The Monte Carlo Method: Overview and Recent Progress

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N-Body vs. Monte Carlo

- (Direct) N-body
  - Directly integrate equations of motion for N stars interacting via Newtonian gravity. Natural timescale is orbital timescale.
  - Makes no simplifying assumptions about system, but is computationally very expensive (generally requiring GRAPEs or GPUs).
- Monte Carlo
  - Evolve stellar orbits in energy and ang. mom. space. Natural timescale is local relaxation time.
  - Assumes spherical symmetry, dynamical equilibrium, diffusive two-body relaxation. Is computationally very cheap.

Relaxation: Henon's Trick

- Scattering angle for a single deflection: $\beta^2 = \dfrac{4G^2(m+m')^2}{b^2 w^4} \equiv \left(\dfrac{2b_0}{b}\right)^2$
- Summed over all stars: $\langle\beta^2\rangle = \displaystyle\int \frac{4b_0^2}{b^2} nw\Delta t\, f(\vec{w}) d^3\vec{w}\, d^3\vec{b}\, f(m')dm'$
- Assume local approximation: $\langle\beta^2\rangle = 4b_0^2 nw\Delta t \displaystyle\int \frac{d^3\vec{b}}{b^2}$
- Assume spherical symmetry: $\langle\beta^2\rangle = 8\pi b_0^2 nw\Delta t \displaystyle\int \frac{db}{b} = 8\pi b_0^2 nw\Delta t \ln\Lambda$

$$\langle\beta^2\rangle = \left(\frac{2b_0}{b_{\rm SE}}\right)^2 \text{ where } b_{\rm SE} = (2\pi w\Delta t n \ln\Lambda)^{-1/2}$$

Anatomy of a Monte Carlo Timestep

- Start with particle positions and velocities.
- Calculate potential under assumption of spherical symmetry.
- Choose timestep to be a fraction of relaxation time (and other relevant physical timescales).
- [Include additional physics]
- Perform "super encounter" (relaxation)
- Calculate new E, J for each star, assuming potential calculated above.
- Choose r for each new orbit by time-weighted averaging.

Code Capabilities

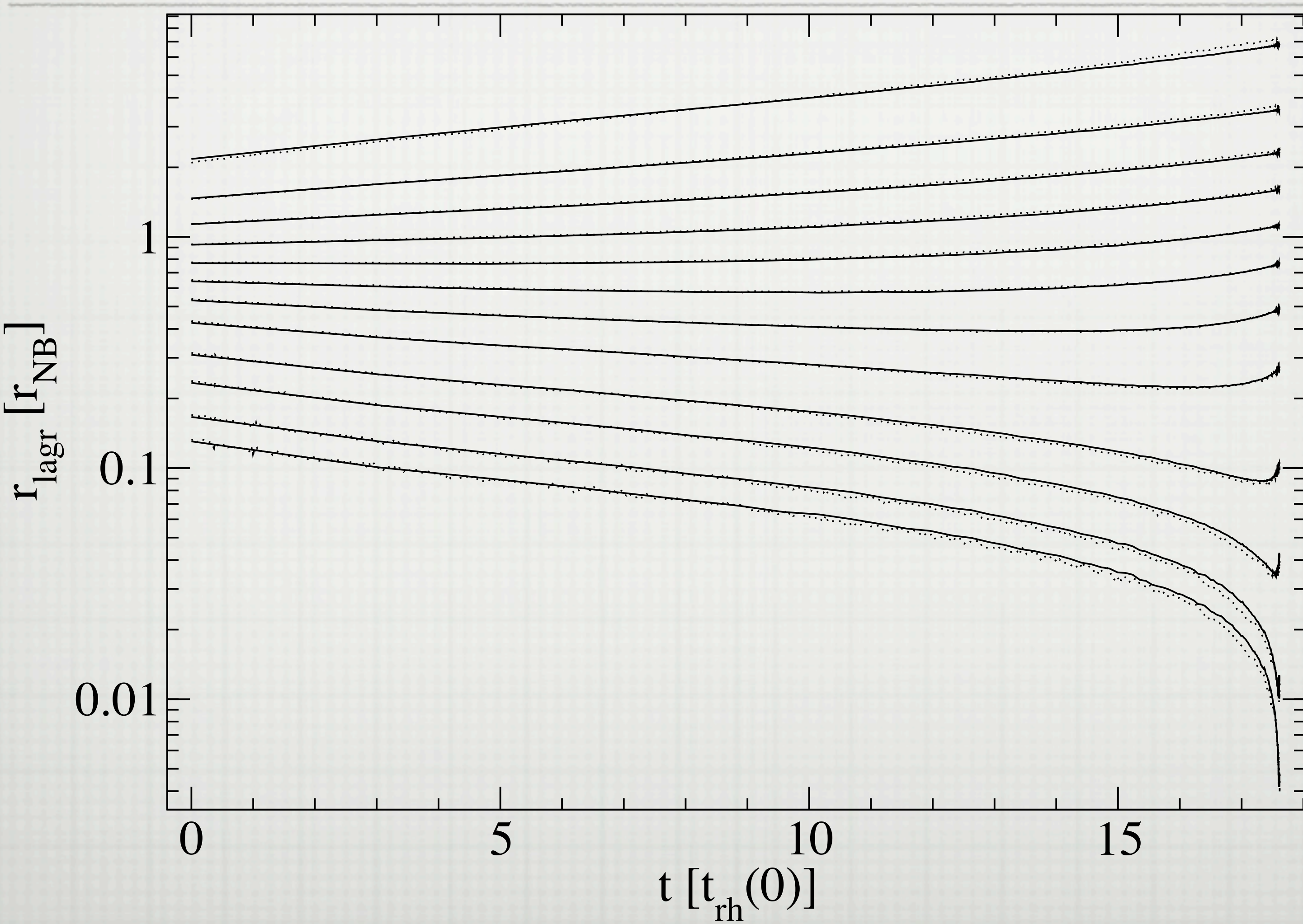
Physics	**NB**	**MC**	**MN**	**F**	**G**	**GS**
two-body relaxation	•	•	•	•	•	•
stellar evolution	•	•	•	◦	◦	
stellar collisions	•	•	•	•		
binary interactions	•	•	•		◦	•
external effects	•	◦	◦	◦	◦	◦
central BH	•	•	•	•		
rotation	•					
violent relaxation	•					
large-angle scattering	•	•				
three-body binaries	•	•			•	•
large N, f_b		•	•	•	•	•

NB=N-body, MC=Monte-Carlo, MN=MIT/Northwestern,
F=Freitag, G=Giersz, GS=Giersz & Spurzem

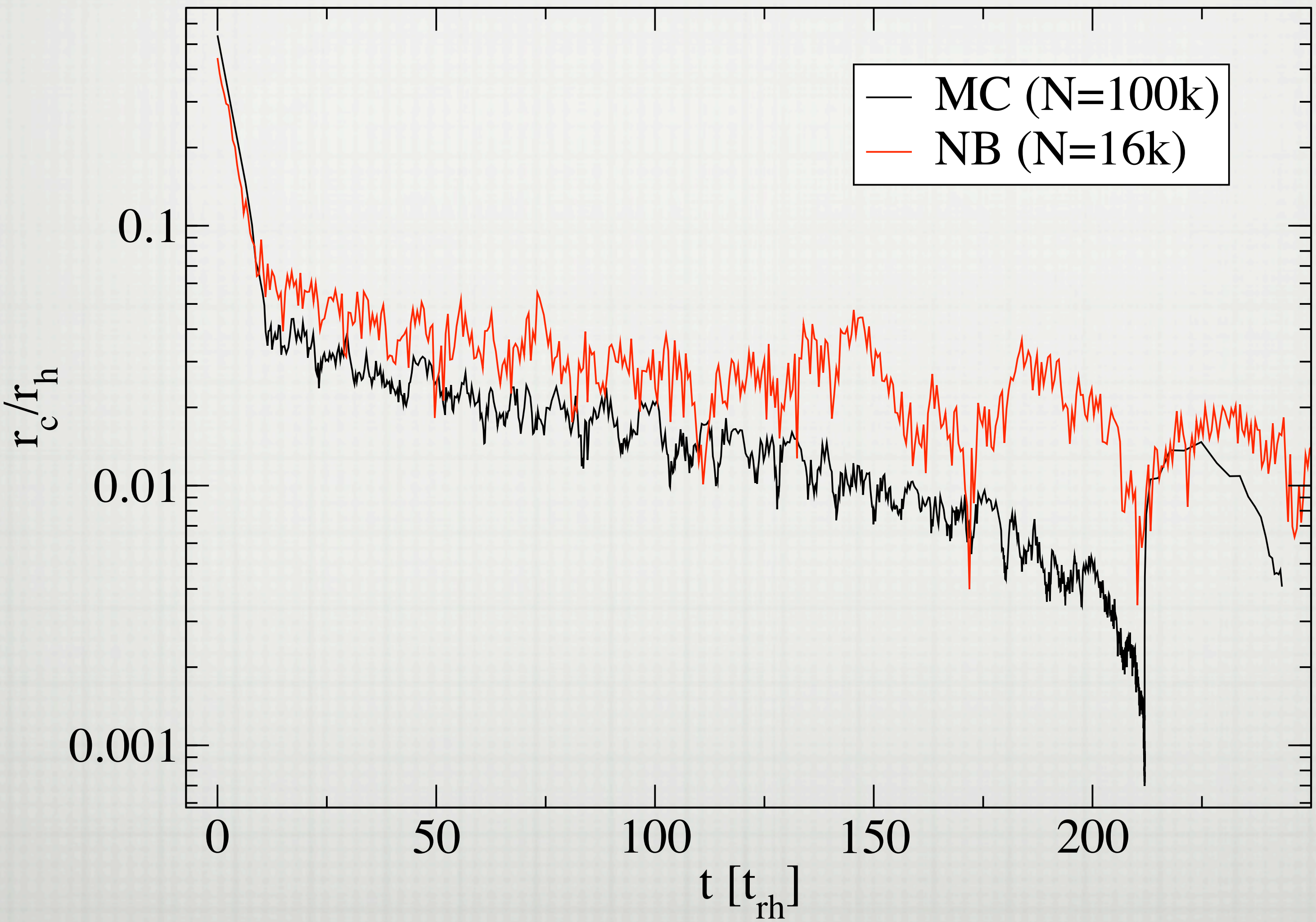
Treatment of Additional Physics

- Binary scattering interactions: integrated directly with Fewbody.
- Stellar collisions: treated in the sticky sphere approximation.
- Single and binary stellar evolution: treated via Hurley's SSE/BSE.
- Central massive black hole: treated via loss cone prescription with ang. mom. diffusion.
- Tidal mass loss: treated via simple apocenter criterion for large N, energy criterion for small N.

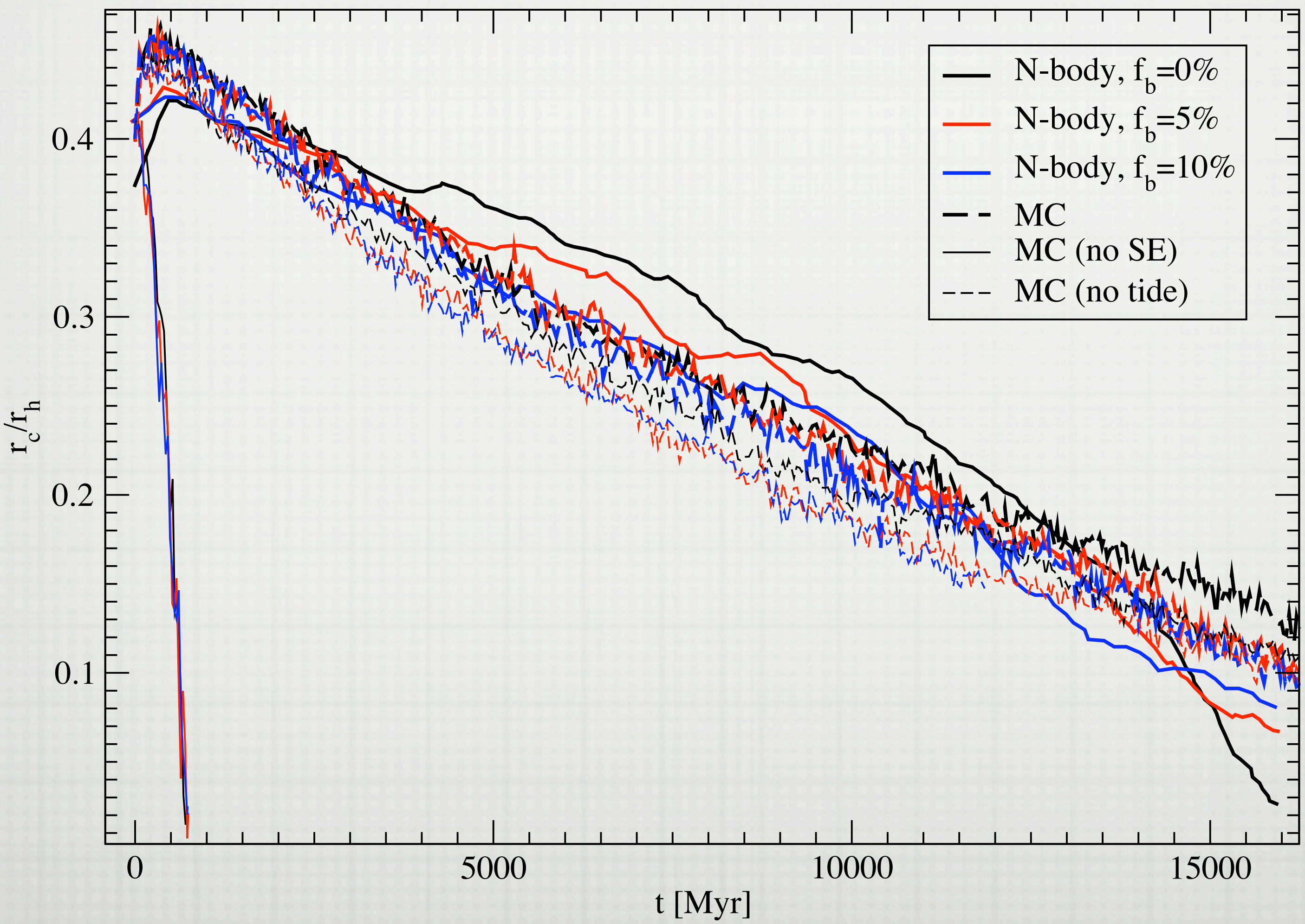
Relaxation Test: Collapse of a Simple Plummer Model



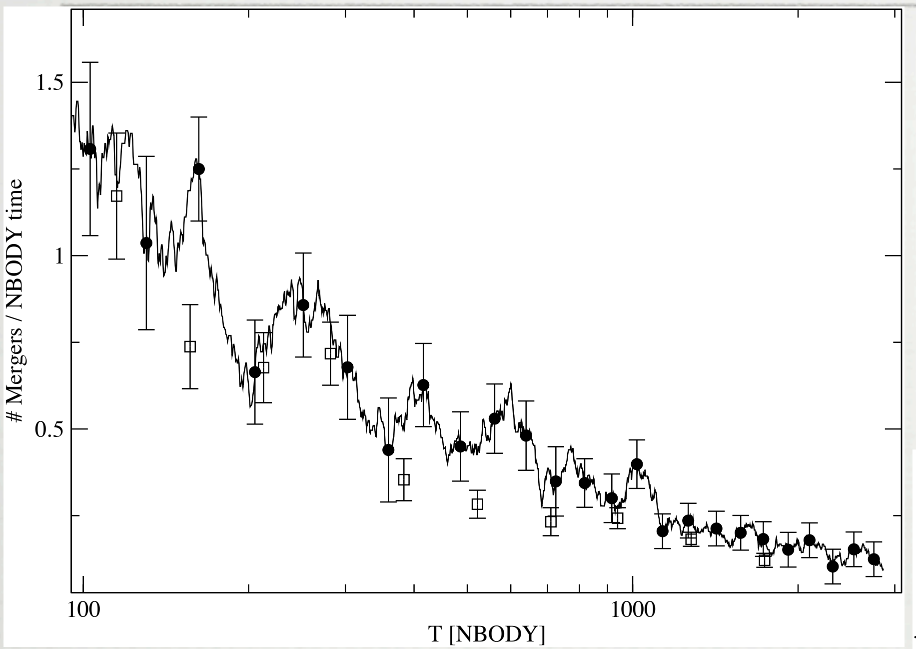
Binary Burning Test

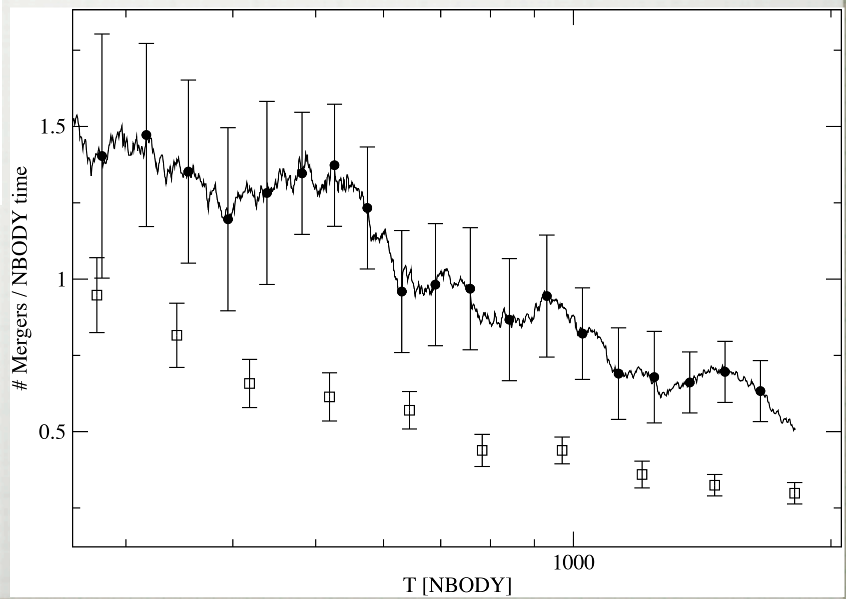


Stellar Evolution Test

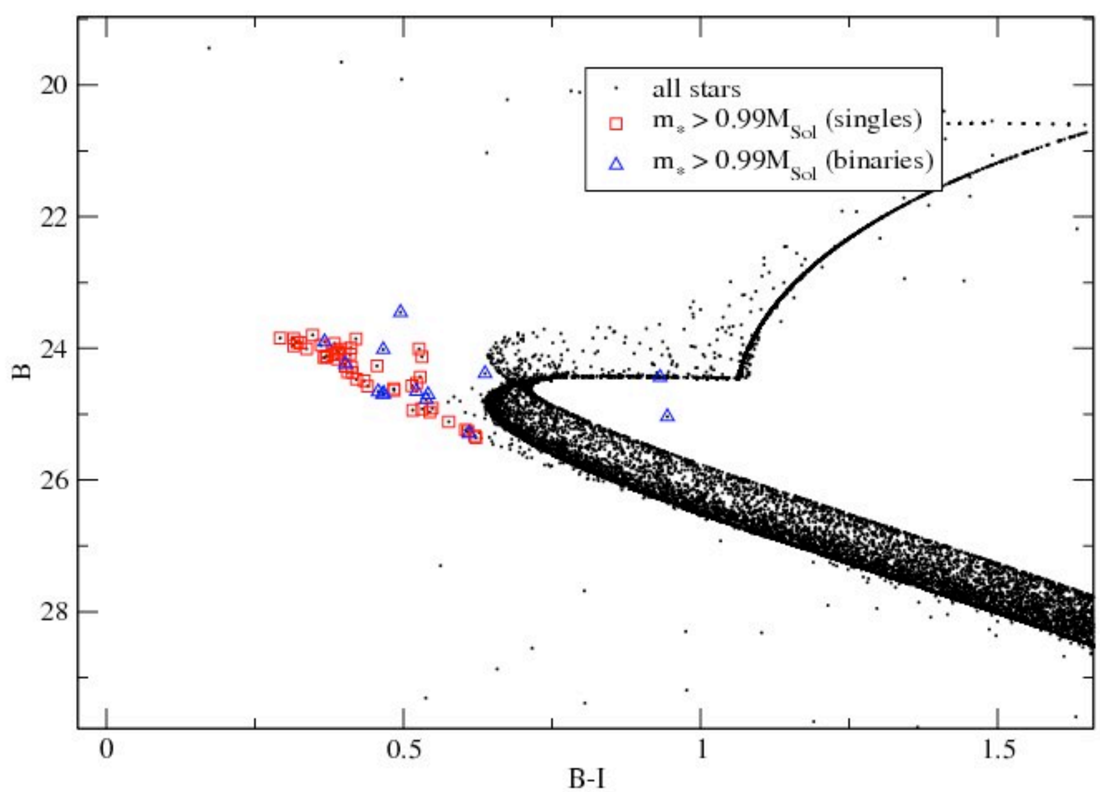


Central Massive BH Test





Blue Straggler Formation (NGC 2419 Model)



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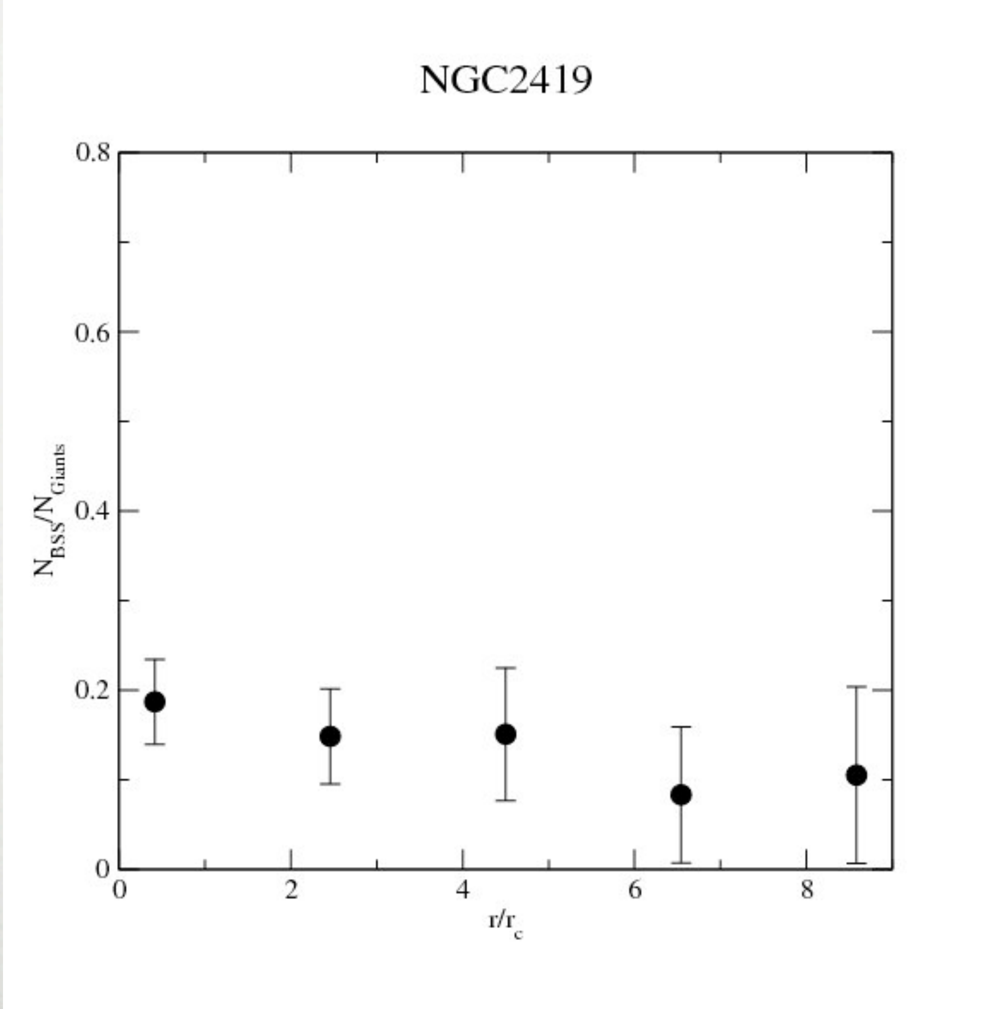

Generic "Core Collapse" Cluster (N=300k, 1% binaries)