Spectral Properties of the SYK Model

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KITP, December 2018

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Acknowledgments

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References

J.J.M. Verbaarschot and M.R. Zirnbauer, Replica Variables, Loop Expansion and Spectral Rigidity of Random Matrix Ensembles, Ann. Phys. **158** , 78 (1984)

A. Garcia-Garcia and J.J.M. Verbaarschot, Spectral and Thermodynamical Properties of the SYK model, Phys. Rev. **D94** (2016) 126010 [arxiv:1610.03816].

A. Garcia-Garcia and J.J.M. Verbaarschot, Spectral and Thermodynamical Properties of the SYK model, Phys. Rev. **D96** (2017) 066012 [arxiv:1701.06593].

A. Garcia-Garcia, Y. Jia and J.J.M. Verbaarschot, Exact moments of the Sachdev-Ye-Kitaev model up to order $1/N^2$, JHEP (2018) (in press) [arxiv:1801].

A. Garcia-Garcia, Y. Jia and J.J.M. Verbaarschot, Universality and Thouless energy in the supersymmetric Sachdev-Ye-Kitaev Model, Phys. Rev. **D87** (2018) 106003 [arxiv:1801.01071].

Y. Jia and J.J.M. Verbaarschot, Free Energy and Moments of the Sachdev-Ye-Kitaev Model JHEP 1811 (2018) 031 [arXiv:1806.03271.

A. Garcia-Garcia, D. Rosa, T. Nosaka and J.J.M. Verbaarschot, On the Maldacena-Qi Model (2018).

A. Garcia-Garcia, Y. Jia and J.J.M. Verbaarschot, Long Range Spectral Correlations of the Sachdev-Ye-Kitaev Model (2018).

Contents

I. Motivation

II. The SYK model

III. Spectral Density of the SYK model

IV. Spectral Correlations

V. Spectral Correlations of the QCD Dirac Operator

VI. Conclusions

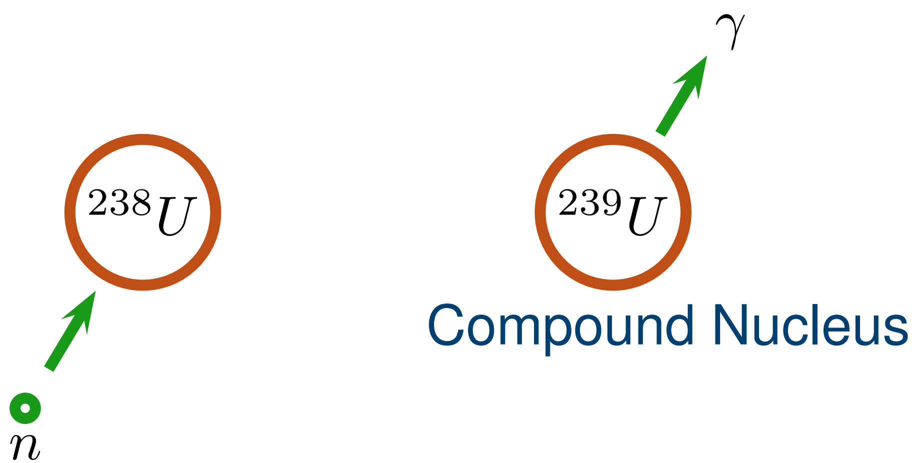
Compound Nuclei and the SYK Model

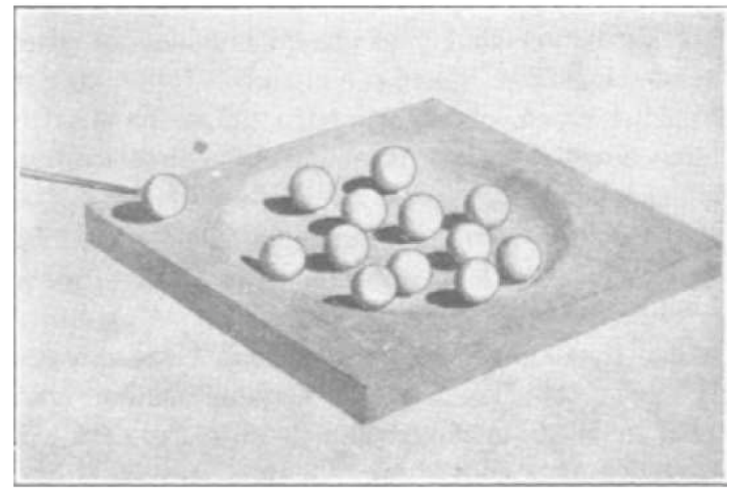
Compound Nucleus

Random Matrix Theory

The Complex SYK Model

Compound Nucleus





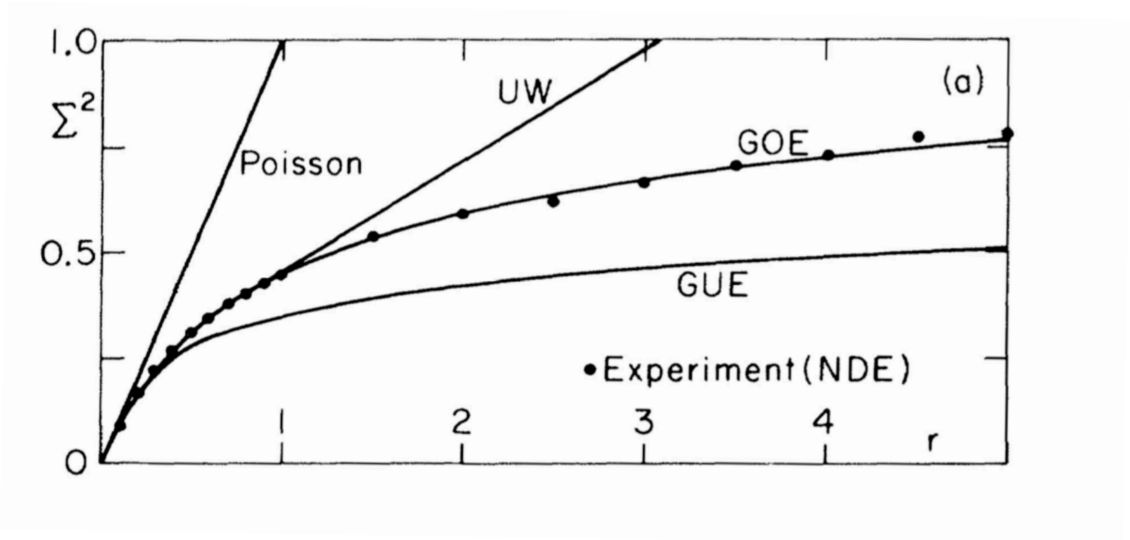
Bohr's Model

- A compound nucleus equilibrates and has to be chaotic. May saturate the quantum bound Maldacena-Shenker-Stanford-2016. Estimate for $T \sim 10\ MeV$ gives $2\pi kT/\hbar \sim 1/[3\text{fm}/c]$.

Basic properties of a Compound Nucleus

- Levels of a compound nucleus are correlated according to Random Matrix Theory.

Chaos and Nuclear Levels



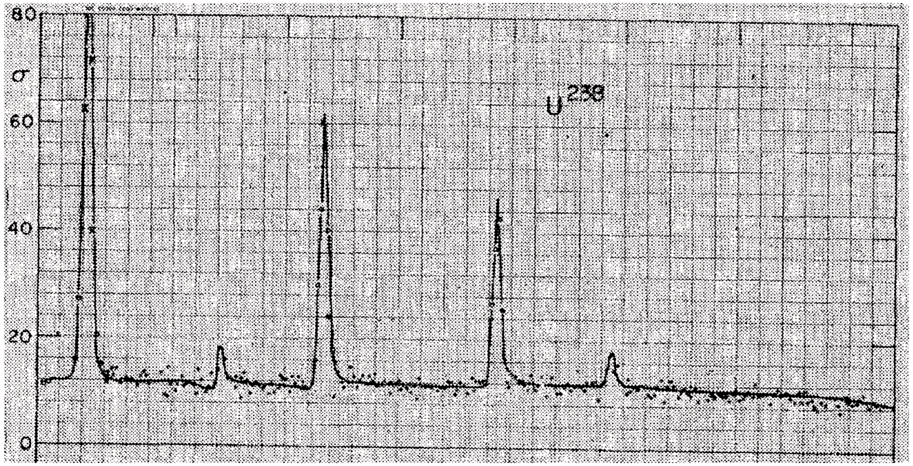
Number variance of nuclear levels for the the Nuclear Data Ensemble as a function of the average level spacing r .

Bohigas-Haq-Pandey, 1985

Basic properties of a Compound Nucleus

- Levels of a compound nucleus are correlated according to Random Matrix Theory.
- Because the system is chaotic, all information on its formation gets lost. Formation and decay of a compound nucleus are independent (Hauser-Feshbach formula).
- A compound nucleus has quantum hair.

Quantum Hair of a Compound Nucleus



Quantum hair of a compound nucleus – Total cross section versus energy (in eV).

Garg-Rainwater-Petersen-Havens,1964

Basic properties of a Compound Nucleus

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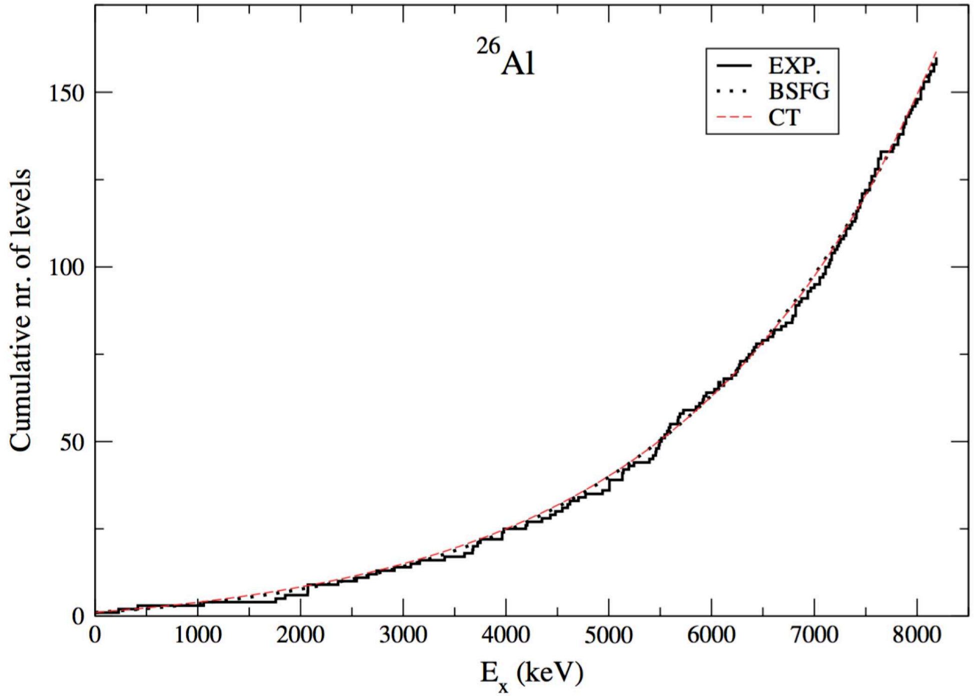
- Formation and decay of a compound nucleus are independent (Hauser-Feshbach formula).
- A compound nucleus has quantum hair.
- The level density of a compound nucleus behaves as $\rho(E) \sim \exp(c\sqrt{E - E_0})$.

Bethe Formula for the Nuclear Level Density

- The nuclear level density behaves as $e^{\alpha\sqrt{E-E_0}}$.



T. von Egidy

Basic properties of a Compound Nucleus

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- A compound nucleus has quantum hair.
- Level of a compound nucleus are correlated according to Random Matrix Theory.
- The level density of a compound nucleus behaves as $\rho(E) \sim \exp(c\sqrt{E - E_0})$.
- The compound nucleus cross-section is holographic.

Holography in Nuclear Physics

Because the compound nucleus is chaotic, the fluctuations of the S-matrix are universal so that the average cross-section, $\langle |S_{ab}|^2 \rangle$ only depends on the average diagonal S -matrix elements

$$\langle |S_{ab}|^2 \rangle = \delta_{ab} |\langle S_{ab} \rangle|^2 + \langle |S_{ab}^{\text{fluc}}|^2 \rangle = F_{\text{universal}}(\langle S_{cc} \rangle)$$

with $F_{\text{universal}}$ a universal function. JV-Weidenmüller-Zirnbauer-1983, Mello-Pereyra-Seligman-1984

The average diagonal S -matrix is obtained by an energy average over large intervals. Therefore the average diagonal S matrix is determined by the fast processes, in which a compound nucleus is not formed, i.e. the physics that takes place at the surface of the nucleus.

Basic properties of a Compound Nucleus

- Levels of a compound nucleus are correlated according to Random Matrix Theory.
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- The level density of a compound nucleus behaveS as $\rho(E) \sim \exp(c\sqrt{E - E_0})$.
- The compound nucleus cross section is holographic.
- The nuclear interaction is mostly a two-body interaction.

The Complex SYK Model

$$H = \sum_{\alpha\beta\gamma\delta} W_{\alpha\beta\gamma\delta} a^\dagger_\alpha a^\dagger_\beta a_\gamma a_\delta.$$

French-Wong-1970

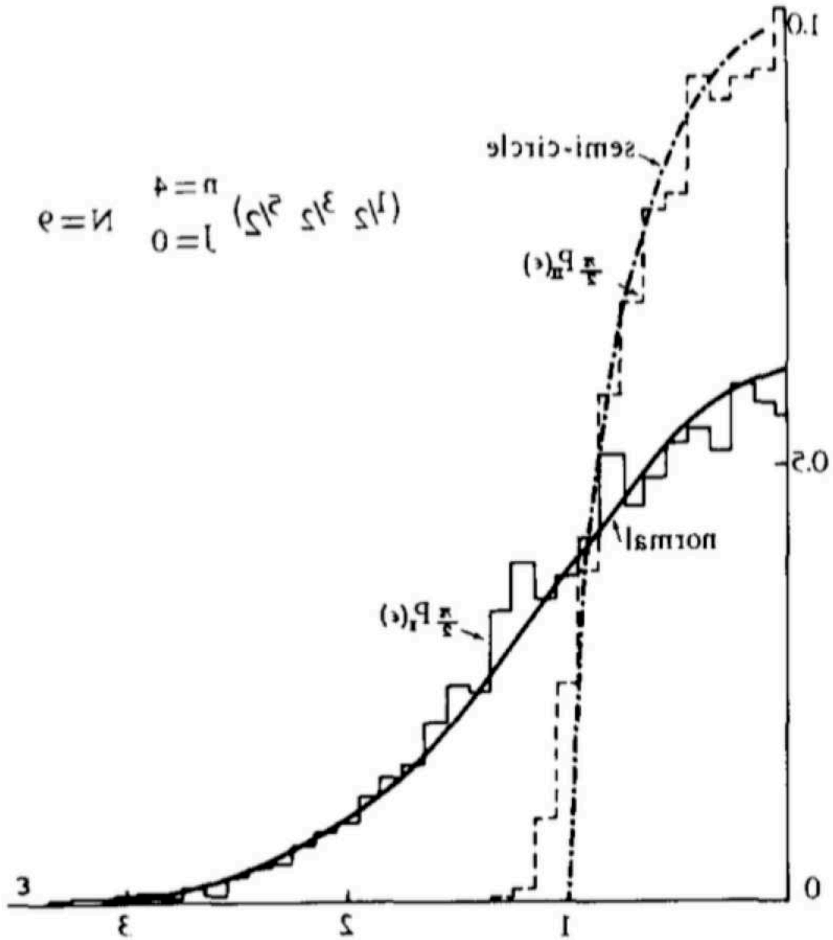
Bohigas-Flores-1971

The labels of the fermionic creation and annihilation operators run over N single particle states. The Hilbert space is given by all many particle states containing m particles with $m = 0, 1, \cdots, N$.

- $W_{\alpha\beta\gamma\delta}$ is Gaussian random.
- The Hamiltonian is particle number conserving.
- The matrix elements of the Hamiltonian are strongly correlated.

Brody-et-al-1981, Brown-Zelevinsky-Horoi-Frazier-1997, Izrailev-1990,Kota-2001,Benet-Weidenmüller-2002,Zelevinsky-Volya-2004, Borgonovi-Izraelev-Santos-Zelevinsky-2016

First Numerical Results



Comparison of the spectral density of the GOE and the two-body random ensemble for the sd-shell. Bohigas-Flores-1971

The Sachdev-Ye-Kitaev Model

The SYK Model

The Sachdev-Ye-Kitaev (SYK) Model

The two-body random ensemble from nuclear physics also has merged into the SYK model, where the fermion creation and annihilation operators are replaced by Majorana operators (in general q of them. For $q = 4$ the model is Sachdev-Ye-1993,Kitaev-2015

$$H = \sum_{\alpha<\beta<\gamma<\delta} W_{\alpha\beta\gamma\delta}\chi_\alpha\chi_\beta\chi_\gamma\chi_\delta, \qquad q = 4.$$

The fermion operators satisfy the commutation relations

$$\{\chi_\alpha, \chi_\beta\} = \frac{1}{2}\delta_{\alpha\beta}.$$

The two-body matrix elements are taken to be Gaussian distributed with variance that is chosen such that the ground state energy scales with N.

Solving the SYK Model

- For $q = 2$ the model can be solved by diagonalizing the two-body matrix elements.
- Moment method. Moments have been calculated to order $1/N^4$ for arbitrary q. Garcia-Garcia-Jia-JV-2018, Jia-JV-2018 Berkooz-Isachenkov-Narovlansky-Torrents-2018
- Formulating the model as a Feynman path integral. This makes it possible to take the large-N limit. Sachdev-Ye-1993,Kitaev-2015,Maldacena-Stanford-2016,Klebanov-Tarnopolsky
- Generating function $\langle \det(H+z) \rangle$, $\langle \det(H+z)/\det(H+z') \rangle$. JV-Zirnbauer-1983, Benet-Weidenmüller-2002,Altland-Bagrets-2017
- Representing the Majorana fermions as γ-matrices in N dimensions. This allows numerical diagonalization up to $N = 42$. Maldacena-Stanford-2015,Garcia-Garcia-JV-2016,Cotler-et-al-2016,Gur-Ari-Mahajan-Vaezi-2018

Scaling Limit of the Spectral Density of the SYK Model

- $N \gg q^2$: the eigenvalue density is point-wise a Gaussian.
  Mon-French-1971, Garcia-Garcia-JV-2016
- $q^2 \gg N$: the eigenvalue density is point-wise a semicirle.
  Mon-French-1971, Benet-Weidenmüller-2002, Liu-Nowak-Zahed-2017
- q^2/N fixed for $N \to \infty$: This is a nontrivial scaling limit where the spectral density converges to the weight function of the Q-Hermite polynomials. Cotler-etal-2016, Garcia-Garcia-JV-2017,Berkooz-Isachenkov-Narovlansky-Torrents-2018.

The Spectral Density of the SYK Model

Bethe Formula

Q-Hermite Approximation

Bethe Formula

The low-temperature limit of the partition function is given by

$$Z(\beta) \sim \frac{2^{N/2} e^{-\frac{\pi^2 N}{4q^2}}}{\beta^{3/2}} e^{c/(2\beta)}.$$

Georges-Parcolet-Sachdev-2000,Bagrets-Altland-Kamenev-2017,
Standford-Witten-2017

The level density is given by the Laplace transform of the partition function.

$$\rho(E) \quad = \quad \int_{r-i\infty}^{r+i\infty} d\beta e^{\beta E} Z(\beta) \sim \sinh(\sqrt{2c(E-E_0)}).$$

This gives the Bethe formula for the nuclear level density.

Bethe-1936

The Q-Hermite Approximation

In the large N limit, we have that ($H = \sum w_\alpha \Gamma_\alpha$)

$$[\Gamma_\alpha W_\alpha, \Gamma_\beta W_\beta] = 0 \quad \text{for} \quad \alpha \cup \beta = 0.$$

Therefore are Wick contractions can be commuted to neighboring Wick contractions. Using that $\Gamma_\alpha^2 = 1$ we obtain Mon-French-1975

$$\langle \text{Tr} H^{2p} \rangle = (2p-1)!! \langle \text{Tr} H^2 \rangle^p.$$

These are the moments of a Gaussian distribution. One actually can do better than this. Γ_α and Γ_β commute or anti-commute depending on the number p of indices they have in common Garcia-Garcia-JV-2016

$$\Gamma_\alpha W_\alpha \Gamma_\beta W_\beta + (-1)^{p+1} \Gamma_\beta W_\beta \Gamma_\alpha W_\alpha = 0.$$

The Q-Hermite approximation is if all such crossings are treated independently.

Spectral Density of the SYK Model

The Q-Hermite approximation to the spectral density is given by

Erdos-2014,Cotler-et-al-2016,Garcia-Garcia-JV-2017

$$\rho_{\rm QH}(E) = c_N \sqrt{1-(E/E_0)^2} \prod_{k=1}^{\infty} \left[1 - 4\frac{E^2}{E_0^2}\left(\frac{1}{2+\eta^k+\eta^{-k}}\right)\right]$$

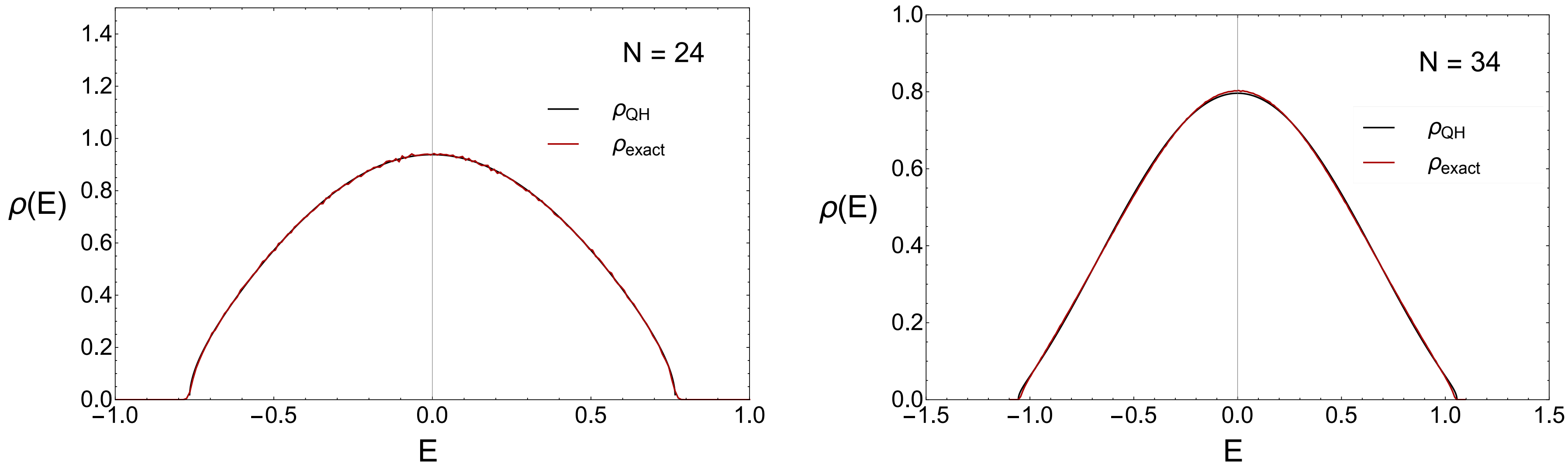
with $E_0^2 = \frac{4\sigma^2}{1-\eta}$, $\eta = \binom{N}{q}^{-1} \sum_p (-1)^p \binom{q}{p}\binom{N-q}{q-p}$ and σ the variance of the spectral density. This approximation neglects correlations that enter in more complicated contractions.

In the large-N limit at fixed q^2/N the Q-Hermite result away from the edge of the spectrum is given by

$$\rho_{\rm asym}(E) \quad = \quad c_N \exp\left[\frac{2\arcsin^2(E/E_0)}{\log \eta}\right], \qquad \eta \approx \exp(-2q^2/N).$$

Garcia-Garcia-JV-2017

Comparison with Numerical Results for $q = 4$



Comparison of the exact spectral density obtained by numerical diagonalization and the Q-Hermite result for the spectral density.

Garcia-Garcia-JV-2017

Spectral Correlations

Unfolding

Correlations

Number Variance

Spectral Form Factor

Unfolding

- The average spectral density is not universal, and for comparison with random matrix theory, the dependence of the spectral fluctuations on the average spectral density have to be eliminated. This is essential.
- This is achieved by unfolding the spectrum, i.e. by mapping the spectrum by a smooth transformation to one with spectral density equal to 1.
- To isolate the correlations one also subtracts to disconnected part of the correlation functions.

Correlations of eigenvalues

Pair correlation function of $\rho\lambda = \sum_k \delta(\lambda - \lambda_k)$

$$\langle\rho(\lambda)\rho(\lambda')\rangle - \langle\rho(\lambda)\rangle\langle\rho(\lambda')\rangle = \langle\rho(\lambda)\rangle\delta(\lambda-\lambda') - \frac{\sin^2(\pi N(\lambda-\lambda')}{(\pi N(\lambda-\lambda'))^2},$$

where we gave the result for the simplest random matrix theory (GUE).

The δ-function always occurs if the spectrum is discrete. It is just the diagonal term in

$$\sum_{kl} \delta(\lambda-\lambda_k)\delta(\lambda'-\lambda_l).$$

The Wigner-Dyson form of the eigenvalue correlations is strongly universal, and is found in systems all over physics.

Agreement with a specific RMT is determined by the non-unitary symmetries of the system.

Correlations and Spontaneous Symmetry Breaking

$$\sum_{k,l}\left\langle\frac{1}{x-E_k-i\epsilon}\frac{1}{y-E_l+i\epsilon}\right\rangle-\sum_k\left\langle\frac{1}{x-E_k-i\epsilon}\right\rangle\sum_l\left\langle\frac{1}{y-E_l+i\epsilon}\right\rangle.$$

Large contributions occur when the phase cancels, i.e. when $x-E_k\approx y-E_l$ or $x-y\approx E_k-E_l$.

Replica symmetry broken as: U(2) $\to$ U(1) $\times$ U(1)
or: U(1,1) $\to$ U(1) $\times$ U(1).

The corresponding low-energy effective theory is unique which is the underlying reason for the universality of random matrix behavior.

Universal behavior arises in a double scale limit. In standard RMTs, $N(x-y)$ is kept fixed, but in the SYK model $2^{N/2}(x-y)$ has to be kept fixed for $N\to\infty$.

Characterization of Universal Random Matrix Behavior

- Short range repulsion of the eigenvalues as S^β .
- Spectral rigidity. The variance of the number of level in an interval containing n eigenvalues on average behaves as $(\beta/2\pi^2)\log n$ rather than n when correlations are absent.
- The classical limit of a theory with spectral correlations given by the Wigner-Dyson ensembles is a chaotic theory (Bohigas-Giannoni-Schmidt conjecture-1984).

The reverse of the Bohigas-Gionnioni-Schmidt conjecture also holds: if the system is not fully chaotic the level correlations deviate from the Wigner-Dyson results Seligman-JV-Zirnbauer-1984.

Number Variance

$$\Sigma^2(n) = \int_0^{\bar{\lambda}} \int_0^{\bar{\lambda}} (\langle \rho(\lambda) - \langle \rho(\lambda))(\rho(\lambda') - \langle \rho(\lambda')\rangle)\rangle d\lambda d\lambda',$$

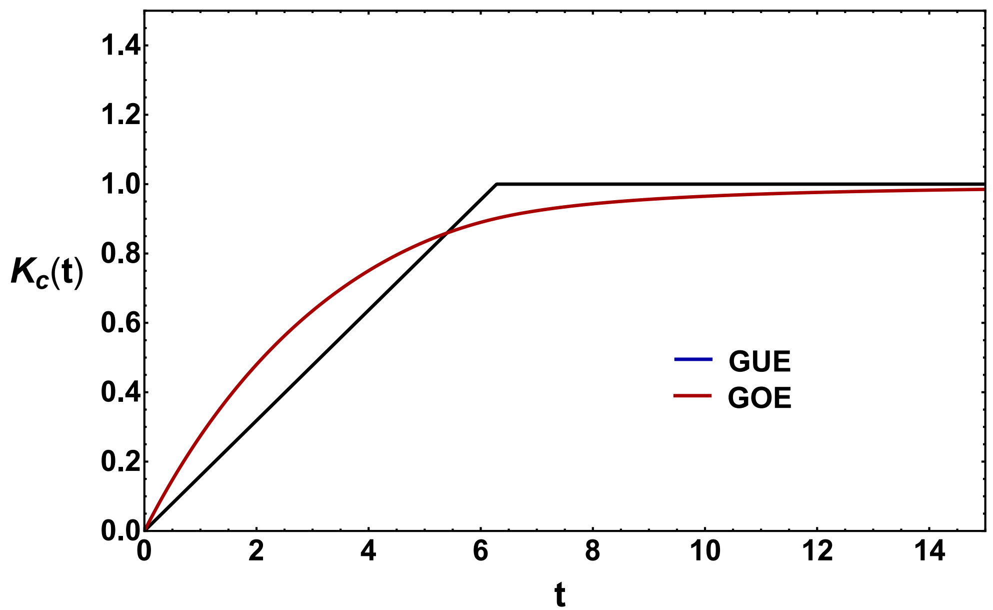
$$\int_0^{\bar{\lambda}} d\lambda \langle \rho(\lambda) \rangle = n.$$

- The δ function results in a linear term, $\Sigma^2(n) \sim n$.
- The $1/(\lambda - \lambda')^2$ term gives a logarithmic term , $\Sigma^2(n) \sim \frac{1}{\pi^2} \log n$.
- For uncorrelated eigenvalues we only have the delta function so that $\Sigma^2(n) = n$.

Spectral Form Factor

$$K_c(t) = \int dxdy e^{it(x-y)} (\langle \rho(x)\rho(y)\rangle - \langle \rho(x)\rangle\langle\rho(y)\rangle) \frac{e^{-\frac{x^2+y^2}{2w^2}}}{\sqrt{\pi} w},$$

where we have added a regulator to remove finite size effects ($w \lesssim 2^{N/2}$) .



Relation Between Spectral From Factor and Number Variance

$$\Sigma^2(n) = \frac{1}{2\pi}\frac{n^2}{\pi}\int_{-\infty}^{\infty} K_c(t)\left(\frac{\sin(nt/2)}{nt/2}\right)^2 .$$

Delon-Jost-Lombardi-1991

Short time behavior of the form factor determines the large n behavior of the number variance:

$K_c(t)$		$\Sigma^2(n)$
$\delta(t)$	$\to$	n^2
Constant	$\to$	$K_c(0)n$

Spectral Correlations of the SYK Model

Questions

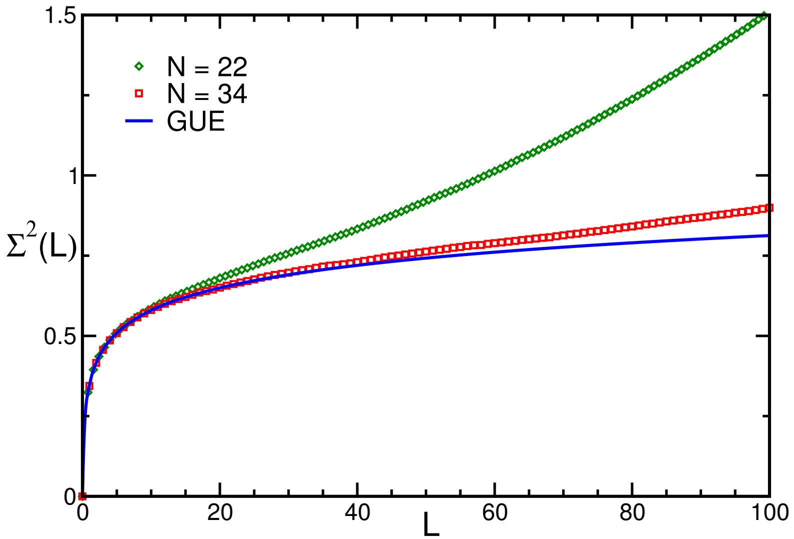
Expansion in Q-Hermite Polynomials

Collective Eigenvalue Fluctuations

Number Variance

Spectral Form Factor

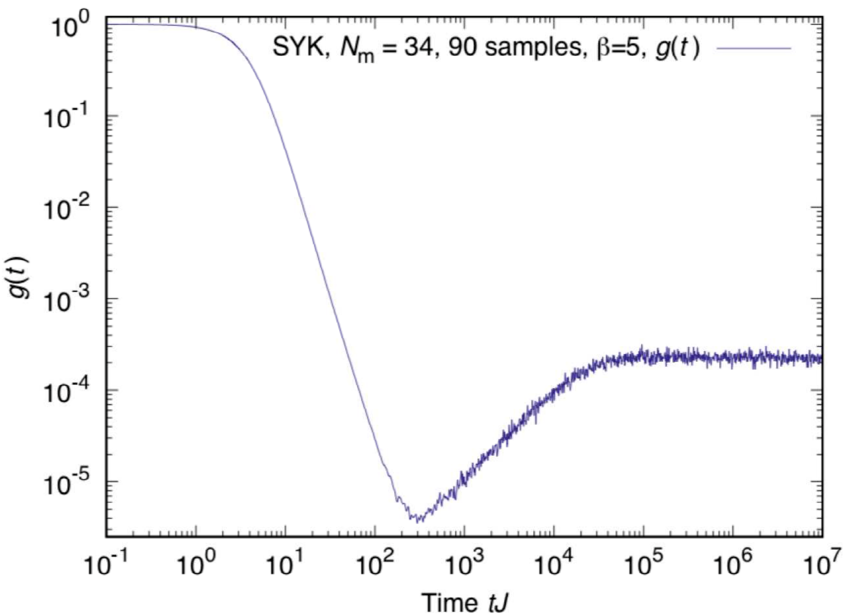
Number Variance and Spectral Form Factor



Number variance versus the length of the interval.

Garcia-Garcia-JV-

arXiv:1610.02363



Spectral from factor versus time.

Cotler-Gur-Ari-Hanada-Polchinski-

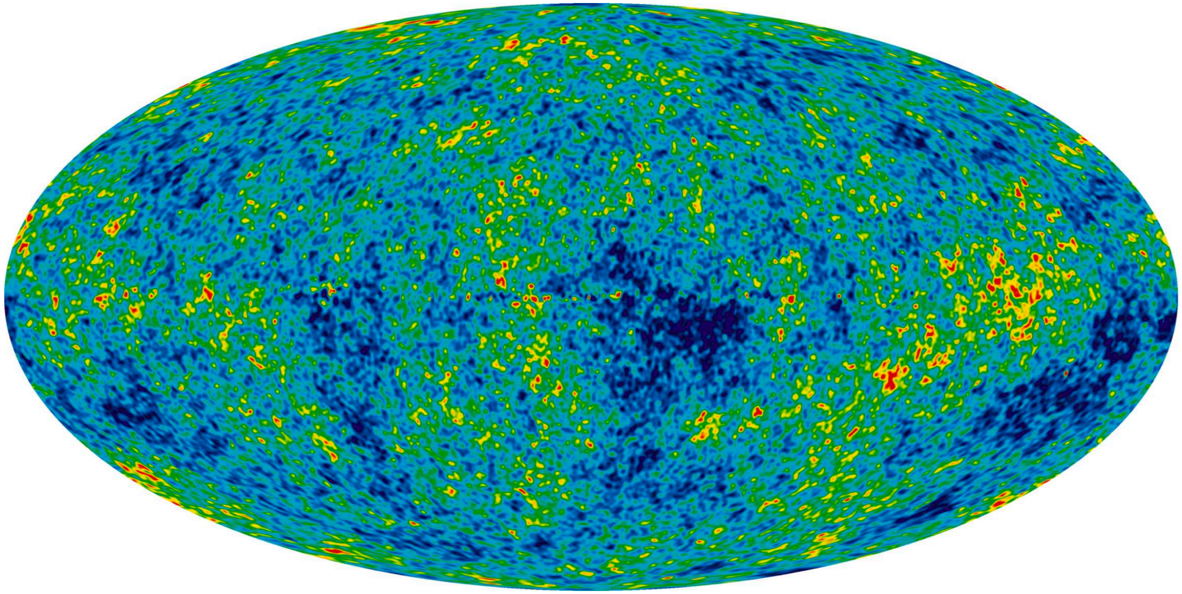
Saad-Shenker-Streicher-Tezuka-

arXiv:1611.04650

Questions

- To what extent is the SYK model chaotic?
- What is the source for the deviations from the Wigner-Dyson ensembles?
- What is the scale at which the deviations occur?
- Why are results for the spectral form factor and the number variance seemingly different?

Separating out the Secular Behavior



After subtracting the spherical harmonics only a thermal spectrum is left. If we unfold configuration by configuration only RMT fluctuations remain.

Expansion in Q-Hermite Polynomials

$$\rho_{\rm SYK}(E) = \rho_{\rm QH}(E)\left[\sum a_k H_k^Q(E/\sigma)\right].$$

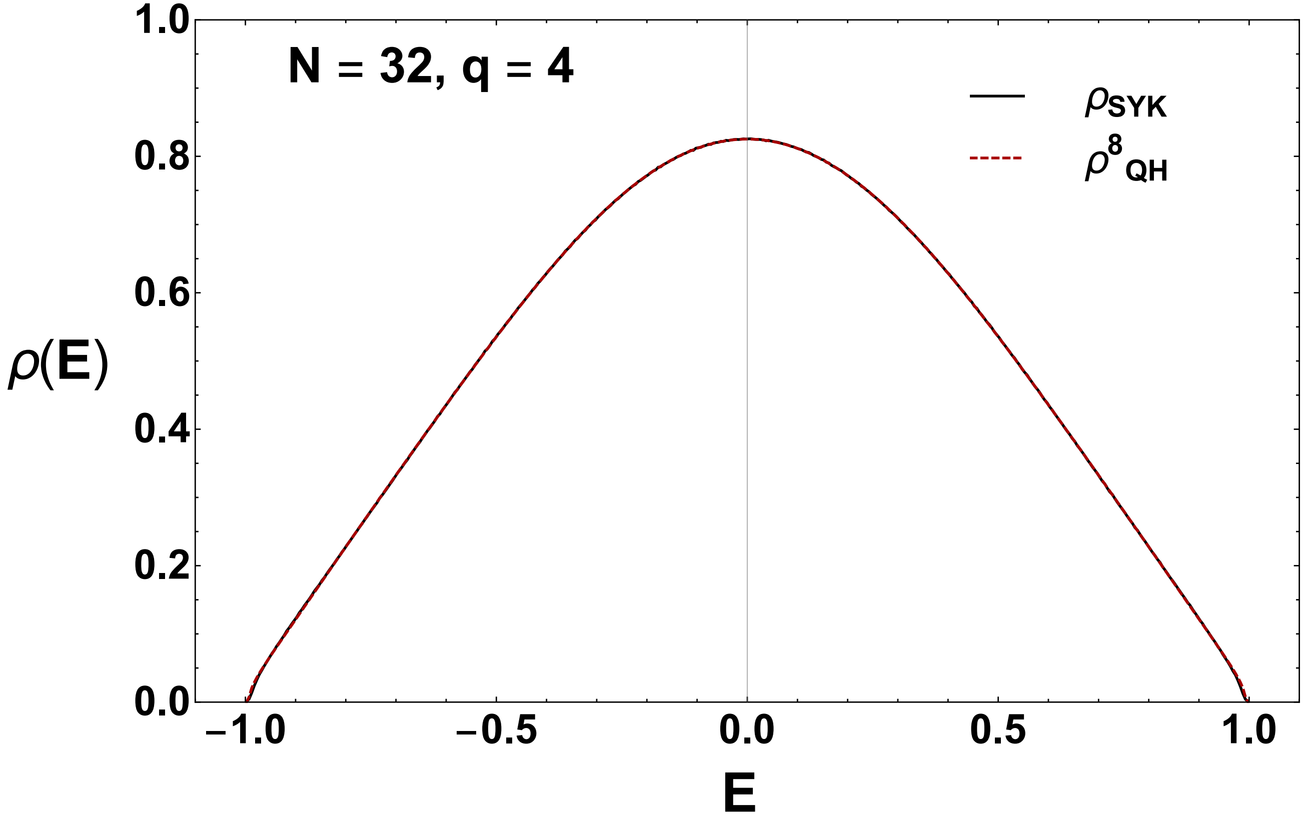
Both the ensemble average and the spectral density for a given configuration can be expanded this way. We have

$$a_0 = 1, \quad a_1 = 0,$$
$$\langle a_2 \rangle = 0, \quad \langle a_4 \rangle = 0, \quad \langle a_{2k+1} \rangle = 0/$$

Numerically, the nonzero $|a_k| < 0.005$ and decreasing for larger k for $N = 32$ and $q = 4$.

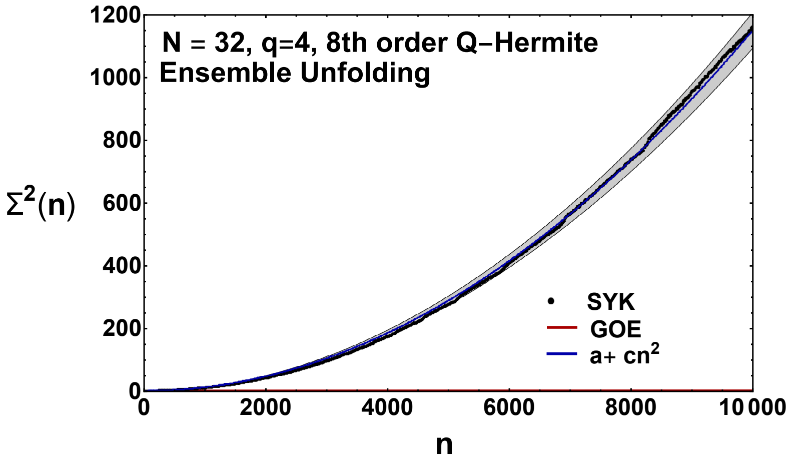
Long-wavelength fluctuations are contained in the fluctuations of the a_k for small k . RMT fluctuations are contained in the coefficients with $k \sim 2^{N/2}$.

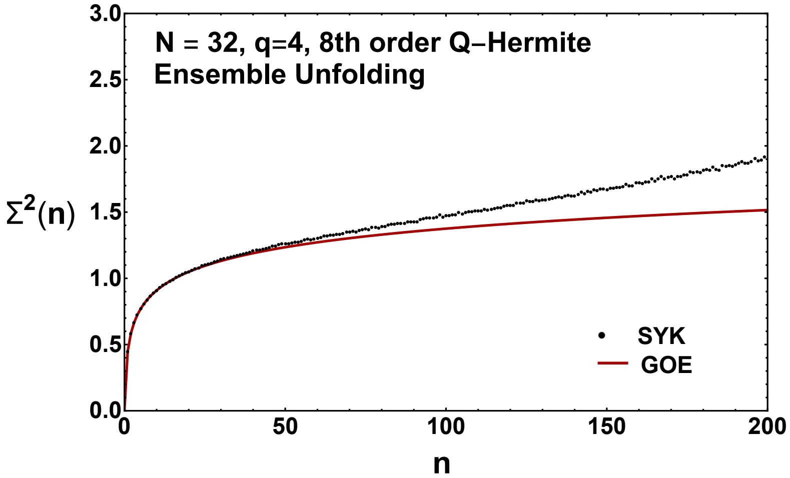
Extreme Unfolding



To study eigenvalue correlations, the unfolding has to be extremely accurate. The 8th order Q-Hermite approximation gives agrees with the ensemble average to about 0.1 level spacing.

Long Range Correlations





- Quadratic term is due to a constant term in the two-point correlator.
- The constant corresponds to overall scale fluctuations from one realization of the ensemble to the next.
- In the spectral form factor this gives a delta function at $t = 0$.

Altland-Bagrets-arXiv:1712.05073,Garcia-Garcia-Jia-JV-arXiv:1801.02696,Gharibyan-Hanada-Shenker-Tezuka-arXiv:1803.08050

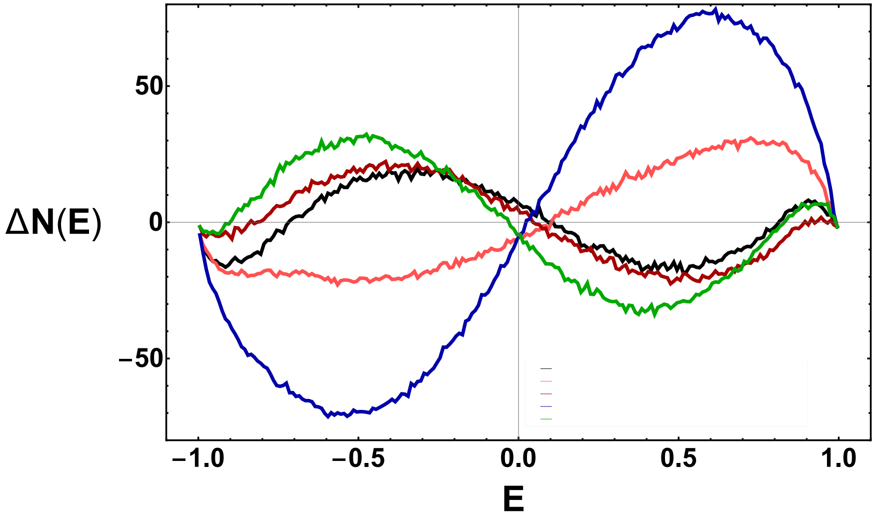
Deviations from Universal RMT Behavior

Deviations from the universal RMT results have two sources:

- Collective fluctuations from one realization of the ensemble to the next.
- For a given realization, at small time scales the spectral correlations are dominated by the details of the system.

The SYK model actually does not have any structure. So we should find RMT fluctuations to very large distance if the eliminate the collective fluctuations. This is achieved by unfolding configuration by configuration.

Fluctuations of the Ensemble



$$\Delta N(E) = \int_{E_0}^{E} (\rho(E') - \langle \rho^8_{\rm QH}(E') \rangle) dE'$$

Difference between the cumulative spectral density (mode number) of a given configuration and the eighth order Q-Hermite fit to the ensemble average of the spectral density. Garcia-Garcia-Jia-JV-2018

Scale Fluctuations

Fluctuations of a_2 correspond to overall scale fluctuations $\lambda_k \to \lambda_k(1+\delta)$. They are determined by the moment French-1973,Flores-Horoi-Müller-Seligman-2000,Hanada-Shenker-Tezuka-2018

$$\langle \mathrm{Tr} H^2 \mathrm{Tr} H^2 \rangle - \langle \mathrm{Tr} H^2 \rangle \langle \mathrm{Tr} H^2 \rangle = 2 \binom{N}{q}^{-1}.$$

Such scale fluctuations contribute to the number variance as

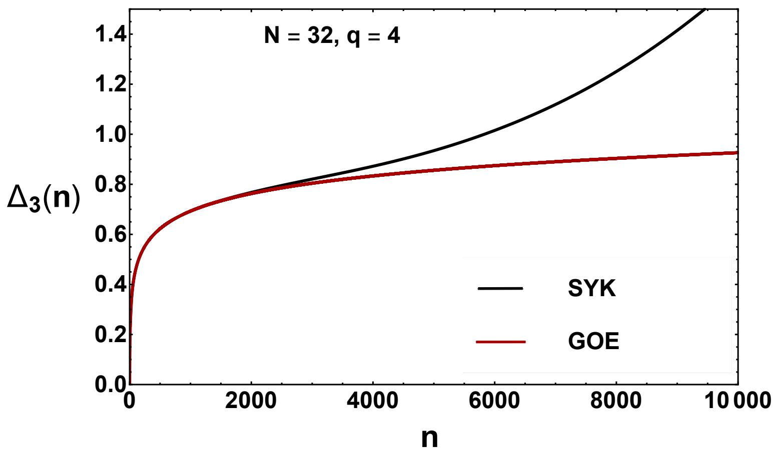
$$\Sigma^2(n) = n^2 \langle \delta^2 \rangle = \frac{1}{2} \binom{N}{q}^{-1} n^2.$$

Numerically this gives $(n/268)^2$ while from fitting the number variance we obtain $(n/294)^2$. For large N this behaves as $n^2 q!/2N^q$, which, for $q=4$, is close to $4\pi n^2/N^4$. Altland-Bagrets-2017

Collective Fluctuations of Higher Multipoles

Long wave-length fluctuations by other a_k contribute to the number variance at much small scales. Patches with an alternating smaller and larger spectral density relative to the ensemble average do contribute to the number variance. These contributions can be estimated from the covariance matrix of the a_k and also behave as n^2/N^4 plus corrections. Garcia-Garcia-Jia-JV-2018

Δ_3 Statistic

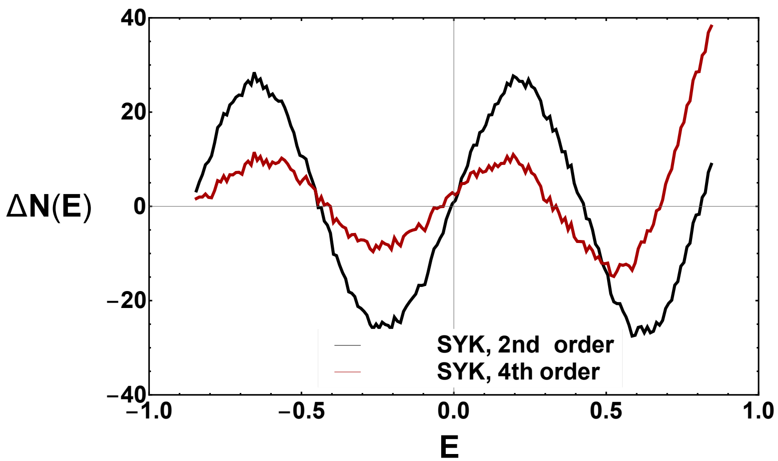


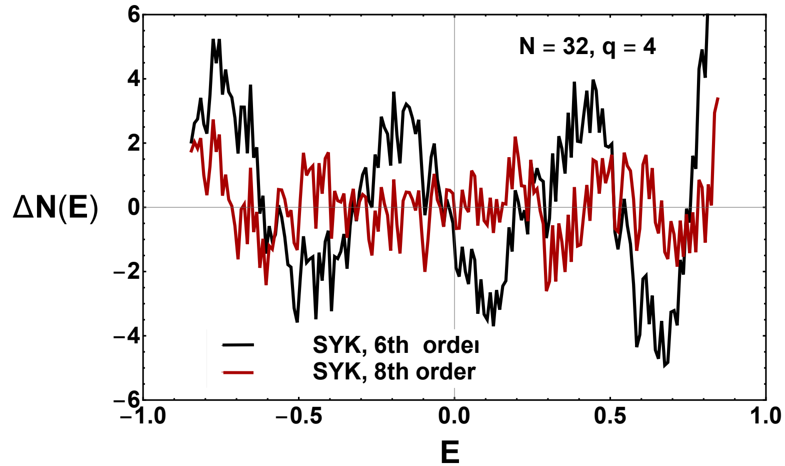
The Δ_3 statistic is obtained by integrating the number variance over a smoothening kernel.

$$\Delta_3(n) = \frac{2}{n^4}\int_0^n dr(n^3 - 2n^2r + r^3)\Sigma^2(r).$$

The kernel maps r^2 to zero.

Wavelength of Collective Fluctuations

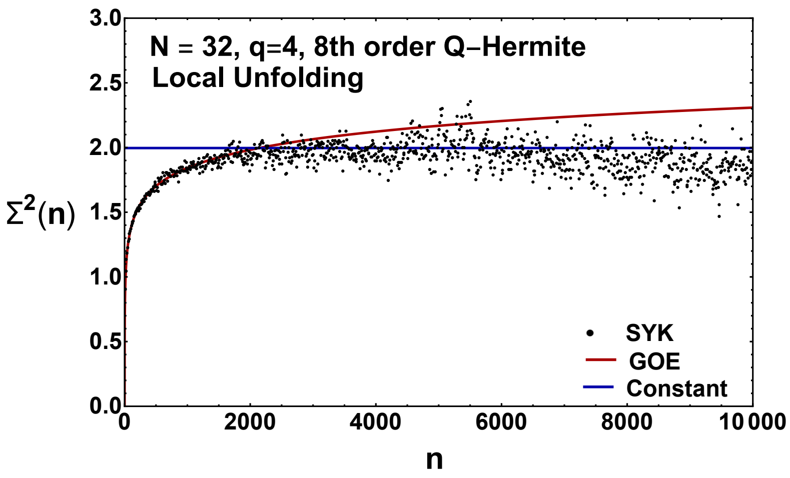


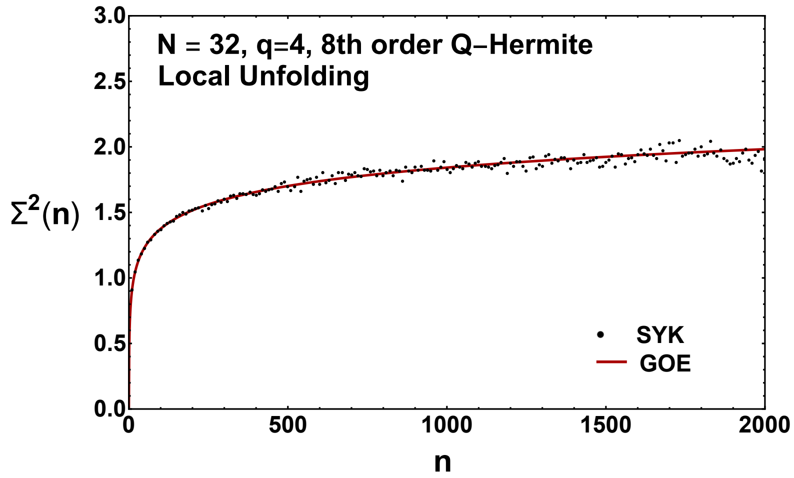


Difference between the exact SYK cumulative spectral density of a single realization and the 2k-th order Q-Hermite approximation.

$$\Delta N(E) = \int_{E_0}^{E} (\rho(E') - \langle \rho^k_{\rm QH}(E') \rangle) dE'$$

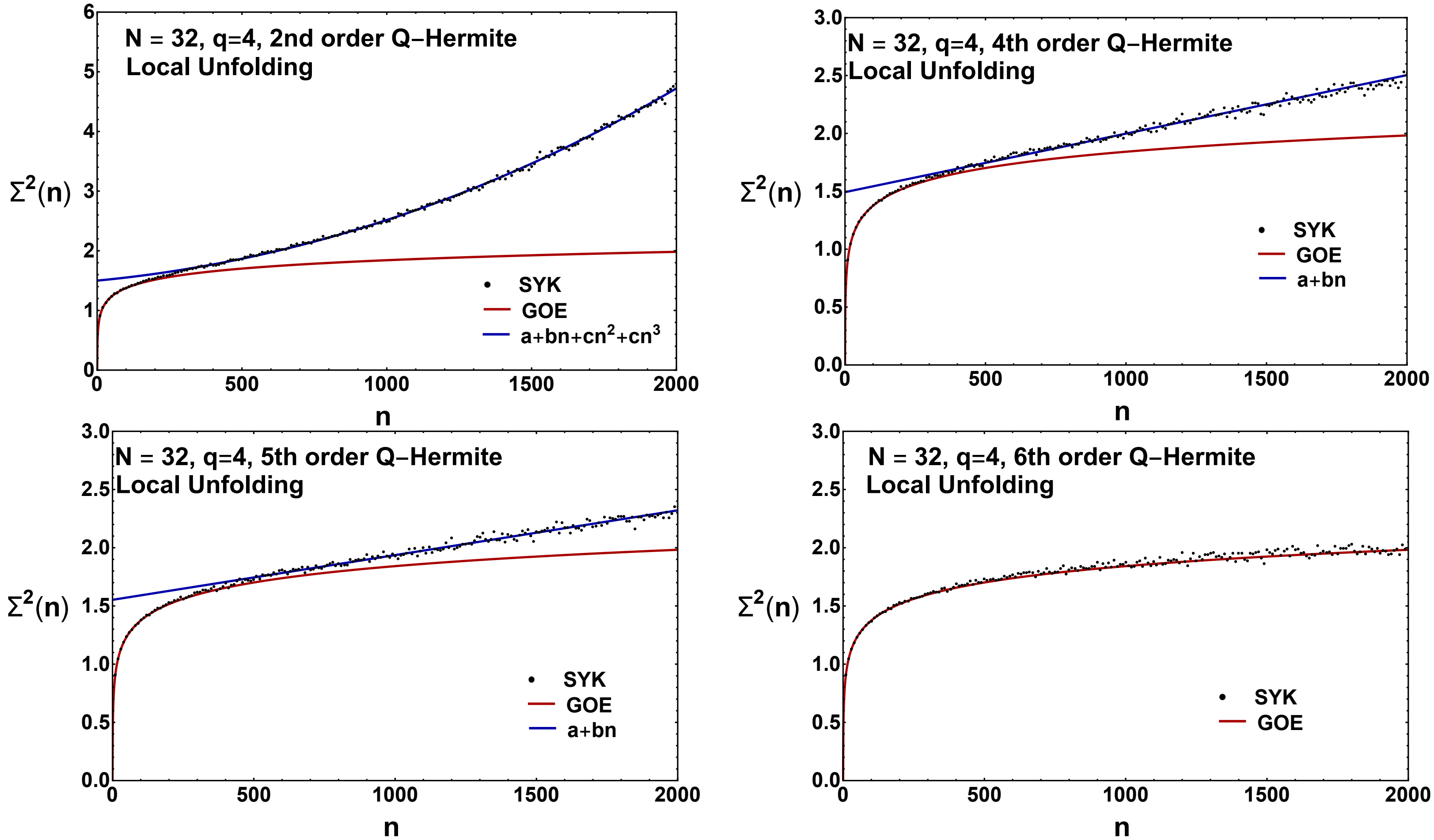
Number Variance for Local Unfolding





Number variance (black points) versus the average number of levels n in the interval.

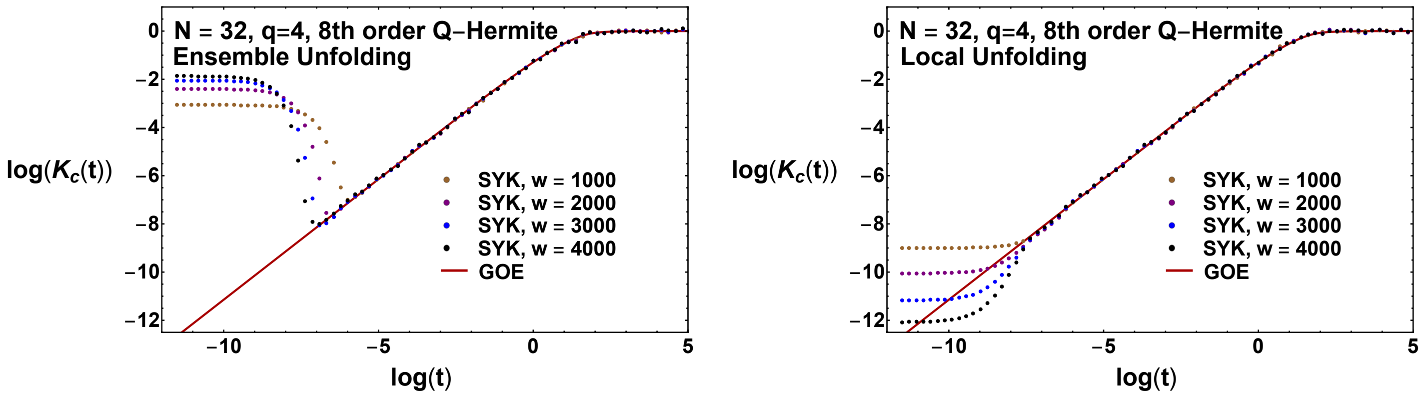
Switching off the Collective Fluctuations



Fluctuations of $a_2, \cdots, a_6$ are responsible for the deviation2 from the universal RMT result.

This means that $a_{k \geq 7}$ fluctuate according to the GOE.

Spectral Form Factor

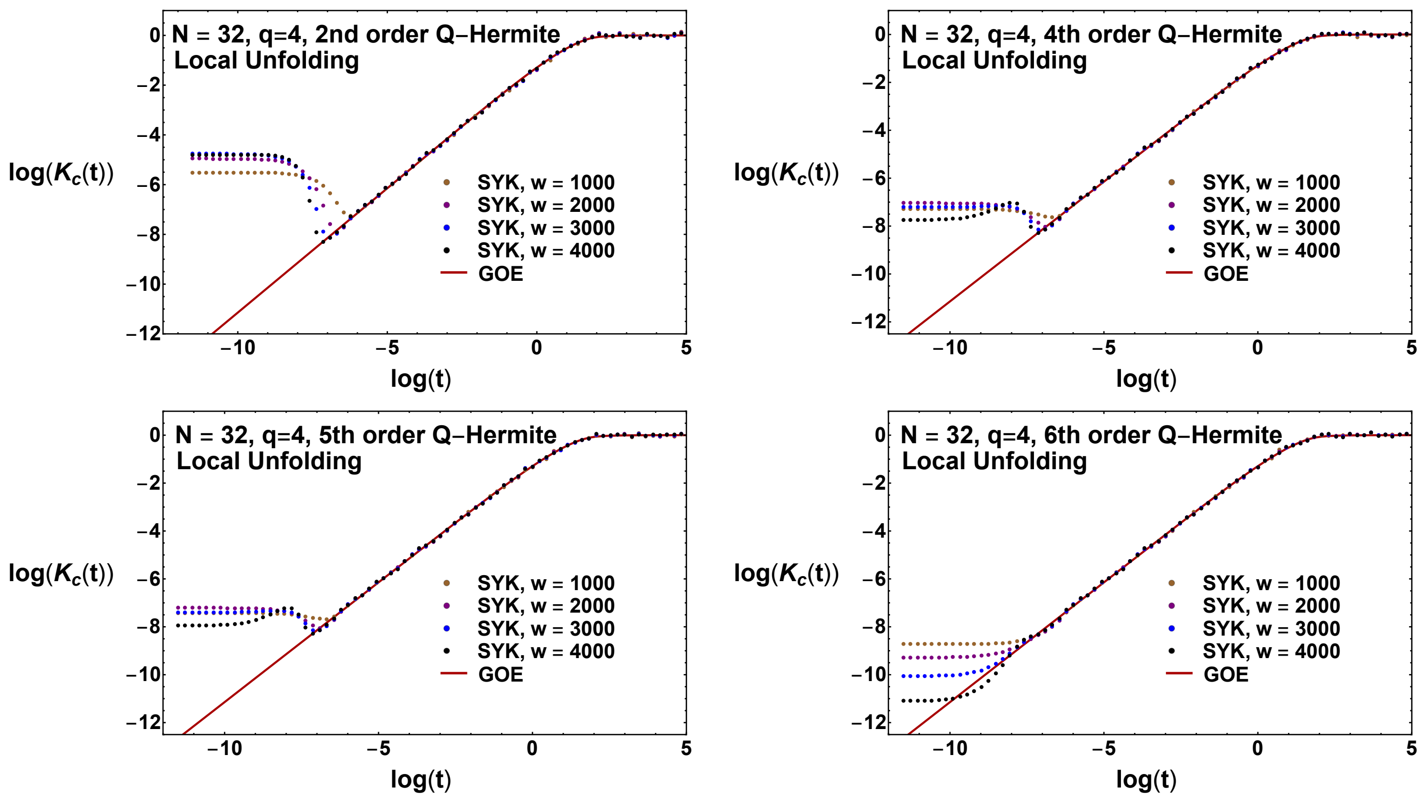


Unfolding with the ensemble average of the spectral density gives a Gaussian peak with width $\approx 1/(\sqrt{2}w)$ centered at zero. Unfolding with eight order Q-Hermite polynomials completely removes this peak, and agreement with the GOE is seen until very short time scales, where finite size effects become important ($\frac{t_{\rm ramp}}{t_{\rm kink}} \approx 1.5 \times 10^{-4}$).

The range of agreement with the GOE in the left figure agrees with the range of GOE behavior in [**] with no unfolding or subtraction of the disconnected part, but the peak at zero is a factor 10^6 lower.

[**] Gharibyan-Hanada-Shenker-Tezuka-arXiv:1803.08050

Spectral Form Factor when Switching off Collective Fluctuations



$$\frac{t_{\rm ramp}}{t_{\rm kink}} \approx 1.5 \times 10^{-4} \approx \frac{\sqrt{N}}{2^{N/2}/2}.$$

Scale Transformations and Spectral Form Factor

$$K_c(t) = \sum_{k,l} e^{it(E_k - E_l)} - \left| \sum_k e^{itE_k} \right|^2$$

A scale tranformation $E_k \to E_k(1+\delta)$ correspond to

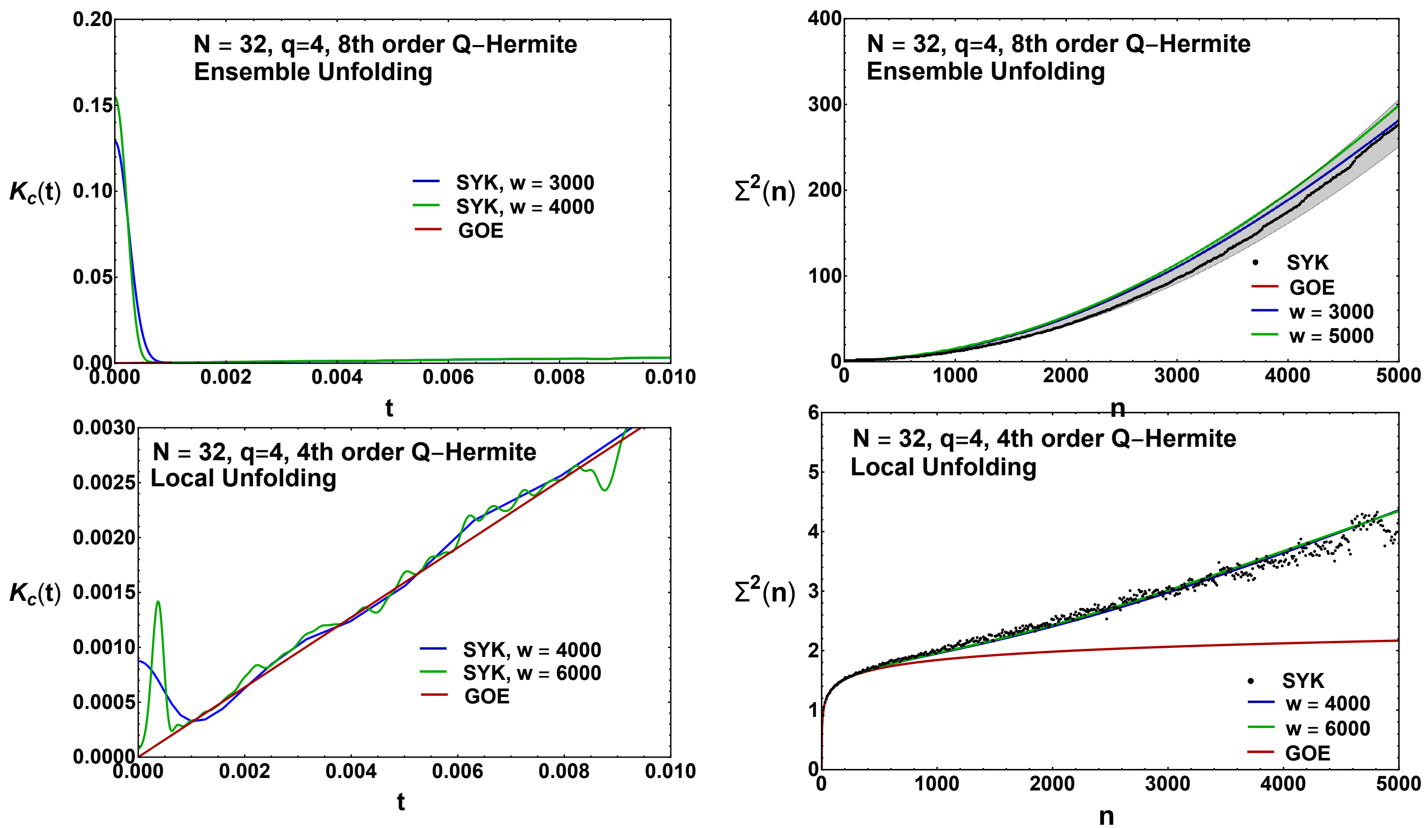
$$K_c(t) \to K_c(t(1+\delta))$$

Averaging,

$$\langle K_c(t(1+\delta))\rangle = K_c(t) + \frac{1}{2} K_c''(t) \langle \delta^2 \rangle$$

Only gives contributions at the kink or when $K_c(t)$ deviates from the random matrix result such that $K_c''(t) \neq 0$.

Consistency Check



Number variance calculated from the spectral form factor.

If the peak near zero of $K_c(t)$ is replaced bu the GOE result (red curve), then the GOE result for the number variance is recovered.

Scale of Universal Random Matrix Fluctuations

- The ensemble fluctuations of the spectral density of the SYK model are long wavelength fluctuation contained in a small number ($O(N)$) coefficients of the Q-Hermite expansions. The covariance matrix of these coefficients can be calculated analytically and is in agreement with the numerical results Garcia-Garcia-Jia-JV-2018.
- Taking out these long-range fluctuations, the spectral correlations of the SYK model becoming universal up to the maximum scale - up to a significant fraction of $2^{N/2}/2$.
- For $N = 32$ we only need a sixth order polynomial to eliminate the collective fluctuations. So $2^{N/2}/N^{0.5-1}$ level spacings would be a good estimate for the energy scale of Wigner-Dyson correlations.

Ergodicity

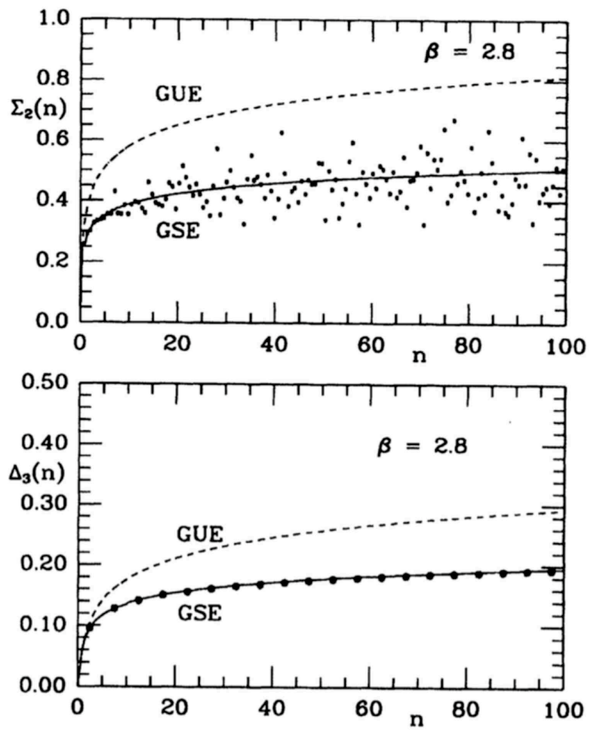
- Level fluctuations relative to the ensemble average deviate from random matrix theory at a scale of $N^{q/2}$ level spacings and mainly due to scale fluctuations. In the spectral form factor these fluctuations give a delta function at time zero.
- The discrepancy in the scale of universal random matrix behavior between the spectral form factor and the number variance was obtained analytically starting from a spectral determinant Altland-Bagrets-2017, see next talk.
- The ensemble average of the number variance is not equal to the spectral average of the number variance.
- This also happens for eigenvalues of the Dirac operator in QCD.

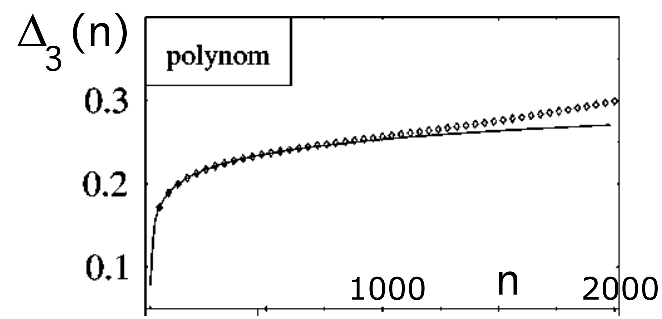
Ergodicity in QCD

Number Variance

Chiral Lagrangian

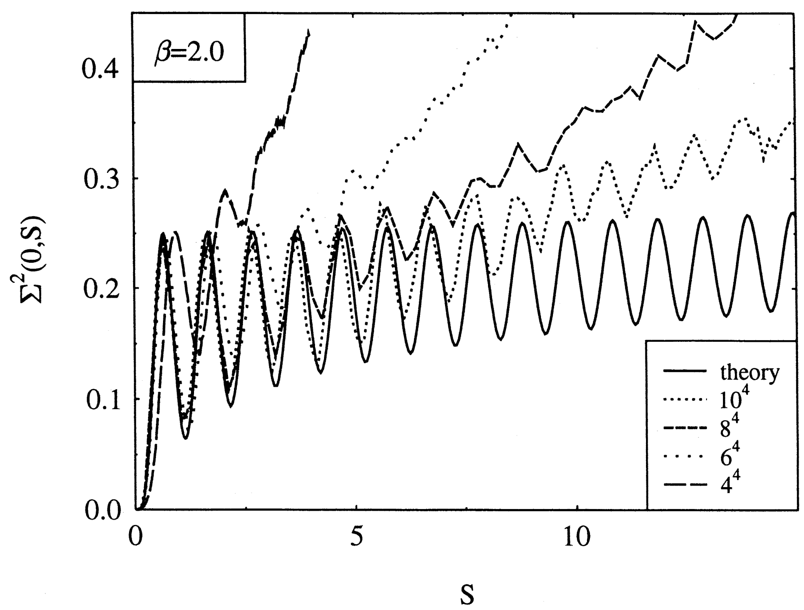
Spectral Correlations in Lattice QCD

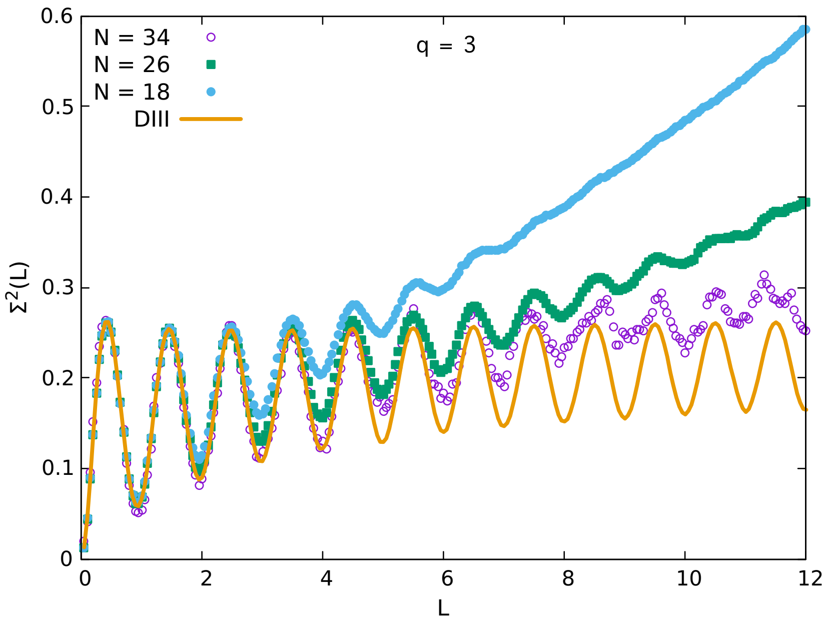




Eigenvalue Correlations of eigenvalues of the Dirac operator for a single configuration (left, Halasz-JV-1996), or obtained by unfolding configuration by configuration (right, Guhr-Ma-Meyer-Wilke-1999), seem to know nothing about QCD. The Dirac eigenvalues follow the RMT prediction up the maximum allowed by statistics.

Number Variance from Zero





Number variance for two-color QCD for various volumes Berbenni-Bitsch-et-al-1998 and the number variance of the SYK model for various numbers of fermions Garcia-Garcia-Jia-JV-2018. The Thouless energy is $\sim \sqrt{V}$ for QCD and $\sim N$ for the SYK model.

Spectral Fluctuations and Chiral Lagrangians

- In QCD the agreement of Dirac spectra and its deviations have been explained completely in terms of a chiral Lagrangian.
- The deviations are due to the kinetic term while the mass term gives the universal random matrix result. It gives the correct value of the pion decay constant.
- Unfolding configuration by configuration gives spectral fluctuation up to the highest scale that can be studied within the available statistics (100 to 1000 level spacings). This suggests that the spectral form factor will follow the universal random matrix result up to very short times.
- The effective Lagrangian that describes spectral fluctuations of the SYK model is an ensemble average. It is not fully understood how it describes the universal Random Matrix results and its deviations, but the linear dependence of the form factor was obtained Saad-Shenker-Stanford-2018.

IV. Conclusions

- The weight function of the Q-Hermite polynomials is very close to the spectral density of the SYK model. The difference is given by a few low-order Q-Hermite polynomials.

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- These results are consistent with the fact that the SYK model is maximally chaotic.
- In a sense a black hole is dual to a compound nucleus.