

- exploring the gravitational side of AdS_3/CFT_2 duality -

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based on arXiv pubs:

1409.6017 (EJM)1705.10844 (EJM, Stefano Massai)1803.08505 (EJM, Stefano Massai, David Turton)work in progress (EJM, Stefano Massai, David Turton)

Branes, Geometry and Entropy

- Standard example of BH μ state counting (Strominger-Vafa '96): String theory compactified on $(S^1)_v \times T^4$
 - ➢ Wrap n₅ NS5-branes along (S¹)_y x T⁴
 - Wrap n₁ F1-strings along (S¹)_y
 - Excite n_p units of momentum P along (S¹)_y

When $R_y \rightarrow \infty$ and $(g_s)_{asymp} \rightarrow 0$

- The near-horizon geometry is $BTZ \times S^3 \times T^4$
- Dual to a thermal ensemble in CFT₂

$$S_{\rm BH} = \frac{A_{hor}}{4G_N} = 2\pi \Big(\sqrt{n_5 n_1 n_p - J^2} + \sqrt{n_5 n_1 \bar{n}_p - \bar{J}^2} \Big) \\ \mathbf{S}_{\rm L} \qquad \mathbf{S}_{\rm R}$$



Long strings and little strings

- In the brane picture, the entropy arises as the log of the number of brane bound states with the given charges
- A cartoon of this entropy accounting enumerates oscillator excitations in an effective "*long string*" sector of these bound states; *eg* in the lift to M-theory:



An **F1** string fractionates into n_5 pieces ("little W-strings" of Little String Theory) whose monodromy can wind into a single effective long string of length $2\pi n_1 n_5 R_y$. Long strings are a special case of little strings

• Is this *long string* physics emergent in the near-horizon structure of this class of BH geometries on the gravity side? How can one see it, since the naïve supergravity solution is featureless at the horizon scale?



- Many puzzles about BH's regarding information storage and retrieval would be resolved if the underlying degrees of freedom which carry the information were *quantum coherent over the horizon scale*.
- The fuzzball proposal posits that string theory naturally generates objects whose entropy is carried by degrees of freedom *delocalized on this scale*.
- Beyond this general notion, there are a variety of suggestions as to how it might be implemented – for example that individual microstates, having zero entropy, should have no horizon: $S_{\mu state} = 0 \iff A_{hor} = 0$
- Trying to realize this idea in supergravity, there is an impressively rich zoology of *bubbled microstate geometries*, wherein brane/flux transitions transform explicit brane sources into smooth horizonless flux geometries (so that topological bubbles w/flux are the horizon scale structures).



The **fuzzball** Idea

- The brane/flux transition can occur down at the bottom of a deep scaling throat, so that the bubbled geometry closely approximates a BH geometry.
- However, the BPS entropy of bubble configurations has $S_{bubbles} \ll S_{BH}$ Thus "empty" bubbled geometries are *not* generic μ states.
- There is more to say here than will fit in the margin of this slide
- Can also excite sugra wiggles in smooth, deep throats of a bubbled geometry – a particular class have been called superstrata. There are reasons to believe their entropy is still not commensurate with BH entropy.
- Perhaps we should not have expected it BH's are highly chaotic; smooth classical geometries are highly coherent states. Microstate geometries are island archipelagos of coherence in a sea of chaos.
- Also, classical sugra solutions satisfy premises of singularity theorems; if sufficiently perturbed, expect collapse leading to horizon formation. *Stringy and/or quantum ingredients are required*.

Two-charge **fuzzballs**

 Nevertheless, these geometries with deep throats are useful starting points. They are *almost* black holes, and if we kick them a little bit, they *become* black holes. What is the route they travel from order to chaos?



- The red line consists of rotating BPS 2-charge (NS5-F1) configurations known as supertubes (Mateos-Townsend '01, Lunin-Mathur '01), which are not the μstates of a macroscopic BTZ BH, but approach extremal BH at small J.
- Look for long string structure in the excitations of these 2-charge states . . .

Putting strings back into string theory

- Typical supertubes are stringy, therefore we would like an exactly solvable worldsheet CFT for the supertube, and remarkably, for particularly nice supertube configurations *this can be achieved*!
- The worldsheet theory is a gauged WZW model (a 2d CFT defined by current algebra symmetries) for the group quotient

 $G/H = \left(\frac{\mathbb{R}^{1,1} \times SL(2,\mathbb{R}) \times SU(2) \times \mathbb{T}^4}{U(1)_L \times U(1)_R}\right)$

- By varying the embedding of *H* into *G*, one finds a variety of supertubes: NS5-P, NS5-F1, as well as spectral flows thereof carrying all three charges (including the nonsupersymmetric "JMaRT" geometries)
- Null gauged WZW models appear to be a valuable tool to explore nearhorizon structure. It is quite rare to have a description of states far from the vacuum that is exact in α'

Overview of the talk





- Supertubes and phase structure
 - A supertube spiral



- Details of the worldsheet construction
- Decorating the supertube: Towards Long string structure in the bulk

- Speculations on horizon/interior structure

Supertubing

• We start with NS5-P supertubes, which generically are fivebranes with a thermal gas of chiral waves on the brane, including the scalars parametrizing the brane embedding, *e.g.*:

$$X^{1} + iX^{2} = \sum_{k} a_{k} \exp\left[\frac{ik}{n_{5}}\left(t + \tilde{y}/\tilde{R}\right)\right]$$



Supertubing

• A particularly simple class of configurations excite only a single harmonic of the scalars parametrizing the brane embedding:

$$X^{1} + iX^{2} = \mathcal{A} \exp\left[\frac{ik}{n_{5}}\left(t + \tilde{y}/\tilde{R}\right)\right]$$



Details, details ...

We build the **worldsheet** CFT for supertube spacetimes in three steps:

- 1) Review known worldsheet CFT's for static NS5's
 - > Naïve nonlinear σ -model
 - Null-gauged WZW model
- 2) Spin them up to add **P** & **J** charge
- 3) Compactify and T-dualize to NS5-F1 rotating supertube



Step 1a: Naïve static NS5 *o*-model

• Consider static NS5 geometry, characterized by a harmonic fn $Z_5 = \frac{n_5 \alpha'}{r^2}$

$$ds^{2} = \left(-dt^{2} + d\tilde{y}^{2} + ds_{\mathbf{T}^{4}}^{2}\right)_{||} + Z_{5}\left[dr^{2} + r^{2}d\Omega_{3}^{2}\right]_{\perp}$$
$$e^{2\Phi} = Z_{5} \qquad H_{\theta\phi\psi}^{(3)} = \epsilon_{\theta\phi\psi}r \,\partial_{r}Z_{5}$$

- Worldsheet string dynamics is exactly solvable (Callan-Harvey-Strominger '91):
- Radial direction: log(r) is a free field w/linear dilaton
- Dilaton blows up at r=0; a perturbative S-matrix does not exist
- Angular sphere: SU(2) WZW model (bosonic level n₅-2). NB: There is no unitary WS description of a single isolated fivebrane throat (n₅=1)

Step 1b: Go to the Coulomb branch

• Separate the n_5 sources onto their Coulomb branch, for instance in a \mathbb{Z}_{n5} symmetric array. The harmonic function becomes

$$Z_5 = \sum_{m=1}^{n_5} \frac{\ell_s^2}{|x_1 + ix_2 - a\,\omega^m|^2 + |x_3 + ix_4|^2}$$

where $\omega^{n_5}=1$

 Strings are now repelled before they get close enough to resolve isolated branes; low-energy dynamics is weakly coupled



• String worldsheet dynamics surprisingly continues to be exactly solvable (Sfetsos '98, Giveon-Kutasov '99), but is now nonsingular because there is no dynamics of perturbative strings near a single isolated fivebrane.

Step 1c: adopt a useful coordinate system

• Change coordinates in transverse space

 $x_1 + ix_2 = a \cosh \rho \, \sin \theta \, e^{i\phi} \, , \quad x_3 + ix_4 = a \sinh \rho \, \cos \theta \, e^{i\psi}$

• The sum over source locations $\phi_m = 2\pi m/n_5$ is a discrete F.T.

$$Z_{5} = \frac{n_{5}\ell_{s}^{2}}{a^{2}(\cosh^{2}\rho - \sin^{2}\theta)} \left[1 + \sum_{k \neq 0} e^{-n_{5}\left(|k|x + ik\phi\right)}\right] \qquad \text{(where } e^{x} = \frac{\cosh\rho}{\sin\theta}\text{)}$$

Smeared source Nonperturbative in α' : $n_{5}=R_{AdS}^{2}/\alpha'$

- Source locations are only distinguished by stringy effects at the bottom of the throat; *supergravity sees only the smeared geometry*
- In fact, the nonperturbative WS effects are not in the metric instead they are encoded in a (Liouville) superpotential term *W* = exp[-n₅(ρ+iφ)] *i.e.* a tachyon condensate, which is a noncompact example of the Calabi-Yau/Landau-Ginsburg correspondence (*more about this later*)

aside: the stringiness of the source

- One does NOT have a geometry which resolves into "little throats" of individual fivebranes on the Coulomb branch (a metric with separated sources), as is suggested by a naive application of BPS solution generating techniques
- The structure of the background near the fivebranes is something much more stringy ...



Step 1d: introduce the GWZW model

• Choose an Euler angle parametrization of $SL(2,R) \simeq SU(1,1)$ and SU(2)

$$g_{sl} = e^{\frac{i}{2}(\tau+\sigma)\sigma_3} \cdot e^{\rho\sigma_1} \cdot e^{\frac{i}{2}(\tau-\sigma)\sigma_3}$$
$$g_{su} = e^{\frac{i}{2}(\psi+\phi)\sigma_3} \cdot e^{i\theta\sigma_1} \cdot e^{\frac{i}{2}(\psi-\phi)\sigma_2}$$

• Gauge the null current $(J_3^{sl} \pm J_3^{su})$; the WZW model has the form (Israel-Kounnas-Pakman-Troost '04) $S = S_{wzw} + \frac{n_5}{4\pi} \int \left[(J_3^{sl} + J_3^{su})\bar{A} + (\bar{J}_3^{sl} - \bar{J}_3^{su})A + (\cosh^2 \rho - \sin^2 \theta)A\bar{A} \right]$

Integrating out A,
$$\overline{A}$$
 yields the appropriate harmonic function for
static fivebranes $Z_5 \propto 1/\Sigma$. Note that the naive singularity at $\rho=0$, $\theta=\pi/2$
is an illusion – the exact worldsheet CFT is smooth.

Step 2: Spinning up the supertube

 Remarkably, the starting point is a *smooth* flux geometry in 10+2d. Null gauging gets us down to 9+1d, and generates the fivebrane harmonic function.

$$G/H = \left(\frac{\mathbb{R}^{1,1} \times SL(2,\mathbb{R}) \times SU(2) \times \mathbb{T}^4}{U(1)_L \times U(1)_R}\right)$$

- In situations where there *appears* to be a fivebrane "singularity" in 9+1d, it is located where the gauge action degenerates. *There is no singularity* in 10+2d geometry (*c.f.* Witten '91), or in low-energy 9+1d string dynamics
- To spin up the system, consider longitudinal NS5 directions ℝ^{1,1} in the WZW group G, parametrized by (t,ỹ); then tilt the gauging into the v = t-ỹ direction, so that *e.g.*

 $\mathcal{J}_{\text{gauge}} = J^{\text{sl}} + J^{\text{su}} + \alpha \partial v$



Step 2: Spinning up the supertube

• The coefficient Σ of $A\bar{A}$ is not affected by the tilting, so we still have a circular ring of fivebranes, but terms $\alpha^2 dv^2/\Sigma$ (momentum) and $\alpha dv d\phi/\Sigma$ (angular momentum) now appear in the effective metric

$$ds^{2} = -dt^{2} + d\tilde{y}^{2} + n_{5} \left[d\rho^{2} + d\theta^{2} + \frac{n_{5}}{\Sigma} \left(ch^{2}\rho \sin^{2}\theta \, d\phi^{2} + sh^{2}\rho \cos^{2}\theta \, d\psi^{2} \right) \right]$$

$$+ \frac{1}{\Sigma} \left[\alpha \, n_{5} \sin^{2}\theta \, d\phi \, (dt - d\tilde{y}) + \alpha^{2} (dt - d\tilde{y})^{2} \right]$$
rotation momentum

 The first line is the metric of Coulomb branch NS5's; the second line is the added effect of tilted gauging. The null-gauged WZW model precisely reproduces the NS5-P supertube geometry

Step 3: Compactify and T-dualize

- After spin-up, n_5 non-compact threads of NS5 spiral along the \tilde{y} - ϕ source cylinder
- Adjust the tilt α to allow periodic identification $\tilde{y} \sim \tilde{y} + 2\pi R_{\tilde{y}}$



• The (discrete) tilt parametrizes the pitch of the source spiral

$$\alpha = \frac{d\phi}{dv} = \frac{k}{n_5 R_{\tilde{y}}}$$



 $n_5 = 2, k = 3$

• If k and n_5 are relatively prime, monodromy wraps the system into one single NS5-P supertube winding the (n_5,k) cycle of the $\tilde{\gamma}$ - ϕ torus.

Step 3: Compactify and T-dualize

- From the CFT viewpoint, R_y = 1/R_ỹ T-duality is trivial, but involves a different effective geometry where substructure is very stringy & hidden in vevs of winding operators
- Integrating out the gauge field now yields the T-dual NS5-F1 geometry w/dilaton

 $e^{-2\Phi} \propto \Sigma = \alpha^2 + \Sigma_0$

where $\Sigma_0 = n_5(ch^2\rho - sin^2\theta)$ is the harmonic function denominator we had before



• Near the NS5-F1 bound state we have a constant dilaton, and the geometry is approximately $(AdS_3 \times S^3)/\mathbb{Z}_k$; far away we have the linear dilaton throat of the fivebranes. Large $\alpha = (k/n_5)R_y = large AdS region$

µ-states and phase structure

 The large k limit approaches the J=0 extremal black hole along the BPS line of supertubes (NB: k is bounded by n₁n₅)



 Can also describe spacetime spectral flowed states (GMS '04, GLMT '12) and nonsusy solutions (JMaRT '05, CTV '15). All this is achieved by varying the choice of L & R null vectors being gauged in 10+2d

Closed string spectrum

• The BRST constraints include the usual (super) Virasoro constraints, as well as the (super) null-gauging constraints

$$Q_{\text{BRST}} = \oint \left(c^z T_{zz} + \gamma^{\theta} G_{z\theta} + \tilde{c} \,\mathcal{J}_z + \tilde{\gamma}_{\theta} \Psi_{\theta} + ghosts \right)$$

• String momenta (and oscillator polarizations) are constrained to be orthogonal to the null vector; for instance for supergravity modes

$$(m_{sl} + m_{su}) - \alpha (E - n_y/R_y) = 0$$

- The gap in the perturbative spectrum is O(1/k) in units of R_{AdS}
- Solving the worldsheet BRST constraints for low-lying states having no winding along the angular directions, one finds an exact match to the supergravity spectrum, including the decoupling limit of unstable ergoregion modes of the nonsusy JMaRT geometry.

Closed string spectrum

- The SL(2) & SU(2) quantum numbers lie in allowed HWR's of the corresponding current algebra. For SL(2), the principal quantum # j_{sl} specifies radial momentum discrete series reps \mathcal{D}^{\pm} describe states bound to the cap; continuous series reps $C_{(1+i\lambda)/2}$ describe scattering states
- For most bound states, the sign of the energy in the AdS_3 cap correlates with the sign of the energy in the asymptotic region – positive frequency modes involve \mathcal{D}^+ , negative frequency modes involve \mathcal{D}^-
- Ergoregion instabilities arise when there are modes that are positive frequency at infinity but negative frequency (*D*⁻ reps) as seen by static observers in the cap. These occur for non-susy JMaRT solutions, but are absent for susy solutions.

Closed string scattering

• The exact tree-level reflection amplitude for scattering states factorizes into a 'supergravity contribution' and a 'stringy contribution'



- The supergravity part has a series of poles at the energies of the \mathcal{D}_j^{\pm} bound states, and for $C_{(1+i\lambda)/2}$ scattering states is a phase. The contribution of this part to the phase shift tends to a constant in the limit of large radial momentum λ as one expects for reflection off a stiff cap.
- The stringy part is the reflection amplitude off the exponential wall of N=2 Liouville theory that describes the stringy fivebrane source.



- The softer Liouville wall allows radial propagation "beyond the cap" in a stringy dual background (The Liouville phase shift results in an additional logarithmic time delay for probes with large radial momentum λ)
- The Liouville linear dilaton signals near-fivebrane physics. Energetic strings can temporarily approach individual fivebrane sources; radial momentum determines the point of closest approach and thus the effective coupling.



• For different probes/observables, different effective backgrounds dominate dynamics (Giveon-Itzhaki-Kutasov '15):

(a) low mom – supergravity behavior; smeared and curved geometry with a cap
(b) high mom – Liouville; flat geometry w/stringy matter density and linear dilaton

 Are we seeing a precursor of "fuzzball complementarity"? Low momentum probes see a wall, while high momentum probes sail through...

Closed winding string spectrum

- Worldsheet spectral flow $\delta J_3 = \frac{1}{2}n_5 w$ generates $AdS_3 \times S^3$ 'giant gravitons'
- A particular linear combination of spectral flows specified by the left/right null currents implements a (large) gauge transformation:

 $V_{gauge} = exp[iq(\mathcal{Y}+\mathcal{Y})]$

$$\partial \mathcal{Y} = -J^{sl} + J^{su} + \alpha \partial (t+y)$$

spectral flows shifts in *E* and w_y

- This large gauge transformation removes q units of 'winding' around the AdS₃ angular direction and adds qk units of winding around the y-circle; at the same time, the energy E of perturbative strings changes by O(qkRy).
 NB: AdS₃ winding is not conserved in correlation functions!
- Energy and winding charge <u>are</u> conserved, but only when contributions from perturbative strings *and the background* are combined (Kim-Porrati '15)

Closed winding string spectrum



- Perturbative strings that wind the AdS_3 angular direction are gauge equivalent to those carrying the F1 charge of the background. Large gauge transformations on the worldsheet mediate brane/flux transitions $\delta n_1 = -k \delta w_v$ that turn branes into background flux and vice versa
- As **F1** charge is stripped away, the supertube shrinks, fivebranes come closer together, and dynamics is more strongly coupled at the tip

The role of **D-branes**

- Low-energy dynamics on (nearly) coincident NS5's is governed by *'little string theory'*. For instance, Maldacena ('96) showed that NS5 thermodynamics is the Hagedorn thermodynamics of little strings.
- On the Coulomb branch, NS5's are separated; *'little W-strings*' are D2-branes stretching between NS5's (or in the T-dual NS5-F1, D3-branes wrapping KK-dipole cycles).
- Thus the 'little W-strings' are heavy; one sees the entropy-carrying degrees of freedom of black holes in a controlled way, by approaching them from the "wrong" phase



The role of **D-branes**

• The 10+2 construction gives a novel perspective on D-branes that end on NS5's:





D2+2 probe in 10+2 flux geometry

The role of **D-branes**

• A *W-brane* pinned between successive windings of the fivebrane helix has monodromy allowing parametrically soft excitations – due to the source helix, open strings live on a *k*-fold cover of the ϕ circle.



One expects these *W-branes* to be precursors of the 'long string sector' which arises in the BH regime of the *spacetime* CFT dual to AdS₃. The *"long string"* sector of the CFT is a particular class of little strings.

the emerging picture:

The geometrical regime of smooth "capped throat" microstate geometries is separated from the BH regime by a phase transition that is similar in many respects to the standard paradigm of gauge/gravity duality:

- > At weak coupling, this is "deconfinement" of nonabelian brane dof's
- > These dof's lead to a breakdown of the supergravity approximation
- > We are starting to see this structure on the **gravity** side of the duality
- > The new dof's *are* the entropic dof's of the BH.
- > The expected scale of their quantum wavefn is the horizon scale

Paradox lost?

• We believe that the incoherent Hawking process is replaced by *coherent* emission from long/little strings, so long as you keep track of their dof's:



• Perhaps we don't need tiny wormholes, non-local interactions, *etc etc*, if we simply accept that the entropic dof's are quantum coherent over the horizon scale ... and radiate from its surface as an ordinary blackbody.

Paradox lost?

- But how is this consistent with causality ???
- Because ... the Penrose diagram is *LEFT-ist propaganda*



Just because *LEFT* probes follow particular trajectories does **NOT** mean that **ALL** objects in the theory evolve that way.

• The *LEFT-ist* Penrose diagram presupposes what is causal evolution of entropic dof's. We already saw an example where *LEFT* geometry of the supertube cap yields a wrong picture of dynamics

Intuition from correspondence transitions

- Perturbative strings below the (Hor-Polch '96) correspondence transition don't source causal horizons even though a classical string of that energy coupled to classical GR would collapse to a BH. The spectrum in this regime consists of highly excited F1 strings and there are no BH states.
- The correspondence transition in AdS_3 differs from flat spacetime: It is a function of R_{AdS}/ℓ_s rather than $E\ell_s$. At the correspondence point $R_{AdS} = \ell_s$ String/BH entropies match for *all* energies (*Giveon-Kutasov-Rabinovici-Sever '05*)
- (BH's are not normalizable states in AdS_3 for $R_{AdS} < \ell_s the$ high energy density of states is the Hagedorn spectrum of perturbative strings)
- The F1 correspondence transition may provide intuition about *little strings*.
 Idea: Long/little strings remain coherent at the horizon scale because they are at their correspondence point their entropy matches S_{BH}

A horizon scale data storage device

- Propose similar behavior for *long/little strings* with the transition point scaled by n_5 due to reduced string tension $1/(n_5\alpha')$. This idea is consistent with BH entropy, and the fact that the AdS₃ curvature radius $(R_{AdS})^2 = n_5\alpha' = (\ell_s^2)_{little}$
- The horizon scale would then characterize the support of the long string wavefunction. We hope to see the formation of this scale from the properties of Coulomb branch *W*-branes near the BH transition
- Is there an analogue of FZZ duality, where localized low-energy F1 probes see the BH geometry of LEFT, while little strings see a smooth geometry with a little string condensate (having no horizon or singularity)?
- Aside: SYK has $R_{AdS} \approx \ell_s$. Can we find similar structures? While there are no objects localized smaller than the string=AdS scale, can still look for "stringy" delocalization of constituent dof's in the radial direction, with quantum coherence over the horizon scale. Need a picture of SYK bulk (non)locality.

Long string properties inferred from LEFT

- Wald formalism applies to the inner horizon. Near extremality, most of entropy lies at the inner horizon $(S_L - S_R \text{ out of the total } S_L + S_R)$ (Cvetic-Larsen '97)
- The *inner horizon is singular* even in GR (Marolf-Ori '12). ST should resolve this.
- While LEFT probes must fall into the inner horizon and get shredded into *little strings*, the *long string* stably lives in the BH interior
- Naive interpretation: S_R, and an equal number of dof's from S_L, lurk near the LEFT "horizon" waiting to become Hawking radiation (Das-Mathur '96, Callan-Maldacena '96)



Aspects of interior design

- The *firewall* question can be recast as an issue of the response function of the little string fluid: Stiff ↔ firewall vs. Soft/elastic ↔ vacuum
- Several facts hint at the existence of the naïve LEFT region behind the horizon, at least down to r ≈ r_:
 - The matching between thermodynamic properties of the little string and geometric quantities involving **both** horizon scales *r*_±
 - > Strings have an aversion to hard scattering when hit with momentum much larger than the scale of the string tension $(1/n_5\alpha')$ for little strings)
 - Scattering off the perturbative string condensate at the supertube cap also shows soft behavior at momentum well above the (F1) string scale
- If the region inside *r* = *r*₊ is dominated by a little string condensate, supergravitons are quasiparticles and do not have a separate "vacuum". The "partner" of a Hawking "particle" is a "hole" in the condensate. The Hawking calculation is *mean field theory*, which loses correlations.

What we have achieved so far...

To summarize:

- Two-charge bound states involving NS5-branes live on the Coulomb branch of *'little string theory'*
- A particularly nice class of examples involves an *exactly solvable* worldsheet CFT based on null-gauged WZW models
- A rich structure of excitations, many hidden from supergravity, is now amenable to quantitative analysis
- Horizon formation is a Coulomb/Higgs type phase transition where 'W-branes' a.k.a. long strings become light and dominate dynamics

Next steps:

- Much to do! Perturbative closed string spectrum now understood
- D-brane spectrum, Closed string S-matrix, absorption/emission processes, etc.