Cluster evaporation in a tidal field

and the related tale of the GCMF

Mark Gieles (ESO)

Definitions

- r_t = King tidal/truncation radius (where density goes to zero)
- r_J = Jacobi radius (of the zero velocity surface)
- Roche-lobe under-filling = ????? ($r_h/r_J < 0.1$)

Assumptions

- Escape time = 0 (but see Fukushige & Heggie 2000; Baumgardt 2001)
- $t_{\rm rh} \propto M^{1/2} r_{\rm h}^{3/2}$, i.e. Coulomb logarithm is constant
- I personally do not care what the initial mass function of the globular clusters was ....

$$\dot{M} = \xi_{\rm e} \frac{M}{t_{\rm rh}}$$

ξ_e = escape fraction

Ambartsumian (1938); Spitzer (1940); Henon (1961); Spitzer (1987)

Assume $r_{\mathrm{h}} \propto r_{\mathrm{J}}$

$$\dot{M} = \xi_{\mathrm{e}} \frac{M}{t_{\mathrm{rh}}}$$

$$\propto \rho_{\mathrm{J}}^{1/2}$$

Jacobi density is set only by the galaxy

$$\rho_{\mathrm{J}}^{1/2} \propto \sqrt{\frac{M_{\mathrm{G}}(R_{\mathrm{G}})}{R_{\mathrm{G}}^{3}}} \propto \frac{V_{\mathrm{G}}}{R_{\mathrm{G}}} = \omega$$

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THE EVOLUTION AND FINAL DISINTEGRATION OF SPHERICAL STELLAR SYSTEMS IN A STEADY GALACTIC TIDAL FIELD

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ABSTRACT

The lifetime for total mass loss is found to be, aside from a slowly varying logarithmic term, proportional to the initial number of stars times the tidal time scale [$\sim 1/(G\rho_t)^{1/2}$], where ρ_t is the tidal density (i.e., mean density within the tidal radius).

$$t_{\mathrm{dis}} \equiv \frac{M}{\dot{M}}$$

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$$\propto M \rho_{\mathrm{J}}^{-1/2}$$

$$\propto M R_{\mathrm{G}}$$

$$\propto M \omega^{-1}$$

cluster galaxy

King (1966)
Lee & Ostriker (1987)
Alguilar, Hut & Ostriker (1988)
Chernoff & Weinberg (1990)
Vesperini & Heggie (1997)
Baumgardt (1998)
Takahashi & Portegies Zwart (2000)
Fall & Zhang (2001)
Baumgardt & Makino (2003)

Eccentric orbits: $t_{\mathrm{dis}}(\varepsilon)=t_{\mathrm{dis}}(0)\times(1+\varepsilon)$

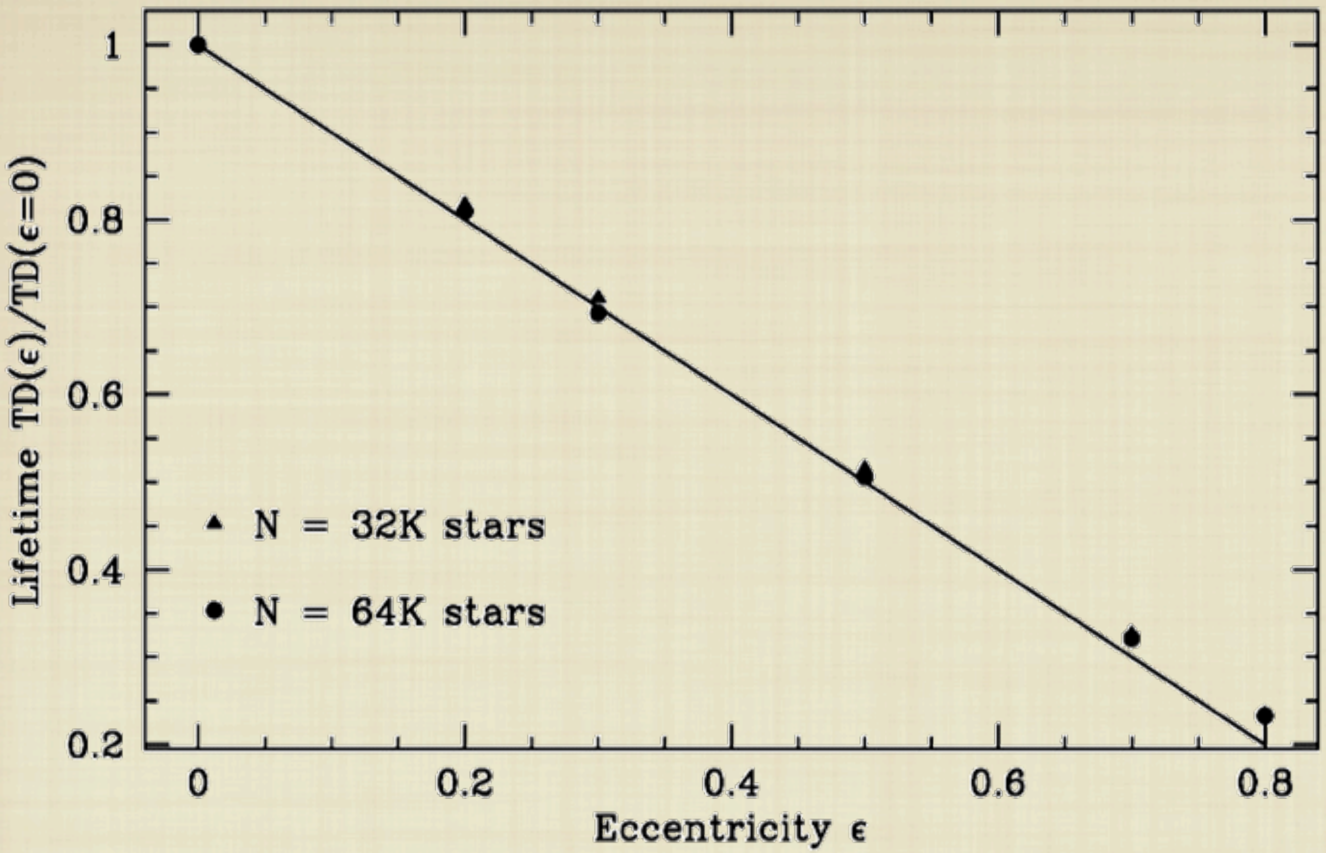


Figure 4. Lifetimes of clusters moving on orbits with different eccentricities ϵ but the same apogalactic distances. Lifetimes are divided by the lifetime of a cluster moving on a circular orbit with radius equal to the apogalactic radius of the clusters on the eccentric orbits. The solid line shows the relation $(1-\epsilon)$, which provides a satisfactory fit for all eccentricities.

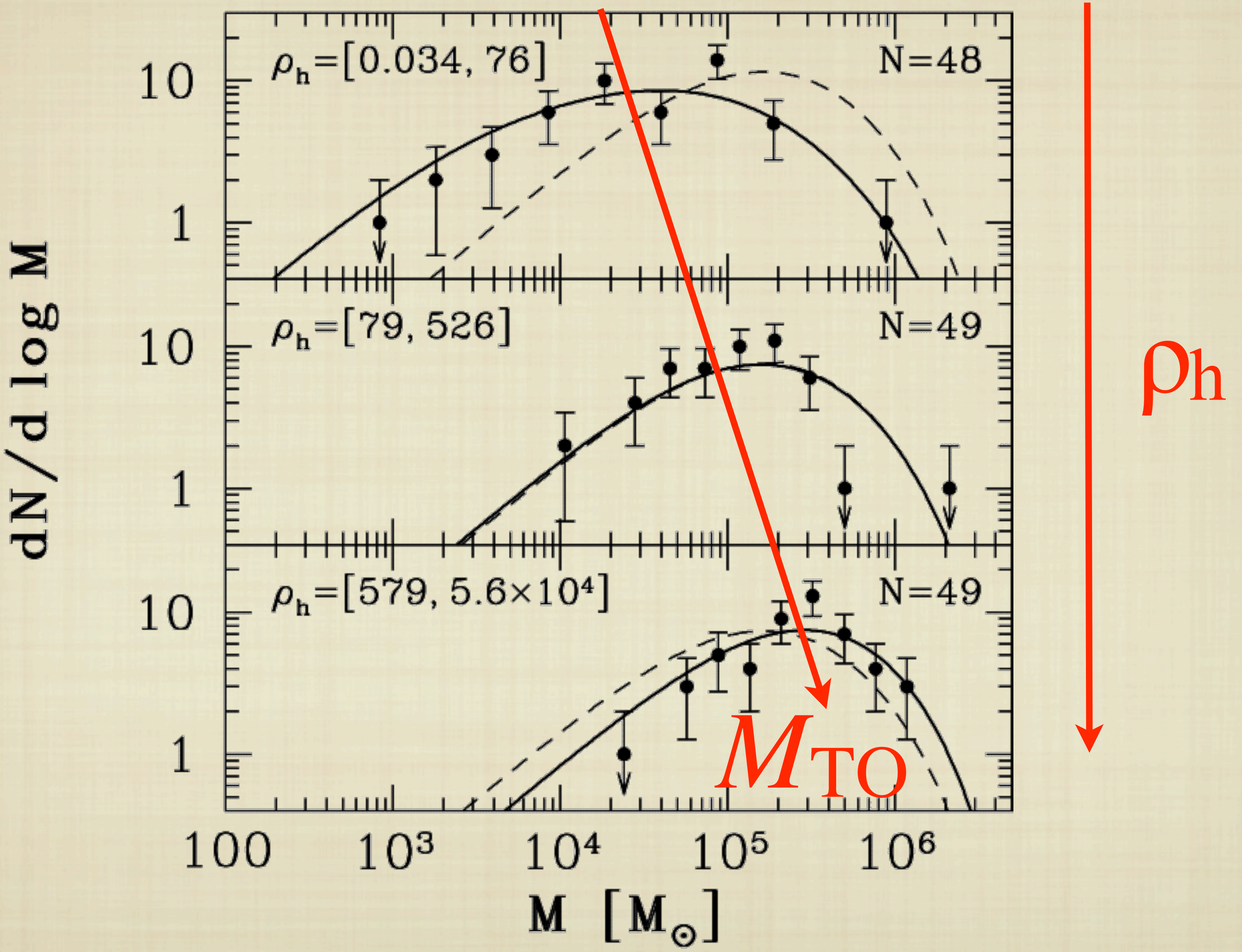
Baumgardt & Makino (2003)

McLaughlin & Fall (2008)

$$\dot{M} = \xi_{\rm e} \frac{M}{t_{\rm rh}}$$

$$\propto \rho_{\rm h}^{1/2}$$

Density dependency of GCMF: $M_{\rm TO} \propto \rho_{\rm h}^{1/2}$



McLaughlin & Fall (2008)

$$\xi_e = \text{constant?}$$

$$\xi_e = \text{constant?}$$

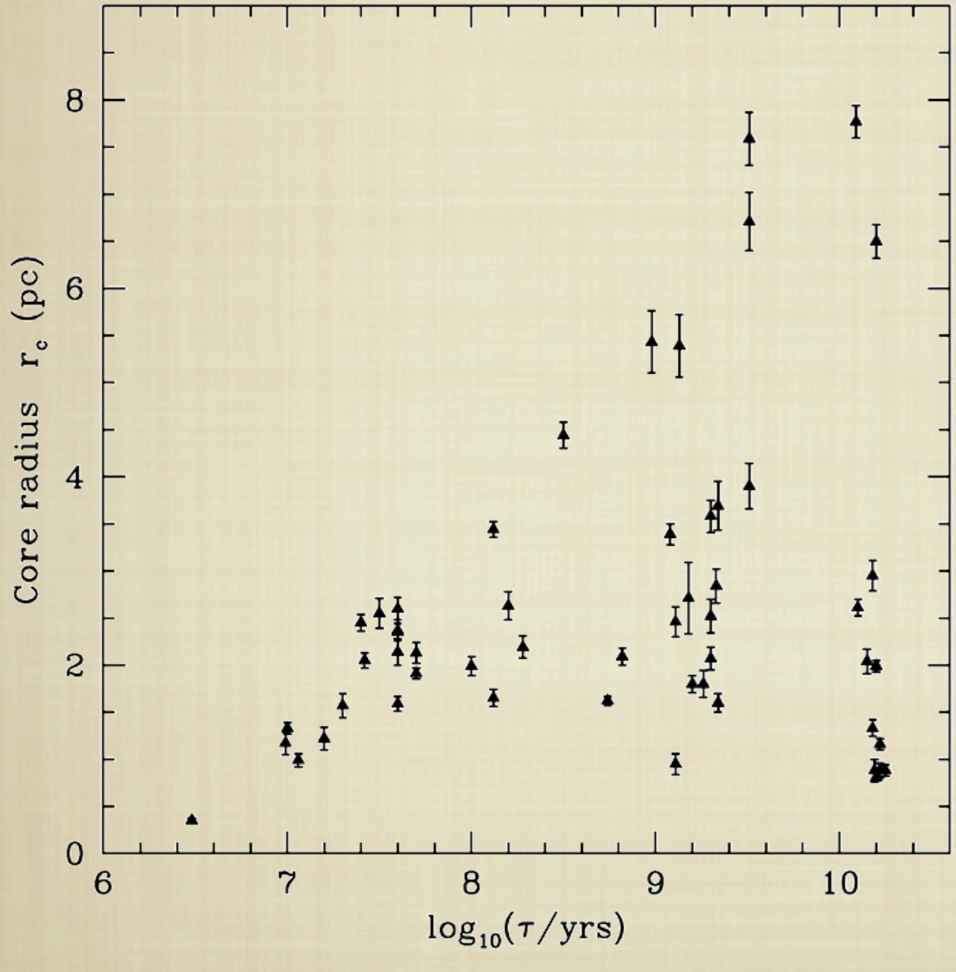
or, do clusters with smaller radii live shorter?

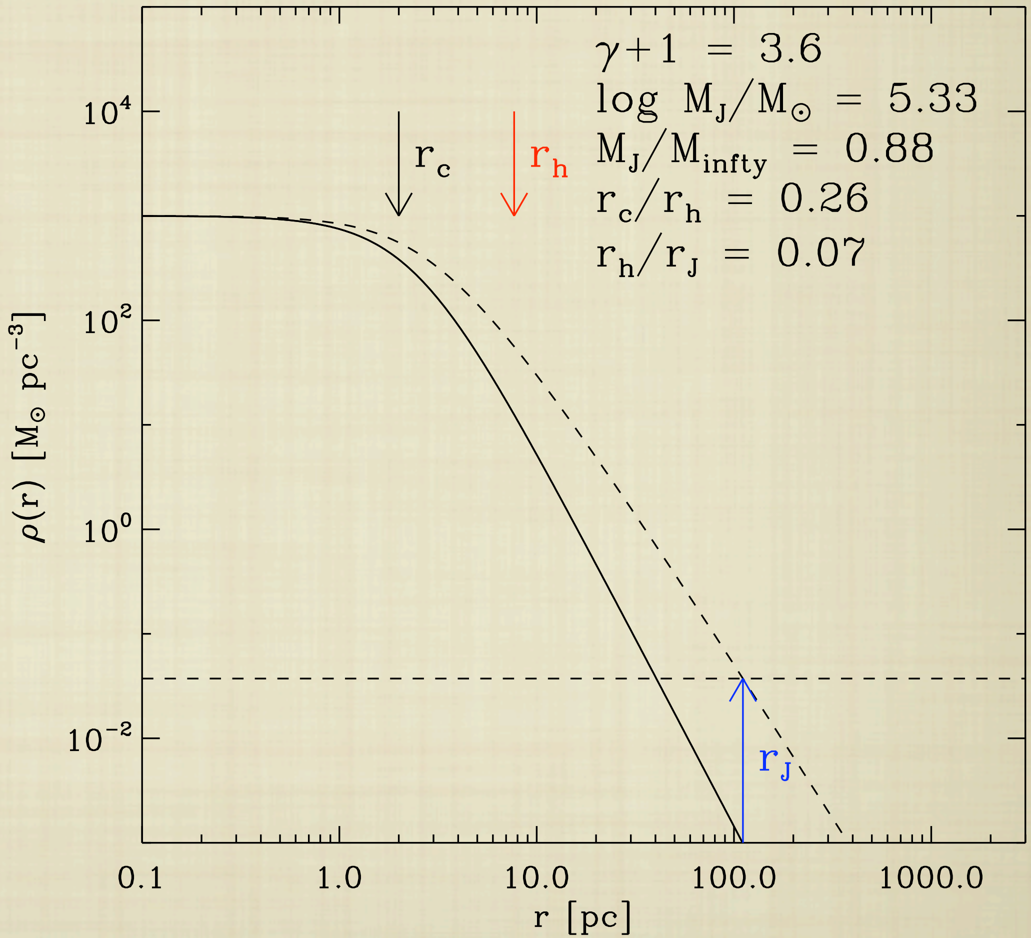
Do we need to consider small $r_{\rm h}/r_{\rm J}$ values?

Parameters for a "typical" globular cluster:

$$
\begin{aligned}
M &= 2\times10^5\,M_\odot & t_{\rm rh} &\simeq 3\,{\rm Gyr}\\
r_{\rm h} &= 5\,{\rm pc} & r_{\rm J} &\simeq 100\,{\rm pc}\\
R_{\rm G} &= 10\,{\rm kpc} & r_{\rm h}/r_{\rm J} &= 0.05
\end{aligned}
$$

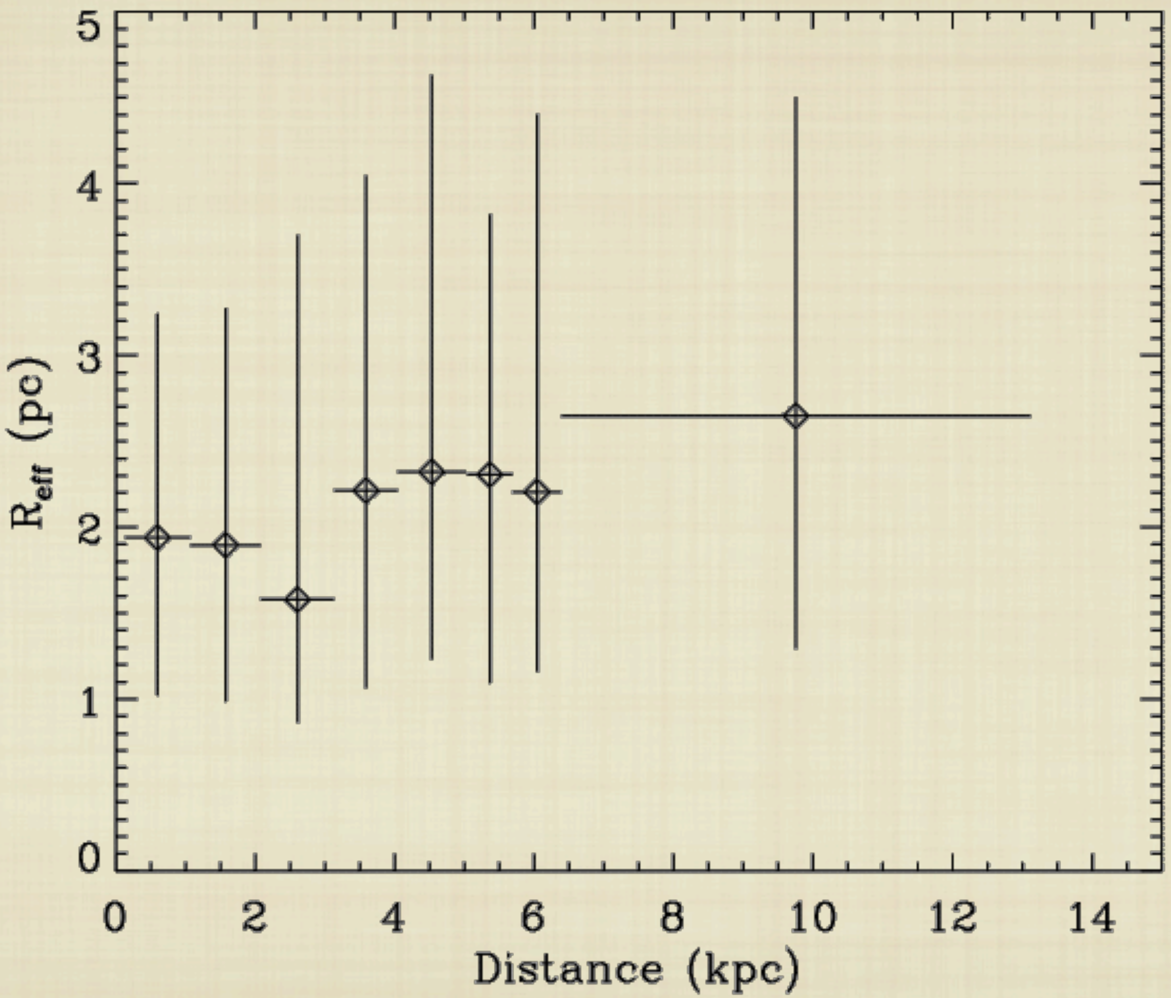
Do we need to consider small r_h/r_J values?





Mackey & Gilmore (2003)

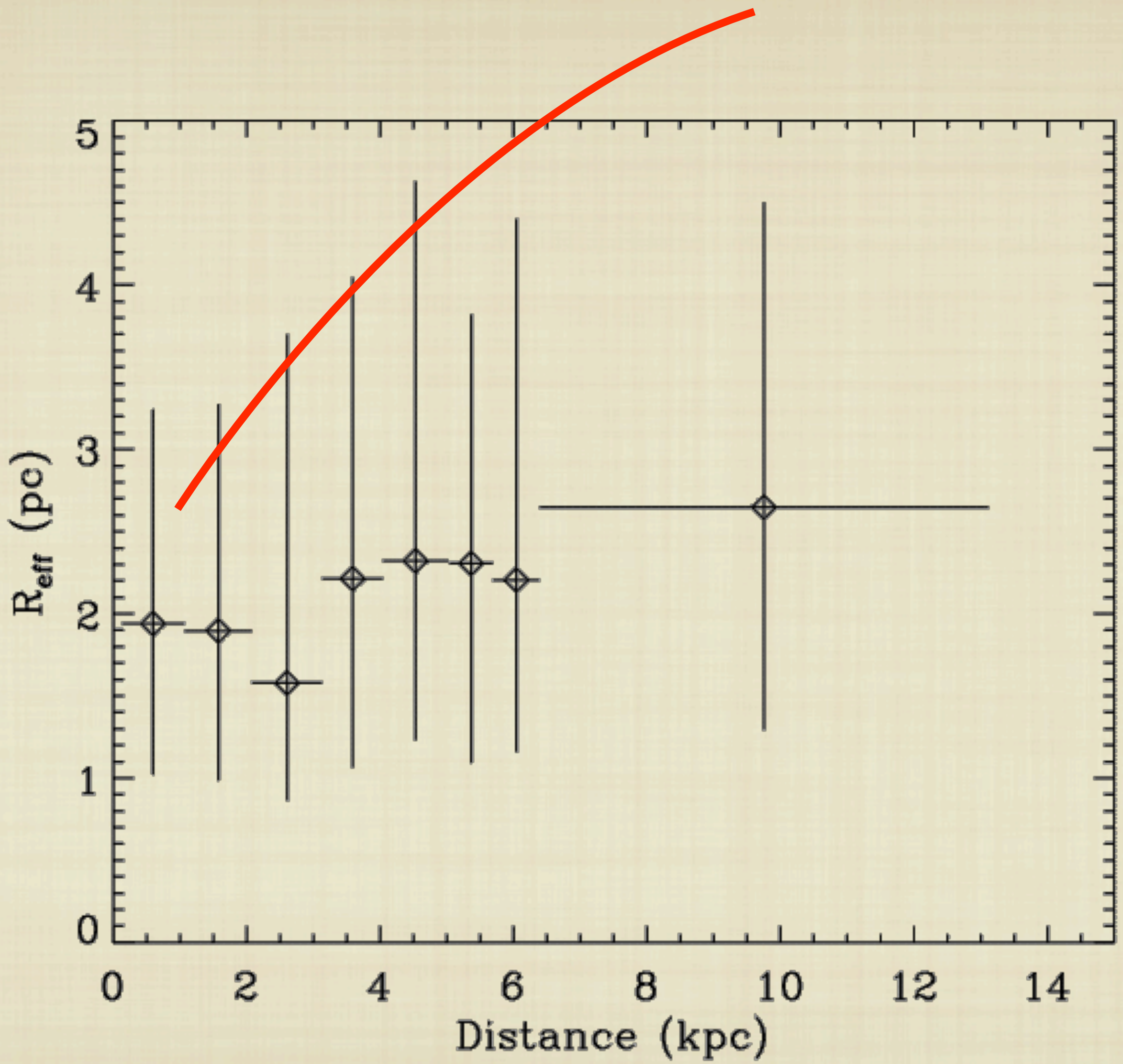
M51



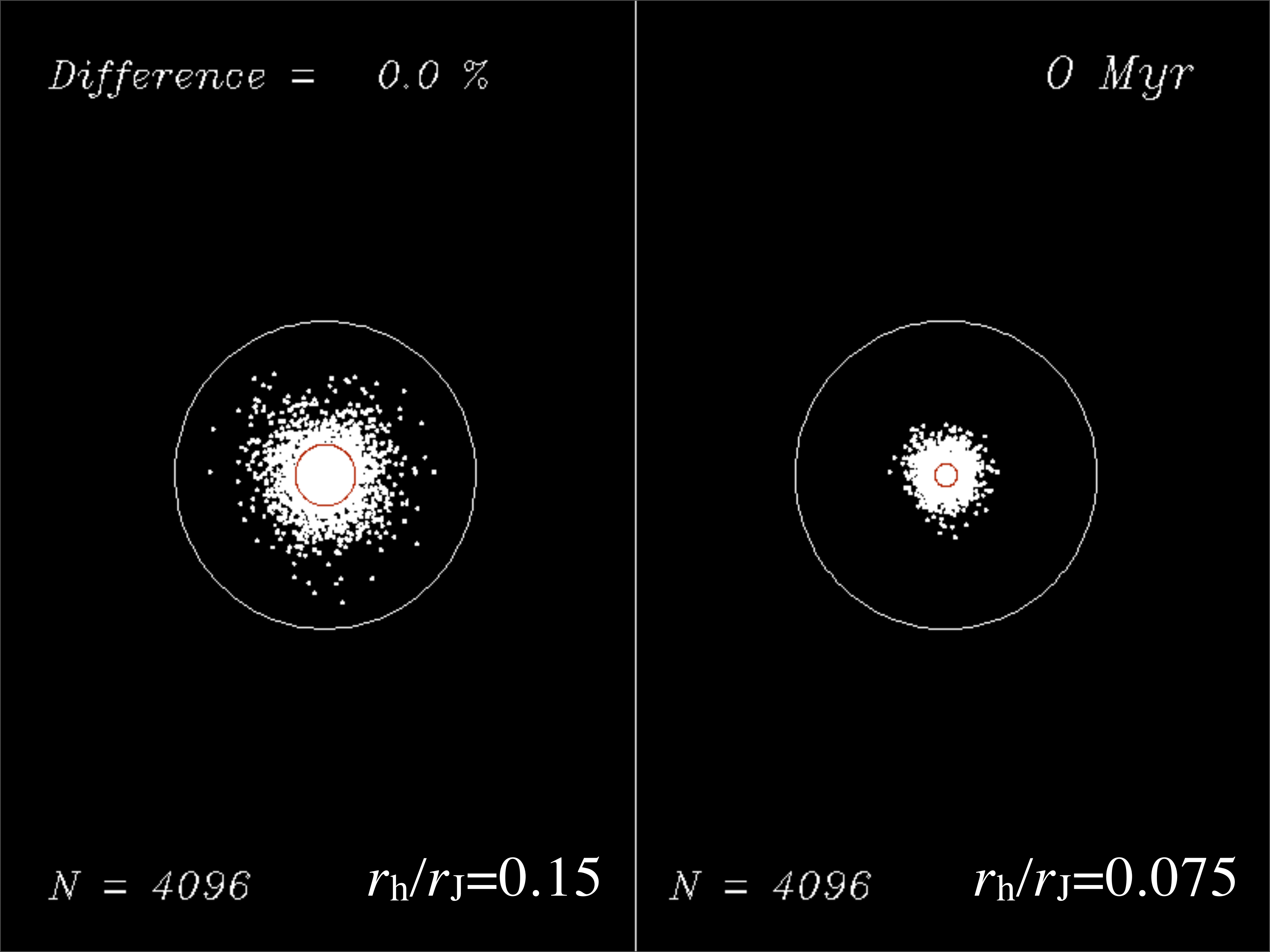
Scheepmaker et al. (2007)

M51

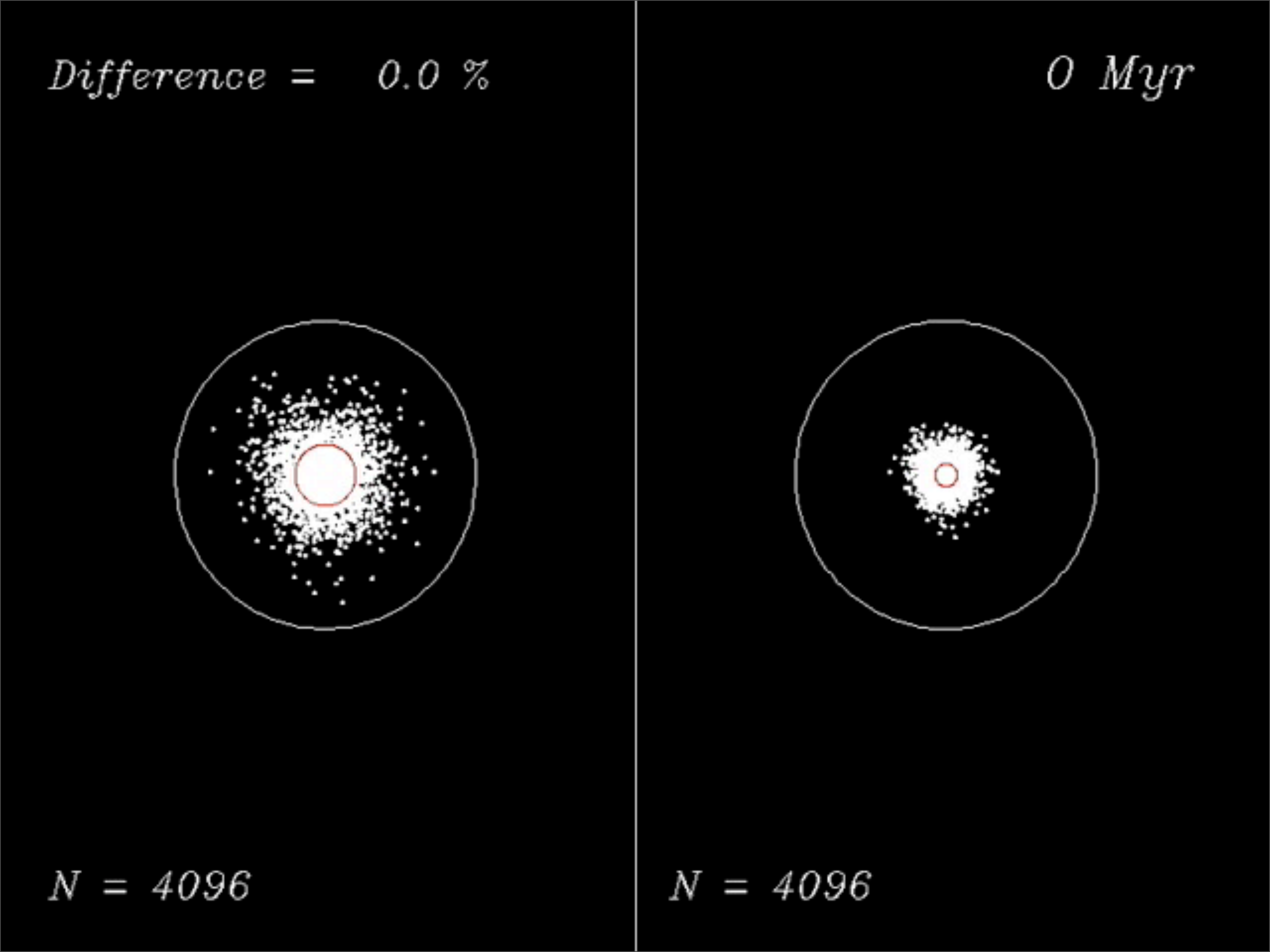
tidal radius: $r_J \propto R_G^{2/3}$

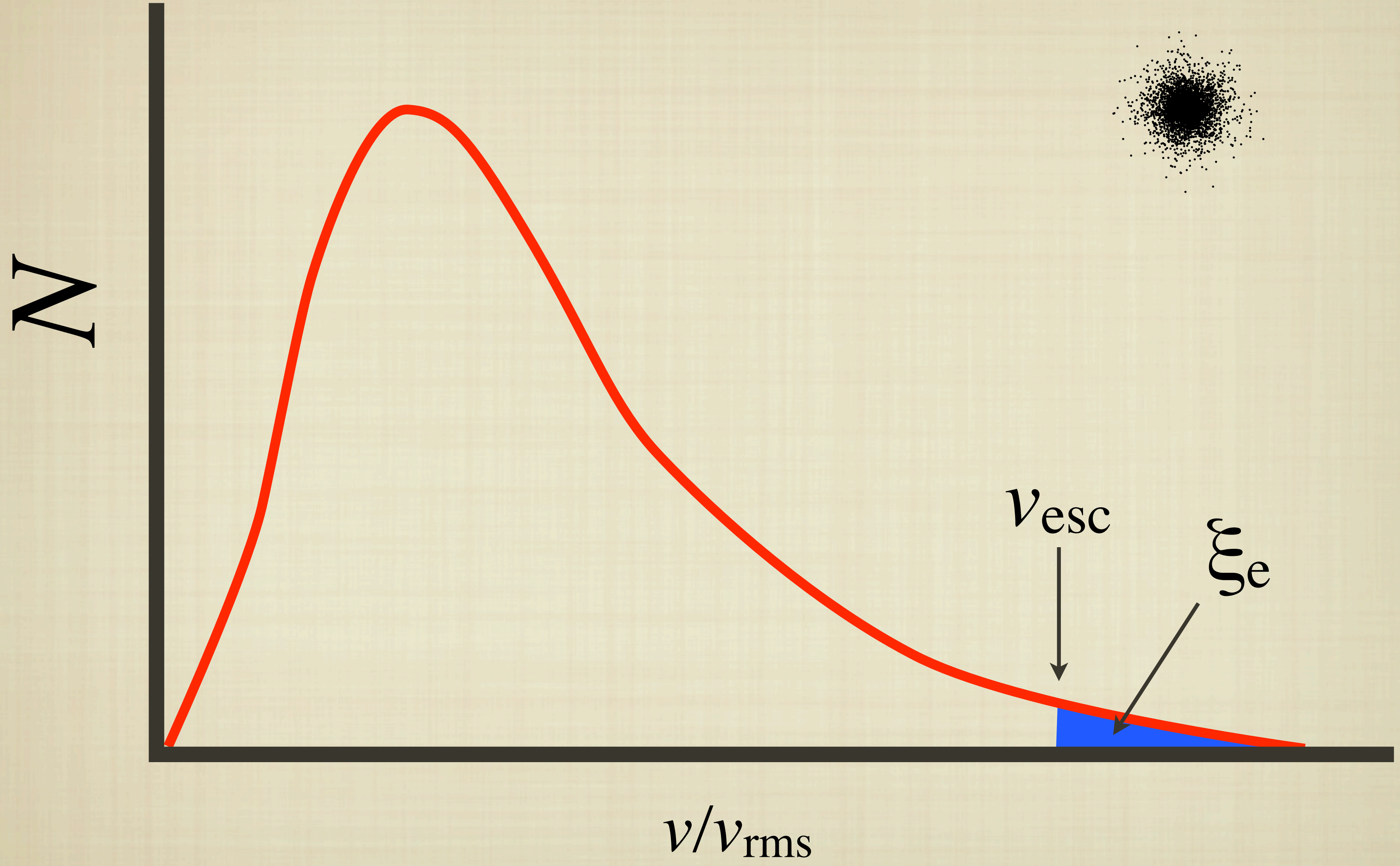


Scheepmaker et al. (2007)

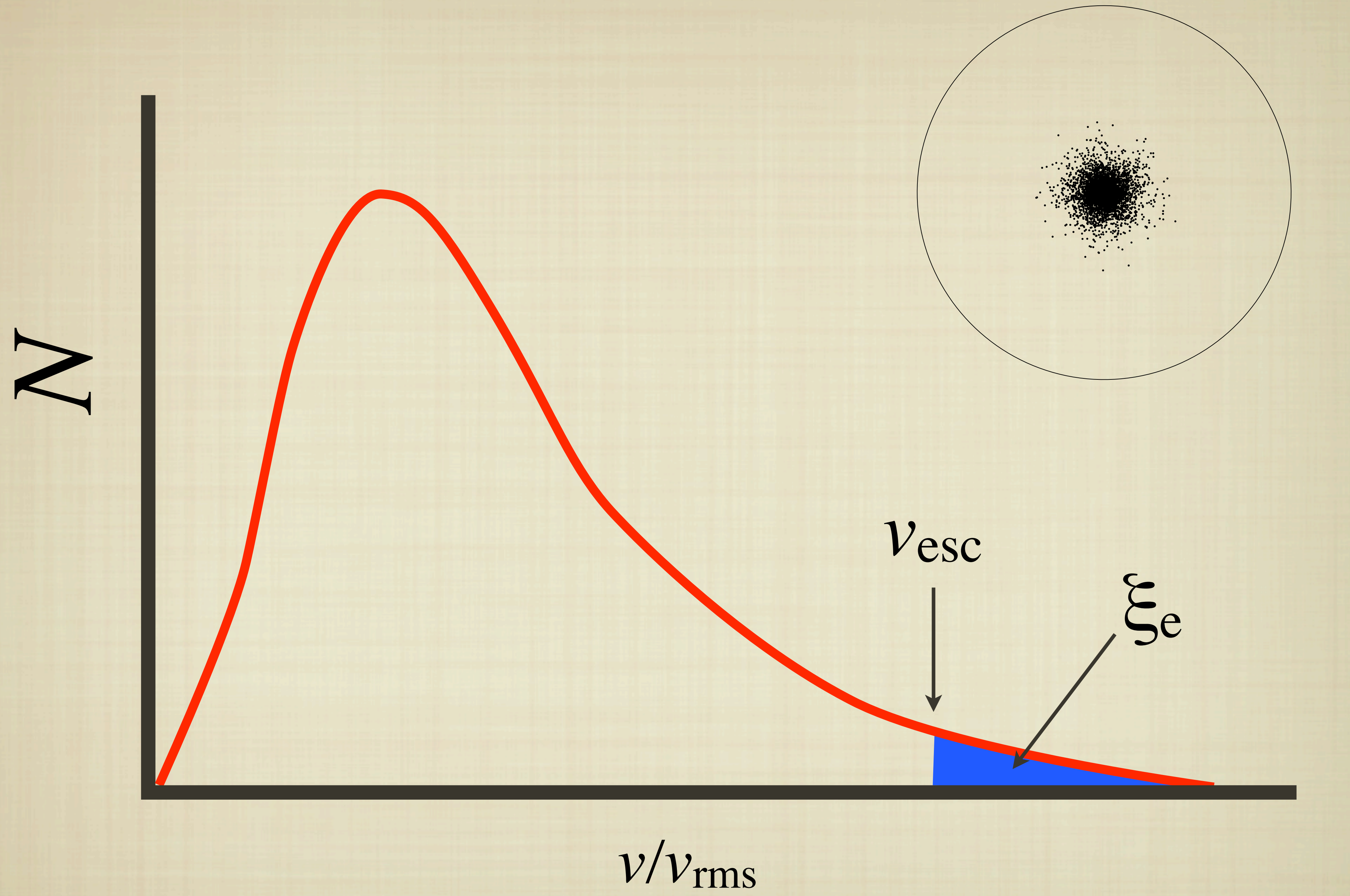


Difference = 0.0 %
0 Myr
N = 4096
r_h/r_J=0.15
N = 4096
r_h/r_J=0.075

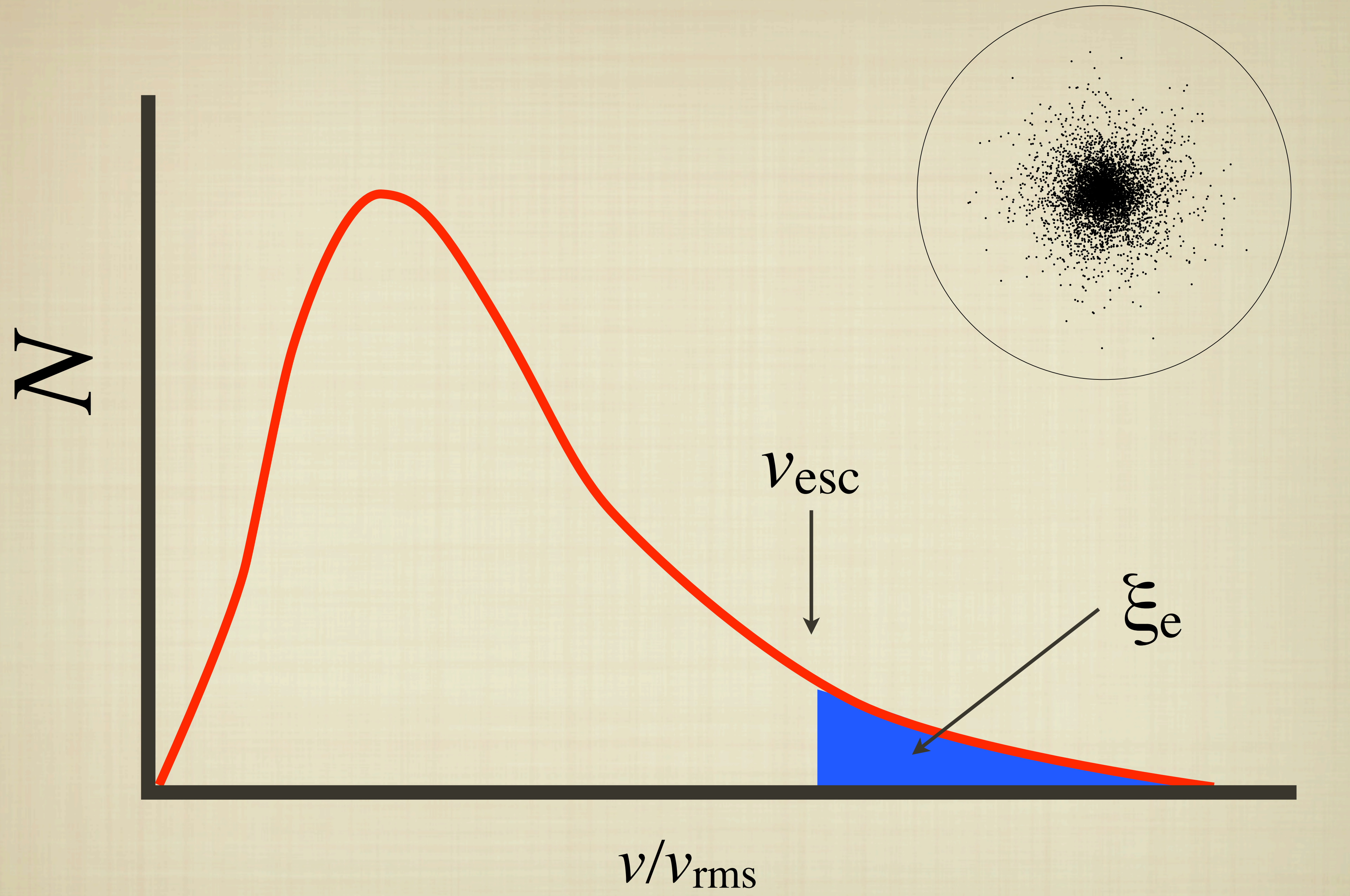

Difference = 0.0 %
0 Myr
N = 4096
N = 4096

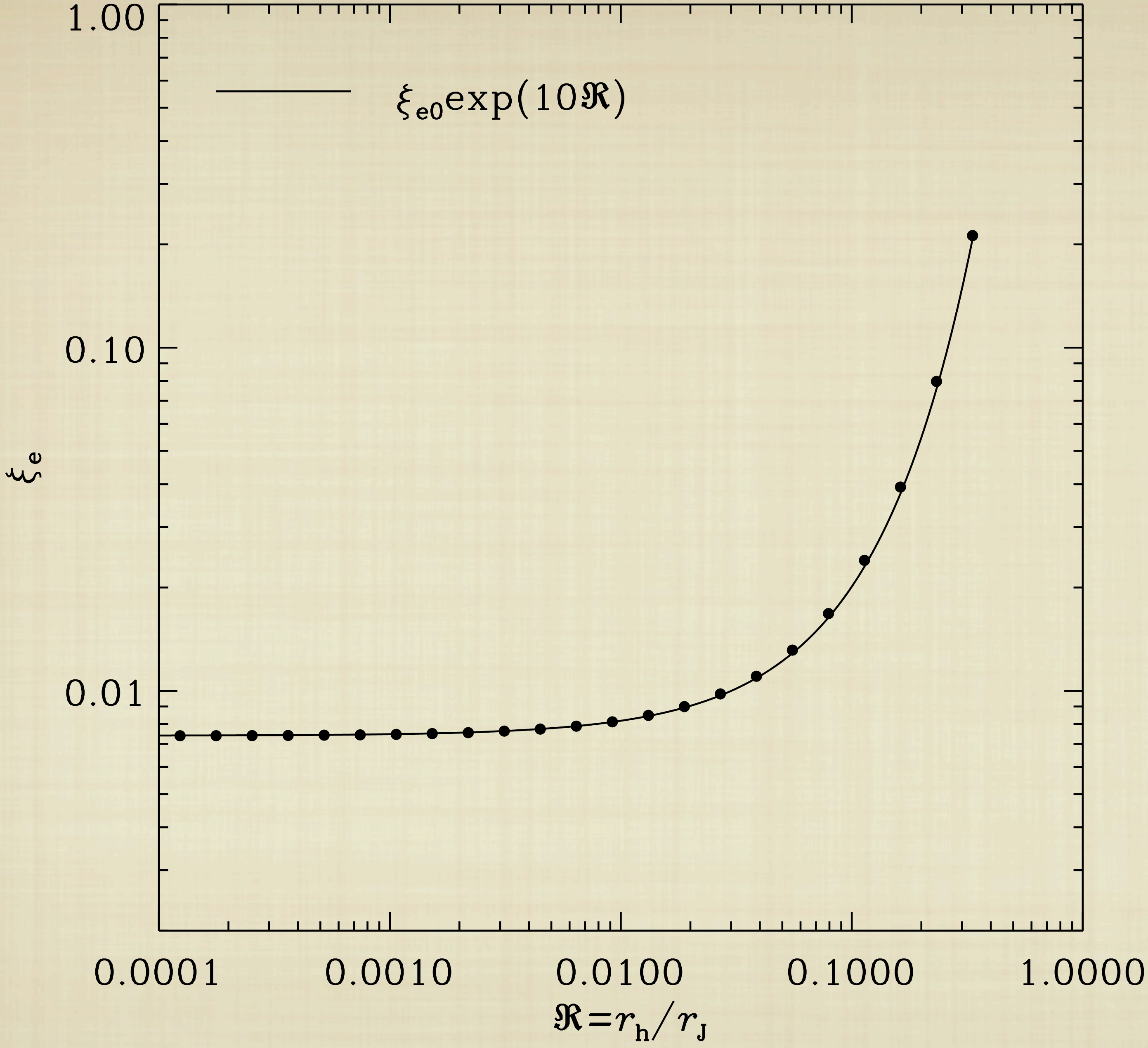

N
v_{esc}
ξ_e
v/v_{rms}

tidal radius: r_J

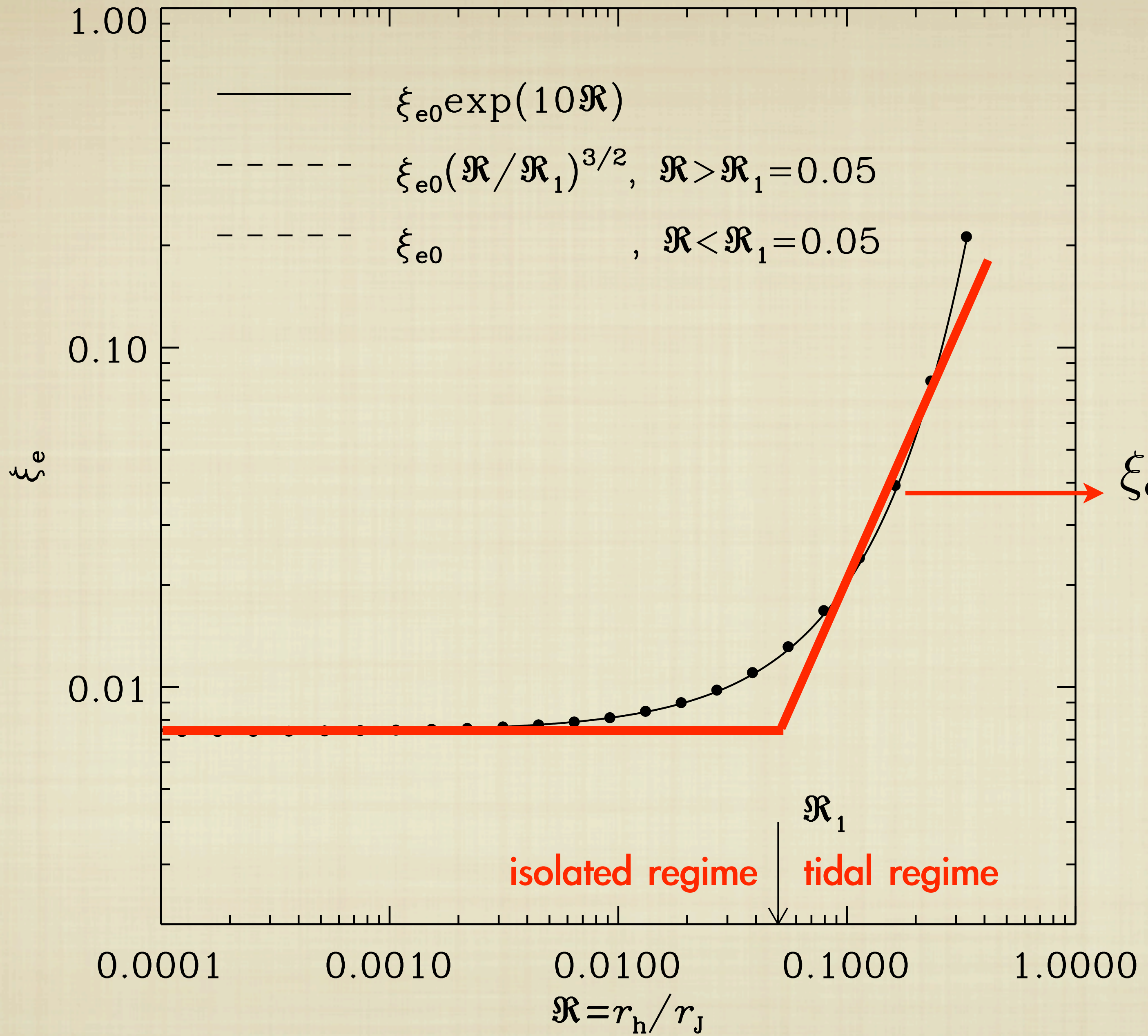


tidal radius: r_J





Gieles & Baumgardt (2008)



$$\xi_e \propto \left(\frac{r_h}{r_J}\right)^{3/2}$$

Gieles & Baumgardt (2008)

<u>See also:</u>

Spitzer & Chevalier (1973)
Aguilar, Hut & Ostriker (1988)
Wielen (1988)

McMillan & Hut (1994)
Baumgardt (1998)
Gnedin, Lee & Ostriker (1999)

$$\dot{M} = \xi_{\rm e} \frac{M}{t_{\rm rh}}$$

ξ_e = escape fraction

Ambartsumian (1938); Spitzer (1940); Henon (1961); Spitzer (1987)

$$\dot{M} = \xi_{\mathrm{e}} \frac{M}{t_{\mathrm{rh}}}$$

$$\propto \left(\frac{r_{\mathrm{h}}}{r_{\mathrm{J}}}\right)^{3/2} \rho_{\mathrm{h}}^{1/2}$$

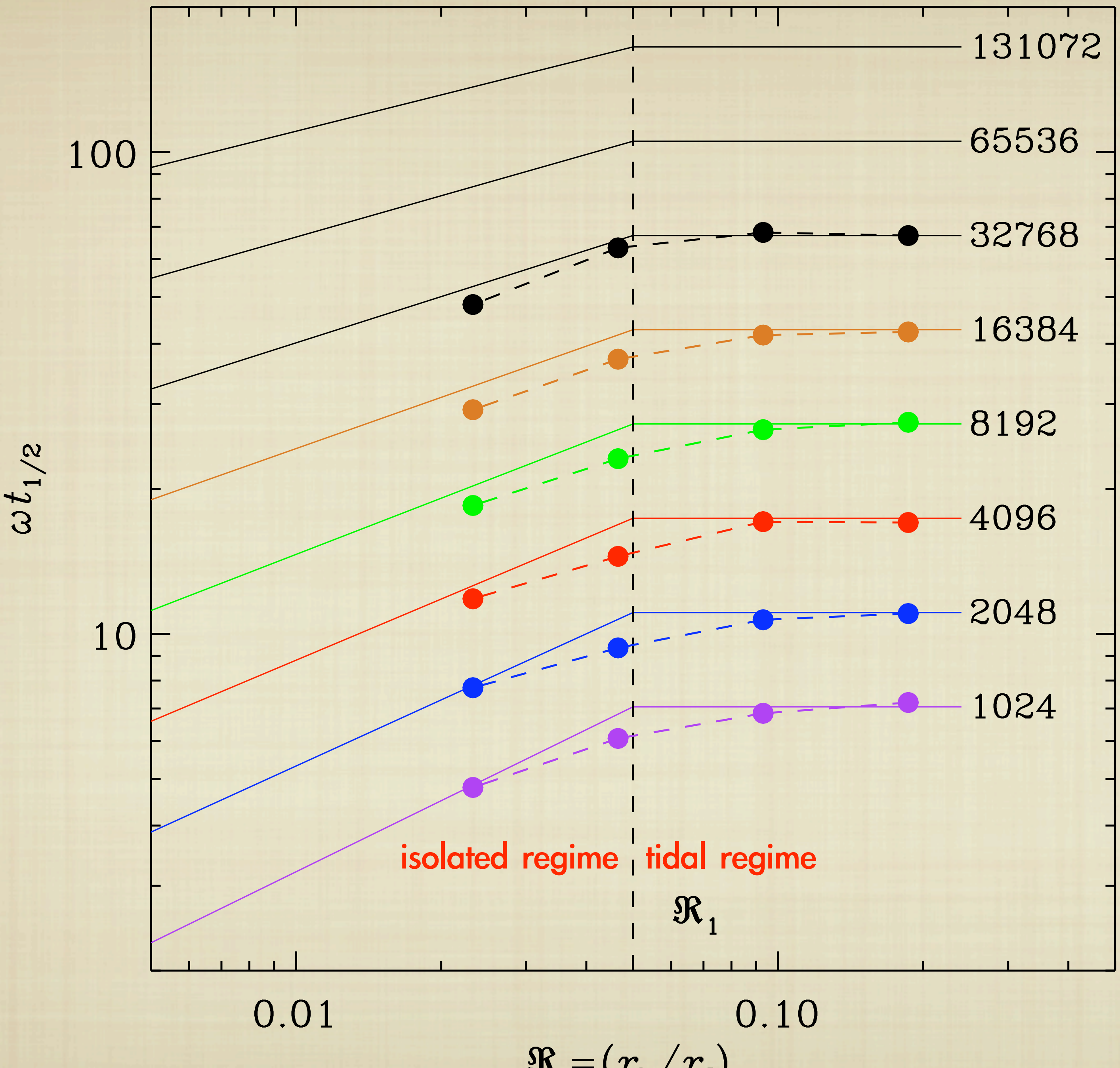
Gieles & Baumgardt (2008)

Tada!

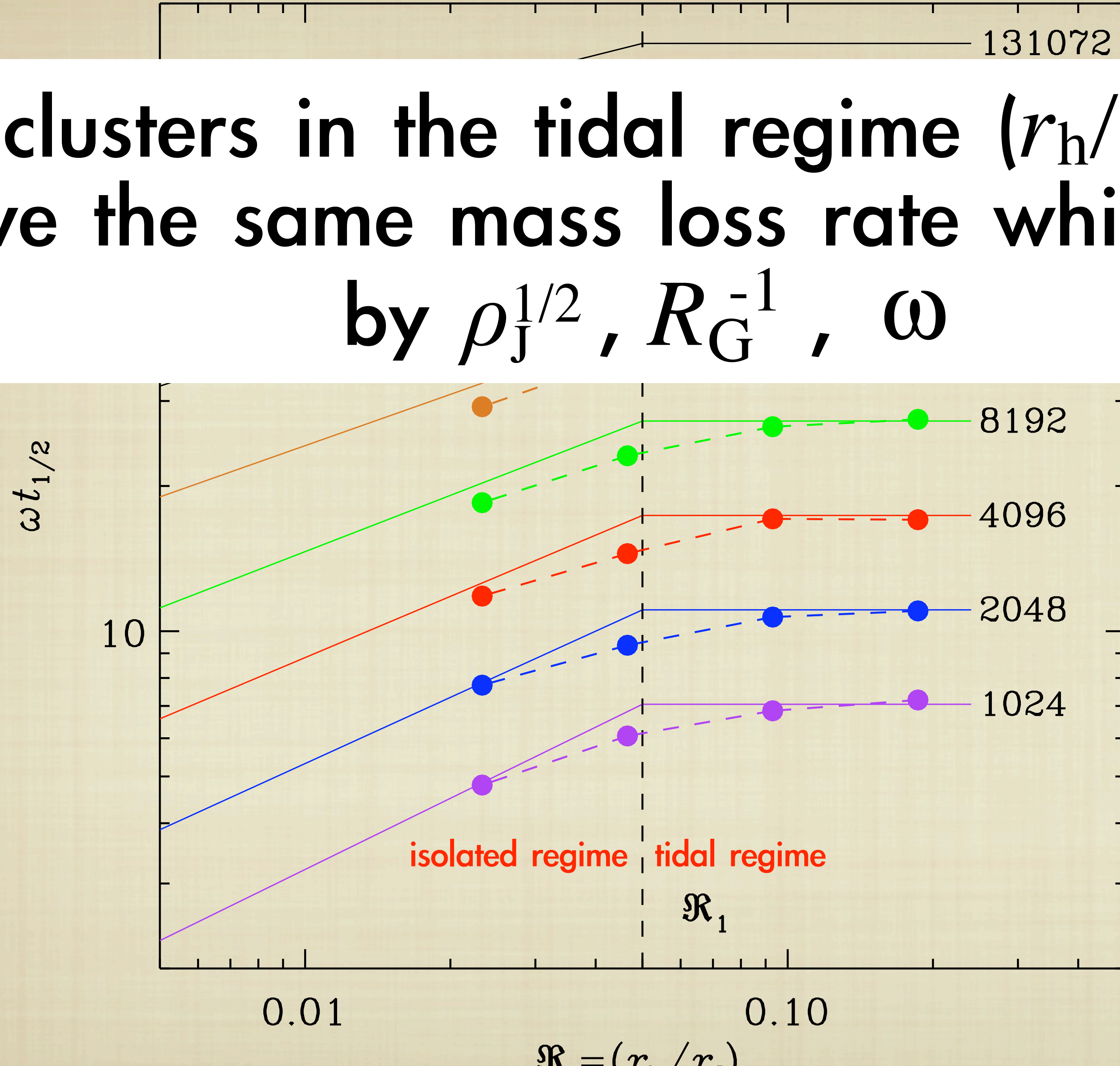
$$\dot{M} = \xi_{\rm e} \frac{M}{t_{\rm rh}}$$

$$\propto \rho_{\rm J}^{1/2}$$

Gieles & Baumgardt (2008)

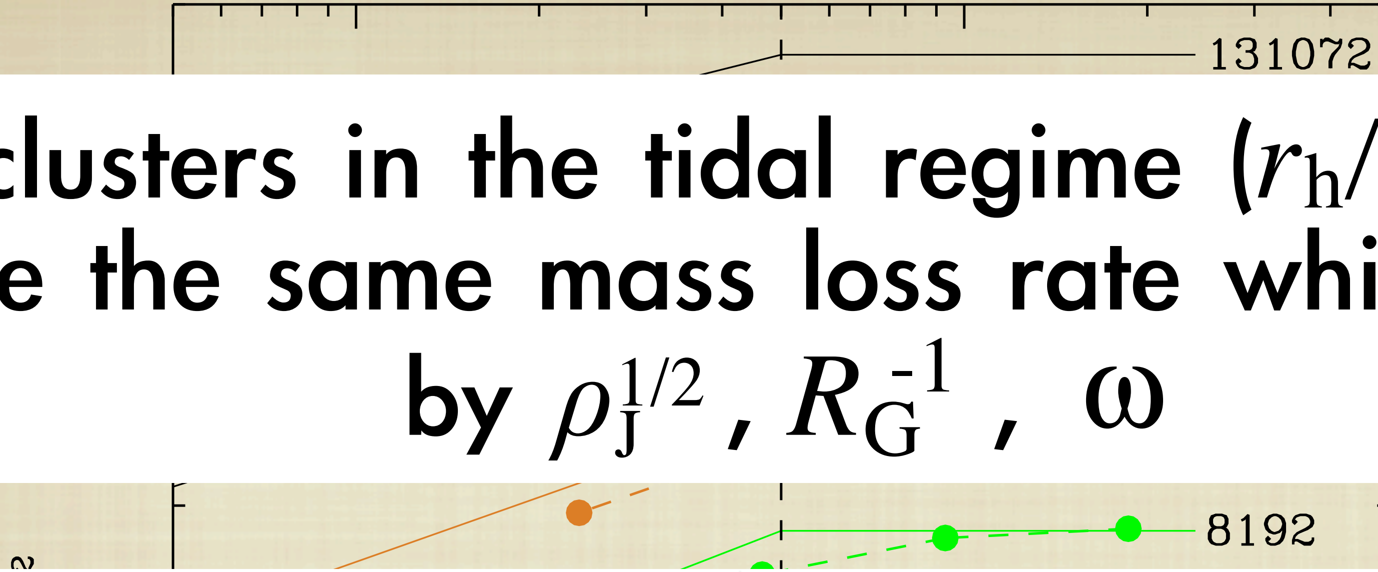


Gieles & Baumgardt (2008)



All clusters in the tidal regime (r_h/r_J >0.05) have the same mass loss rate which is set by $\rho_J^{1/2}$, R_G^{-1}, ω

Gieles & Baumgardt (2008)



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The Structure of Star Clusters. III. Some Simple Dynamical Models

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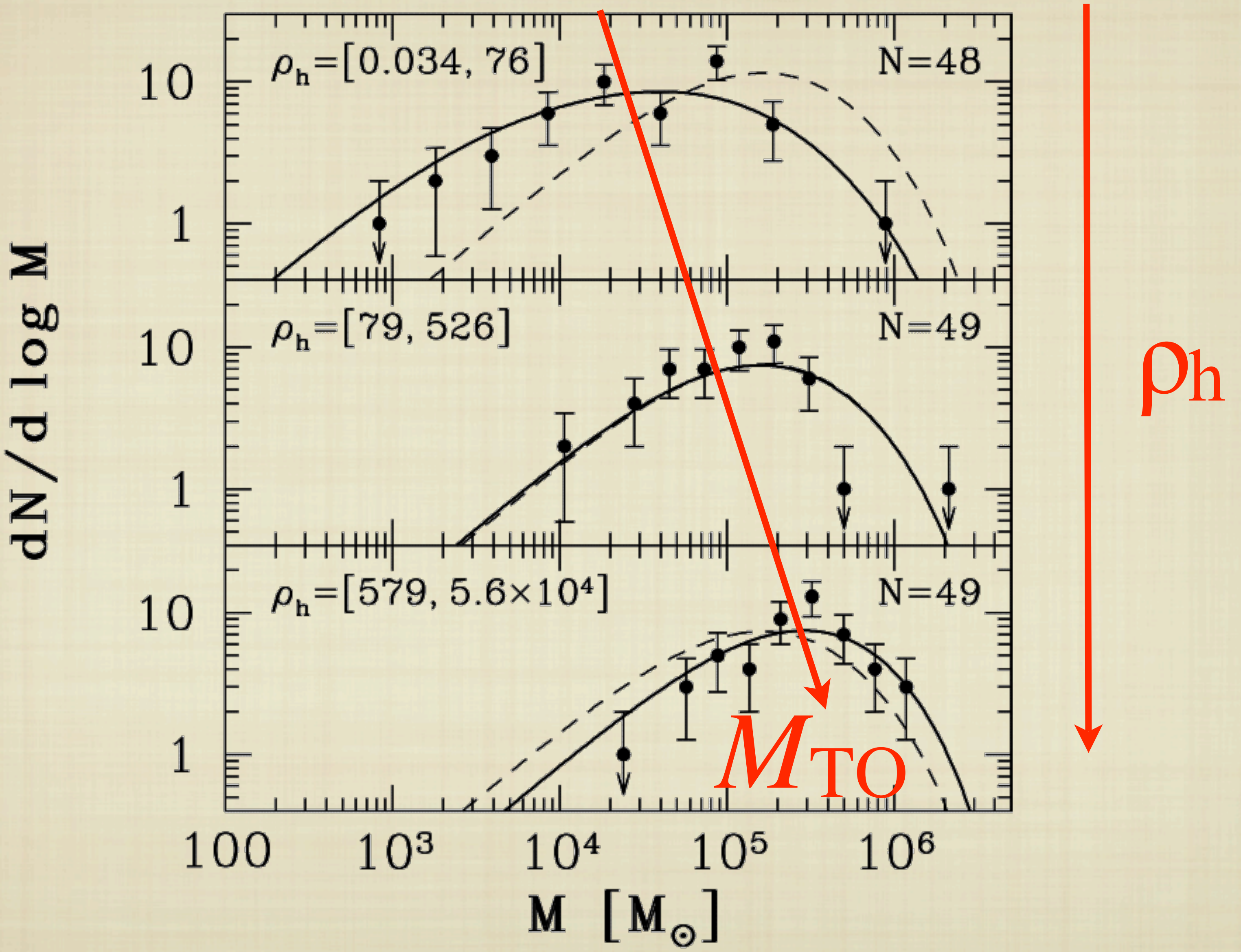
Berkeley Astronomy Department, University of California
(Received 20 October 1965)

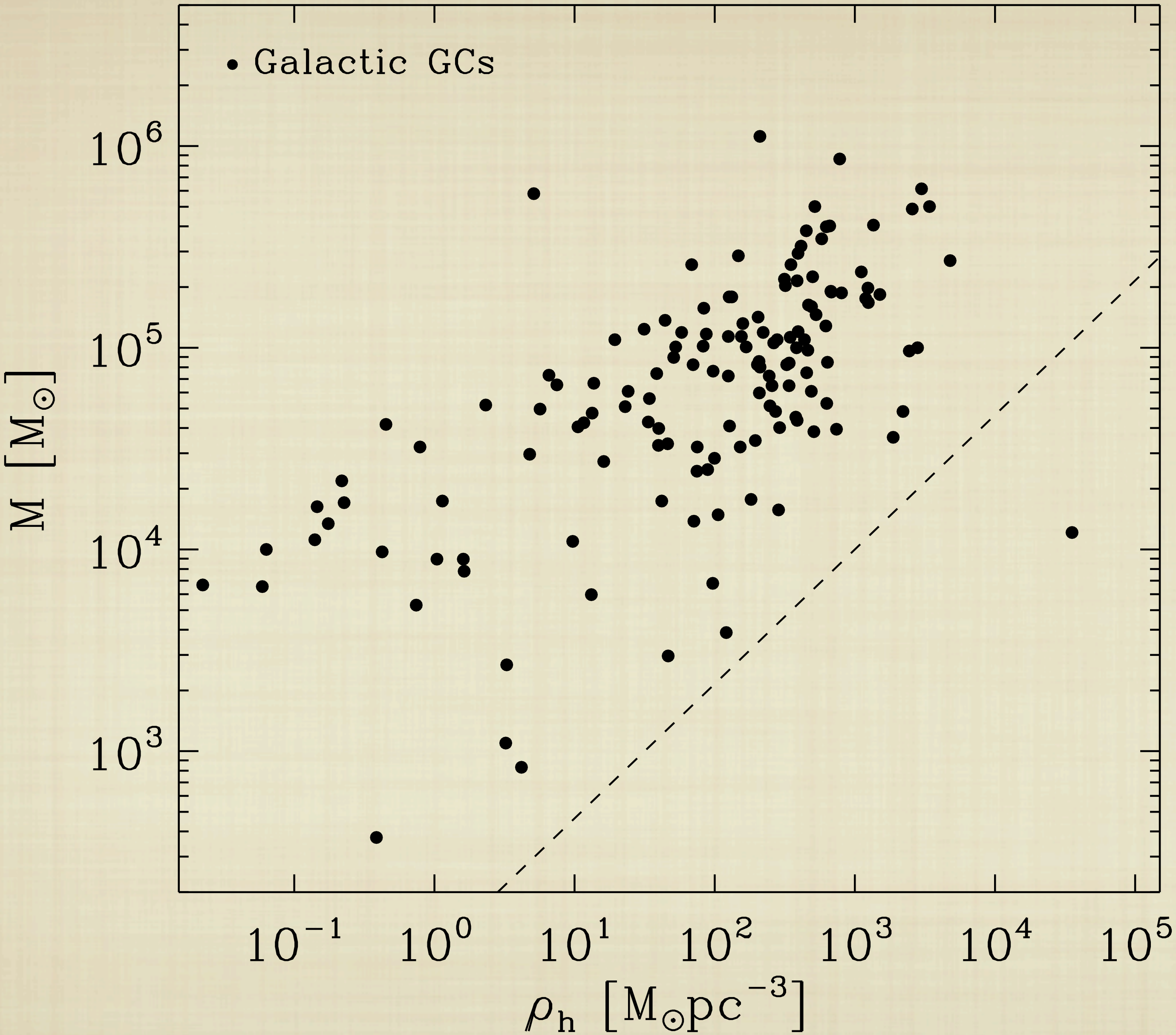
Thus within a wide range of central concentrations the escape rate of stars from a cluster depends only on the number of stars and the tidal field in which the cluster finds itself. There is no obvious physical reason for this simplicity; it seems to arise from a fortuitous compensation of opposing effects.

3)

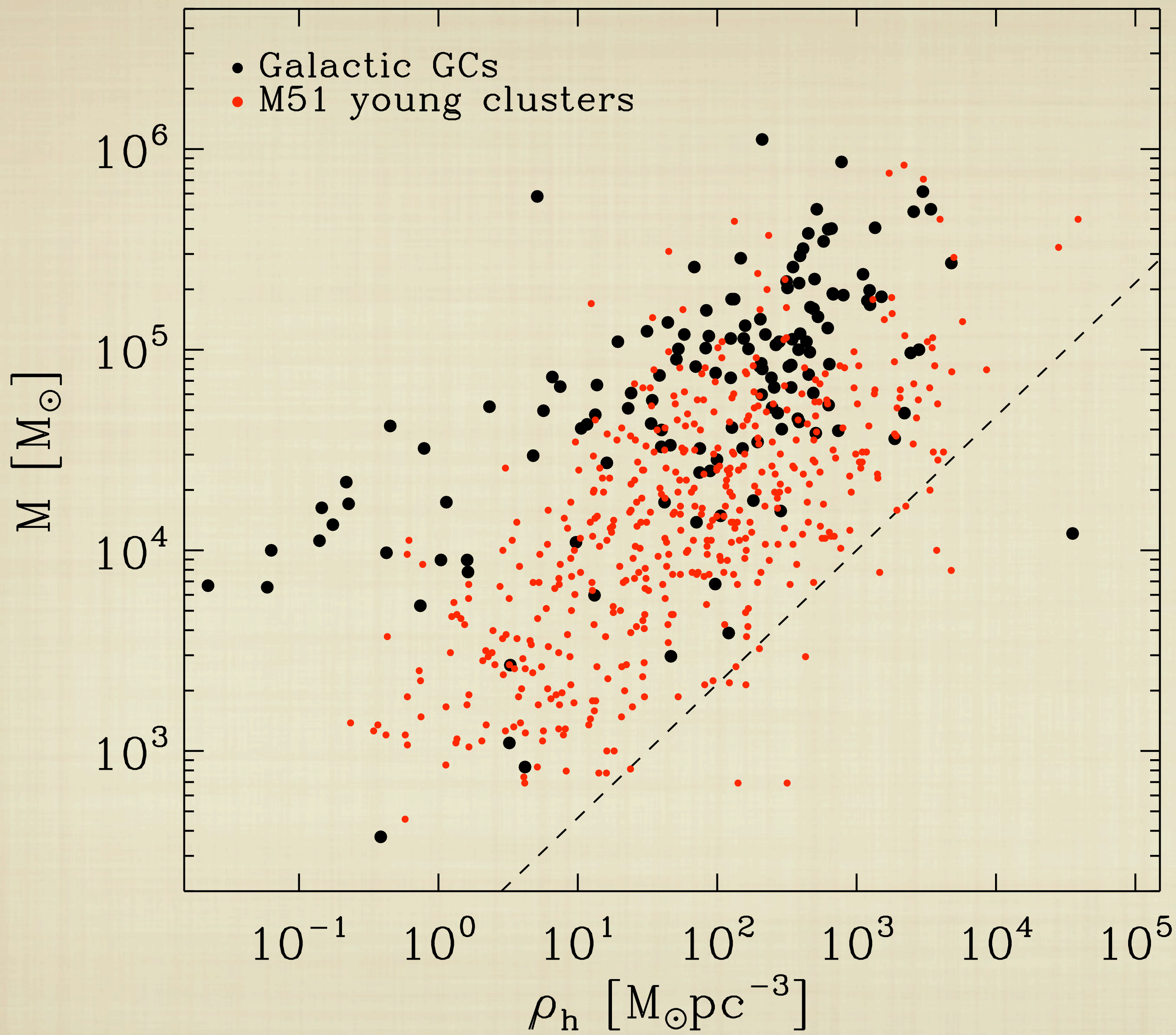
Density dependency of GCMF



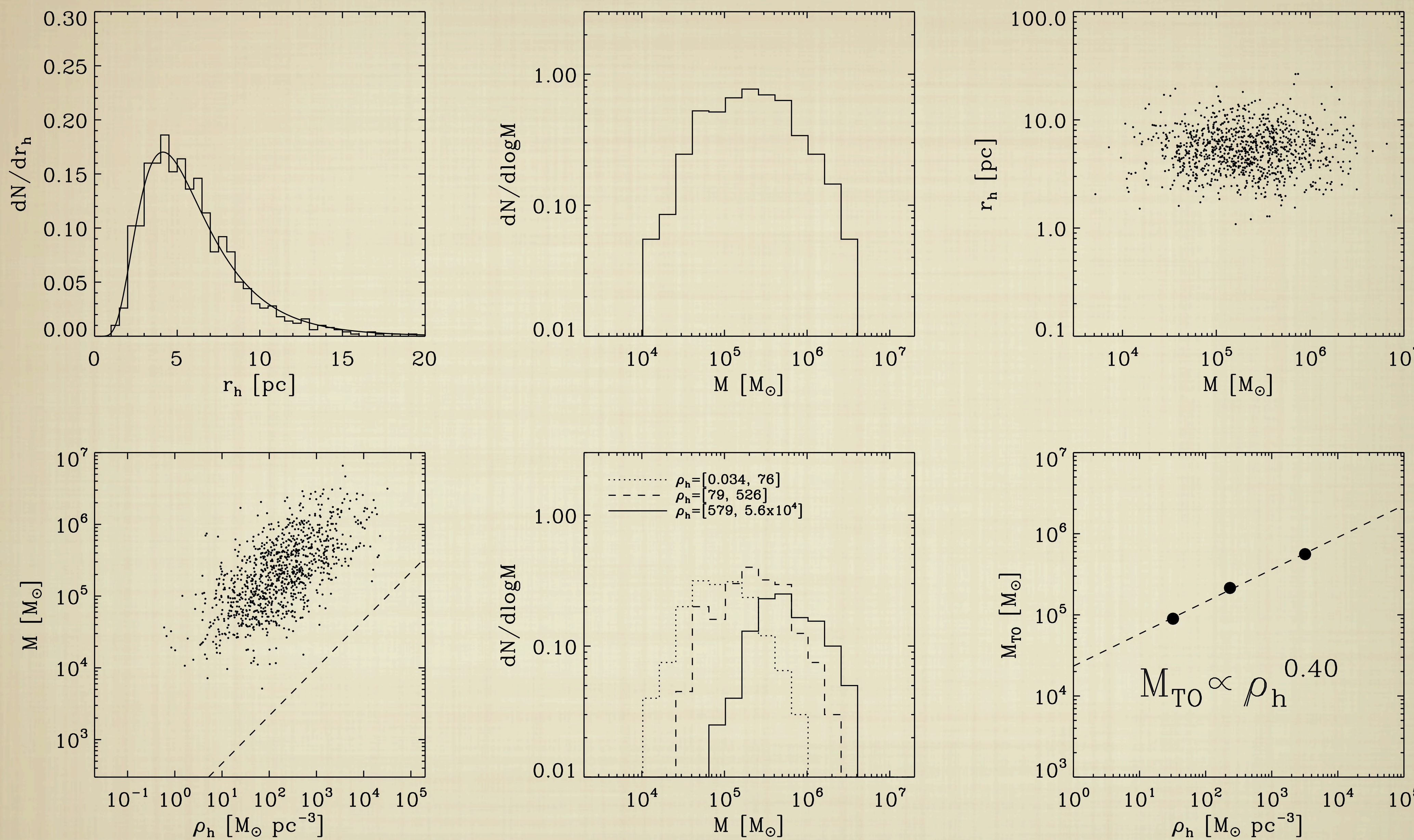
McLaughlin & Fall (2008)



GCs: Harris (2003)



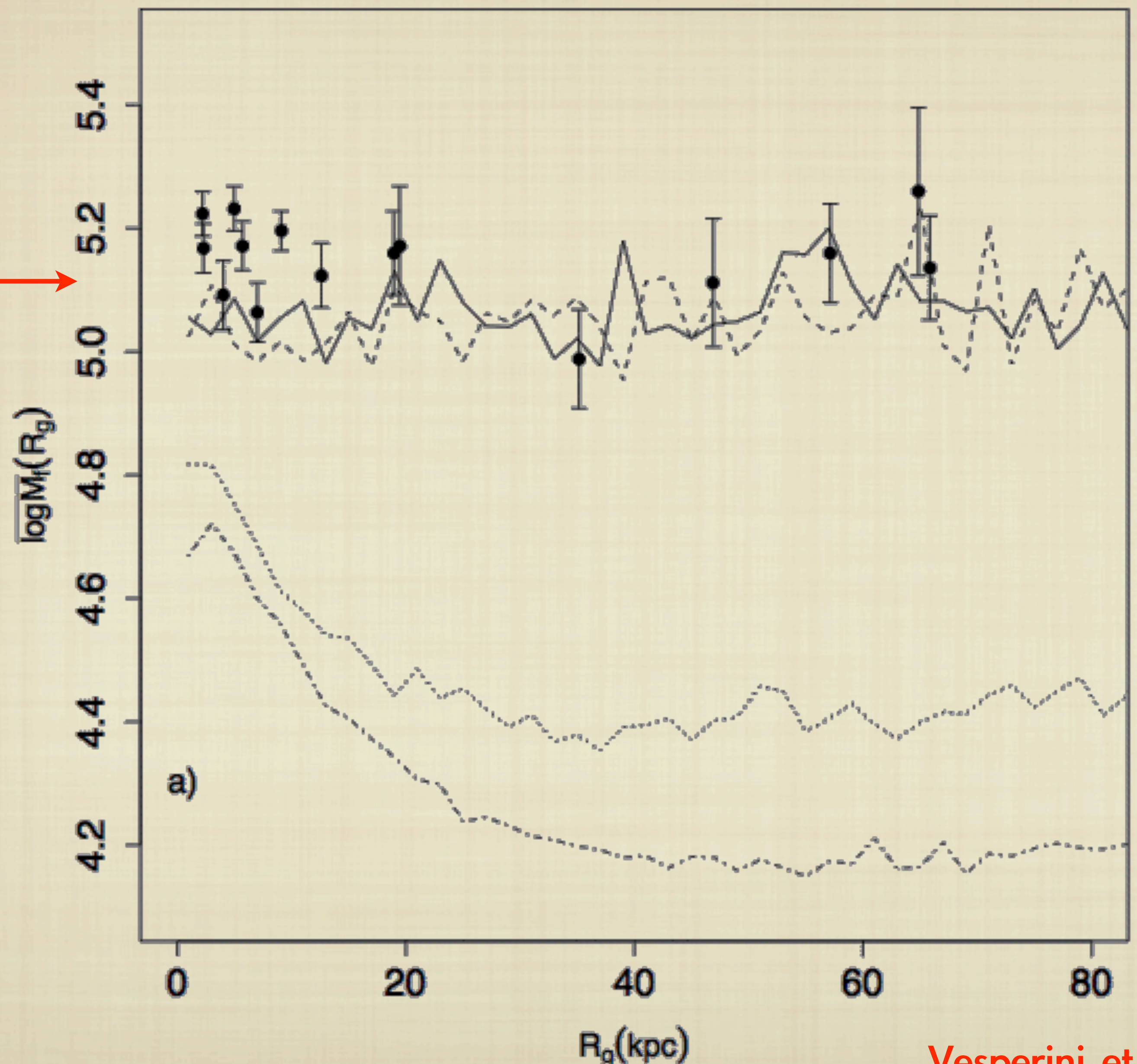
GCs: Harris (2003)
M51: Scheepmaker et al. (2007)



dN/dr_h
r_h [pc]
dN/dlogM
M [M_⊙]
r_h [pc]
M [M_⊙]
M [M_⊙]
ρ_h [M_⊙ pc^-3]
ρ_h=[0.034, 76]
ρ_h=[79, 526]
ρ_h=[579, 5.6x10^4]
dN/dlogM
M [M_⊙]
M_TO [M_⊙]
M_TO ∝ ρ_h^0.40
ρ_h [M_⊙ pc^-3]

M87

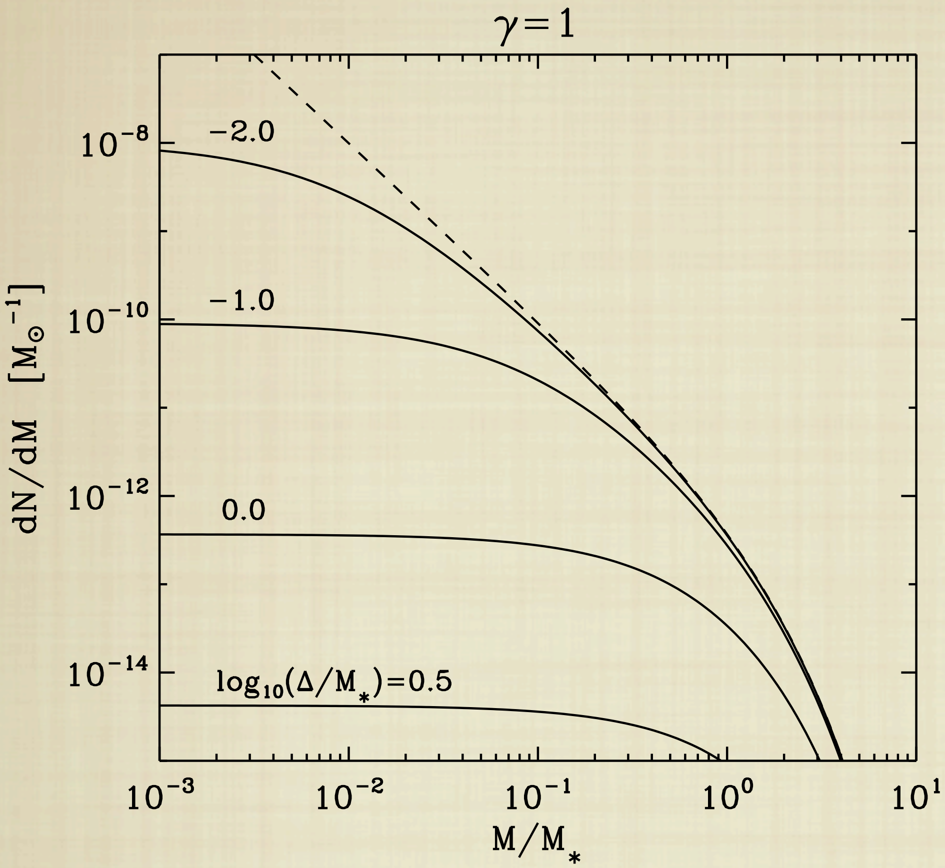
M_{TO} →

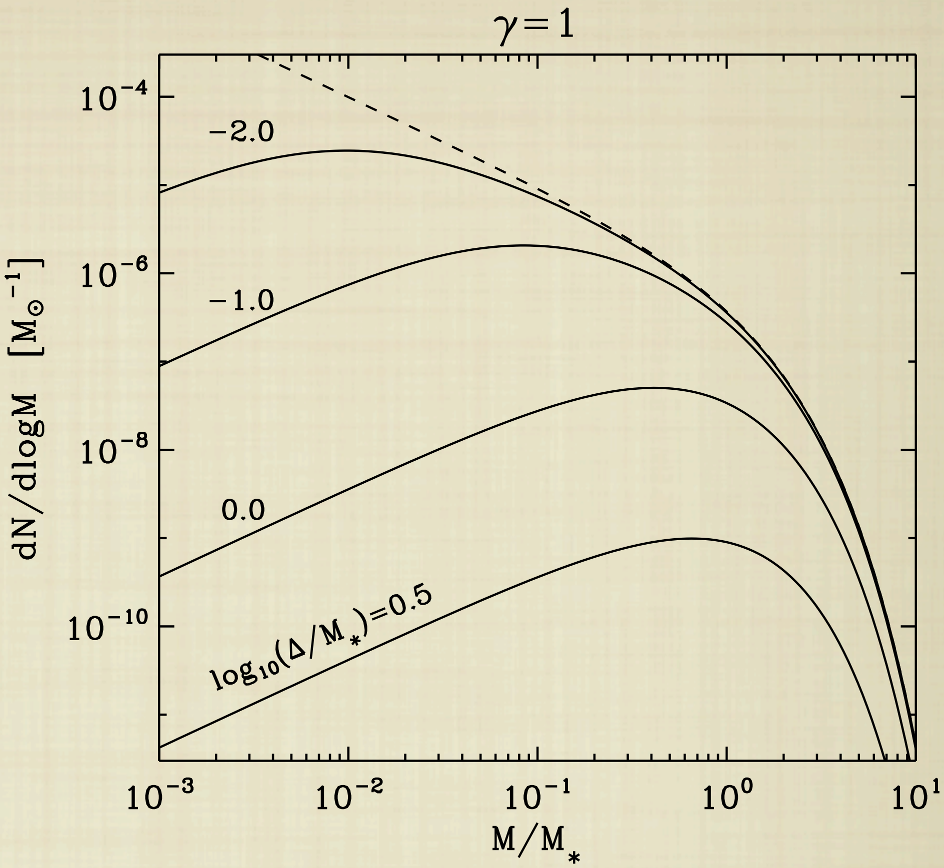


Vesperini et al. (2003)
see also Jordán et al. (2007)

For Fred:

dN/dM $\quad$ $dN/dlogM$





$$\begin{aligned} M_{\rm TO} \propto \Delta &= t/t_0 \\ &\propto t\,\rho_{\rm J}^{1/2} \\ &\propto t\,R_{\rm G}^{-1} \\ &\propto t\,\omega \end{aligned}$$

Jordán et al. (2007)
Gieles (2009)

How to get a constant M_{TO}

$$
\begin{aligned}
M_{\mathrm{TO}} \propto \Delta &= t/t_0 \\
&\propto t\,\rho_{\mathrm{J}}^{1/2} \\
&\propto t\,R_{\mathrm{G}}^{-1} \\
&\propto t\,\omega
\end{aligned}
$$

= constant

How to get a constant M_{TO}

$$
\begin{aligned}
M_{\rm TO} \propto \Delta \quad &= \quad t/t_0 \\
&\propto \quad t\,\rho_{\rm J}^{1/2} \\
&\propto \quad t\,R_{\rm G}^{-1} \\
&\propto \quad t\,\omega
\end{aligned}
\qquad = \text{constant}
$$

To get the turn-over at the same place everywhere, all clusters in the Universe need to pass through a universal phase of tidal evolution with the product $t\omega$ constant

Conclusions

- Mass loss rate scales with $\rho_{\rm J}^{1/2}$, not with $\rho_{\rm h}^{1/2}$
- In the tidal regime ($r_{\rm h}/r_{\rm J}$ > 0.05) the moss loss rate is independent of how the stars are distributed within the Jacobi surface
- To evolve a power-law initial cluster mass function to a peaked GCMF with constant $M_{\rm TO}$ by only 2-body relaxation in a tidal field you need a constant Δ for all clusters
- The similarity of between young and old clusters in the $r_{\rm h}$ vs. M plane suggests that $r_{\rm h}$ is (largely) imprinted by formation, not evolution