Simple generic model of intra-seasonal oscillations in the Earth atmosphere

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Intra-seasonal oscillations – main facts:

- Periods: 10-100 days
- Location: everywhere (SH, NH; mid-latitudes, tropical zone)
- Data: AAM (atmospheric angular momentum, zonal winds,)
- Reason (NH, m.-l. – topography? SH, tropics – ?...)
- Modeling (no general model, particular numerical models, e.g. m.-l. IOs – reproduced with and without topography)
- Open questions – many (later)

Nonlinear resonance theory (NRT)

$$\begin{cases} \omega(\mathbf{k}_1) + \omega(\mathbf{k}_2) - \omega(\mathbf{k}_3) = \Omega, \\ \mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 = 0, \end{cases} \qquad \Omega \geq 0$$

- Not all modes interact resonantly
- Resonant modes form small dynamically independent clusters
- Form of clusters, if any, is defined by boundary conditions
- Resonant interactions are not local in Kolmogorov's sense
- Quasi-resonances possess similar properties

EK: Amer.Math.Soc. (1998); PRL (1994,2007)

Planetary waves on a sphere

$$\partial \triangle \psi / \partial t + 2\, \partial \psi / \partial \lambda = -J(\psi, \triangle \psi)$$

$$\omega_j = -2m_j/\ell_j(\ell_j+1)$$

$$\begin{cases} N_1 \dot{A}_1 = 2Z(N_3 - N_2) A_3 A_2^*, \\ N_2 \dot{A}_2 = 2Z(N_1 - N_3) A_1^* A_3, \\ N_3 \dot{A}_3 = 2Z(N_2 - N_1) A_1 A_2, \end{cases}$$

All clusters in m,l <=21
4 triads,
3 butterflies,
1 cluster of 6 connected triads

Maximum Z at „interaction latitude“:

We can interpret the latitude φ_0 as follows. The overlap of three wave functions in a triad $Z(\lambda, \varphi)^{m_1}_{\ell_1} \times (\lambda, \varphi) Y^{m_2}_{\ell_2}(\lambda, \varphi) Y^{m_3}_{\ell_2}(\lambda, \varphi)$ shows a contribution to the interaction coefficient $Z \propto \int Z(\lambda, \varphi) d\lambda d\varphi$ from a particular location on the sphere. The overlap $Z(\lambda, \varphi)$ has a maximum at a particular latitude φ_0, and a narrow latitudinal belt around φ_0 gives the main contribution to the global interaction amplitude Z. That is why φ_0 can be understood as the interaction latitude.

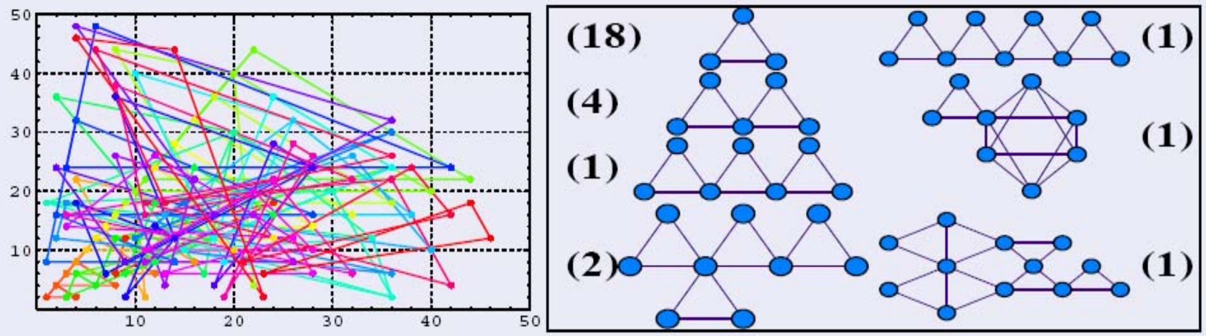
EK, L.Piterbarg, G.Reznik, Oceanolgy (1990)
EK, V.L'vov, PRL (2007);

Main idea: IOs are due to resonance clusters of spherical Rossby waves

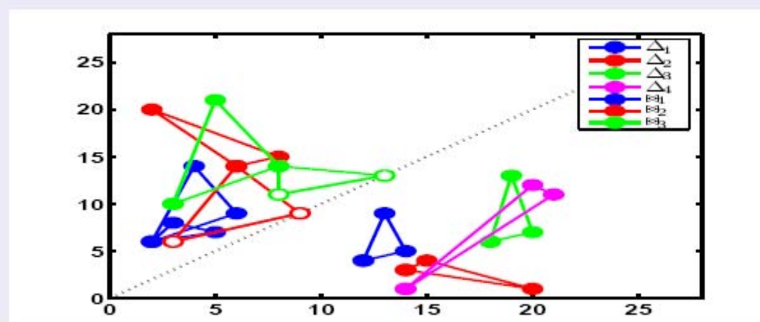
Galerkin:
2500 Fourier harmonics ($m, n <= 50$)

Clipping:
Only 128 are important

Structure of resonances in spectral space ≤ 50



Spectral domain ≤ 21

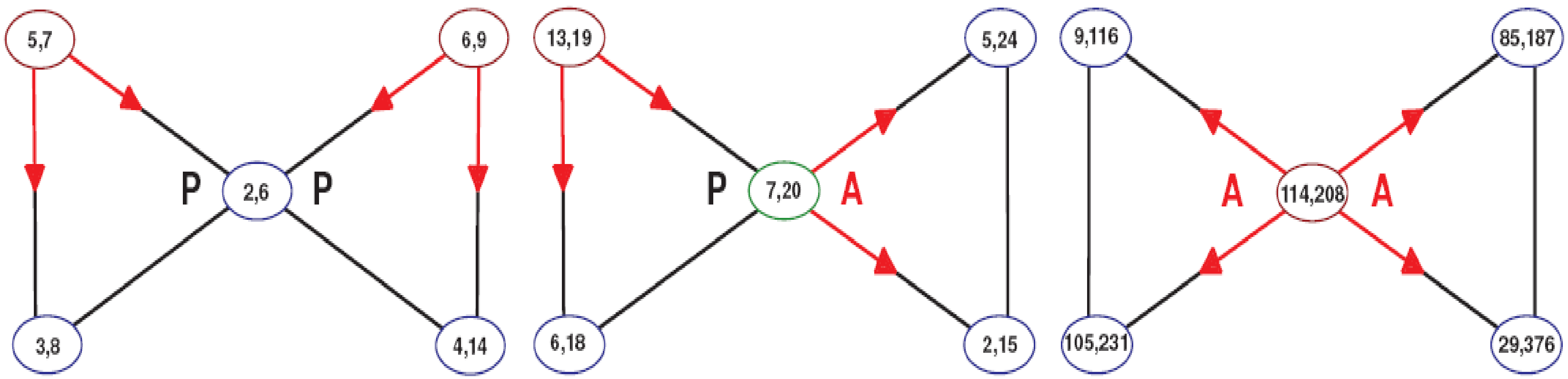


and 1 group of 6 connected triads.

EK, G.Mayrhofer. *Physica A* (2007)

Connection types within a cluster

$$\omega_j = -2m_j/\ell_j(\ell_j+1)$$



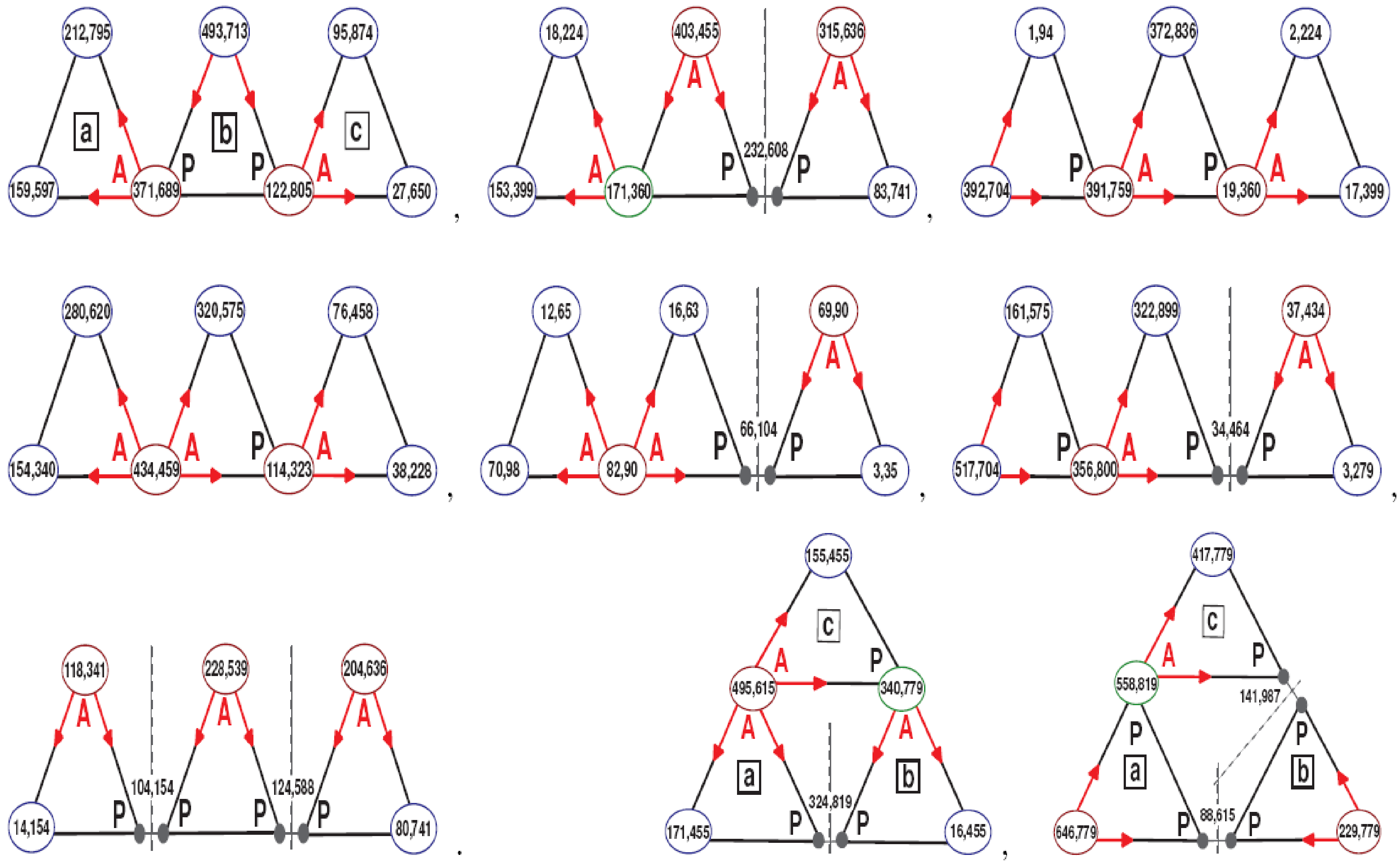
$$\frac{4}{14\cdot 15}+\frac{2}{6\cdot 7}=\frac{6}{9\cdot 10}$$
$$\frac{4}{8\cdot 9}+\frac{2}{6\cdot 7}=\frac{5}{7\cdot 8}$$

$$\frac{13}{19\cdot 20}=\frac{7}{20\cdot 21}+\frac{6}{18\cdot 19}$$
$$\frac{7}{20\cdot 21}=\frac{5}{24\cdot 25}+\frac{2}{15\cdot 16}$$

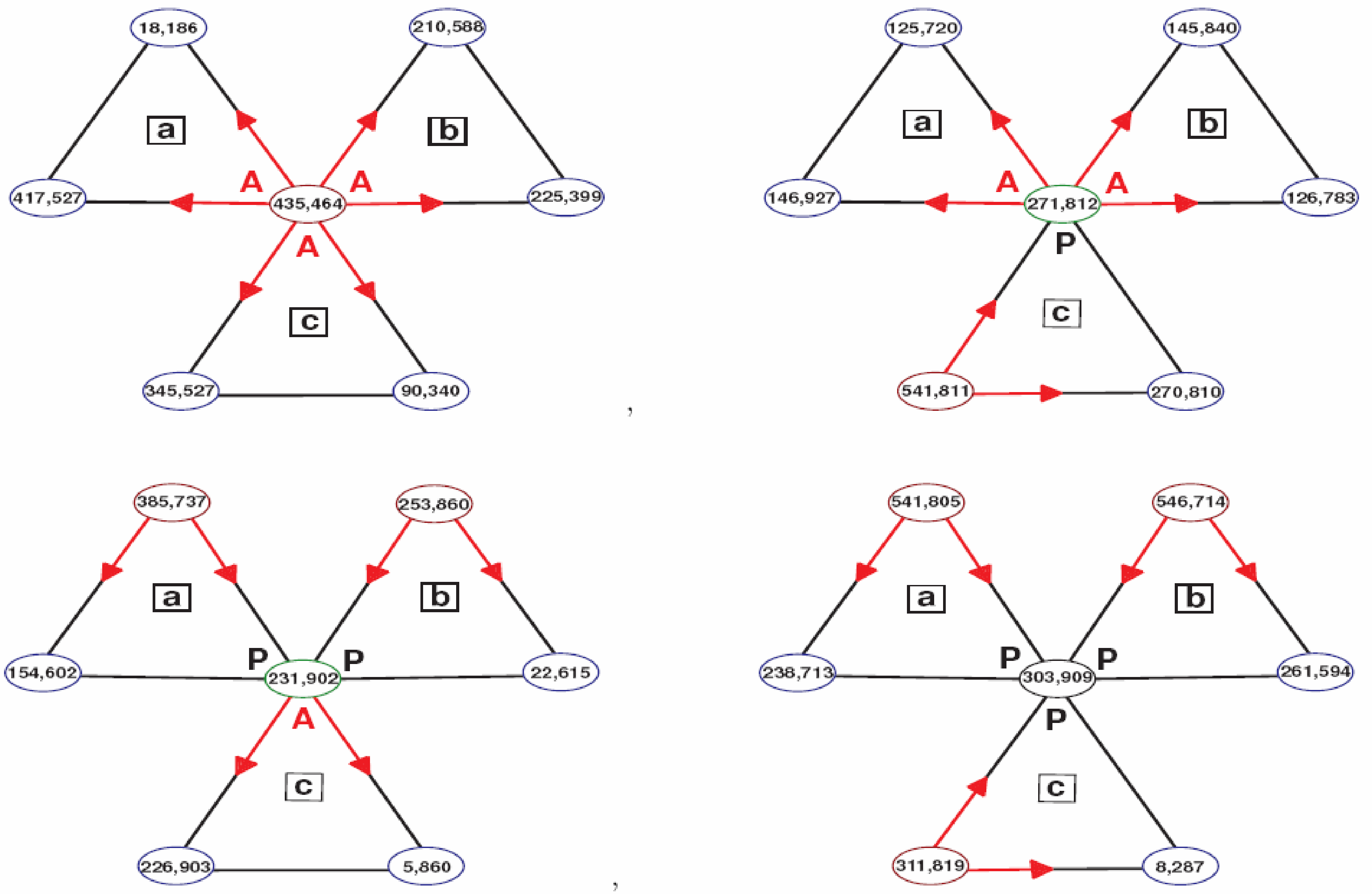
$$\frac{114}{208\cdot 209}=\frac{9}{116\cdot 117}+\frac{105}{231\cdot 232}$$
$$\frac{114}{208\cdot 209}=\frac{85}{187\cdot 188}+\frac{29}{376\cdot 377}$$

EK, V.L'vov, EPL (2008)

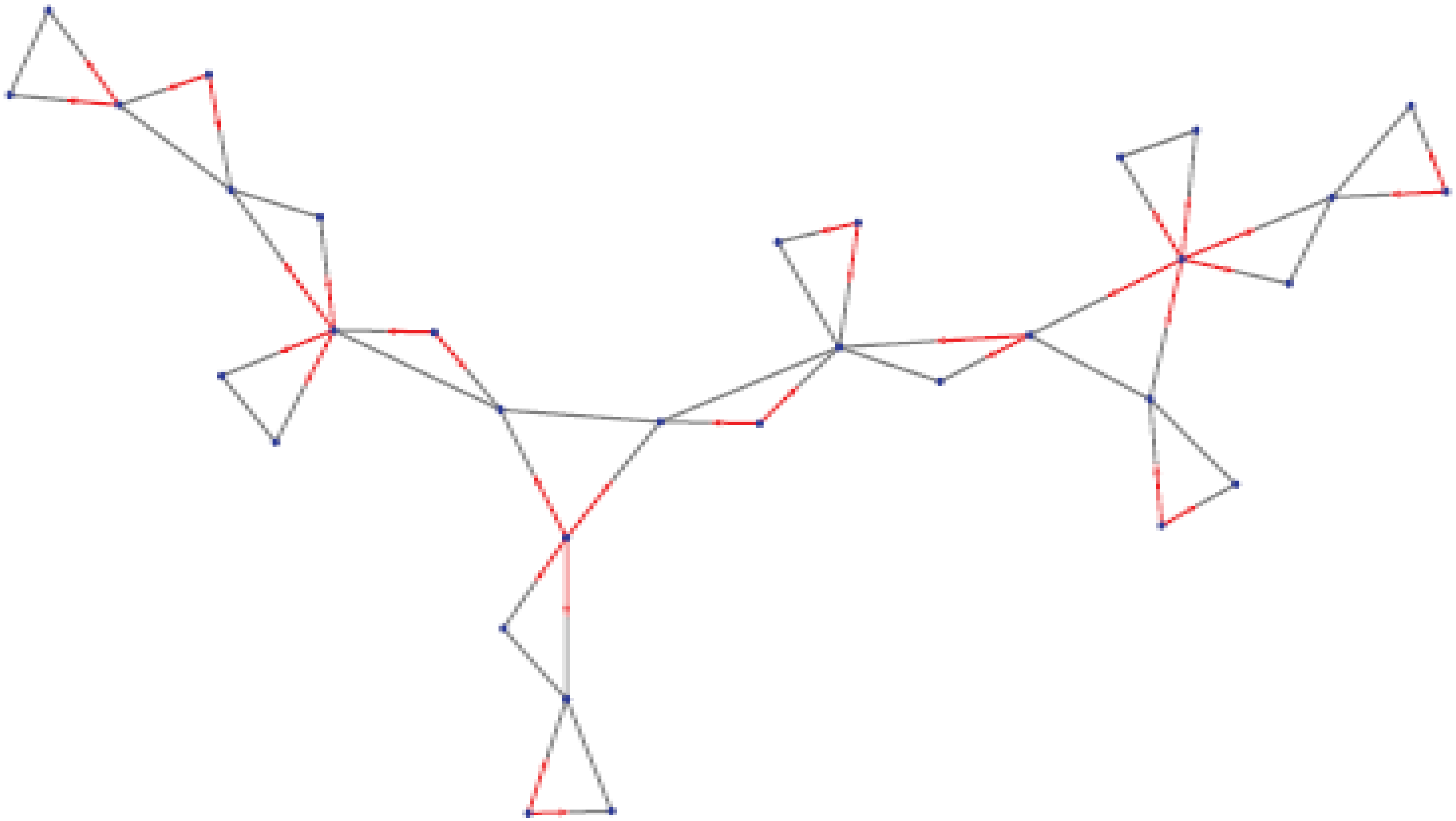
Connection types (continuation)



Connection types (continuation)



Connection types (continuation)



16-triad's PP-reducible cluster

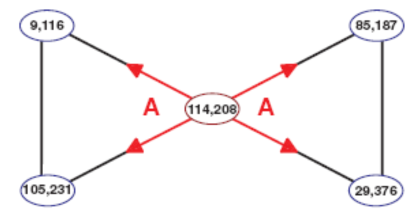
Examples of dynamical systems

1.

$$\begin{cases} \dot{B}_{1a} = Z_a B^*_{2a} B_{3a} \,, & \dot{B}_{1b} = -Z_b B^*_{2b} B_{3a} \,, \\ \dot{B}_{2a} = Z_a B^*_{1a} B_{3a} \,, & \dot{B}_{2b} = Z_b B^*_{1b} B_{3a} \,, \\ \dot{B}_{3a} = -Z_a B_{1a} B_{2a} - Z_b B_{1b} B_{2b} \,. \end{cases}$$

$$\begin{cases} I_{12a} = |B_{1a}|^2 - |B_{2a}|^2 \,, \quad I_{12b} = |B_{1b}|^2 - |B_{2b}|^2 \,, \\ I_{ab} = |B_{1a}|^2 + |B_{3a}|^2 + |B_{3b}|^2 \,. \end{cases}$$

AA-butterfly

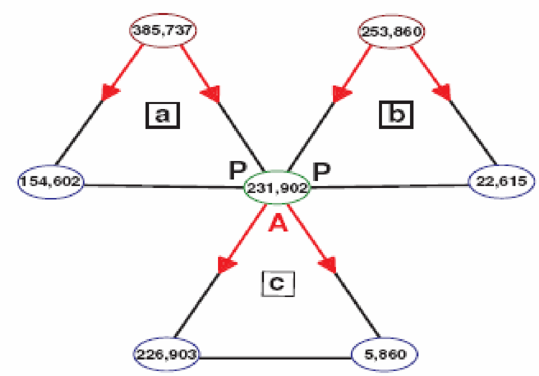


2.

$$\begin{cases} \dot{B}_{3a} = -Z_a B_{1a} B_{2a} \,, & \dot{B}_{2a} = Z_a B_{1a} B_{3a} \,, \\ \dot{B}_{3b} = -Z_b B_{1a} B_{2b} \,, & \dot{B}_{2b} = Z_a B_{1b} B_{3a} \,, \\ \dot{B}_{1c} = Z_c B^*_{2c} B_{1a} \,, & \dot{B}_{2c} = Z_c B_{1a} B_{3a} \,, \\ \dot{B}_{1a} = Z_a B^*_{2a} B_{3a} + Z_b B^*_{2b} B_{3b} - Z_c B_{1c} B_{2c} \,. \end{cases}$$

PPA-star

$$\begin{cases} \text{PPA-star:} \qquad I_{12a} = |B_{1a}|^2 - |B_{2a}|^2 \,, \\ I_{23b} = |B_{2b}|^2 + |B_{3b}|^2 \,, \quad I_{23c} = |B_{2c}|^2 + |B_{3c}|^2 \,, \\ I_{abc} = |B_{1a}|^2 + |B_{3a}|^2 + |B_{3b}|^2 + |B_{3c}|^2 \,. \end{cases}$$



CLs constructed as combinations of known CLs for a triad

New Conservation Laws, PP-butterfly :

$$\begin{cases} \dot{B}_{1a} = Z_a B_{2a}^* B_{3a} + Z_b B_{2b}^* B_{3b}, \; B_{1a} = B_{1b}, \, , \\ \dot{B}_{2a} = Z_a B_{1a}^* B_{3a} \, , \quad \dot{B}_{2b} = Z_b B_{1a}^* B_{3b} \, , \\ \dot{B}_{3a} = -Z_a B_{1a} B_{2a} \, , \; \dot{B}_{3b} = -Z_b B_{1a} B_{2b} \, . \end{cases}$$

$$\begin{cases} I_{23a} = |B_{2a}|^2 + |B_{3a}|^2 \, , \quad I_{23b} = |B_{2b}|^2 + |B_{3b}|^2 \, , \\ I_{ab} = |B_{1a}|^2 + |B_{3a}|^2 + |B_{3b}|^2 \, , \\ I_0 = \mathrm{Im}(Z_a B_1 B_{2a} B_{3a}^* + Z_b B_1 B_{2b} B_{3b}^*) \, , \end{cases}$$

Reduction: from 10 equations to 4:

$$\frac{dC_{3a}}{dt} = -Z_a C_1 C_{2a} \cos\varphi_a \, ,$$
$$\frac{dC_{3b}}{dt} = -Z_b C_1 C_{2b} \cos\varphi_b \, ,$$
$$\frac{d\varphi_a}{dt} = Z_a C_1 \left(\frac{C_{2a}}{C_{3a}} - \frac{C_{3a}}{C_{2a}} \right) \sin\varphi_a - \frac{I_0}{(C_1)^2} \, ,$$
$$\frac{d\varphi_b}{dt} = Z_b C_1 \left(\frac{C_{2b}}{C_{3b}} - \frac{C_{3b}}{C_{2b}} \right) \sin\varphi_b - \frac{I_0}{(C_1)^2} \, .$$

$$B_j = C_j \exp(i\theta_j)$$

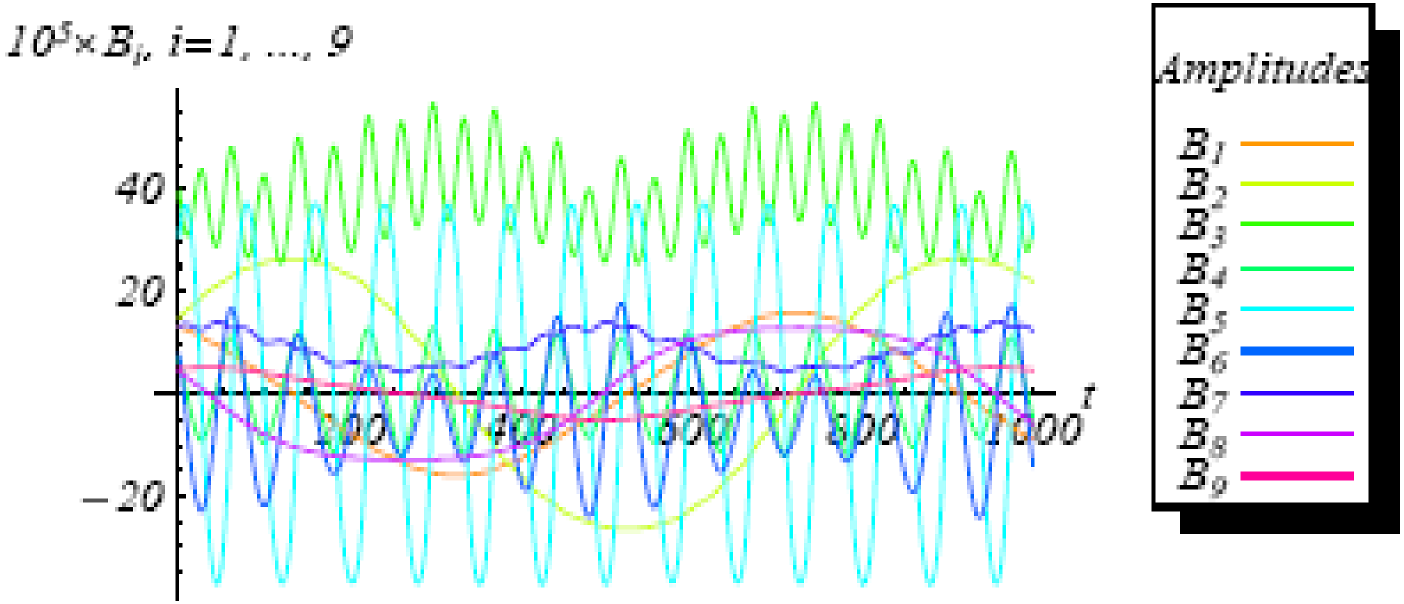
$$\varphi_a = \theta_{1a} + \theta_{2a} - \theta_{3a}$$

$$\varphi_b = \theta_{1b} + \theta_{2b} - \theta_{3b};$$

Integrable cases of butterflies:

1. Real amplitudes; 2. $I_0 = 0$; 3. $Z_a = Z_b$; 4. Some more

4-triad's chain, real amplitudes



Open questions (M. Ghil, 2004)

- Cause of IOs in the SH? The same
- NH oscillation: why by some researchers – 40 days, by others – 20-30 days. Period is inversly proportional to the energy, more energy -> shorter period
- Why IOs are better observable in winter data? For larger energies appr. resonances may destroy the clusters
- How do tropical and mid-latitude IOs interact? Two mechanisms are possible:

1) One cluster, different latitudinal belts;
2) Via mode which is quasi-resonant for one triad and exact for another

- How to predict IOs in data? Obvious (spher. Harmonics with known wavenumbers)

Energy flux between latitudinal belts

Triad	Modes $[m,\ell]$	Z	φ_0	$K[\mu]$	$10^6 E_0$	T_0
Δ_1	[4,12] [5,14] [9,13]	7.82	34	1.62	14.4	24
Δ_2	[3,14] [1,20] [4,15]	37.46	19	1.14	5.4	5
Δ_3	[6,18] [7,20] [13,19]	13.66	34	1.74	32.0	19
Δ_4	[1,14][11,21][12,20]	47.67	28	1.21	0.58	13
$\bowtie_1$	[2,6] [3,8] [5,7]	3.14	35	1.64	5.08	30
	[2,6] [4,14] [6,9]	14.63	37	1.61	0.395	10
$\bowtie_2$	[6,14] [2,20] [8,15]	69.25	31	1.13	0.61	8
	[3,6] [6,14] [9,9]	11.31	—	1.17	0.360	13
$\bowtie_3$	[3,10] [5,21] [8,14]	61.99	31	1.27	0.133	7
	[8,11] [5,21] [13,13]	8.71	—	1.36	0.784	24
$\boxtimes$	[1,6] [2,14] [3,9]	28.98	17	1.38	0.247	6
	[2,7] [11,20] [13,14]	2.77	42	1.08	1.78	26
	[1,6] [11,20] [12,15]	15.08	29	1.06	0.262	11
	[9,14] [3,20] [12,15]	74.93	50	1.36	0.487	8
	[3,9] [8,20] [11,14]	32.12	40	1.11	0.251	9
	[2,14][17,20][19,19]	11.05	—	1.05	3.33	24

the mode (13,19) is near-resonant for Δ_4 (with resonance discrepancy $\delta = 0.16$) and is resonant for Δ_3.

Thank you!