Turbulent collision-coalescence of cloud droplets and its impact on warm rain initiation

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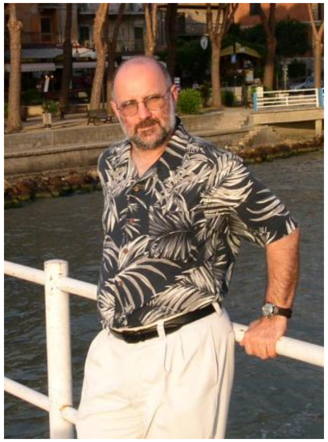

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Why clouds?



Clouds of various colors, shapes, sizes

An essential link for the water cycle

Weather & climate change: direct and indirect effects

(1) **Reflecting the short-wave solar radiation and trapping long-wave radiation energy from the surface**

(2) **Removing aerosols; Removing water vapor**

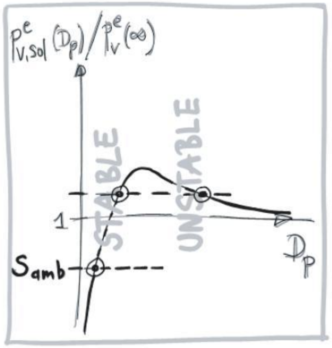
(3) **Clouds represent a source of significant uncertainty in numerical weather prediction and climate models.**

Air pollution and global warming

A natural multi-scale problem

Thermodynamics, chemical and phase equilibrium, fluid dynamics …..

Droplet activation and condensational growth – 1st and 2nd microphysical steps



- ❖ Aerosols (before activation, interstitial aerosols): 10 nm to 1 μm
  - sea salt particles [NaCl] over ocean
  - ammonium sulfate [$(NH_4)_2SO_4$] over continental
- ❖ Activated aerosols (CCN): ~ 100 nm to 10 μm

The Koehler theory: kinetic theory, thermodynamics & chemistry

$$p^e_{v,solution} = p^e_v(\infty)\exp\left[\frac{4\sigma\, \mathrm{v}_l}{kTD_p} - \frac{6n_s \mathrm{v}_l}{\pi D_p^3} - \text{effects due to pollutants}\right]$$

Pollutants reduce the effective interface area, alter intermolecular force & introduce surfactants.

- ❖ Condensational growth (diffusion theory): $\frac{dr}{dt} = \frac{f_{vent} A S}{r}$

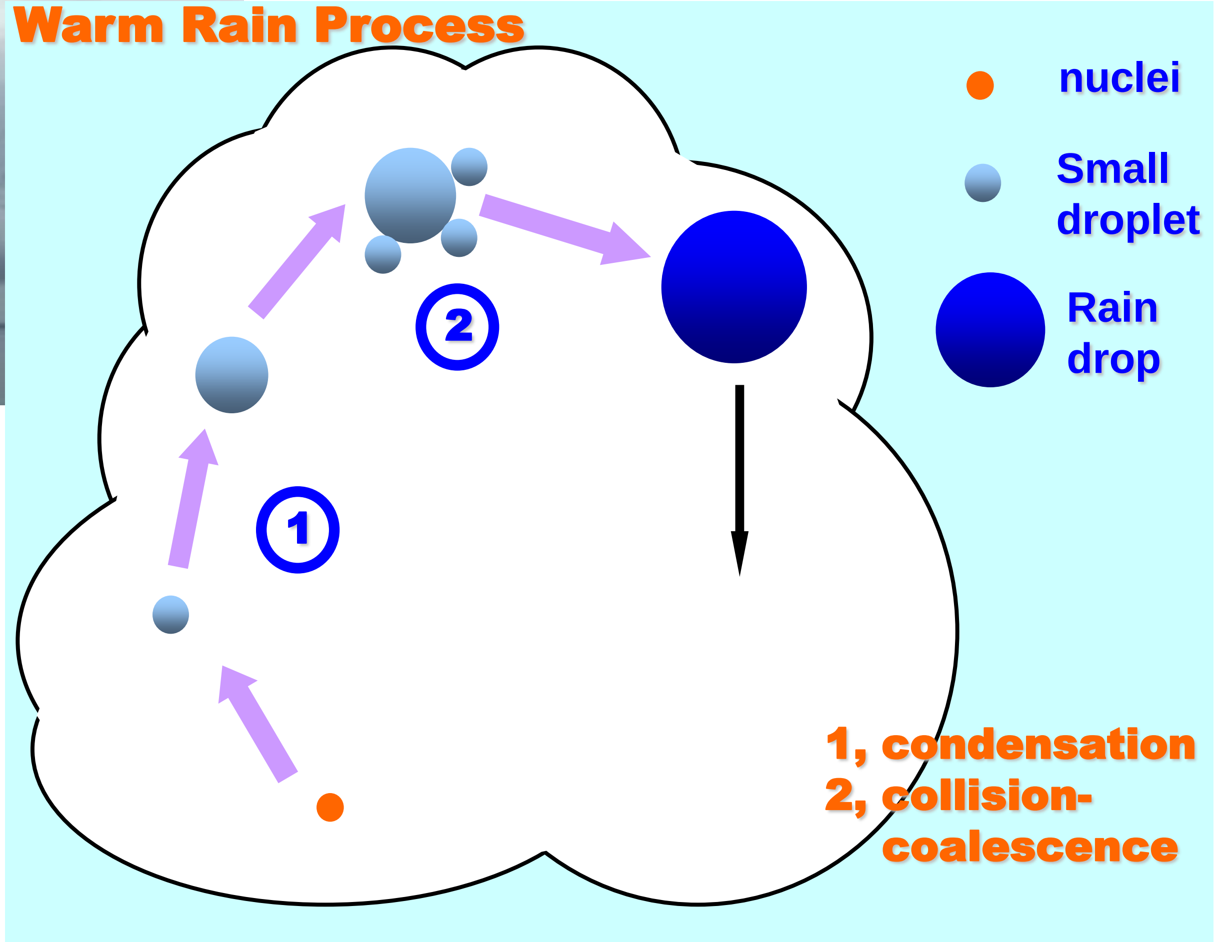
large-scale turbulent fluctuations or stochastic condensation

$$S = \frac{P^e_v(\infty)}{P^e_{v,solution}} - 1, \quad f_{vent} = \text{ventilation coefficient}, \quad A \approx 10^{-10}\, m^2 s^{-1}$$

Kenneth V. Beard & Harry T. Ochs III, An overview …., *J. Applied Meteor.* 32: 608-624 (1993).
F. Raes, Take a glass of water …., *J. Phys. IV France* 139: 63-80 (2006).

Collision-coalescence: effects of small-scale turbulence
The 3rd microphysical step

Growth of cloud droplets



How does air turbulence affect the collision rates and collision efficiency of cloud droplets?

What is the impact on warm rain initiation?

Rapid onset of precipitation in shallow cumulus clouds

?→

- ❖ Formation of drizzles (>100 μm) from cloud droplets: 15 ~ 30 minutes.
  - Hawaiian rainband clouds: Szumowski *et al*. (1997)
  - Florida cumulus (SCMS): Knight *et al*. (2002), Blyth *et al*. (2003)
- ❖ Different widths between predicted and measured size distributions

Possible explanations

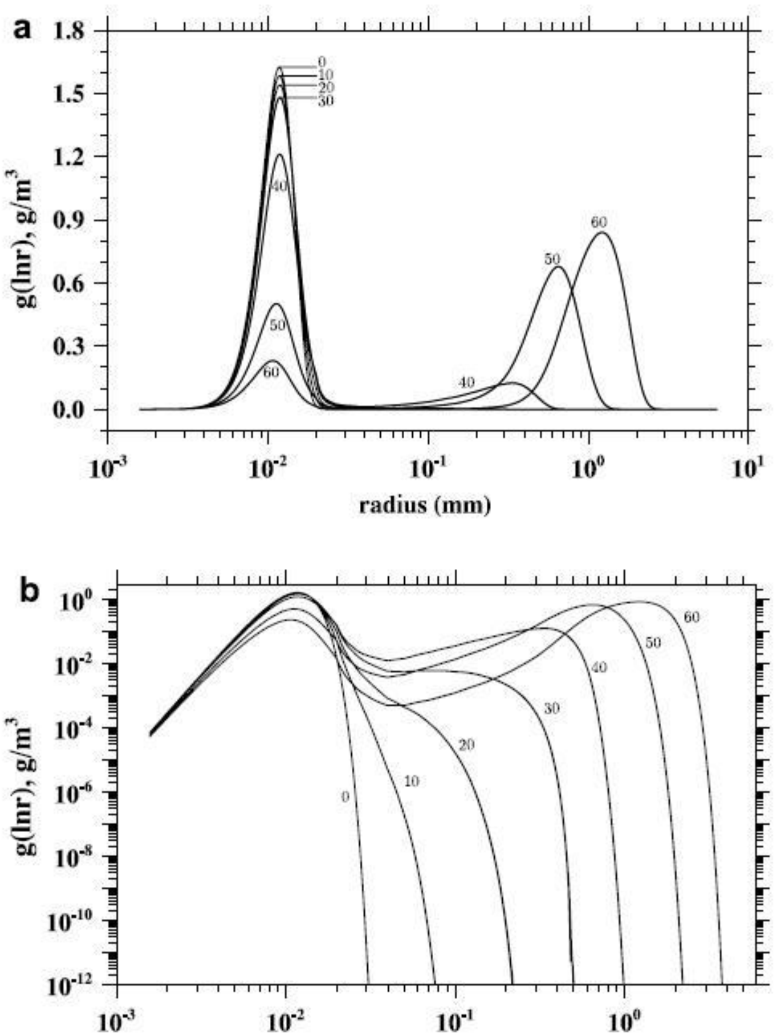
The size gap or bottleneck problem: 10 to 50 μm

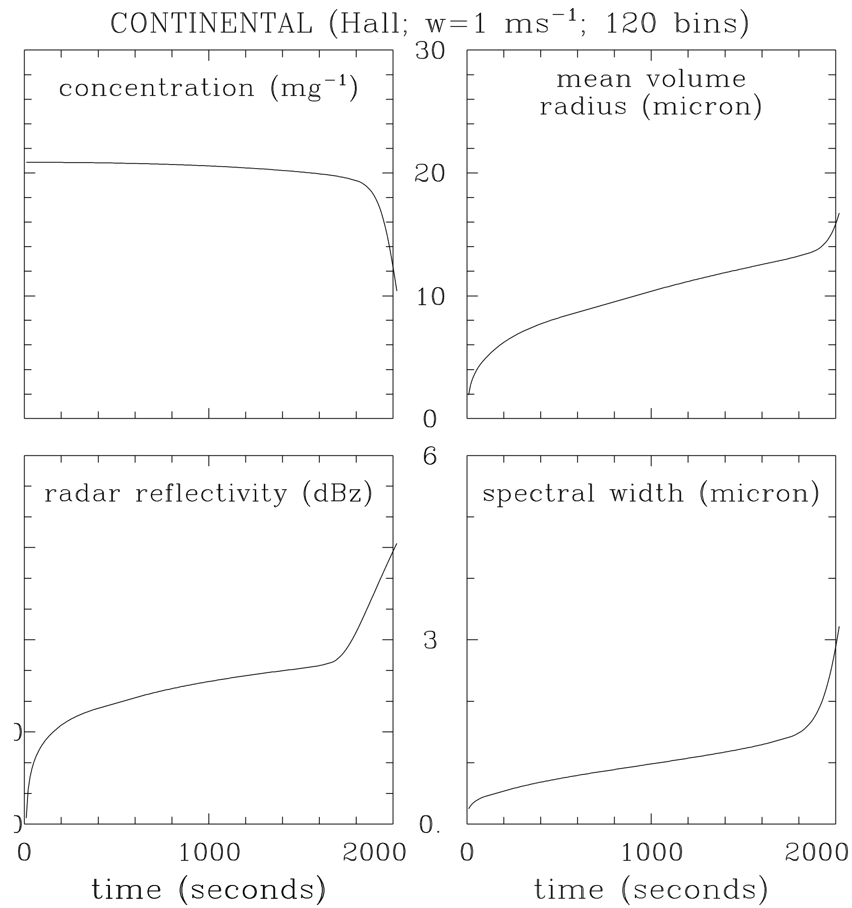
- ❖ Growth by giant and ultragiant particles
- ❖ Effects of air turbulence (on condensations and collision-coalescence)
- ❖ Entrainment-induced spectral broadening
- ❖ Effects of preexisting clouds

Arenberg 1939: Turbulence as a major factor in the growth of cloud droplets. Bull. Amer. Met. Soc. 20, 444-445.

Warm rain initiation = "colloidal" instability = explosive growth

What causes formation of droplets large enough for the explosive growth by coalescence?



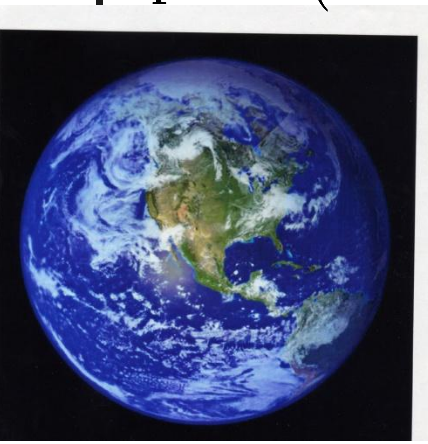


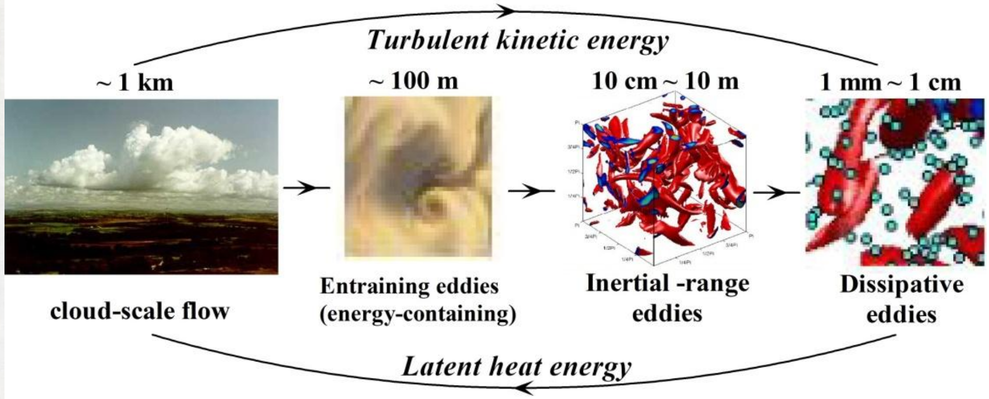
Clouds and warm-rain microphysics: enormous range of length and time scales

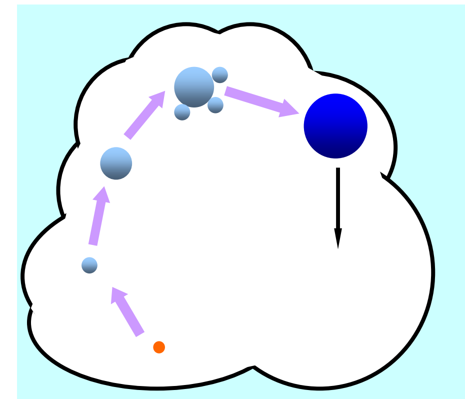
up to $O\left(10^{7} m\right)$

Multi-scale approach

$O\left(10^{-6} m \sim 10^{-4} m\right)$





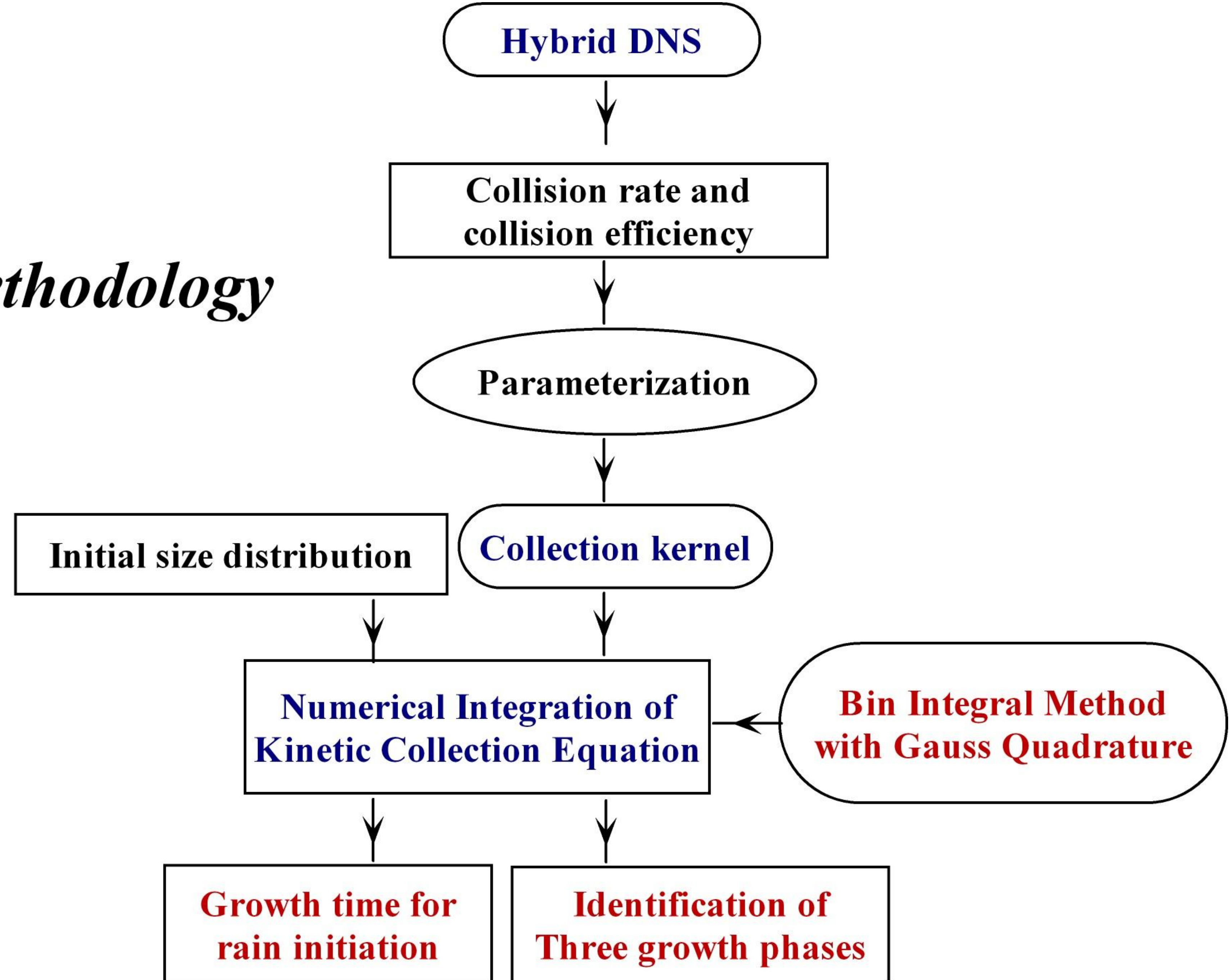


Numerical Modeling vs Numerical Simulations

- ❖ Global climate model or General circulation model (GCM): $\Delta x = 50 \sim 100\ \mathrm{km}$
- ❖ Numerical weather prediction: $\Delta x = 1 \sim 10\ \mathrm{km}$
- ❖ Cloud-resolving model / LES: $\Delta x = 1 \sim 50\ \mathrm{m}$
- ❖ Direct numerical simulations of small-scale turbulence and collision-coalescence:

$$\Delta x = 1\ \mathrm{mm}$$

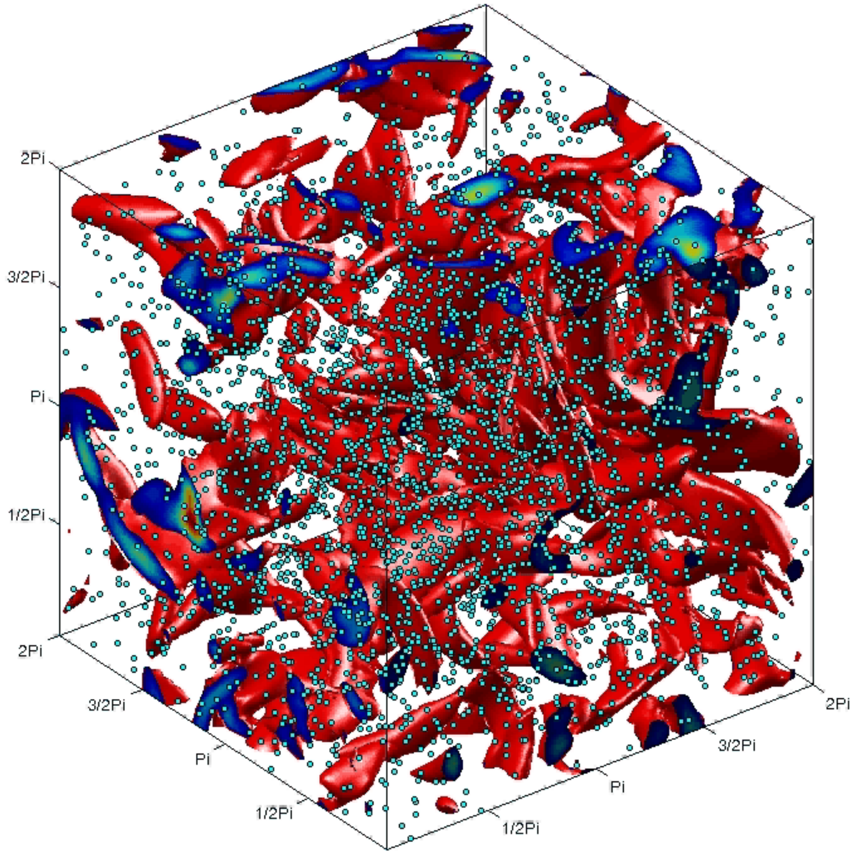
Methodology



Part 1.

Hybrid Direct Numerical Simulation

DNS of small scale turbulence and collision-coalescence



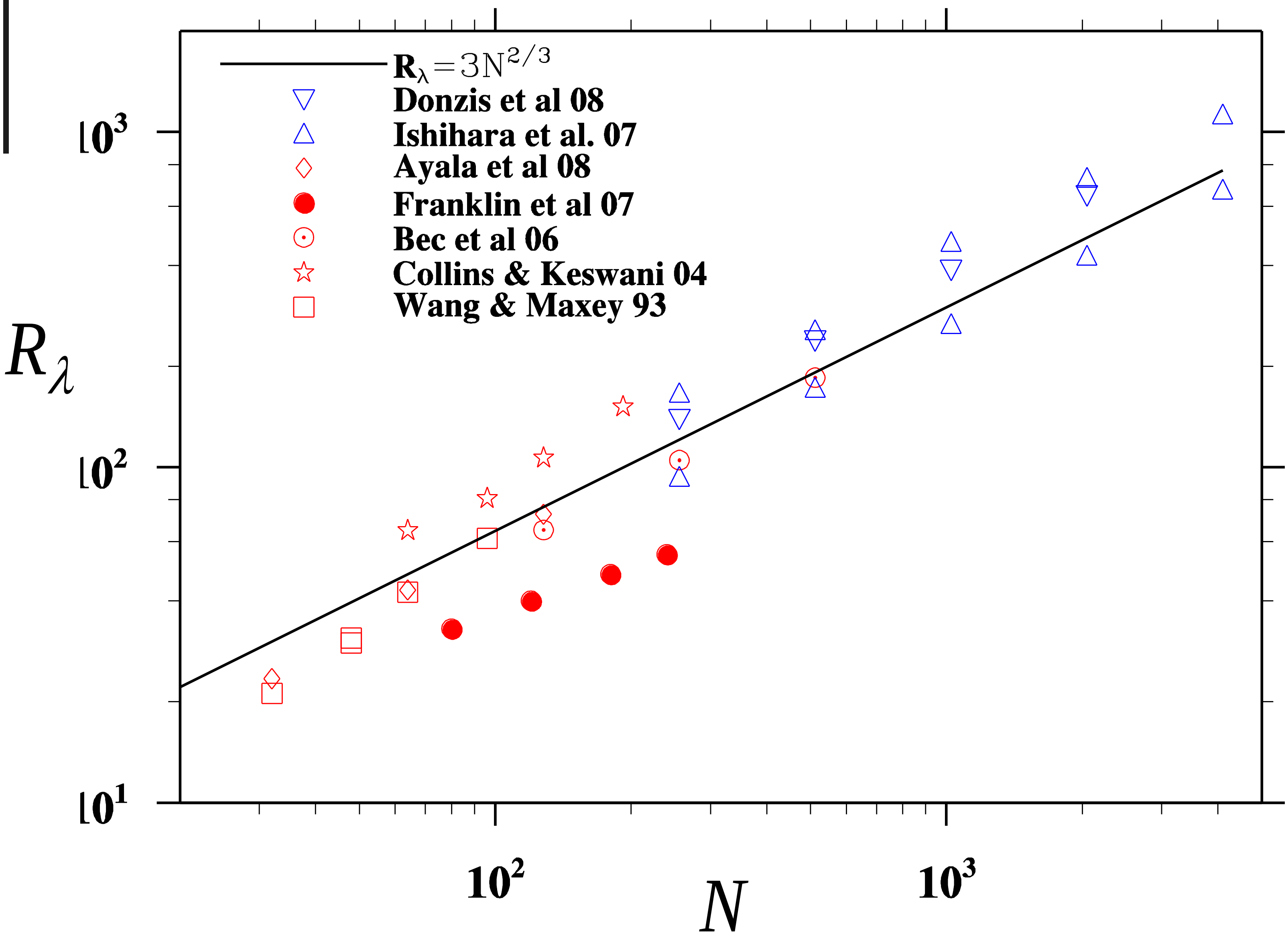
Flow by pseudo-spectral method
Gravitational settling
Finite-inertia droplets
Lagrangian tracking of droplets
Efficient collision detection

Issues addressed:

- Enhanced relative motion
- Preferential concentration
- Enhanced settling rates

Grid spacing ~ 1 mm, flow around cloud droplets are not directly resolved.
Domain size 10 to 20 cm

Taylor microscale flow Reynolds number



Wang et al. 2008, Submitted to *Int. J. Multiphase flow*

Direct simulation of small-scale air turbulence

Flow field
$$\frac{\partial \vec{U}}{\partial t} = \vec{U} \times \vec{\omega} - \nabla\left(\frac{P}{\rho} + \frac{1}{2}U^2\right) + \nu \nabla^2 \vec{U} + \vec{f}(\vec{x},t)$$

solved with $\nabla \cdot \vec{U} = 0$ in a periodic box

isotropic and homogeneous: $\langle \vec{U}(\vec{x},t) \rangle = 0$

Kolmogorov scales:
$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}; \quad \tau_k = \left(\frac{\nu}{\varepsilon}\right)^{1/2}; \quad v_k = (\nu\varepsilon)^{1/4}$$
Primary effect

Effect of large scales:
$$u' = \sqrt{\frac{\langle \vec{U} \cdot \vec{U} \rangle}{3}} \quad \text{or} \quad \mathrm{R}_\lambda = \sqrt{15}\left(\frac{u'}{v_k}\right)^2$$
Secondary effect

Small-scale flow of adiabatic cumulus cloud core is assumed to be nearly homogeneous and isotropic. (Vaillancourt and Yau 2000)

One-way coupling: Loading $= O(10^{-3})$ by mass or $O(10^{-6})$ by volume.

Equation of motion for droplets

$$\frac{d\vec{V}^{(\alpha)}(t)}{dt} = -\frac{\vec{V}^{(\alpha)}(t) - \left[(U(\vec{Y}^{(\alpha)}(t),t) + \vec{u}(\vec{Y}^{(\alpha)},t)\right]}{\tau_p^{(\alpha)}} - \vec{g}$$

$$\frac{d\vec{Y}^{(\alpha)}(t)}{dt} = \vec{V}^{(\alpha)}(t)$$

Where $\tau_p^{(\alpha)} = 2\rho_p \left(a^{(\alpha)}\right)^2 /(9\mu),\qquad \mathrm{W}^{(\alpha)} = \tau_p^{(\alpha)} g$

If hydrodynamic interaction is considered: $\vec{u}(\vec{Y}^{(\alpha)},t) \neq 0$

Self-consistent: no ambiguity in defining undisturbed fluid velocity

Typically tracking 200,000 droplets with hydrodynamic interactions.

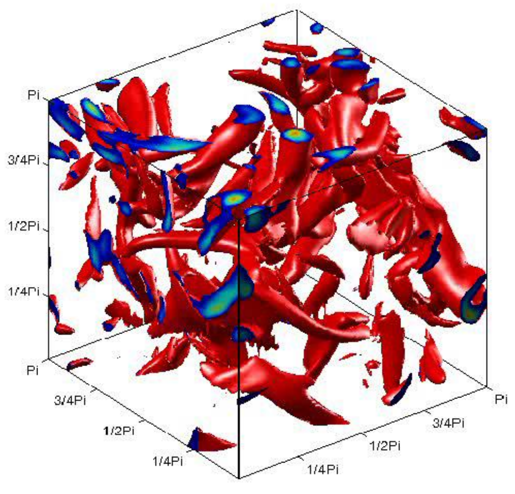
A lot of quantitative information can be extracted!

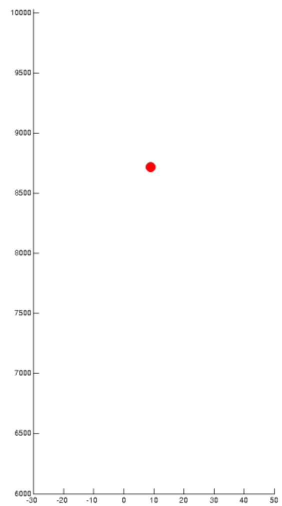
The hybrid DNS approach: disturbance flows due to droplets

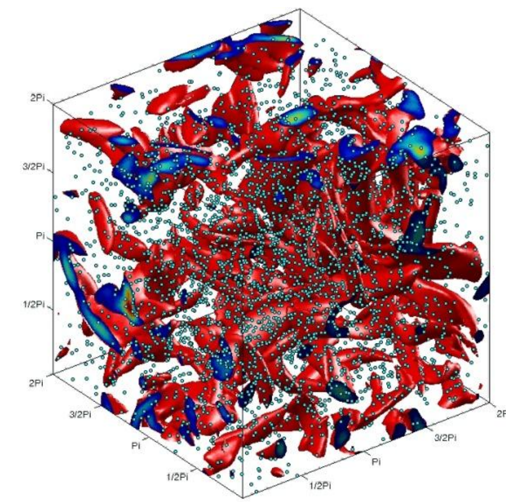
$$\vec{U}(\vec{x},t)+\sum_{k=1}^{N_p}\vec{u}_s(\vec{r}_k;a_k,\vec{V}_k-\vec{U}(\vec{Y}_k,t)-\vec{u}_k)$$

Background turbulent flow

Disturbance flows due to droplets



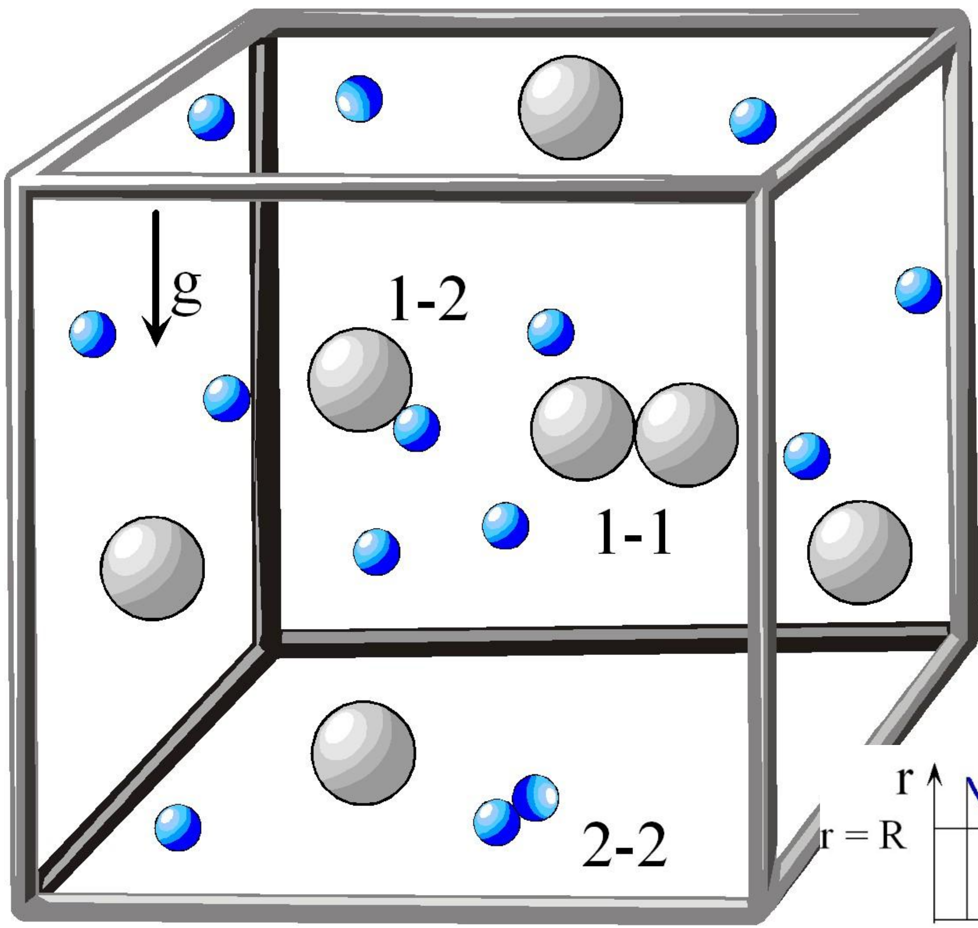




Features: Background turbulent flow can affect the disturbance flows;
No-slip condition on the surface of each droplet is satisfied on average;
Both near-field and far-field interactions are considered.

Wang, Ayala, and Grabowski, J. Atmos. Sci. 62(4): 1255-1266 (2005).
Ayala, Wang, and Grabowski, J. Comp. Phys, 225, 51-73 (2007).

Dynamic collision detection



Dynamic collision kernels:

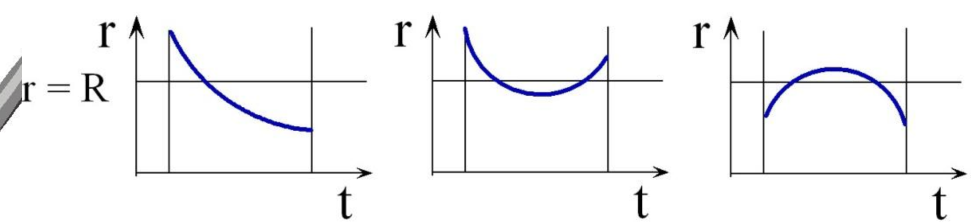
$$K_{12}^{D} = \langle \dot{N}_{12} \rangle / (n_1 n_2)$$

$$K_{11}^{D} = \langle \dot{N}_{11} \rangle / (n_1^2 / 2)$$

$$K_{12}^{D} = \langle \dot{N}_{22} \rangle / (n_2^2 / 2)$$

where

$$n_1 = N_1 / V_B, \qquad n_2 = N_2 / V_B$$



General kinematic collision kernel: Finite-inertia droplets in a turbulent flow

$$K_{12} = 2\pi R^2 \langle | w_r (r = R) | \rangle g_{12}(r = R)$$

Radial relative velocity

Radial distribution function

$$\langle | w_r | \rangle = \frac{1}{N_{pair}} \sum_{all\ pairs} \left| \vec{r} \cdot \frac{(\vec{V}_1 - \vec{V}_2)}{r} \right|$$

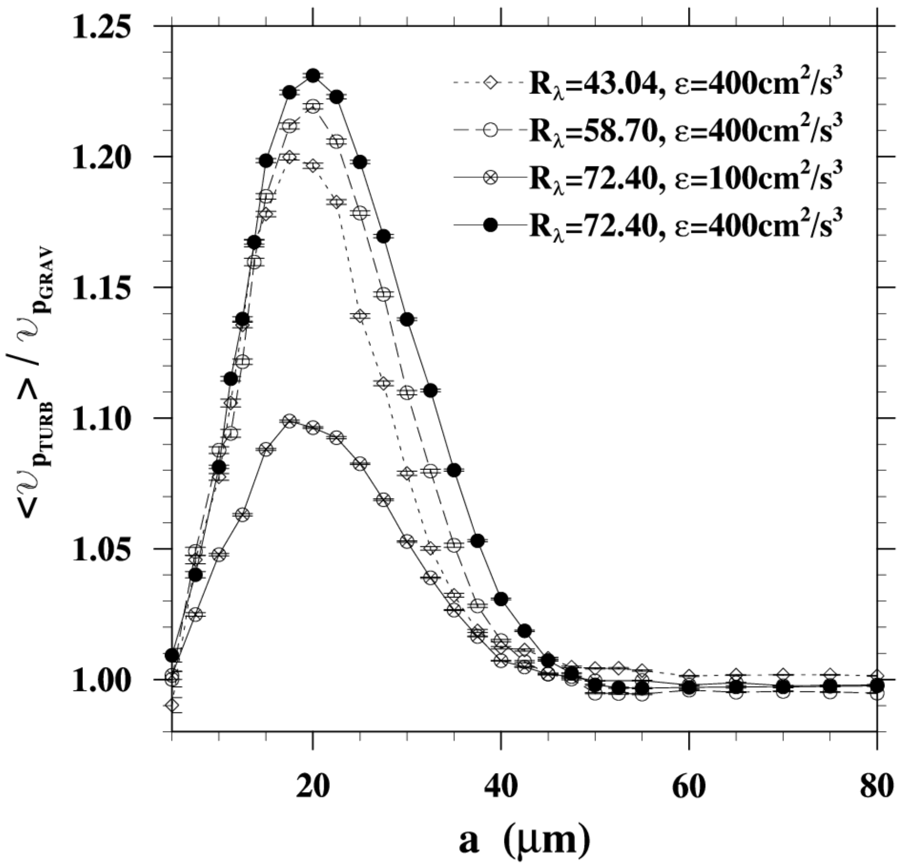
$$g_{12}(R) = \lim_{\delta << r} \frac{N_{pair}(r - \delta \le d \le r + \delta) / 4\pi [(r+\delta)^3 - (r-\delta)^3]}{N_1 N_2 / V_B}$$

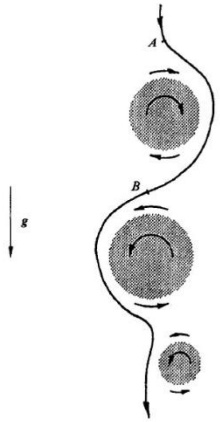
- Based on the spherical formulation
- Confirmed by DNS for all different situations
- Non-overlap corrections when hydrodynamic interactions are considered
- Easy to calculate in DNS, but could be very difficult to measure!!!

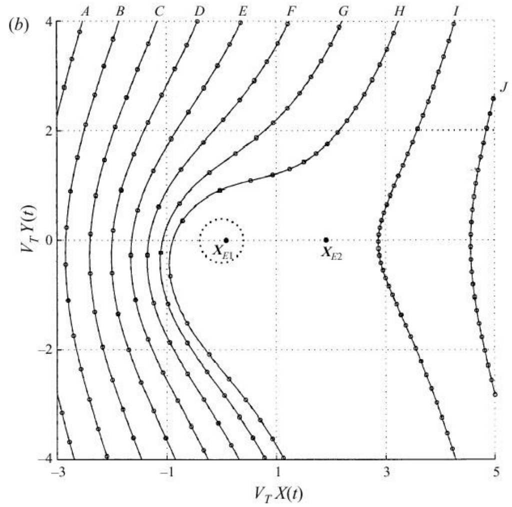
Important for parameterization of collection kernel.

Sundaram & Collins, J. Fluid Mech. 335: 75-109 (1997).
Wang, *et al.* J. Atmos. Sci. 62: 2433-2450 (2005).

The mean settling velocity of droplets in turbulent flow





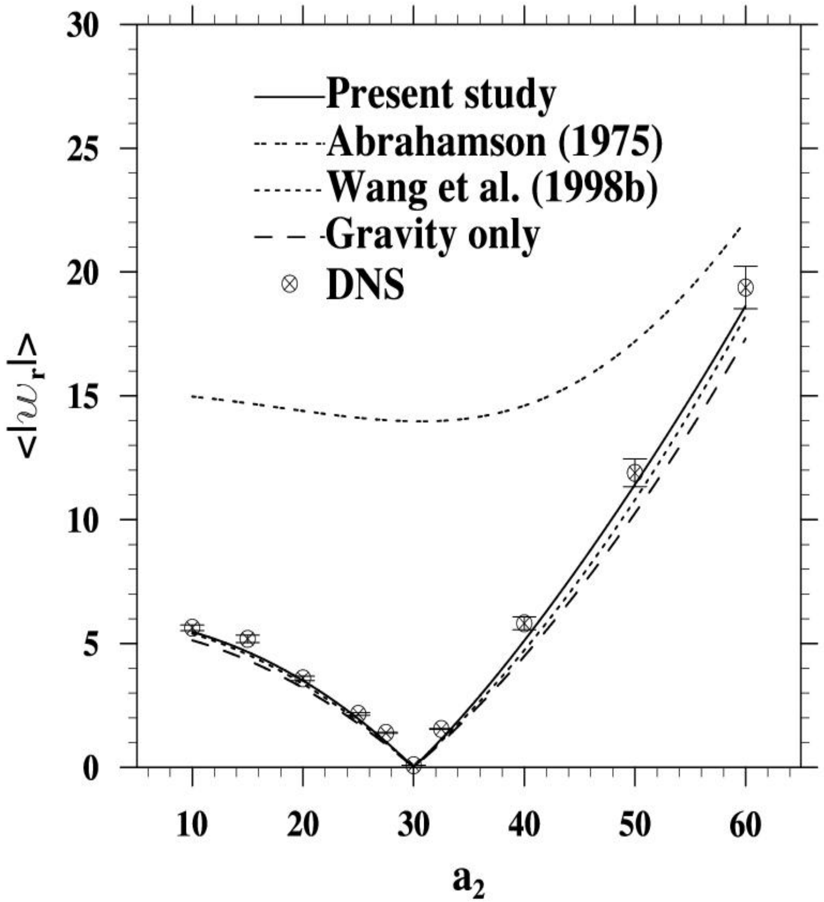


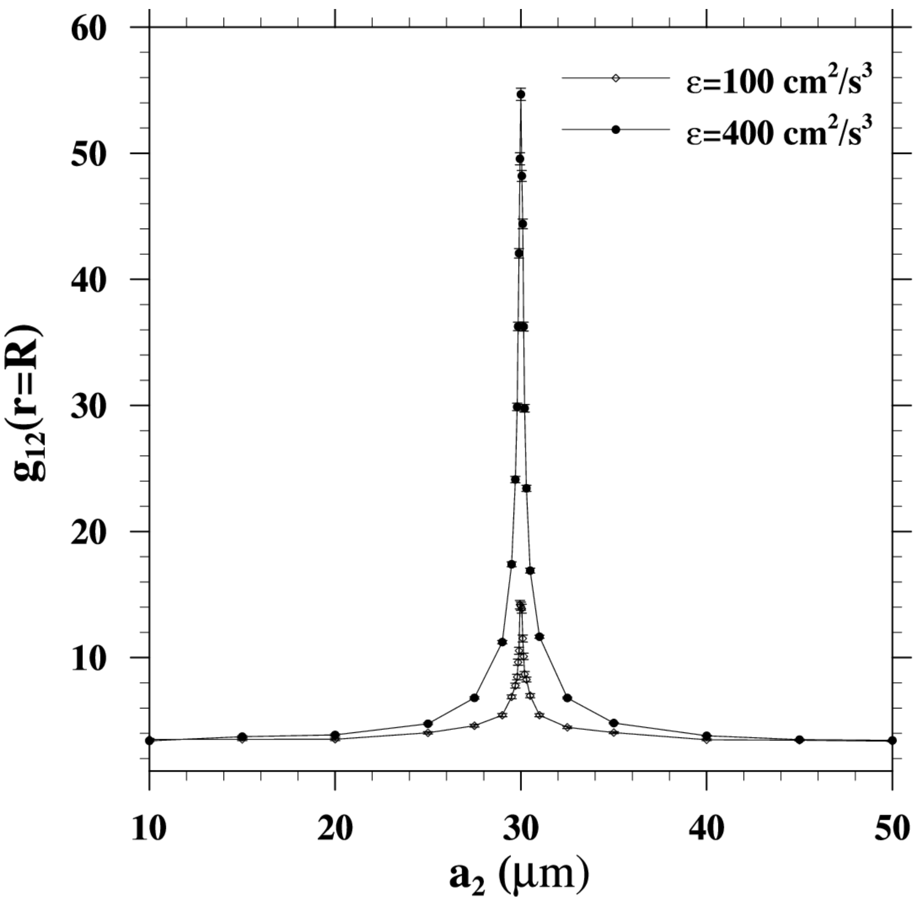
$$\tau_p \sim \frac{\Gamma_{vort}}{W_p^2} \sim \frac{v_k \eta}{(\tau_p g)^2} \sim \frac{\nu}{(\tau_p g)^2} \qquad a \sim 20\ \mu m$$

Davila & Hunt, J. Fluid Mech. 440: 117-145 (2001).
Wang & Maxey, J. Fluid Mech. 256: 27-68 (1993).
Ayala et al., New J. Physics, in press (2008)

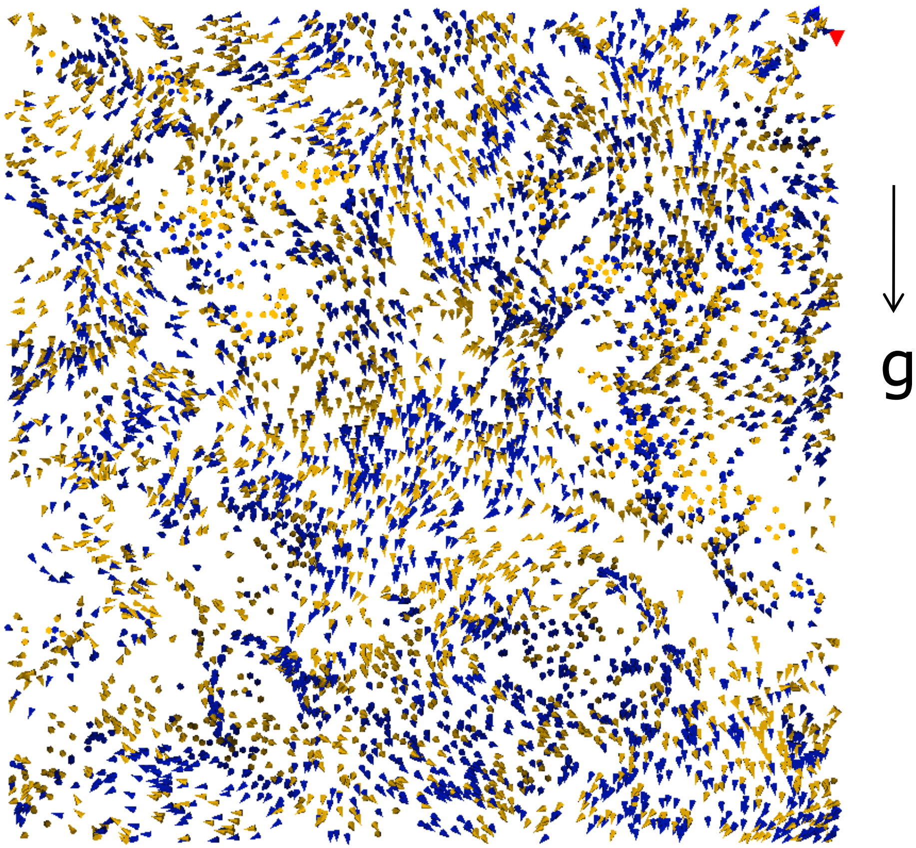
Geometric collision: radial relative velocity and radial distribution function

$$a_1 = 30\mu m, \quad R_\lambda = 72.4, \quad \varepsilon = 400\, cm^2/s^3$$



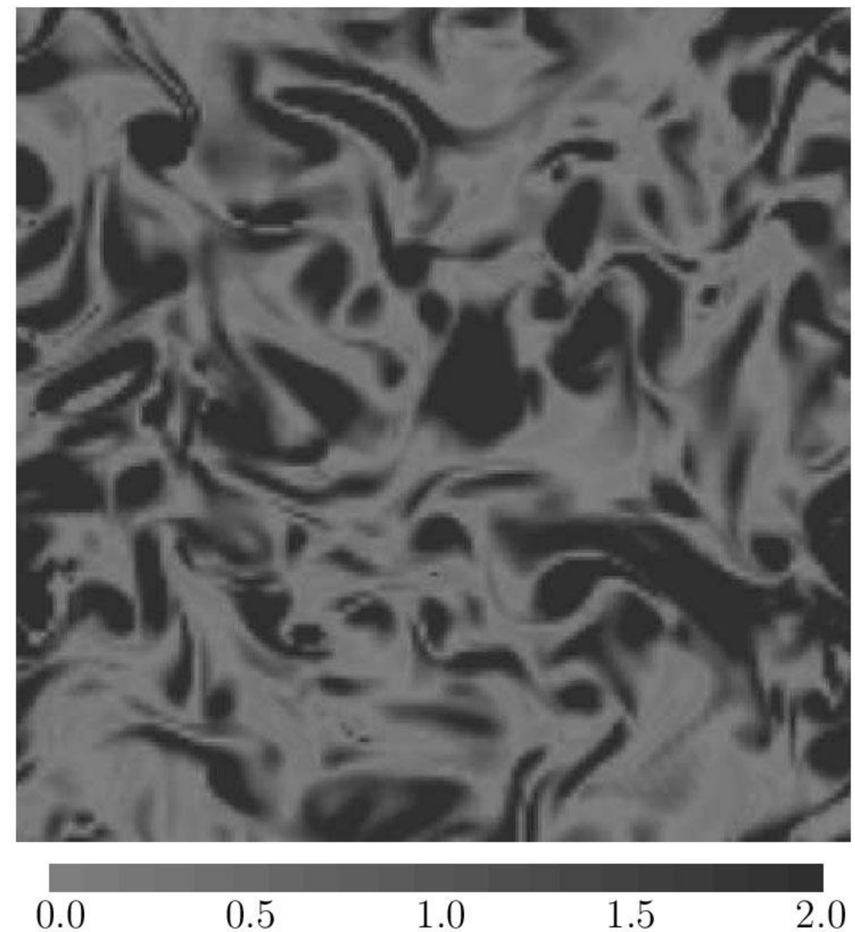


Droplet positions (a thin 2D slice):
black cones for 30 μm (St=0.570)
gray cones for 20 μm (St=0.253)



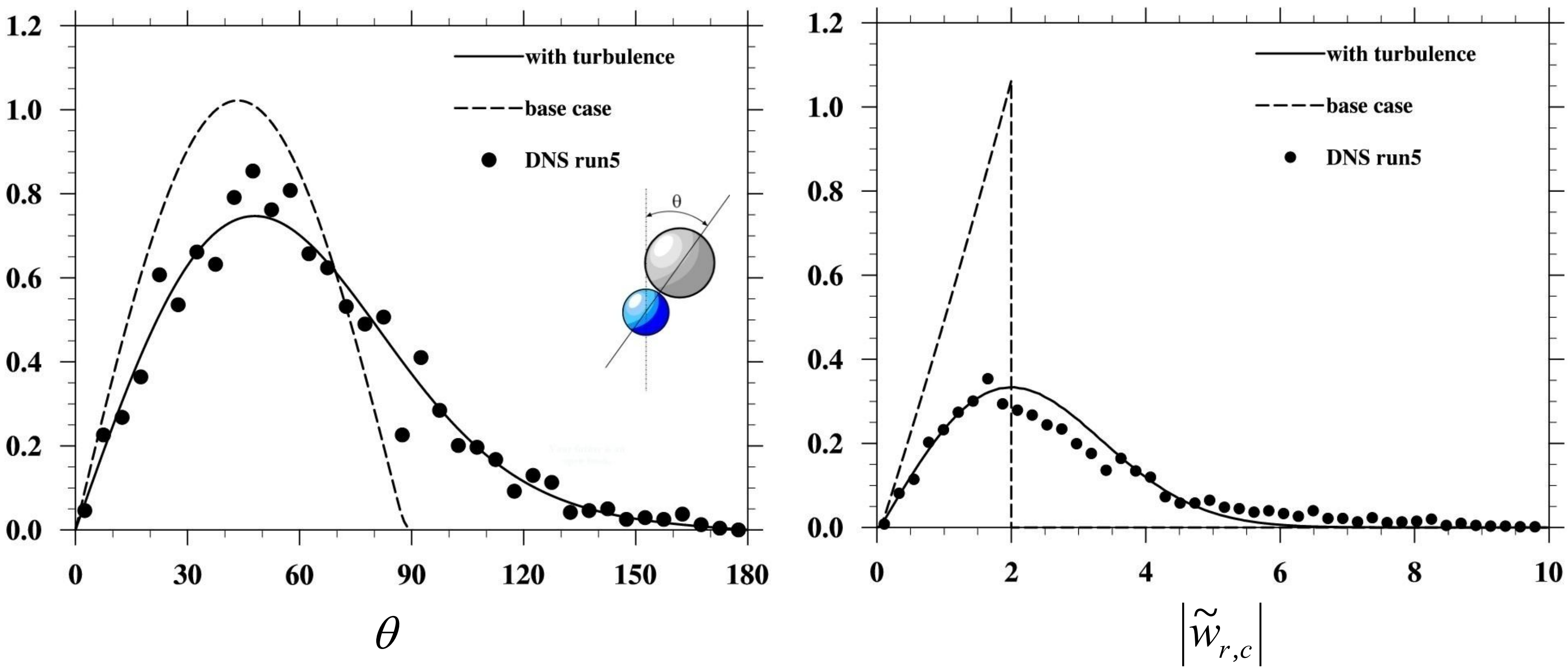
$$g_{12} = 1.125,$$
$$g_{11} = 16.824,$$
$$g_{22} = 5.087$$

Flow enstrophy field



$$R_\lambda = 72, \quad \varepsilon = 400\, cm^2 / s^3$$

PDF of angle-of-approach and radial relative velocity (colliding pairs)



Wang, *et al.* J. Atmos. Sci. 63: 881-900 (2006).

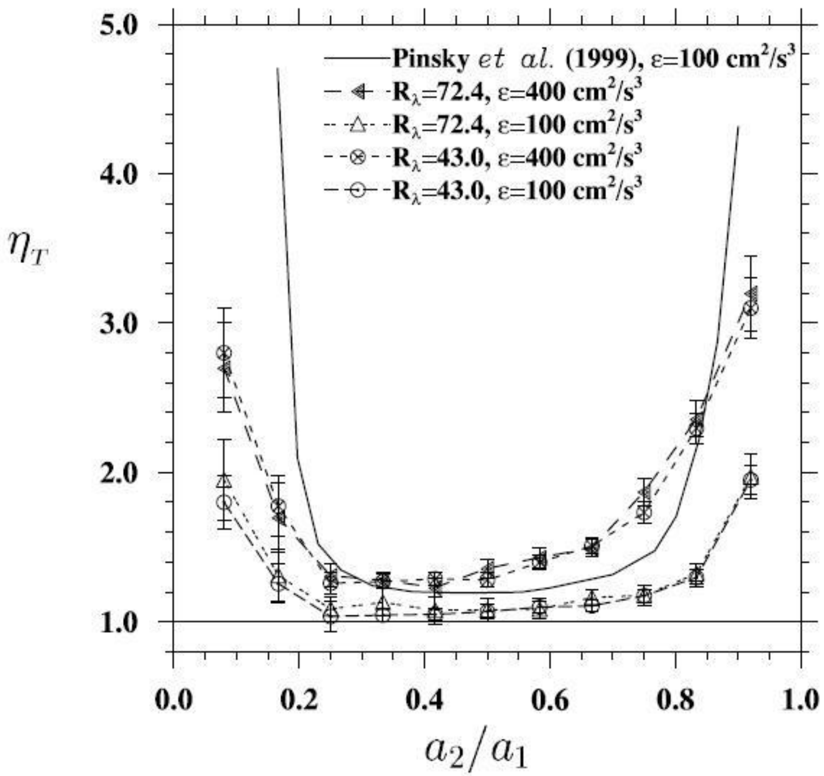
Typical enhancement factors by air turbulence

TABLE II. A case study: $a_1 = 25\ \mu m$, $a_2 = 20\ \mu m$, $R_\lambda = 40$

$\epsilon\ (cm^2/s^3)$		No HI	HI	E	$\eta/\eta_G/\eta_E$
0	$\Gamma_{12}^{D}/\Gamma_{12}^{g}$	1.000 ± 0.026	0.257 ± 0.012	0.257	
	$\Gamma_{12}^{K}/\Gamma_{12}^{g}$	1.026 ± 0.046	0.286 ± 0.023	0.279	
	$< \|w_r\| >^{K} /\Delta W$	0.516 ± 0.013	0.132 ± 0.005		
	g_{12}^{K}	0.99 ± 0.04	1.08 ± 0.04		
100	$\Gamma_{12}^{D}/\Gamma_{12}^{g}$	1.117 ± 0.032	0.315 ± 0.016	0.282	1.23/1.12/1.10
	$\Gamma_{12}^{K}/\Gamma_{12}^{g}$	1.180 ± 0.130	0.302 ± 0.048	0.256	
	$< \|w_r\| >^{K} /\Delta W$	0.533 ± 0.020	0.129 ± 0.011		
	g_{12}^{K}	1.11 ± 0.08	1.17 ± 0.09		
400	$\Gamma_{12}^{D}/\Gamma_{12}^{g}$	1.420 ± 0.032	0.584 ± 0.021	0.411	2.27/1.42/1.60
	$\Gamma_{12}^{K}/\Gamma_{12}^{g}$	1.544 ± 0.109	0.656 ± 0.052	0.425	
	$< \|w_r\| >^{K} /\Delta W$	0.561 ± 0.015	0.218 ± 0.007		
	g_{12}^{K}	1.38 ± 0.06	1.50 ± 0.07		

Wang, *et al*. J. Atmos. Sci. 62: 2433-2450 (2005).

Enhancement factors (Turbulent kernel / gravitational kernel)



turbulent fluctuation $R(v_k/\eta) \sim (v_{p1} - v_{p2})$

aerodynamic interaction time $\dfrac{R}{(v_{p1} - v_{p2})} \sim \tau_{p2}$

inertial response $\tau_p \sim \tau_k$ flow Kolmogrov time

$\tau_p \sim \dfrac{\Gamma_{vort}}{W_p^2} \sim \dfrac{\nu}{(\tau_p g)^2}$ eddy - particle interaction

favor

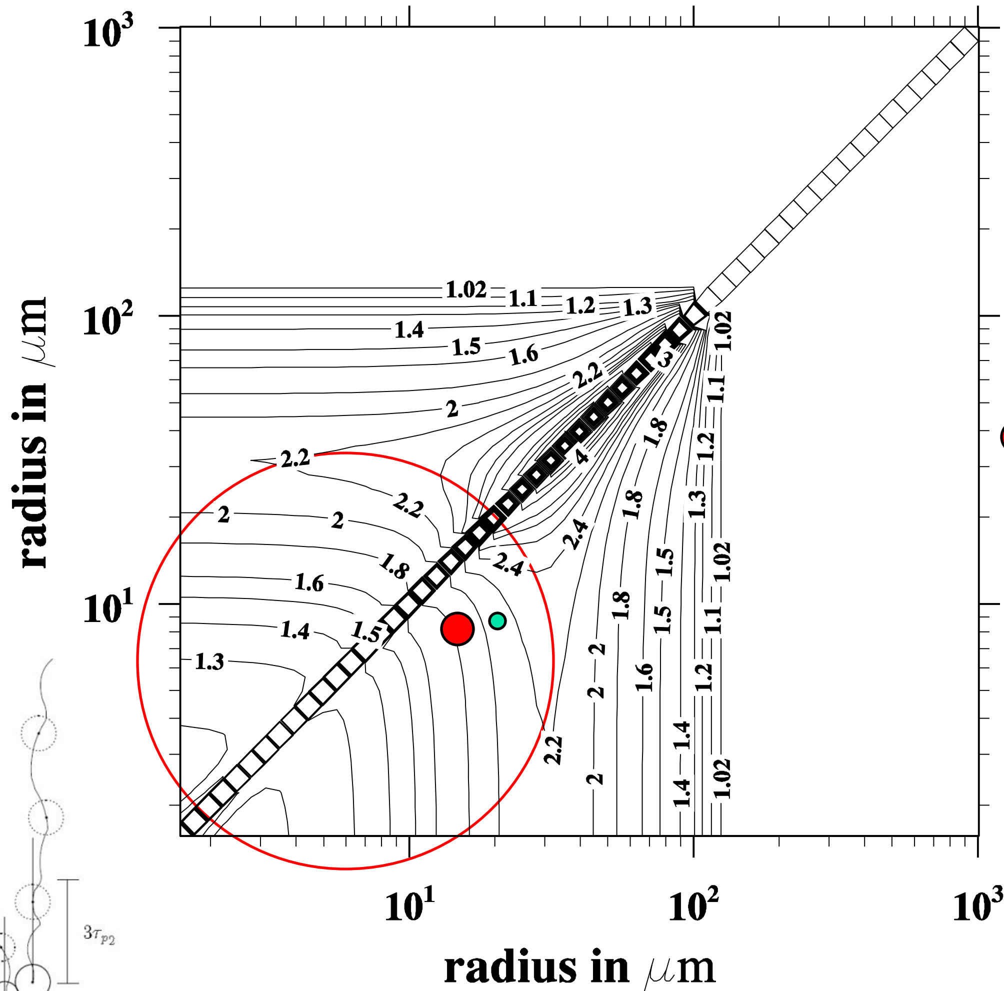
$a_1 \approx a_2$

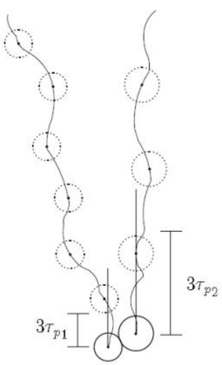
$\dfrac{a2}{a1} \rightarrow 0$ for a given a_1

a_1 is in the bottlebeck range

Overall enhancement factor on geometric collision kernel

Ayala kernel $\varepsilon = 300\, cm^2/s^3; \quad u' = 202\ cm/s$





- Franklin et al. (2005): 10μm-20μm
  a factor of 1.2 to 2.1 for

  $280\, cm^2/s^3 \le \varepsilon \le 1535\, cm^2/s^3;$
  $9\ cm/s \le u' \le 21\ cm/s$

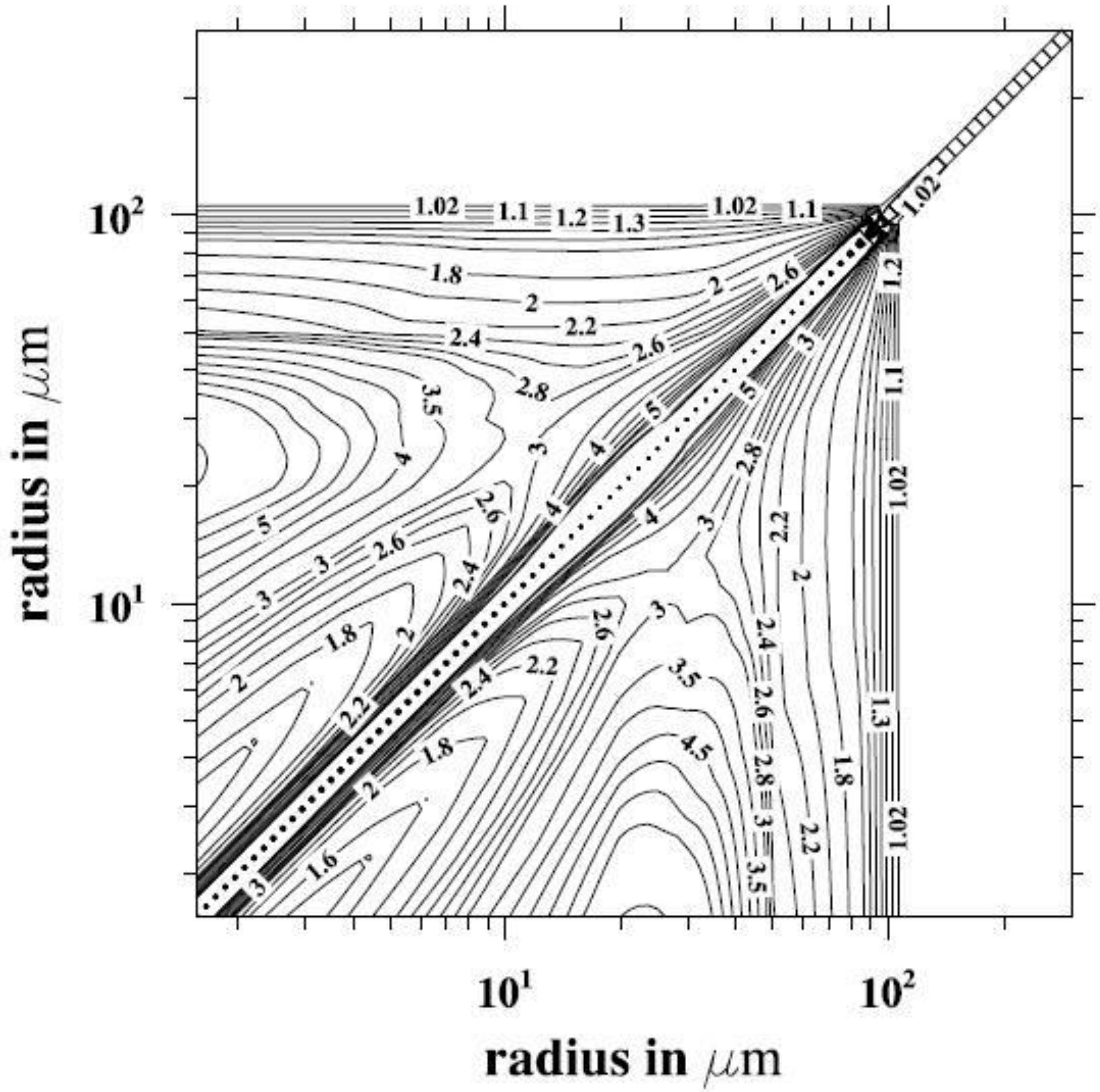
- Pinsky et al. (2006): 10μm-15μm, a factor of 1.6 for
  $\varepsilon = 1000\, cm^2/s^3$

Pinsky et al. (1997): a factor of 10 for

$\varepsilon = 600\, cm^2/s^3$

Ayala, *et al.*, Effects of turbulence on the geometric collision rate of sedimenting droplets: Part 2. Theory and parameterization. *New J. Physics*, July, 2008.

Turbulent kernel / Hall kernel (with turbulent collision efficiency)



The overall enhancement factor of collection kernel by air turbulence.

Flow dissipation rate is 300 cm^2/s^3; r.m.s. velocity 202 cm/s.

Wang, *et al.*, Turbulent collision efficiency of heavy particles relevant to cloud droplets. *New J. Physics*, July, 2008.

Part 2.

Impact of turbulent collection kernel on warm rain initiation

Kinetic collection equation

$$\frac{\partial n(x,t)}{\partial t} = \frac{1}{2}\int_{x_0}^{x-x_0} n(x-y,t)n(y,t)K(x-y,y)dy - n(x,t)\int_{x_0}^{\infty} n(y,t)K(x,y)dy$$

x_0 is the mass of the smallest droplet in the system.
n(x,t) is continuous number density distribution
K(x,y) is the collection kernel, a non-negative, symmetric function of x and y

Initial distribution

$$n(x,t) = \frac{N_0}{\bar{x}_{f0}}\exp\left(-\frac{x}{\bar{x}_{f0}}\right)$$

Numerical method: Bin Integral Method with Gauss Quadrature
Converged solution without numerical diffusion/dispersion errors

Wang, Xue & Grabowski 2007, *J. Comp. Physics*, 226, 59-88.

A general initial size distribution

$$n(x, t=0) = A\frac{L_0}{\bar{x}_0^2}\exp\left[-\left(\frac{Bx}{\bar{x}_0}\right)^\alpha\right]$$

$\bar{x}_0$ Initial average mass of cloud droplets, 3.3e–9 g (9.3 μm)

L_0 The cloud liquid water content, 1 g/m^3

α	A	B	γ
1	1	1	0.3634
2	0.636618	0.564189	0.3015
3	0.566034	0.505479	0.2825

γ Initial relative radius dispersion
= standard deviation of radius / mean radius

Observations: $0.1 < \gamma < 0.4$

Pawlowska *et al.* 2006 *Geophys. Res. Lett.* 33: L19810

Five collection kernels considered

1. **Hall kernel: Hydrodynamical gravitational kernel without effect of air turbulence. Base case.**

$$K_{ij} = 2\pi\, R^2 E_{ij}^s \left\{ \frac{2}{\pi} \left[\frac{\pi}{8} (v_{pi} - v_{pj})^2 \right] \right\}^{1/2} = \pi\, R^2 E_{ij}^s \, | v_{pi} - v_{pj} |$$

2. **mZWWa kernel (Zhou et al. 2001): Without gravitational settling**

$$K_{ij} = 2\pi R^2 E_{ij}^s \left[\frac{2}{\pi} (\langle w_{r,accel}^2 \rangle + \langle w_{r,shear}^2 \rangle) \right]^{1/2} g_{ij}^2 (r = R)$$

3. **mZWWb kernel: Turbulence+gravity**

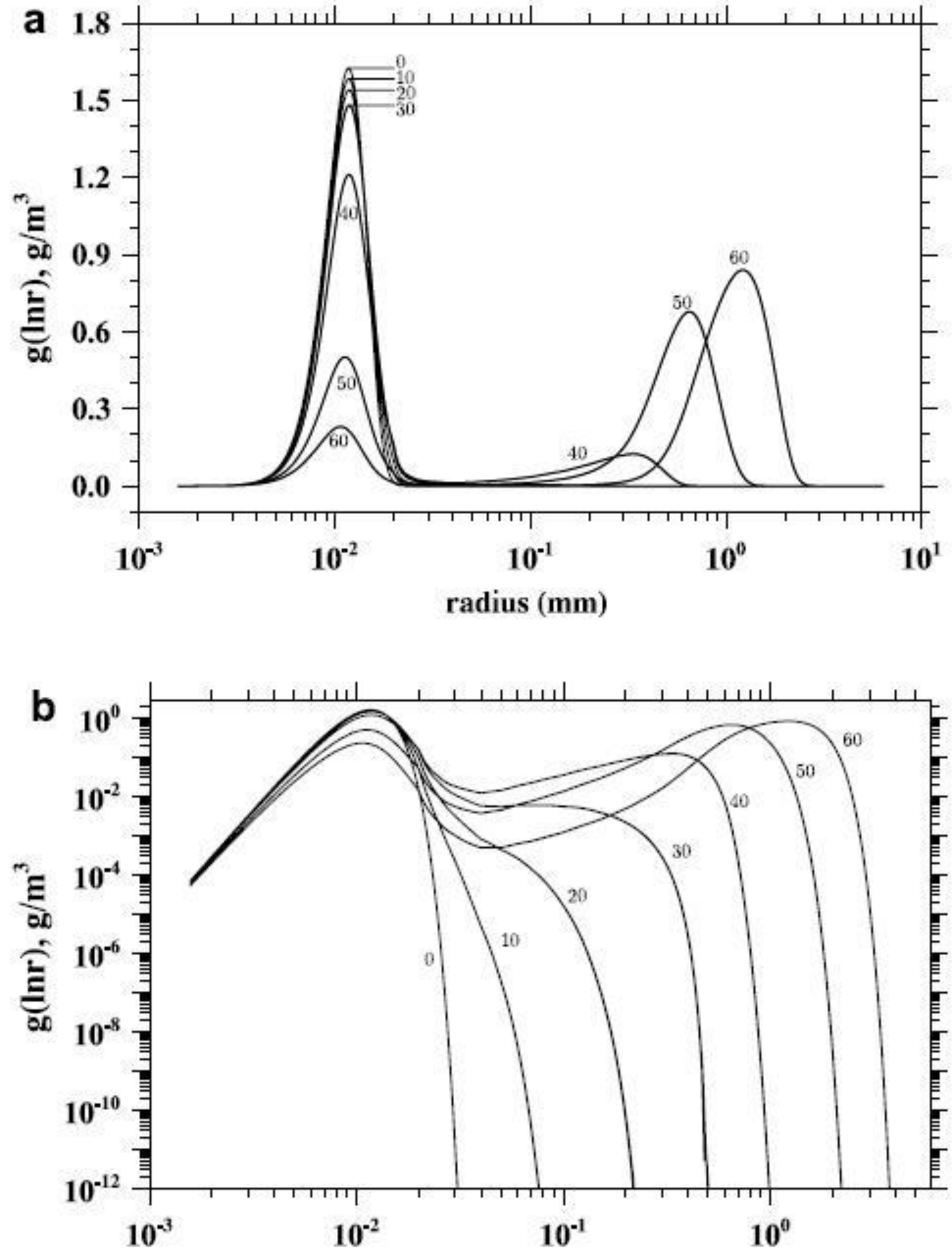
$$K_{ij} = 2\pi R^2 E_{ij}^s \left\{ \frac{2}{\pi} \left[\left(\langle w_{r,accel}^2 \rangle + \langle w_{r,shear}^2 \rangle \right) g_{ij}^2 (r = R) + \frac{\pi}{8} g^2 (\tau_{pi} - \tau_{pj})^2 \right] \right\}^{1/2}$$

4. **Ayala (Ayala 2005) kernel: Turbulence+gravity**

$$K_{ij} = 2\pi R^2 E_{ij}^s \left\{ \frac{2}{\pi} \left[\sigma^2 + \frac{\pi}{8} g^2 (\tau_{pi} - \tau_{pj})^2 \right] \right\}^{1/2} g_{ij} (r = R)$$

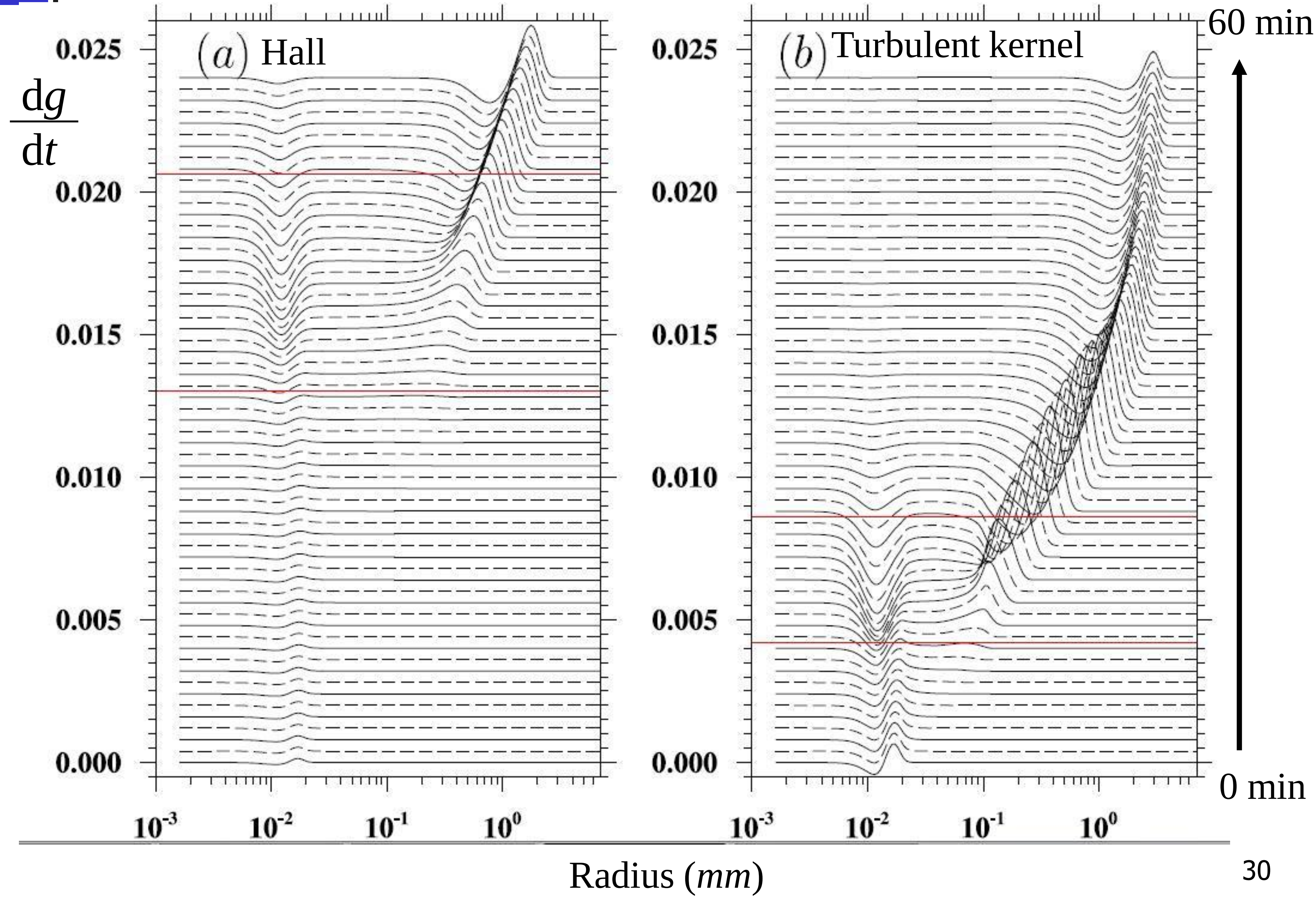
5. **ZWW-RW kernel (Riemer and Wexler 2005):**
   Turbulence+gravity or Hall+mZWWa

Mass distributions based on the Hall kernel

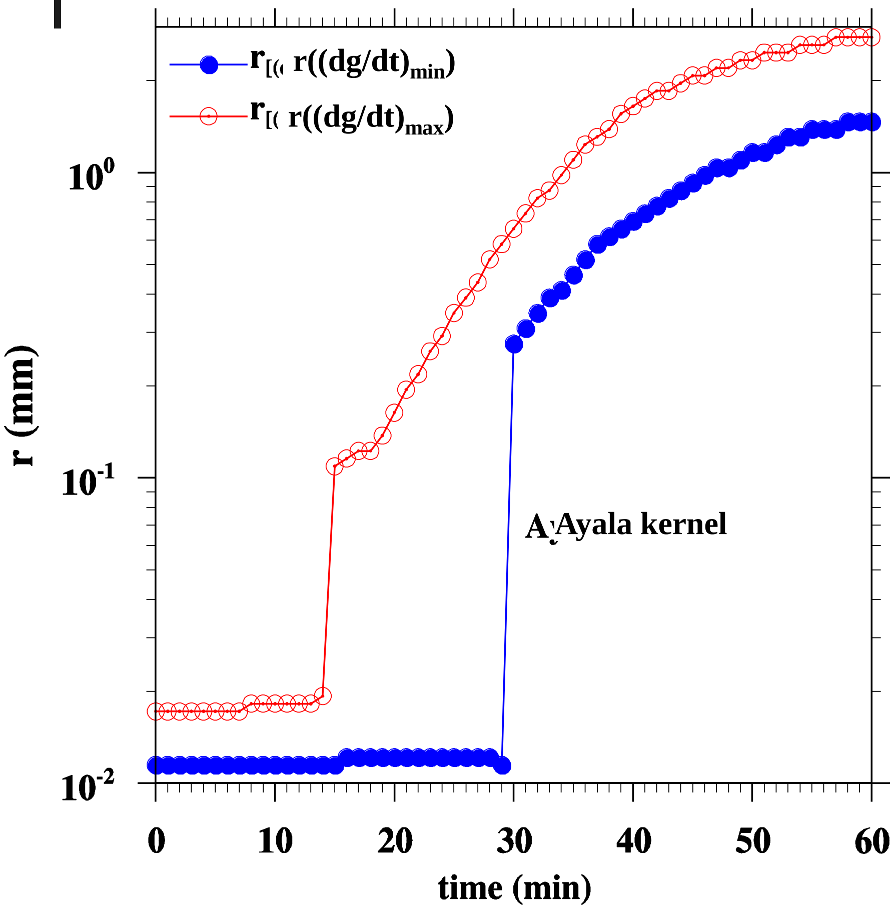


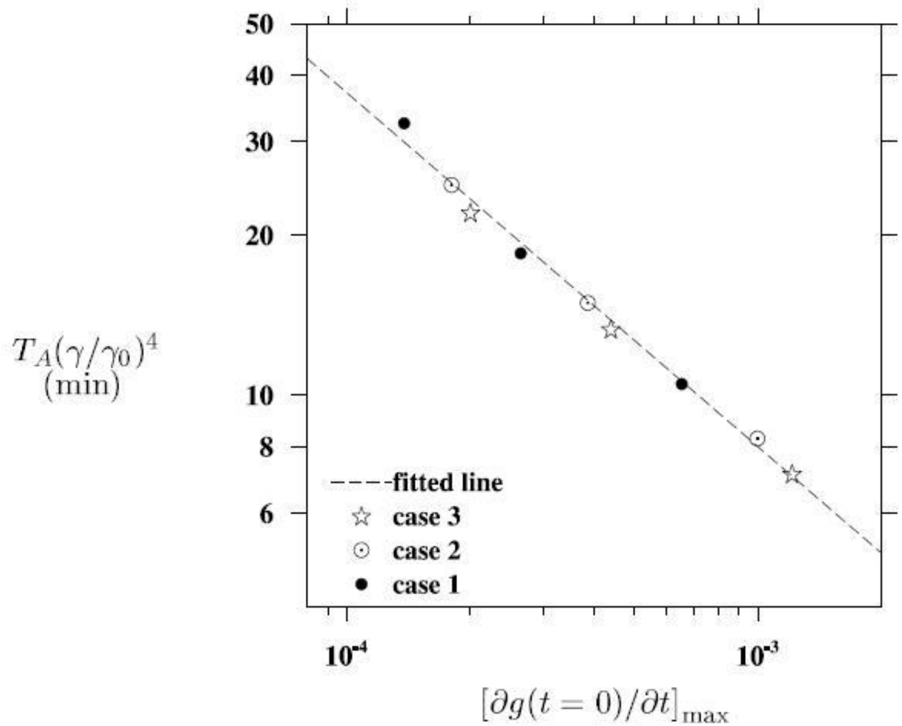
Wang, Xue & Grabowski 2007
J. Comp. Physics, 226, 59-88.

1. Autoconversion; 2. Accrection; 3. Hydrometeor self-collection (Berry and Reinhardt, 1974)



Method to identify the three phases

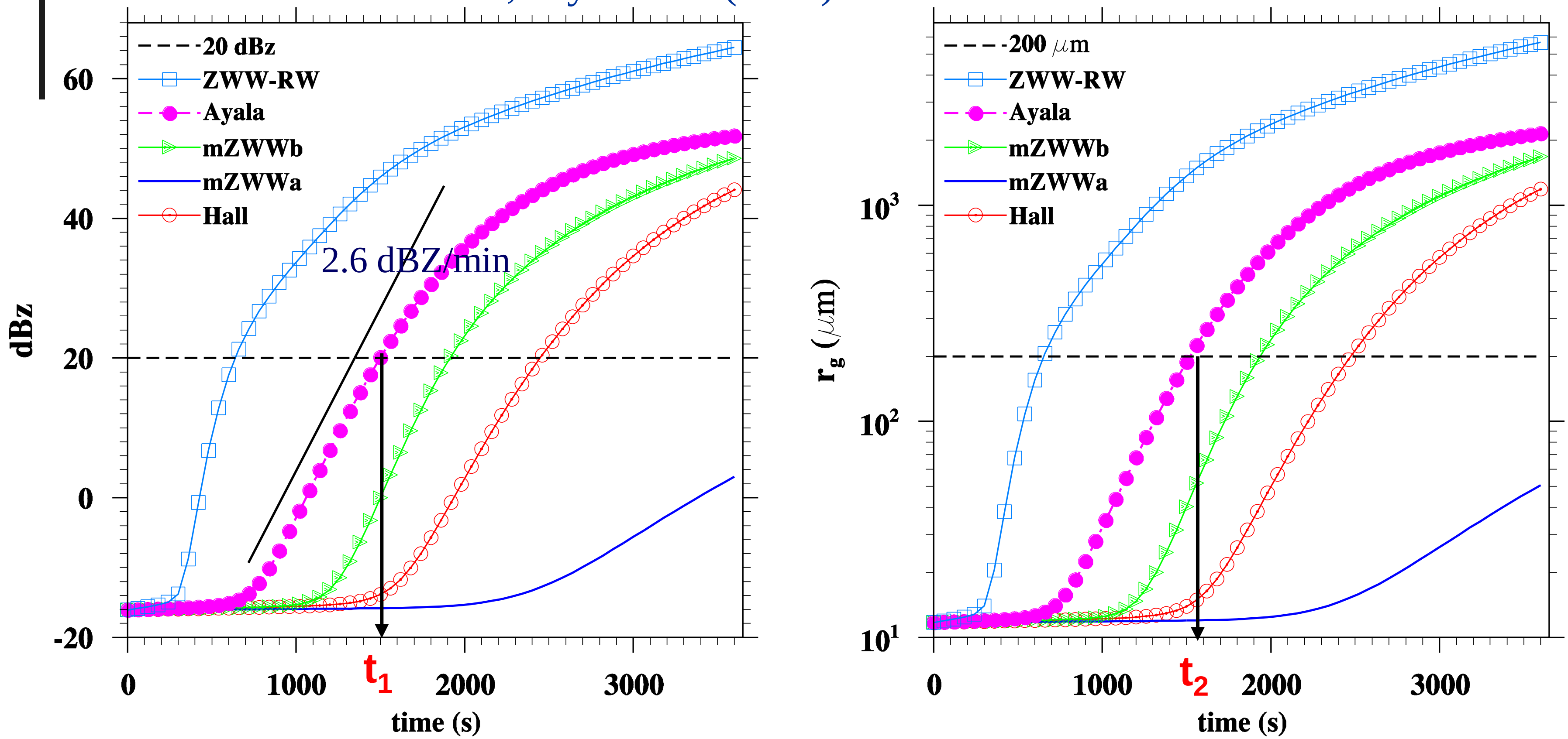




$$T_A \approx 0.08 \times \left(\frac{\partial g(\ln r, t=0)}{\partial t}\right)_{\max}^{-2/3} \times \left(\frac{\gamma}{\gamma_0}\right)^{-4}$$

Characteristic growth times: t_1 and t_2

SCMS: 0.9~5.0 dBZ/min, Blyth et al. (2003)



	Hall	mZWWb	Ayala	ZWW-RW
t_1 (s)	2448	1913	1498	640
t_2 (s)	2474	1935	1519	653

ε= 300 cm^2/s^3

u' = 202 cm/s

Dependence of growth time on ε, u'

For Ayala kernel

ε cm^2/s^3	u' cm/s	t_1 s	t_2 s	Δt_1 %	Δt_2 %
100	100	1949	1972	20.4	20.3
	150	1832	1855	25.2	25.0
	202	1738	1761	29.0	28.8
200	100	1816	1837	25.8	25.7
	150	1685	1707	31.2	31.0
	202	1584	1605	35.3	35.1
300	100	1736	1757	29.1	28.9
	150	1602	1623	34.6	34.4
	202	1498	1519	38.8	38.6
400	100	1681	1702	31.3	31.2
	150	1547	1568	36.8	36.6
	202	1443	1464	41.1	40.8
With turbulent collision efficiency: 400	202	1230	1250	49.8	49.4

Reduction relative to the Hall kernel

Xue, Wang & Grabowski 2007, Growth of cloud droplets by turbulent collision-coalescence, *J. Atmos. Sci*, 65: 331-356.

Impact study using a parcel model

Conservation of the moist static energy and total water in a rising adiabatic parcel

$\phi^{(i)} dr$ droplet concentration in a bin

T parcel temperature

P parcel pressure

q_v water vapor mixing ratio

w rising velocity (prescribed)

$$C_p \frac{dT}{dt} = -gw + LC$$

$$\frac{dq_v}{dt} = -C$$

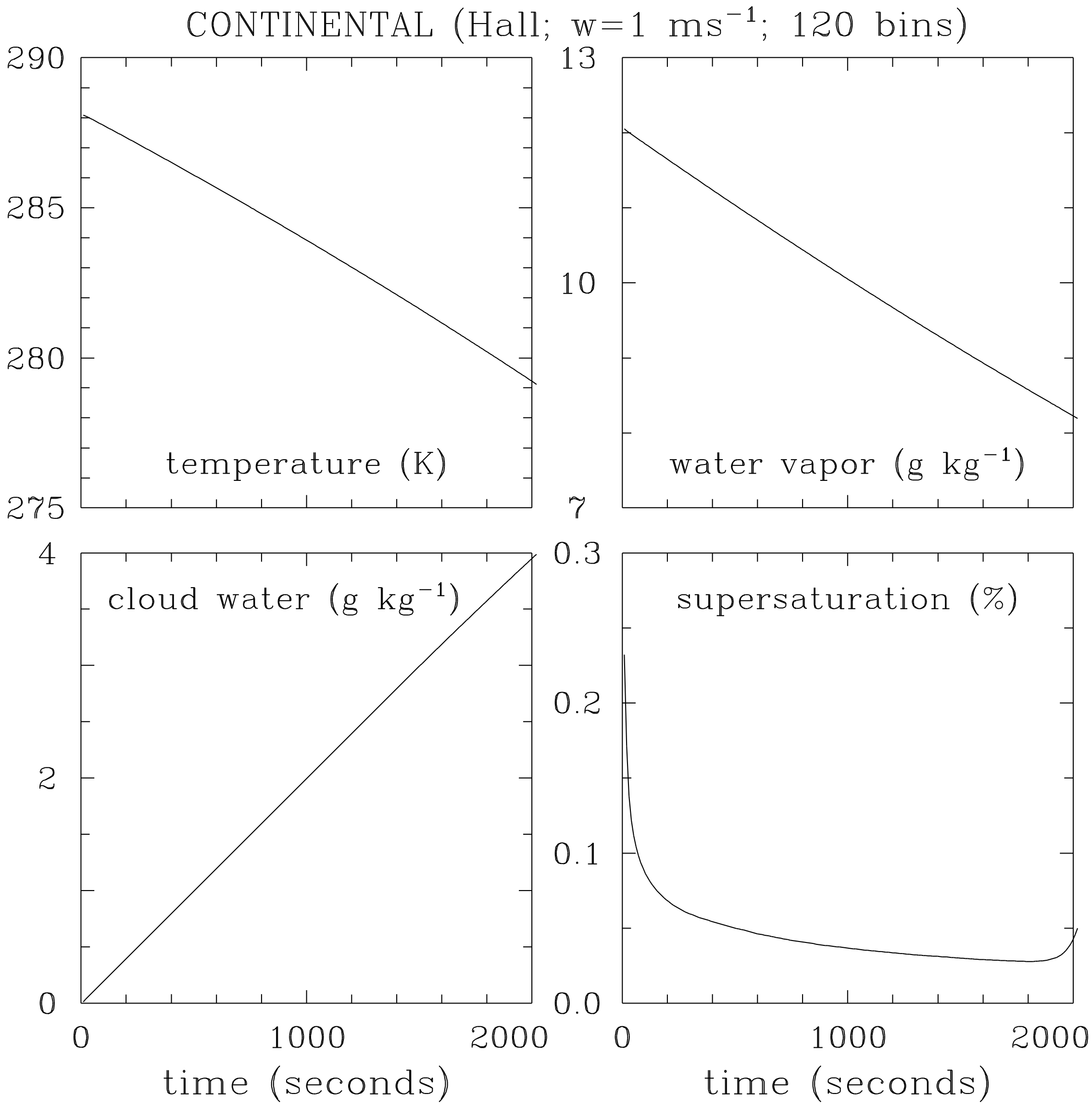
$$\frac{dp}{dt} = -\rho_0 wg$$

$$\frac{\partial \phi^{(i)}}{\partial t} = \left(\frac{\partial \phi^{(i)}}{\partial t} \right)_{activation} + \left(\frac{\partial \phi^{(i)}}{\partial t} \right)_{condensation} + \left(\frac{\partial \phi^{(i)}}{\partial t} \right)_{coalescence}$$

supplemented with an activation model

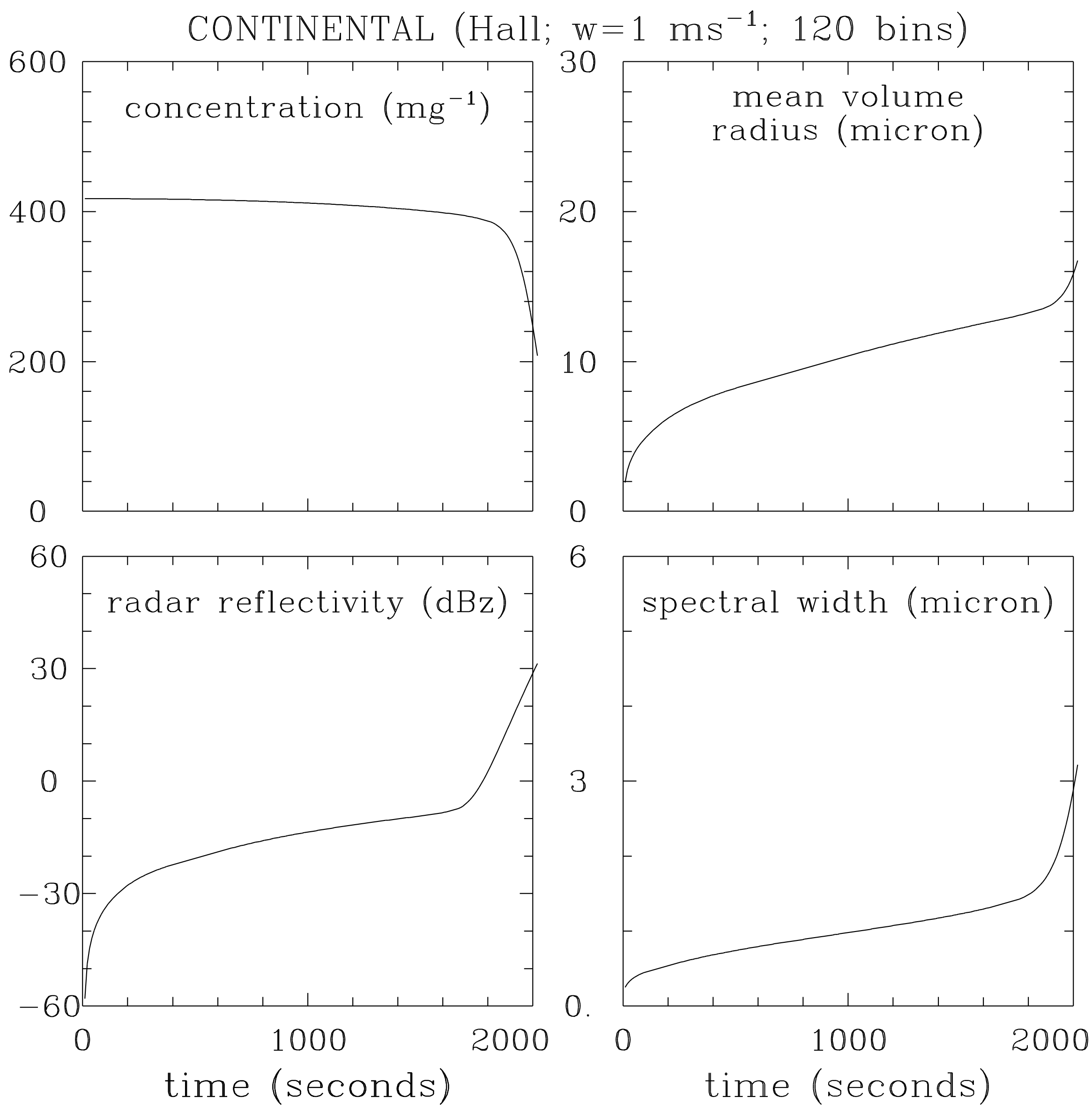
$$N_{CCN} = C_0 (100S)^k$$

Time evolution (continental, Hall gravitational kernel)



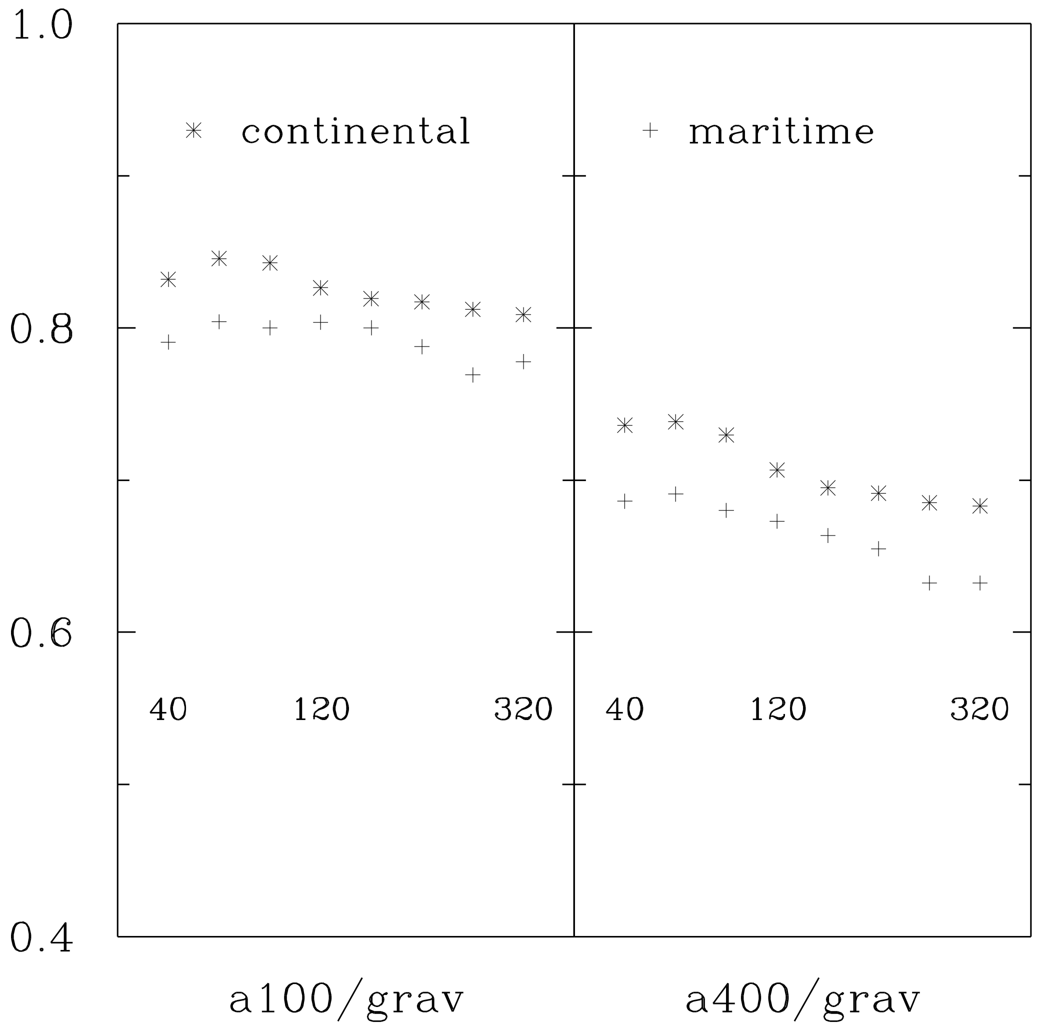
w=1 m/s

Time evolution (continental, Hall gravitational kernel)



w=1 m/s

The speed up factor by air turbulence



w=1 m/s

Grabowski & Wang 2008, Diffusional and accretional growth of water drops in a rising adiabatic parcel: effects of turbulent collision kernel, submitted to *Atmos. Chem. Phys.*

Summary:

Cloud turbulence seems to have appreciable effect on droplet growth by collision/coalescence.

Next steps:

- **High-resolution HDNS simulations**
- **More accurate parameterizations**
- **Including turbulent collision kernels in a dynamic cloud model**

More information (papers published, in press, accepted) at:

http://research.me.udel.edu/~lwang/publications.html



Questions?