Turbulent suspensions of heavy particles

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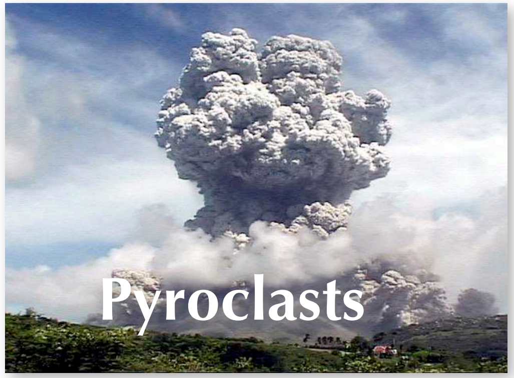
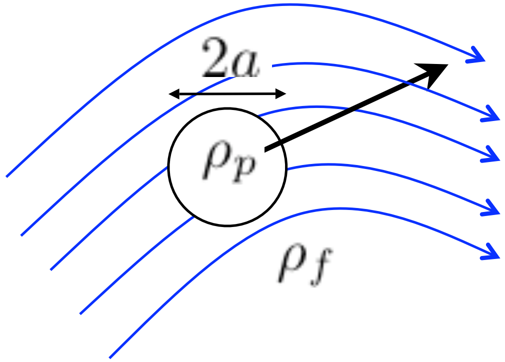
Particle laden flows



- **Finite-size** and **mass** impurities transported by turbulent flow

Dispersed particles

- **Passive suspensions:** no feedback of the transported particles onto the fluid flow.
- **Rigid spherical particles** that are assumed
  - ∗ much smaller than the smallest active scale of the flow (Kolmogorov η)
  - ∗ associated with a very small Reynolds number



$$m_p = \frac{4}{3}\pi\rho_p a^3$$

$\Rightarrow$ **Surrounding flow = Stokes flow**
Maxey & Riley (1983)

$$m_p\ddot{\boldsymbol{X}} = m_f\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t}(\boldsymbol{X},t) - 6\pi a\mu[\dot{\boldsymbol{X}} - \boldsymbol{u}(\boldsymbol{X},t)] - \frac{m_f}{2}\left[\ddot{\boldsymbol{X}} - \frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{u}(\boldsymbol{X},t))\right] - \frac{6\pi a^2\mu}{\sqrt{\pi\nu}}\int_0^t \frac{\mathrm{d}s}{\sqrt{t-s}}\frac{\mathrm{d}}{\mathrm{d}s}[\dot{\boldsymbol{X}} - \boldsymbol{u}(\boldsymbol{X},s)].$$

Very heavy particles

- Spherical particles much smaller than the Kolmogorov scale η, much heavier than the fluid, feeling no gravity, evolving with moderate velocities: **one of the simplest model**

$$\ddot{\boldsymbol{X}} = -\frac{1}{\tau}\left(\dot{\boldsymbol{X}} - \boldsymbol{u}(\boldsymbol{X}, t)\right)$$

↑ Prescribed velocity field (random or solution to NS)

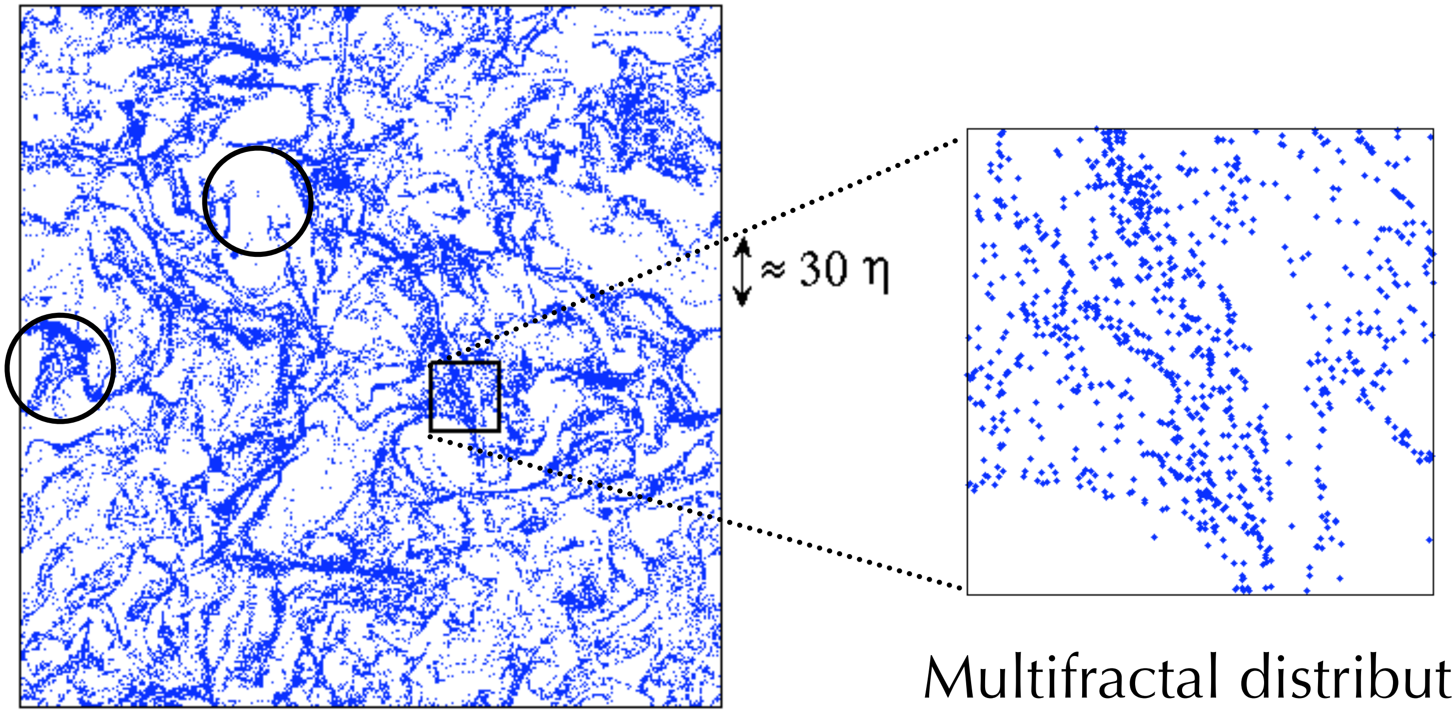
2 parameters

$$\begin{cases} \mathrm{St} = \tau/\tau_\eta \\ \mathrm{Re} = UL/\nu \end{cases}$$

- **Dissipative dynamics** (even if $\boldsymbol{u}(\boldsymbol{x}, t)$ is incompressible) Lagrangian averages correspond to an SRB measure that depends on the realization of the fluid velocity field.

Clustering of inertial particles

- **Important for**
  - ∗ the rates at which particles interact (collisions, chemical reactions, gravitation…)
  - ∗ the fluctuations in the concentration of a pollutant
  - ∗ the possible feedback of the particles on the fluid



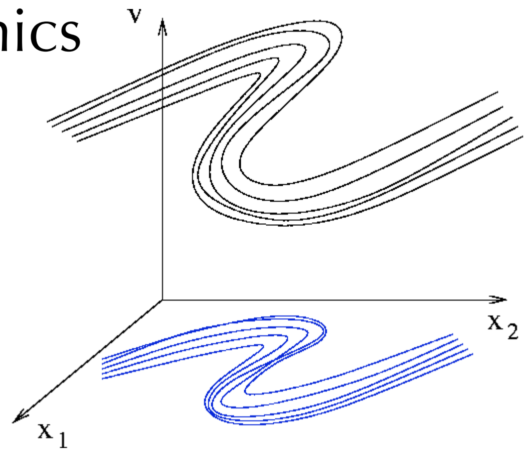
Inertial-range clusters and voids

Multifractal distribution at dissipative scales

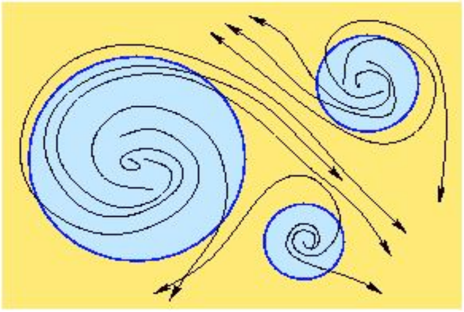
Phenomenology of clustering

- **Different mechanisms:**

Dissipative dynamics
⇒ **attractor**



Ejection from **eddies** by centrifugal forces



- **Theory:** requires elaborating models to disentangle these two effects. For instance:
  - flows with no structures (uncorrelated in time) to isolate the effects of a dissipative dynamics
  - coarse-grained closures to understand ejection from eddies

- Numerics show that these effects act at different scales

Summary of DNS

- Pseudo-spectral code, normal viscosity, parallel code (MPI+FFTW)
- Spatial resolutions 128^3, 256^3, 512^3

R_λ	$u_{\rm rms}$	ε	ν	η	L	T_E	τ_η	T_{tot}	T_{tr}	Δx	N^3	N_t	N_p	N_{tot}
185	1.4	0.94	0.00205	0.010	π	2.2	0.047	14	4	0.012	512^3	$5 \cdot 10^5$	$7.5 \cdot 10^6$	$12 \cdot 10^7$
105	1.4	0.93	0.00520	0.020	π	2.2	0.073	20	4	0.024	256^3	$2.5 \cdot 10^5$	$2 \cdot 10^6$	$32 \cdot 10^6$
65	1.4	0.85	0.01	0.034	π	2.2	0.110	29	6	0.048	128^3	$3.1 \cdot 10^4$	$2.5 \cdot 10^5$	$4 \cdot 10^6$

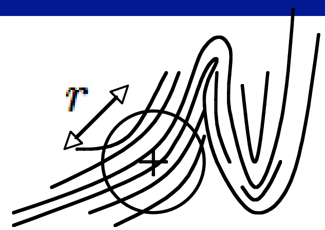
- Particle positions, velocities, fluid velocity at particle positions, fluid gradient, stored at two different rates
  - every 0.1 τ_η for $5\ 10^5$ particles / Stokes time
  - every 10 τ_η for $7.5\ 10^6$ particles / Stokes time

- Data available on the **iCFDdatabase** (http://cfd.cineca.it)

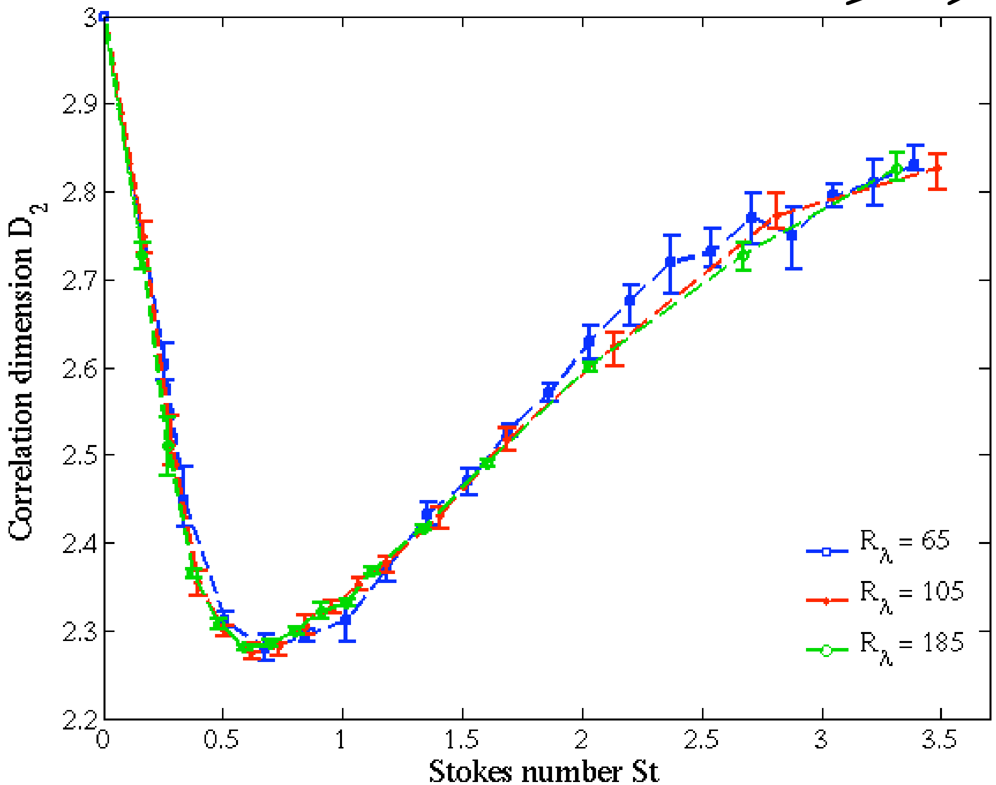
Small-scale clustering

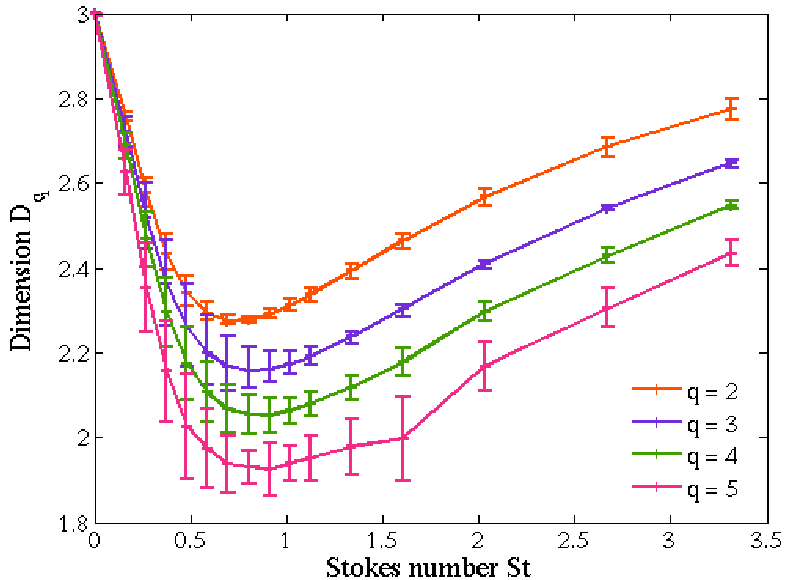
- **Fractal dimensions** $r \ll \eta$

  Coarse-grained density $\bar{\rho}_r$



$$\langle \bar{\rho}_r^p \rangle \sim r^{p(\mathcal{D}_{p+1}-d)}$$



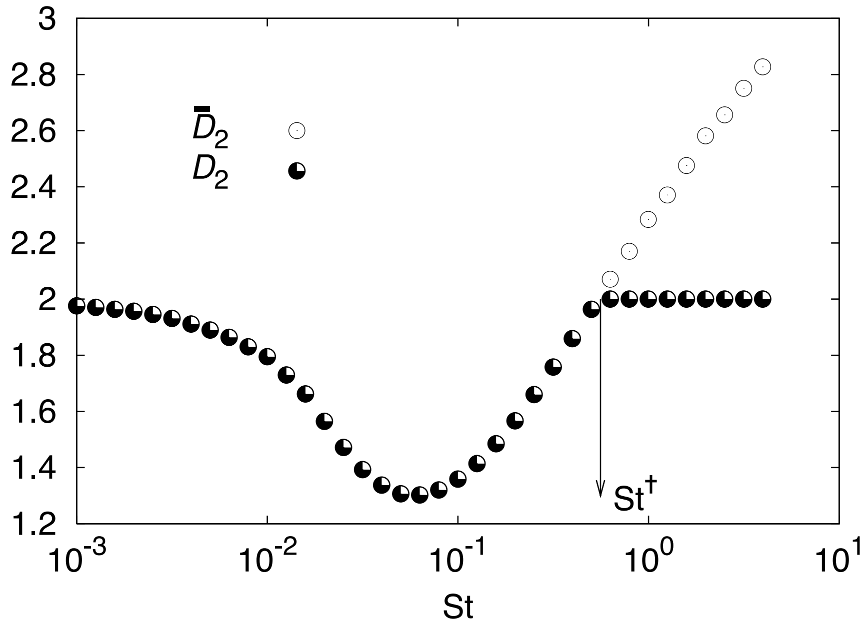


- Spectrum $\mathcal{D}_p$ is a function of St but does not depend on Re

  PDF local dimension $\delta_r = \dfrac{\ln \bar{\rho}_r}{\ln r}$ $\qquad p_r(\delta) \propto r^{\mathcal{S}(\delta,\mathrm{St})}$

Analytic attempts

- **Two-point motion:** carrier flow = smooth Kraichnan



$\mathcal{D}_2(\mathrm{St})$ has the same qualitative shape as in real flows

No analytic form yet, not even for the Lyapunov exponents

(Piterbarg, Wilkinson & Mehlig, Falkovich et al., Horvai & Fouxon)

- **Only solved case = 1D** (Derevyanko, Falkovich, Turitsyn & Turitsyn 2007)

- **Small-Stokes number asymptotics**

  WKB (Wilkinson & Mehlig 2004)

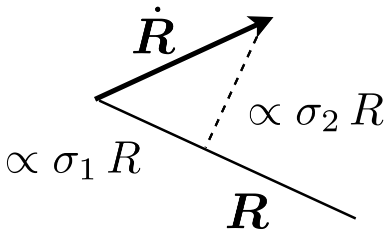
  Stochastic averaging techniques (JB, Cencini, Hillerbrand & Turitsyn 2007)

$$\mathcal{D}_2 = d - 2(d+1)(d+2)\,\mathrm{St} + O(\mathrm{St}^2)$$

Problem = non relevant limit + diverging series (singular limit)

Reduced dynamics

- Two-point motion can be written as a system of SDE with **additive noise** (Piterbarg, Wilkinson *et al.*)



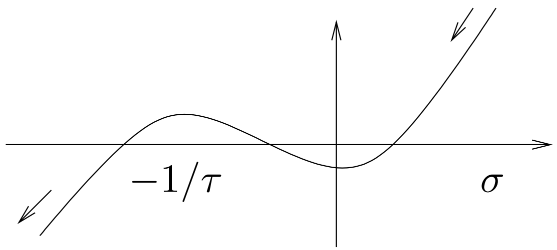
$$\sigma_1 = \boldsymbol{R} \cdot \dot{\boldsymbol{R}} / R^2 \qquad \dot{R} = \sigma_1 R$$

$$\sigma_2 = |\boldsymbol{R} \times \dot{\boldsymbol{R}}| / R^2$$

- One dimension ($d = 1$)

$$\dot{\sigma} = -\sigma/\tau - \sigma^2 + \sqrt{C}\eta$$

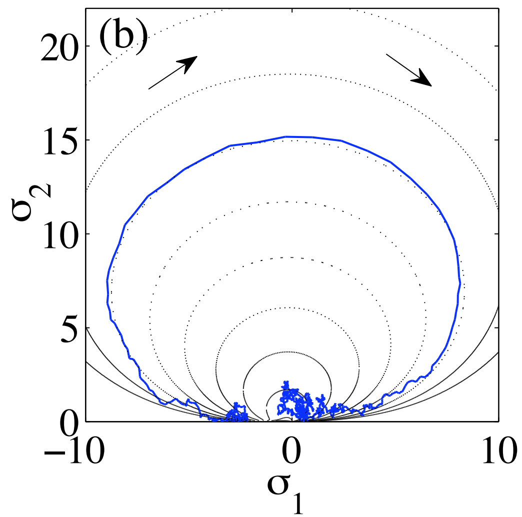
$\approx$ Anderson localization



- Two dimensions ($d = 2$)

$$\begin{cases} \dot{\sigma}_1 = -\sigma_1/\tau - [\sigma_1^2 - \sigma_2^2] + \sqrt{C}\eta_1, \\ \dot{\sigma}_2 = -\sigma_2/\tau - 2\sigma_1\sigma_2 + \sqrt{3C}\eta_2. \end{cases}$$

$Z = \sigma_1 + i\,\sigma_2$ complex potential



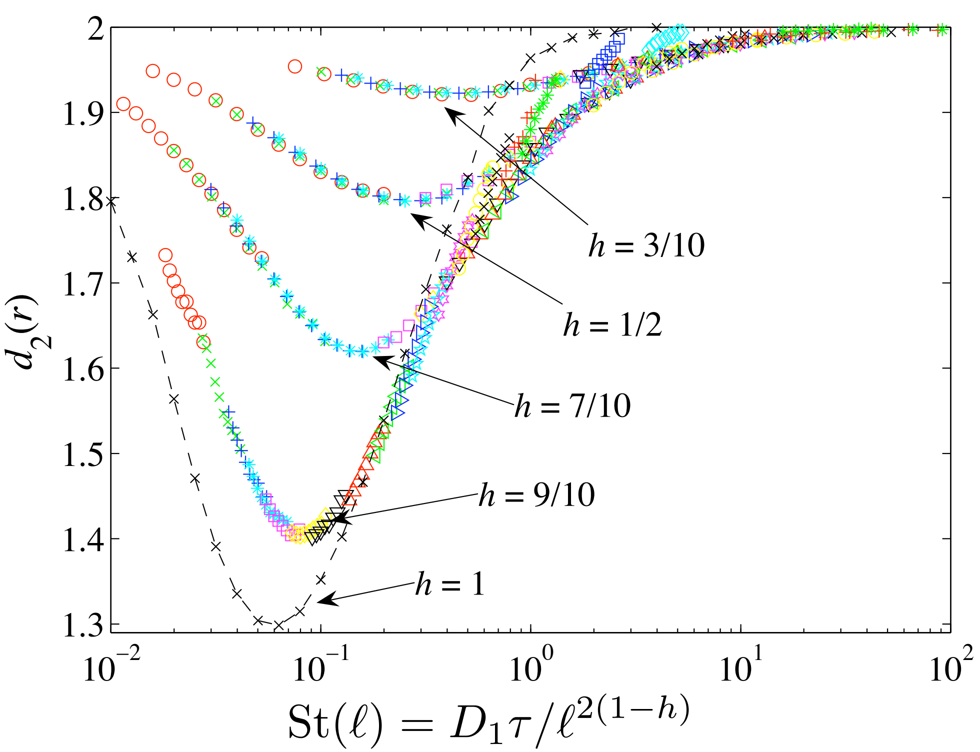
Caustics = loops

Inertial-range clustering ?

- Case of *non-differentiable* Kraichnan: particle dynamics at scale ℓ depends on a **local (scale-dependent) Stokes number**

$$\mathrm{St}(\ell) = \tau/\tau_\ell = \varepsilon^{1/3}\tau/\ell^{2/3}$$

Falkovich, Fouxon, Stepanov 2003
JB, Cencini, Hillerbrand 2007



Both the scale-invariance of the fluid flow and that of the particle distribution are broken

$\ell \to \infty \quad \mathrm{St}(\ell) \to 0$

inertia becomes negligible

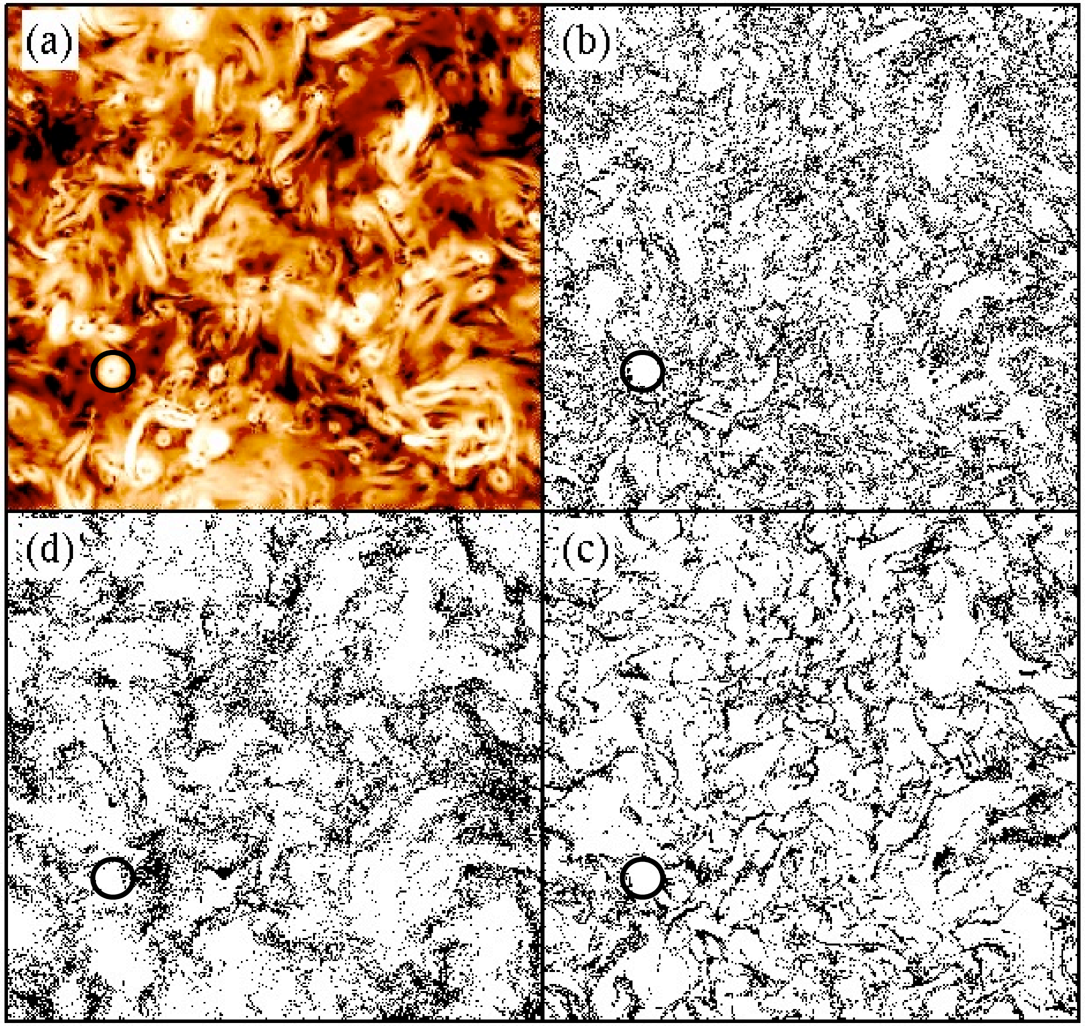
$\ell \to 0 \quad \mathrm{St}(\ell) \to \infty$

particles move almost ballistically

Particles in turbulent flow

Real flow have structure and particle distribution correlates with the acceleration field

Modulus of acceleration



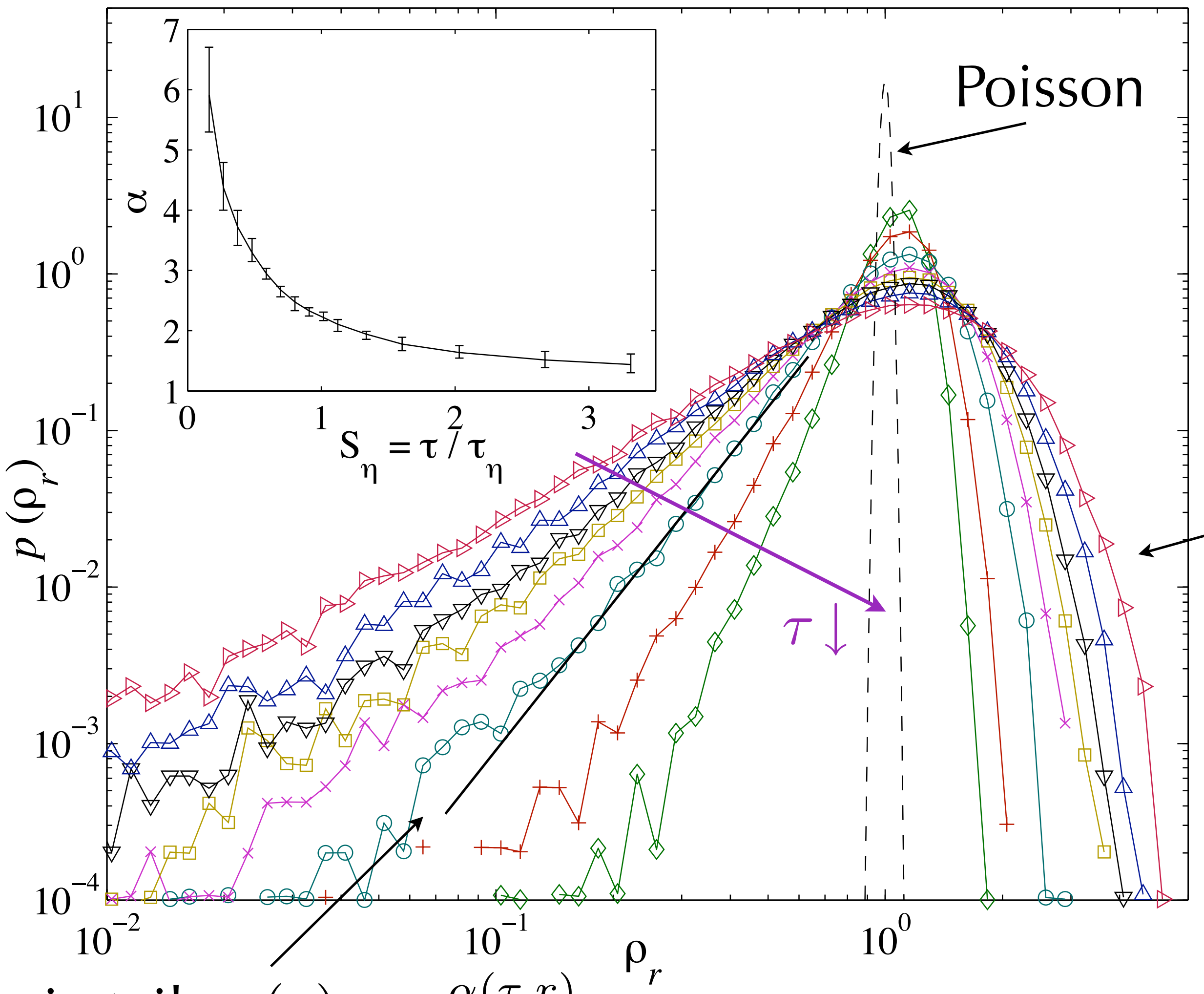
$St = 0.16$

$R_\lambda = 185$

$St = 3.3$

$St = 0.8$

Coarse-grained density



Tails faster than exponential

Algebraic tails $p(\rho) \propto \rho^{\alpha(\tau,r)}$
(signature of voids)

Time scales of clustering

- The local Stokes number $\mathrm{St}(\ell) = \varepsilon^{1/3}\tau/\ell^{2/3}$ is not relevant
- **Non dimensional contraction rate**

When inertia is very weak: Maxey's approximation

$$\dot{\boldsymbol{X}} \approx \boldsymbol{v}(\boldsymbol{X},t) = \boldsymbol{u}(\boldsymbol{X},t) - \tau\left[\partial_t \boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\right]$$

Rate at which a particle blob with size r is contracted

$$\Gamma_{r,\tau} = \frac{1}{r^3}\int_{|\boldsymbol{x}|<r} \nabla\cdot\boldsymbol{v}\,\mathrm{d}^3x \simeq \frac{\tau}{r^2}\delta_r p$$

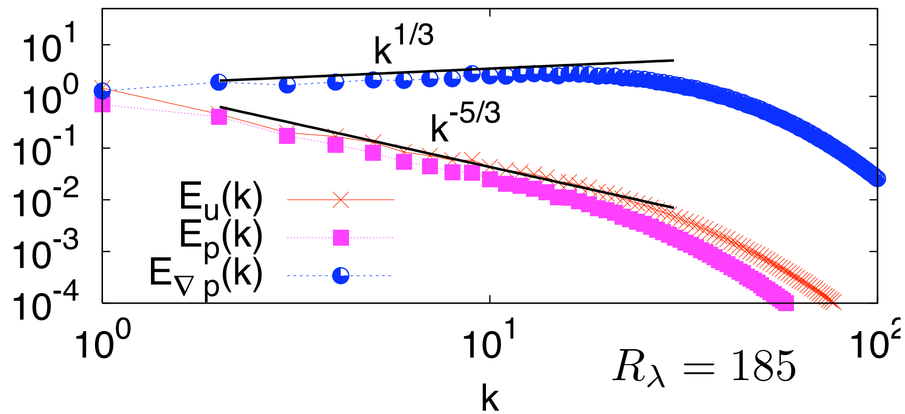
The question of pressure scaling has (at least) two answers

K41: $\delta_r p \propto (\varepsilon r)^{2/3}$

$$\Gamma_{r,\tau} \sim \tau/r^{4/3}$$

Sweeping $\delta_r p \sim \mathrm{U}(\varepsilon r)^{1/3}$

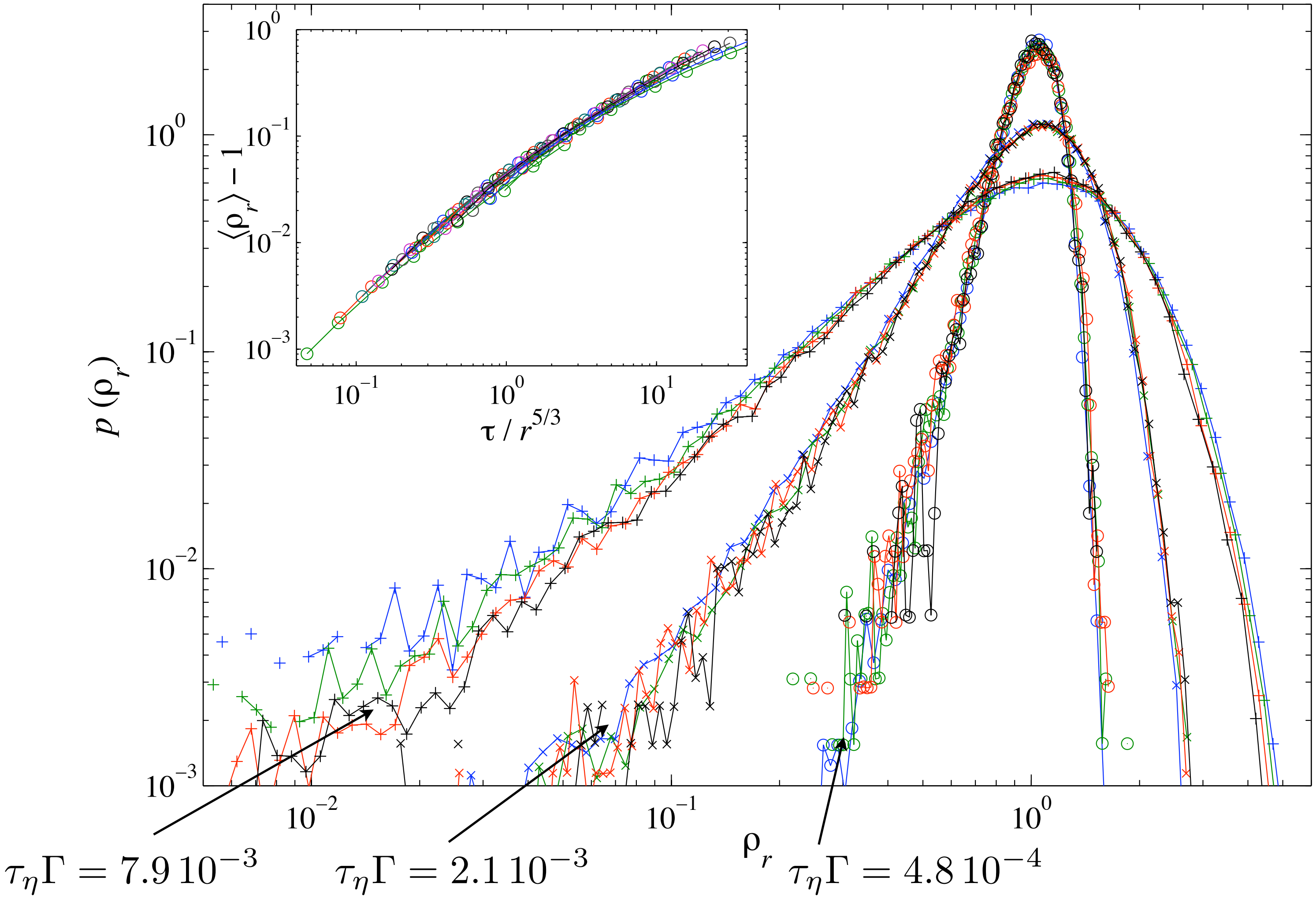
$$\boxed{\Gamma_{r,\tau} \sim \tau/r^{5/3}}$$



Scalable deviations from uniformity

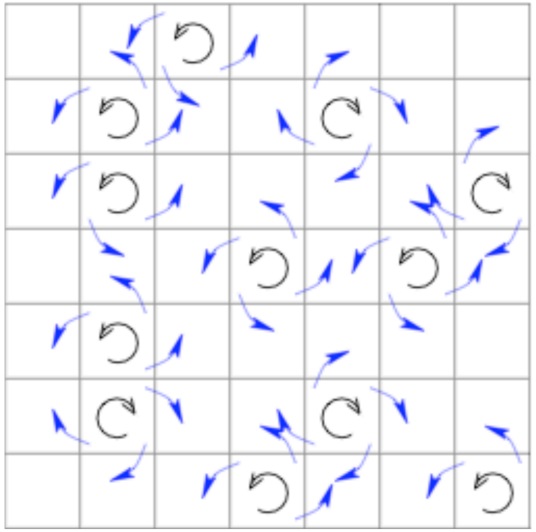
- Mass distribution depends only on

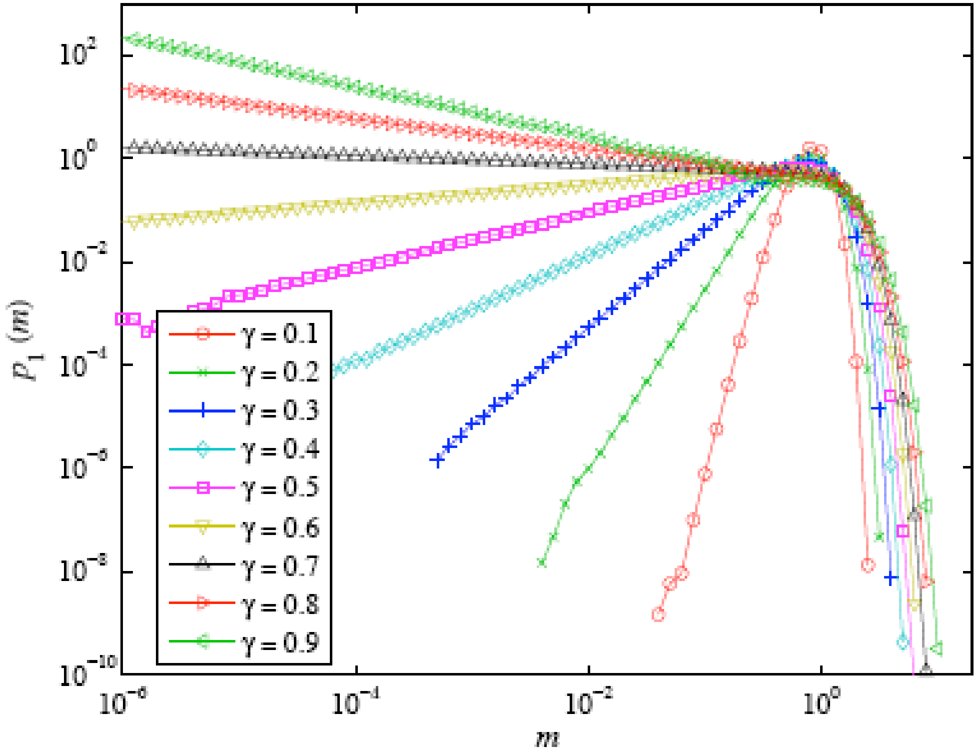
$$\tau_\eta \Gamma_{r,\tau} \sim \mathrm{Re}^{1/4}\, \mathrm{St}(r/\eta)^{5/3} \sim \mathrm{Re}^{-1}\, \mathrm{St}(r/L)^{5/3}$$



Mass transport model

- Find models belonging to the same universality class
- Discreteness in time and space
- At each time step some (randomly chosen with probability p) cells eject a fraction of their mass to their neighbors
- Parameter = γ ejection rate

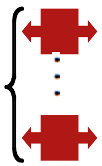




JB, R. Chétrite 2007

Tails

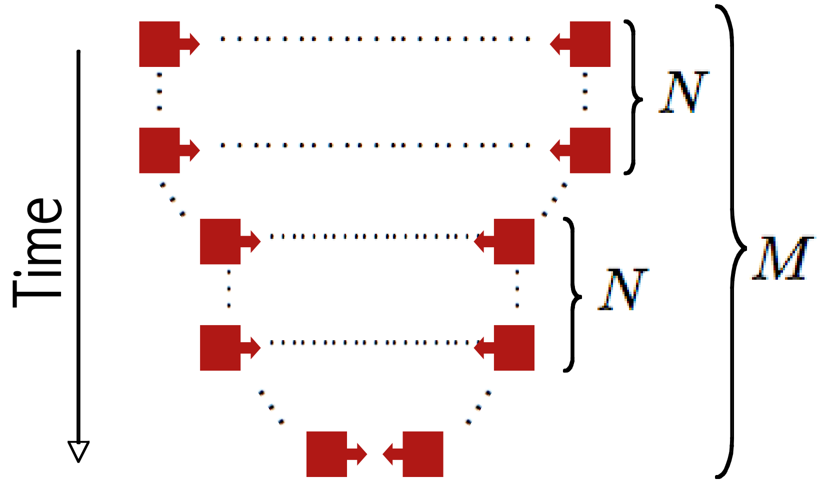
- Right tail = algebraic $p(m) \propto m^{\alpha(\gamma)}$

N times  $m_0 \approx 1$, $m_N \approx (1-\gamma)^N$

$$\text{Prob} = p^N (1-p)^{2N}$$

$$\Rightarrow \alpha(\gamma) = \frac{\ln p(1-p)}{\ln(1-\gamma)}$$

- Left tail = super-exponential



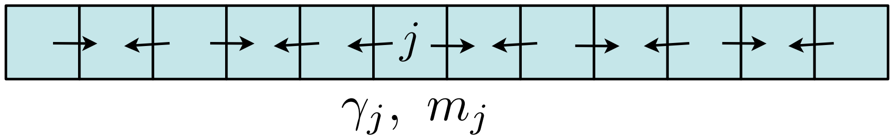
$$\text{Prob} = [p^2(1-p)]^{N\,M}$$

$$m_{N\,M} = \frac{1-[1-(1-\gamma)]^N]^M}{(1-\gamma)^N}$$

$$\Rightarrow p(m) \propto \exp(-C\,m \ln m)$$

Relation with RWRE

- Ejection rate depends on space



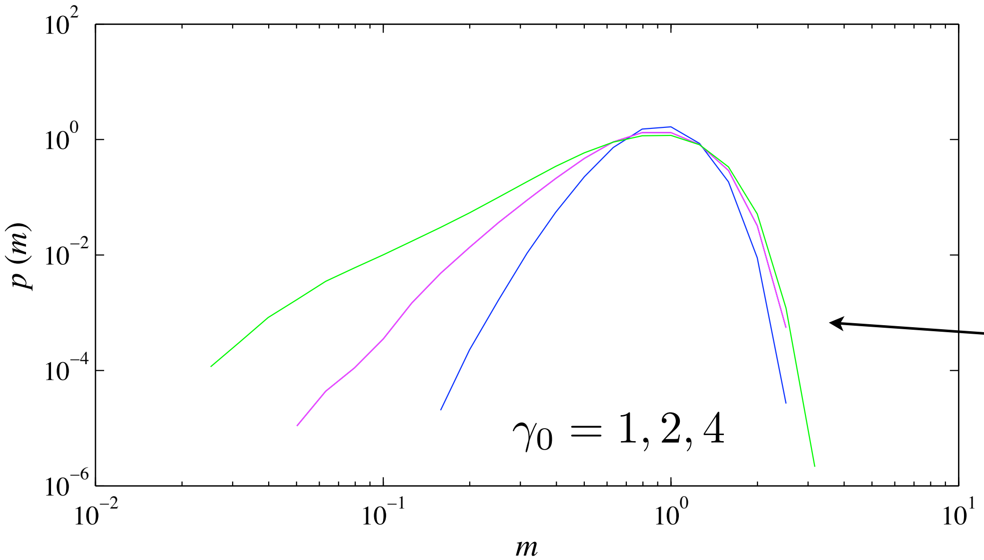
- Evolution of mass:

$$m_j(t+\delta t) = (1-2\gamma_j)m_j(t) + \gamma_{j-1}m_{j-1}(t) + \gamma_{j+1}m_{j+1}(t)$$

$\delta t, \ \delta x \to 0$

$$\partial_t m = \partial_x^2[\gamma(x,t)\, m]$$

one-point distribution for a random walk in the time-dependent environment $\gamma(x,t)$



$\gamma(x,t)$ iid uniformly in $[0, \gamma_0]$

Conclusions

Clustering

- Of two kinds, depending on the observation scale: multifractal in the dissipative range, dependent only on a rescaled contraction rate in the inertial range. Some attempts to get analytical forms for the mass distribution.
- Use of more refined cluster analysis tools to study the dynamics of particle clusters: how do they form, how long do they live?
- Correlation of particle positions with the flow structures requires to understand the inertial-range distribution of acceleration.

Collisions / Velocity statistics

- Clean-up the scaling properties of particle velocity differences
- Understand the limit of validity of the ghost-collision approach