We need “pV=nRT” for climate

(J. Harte, May 6, 2008)

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Outline

1. Conceptual overview
2. Implementation of ideas
3. Modelling results
4. Observations

Overview -- the problem:
Oceans, lakes and (most) duck ponds are too big.

See 10^{24} to 10^{30} excited degrees of freedom. Get a bigger computer? Even biggees care state vectors of maybe 10^{10}. For every variable resolved, one must guess dependence 10^{15} unknowns. Rethink!

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Dependent variables as expectations:

$\mathbf{y}$=state vector (temp, salin, veloc, ...) $[\mathbf{y}] \sim 10^{30}$

for this $\mathbf{y}$ textbooks give us $d\mathbf{y}/dt=\mathbf{f}(\mathbf{y})+\mathbf{g}$

$dp=p(\mathbf{y})dy$: probability actual $\mathbf{y}'$ within dy of $\mathbf{y}$

expectations $\mathbf{Y}=\int \mathbf{y}dp$, $\mathbf{R}=\int \mathbf{r}(\mathbf{y})dp$. $[\mathbf{Y}]$ can be small

$d\mathbf{Y}/dt=\mathbf{F}(\mathbf{Y})+\mathbf{G}+$"more". "more" because $\mathbf{F} \neq \int \mathbf{f}dp$

what to do about "more"?

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d$\mathbf{Y}$/dt=$\mathbf{F}$($\mathbf{Y}$)+$\mathbf{G}$+“more”. “more” because $\mathbf{F}\neq\int\mathbf{f}$dp

what to do about “more”? entropy H=-∫dp log(p)

three choices:

a) forget d/dt, forcing, dissip. let **Y** maximise H

b) “more” are such to maximise production of H

c) “entropic force”: $d\mathbf{Y}/dt=\mathbf{F}(\mathbf{Y})+\mathbf{G}+$

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$\mathbf{C}\cdot\partial_{\mathbf{Y}}H$ has two parts: **C** and $\partial_{\mathbf{Y}}H$. *n.b*: "accessible"

$\mathbf{C}\cdot\partial_{\mathbf{Y}}H \sim \mathbf{C}\cdot\partial_{\mathbf{Y}}\partial_{\mathbf{Y}}H\cdot(\mathbf{Y}-\mathbf{Y}^*) = \mathbf{K}\cdot(\mathbf{Y}-\mathbf{Y}^*)$ where

$\mathbf{Y}^*$ only needs be evaluated at "small" $\partial_{\mathbf{Y}}H$

(*n.b*: you still need **K**)

Wooly words! See explicit *e.g.* $\partial_t \nabla^2\psi + J(\psi, \nabla^2\psi + h) = ...$

expand $\psi = \sum \psi_n(t)\phi_n$, $h = \sum h_n \phi_n$ on eigenfunctions $\nabla^2\phi_n + q_n^2\phi_n = 0$

Conserved quadratics are $E = \frac{1}{2}\sum q_n^2 |\psi_n|^2$ and $\Omega = \frac{1}{2}\sum |-q_n^2\psi_n + h_n|^2$ (circulation=0 here)

Maximise $S = -\int dp \log p$ subject to $\langle E\rangle = E_0$, $\langle \Omega \rangle = \Omega_0$ and $\langle 1 \rangle = 1$.

$\delta \int \mathbf{dx}(p\log p + \alpha E p + \beta\Omega p + \gamma p) = 0$ hence $\log p + 1 + \alpha E + \beta\Omega + \gamma = 0$

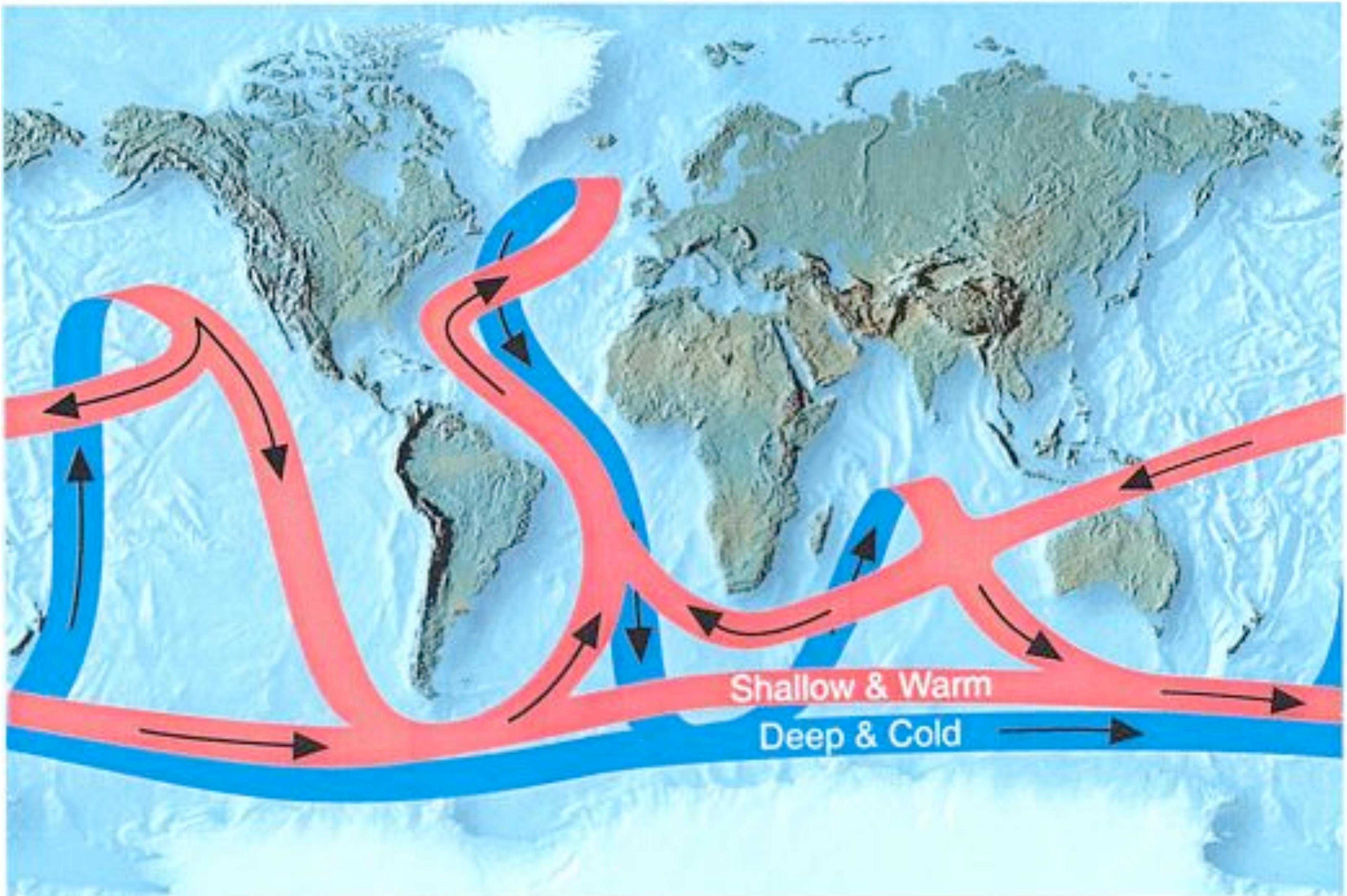
$$p = \exp\{-1-\gamma\}\exp\left\{-\sum\left\{q_n^2(\alpha + \beta q_n^2)|\psi_n|^2 - 2\beta q_n^2 \operatorname{Re}\psi_n h_n + \beta|h_n|^2\right\}\right\}$$

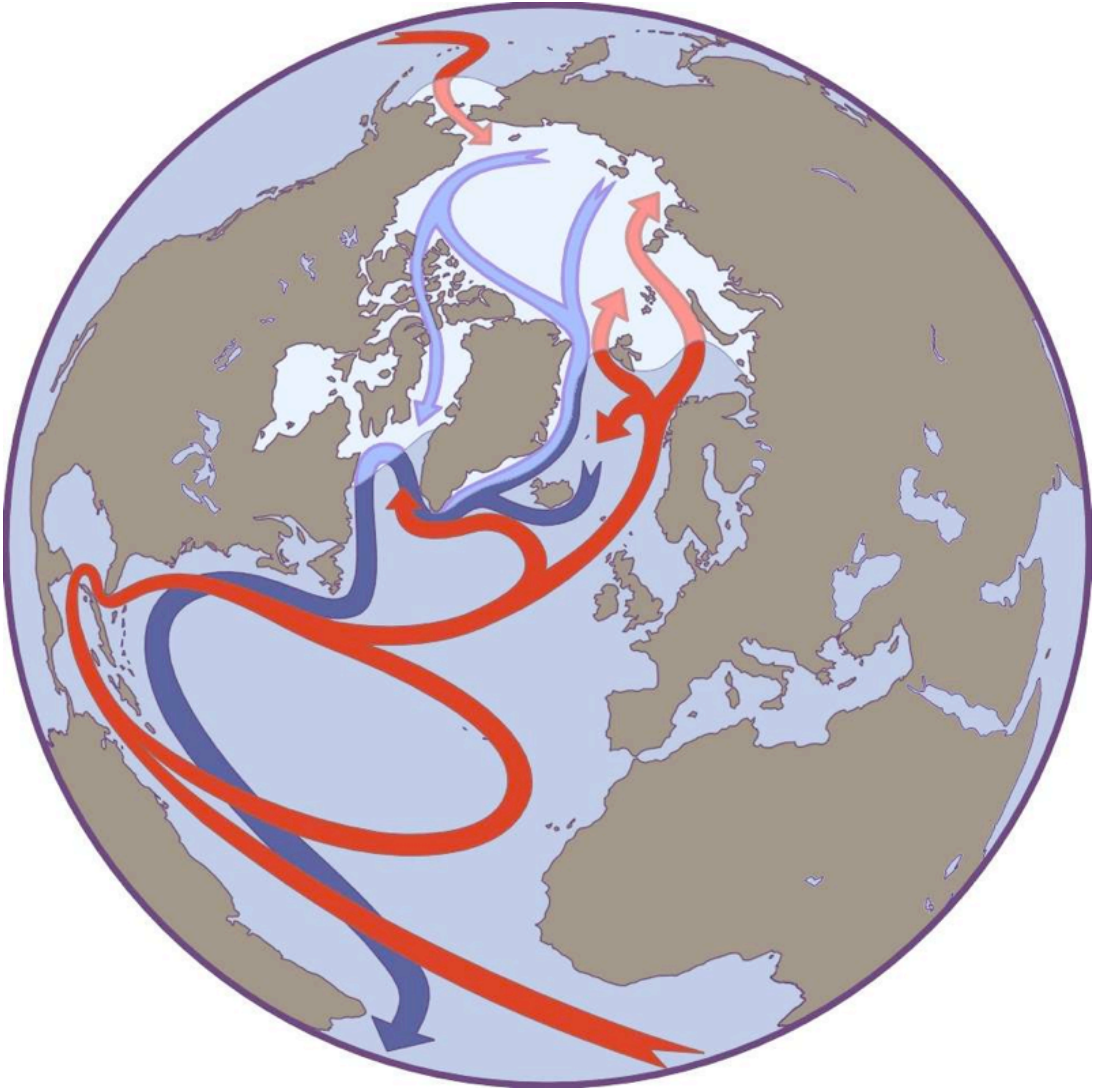
$= \Gamma \exp\left\{-\sum q_n^2(\alpha + \beta q_n^2)|\psi_n - \hat{\psi}_n|^2\right\}$ where $\hat{\psi}_n = \beta h_n / (\alpha + \beta q_n^2)$ or $(\alpha/\beta - \nabla^2)\hat{\psi} = h$

If resolved scales are larger than $\lambda = \sqrt{\beta/\alpha}$, drop ∇^2 and simply $\hat{\psi} = \lambda^2 h$

K·(**Y**-**Y***) with **Y***=-fL^2D, **K**=A∇^2 : "neptune"

The Global Ocean Conveyor Belt





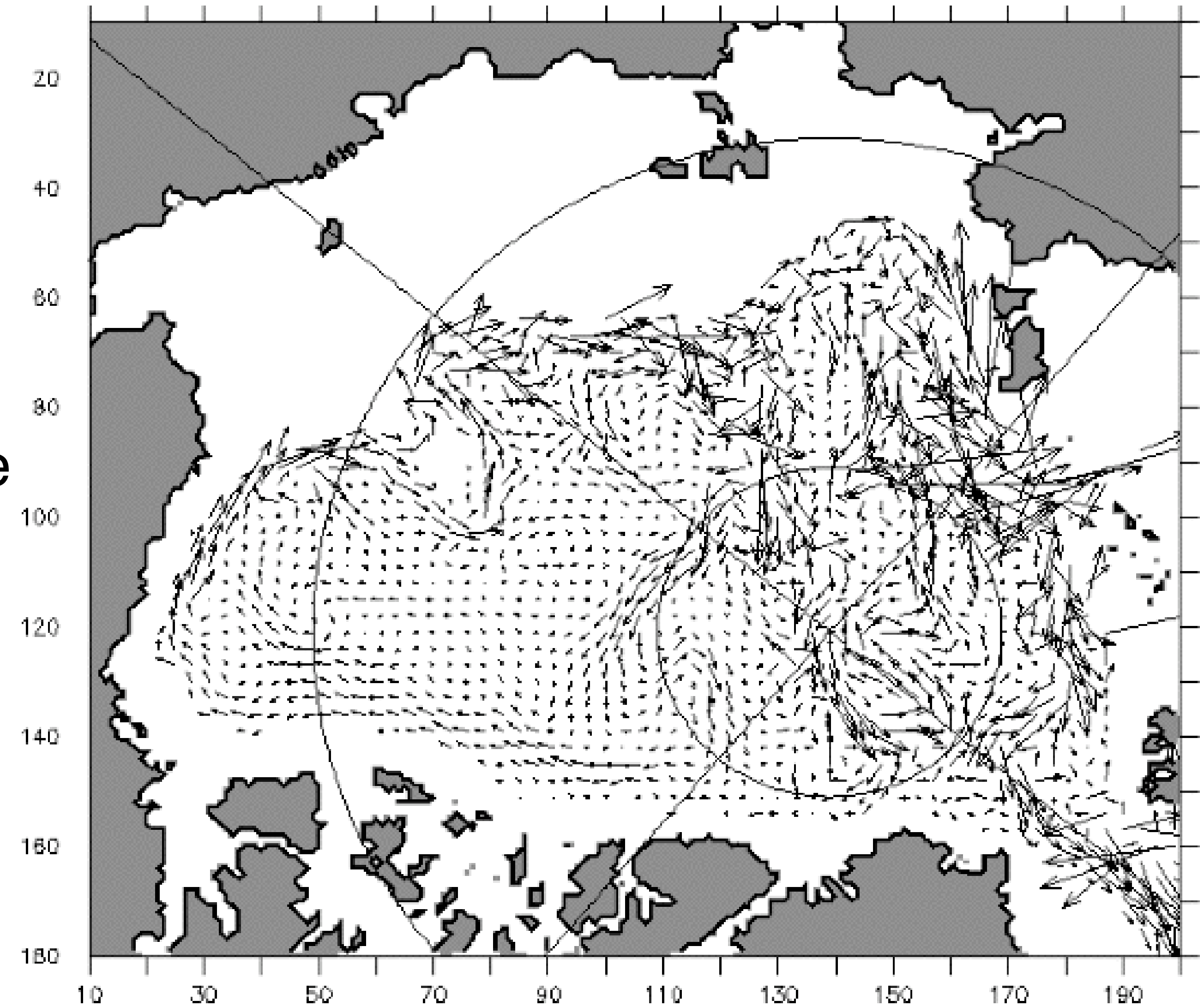
Arctic Ocean Models Intercomparison Project: To compare models, T and S are simple. Average, make heat or "freshwater" storage, etc. What to do about **V**?

Define "topostrophy"

$\tau \equiv \mathbf{f} \times \mathbf{V} \cdot \nabla D$, a scalar that averages like T or S. Normalize

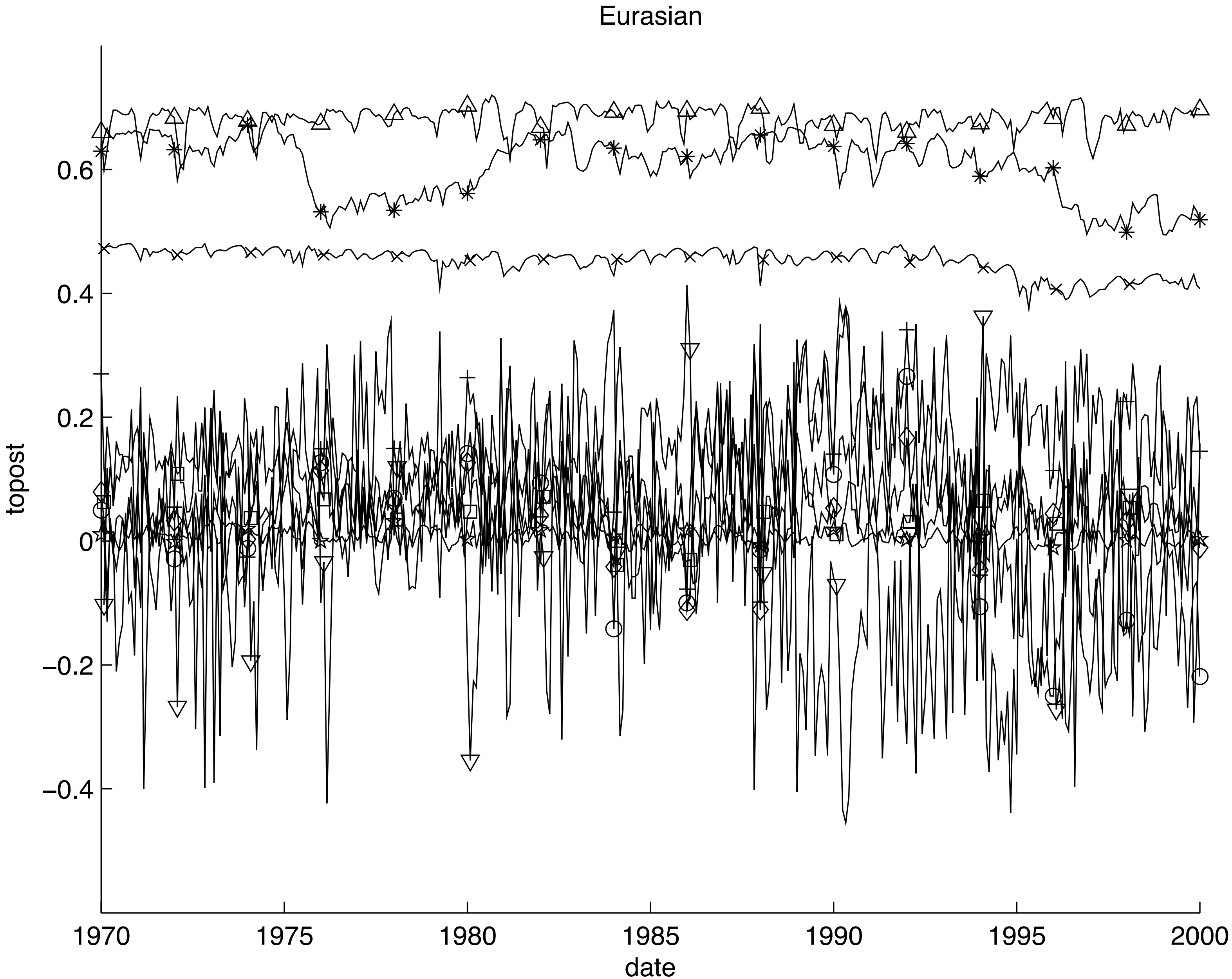
$$\tau \equiv \frac{\langle \mathbf{f} \times \mathbf{V} \cdot \nabla D \rangle}{\sqrt{\langle |\mathbf{f} \times \mathbf{V}|^2 \rangle \langle |\nabla D|^2 \rangle}}$$

then $-1 \le \tau \le +1$



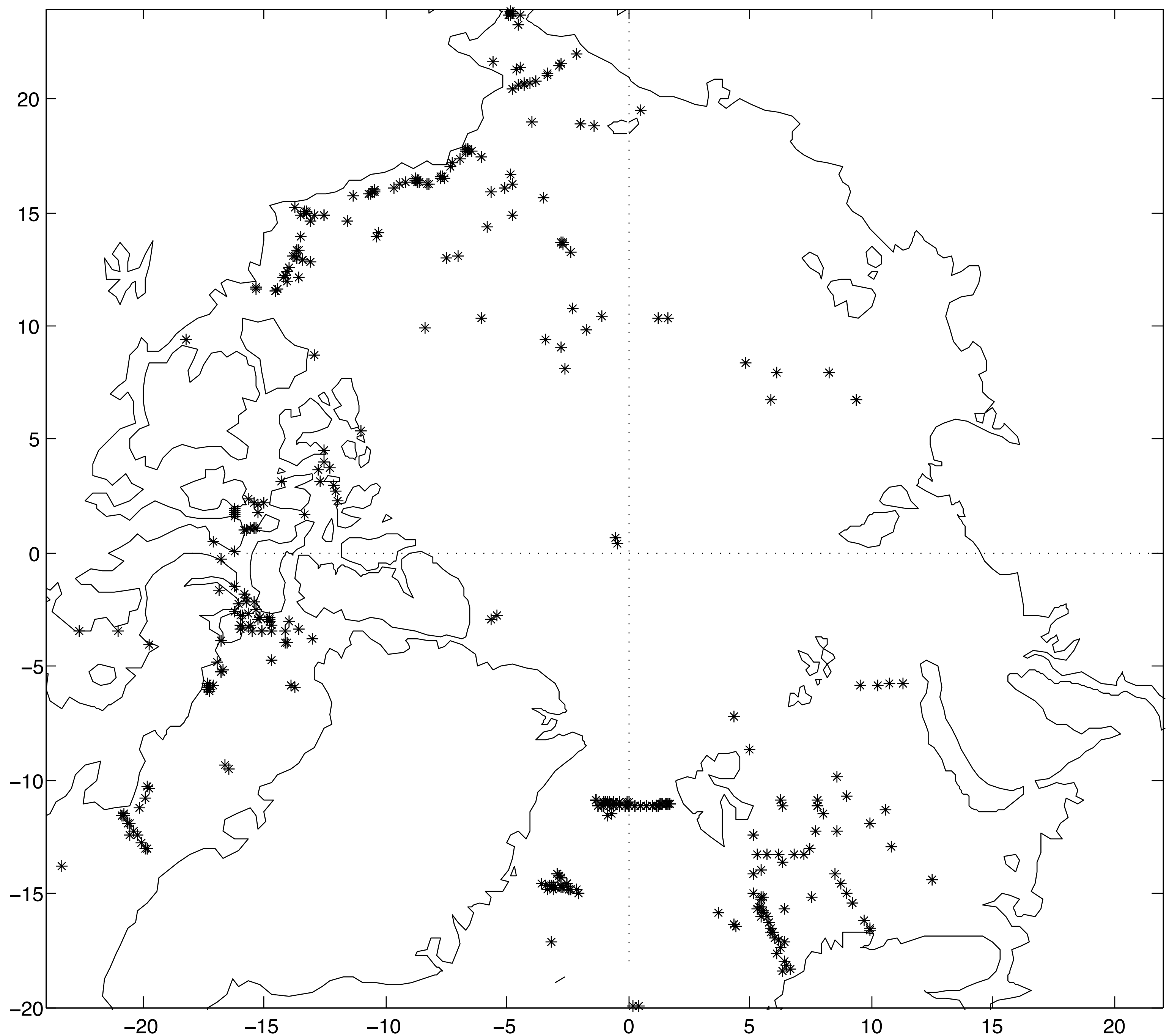
Arctic observers refer to prevalent "cyclonic rim currents", large + τ

Topostrophy averaged over Eurasian basin

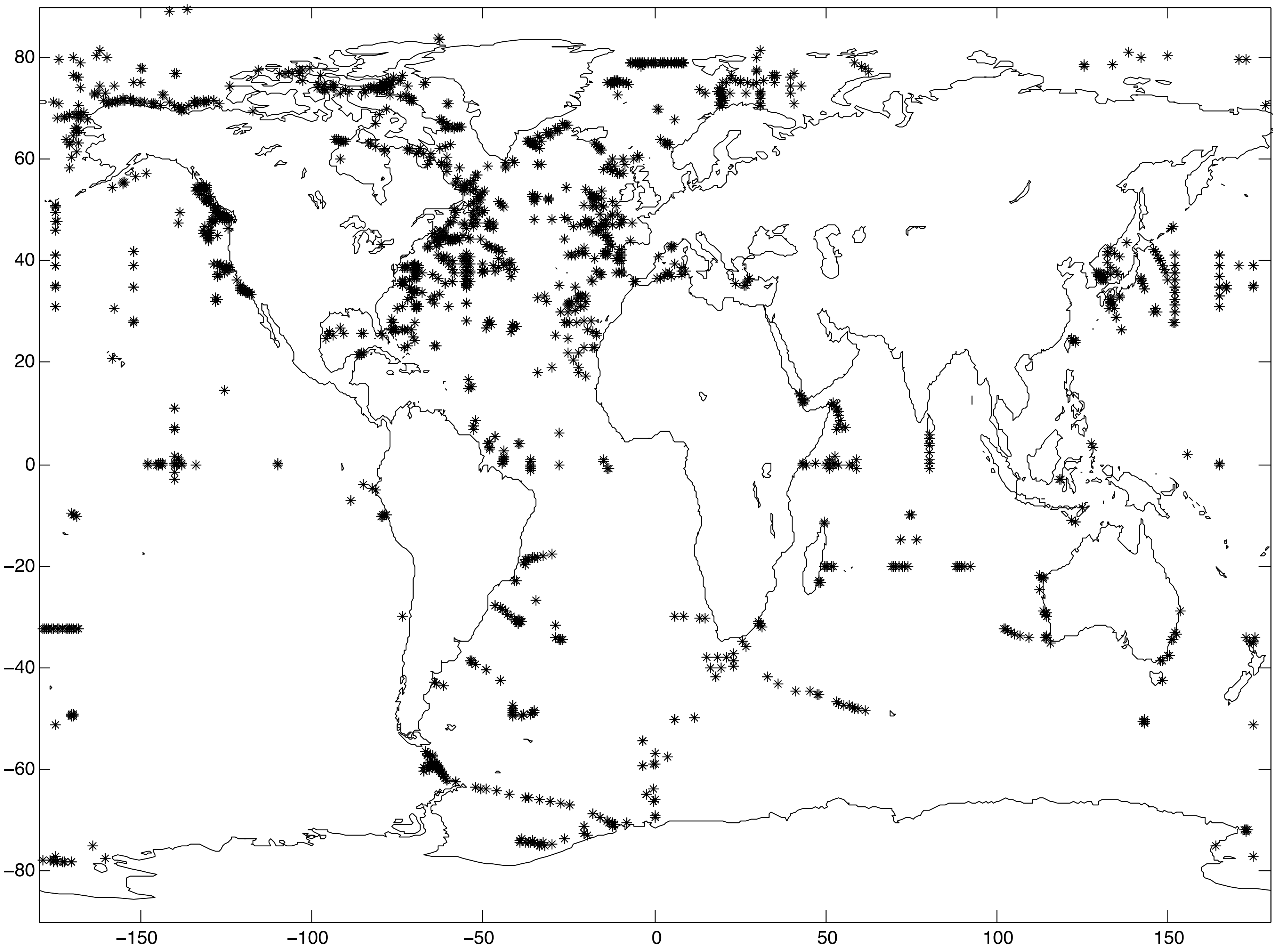


Interesting, but what is observed?

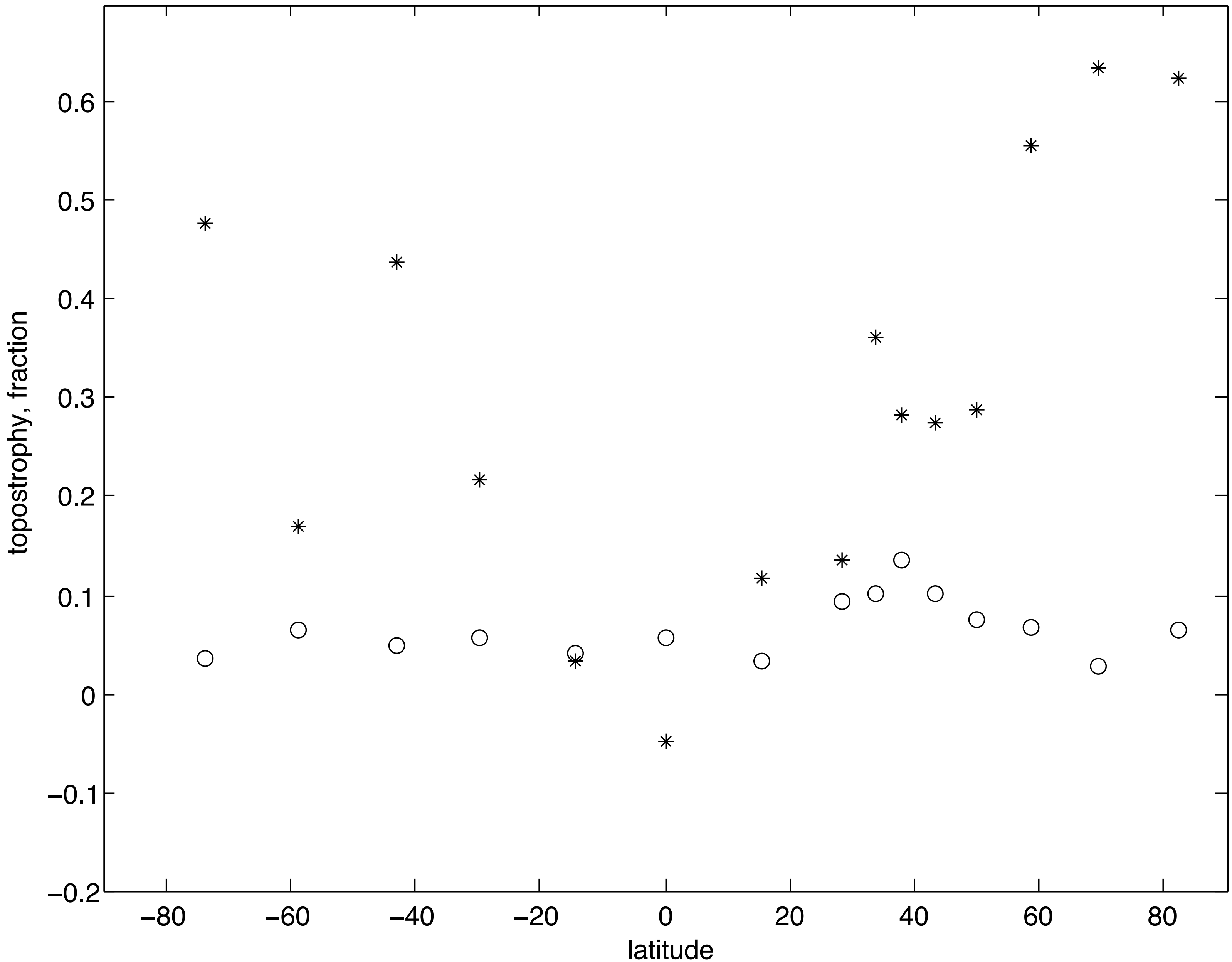
Can we estimate topostrophy from current meter records?



17120 CM records, 83087 months later ...



vs latitude nonuniform bins, here omitting top 500m



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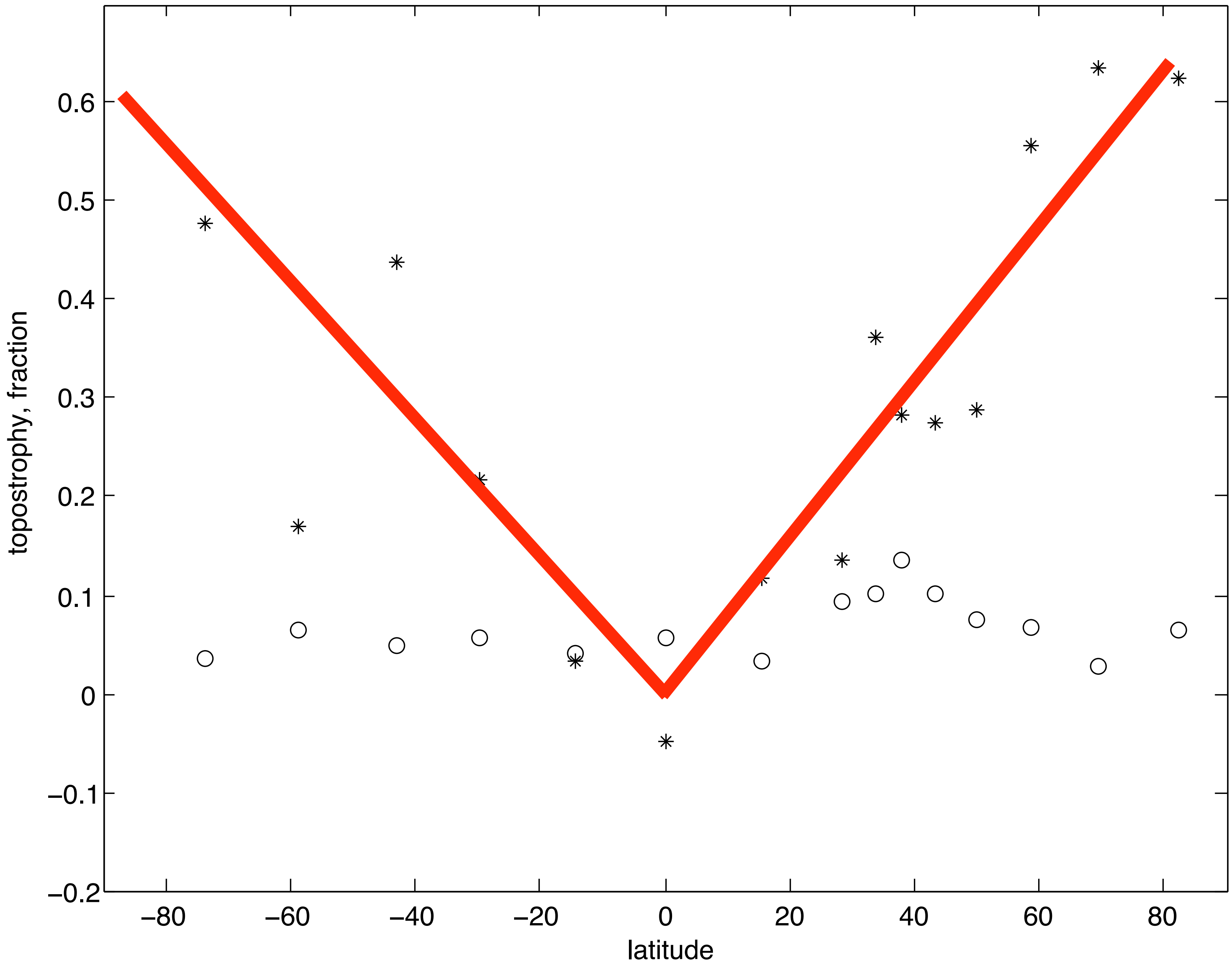
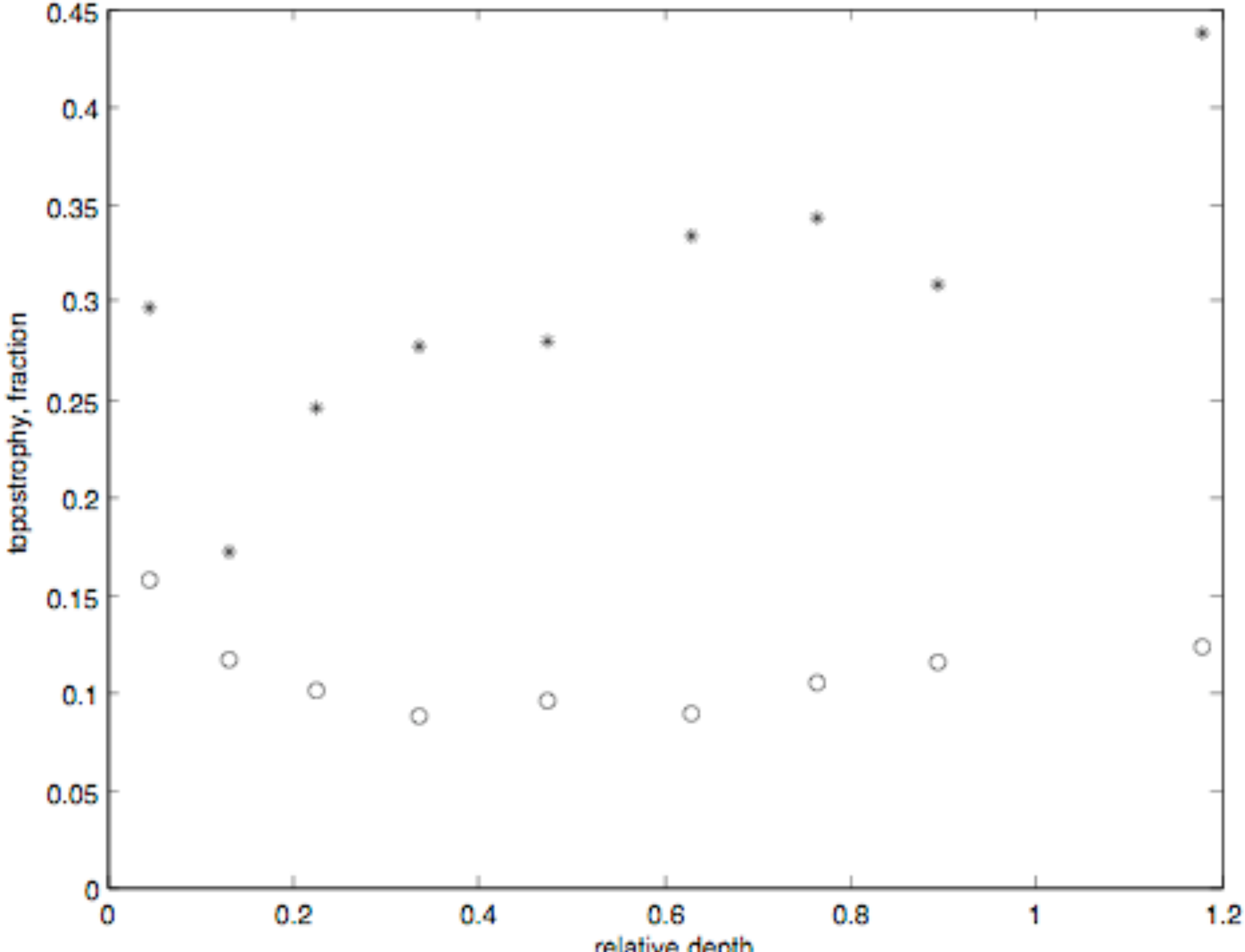


All latitudes, vs. relative depth



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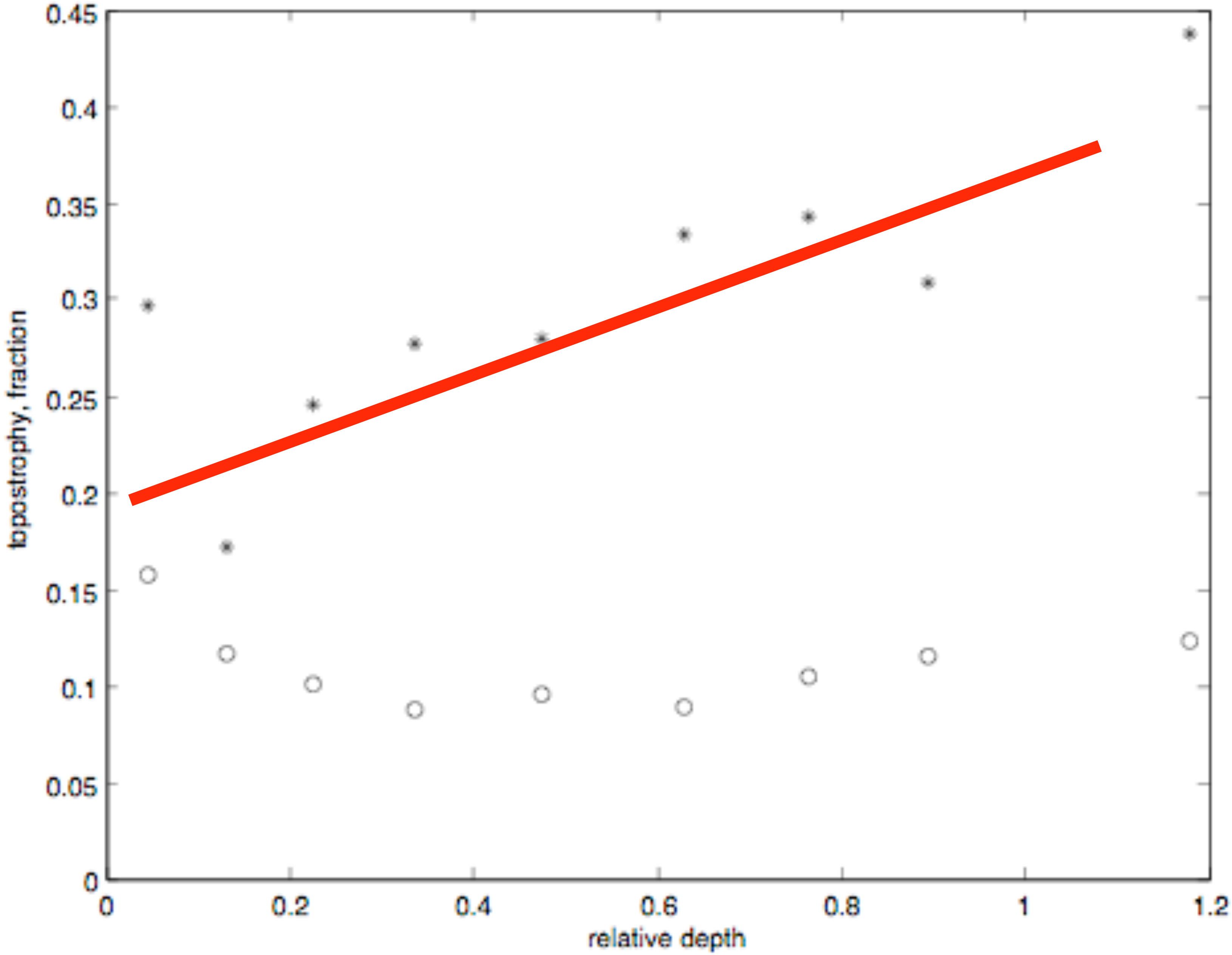
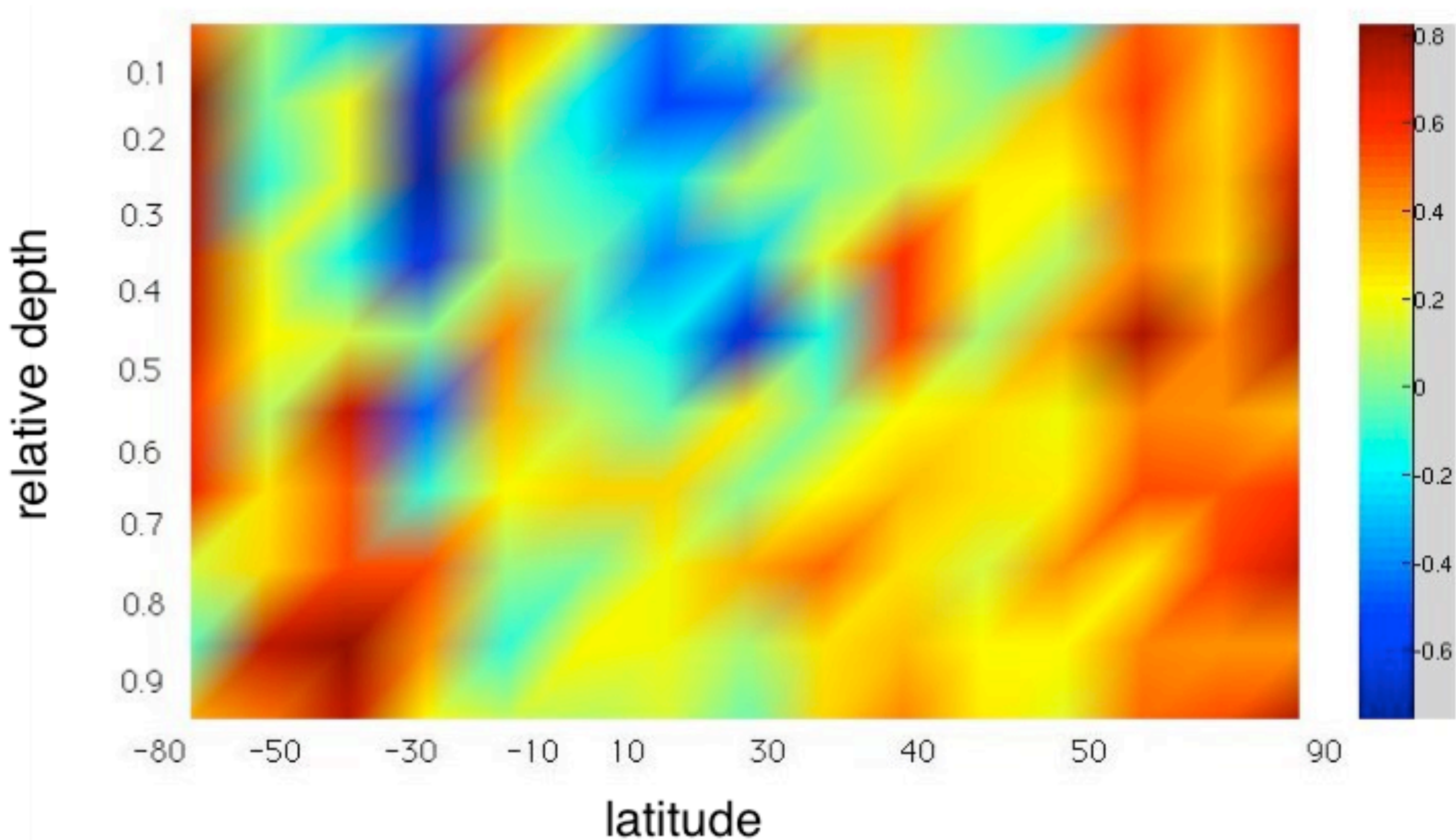


Topostrophy vs. latitude and relative depth



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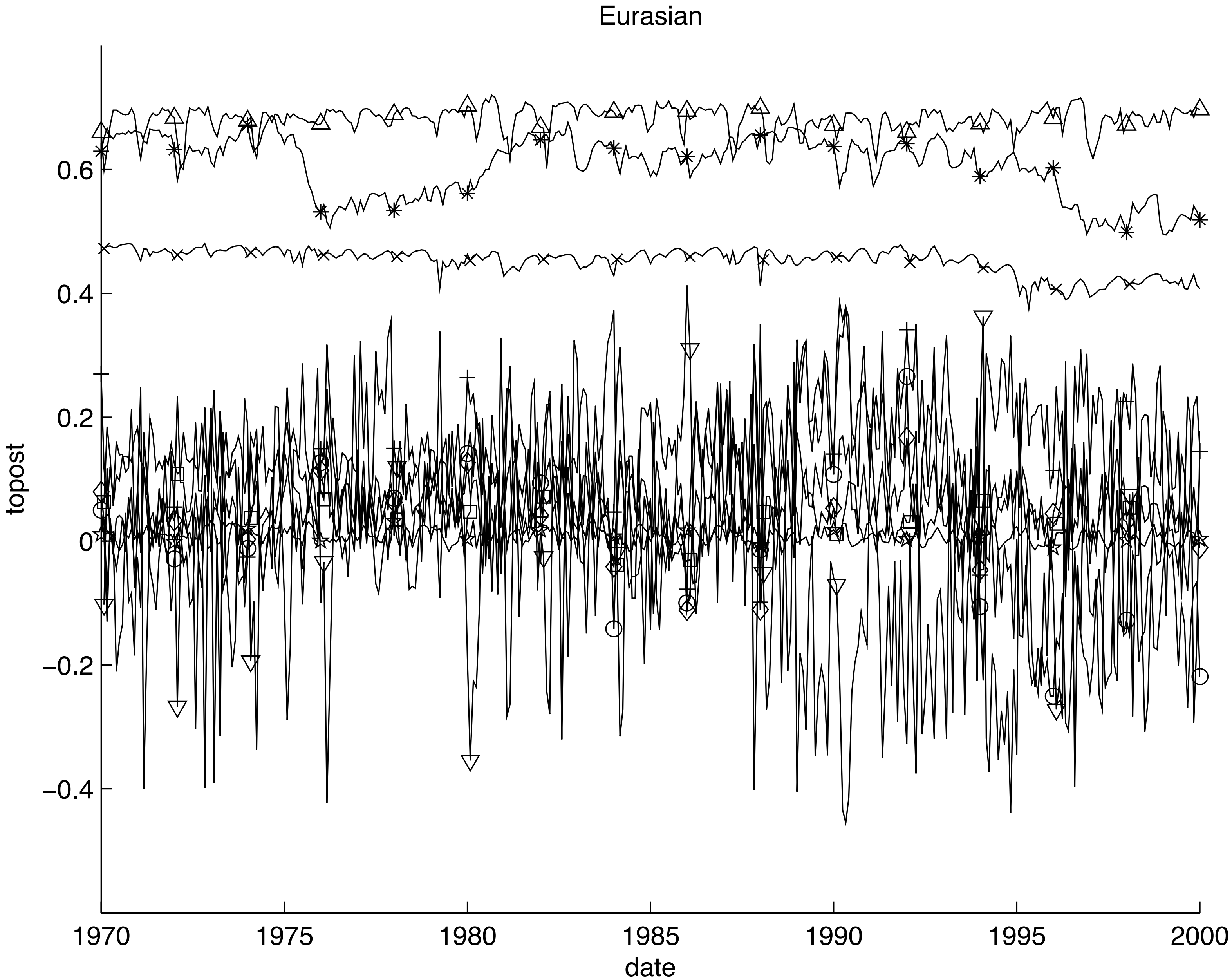

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In plain words --

1) entropy (-∫log(p)dp) is "starved" at short scales

2) simplest enstrophy $(\zeta + h)^2 = \zeta^2 + 2\zeta h + h^2$

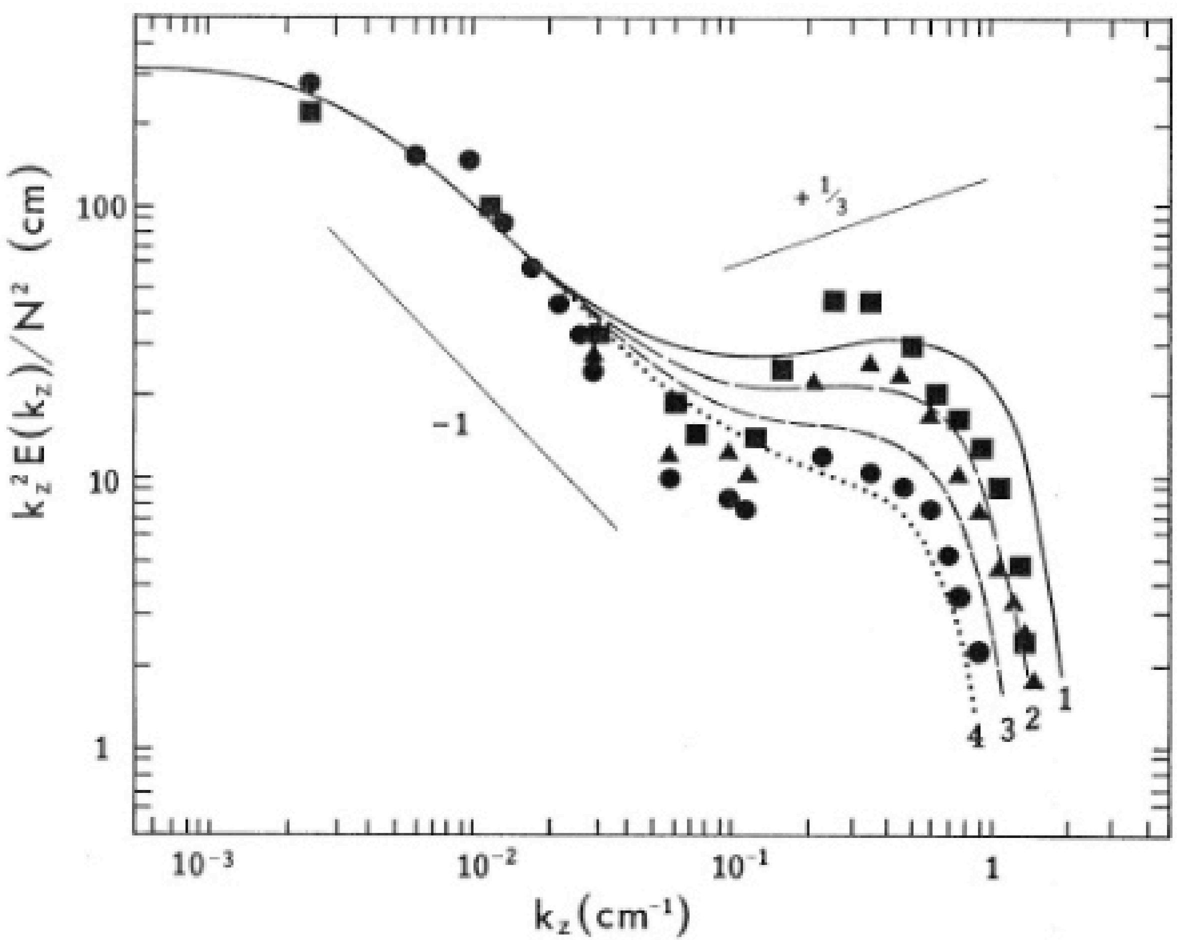
3) organizing a little $\zeta h < 0$ (losing entropy)

4) generates ζ^2 (=short scales, gaining entropy)

5) hence "entropic forcing" drives $\zeta \Rightarrow -h$

or $\mathbf{V} \Rightarrow -\mathbf{f} \times \nabla D$ or $\tau > 0$

change subject, change scale, change physics:



1. internal waves => “buoyancy range” => “turbulence” => dissip

2. where does downward buoyancy mixing occur?
puzzle: persistent countergradient fluxes (“PCG”s) -- why?

one integral: total (KE + PE) energy = waves + vortical energy

Y*: at each α,β wave energy = 2x vortical, KE = 2x PE

with forcing & dissip, much more energy at low α,β

C·$\partial_{\mathbf{Y}}$H meets 2 demands: 1) transfer energy to high α,β

2) seek KE = 2x PE at each α,β

transfer depends on $\theta_{kpq} = \left(\mu_k + \mu_p + \mu_q\right) \Big/ \left(\left(\mu_k + \mu_p + \mu_q\right)^2 + \left(\omega_k + \omega_p + \omega_q\right)^2\right)$

$\mu \ll \omega$ see resonant wave interactions, $\mu \gg \omega$ see turbulence

$\theta \approx \tau^{-1} \Big/ \left(\tau^{-2} + N^2\right)$ where $\tau \approx \varepsilon^{-1/3} k^{-2/3} \Rightarrow D_U \approx \theta \tau^{-2} k^2 \Rightarrow U \approx N^2 k^{-3} + \varepsilon^{2/3} k^{-5/3}$

transfer of veloc variance (KE) is less efficient than tracer var (PE),

KE > 2xPE at lower α,β , KE < 2xPE at higher α,β

vertical buoyancy flux F=w'b' converts: ∂_tKE = - ∂_tPE = F

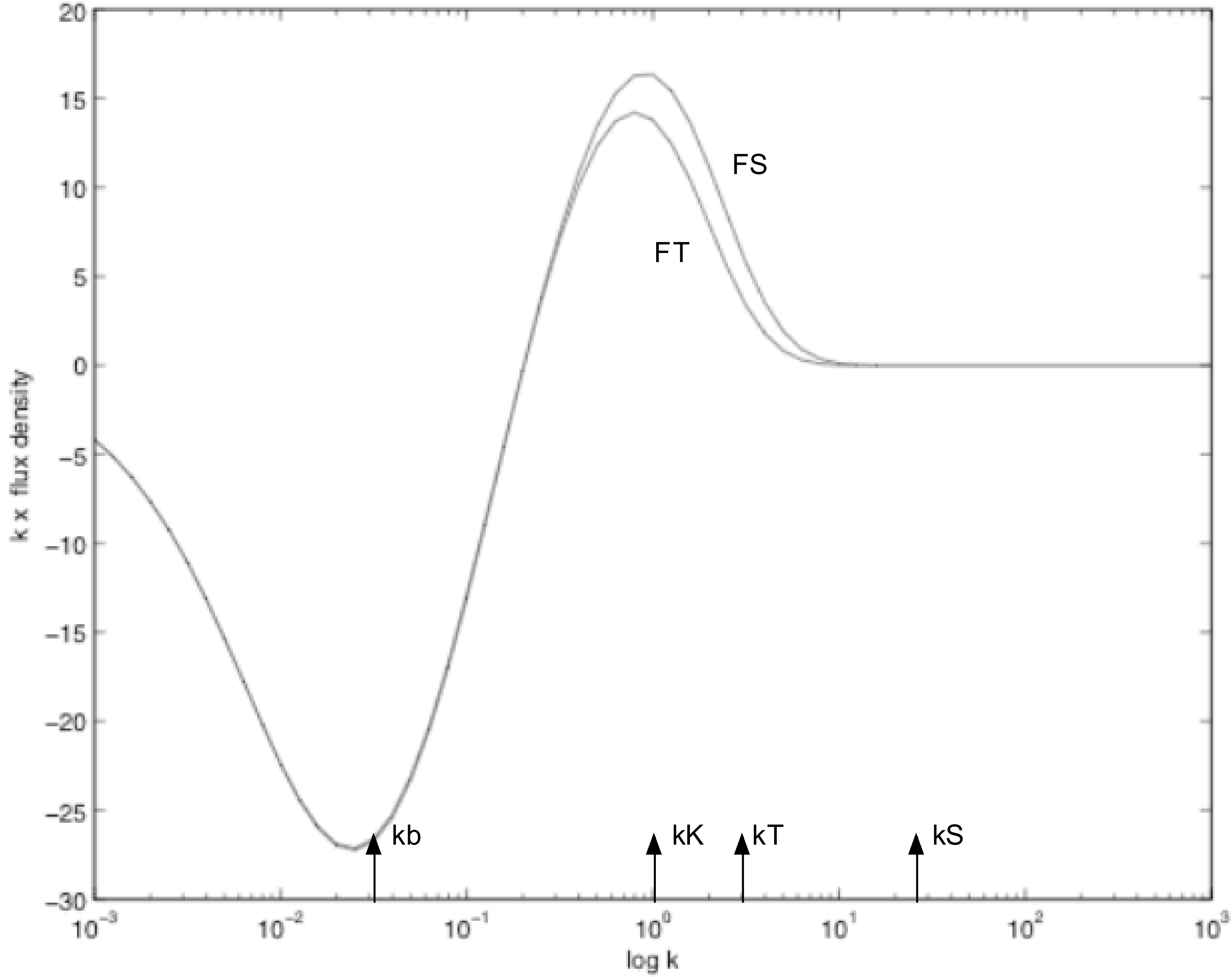


Figure 4. FT and FS corresponding to Fig. 3.

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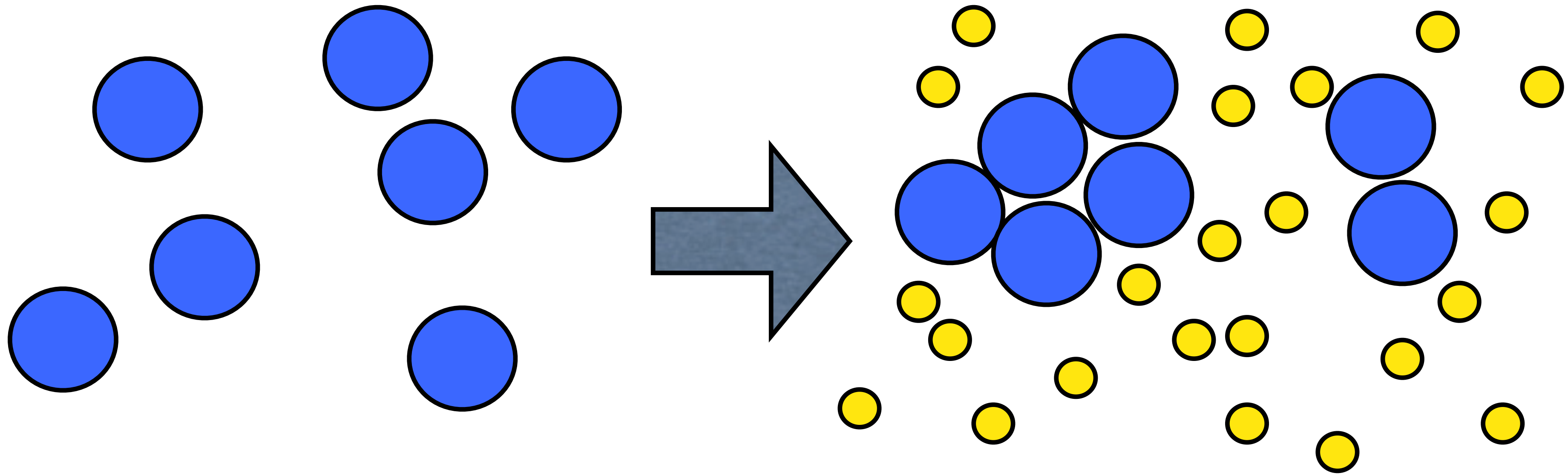
Summary

1. See dependent variables as expectations
2. Entropy gradients force expectations
3. *E.g*: eddy forcing mean flow along slopes
   with secondary upwelling

   E.g: internal waves / vortical => mixing
   with persistent countergrad fluxes

Outlook

1. Work at less fudge
2. Alternatives (max entropy production, ...?)
3. Further applications (sea ice, ...?)



Examples from nanoworld (colloids, ‘machines’, microbiol): The only explicit physics is repulsion among balls, and from walls. “See” attraction. “Entropic forcing” in the lab!