Flat Entanglement Spectra in Fixed-Area States of Quantum Gravity

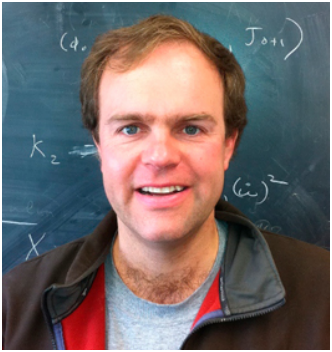
Xi Dong

UC SANTA BARBARA

December 12, 2018

Order from Chaos, KITP

This talk is based on recent work with Daniel Harlow and Don Marolf [arXiv:1811.05382]:



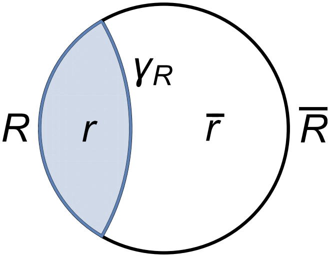
Plan

- Review:
  - Ryu-Takayanagi (RT) formula
  - Holography and quantum error correction (QEC)
  - Tensor network models of holographic codes
- A discrepancy in entanglement structure between tensor networks and holography
- Fixed-area states in gravity have flat entanglement spectrum
- Quantum error correction interpretation
- Strengthened JLMS formula and implications for bulk reconstruction

Ryu-Takayanagi Formula

The von Neumann entropy of a boundary spatial subregion R in any semiclassical state ρ:

$$S(\rho_R) = \frac{A[\gamma_R]}{4G}$$



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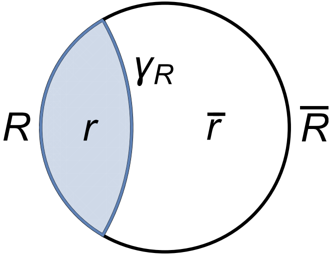

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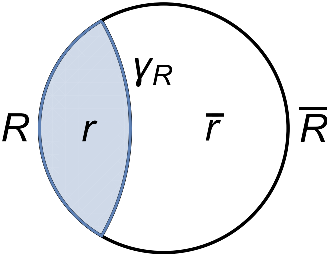

- γ_R is the Hubeny-Rangamani-Takayanagi (HRT) surface: the (minimal) extremal surface homologous to R.
- Works at leading order in the semiclassical expansion in G.

Quantum RT formula

At next order in G, the RT formula receives quantum corrections from bulk fields: [Faulkner, Lewkowycz, Maldacena]

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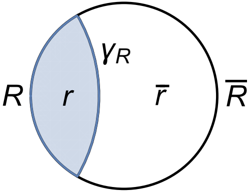

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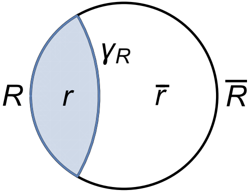

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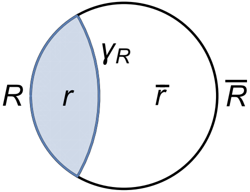

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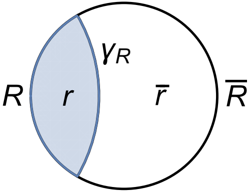

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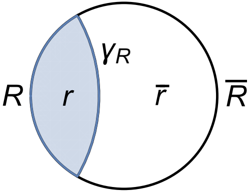

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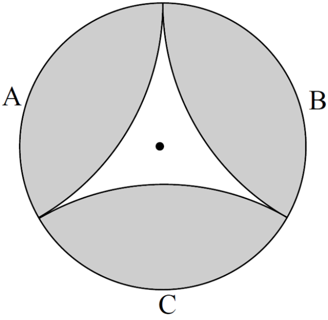
- At all orders in G, promote γ_R to "quantum extremal surface" [Engelhardt, Wall; XD, Lewkowycz].

AdS/CFT = Quantum Error Correction

Bulk operators can be reconstructed on different boundary subregions

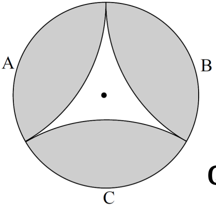
=

Protected quantum information can be recovered in different ways after partial erasures in a quantum error-correcting code



[Almheiri, XD, Harlow]

Dictionary between AdS/CFT and QEC



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- Semiclassical bulk states

Quantum Error Correction

- States in the code subspace

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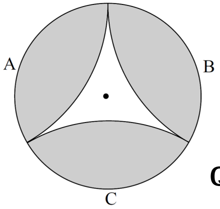

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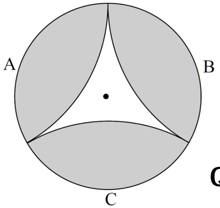

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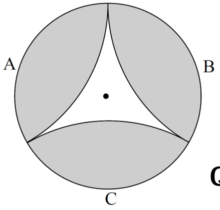

AdS/CFT

Quantum Error Correction

AdS/CFT	Quantum Error Correction
Semiclassical bulk states	States in the code subspace
Different CFT representations of a bulk operator	Redundant implementation of the same logical operation
Algebra of bulk operators	Algebra of protected operators acting on the code subspace
Radial distance	Level of protection

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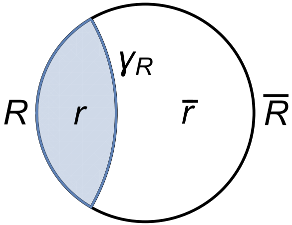

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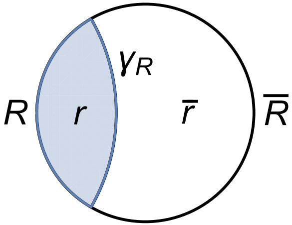

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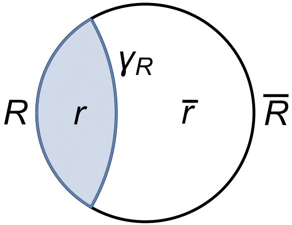

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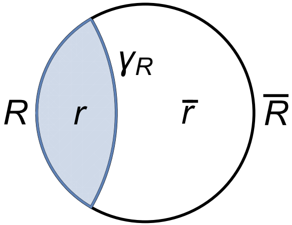
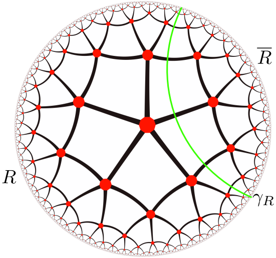


- Subalgebra: bulk operators in the entanglement wedge of R are precisely those recoverable on R.
- Complementary recovery: bulk operators commuting with those recoverable on R can be recovered on $\bar{R}$.
- Derived from the quantum RT formula. [XD, Harlow, Wall; Jafferis, Lewkowycz, Maldacena, Suh]

Networks made from perfect tensors:
(or random tensors)

[Pastawski, Yoshida, Harlow, Preskill; Hayden, Nezami, Qi, Thomas, Walter, Yang]



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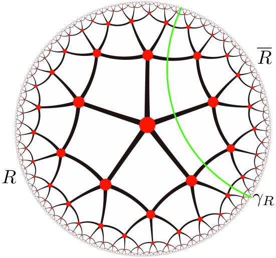

- They are subalgebra codes with complementary recovery – in fact, special ones where the code subspace factorizes:

$$\mathcal{H}_{code} = \mathcal{H}_r \otimes \mathcal{H}_{\overline{r}}$$

and the subalgebra recoverable on R simply consists of all operators acting on the subsystem r of the code subspace.

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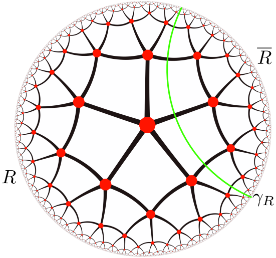

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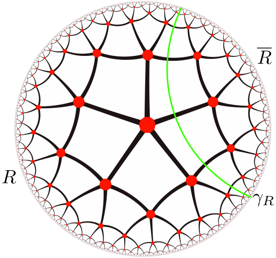

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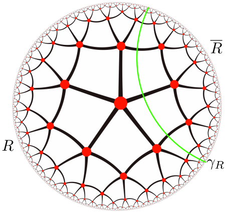
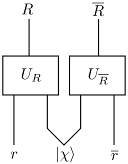
  and the subalgebra recoverable on R simply consists of all operators acting on the subsystem r of the code subspace.
- Tensor network with edge modes [Donnelly, Michel, Marolf, Wien] does not obey this factorization. We will return to it later.
- Other tensor networks (such as MERA) are useful for different purposes. We focus on the ones above because they are nice codes.

Interestingly, these tensor networks also satisfy the quantum RT formula:

$$S(\rho_R) = \frac{\langle A[\gamma_R] \rangle}{4G} + S(\rho_r)$$





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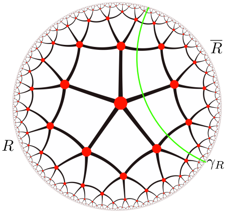

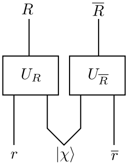

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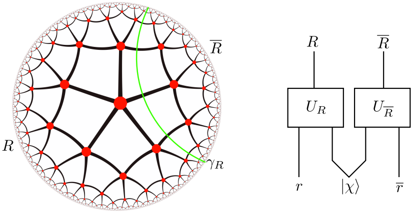

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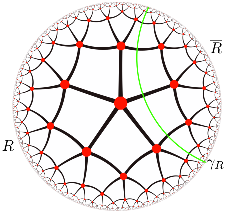

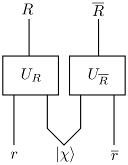

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$$S(\rho_R) = S(\chi_R) + S(\rho_r)$$

- χ_R: restriction of $|\chi\rangle$ to R.
- $S(\chi_R)$: proportional to the number of links cut by γ_R. Area term in the quantum RT formula!

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Any subalgebra code with complementary recovery satisfies a "quantum RT formula". [Harlow]

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Should we congratulate ourselves? Not yet...

A discrepancy in entanglement structure

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- Useful way of computing von Neumann entropy by taking $n \to 1$.
- Interesting on their own: n-dependence probes much more information about ρ, in principle allowing to extract the entanglement spectrum.

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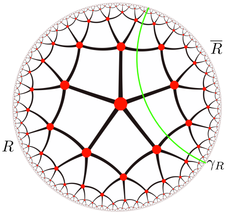
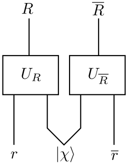
$$S_n(\rho_R) = S_n(\chi_R) + S_n(\rho_r)$$





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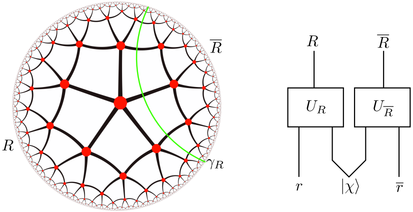

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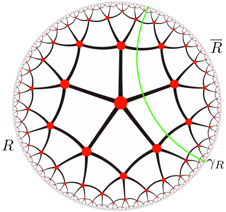

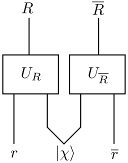

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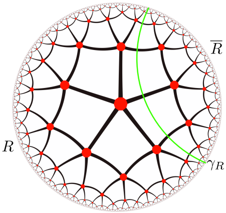

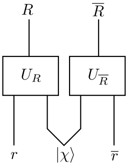

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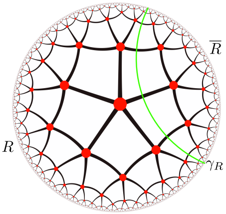

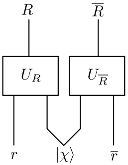

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- But this is not what gravity predicts!

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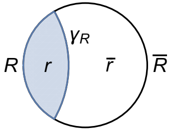
- $\gamma_{R,n}$: a cosmic brane replacing the HRT surface.
- Has tension $(n-1)/(4nG)$ and backreacts on the bulk geometry.
- $\widetilde{S}_n(\rho_R)$ depends on n nontrivially, and so does $S_n(\rho_R)$.



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- A general semiclassical state is a superposition of many fixed-area states, and its Renyi entropy is determined by integrating over area.
- For given n, the integral is dominated by a single value of the area, but this value changes with n, reflecting the nontrivial backreaction.

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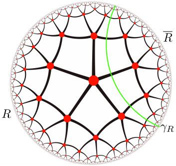
Now, let us define fixed-area states precisely.

- First, recall that in tensor networks:

$$\mathcal{H}_{code} = \mathcal{H}_r \otimes \mathcal{H}_{\bar{r}}$$



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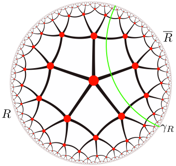
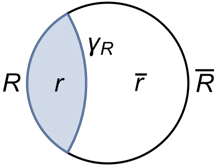
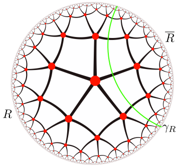


- In AdS/CFT, $\mathcal{H}_{code}$ consists of semiclassical bulk states, and r, $\bar{r}$ are the entanglement wedges of R, $\overline{R}$.





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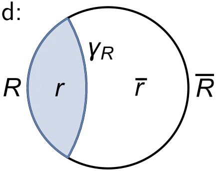

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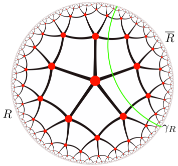

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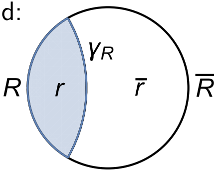

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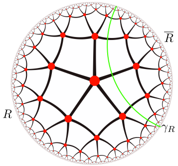
- More precisely, α labels degrees of freedom in the shared "center" of two von Neumann algebras that are commutants of each other and consist of operators acting on r, $\bar{r}$ respectively.



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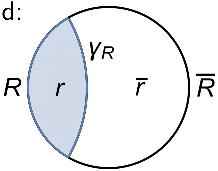

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- More precisely, α labels degrees of freedom in the shared "center" of two von Neumann algebras that are commutants of each other and consist of operators acting on r, $\overline{r}$ respectively.
- In Maxwell theory, α can be chosen as the gauge potential on the shared boundary (modulo gauge transformations).

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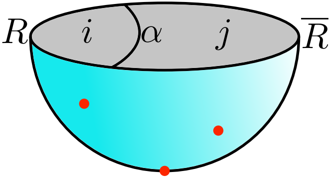

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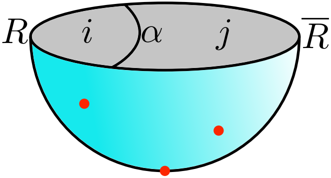

- Define a fixed-area state $|\psi_{\hat{A}}\rangle$ by the same bulk path integral but only configurations where the area of γ_R is $\hat{A}$ are integrated over.

- The norm of such a fixed-area state is calculated semiclassically by a saddle-point geometry g_1 with a conical defect on γ_R. The area of γ_R is fixed, so g_1 is not required to satisfy the EOMs there.

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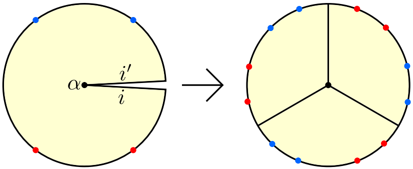

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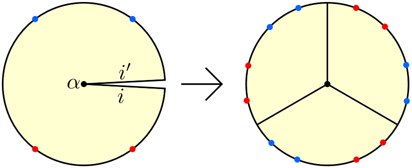

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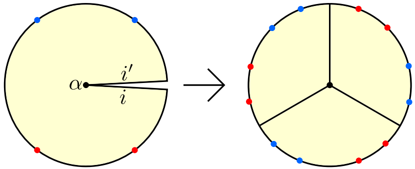

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- This is reminiscent of the lack of backreaction in tensor networks.

Flat entanglement spectrum

- Not surprisingly, it can be shown that Renyi entropy in a fixed-area state $|\psi_{\hat{A}}\rangle$ does not depend on n:

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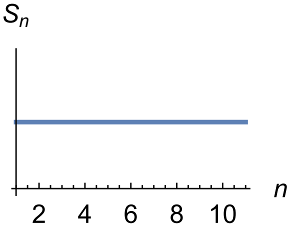

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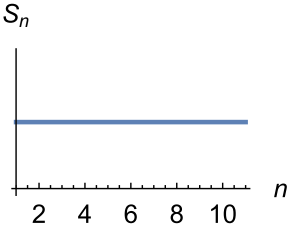

- A quick way to see this: the refined Renyi entropy $\widetilde{S}_n(\rho_R)$ is given by the cosmic brane area. Since the area is fixed, it is independent of n. The Renyi entropy is obtained by an integral

$$S_n(\rho_R) = \frac{n}{n-1} \int_1^n \frac{\widetilde{S}_{n'}(\rho_R)}{n'^2} dn'$$

and therefore also n-independent.

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- Actually, a stronger statement exists: in a certain sense, the flatness result applies to next order in G as well. We will come back to this later.

Origin of n-dependence in general semiclassical states

- We started with a general semiclassical state $|\psi\rangle$ prepared by a bulk path integral, projected it to a fixed area to obtain a new state $|\psi_{\hat{A}}\rangle$, and showed that the Renyi entropy in $|\psi_{\hat{A}}\rangle$ is independent of n.

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- This final integral is dominated by a saddle point A_n that depends on n. This is the origin of n-dependence in holographic Renyi entropy!

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- Gravity is automatically equipped with diffeomorphism invariance and a nontrivial center. A general gravitational state spans many superselection sectors, and different sectors dominate Renyi entropies at different values of n.
- We should build and study better tensor network models of holographic codes by adding a nontrivial center!

Quantum error correction interpretation

- Consider a general subalgebra code with complementary recovery.
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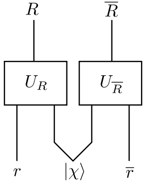

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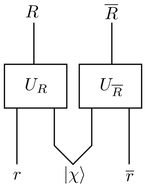

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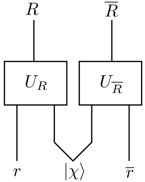

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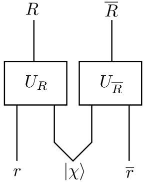

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- This property, together with the existence of a nontrivial center, provides a new condition for quantum error-correcting codes to be truly holographic!
- As in bulk gravity, a general state in $\mathcal{H}_{code}$ spans multiple α sectors and can therefore have n-dependent Renyi entropy.

Strengthened JLMS formula

- Any subalgebra code satisfying the quantum RT formula (or equivalently complementary recovery), including AdS/CFT, obeys a version of the Jafferis-Lewkowycz-Maldacena-Suh (JLMS) formula:

$$P_c K_R P_c = \frac{A[\gamma_R]}{4G} + K_r$$

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- This strengthened formula can be exponentiated to equate bulk and boundary modular flows, which was essential in an explicit form of entanglement wedge reconstruction [Faulkner, Lewkowycz].
- Q: what does this have to do with flat entanglement spectra?
- A: the strengthened formula holds if $|\chi\rangle$ has a flat entanglement spectrum within each superselection sector, through order G^0.

Summary

- Tensor network models of holographic codes satisfy the RT formula, but they do it "too well" and obey RT even for Renyi entropy. The resulting Renyi entropy is independent of n, in contradiction with the prediction of AdS/CFT.

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- We interpreted all of this as a new condition for quantum error-correcting codes to be truly holographic.
- In particular, the state $|\chi\rangle$ must have a flat entanglement spectrum within each superselection sector.
- This turns out to be precisely what we need for a strengthened JLMS formula and one form of entanglement wedge reconstruction.

Questions

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3. What other surprises are there for us in the realm of quantum gravity and quantum information?