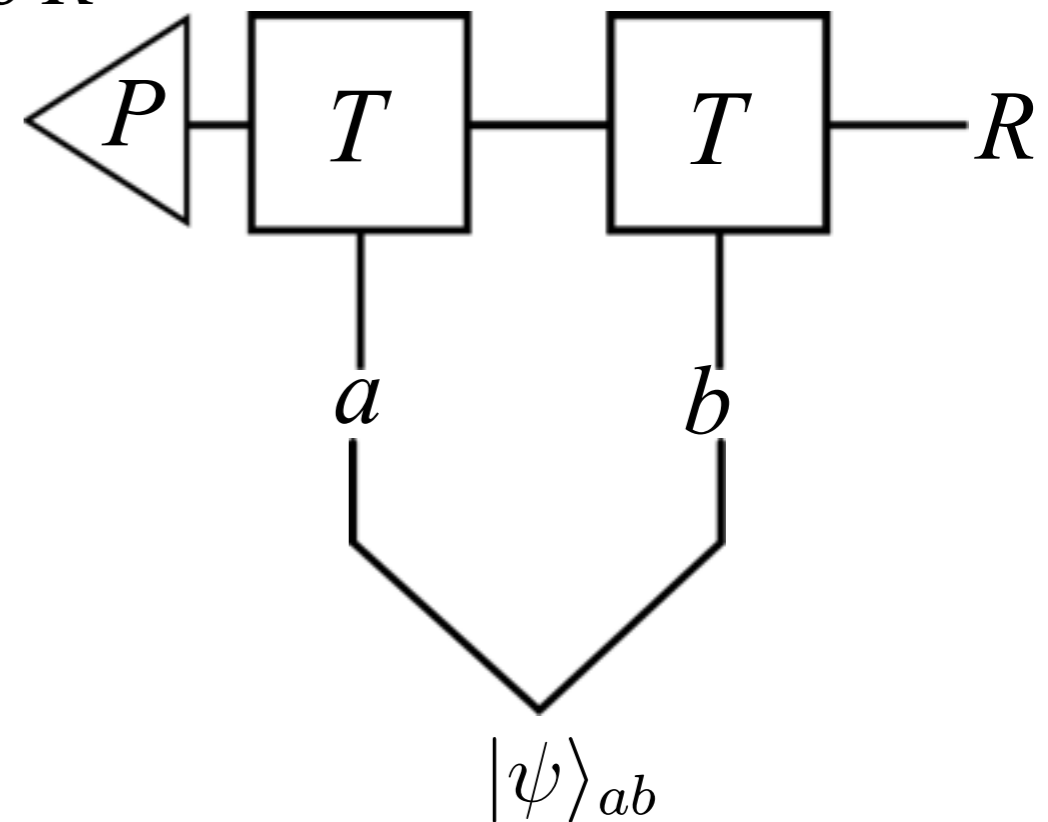
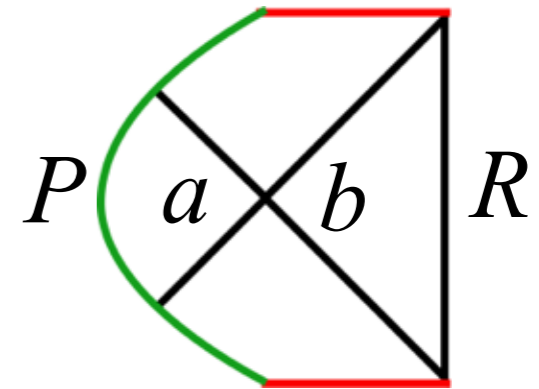


Subregion-Subregion Duality

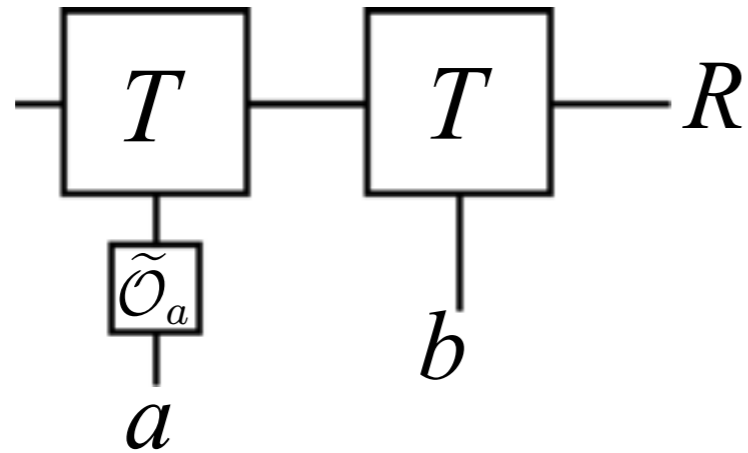


Projected BH Code

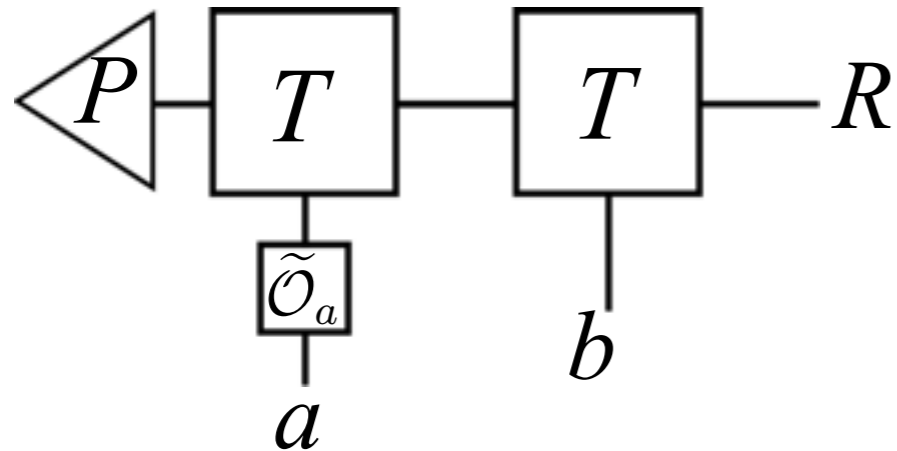
- Projected BH has ‘same’ geometry.
- Project on original tensor network.
- We want to understand the conditions on P such that operators acting on a map to R



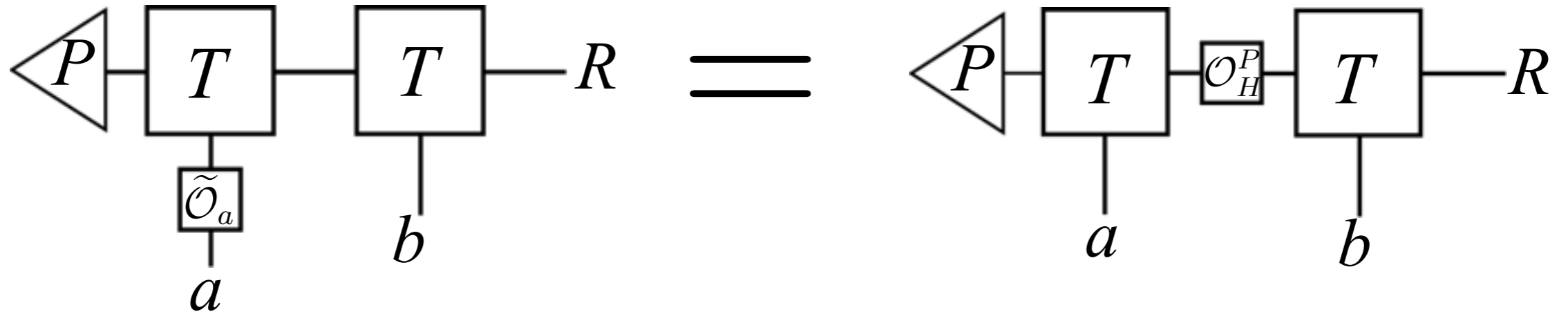
Condition on the Projector



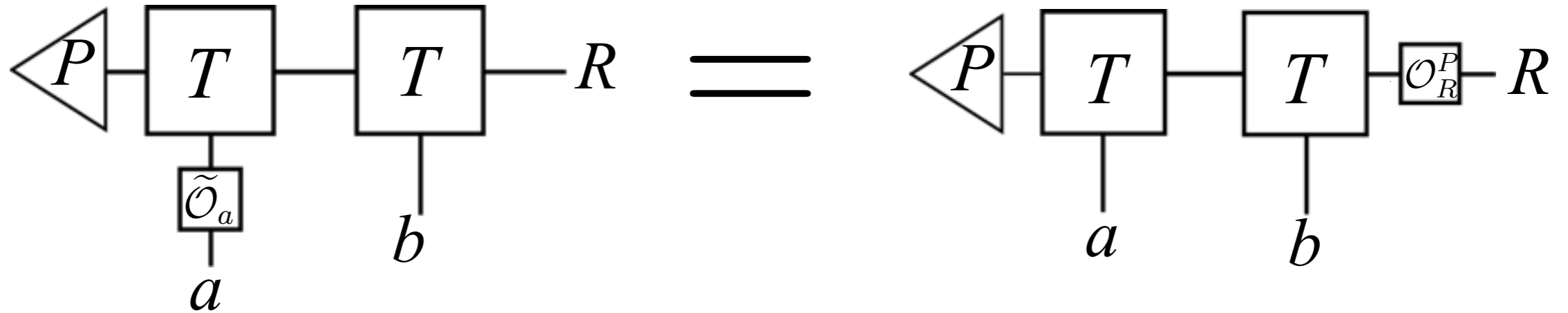
Condition on the Projector



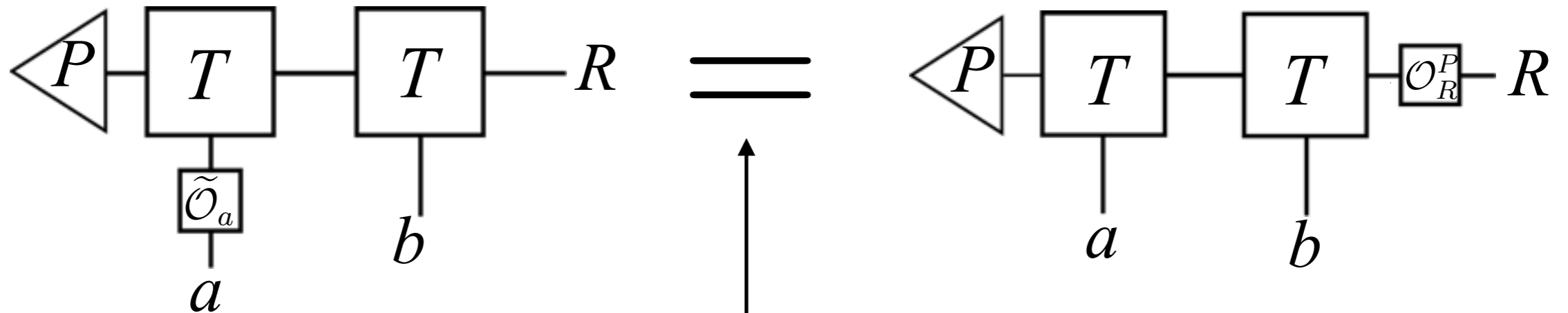
Condition on the Projector



Condition on the Projector

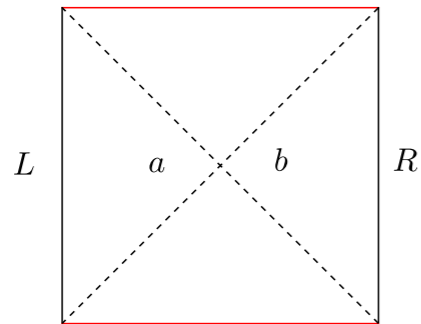
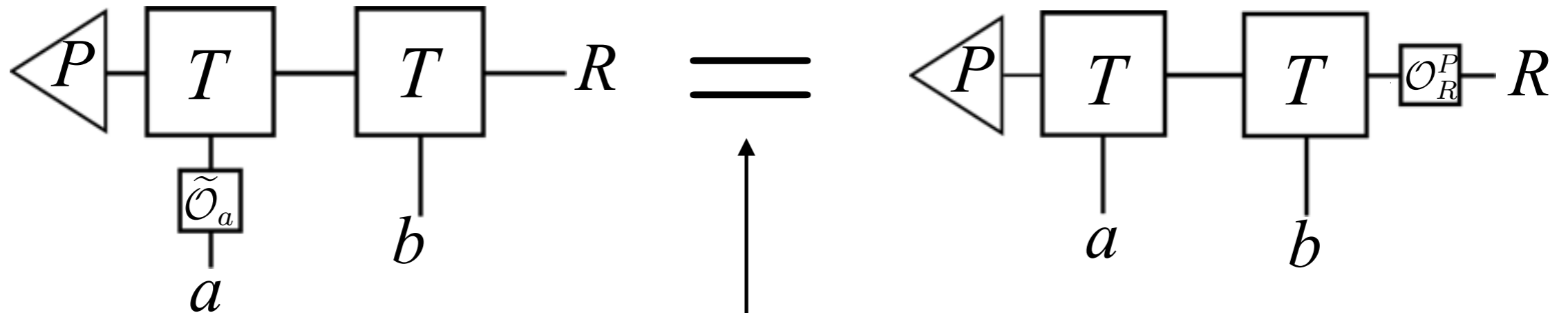


Condition on the Projector



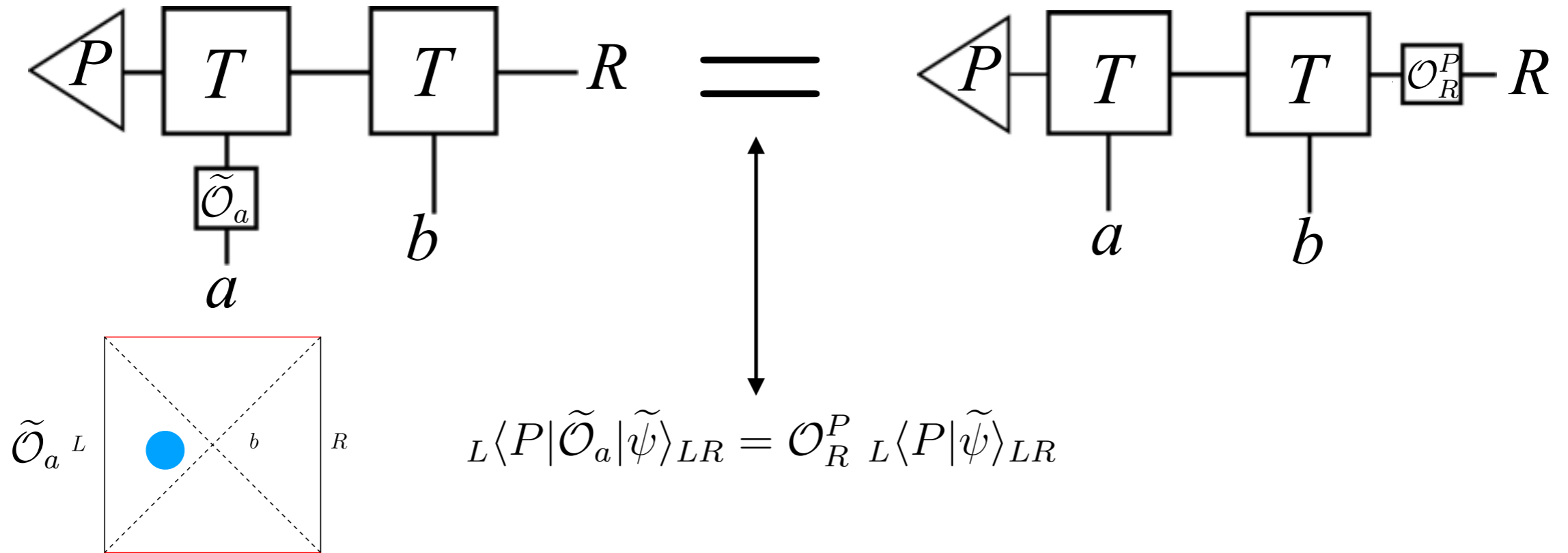
$${}_L\langle P|\tilde{\mathcal{O}}_a|\tilde{\psi}\rangle_{LR} = \mathcal{O}_R^P {}_L\langle P|\tilde{\psi}\rangle_{LR}$$

Condition on the Projector

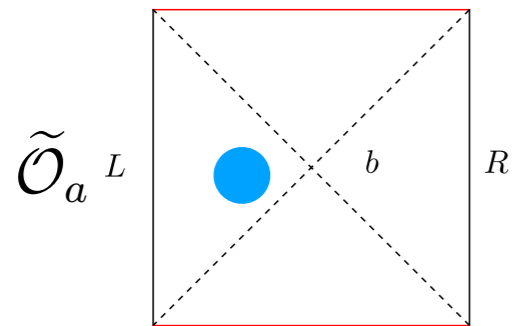
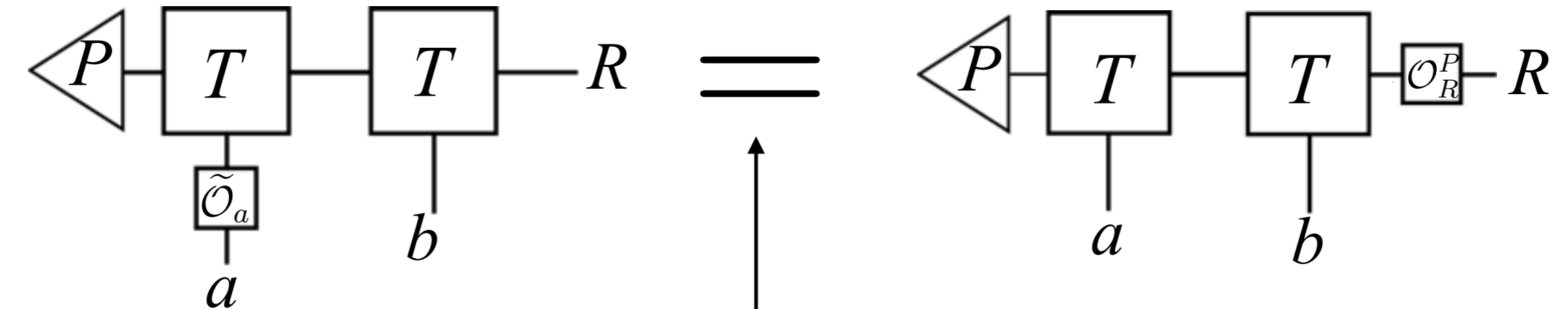


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Condition on the Projector

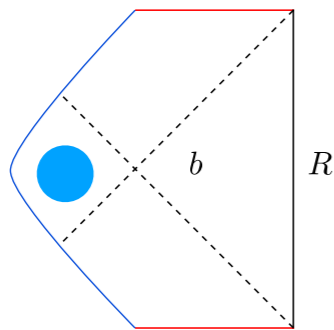


Condition on the Projector

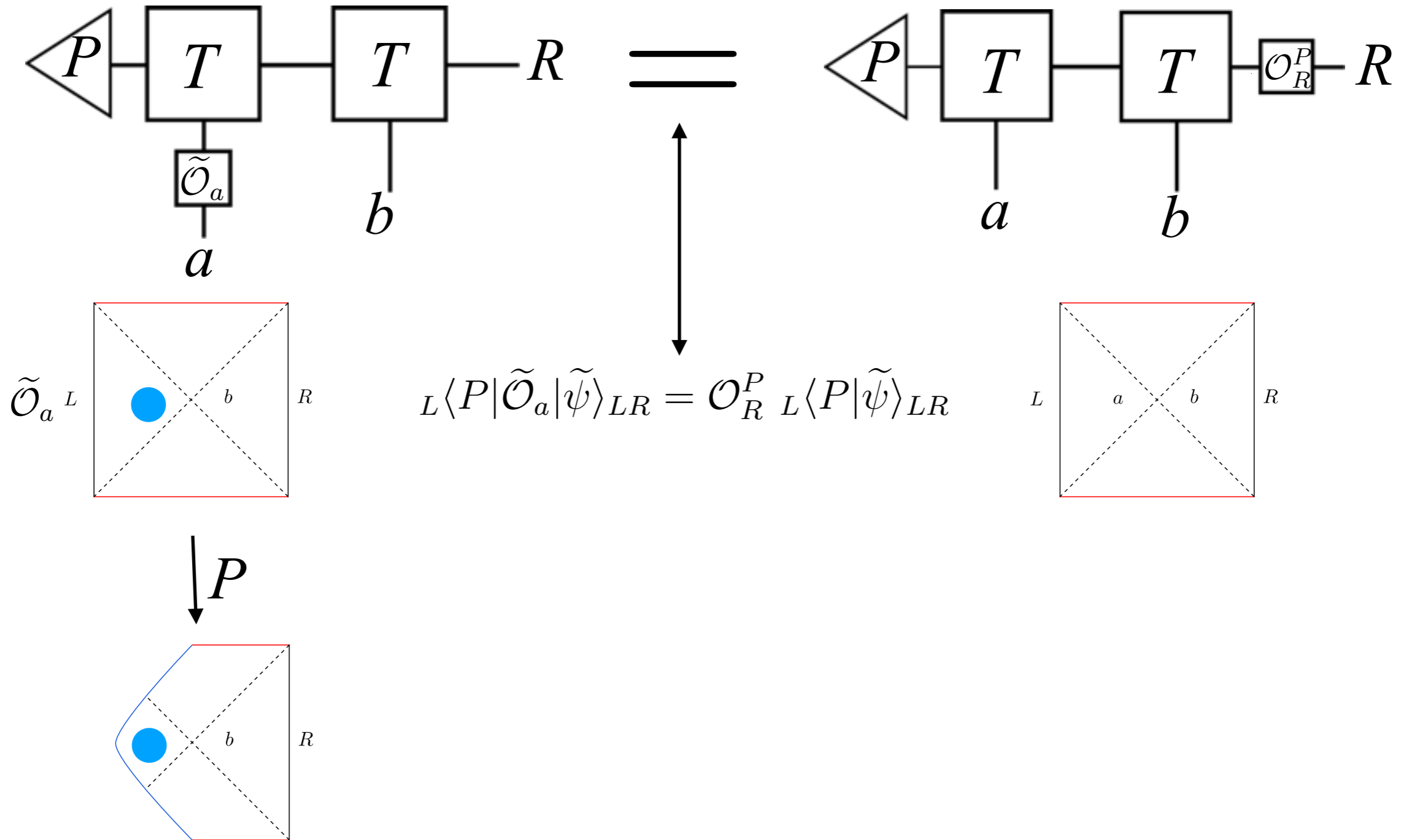


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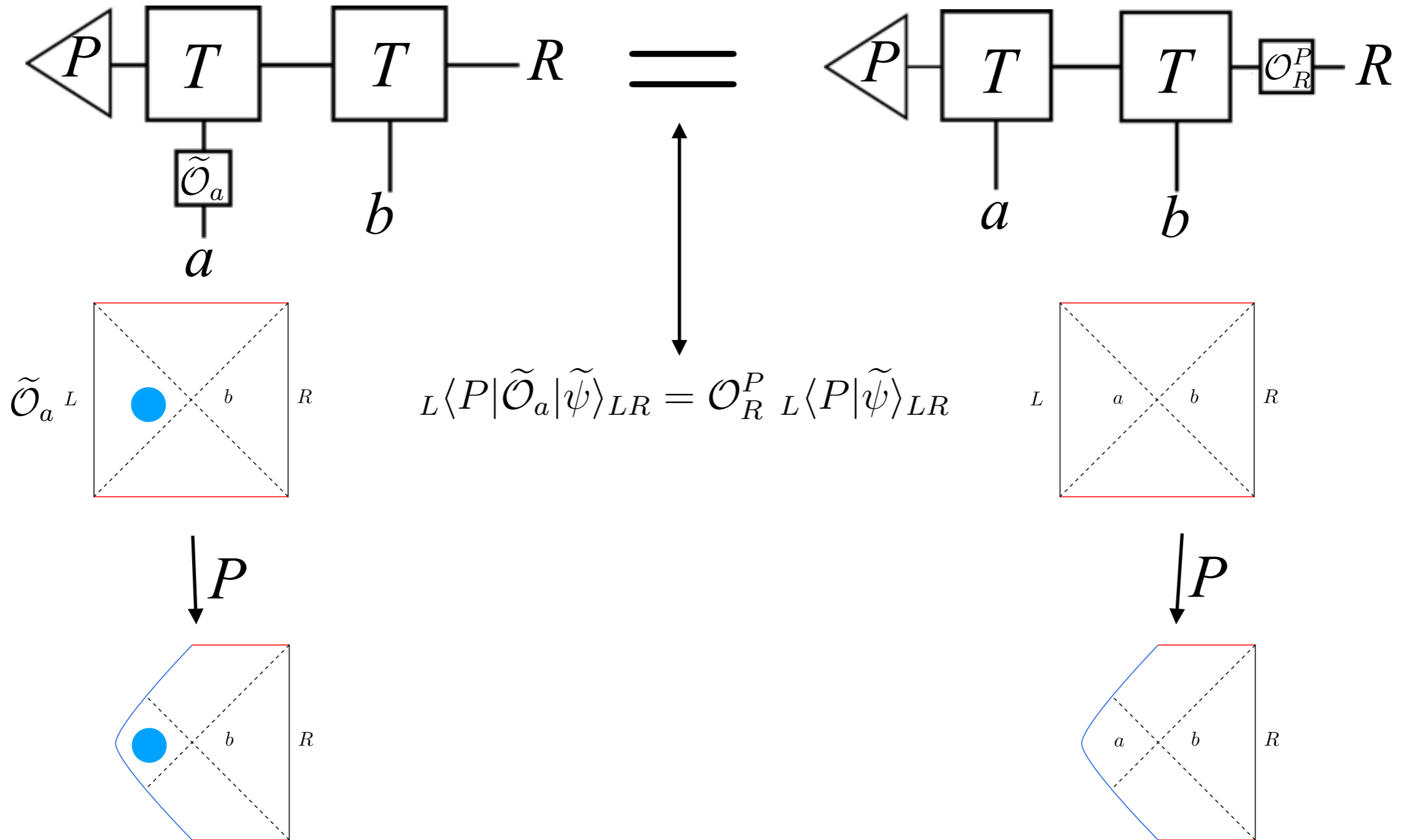
$\downarrow P$



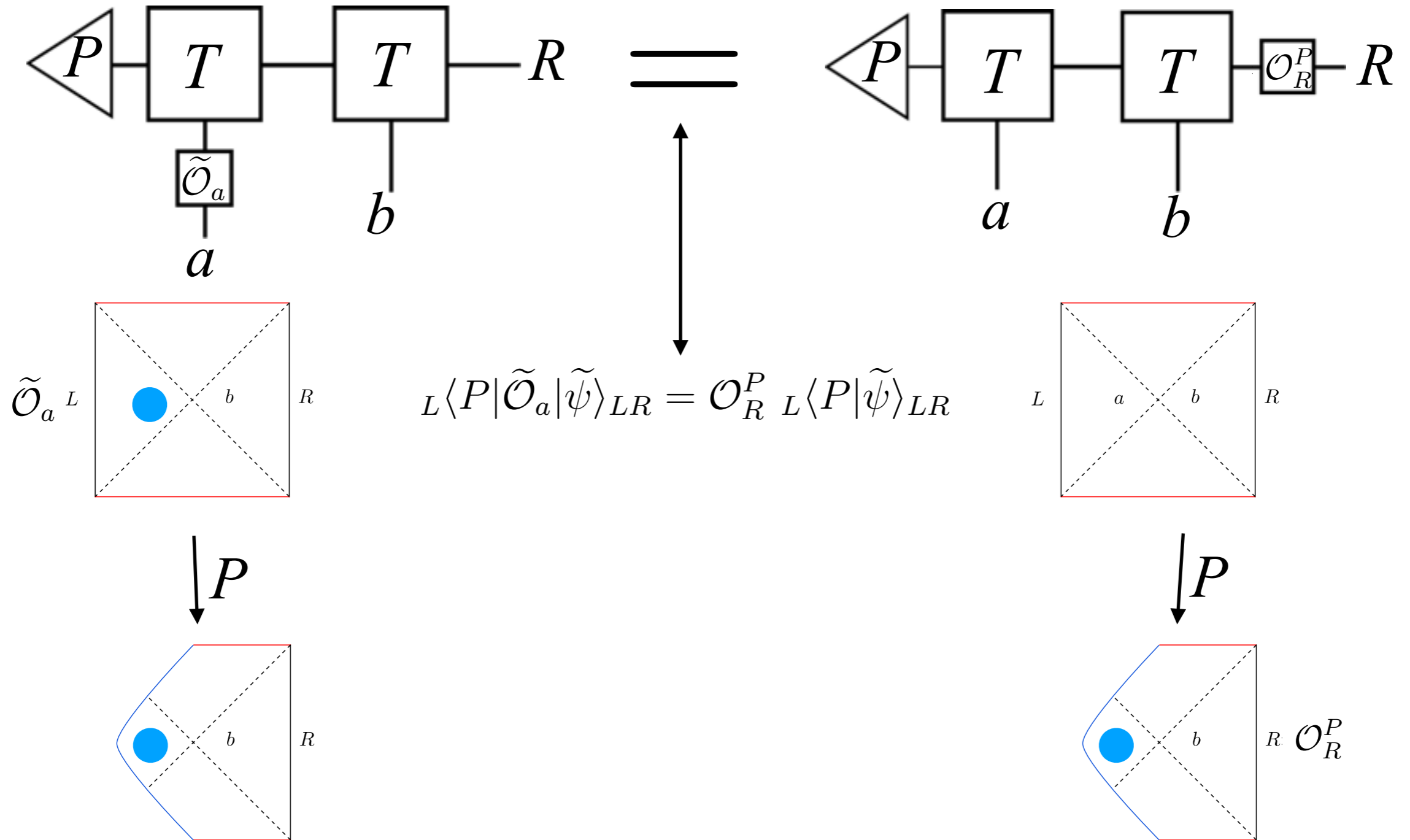
Condition on the Projector



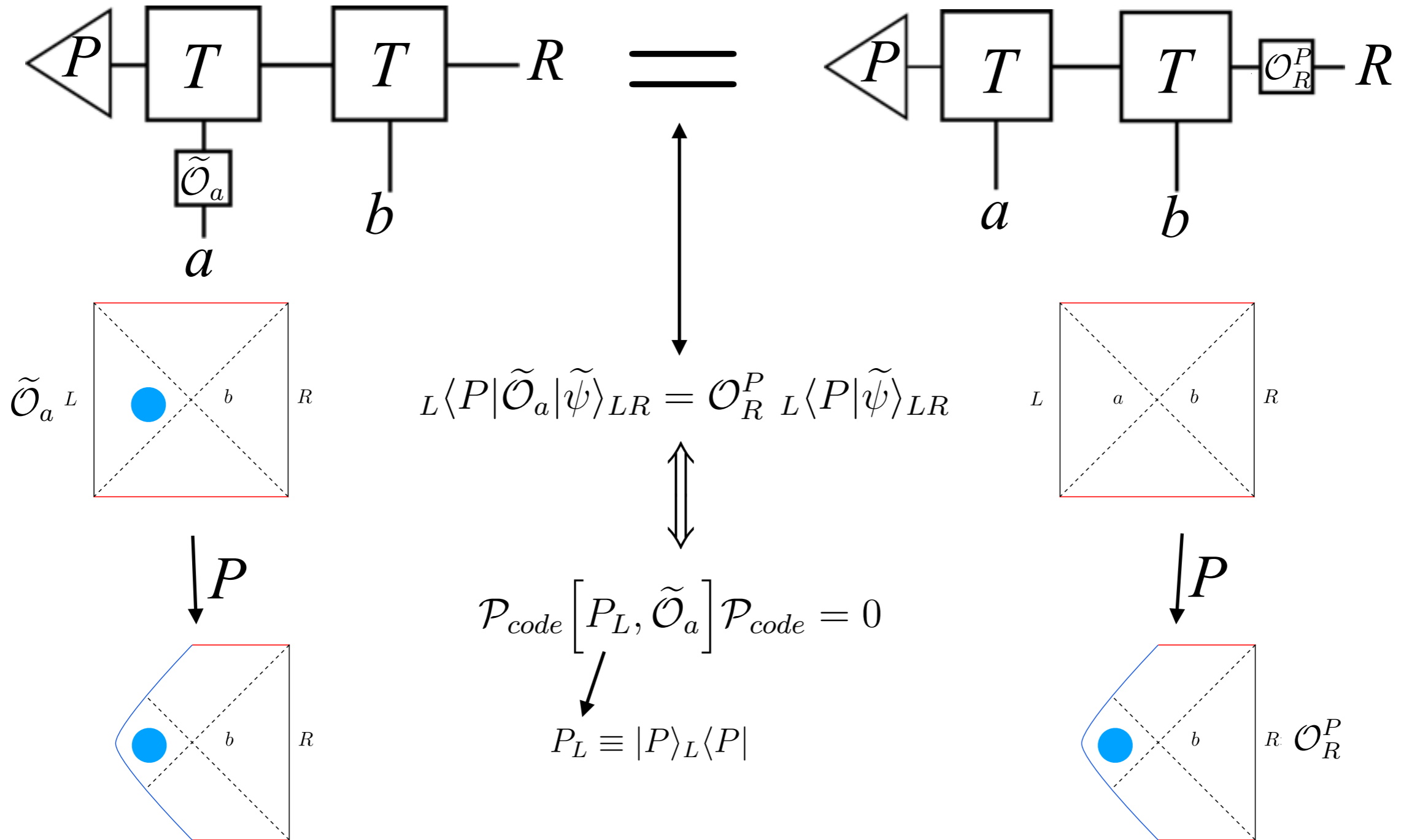
Condition on the Projector



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Condition on the Projector



Properties of the Dictionary

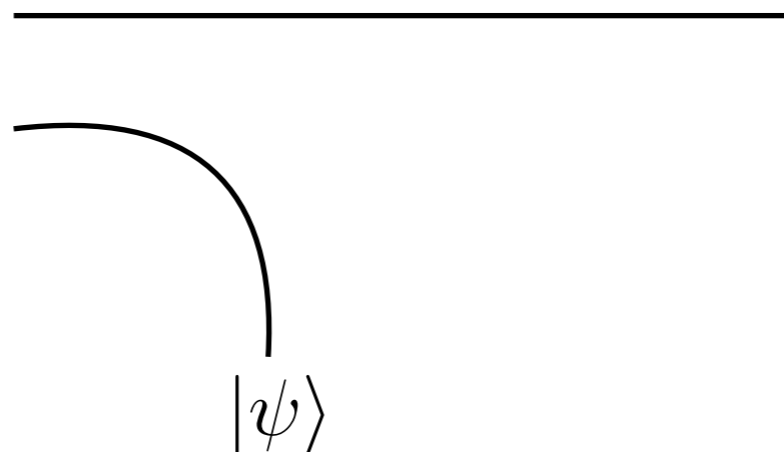
$${}_L\langle P|\tilde{\mathcal{O}}_a|\tilde{\psi}\rangle_{LR} = \mathcal{O}_R^P {}_L\langle P|\tilde{\psi}\rangle_{LR}$$

- Operators \mathcal{O}_R^P are a dictionary for the interior.
- These are state dependent!

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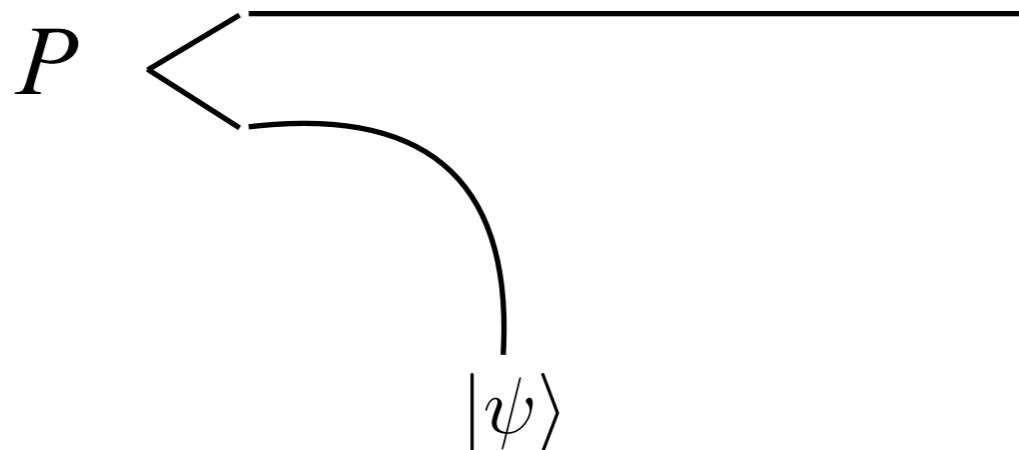
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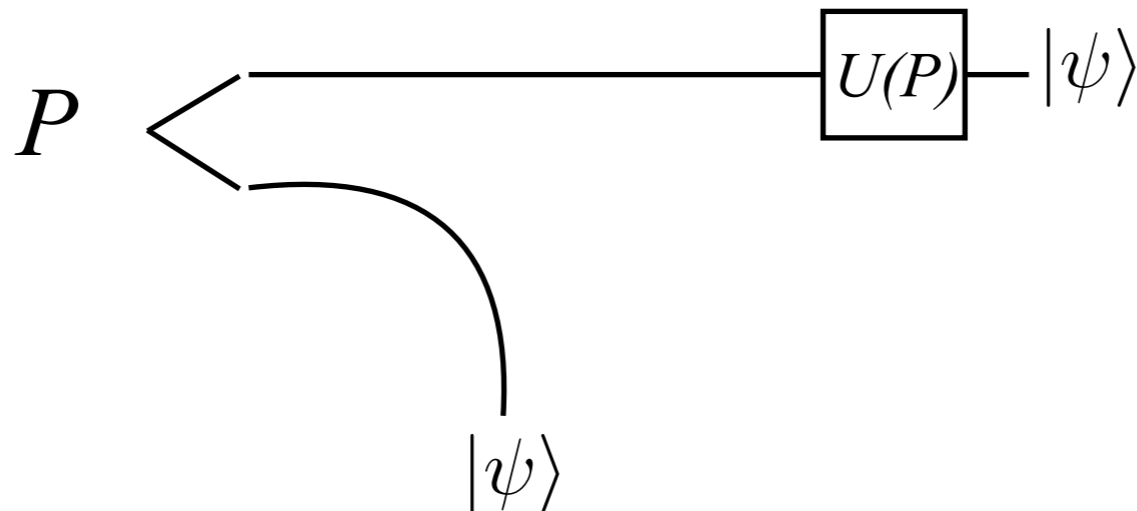
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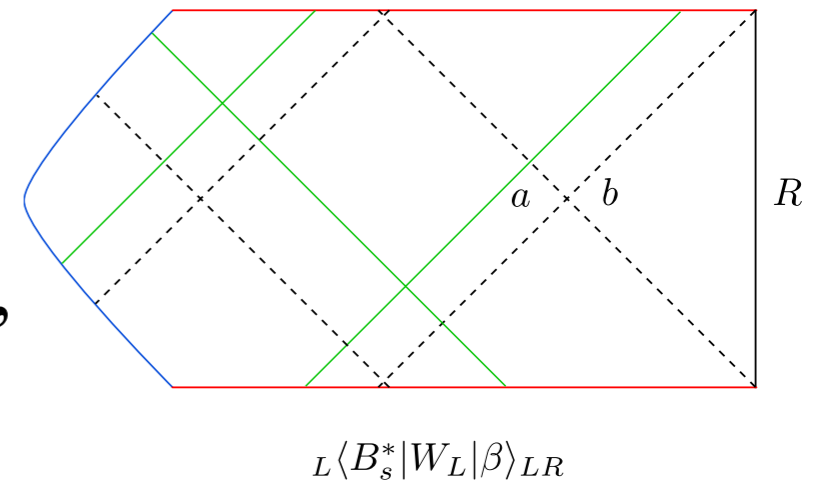
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Relation to PR

- Problem with PR: Consider Typical Microstates
- All simple operators have thermalized
- Mirror construction would predict ‘nothing’ behind the horizon.

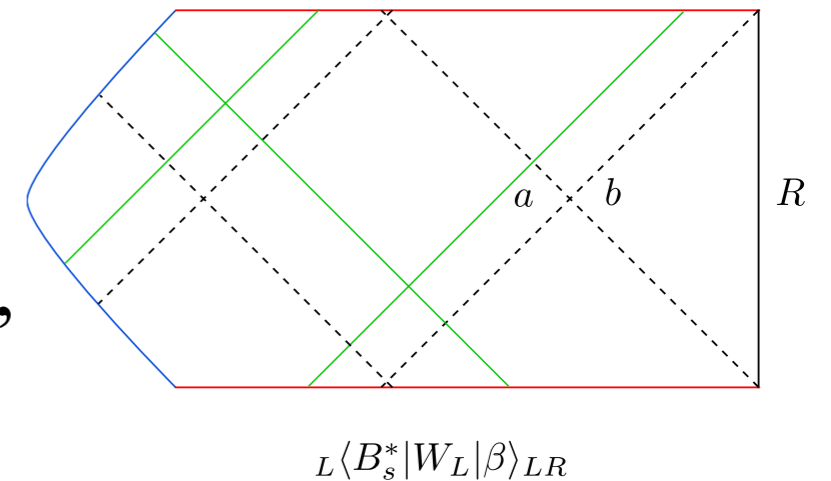


Relation to PR

- Problem with PR: Consider Typical Microstates

- All simple operators have thermalized

- Mirror construction would predict ‘nothing’ behind the horizon.



- The QEC construction is not a statement about typical states, but states constructed in a special way.
- It IS sensitive to what happens behind the horizon.

Thank You!

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