Potential Application of "SYK Physics" in Condensed Matter Systems

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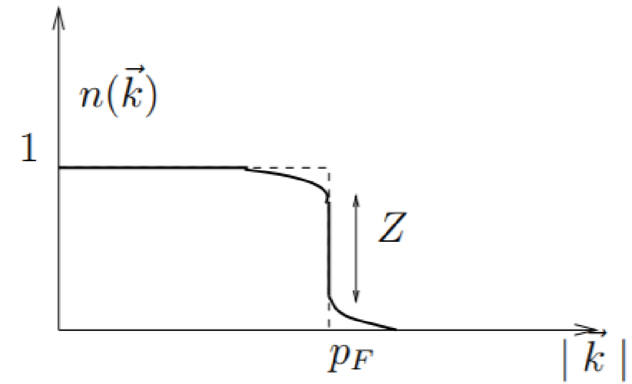
Introduction:

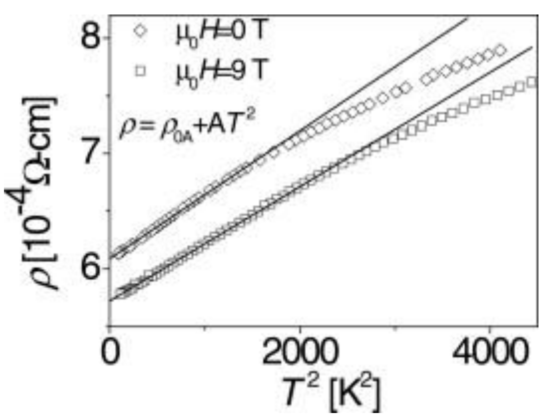
Fermi liquid: well-defined fermi surface, quasi-particles are more and more like noninteracting fermions closer to the Fermi surface.



$$G^c(\omega, k) = \frac{Z_k}{\omega - \varepsilon_k^{(0)} + i\delta \,\mathrm{sign}\,\omega} +$$

$$\varepsilon_k^{(0)} = \varepsilon_k - \mu + \mathrm{Re}\,\Sigma_{\varepsilon_k^{(0)}}(k) \qquad |\,\mathrm{Im}\,\Sigma_{\varepsilon_k^{(0)}}(k)| \ll \varepsilon_k^{(0)}$$

The IR fixed point of FL has a lot of conserved quantities: particle number at momentum k (at the fermi surface) is conserved.
Phenomenological signature: $\rho \sim \rho_0 + T^2$.



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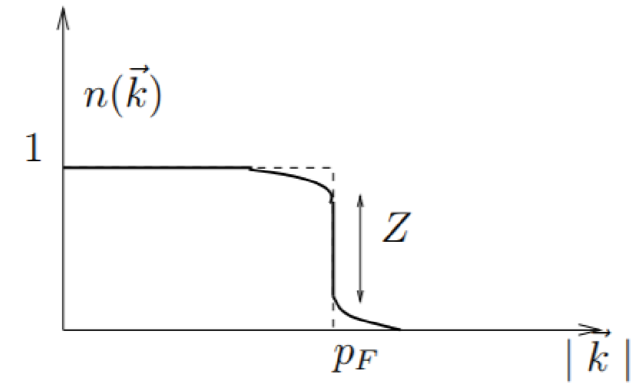

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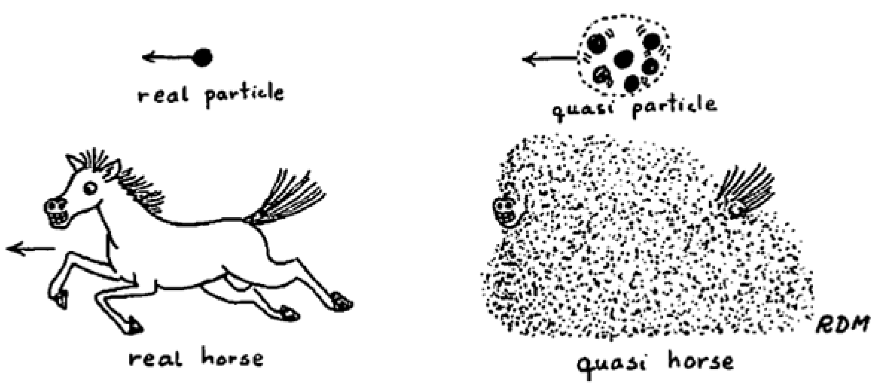

Fig. 0.4 *Quasi Particle Concept*

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Shankar RG:

Focus on 2d. Considering particles 1,2 interact and scatter into particle 3,4, By rescaling the energy scale (energy shell) near a fermi surface, in the IR only the following cases survive:

Case I: $\mathbf{\Omega}_3 = \mathbf{\Omega}_1$ (hence $\mathbf{\Omega}_2 = \mathbf{\Omega}_4$) ,

Case II: $\mathbf{\Omega}_3 = \mathbf{\Omega}_2$ (hence $\mathbf{\Omega}_1 = \mathbf{\Omega}_4$) ,

Case III: $\mathbf{\Omega}_1 = -\mathbf{\Omega}_2$ (hence $\mathbf{\Omega}_3 = -\mathbf{\Omega}_4$)

$$\frac{du}{dt} = \frac{u^2}{2\pi}$$

Case I and II do not change the particle number at each momentum at the FS, case III turns out to be marginally relevant after one-loop correction, which leads to BCS superconductor instability. In other words, we can say that the Fermi liquid is the "parent state" of ordinary BCS superconductor (what about SYK state?).

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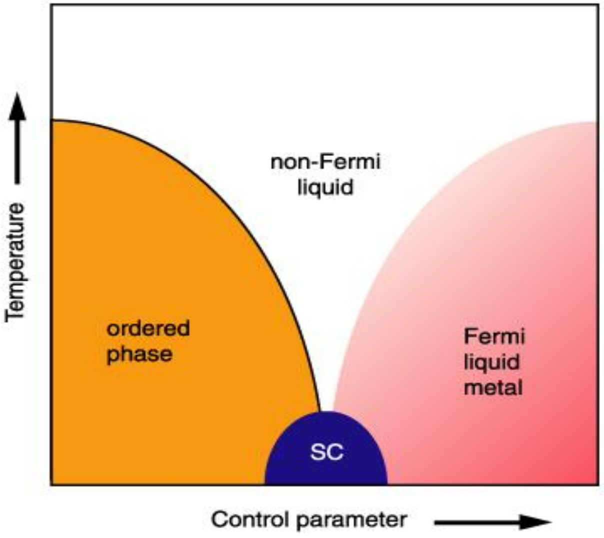
What is non-fermi liquid? Why is it important?

Human nature demands us pursue states beyond fermi liquid: non-fermi liquid

no well-defined fermi sea; fermions are never noninteracting; appears to be the parent state of many superconductors.



One way to get non-fermi liquid: start with fermi liquid, increase its interaction with other "particles", such as quantum critical fluctuations.

The hope is that, the interaction between quantum critical fluctuation and quasi-particle will make it short-lived even close to the fermi energy. But this is a notoriously difficult (but fun!) problem.

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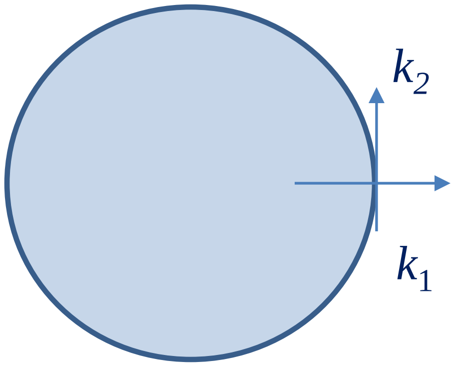
$$S = \sum_{s=\pm}\sum_{j=1}^{N}\int \frac{d^3k}{(2\pi)^3}\psi^\dagger_{s,j}(k)\left[ik_0 + sk_1 + k_2^2\right]\psi_{s,j}(k)$$
$$+ \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\left[k_0^2 + k_1^2 + k_2^2\right]\phi(-k)\phi(k)$$
$$+ \frac{e}{\sqrt{N}}\sum_{s=\pm}\sum_{j=1}^{N}\int \frac{d^3k d^3q}{(2\pi)^6}\,\lambda_s\,\phi(q)\,\psi^\dagger_{s,j}(k+q)\psi_{s,j}(k).$$



Power-counting: the boson fermion coupling constant *e* has scaling dimension ½, it is a relevant perturbation on the noninteracting theory.

Can *e* flow to a interacting fixed point? A controlled expansion needed.

1, fix *N*, change the dispersion of the boson field to a nonanalytic form (Nayak, Wilczek, 1994): $|k|^{1+\epsilon}|\phi(k)|^2$

Now the scaling dimension of *e* is ε/2, expanding with "small" ε, eventually extrapolating ε back to 1.

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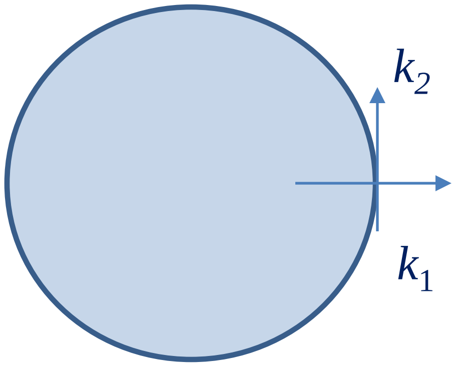

Power-counting: the boson fermion coupling constant *e* has scaling dimension ½, it is a relevant perturbation on the noninteracting theory.

Can *e* flow to a interacting fixed point? A controlled expansion needed.

2, large-*N*, change the dispersion of the boson field to a nonanalytic form:

$$|k|^{1+\epsilon}|\phi(k)|^2$$

Fix *Nε* a constant, a double expansion 1/*N* and *ε*. (Mross, et.al. 2010)

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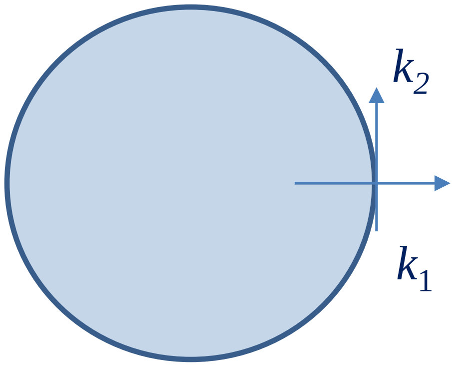
$$
\begin{aligned}
S =& \sum_{s=\pm}\sum_{j=1}^{N}\int \frac{d^3k}{(2\pi)^3}\psi^{\dagger}_{s,j}(k)\left[ik_0 + sk_1 + k_2^2\right]\psi_{s,j}(k) \\
&+ \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\left[k_0^2 + k_1^2 + k_2^2\right]\phi(-k)\phi(k) \\
&+ \frac{e}{\sqrt{N}}\sum_{s=\pm}\sum_{j=1}^{N}\int \frac{d^3k d^3q}{(2\pi)^6}\,\lambda_s\,\phi(q)\,\psi^{\dagger}_{s,j}(k+q)\psi_{s,j}(k).
\end{aligned}
$$



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Can *e* flow to a interacting fixed point? A controlled expansion needed.

3, Find the upper critical dimension d_c, (just like ordinary Wilson-Fisher fixed point has upper critical dimension $d_c = 3$ (D = 3+1), here $d_c = 5/2$), do a $\varepsilon = d_c - d$ expansion (Lee, 2013).

Many other important works, like Polchinski, metlitski&Sachdev…...

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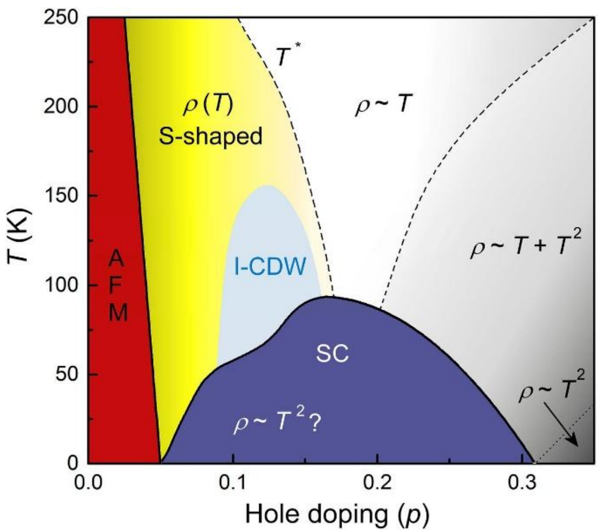
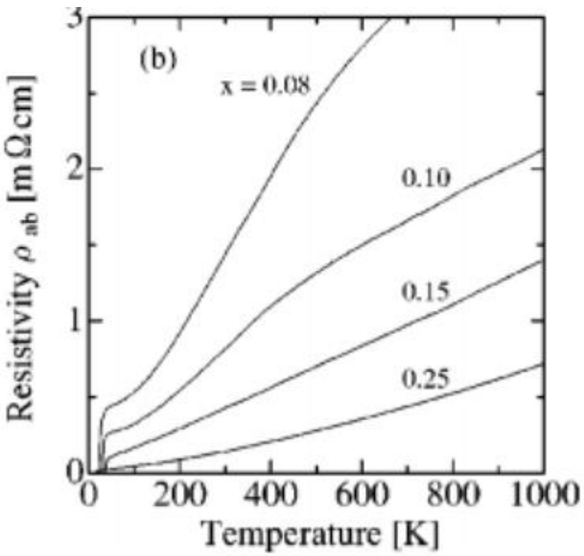
All the previous efforts start with Fermi liquid, and perturb on it with a (relevant) interaction with quantum critical fluctuation, and get a state non-Fermi liquid.





The most famous non-fermi liquid is the strange metal phase in High Tc cuprates. Could it be a generic non-Fermi liquid state which is the parent state of High Tc SC?

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We may need a model for a generic non-Fermi liquid state to explain the strange metal (the original motivation of S and Y). SYK type of model falls precisely in this category.

Single cluster SYK model:

$$G_{free}(\tau) = \frac{1}{2}\mathrm{sgn}(\tau) \ , \qquad G_{free}(\omega) = -\frac{1}{i\omega}$$

$$G(\tau) = \left(\frac{1}{4\pi}\right)^{1/4} \frac{\mathrm{sgn}(\tau)}{|J_4\tau|^{1/2}}, \qquad G(i\omega) = \pi^{1/4} \frac{i\mathrm{sgn}(\omega)}{|J_4\omega|^{1/2}}.$$

Survey: what are the most "unrealistic" ingredients of the SYK model?

1, fully random interaction; 2, all-to-all nonlocal interaction.

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Let's temporarily ignore the "unrealistic" features…

Can SYK-like models be the parent state of superconductor?

First consider a toy model perturbed away from SYK_4:

$$H = \frac{J_{ijkl}}{4!}\chi_i\chi_j\chi_k\chi_l + \frac{u}{2}C_{ij}C_{kl}\chi_i\chi_j\chi_k\chi_l$$

$$\begin{aligned} \mathcal{S}_{eff} &= \int d\tau \frac{1}{2}\sum_i \chi_i\partial_\tau\chi_i + \sum_{ijkl}\left\{\frac{J_{ijkl}}{4!}\chi_i\chi_j\chi_k\chi_l + \frac{u}{2}C_{ij}C_{kl}\chi_i\chi_j\chi_k\chi_l\right\} \\ &= \int d\tau \left(\frac{1}{2}\chi_i\partial_\tau\chi_i + \frac{u}{2}b^2 - iuC_{jk}b\chi_j\chi_k\right) + \frac{J_{ijkl}}{4!}\chi_i\chi_j\chi_k\chi_l \end{aligned}$$

Approach #1: Convert to G, Σ action, follow mean field procedure, looking for a long range correlation (ordered phase with spontaneous *T*-symmetry breaking) $\langle b(\tau_1)b(\tau_2)\rangle = Nw^2$

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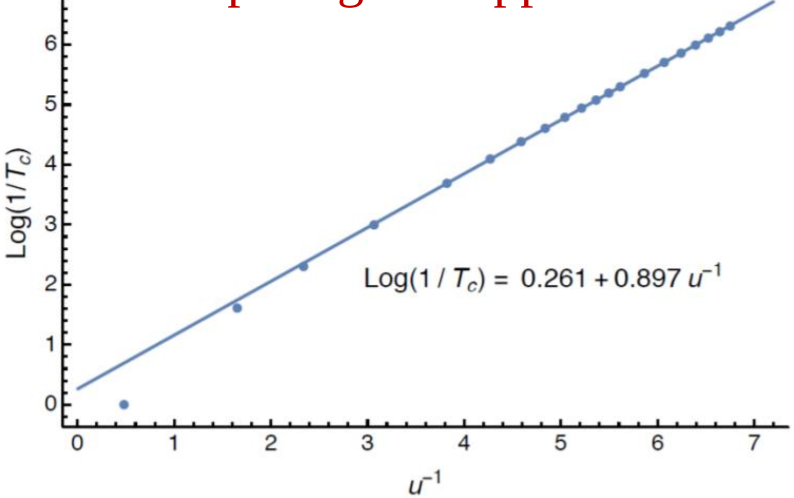
Approach #2: one-loop RG like the Shankar RG:

$$\beta(u) = \frac{du}{d\ln l} = \frac{2J^2}{\sqrt{\pi}J_4}u^2.$$

RG equation defines a "transition energy scale":

$$T_c \sim \Lambda \exp\left(-\frac{\sqrt{\pi}J_4}{2J^2 u}\right)$$

Comparing two approaches:



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Approach #3: exact diagonalization of small system size, consistent with the previous two approaches.

This toy model shows that the SYK model is instable against marginally relevant four fermion interaction perturbations, which may lead to fermion "pair condensate" which spontaneously breaks some symmetry. Thus the SYK-like state has the potential of becoming the parent state of superconductor.

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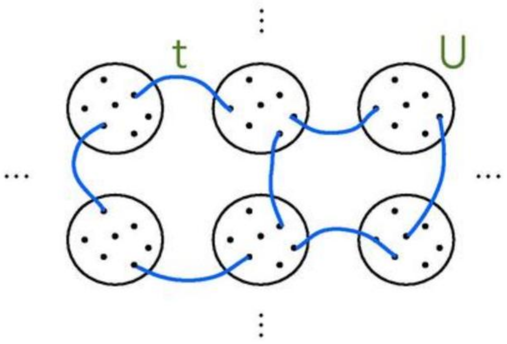
What about the linear-*T* resistivity of strange metal?

$$\mathcal{H} = \sum_{x} \sum_{i<j,k<l} U_{ijkl,x} c^{\dagger}_{ix} c^{\dagger}_{jx} c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^{\dagger}_{i,x} c_{j,x'}$$

A toy model with A random single particle hopping connected SYK clusters, with large-*N* number of complex fermions per cluster. (Song, Jian, Balents, 2017)



Assuming *U* dominates *t*, the physics for a large energy range is dominated by the SYK physics, i.e. the scaling dimension of fermion operator is 1/4, so the electric current operator (a fermion bilinear) has scaling dimension 1/2. The Kubo formula directly leads to the linear–*T* dc resistivity in the temperature window $T \sim (t^2/U, U)$.

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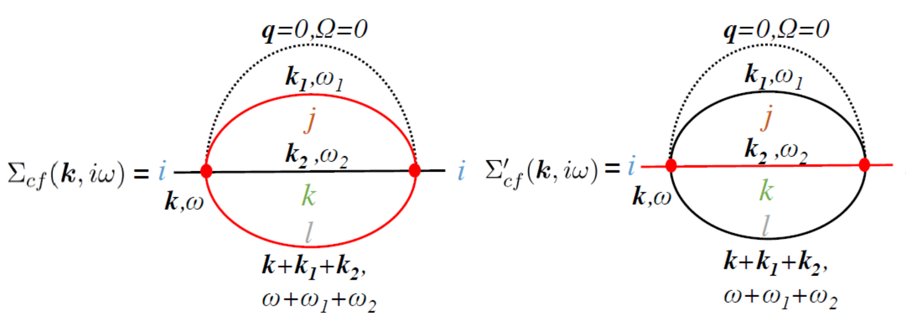
Since then, a plethora of models based on connected SYK clusters were proposed:

Examples, model for marginal fermi liquid: itinerant electron interacting with localized SYK fermions, Patel, et.al. 2017, Chowdhury, et.al. 2018. The latter model has random interaction but translation invariance.

$$
\begin{aligned}
H = & -t \sum_{\langle rr' \rangle;\ i=1}^{M} \left(c_{ri}^{\dagger} c_{r'i} + \mathrm{h.c.} \right) - \mu_c \sum_{r;\ i=1}^{M} c_{ri}^{\dagger} c_{ri} - \mu \sum_{r;\ i=1}^{N} f_{ri}^{\dagger} f_{ri} \\
& + \frac{1}{NM^{1/2}} \sum_{r;\ i,j=1}^{N} \sum_{k,l=1}^{M} g_{ijkl}^{r} f_{ri}^{\dagger} f_{rj} c_{rk}^{\dagger} c_{rl} + \frac{1}{N^{3/2}} \sum_{r;\ i,j,k,l=1}^{N} J_{ijkl}^{r} f_{ri}^{\dagger} f_{rj}^{\dagger} f_{rk} f_{rl} .
\end{aligned}
$$



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$$H = -t \sum_{\langle rr' \rangle;\ i=1}^{M} (c^\dagger_{ri} c_{r'i} + \text{h.c.}) - \mu_c \sum_{r;\ i=1}^{M} c^\dagger_{ri} c_{ri} - \mu \sum_{r;\ i=1}^{N} f^\dagger_{ri} f_{ri}$$

$$+ \frac{1}{NM^{1/2}} \sum_{r;\ i,j=1}^{N} \sum_{k,l=1}^{M} g^r_{ijkl} f^\dagger_{ri} f_{rj} c^\dagger_{rk} c_{rl} + \frac{1}{N^{3/2}} \sum_{r;\ i,j,k,l=1}^{N} J^r_{ijkl} f^\dagger_{ri} f^\dagger_{rj} f_{rk} f_{rl}.$$

$$\Sigma^c(i\omega_n) \to \frac{i g^2 \nu(0)}{2J \cosh^{1/2}(2\pi\mathcal{E}) \pi^{3/2}} \omega_n \ln\left(\frac{|\omega_n| e^{\gamma_E - 1}}{J} \right)$$

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Battle between solubility and reality:
why SYK model (and its generalizations) seems unrealistic to cuprates?

1, fully random (Gaussian distributed) interaction;

2, large-N number of degrees of freedom on each unit cell, while in cuprates every site only has one active orbital of electron;

Can we construct a model that is "close" to real systems, i.e. no random interaction, one orbital of electron on each site, but may still have all the desired physics of the SYK model and its generalizations at least in a finite energy window?

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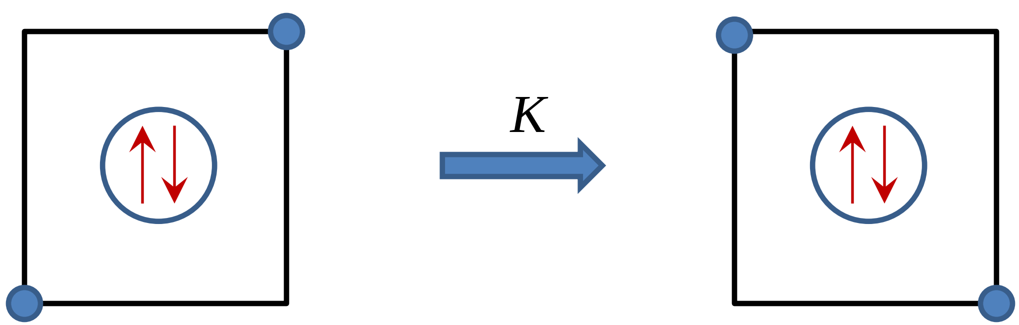
The key term of the minimal version of our model:

$$H = \sum_j H_j,$$

$$H_j = U\hat{n}_j^2 + \sum_{\hat{e}=\hat{x},\hat{y}} J\left(\vec{S}_j \cdot \vec{S}_{j+\hat{e}} - \frac{1}{4}\hat{n}_j\hat{n}_{j+\hat{e}}\right)$$

$$- K\left(\epsilon_{\alpha\beta}\epsilon_{\gamma\sigma}c^\dagger_{j,\alpha}c^\dagger_{j+\hat{x}+\hat{y},\beta}c_{j+\hat{y},\gamma}c_{j+\hat{x},\sigma} + H.c.\right)$$

In addition to the standard Hubbard and Heisenberg interaction, this term has a single plaquette spin singlet ring exchange. It has one orbital/site, and preserves all the symmetries of square lattice,



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&\quad - K\left(\epsilon_{\alpha\beta}\epsilon_{\gamma\sigma}c^\dagger_{j,\alpha}c^\dagger_{j+\hat{x}+\hat{y},\beta}c_{j+\hat{y},\gamma}c_{j+\hat{x},\sigma} + H.c.\right)
\end{aligned}
$$

In addition to the standard Hubbard and Heisenberg interaction, this term has a single plaquette spin singlet ring exchange. It has one orbital/site, and preserves all the symmetries of square lattice, With certain relation between U, J, K, the Hamiltonian takes a new form (with N=3, M=2).

$$
\begin{aligned}
H = \sum_j \sum_{r,r'=-(N-1)/2}^{(N-1)/2} \sum_{\alpha,\beta,\gamma,\sigma=1}^{M} -\frac{g\eta_{r,r'}}{N\sqrt{M}} \\
\times\ \mathcal{J}_{\alpha\beta}\mathcal{J}_{\gamma\sigma}c^\dagger_{j_x,j_y,\alpha}c^\dagger_{j_x+r,j_y+r',\beta}c_{j_x,j_y+r',\gamma}c_{j_x+r,j_y,\sigma}
\end{aligned}
$$

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In the large-*N*, *M* limit……

$$H=\sum_{j}\sum_{r,r'=-(N-1)/2}^{(N-1)/2}\sum_{\alpha,\beta,\gamma,\sigma=1}^{M}-\frac{g\eta_{r,r'}}{N\sqrt{M}}$$
$$\times\ \mathcal{J}_{\alpha\beta}\mathcal{J}_{\gamma\sigma}c^{\dagger}_{j_x,j_y,\alpha}c^{\dagger}_{j_x+r,j_y+r',\beta}c_{j_x,j_y+r',\gamma}c_{j_x+r,j_y,\sigma}$$

has the similar solution as a three index tensor fermion model:

$$H_2^t\ =\ -\frac{g}{(N_aN_bN_c)^{1/2}}\mathcal{J}_{c_1,c_1'}\mathcal{J}_{c_2,c_2'}c^{\dagger}_{a_1b_1c_1}c^{\dagger}_{a_2b_2c_1'}c_{a_1b_2c_2}c_{a_2b_1c_2'},$$

Similar large-*N* solution as the SYK model, localized Green's function (center of mass conservation), local QCP scaling.

$$G(\tau)\ =\ -\mathcal{B}(\theta)\,e^{-2\pi T\mathcal{E}\tau}\sqrt{\frac{\pi T}{2g\sin(\pi T\tau)}},$$

$$G(i\omega)_{T=0}\ =\ \frac{\mathcal{B}(\theta)}{\sin\left(\frac{\pi}{4}+\theta\right)}\frac{e^{-i\mathrm{sgn}[\omega]\left(\frac{\pi}{2}+\theta\right)}}{|2g\omega|^{\frac{1}{2}}},$$

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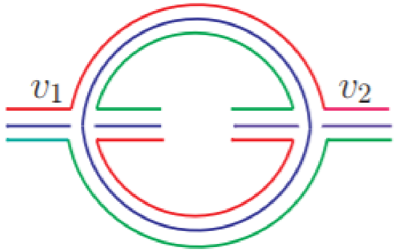
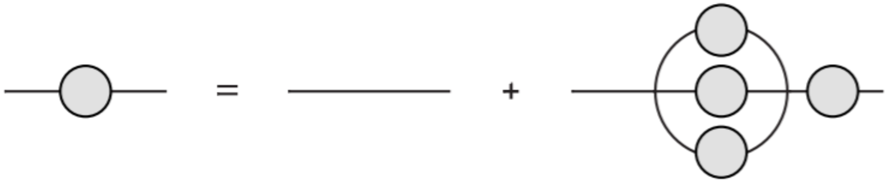
3-index fermion tensor model (simplest Majorana fermion version Gurau 2009, Witten 2016, Klebanov, Tarnopolsky 2016):

$$S = \int dt \Big(\frac{i}{2} \psi^{abc} \partial_t \psi^{abc} + \frac{1}{4} g \psi^{a_1 b_1 c_1} \psi^{a_1 b_2 c_2} \psi^{a_2 b_1 c_2} \psi^{a_2 b_2 c_1} \Big)$$





Leading large-*N* order diagrams can be summed exactly.
1/*N* correction, diagrammatic, Goldstone modes…...?
The original SY model has a finite temperature phase governed by the large-*N* solution until $T_c^f \sim J e^{-\sqrt{N}}$ (Georges, Parcollet, Sachdev 2001)

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If the "SYK physics" persists in a finite energy window:

1, Like many other non fermi liquid state constructed with SYK clusters, linear-*T* resistivity is expected using the large-*N* solution.

Define the real space current operator by turning on a hopping term (with both nearest neighbor and next neighbor), and then couple to the external EM gauge field:

$$\mathrm{Re}\sigma\left(\omega\right)=\frac{\sqrt{\pi}t^{2}}{4gT}\Upsilon_{\sigma}\left(\mathcal{Q},\omega/T\right)$$

$$\Upsilon_{\sigma}\left(\mathcal{Q},\omega/T\right)=\sqrt{\cos\left(2\theta\left(\mathcal{Q}\right)\right)}\frac{\tanh\left(\omega/2T\right)}{\omega/2T}$$

Similar local spin/charge density correlation as the SY model, and marginal FL.

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2, with extra four-fermion interactions, say a nearest neighbor interaction as following (mixture of Heisenberg spin and charge density interaction), there is a "pairing instability"

$$H_u = \sum_{<i,j>} -\frac{u}{2M}\left(\Delta^\dagger_{i,j}\Delta_{i,j} + \Delta_{i,j}\Delta^\dagger_{i,j}\right) \qquad \Delta_{i,j} = \mathcal{J}_{\alpha\beta} c_{i,\alpha} c_{j,\beta}$$

Generalizing to the fermion tensor model, it reads

$$H_u \sim -\frac{u}{M}\left(\Delta^\dagger_{a_1 b_1, a_2 b_2}\Delta_{a_1 b_1, a_2 b_2} + \Delta_{a_1 b_1, a_2 b_2}\Delta^\dagger_{a_1 b_1, a_2 b_2}\right)$$

$$\Delta = \mathcal{J}_{c_1, c_1'} c_{a_1 b_1 c_1} c_{a_2 b_2 c_1'}$$

(a_1, b_1), (a_2, b_2) are neighbors. Evaluated at the "SYK" fixed point, it is marginally relevant (u>0) due to the most dominant one loop diagram (BCS diagram) with large N_a, N_b, N_c (M).

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$$\frac{du}{d\ln l} = \frac{u^2}{\sqrt{g^2\pi\cos(2\theta)}}.$$

u becomes nonperturbative and lead to pairing instability at temperature *T**

$$T^* \sim g\exp\left(-\sqrt{\pi\cos(2\theta)}\frac{g}{u}\right)$$

In the SYK model, changing sign of a similar *u* term drives a transition between the maximally chaotic phase to a weakly chaotic phase (Bi, et.al. 2017, Jian, et.al. 2017).

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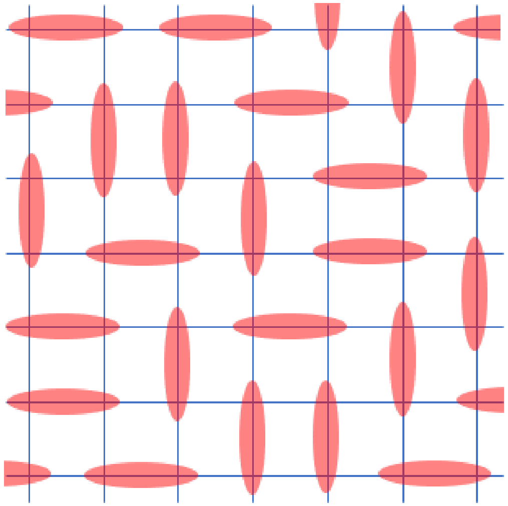
3, when both the *u*-interaction term and the hopping term are treated as perturbations on the *g*-term, there is competition between them.

$$H_u = \sum_{<i,j>} -\frac{u}{2M}\left(\Delta^\dagger_{i,j}\Delta_{i,j} + \Delta_{i,j}\Delta^\dagger_{i,j}\right) \qquad H_t = \sum_{<i,j>,\alpha} -tc^\dagger_{i,\alpha}c_{j,\alpha}$$

When the *u* term is renormalized strong before the hopping term, the system forms pairing before global phase coherence.

Similar to the Rokhsar's theorem for SU(*M*) spin systems, in the large-*M* limit, ground states of the H_u term are the "maximal nearest neighbor dimer covering" of Sp(*M*)-singlet, i.e. $\langle\Delta_{ij}\rangle = \Delta$, and highly degenerate. We identify this phase as the pseudo-gap phase.



Potential Application of "SYK Physics" in Condensed Matter Systems

3, when both the *u*-interaction term and the hopping term are treated as perturbations on the *g*-term, there is competition between them.

$$H_u = \sum_{<i,j>} -\frac{u}{2M}\left(\Delta^\dagger_{i,j}\Delta_{i,j} + \Delta_{i,j}\Delta^\dagger_{i,j}\right) \qquad H_t = \sum_{<i,j>,\alpha} -tc^\dagger_{i,\alpha}c_{j,\alpha}$$

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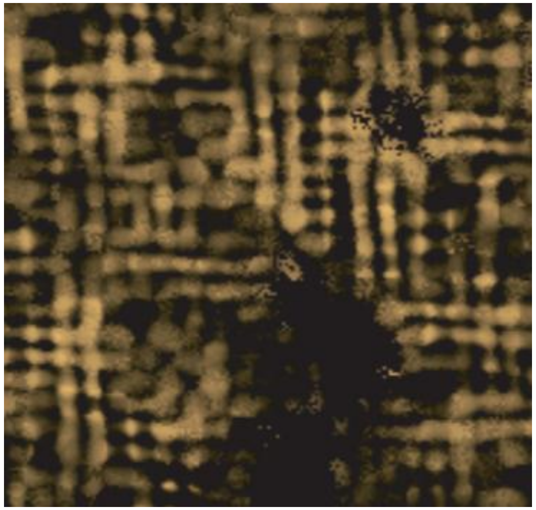

Kohsaka, et.al. 2008

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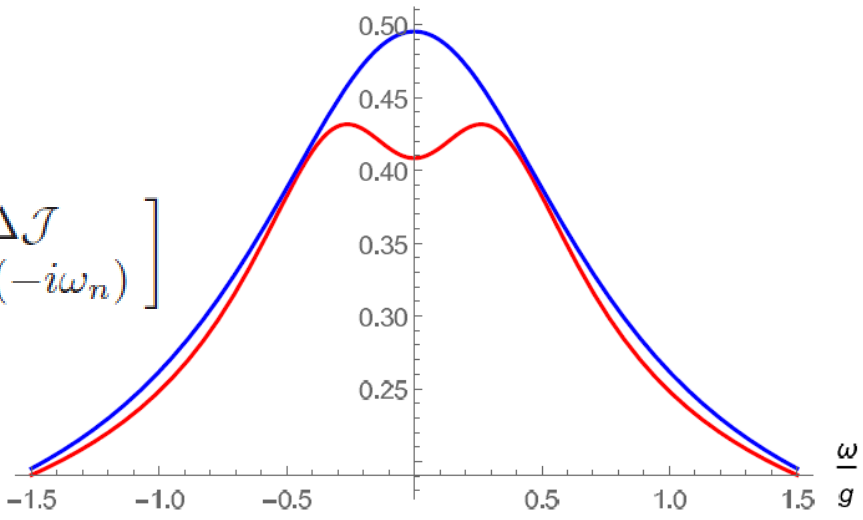
4, In the pseudo gap phase, the center of mass is still conserved, the fermion Green's function is localized on a link, and then the fermion Green's function can be computed on each link occupied by a "dimer", and the local density of states explicitly shows a "pseudo gap".

$$\Psi = \left(c_{1,\alpha}\ ,\ c_{2,\alpha}^{\dagger}\right)$$

$$\mathcal{G}^{-1}\left(i\omega_n\right) = \begin{bmatrix} G^{-1}\left(i\omega_n\right) & \frac{u}{M}\Delta\mathcal{J} \\ \frac{u}{M}\Delta^{*}\mathcal{J}^{\mathrm{T}} & -G^{-1}\left(-i\omega_n\right) \end{bmatrix}$$



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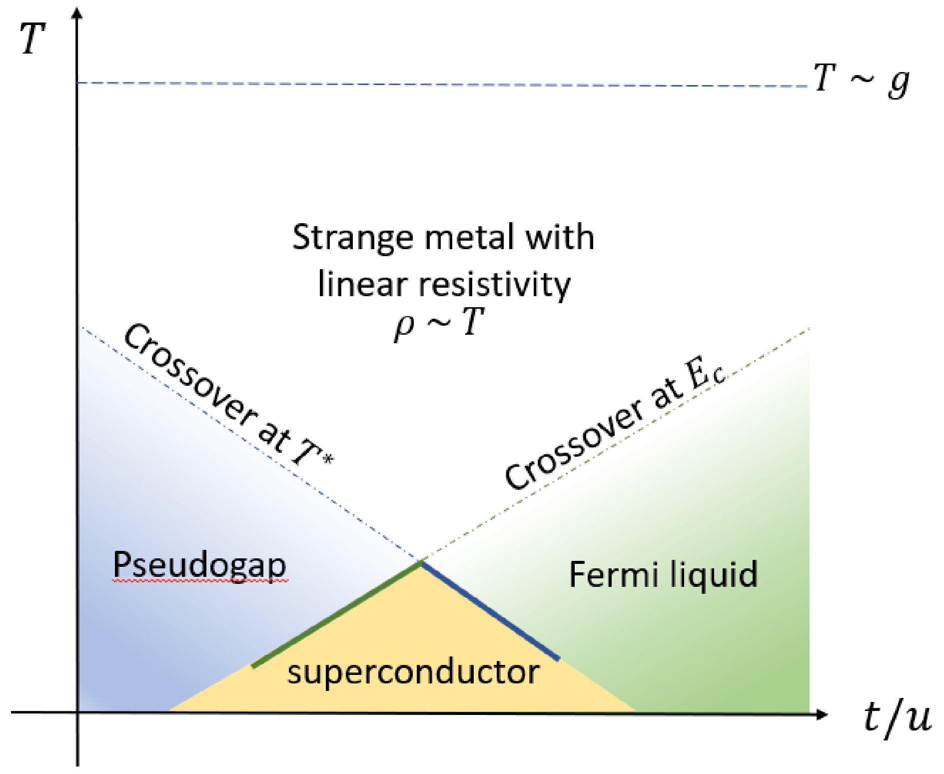
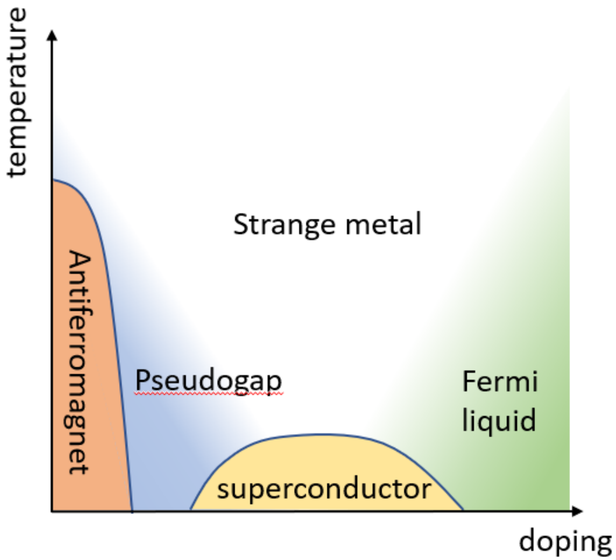
5, when the hopping term becomes relevant first lowering temperature, then the *u* term will again be marginally relevant using the standard RG of Fermi liquid, and lead to a *d*-wave SC on the square lattice.

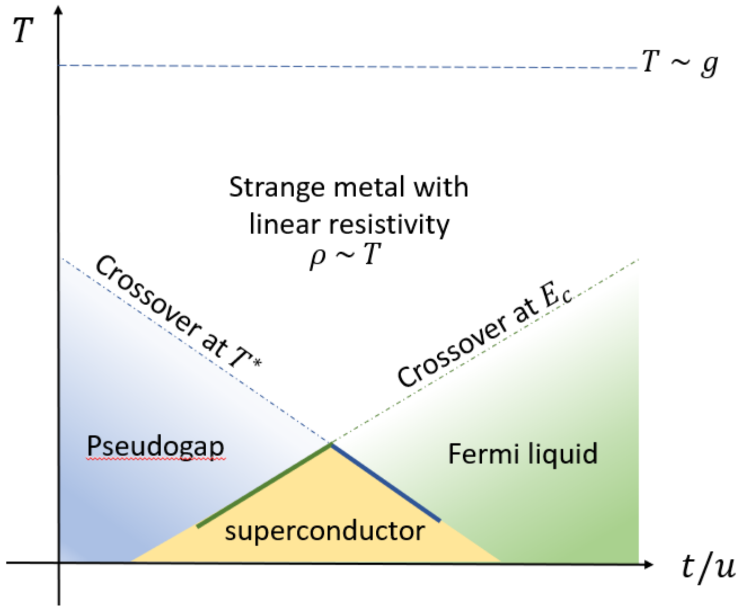


Potential Application of "SYK Physics" in Condensed Matter Systems

Summary:

A relatively realistic model (free of randomness, one orbital per site) that potentially possesses the desired SYK like physics. It may describe the strange metal phase in a finite energy (temperature) window, and it is the parent state of high T_c SC, and pseudogap phase arXiv:1802.04293





Potential Application of “SYK Physics” in Condensed Matter Systems

To do:

1, Fermi arc?

2, scaling between resistivity and magnetic field?

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