Coarse-Graining = Area Laws

Netta Engelhardt

Princeton University

Based on 1805.08891 w/ S. Fischetti, 1806.01281 & 1706.02038 w/ A. Wall, and work in progress

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- This requires a microscropic description of the d.o.f. of QG.
- So AdS/CFT is our best bet.

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3. Can try to use CFT intuition "what is a useful measure of irreversibility of coarse-graining?"
4. Of course, QG might be playing mind games with us, and intuition would be completely useless...

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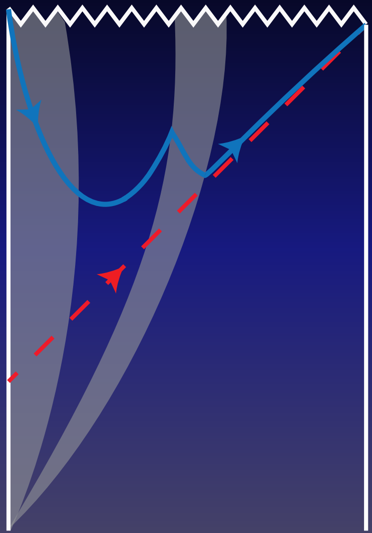
II. Boundary-Motivated Coarse-Graining (w/ Fischetti)

III. Conclusions & Advertisement

Intuition from Area Laws



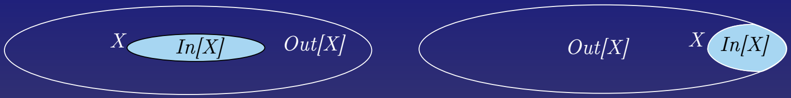
- Hawking area theorem Hawking '71: area of the event horizon is non-decreasing (null energy condition)

- Holographic screen area theorem Hayward '93, Ashtekar-Krishnan '05, Bousso-NE '15: area of holographic screen (generalization of apparent horizon) is non-decreasing

Intuition: Growing Ignorance

Basic Intuition

Area increase theorems correspond to growing ignorance about the interior of a spacetime region, provided we know everything about its complement.



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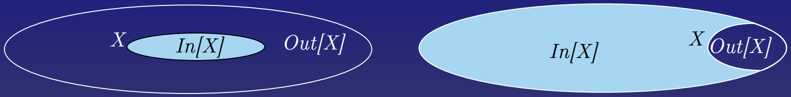

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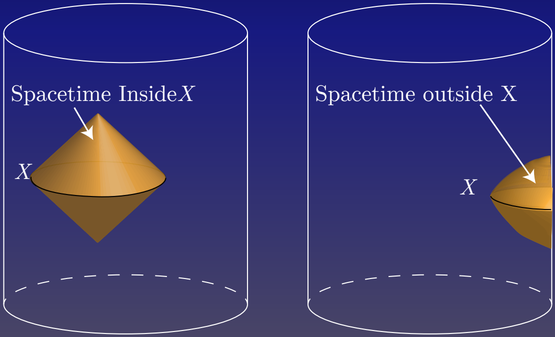

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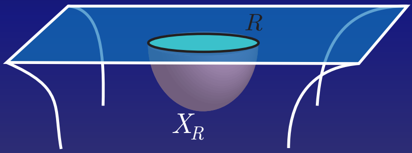
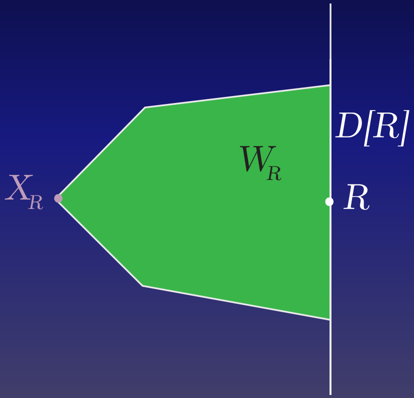
$$S_{vN}[\rho_R] = \frac{\text{Area}[X_R]}{4G\hbar} + \cdots$$





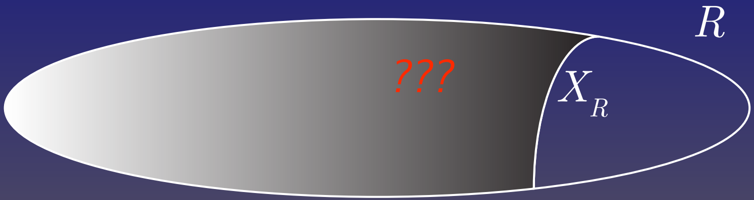
Knowing ρ_R, we can reconstruct W_R. [Van Raamsdonk; Czech et al.; Wall; Hubeny et al.…; proved by Dong, Harlow, Wall; explicit construction by Faulkner, Lewkowycz]

HRT: A Precise Realization

Let X_R be the HRT surface of some boundary region R in some fixed global state ρ. Then

$$\frac{\text{Area}[X_R]}{4G\hbar} = \max_{\mathcal{H}} S[R]$$

where $\mathcal{H}$ is the space of all ρ whose (semiclassical) dual contains $W_E[R]$.



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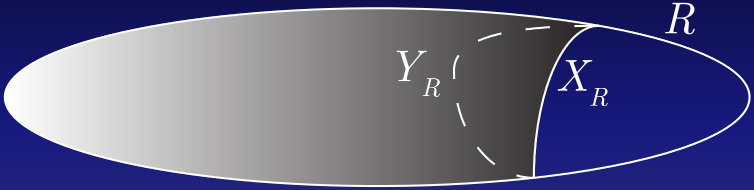


$$\text{Area}[X_R] > \text{Area}[Y_R]$$

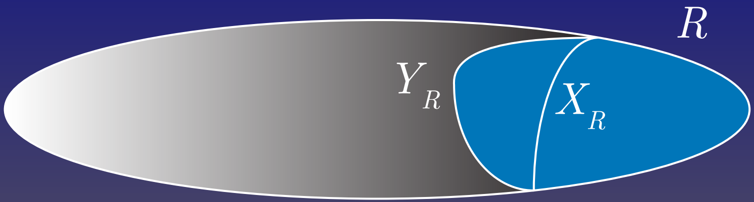
Replacing the region behind X_R allows reconstruction of a larger spacetime volume (corresponding to reducing the entanglement between R and $\overline{R}$). Case where X_R is compact and $Y_R = \varnothing$ is well-studied, see literature starting with

[Kourkoulou, Maldacena]

Beyond HRT: Non-minimal extremal

Immediate Consequence

If Y_R is the HRT surface, and $X_R \subset In[Y_R]$ is a non-minimal extremal surface, then by modifying $Out[X_R]$, we can construct a state in which X_R **is** an HRT surface. The result is a "coarse-grained" spacetime, in which the entanglement wedge contains less information.



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No.

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No. It is enough that a surface X_R be *minimar*:

If X_R satisfies the following:

1. It is the minimal area surface on a slice of its exterior
2. It is "marginal" towards its exterior: variations of X_R along the outwards-null direction do not change its area to first order.
3. bunch of technical assumptions

then

$$\frac{\text{Area}[X_R]}{4G\hbar} = \max_{\mathcal{H}} S[R]$$

Beyond HRT: Non-extremal marginal

For minimar surfaces:

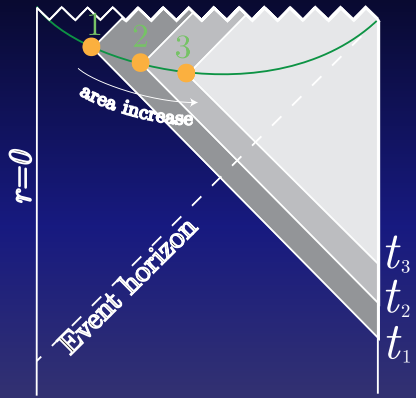
$$\frac{\text{Area}[X_R]}{4G\hbar} = \max_{\mathcal{H}} S_{vN}[R]$$

where $\mathcal{H}$ is the set of all semiclassical states containing $\text{Out}[X_R]$.

Example: apparent horizons are minimar.

Coarse-Graining=Area Law

Compact case:



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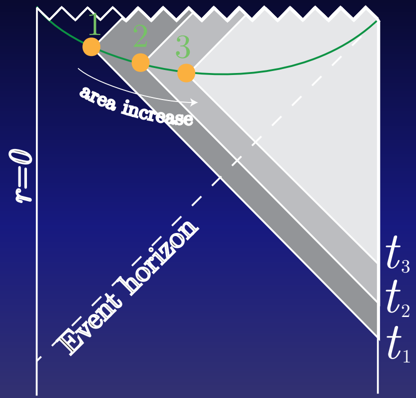

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Open Q: mixed signature law cannot be explained this way

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Hawking area law applies to surfaces whose area *increases* under variations in the outwards null direction.

No Area-Coarse-Graining Interpretation

If the area of X_R is increasing along both outwards null deformations, then:

$$\frac{\text{Area}[X_R]}{4G\hbar} > \max_{\mathcal{H}} S_{vN}[R]$$

Conversely, for certain surfaces X_R where the area is decreasing along both future null deformations, then:

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This intuition isn't sufficient.

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$$I[A:BC] \geq I[A:B]$$

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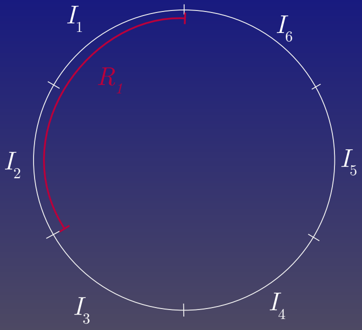
Natural quantity to look for: object defined by a subdivision of boundary into subregions, which is monotonic under SSA as we shrink those boundary regions.

Differential Entropy Review

Differential entropy of Balasubramanian et al.; Headrick et al...:

$$S^{disc}_{diff}[\{R_i\}^n_{=1}] = \sum_{i=1}^{n} (S[R_i] - S[R_i \cap R_{i+1}])$$

where $R_{n+1} = R_1$. (technically cheating: discrete version here for conceptual simplicity)



Monotonicity Under Coarse-Graining

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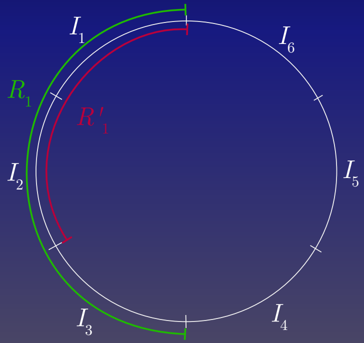
$$S_{diff}[\{R_i\}] = \sum S[I_i I_{i+1} I_{i+2}] - S[I_{i+1} I_{i+2}]$$

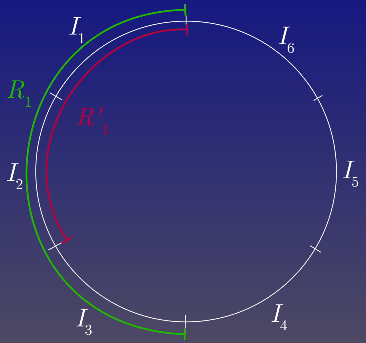
$$S_{diff}[\{R'_i\}] = \sum S[I_i I_{i+1}] - S[I_{i+1}]$$



Monotonicity Under SSA

How does S_{diff} behave under progressive coarse-graining?
When we shrink $R_i \to R_i'$:

$$S_{diff}[\{R_i'\}] - S_{diff}[\{R_i\}] = \sum S[I_i I_{i+1}] + S[I_i I_{i+1} I_{i+2}] - S[I_{i+1}] - S[I_{i+1} I_{i+2}] \overset{\text{SSA}}{\geq} 0$$



Now for the Bulk

Differential Entropy and Area (Balasubramnian et al; Headrick et al)

In 3 bulk dimensions, there exists a bulk surface σ constructed from the entanglement wedges of the R_α such that

$$S_{diff}[\{R_\alpha\}] = \frac{\text{Area}[\sigma]}{4G\hbar}$$

This means that coarse-graining under SSA via shrinking the regions implies:

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where r is a parameter indexing the continuous shrinking of the boundary regions. **Bulk consequence is an area monotonicity theorem.**

For Poincaré invariant states, this is simply the c theorem.

Upshot

Area Law from SSA

The process of removing long-range entanglement in the boundary theory (effectively coarse-graining over a set of constraints on the state) maps precisely to an area law in the bulk.

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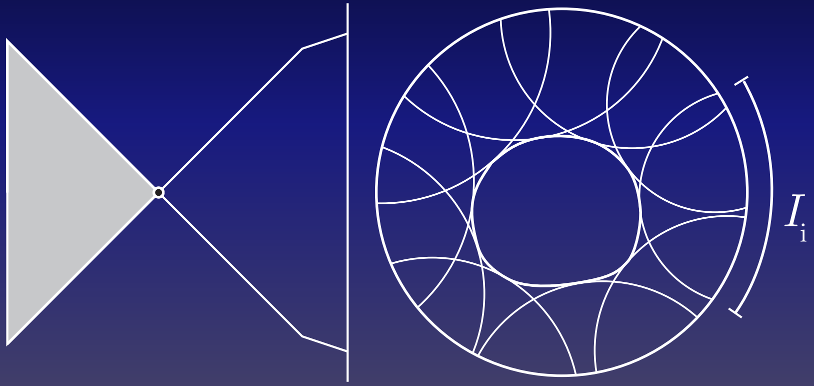
What does this area law look like?

Area Law: Surfaces

First, which area does the differential entropy compute?
Derived in broad generality by Headrick, Myers, Wien



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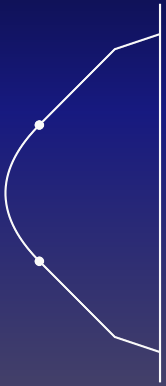

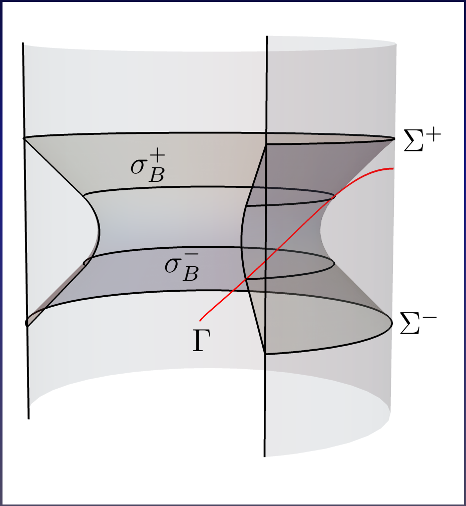

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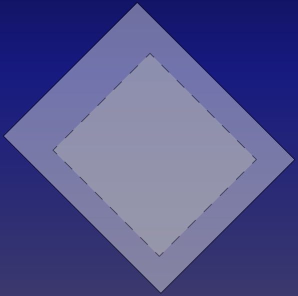

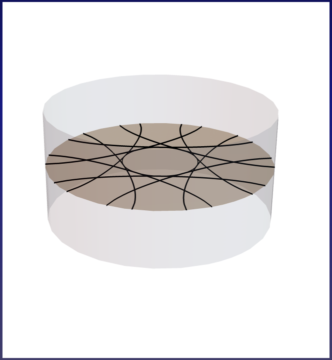

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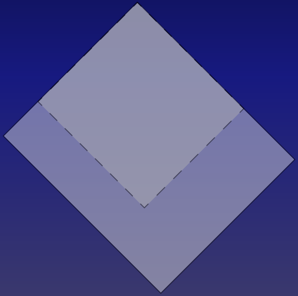
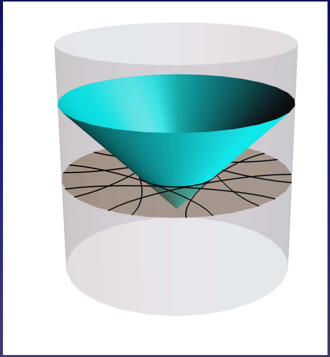
2. Null Law:





Area Laws: Hypersurfaces

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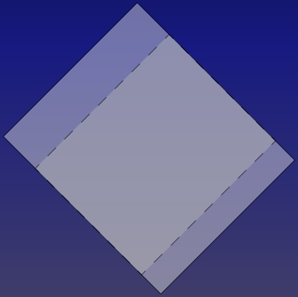
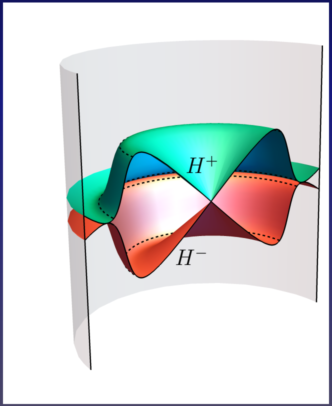
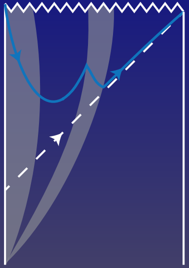
3. Mixed-Signature Law:

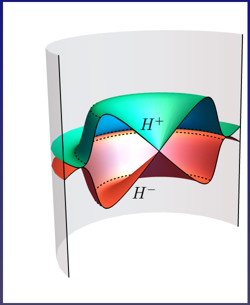




Connection to Black Hole Laws

The hypersurfaces we recover are not the same (although for certain far out of equilibrium black holes, we *do* get the Hawking law), but this is the first entropic explanation for dynamical causal horizons and mixed-signature hypersurfaces.





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- The gravitationally-motivated coarse-graining is clearer, but does not account for area laws of general, mixed signature

- The CFT-motivated coarse-graining is a bit mysterious in that while the coarse-graining procedure is clear, it is not obvious *what* ignorance is increasing. However, this can accommodate arbitrary signature area laws.

Questions

- Dual of S_{diff} for understanding the coarse-graining when the area law isn't simply the c theorem (possibly constrained quantum state merging? Czech et al.)
- Higher-dimensional case? Can we "guess" at the analogue of differential entropy in other dimensions based on a- and F-theorems?
- Universality of coarse-grainings: which ones are "good" and which ones are meaningless (for semiclassical gravity)?
- Quantum Corrections

On the subject of quantum corrections...

Area laws require classical focusing, which can be violated by quantum corrections. The quantum corrected version (Generalized Second Law) follows from the quantum focusing **conjecture** [Bousso, Fisher, Leichenauer, Wall].

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by quantum co... ersion
(Generalized Sec... n
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