

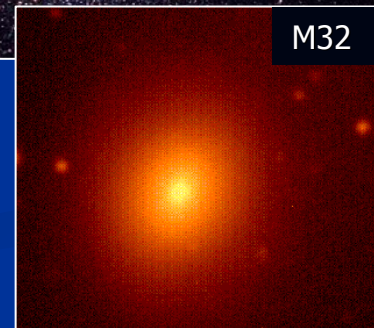
# Dynamical Schwarzschild models of globular clusters

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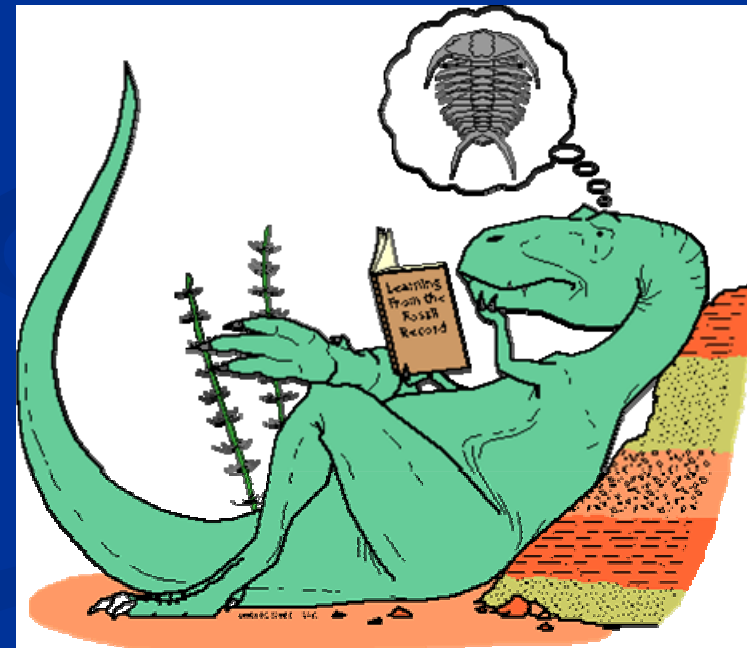
# Stellar systems



# Fossil record

- How do stellar systems form and evolve?
- Wealth of structure in morphology and kinematics
- Range in stellar population properties (age, metallicity, ...)
- Link between dynamics and stellar populations?

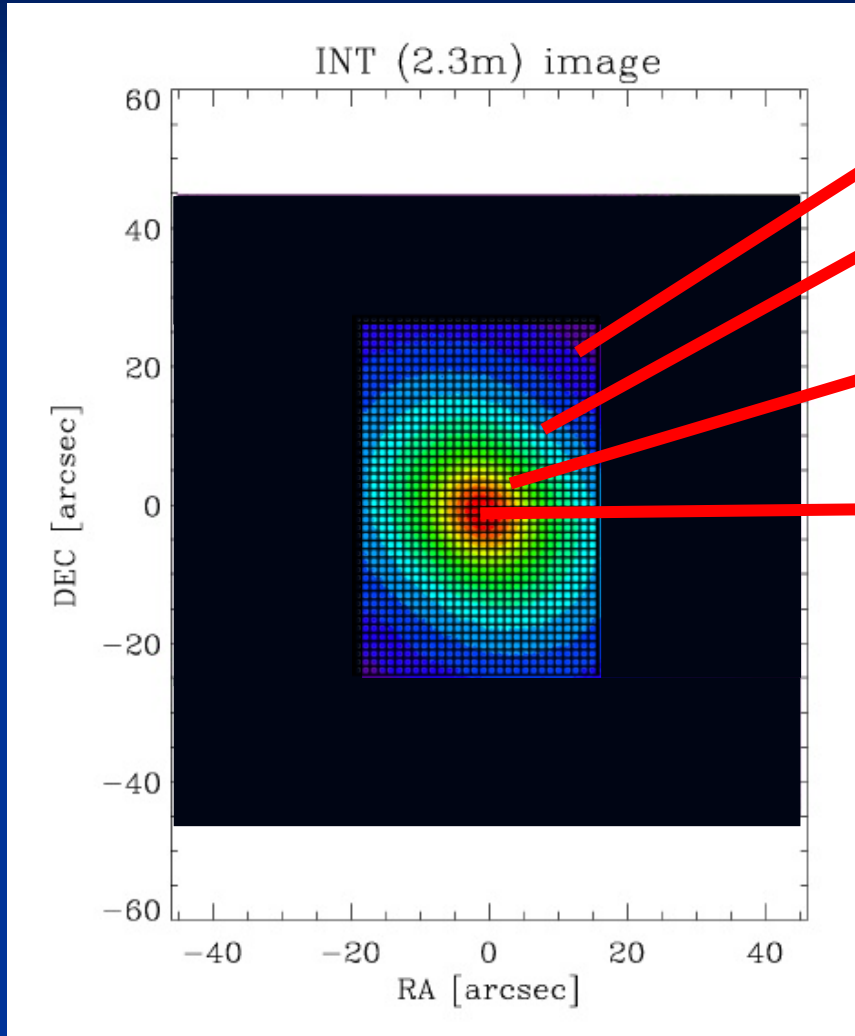
- Clean fossil record in early-type galaxies and globular clusters
- Galaxies: integral-field spectroscopy
- Globular clusters: discrete stellar kinematics + population properties



# Early-type galaxies

- Integral-field spectroscopy
- Schwarzschild's method
- Triaxial dynamical model of NGC4365

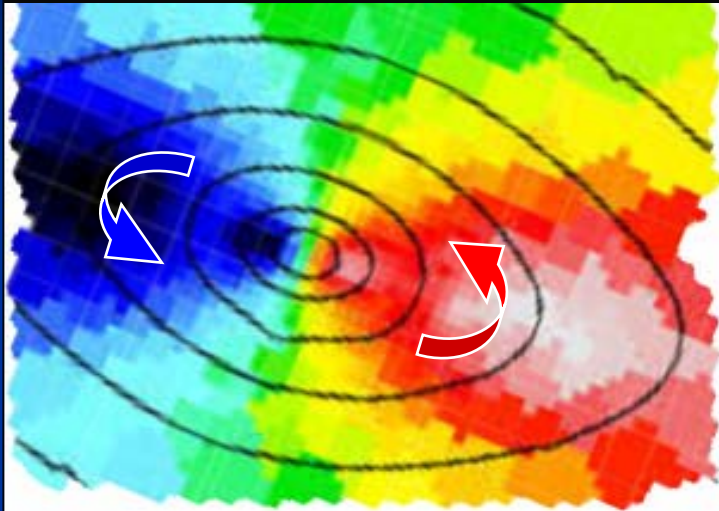
# Integral-field spectroscopy



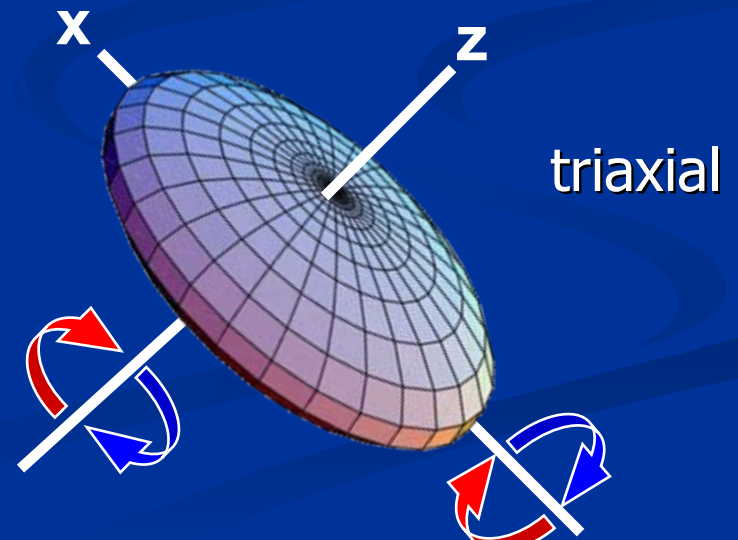
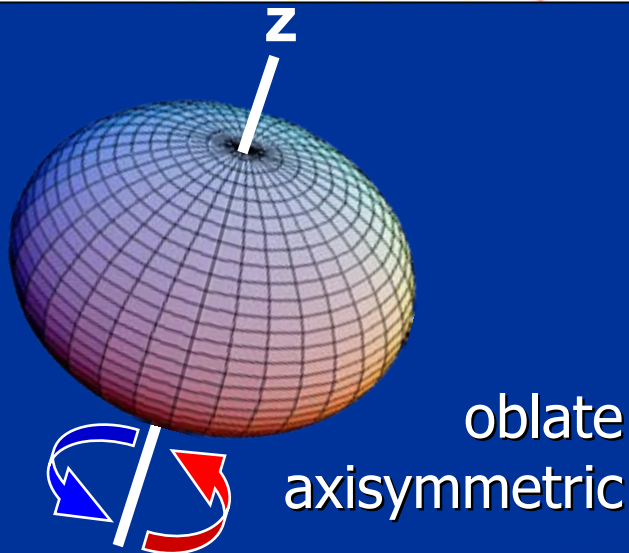
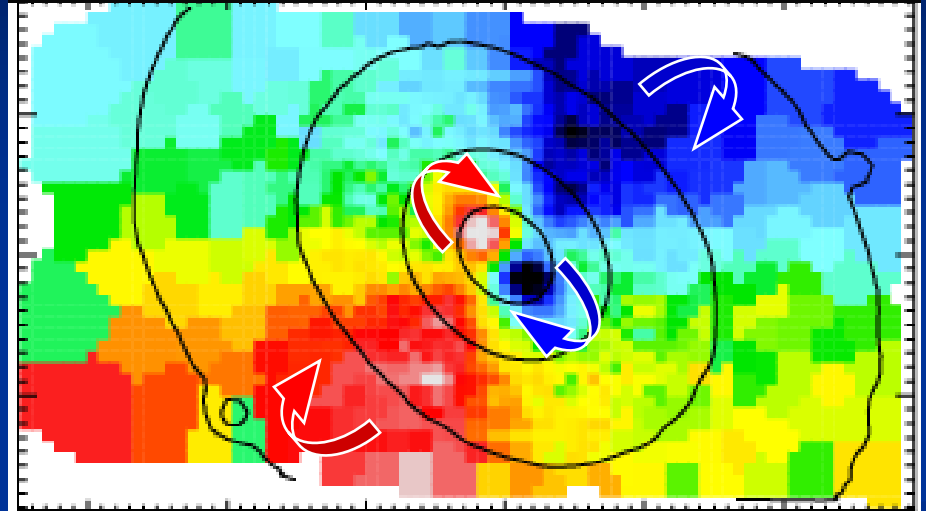
... a spectrum at every position on the plane of the sky

# Stellar velocity fields

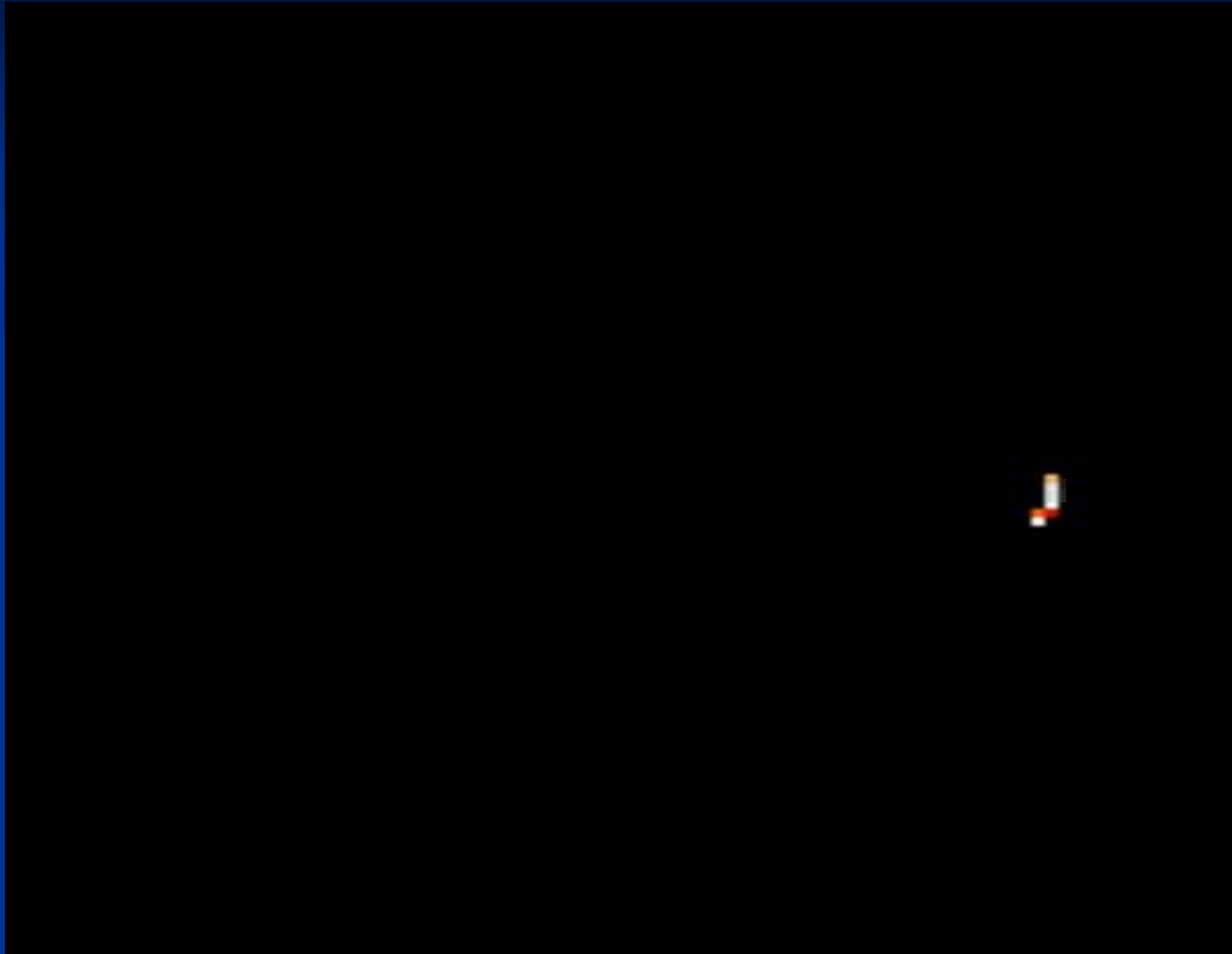
NGC 4660 [-150/+150 km/s]



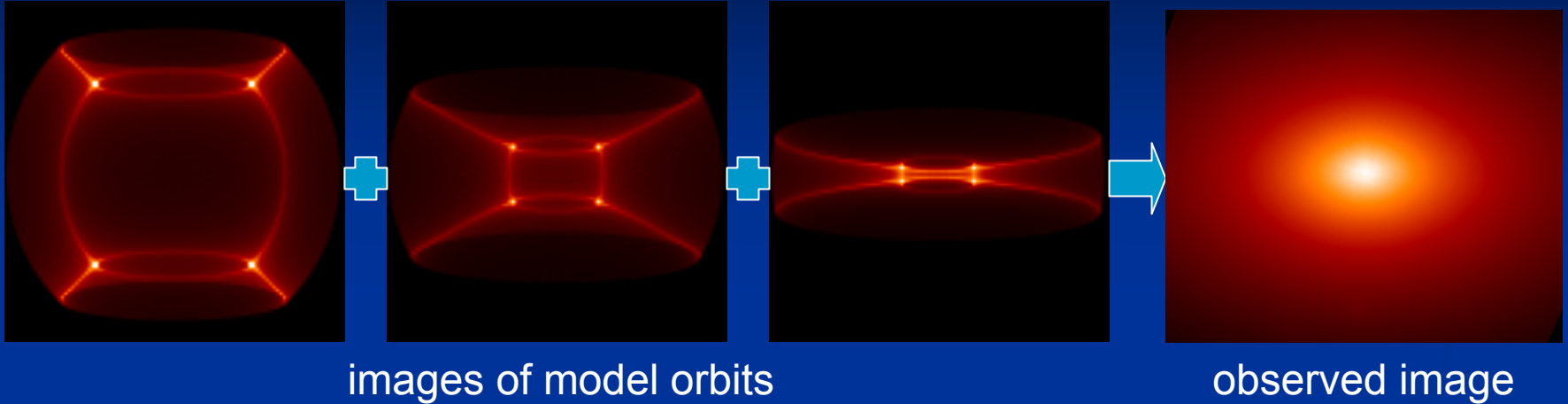
NGC 4365 [-58/+58 km/s]



# Image of numerical orbit



# Schwarzschild's method

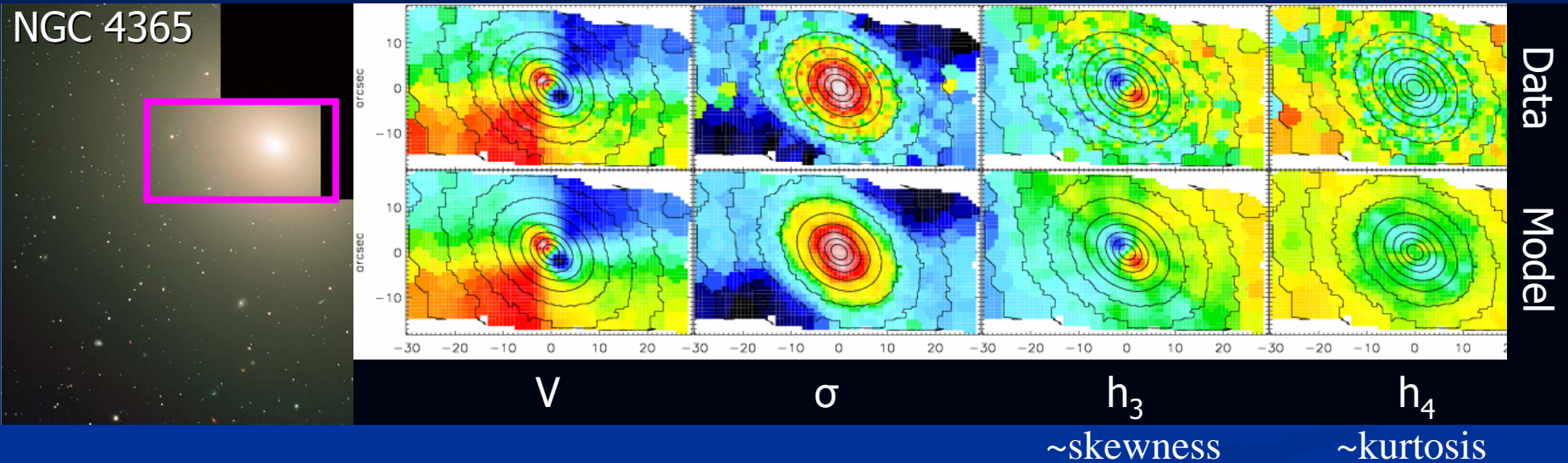


- Surface brightness  $\rightarrow$  (MGE) gravitational potential
- Grid of  $(E, I_2, I_3)$   $\rightarrow$  initial orbit conditions  $\rightarrow$  orbit library
- Weighted superposition of orbits that best fits photometry and kinematics  $\rightarrow$  dynamical model

Schwarzschild (1979), Richstone & Tremaine (1988), Rix et al. (1997), van der Marel (1998), Cappellari et al. (2002), Gebhardt et al. (2003), ...



# Dynamical model elliptical galaxy



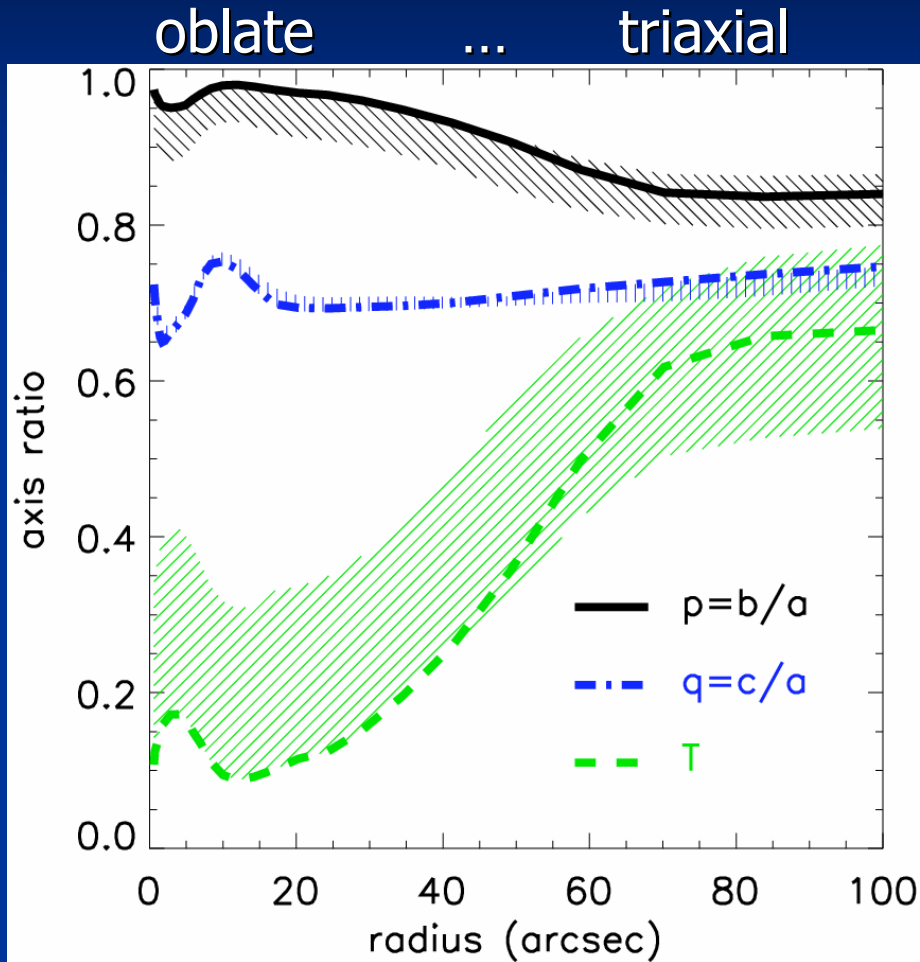
Fitting simultaneously photometry and kinematics in triaxial geometry, including a possible black hole and/or dark matter halo

van den Bosch, van de Ven, et al. (2008, MNRAS, 385, 647)

van de Ven, de Zeeuw & van den Bosch (2008, MNRAS, 385, 614)

van den Bosch & van de Ven (2008, MNRAS, arXiv:0811.3474)

# Intrinsic shape

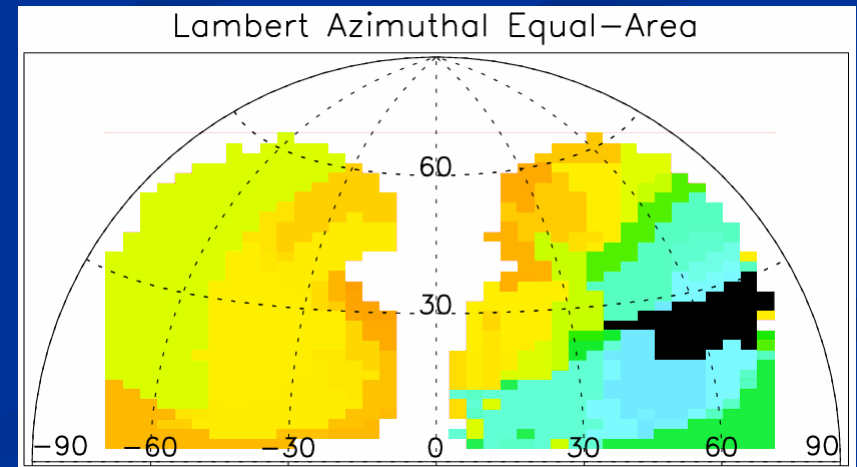


$$T = (1-p^2)/(1-q^2)$$

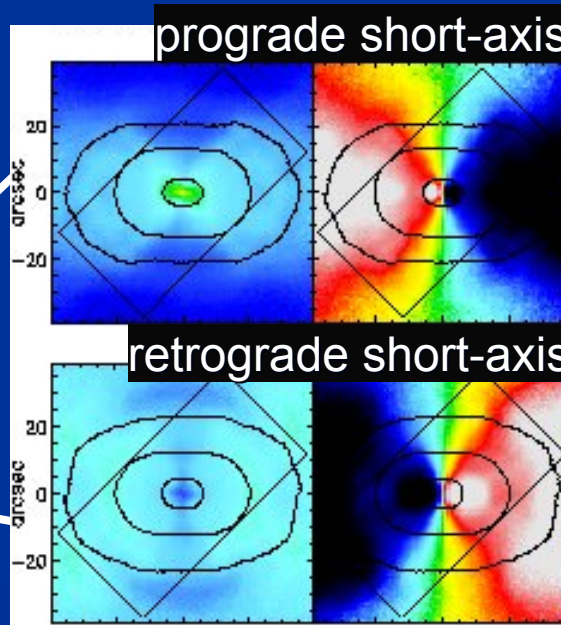
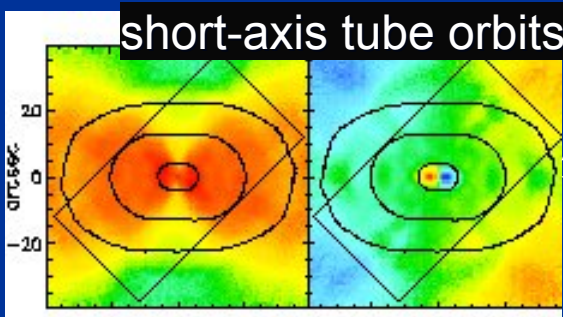
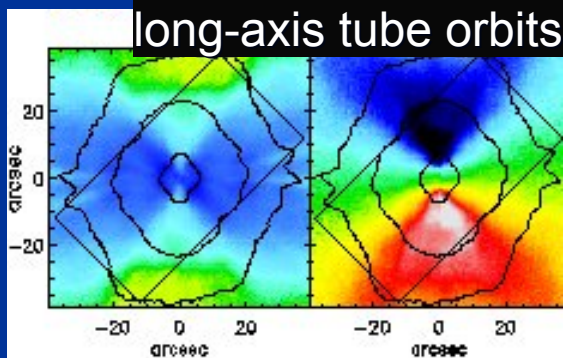
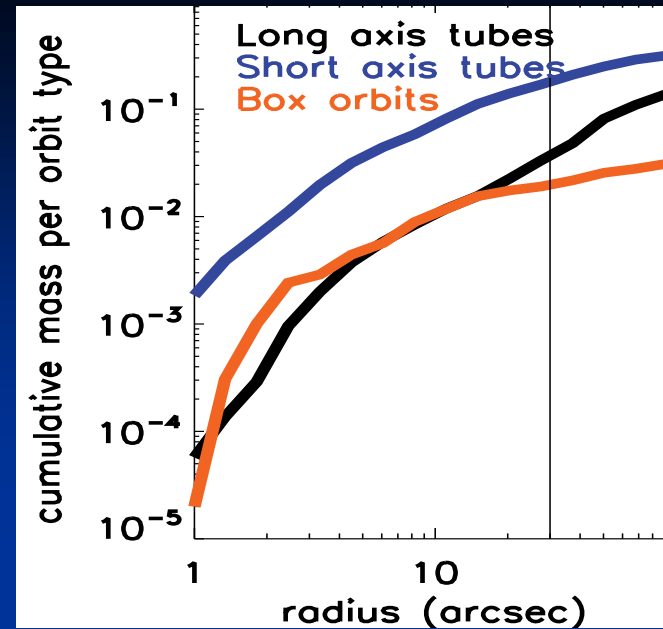
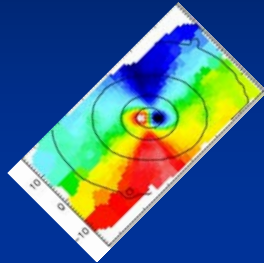
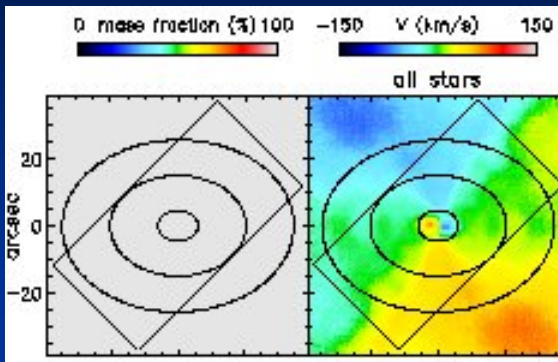
Oblate:  $p=1$  ( $a=b>c$ ),  $T=0$

Prolate:  $p=q$  ( $a>b=c$ ),  $T=1$

... and viewing direction  
 $(\theta, \phi, \psi) = (68^\circ, 73^\circ, 91^\circ)$



# Orbital decomposition



rotation cancels  
except center

# Globular clusters: $\omega$ Cen

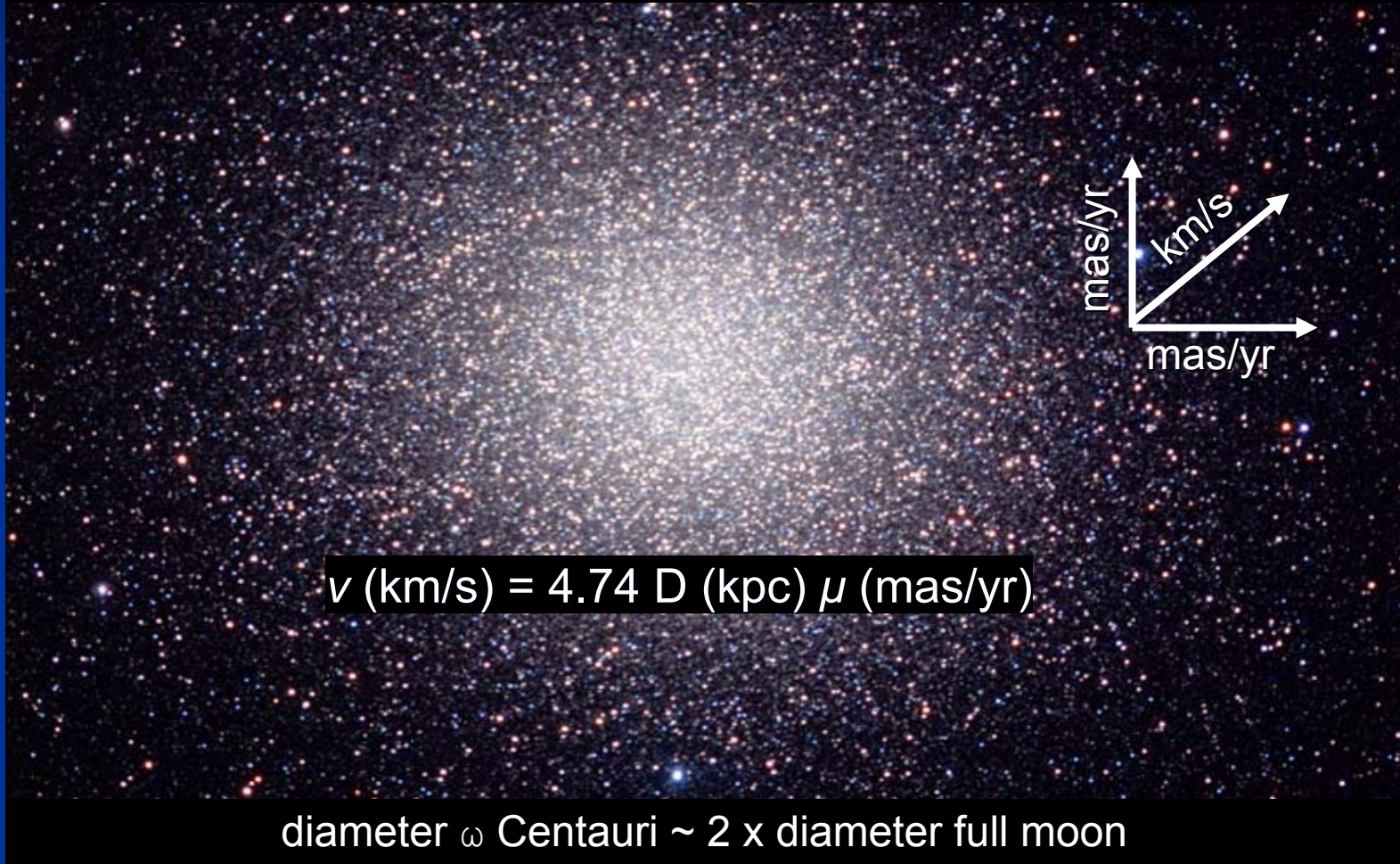
- discrete kinematics: proper motions and line-of-sight velocities
- distance, inclination, M/L
- inner disk and tidal striping

van de Ven et al. (2006, A&A, 445, 513)

# $\omega$ Cen (NGC 5139)

proper motions  $\sim 10000$  (van Leeuwen et al. 2000)

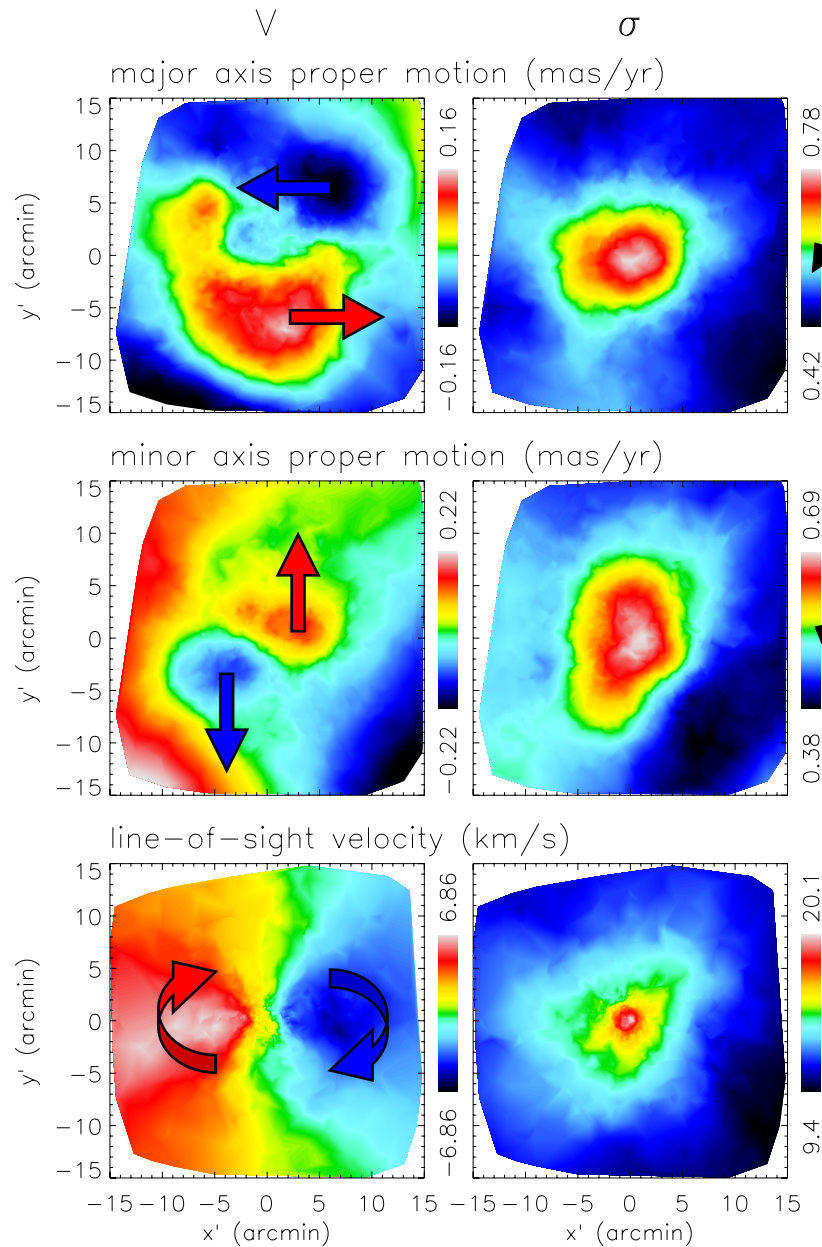
l.o.s. velocities  $\sim 4000$  (4 data-sets)



Loke Kun Tan (StarryScapes)

Glenn van de Ven, "Schwarzschild models GCs"

# Smooth velocity and dispersion fields

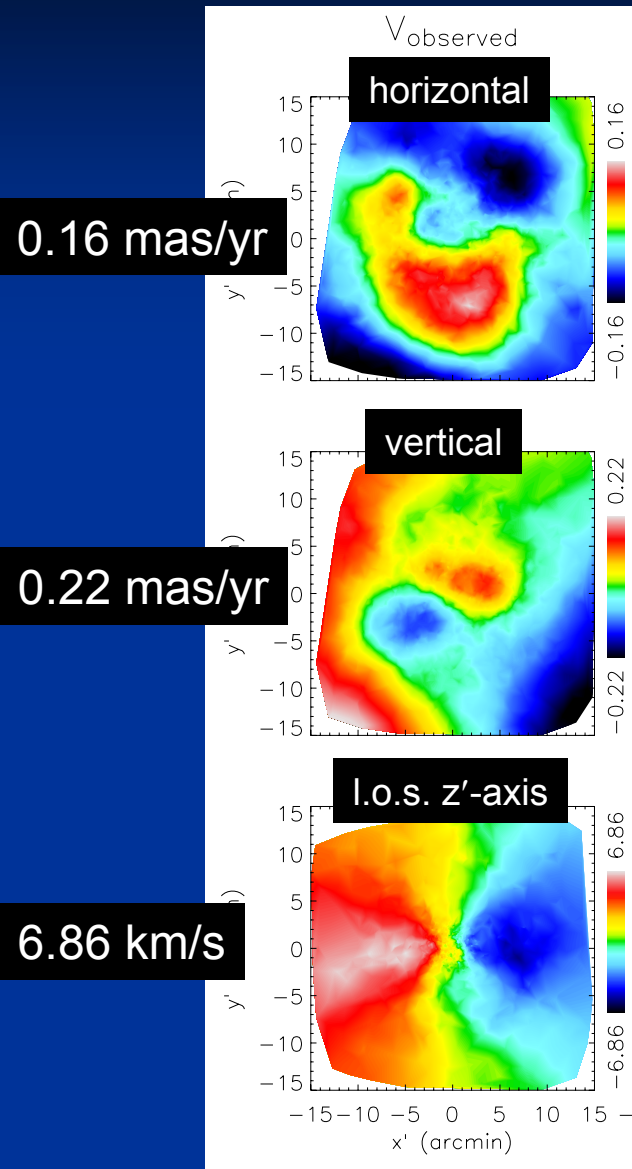


horizontal: in direction of major  $x'$ -axis

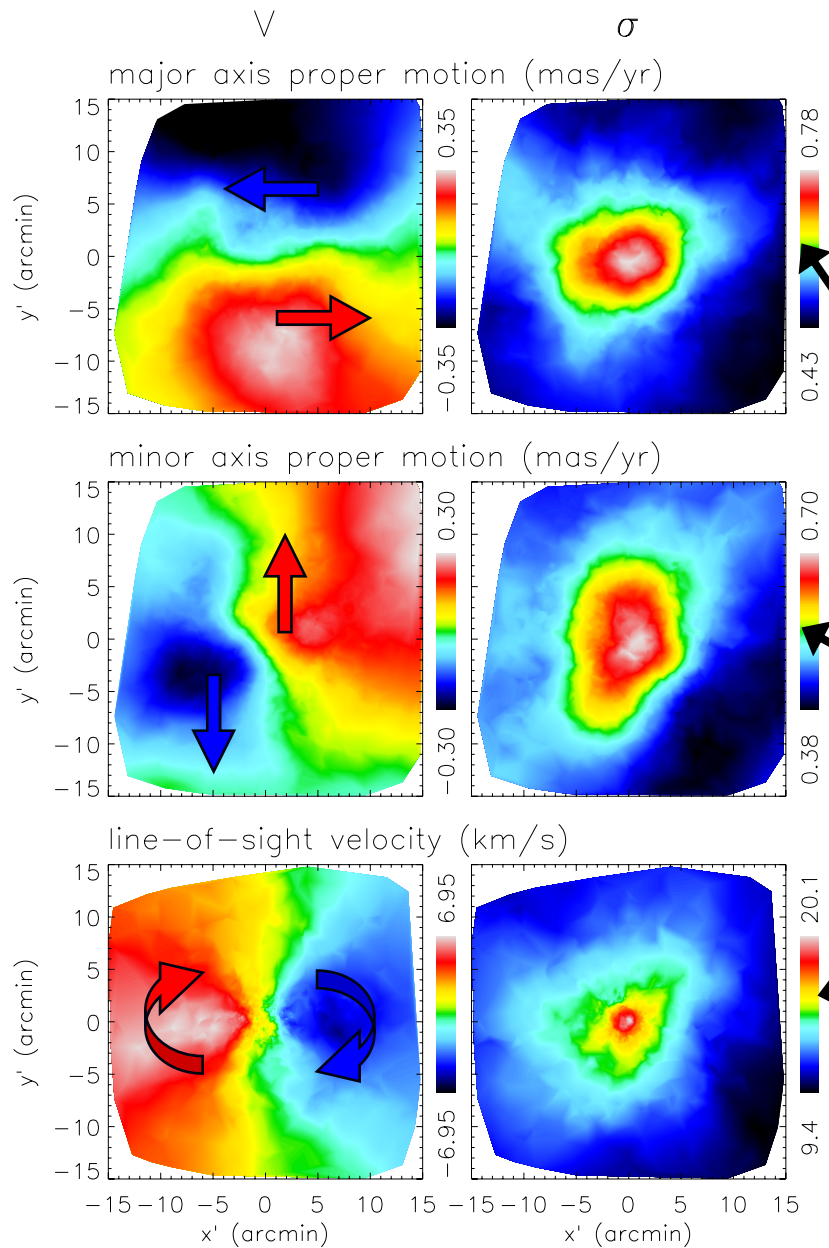
vertical: in direction of minor  $y'$ -axis

in direction of l.o.s.  $z'$ -axis

# Corrected velocity field



# Smooth velocity and dispersion fields



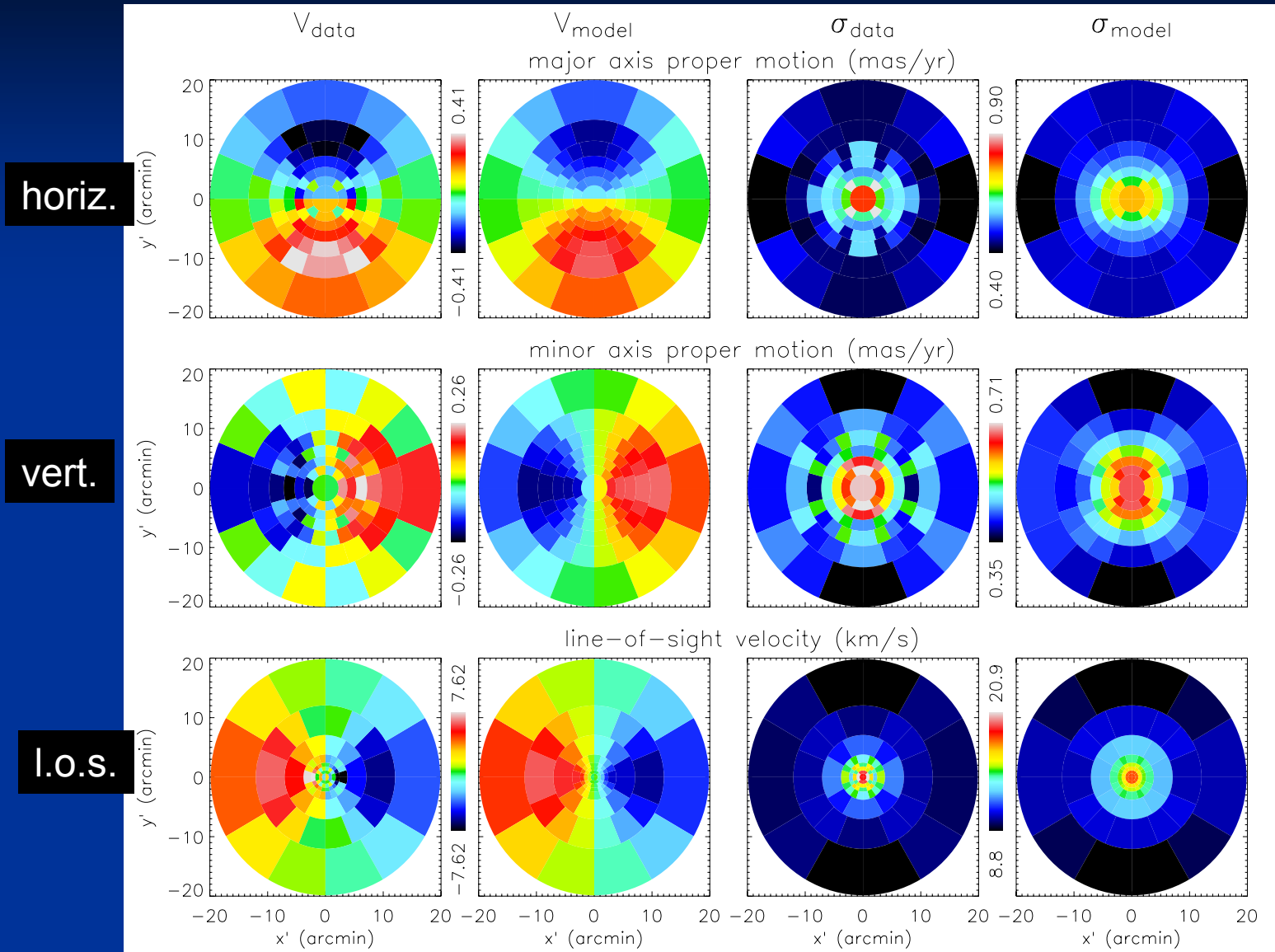
horizontal: in direction of major axis

vertical: in direction of minor axis

in direction of line-of-sight axis

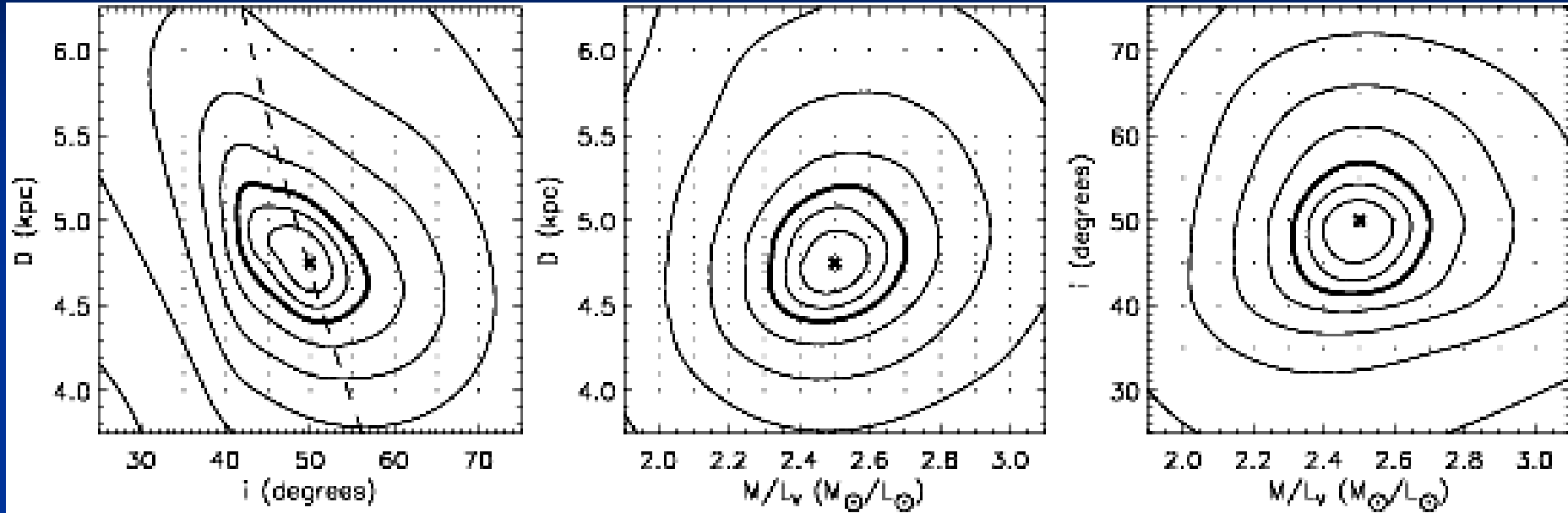


# Schwarzschild model



1 arcmin  $\sim$  1.45 pc

# Best-fit parameters



$$D = 4.8 \pm 0.3 \text{ kpc,}$$

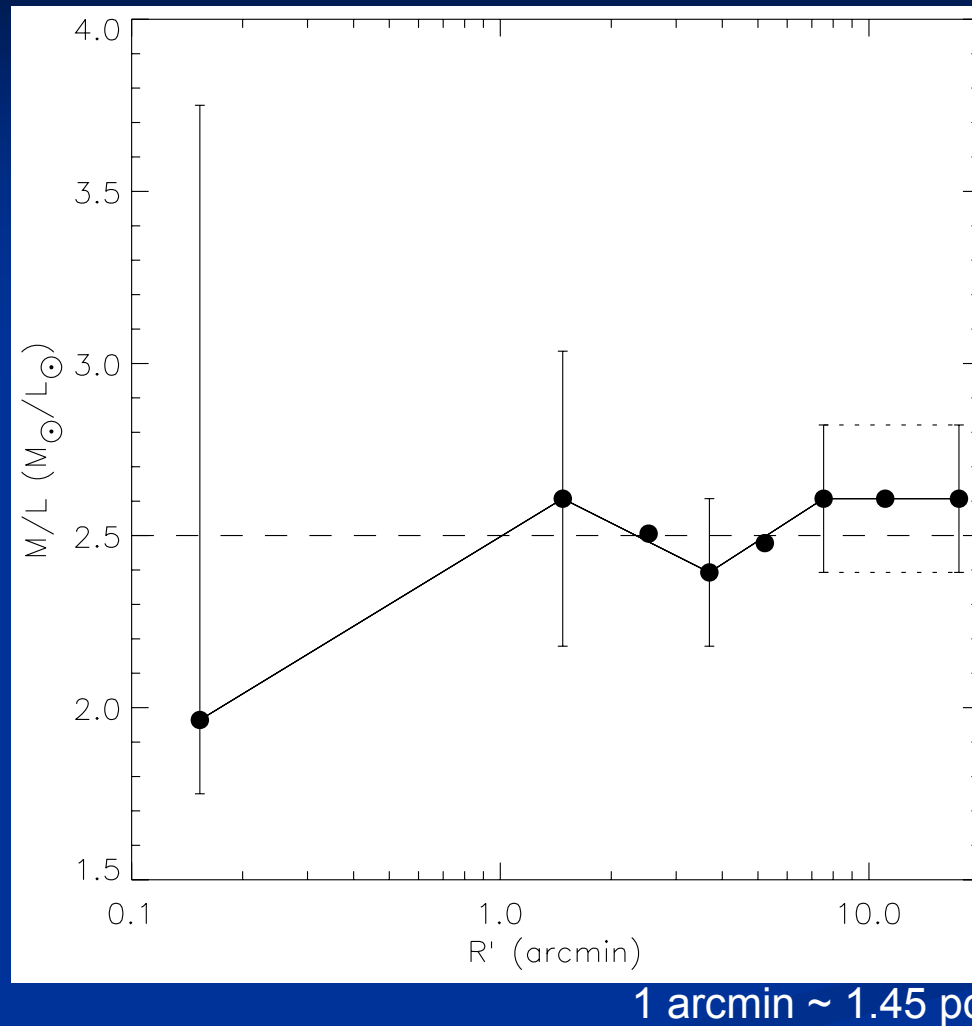
$$i = 50^{\circ} \pm 4^{\circ}$$

$$M/L_V = 2.5 \pm 0.1 \text{ } M_{\odot}/L_{\odot}$$

$$L_V = 1.0 \pm 0.1 \times 10^6 L_{\odot}$$

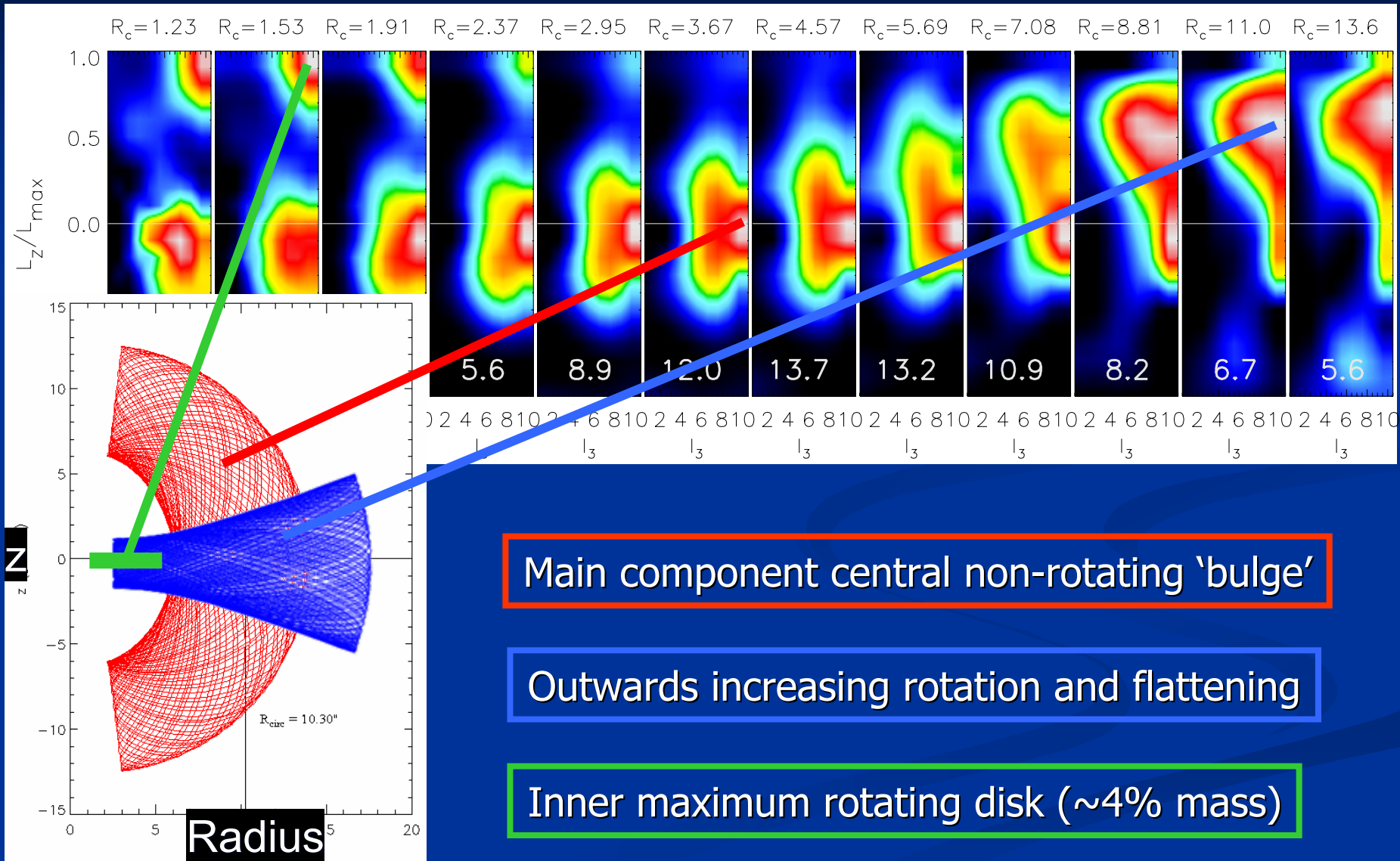
$$M = 2.5 \pm 0.3 \times 10^6 M_{\odot}$$

# M/L variation with radius?



Consistent with constant  $M/L_V = 2.5 \pm 0.1 M_{\odot}/L_{\odot}$

# Phase-space distribution



# Tidal interaction and multiple populations

- $\omega$  Centauri mean rotation ( $L_z > 0$ ) and orbit around Milky Way center opposite  $\rightarrow$  prograde orbits ( $L_z < 0$ ) tidally removed?
- Impulse approximation:  $|\Delta v| \sim \sigma$  around 16 arcmin (tidal radius around 45 arcmin) Dinescu et al. (1999)
- Multiple stellar populations (Freeman & Rodgers 1975, Norris et al. 1997, Pancino et al. 2003, ...)

Metal-rich  $[Ca/H] > -1.2$

- Centrally concentrated
- No apparent rotation
- Nearly round



Non-rotating 'bulge' ?

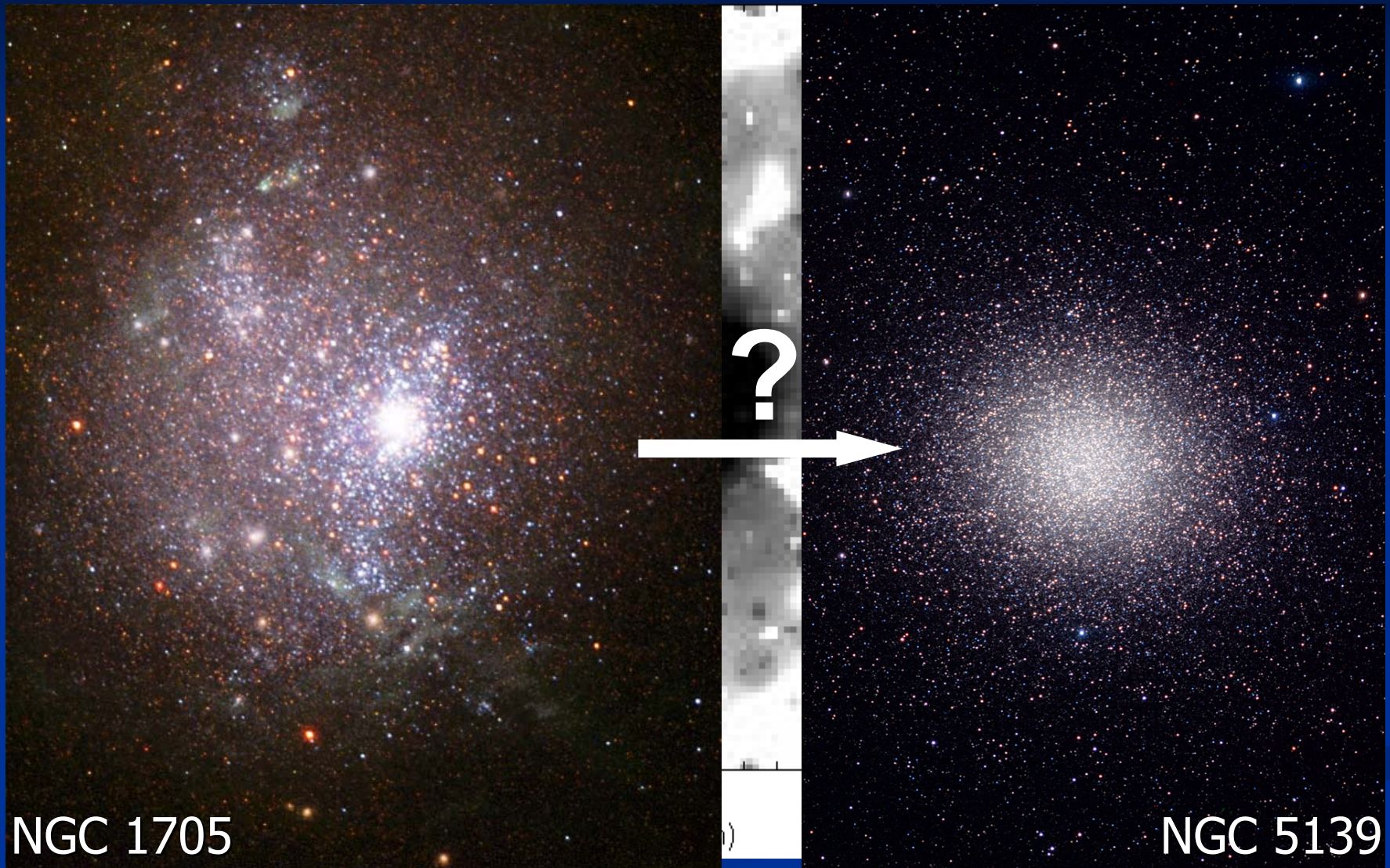
Metal-poor  $[Ca/H] < -1.2$

- Throughout galaxy
- Rapidly rotating
- Flattened



Rotating flattened component?

# Tidally stripped dwarf galaxy?



NGC 1705

NGC 5139

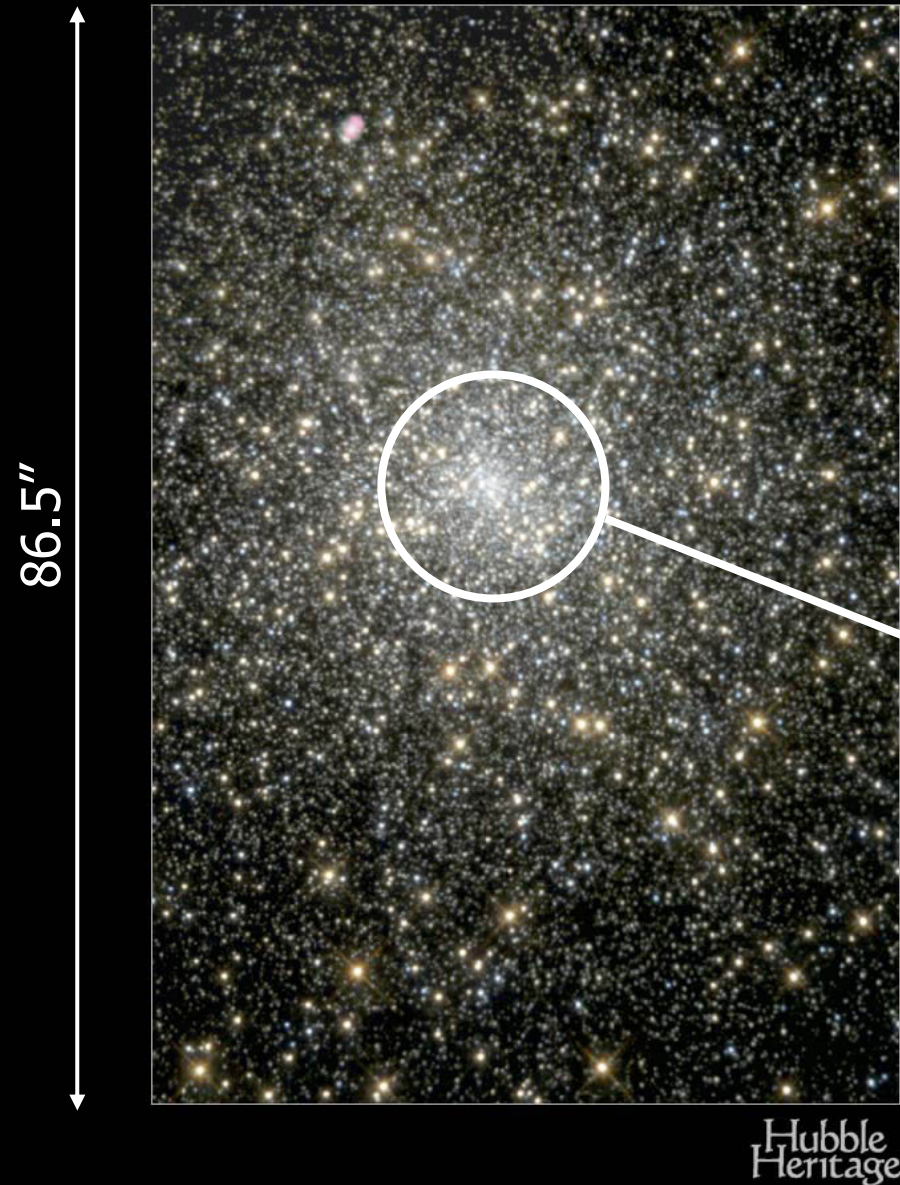
Leon, Meylan & Combes (2000)

# Globular clusters: M15

- mass-to-light ratio
- IMBH or dark remnants?

van den Bosch et al. (2006,ApJ,641,852)

Globular Cluster M15



# M15 (NGC 7078)

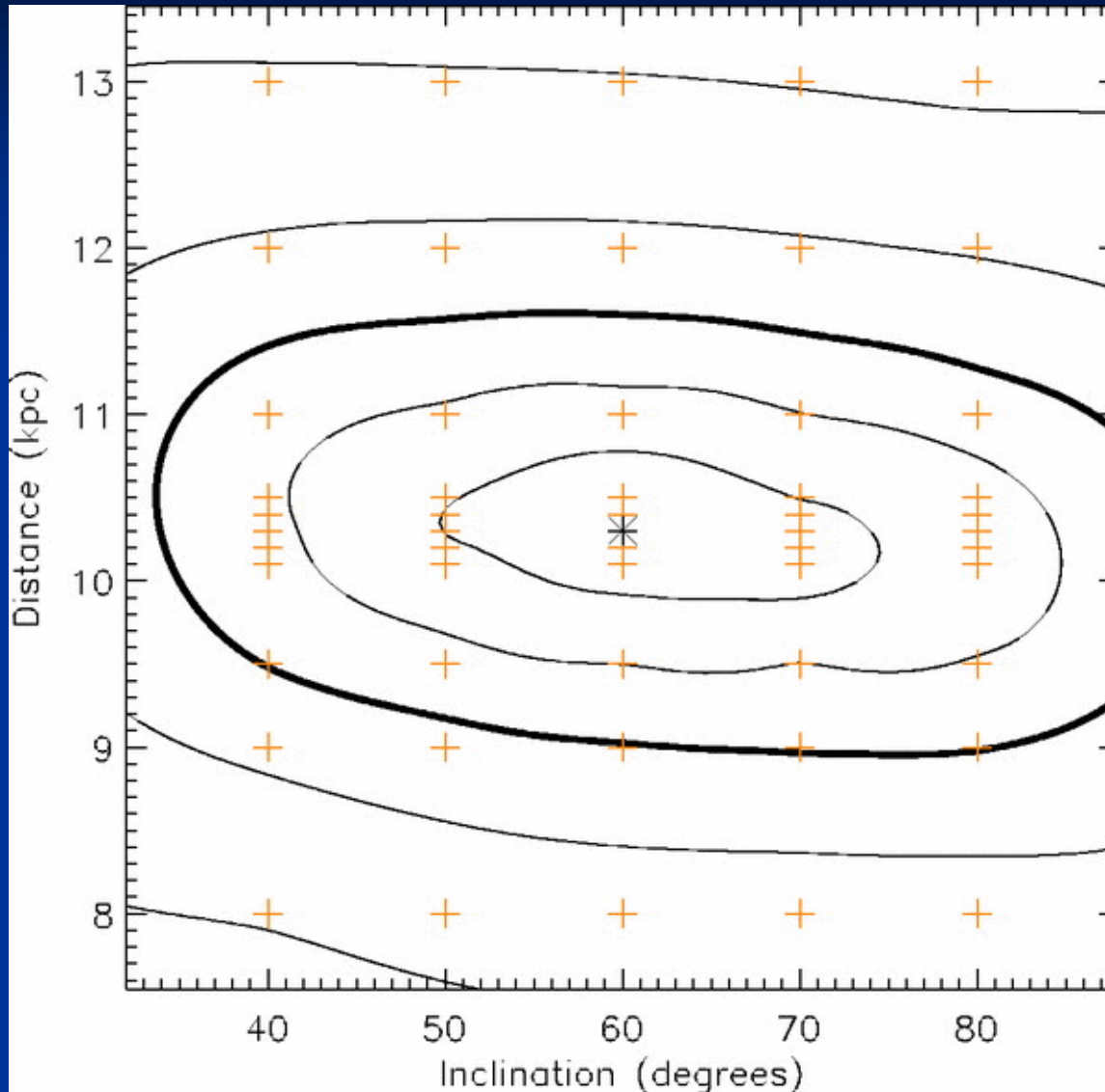
- $D=10$  kpc ( $1''=0.05$  pc)
- SB profile:  $r_c=0.05$  pc, central slope= $-0.62\pm 0.06$  (Noyola & Gebhardt 2006, AJ, 132, 447)
- 1540 l.o.s. velocities (Gebhardt et al. 2009, AJ, 119, 1268)
- 703 HST proper motions (McNamara et al. 2003, ApJ, 595, 187)

Hubble  
Heritage

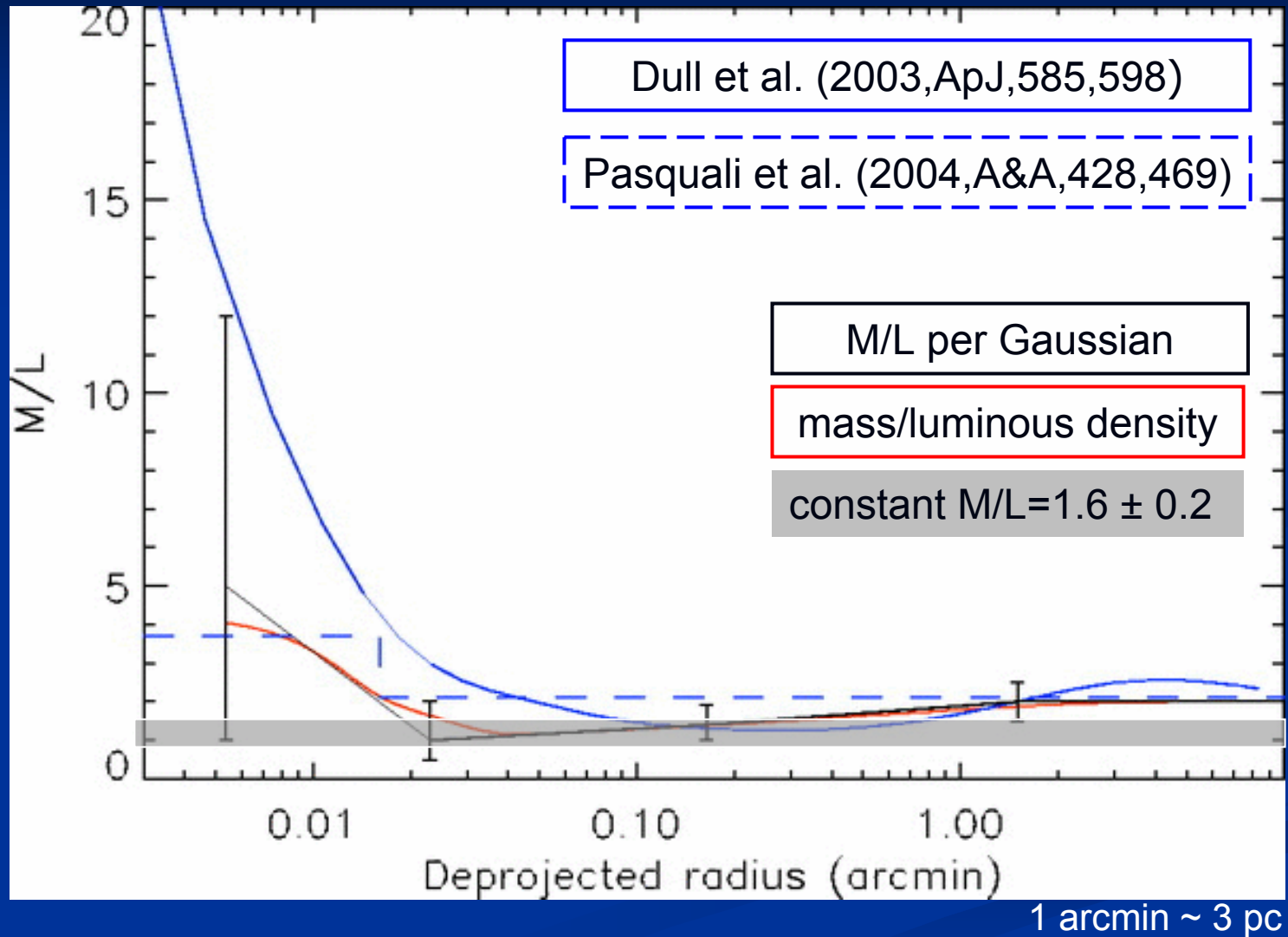
NASA and the Hubble Heritage Team (STScI/AURA · Hubble Space Telescope WFPC2 · STScI-PRC00-25)



# Distance and inclination

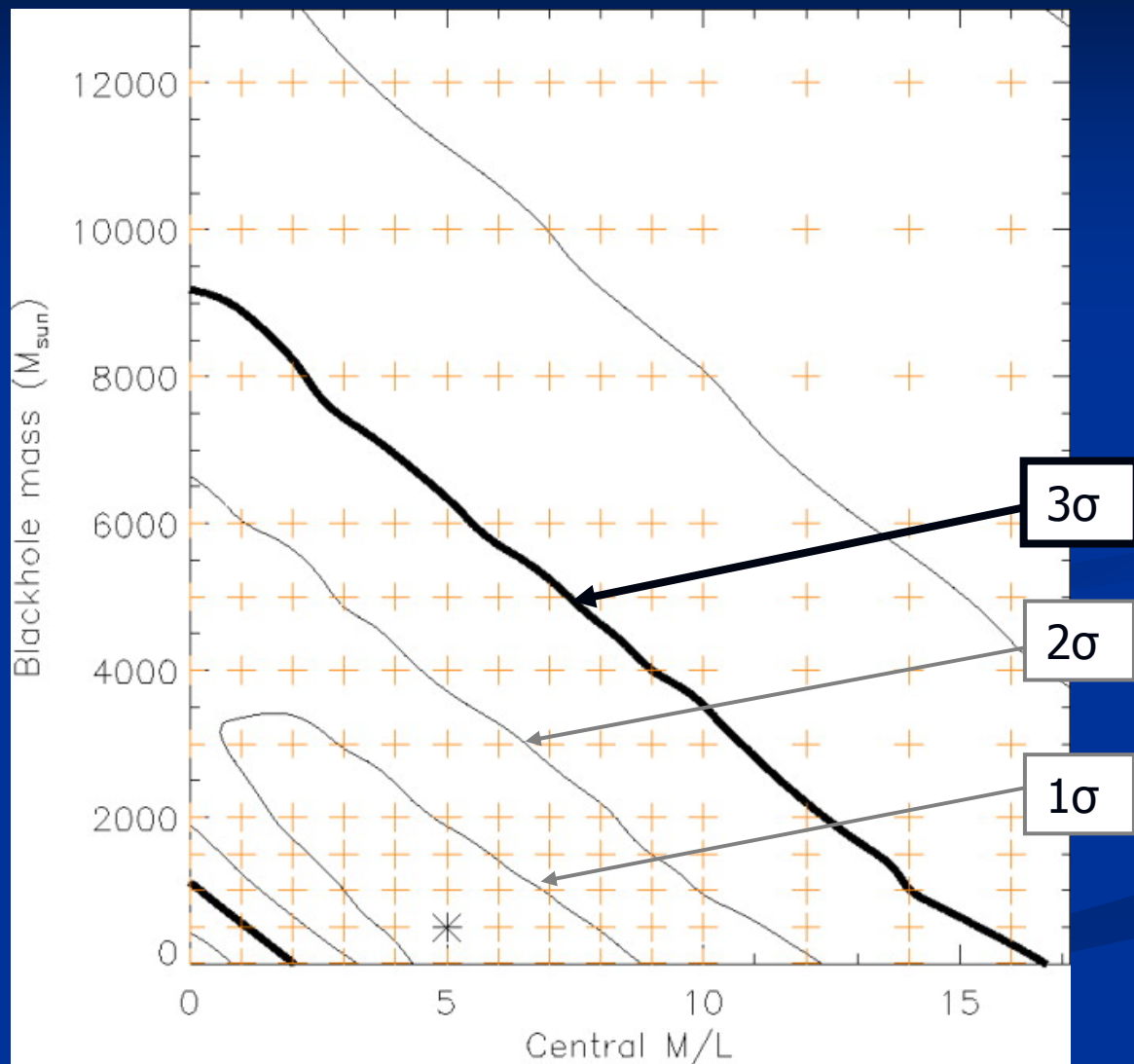


# M/L variation with radius



# IMBH or dark remnants?

- $D = 10.3 \pm 0.05$  kpc
- $i = 60 \pm 15$  degrees
- From  $M_{\text{BH}}-\sigma$  relation:  
 $\sim 10^3 M_{\odot}$  ( $\sigma \sim 11$  km/s)
- $r_{\text{BH}} \sim 0.5'' \sim 0.025$  pc
- Within  $r_c = 0.05$  pc  
 $M_c = 3.4 \times 10^3 M_{\odot}$   
 $\rho_c = 7.4 \times 10^6 M_{\odot}/\text{pc}^3$
- $M_{\text{tot}} = 4.4 \times 10^5 M_{\odot}$

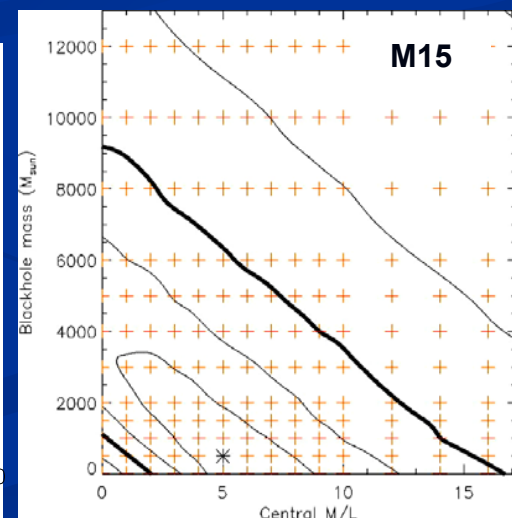
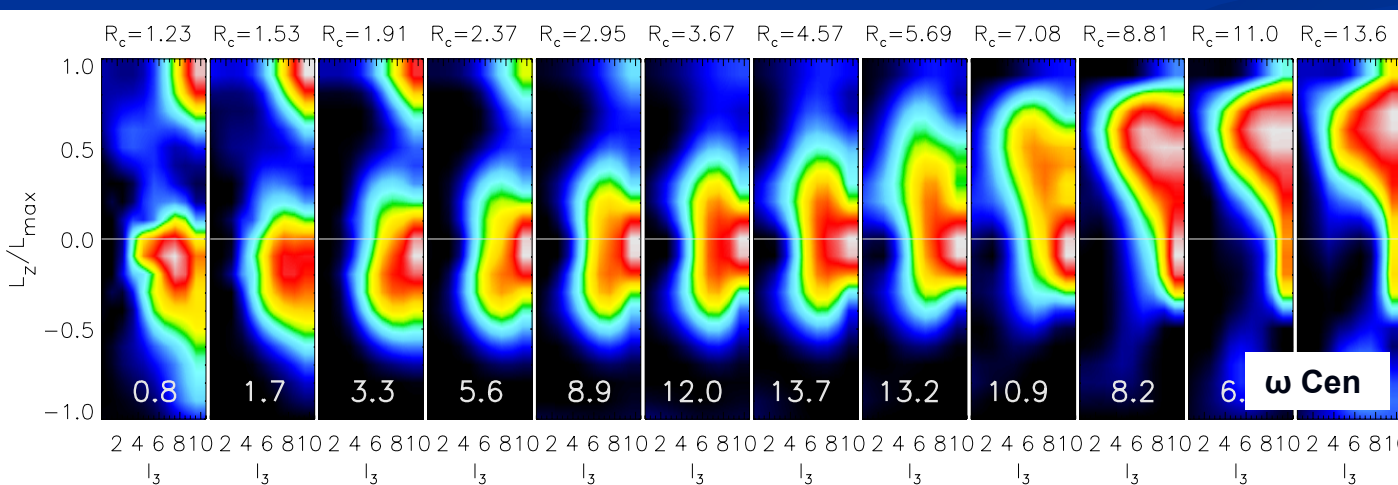
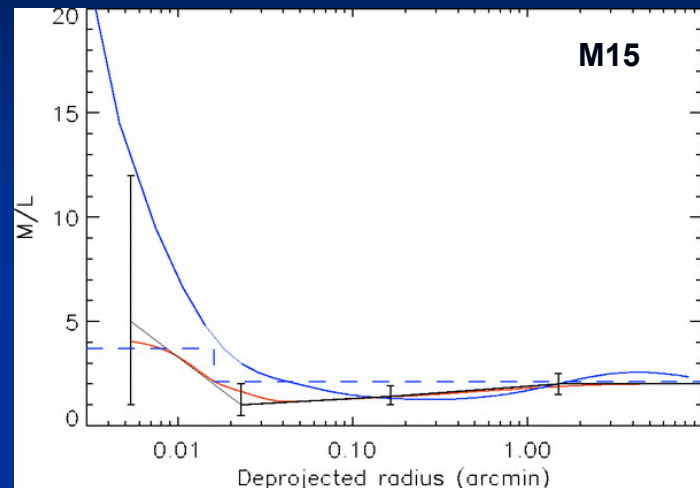
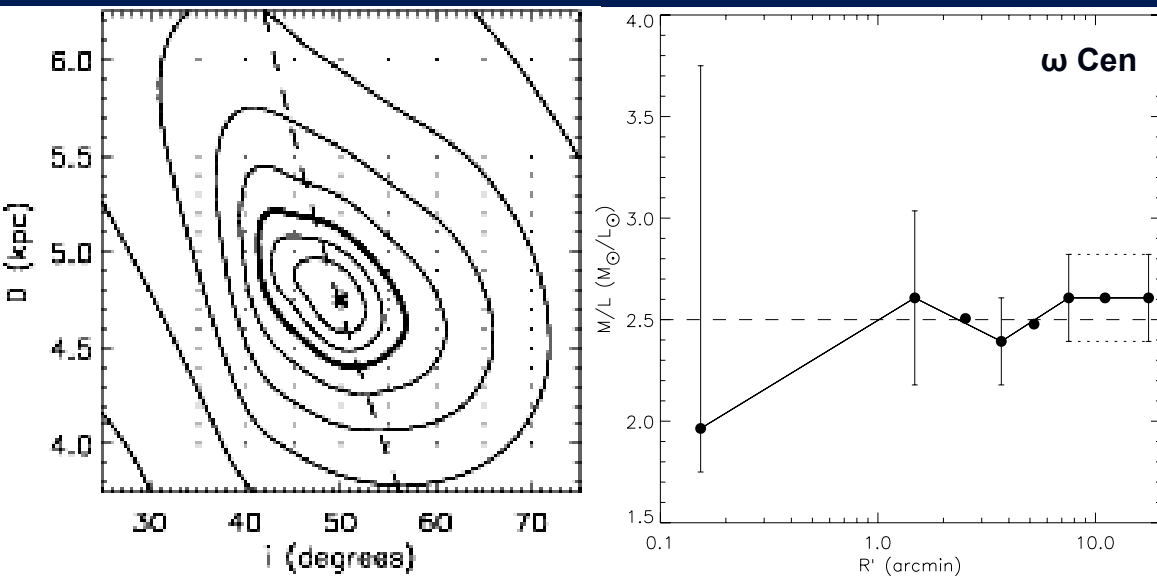


# Outlook and summary

# Next steps

- HST proper motions: IMBH?
- Correlated and higher order velocity moments, or...
- Fitting directly discrete kinematics with Max. Likelihood methods
- Including color, metallicity and age indicators, etc.:  
link kinematics and stellar properties in single model
- Synergy with particle-based models?

# Summary in figures



...the end





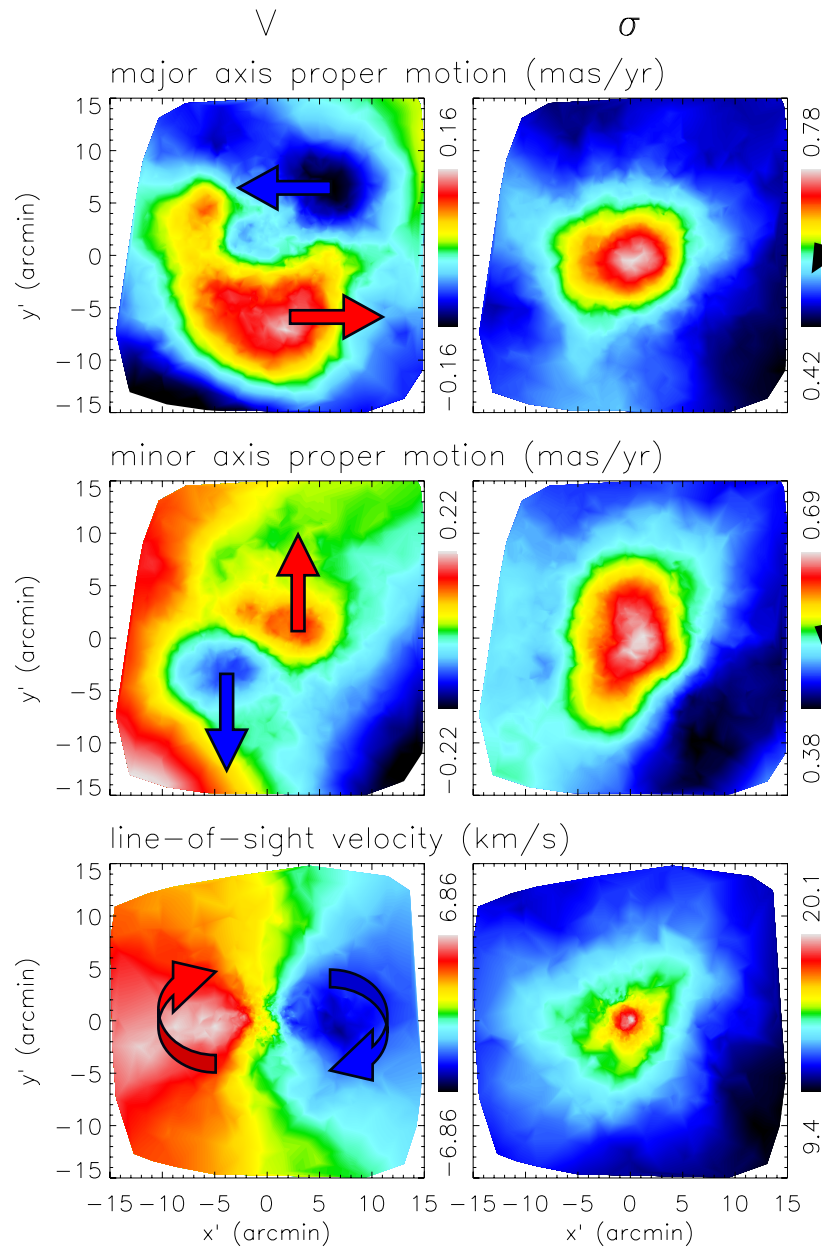
# Extra: $\omega$ Cen

# Some properties

- Most massive GC in Milky Way:  $M \sim 2-5 \times 10^6 M_{\odot}$
- One of most flattened GCs:  $q' \sim 0.9$
- Relatively loosely bound:  
tidal radius  $r_t \sim 45'$ , core radius  $r_c \sim 2.6'$   $\log(r_t/r_c) \sim 1.24$
- Small heliocentric distance:  $D \sim 5$  kpc  
RR Lyrae and eclipsing binary
- Complicated composition with multiple stellar populations
  - Self-enrichment (isolated cluster/nucleus dwarf galaxy)?
  - (Subsequent) interaction/merger GCs?

F. van Leeuwen, G. Piotto & J. Huyghes, 2001, ASP Conf. Ser. 265

# Smooth velocity and dispersion fields

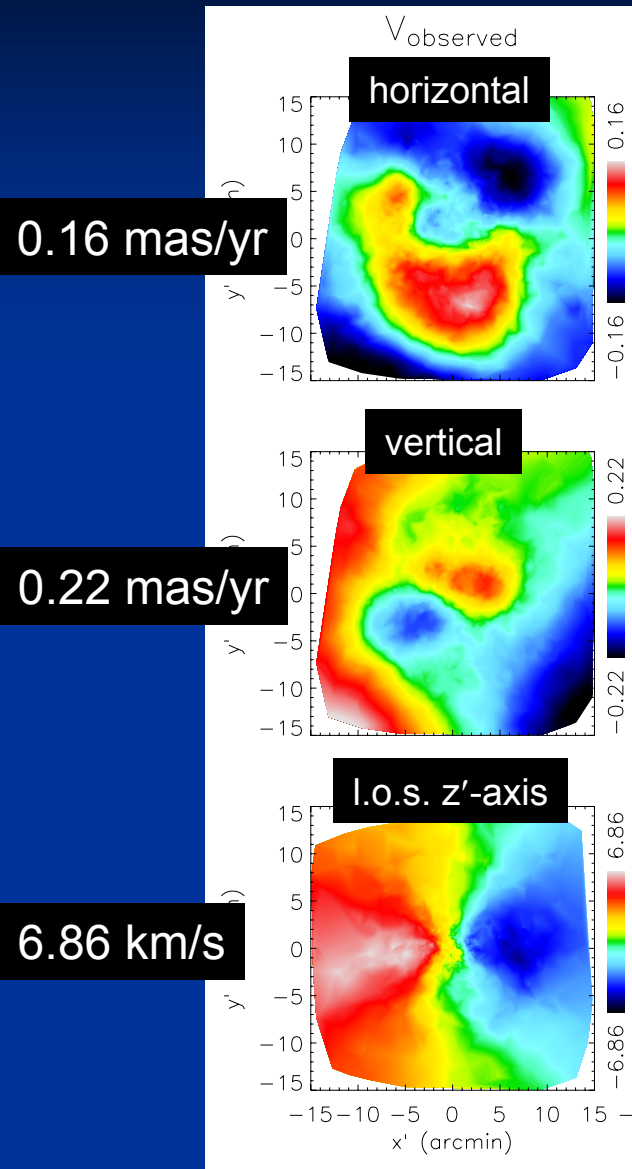


horizontal: in direction of major  $x'$ -axis

vertical: in direction of minor  $y'$ -axis

in direction of l.o.s.  $z'$ -axis

# Corrected velocity field



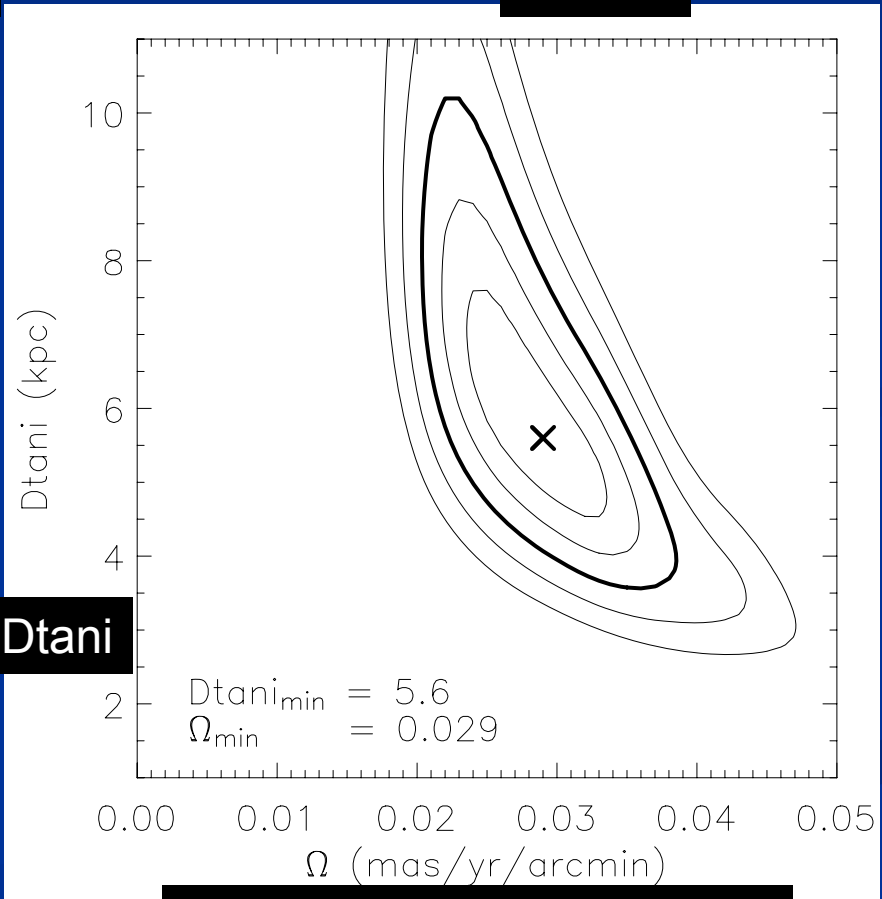
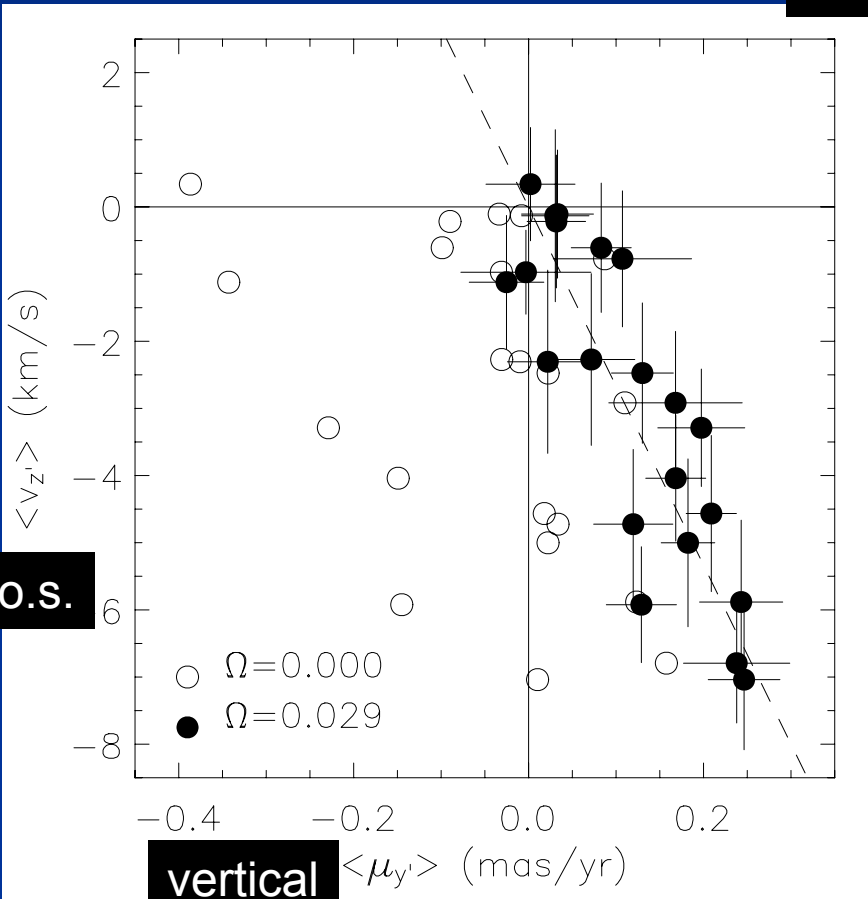
# Measuring 'solid-body' rotation

Any axisymmetric object:

$$\langle v_{z'} \rangle(x', y') = -4.74 D \tan i \langle \mu_{y'} \rangle(x', y')$$

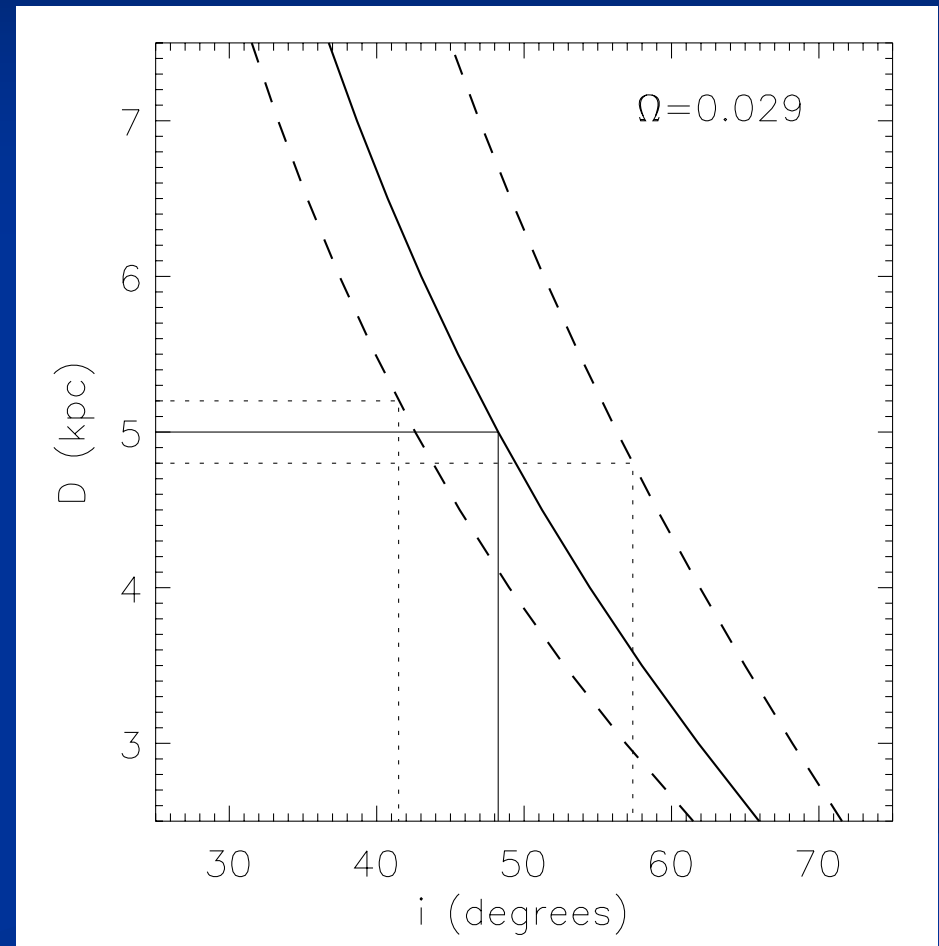
I.O.S.

vertical



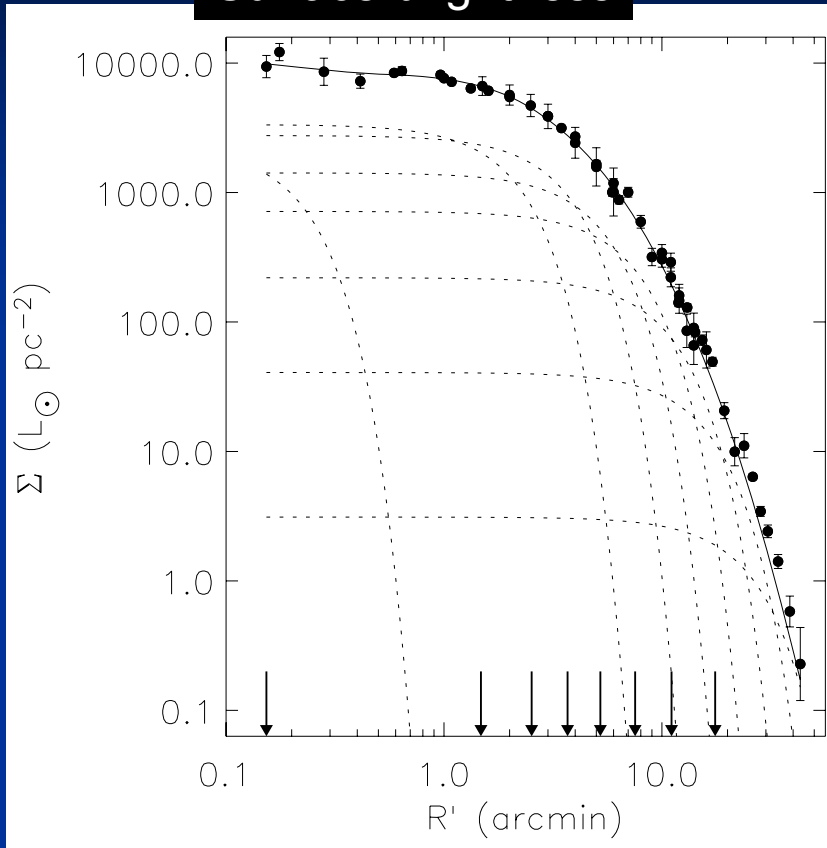
# Constraint on inclination

- Best-fit solid-body rotation  
 $sbr = 0.029 \text{ mas/yr/arcmin}$
- canonical  $D = 5.0 \pm 0.2 \text{ kpc}$   
inclination  $i = 41 - 57^\circ$
- flattening:  
observed  $q' = 0.88 \pm 0.01$   
intrinsic  $q = 0.78 \pm 0.03$

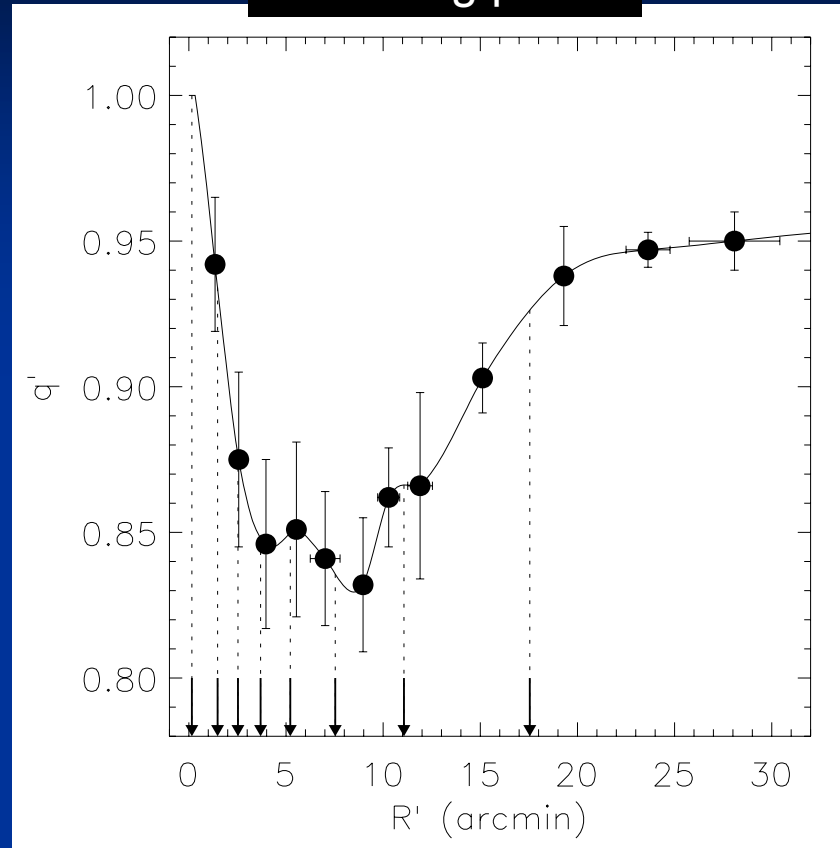


# MGE mass model

## Surface brightness



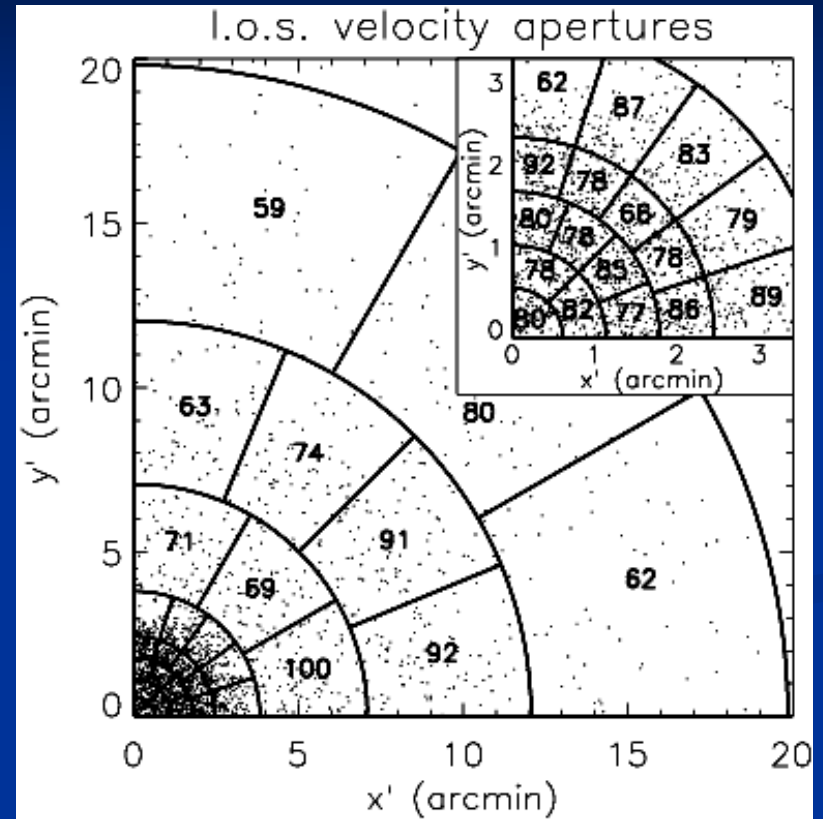
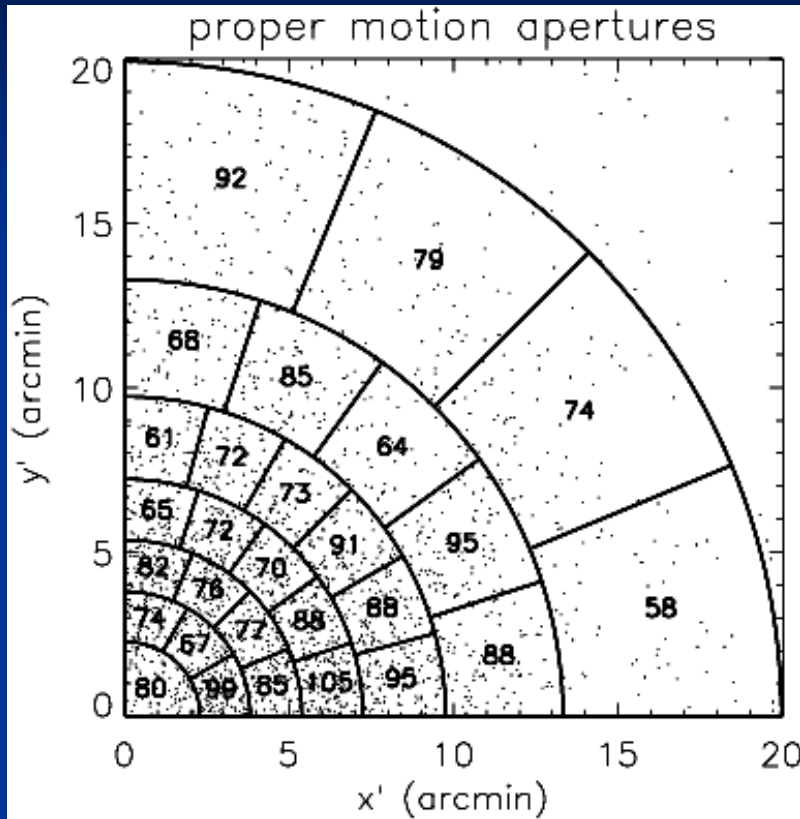
## Flattening profile



1 arcmin  $\sim$  1.45 pc

- 1D: 8 Gaussians  $\rightarrow$  2D: flattening profile Geyer et al. (1983)
- $E(B-V) = 0.11$ ,  $D = 5.0 \pm 0.2$  kpc:  $L_V \sim 1.0 \pm 0.1 \times 10^6 L_{\odot}$

# Polar grid of apertures



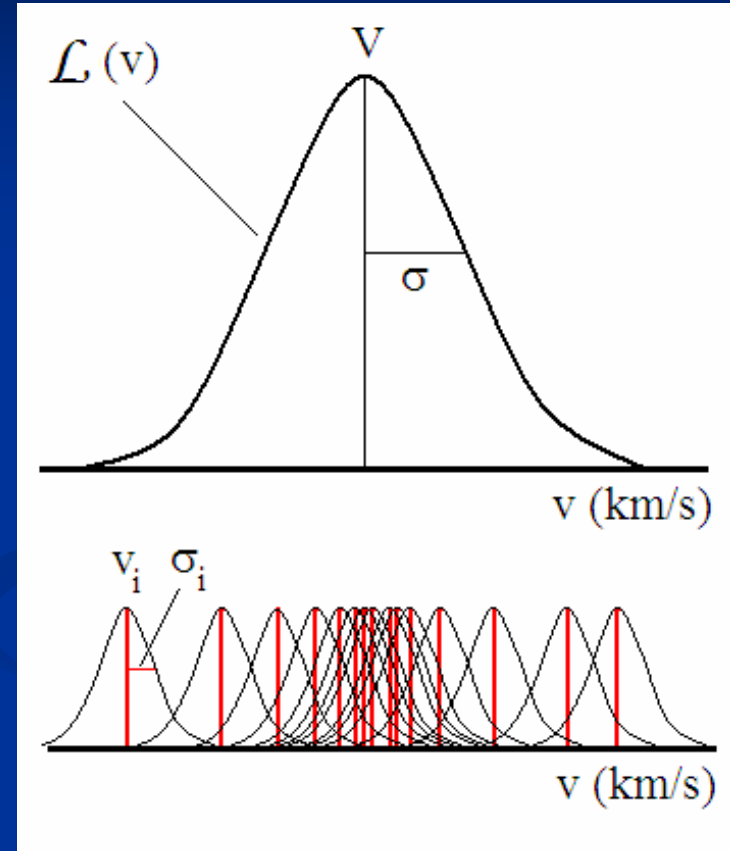
Reflected to first quadrant, around 80 stars per aperture

- Proper motions: 28 apertures, total 2295 stars
- L.o.s. velocities: 27 apertures, total 2223 stars



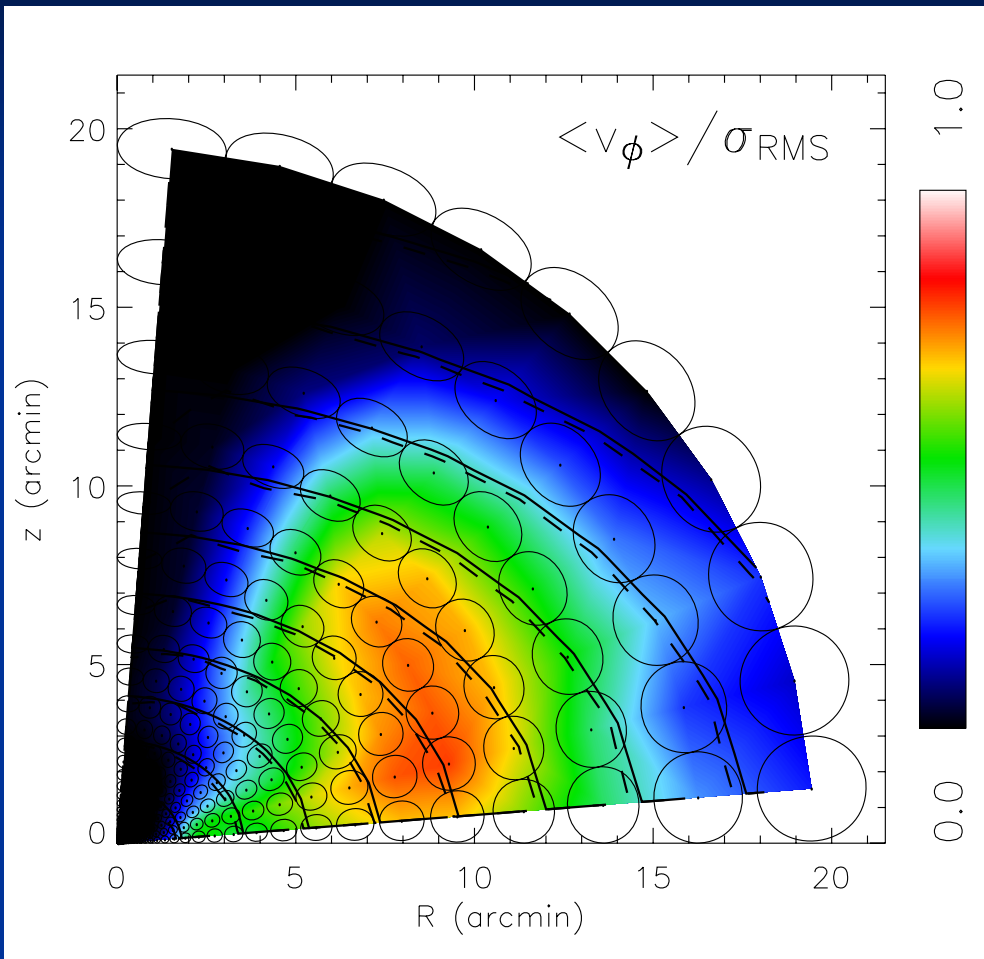
# Averaged kinematics

- Fitting to average kinematics ( $V, \sigma, \dots$ ) of stars within aperture:
  - Linear method ensures global best-fit
  - Faster than using discrete velocities
- How to extract velocity moments?
  - Gaussian fit to velocity histograms
  - Instrumental dispersion:  $\sigma_{\text{fit}}^2 = \sigma^2 + \sigma_{\text{ins}}^2$
  - Maximum likelihood estimation:



$$L(V, \sigma, \dots) = \prod_{i=1}^n \int_{-\infty}^{\infty} \mathcal{L}(v) \frac{e^{-\frac{1}{2} \left( \frac{v_i - v}{\sigma_i} \right)^2}}{\sqrt{2\pi} \sigma_i} dv$$

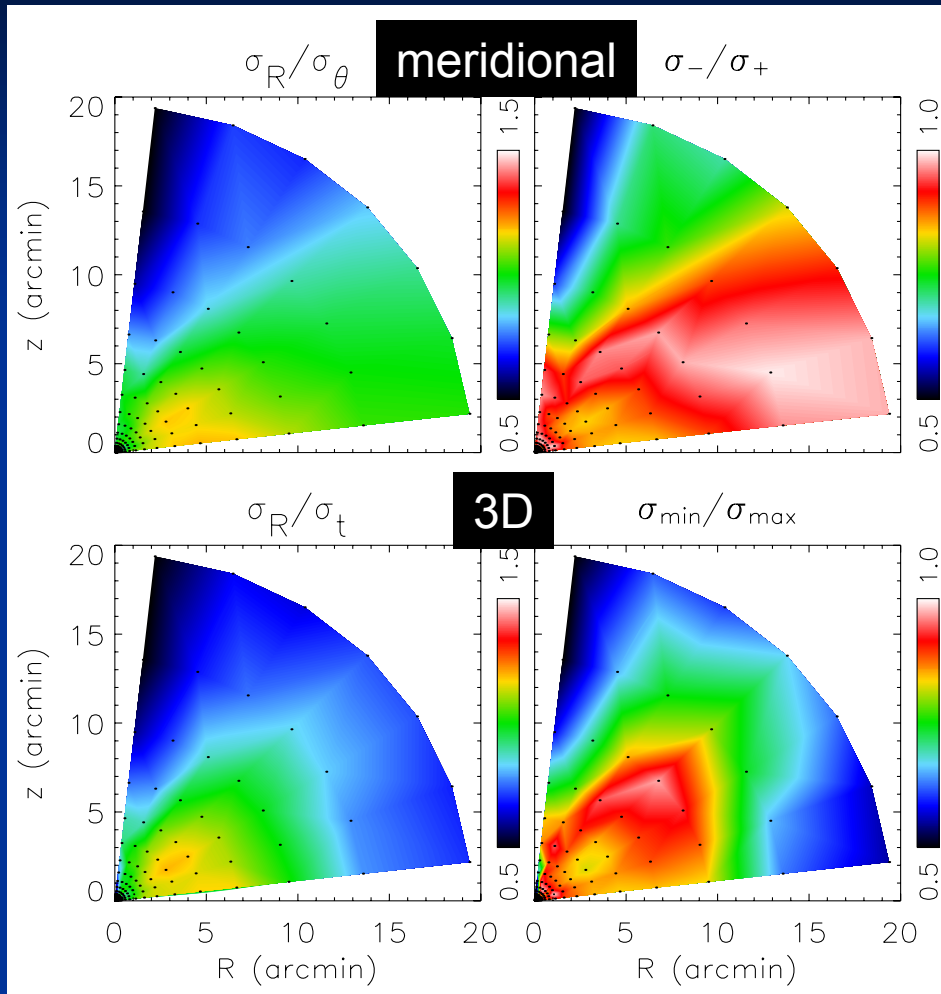
# Rotational/pressure supported



- max  $V/\sigma$  at  $R \sim 8$  arcmin  
~ at maximum  $v_{los}$
- $V/\sigma > 0.5$  above isotropic oblate rotator in  $(V/\sigma, \epsilon)$   
~ rotational support
- Outwards (partly)  
pressure supported

$$\sigma_{RMS}^2 = (\sigma_R^2 + \sigma_\theta^2 + \sigma_\phi^2)/3$$

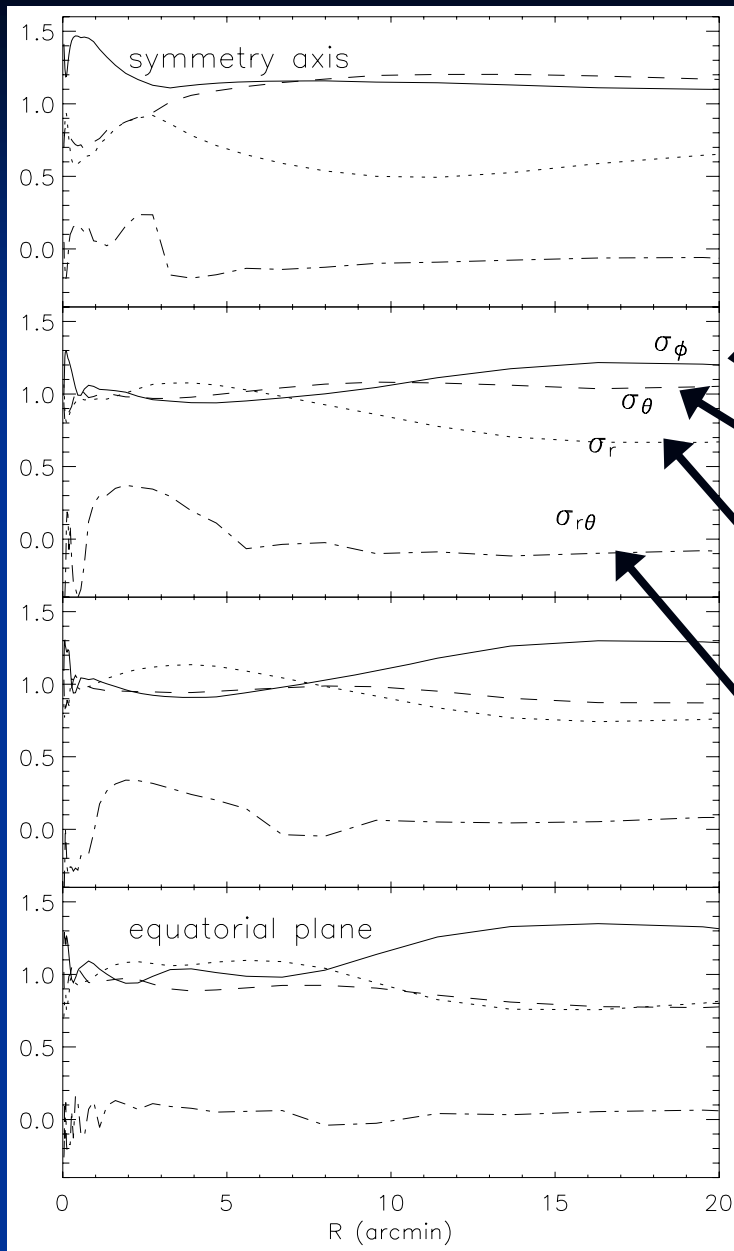
# Anisotropy



- Principal axis velocity ellipsoid  $\sigma_+$ ,  $\sigma_-$ , and  $\sigma_\phi$
- In meridional plane:
  - Almost isotropic near equatorial plane
  - Tangential anisotropic towards symmetry axis
- 3D (including azimuthal)
  - Radial anisotropic center
  - Tangential anisotropic in outer parts
- Not two-integral  $F(E, L_z)$

$$\sigma_t^2 = (\sigma_\theta^2 + \sigma_\phi^2)/2$$

# Intrinsic velocity moments



$$\sigma_{\phi}^2 = \langle v_{\phi}^2 \rangle - \langle v_{\phi} \rangle^2$$

$$\sigma_{\theta}^2 = \langle v_{\theta}^2 \rangle$$

$$\sigma_R^2 = \langle v_R^2 \rangle$$

$$\sigma_{R\theta}^2 = \langle v_R v_{\theta} \rangle$$

Axisymmetric:

$$\langle v_R \rangle = \langle v_{\theta} \rangle = \langle v_R v_{\phi} \rangle = \langle v_{\theta} v_{\phi} \rangle = 0$$

# Conclusions $\omega$ Centauri

- Significant perspective and residual solid-body rotation
- Amount solid-body rotation and  $D \cdot \tan(i)$  directly from data
- Axisymmetric anisotropic Schwarzschild model:  
 $D = 4.8 \pm 0.3$  kpc,  $M/L_V = 2.5 \pm 0.1 M_{\odot}/L_{\odot}$ ,  $M = 2.5 \pm 0.3 \times 10^6 M_{\odot}$
- Substructure in distribution function:
  - Main component center non-rotating 'bulge'
  - Outwards increasing rotation and flattening
  - Inner ( $\sim 1-3$  arcmin) maximum rotating disk  $\sim 4\%$  mass

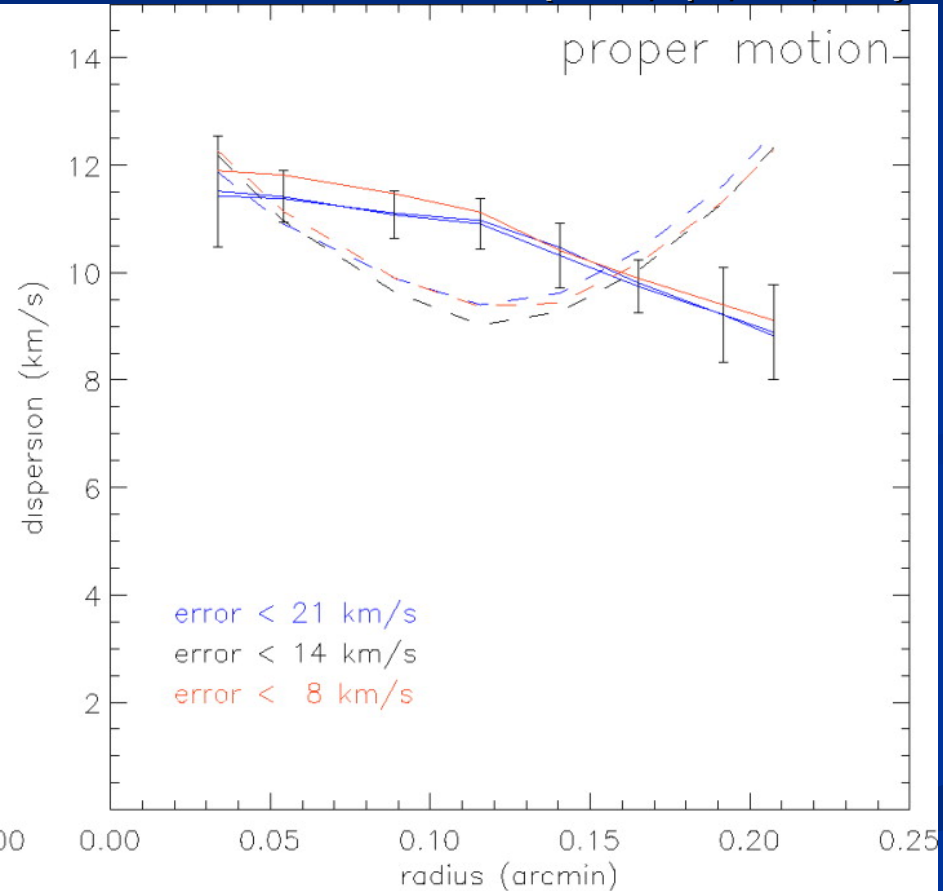
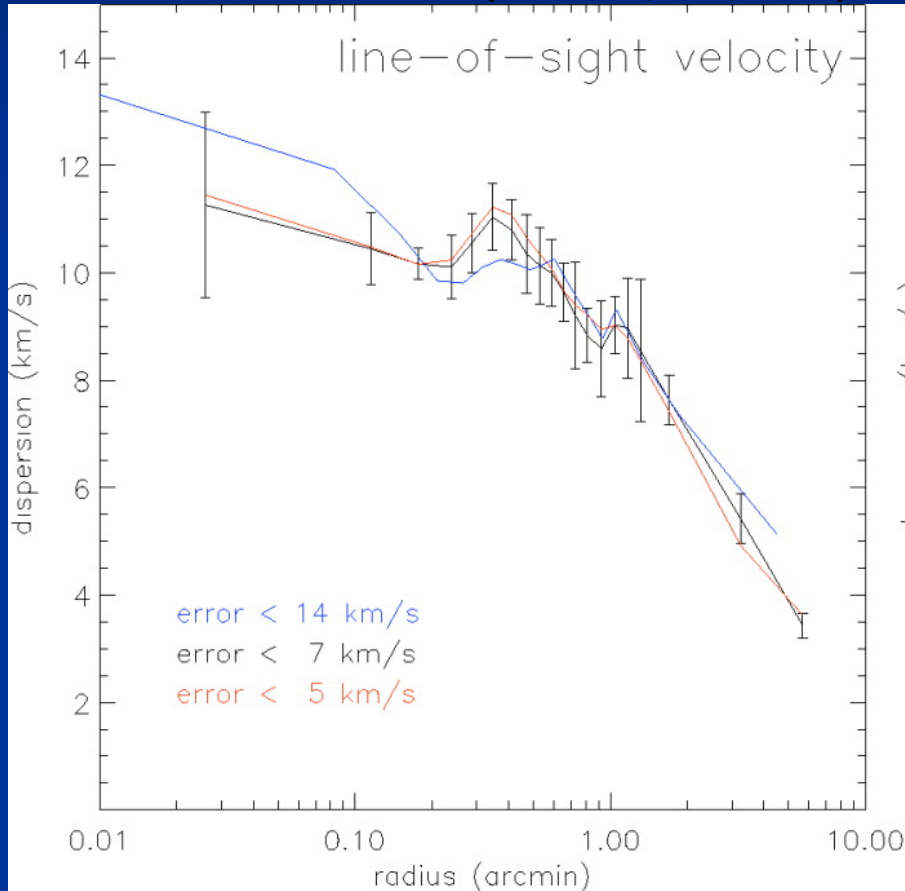
... linked with multiple stellar populations?  
... tidally stripped dwarf galaxy?

# Extra: M15

# Dispersion profiles

#1773 Gebhardt et al. (2009, AJ, 119, 1268)

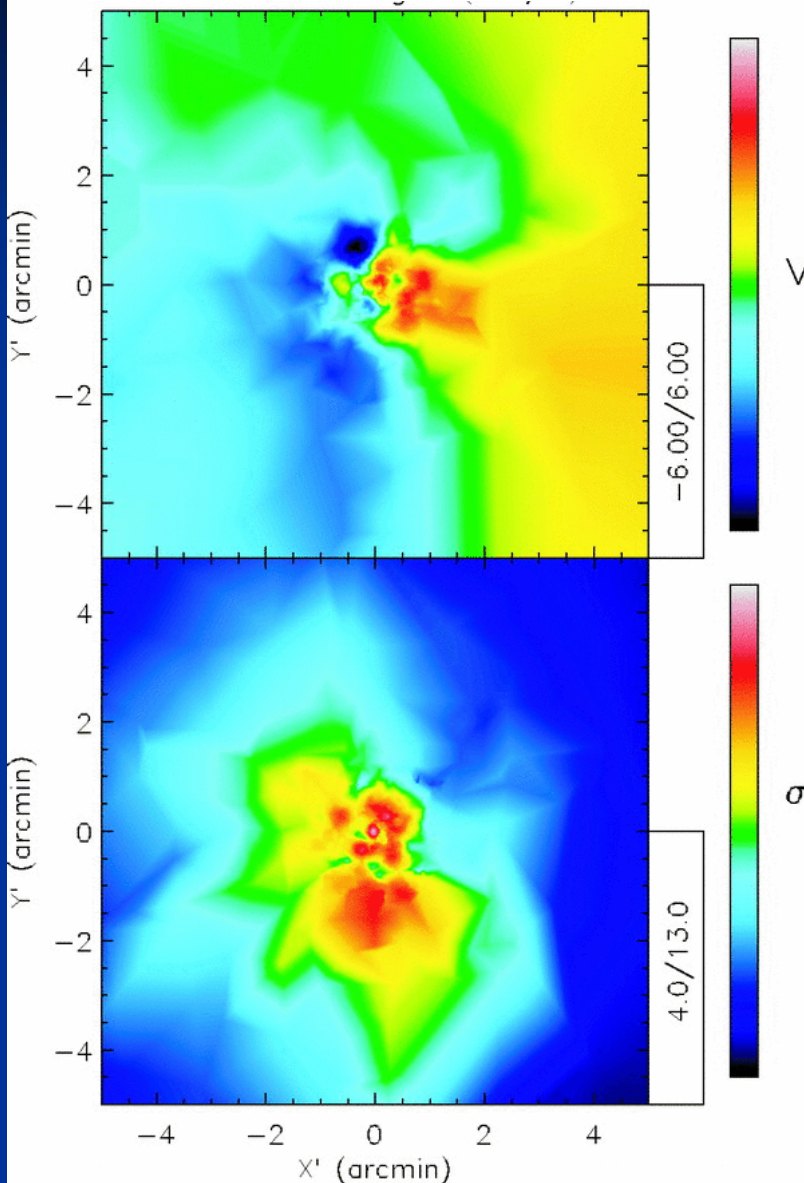
#703 McNamara et al. (2003, ApJ, 595, 187)



1 arcmin  $\sim$  3 pc

# 1540 I.o.s. velocities

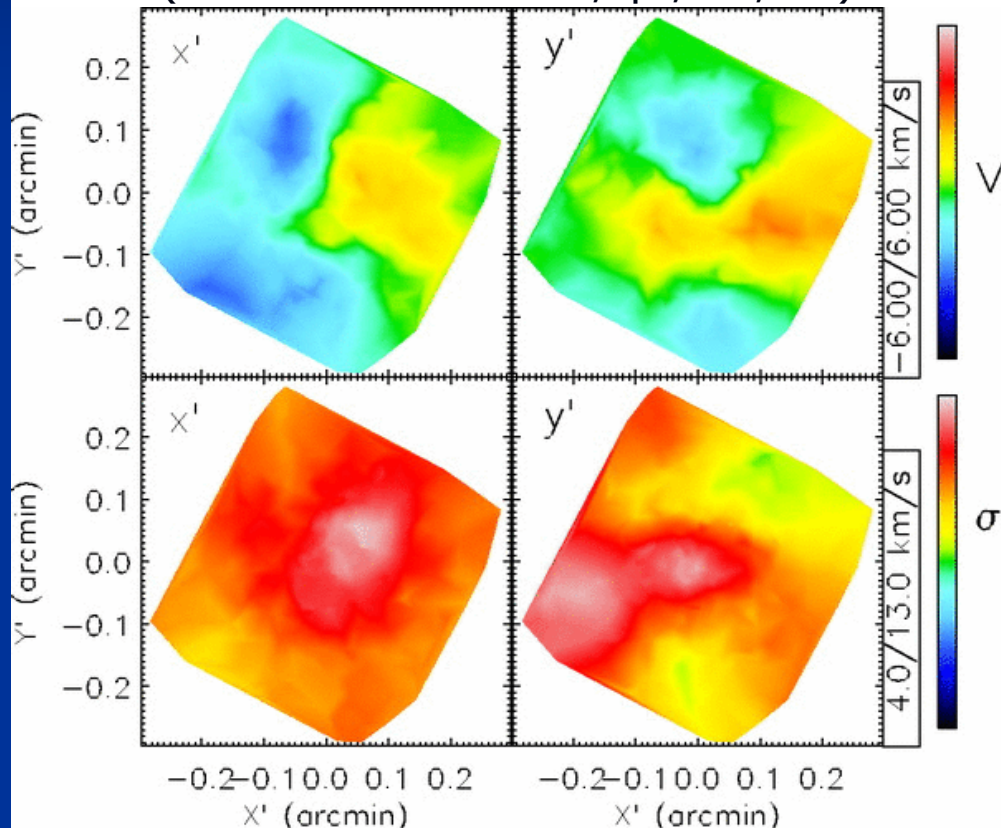
(Gebhardt et al. 2009, AJ, 119, 1268)



# Kinematic maps

## 703 HST proper motions

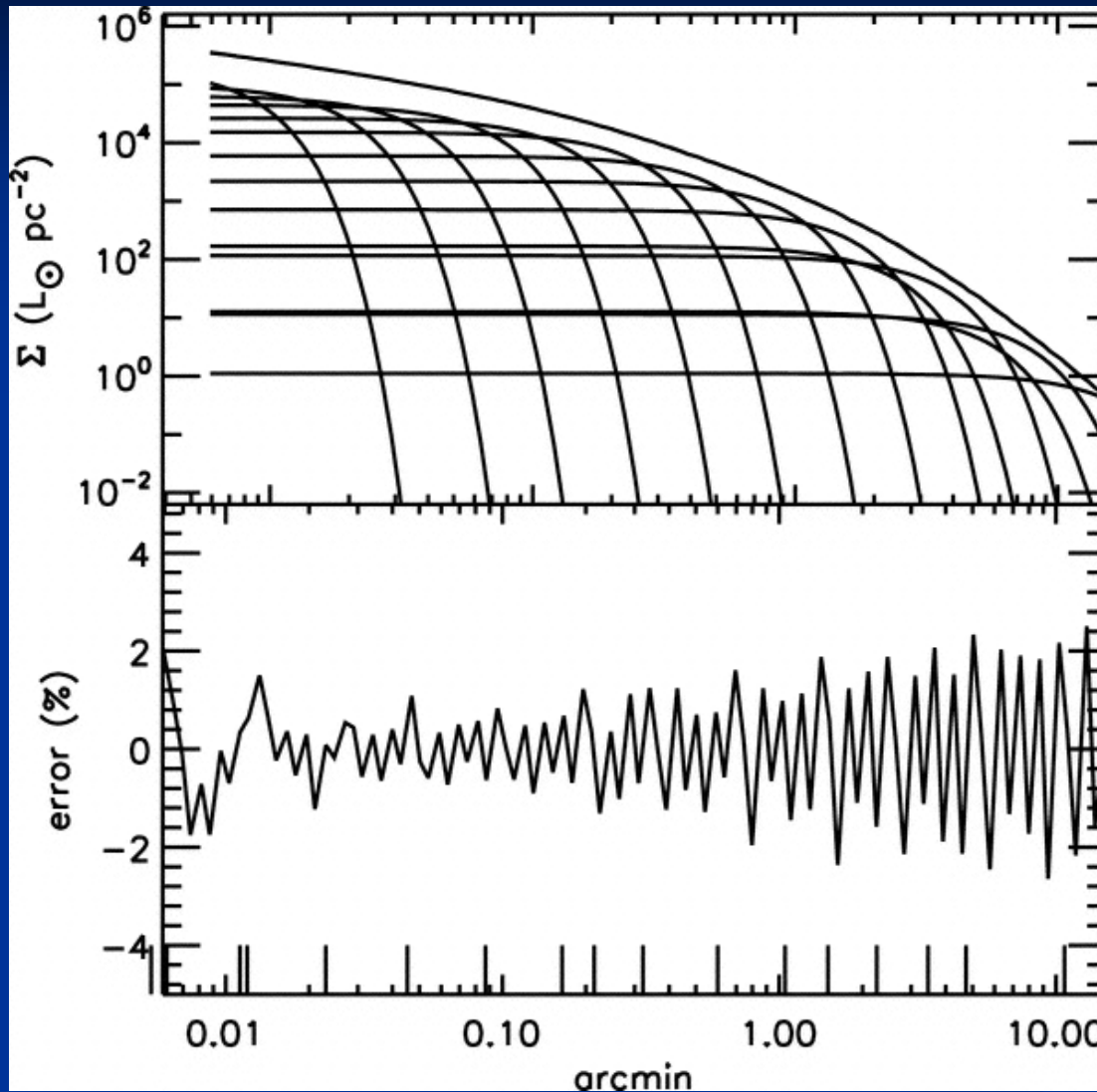
(McNamara et al. 2003, ApJ, 595, 187)



1 arcmin  $\sim$  3 pc



# MGE mass model



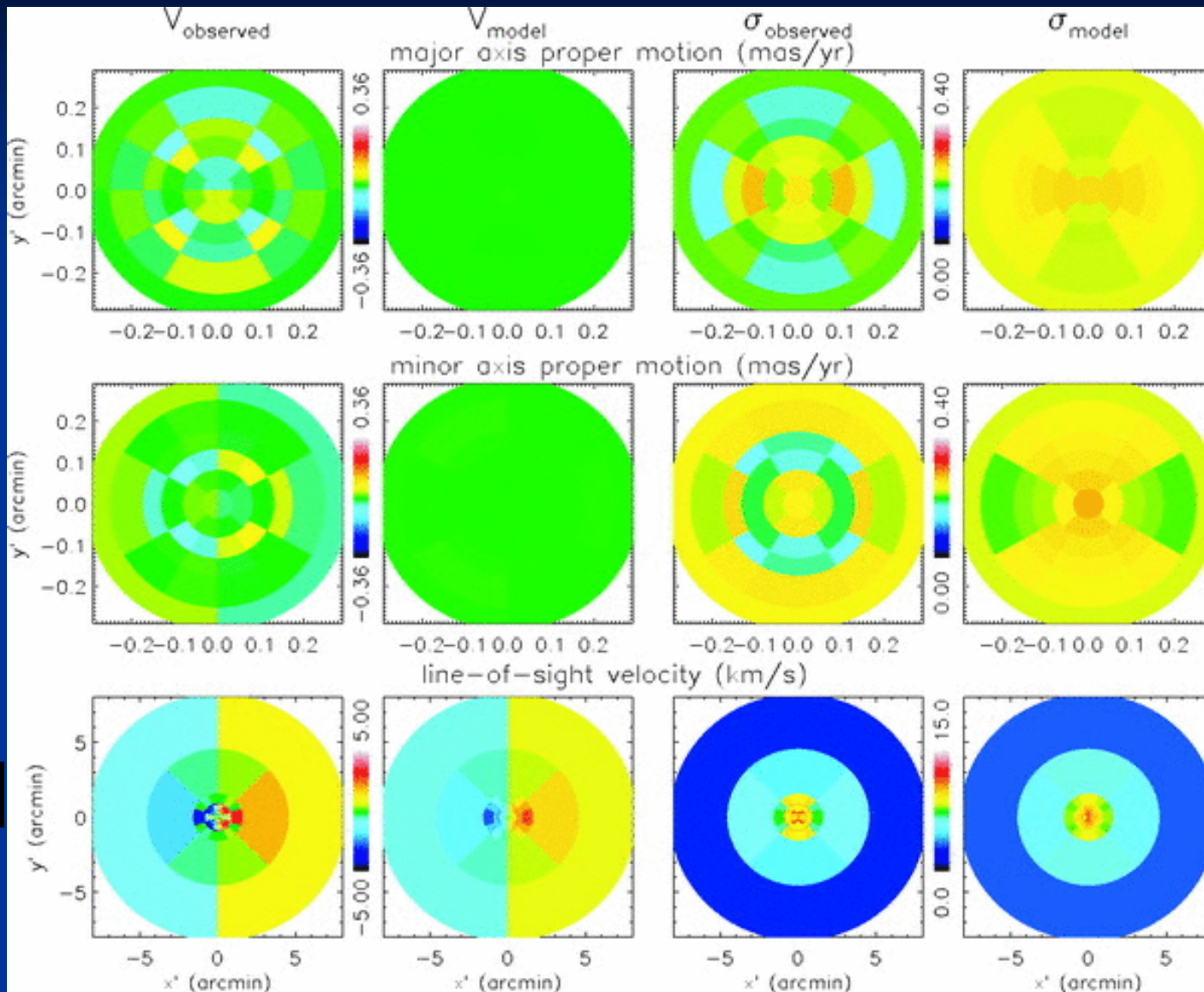
Noyola & Gebhardt (2006, AJ, 132, 447)      1 arcmin  $\sim$  3 pc

# Schwarzschild model

horiz.

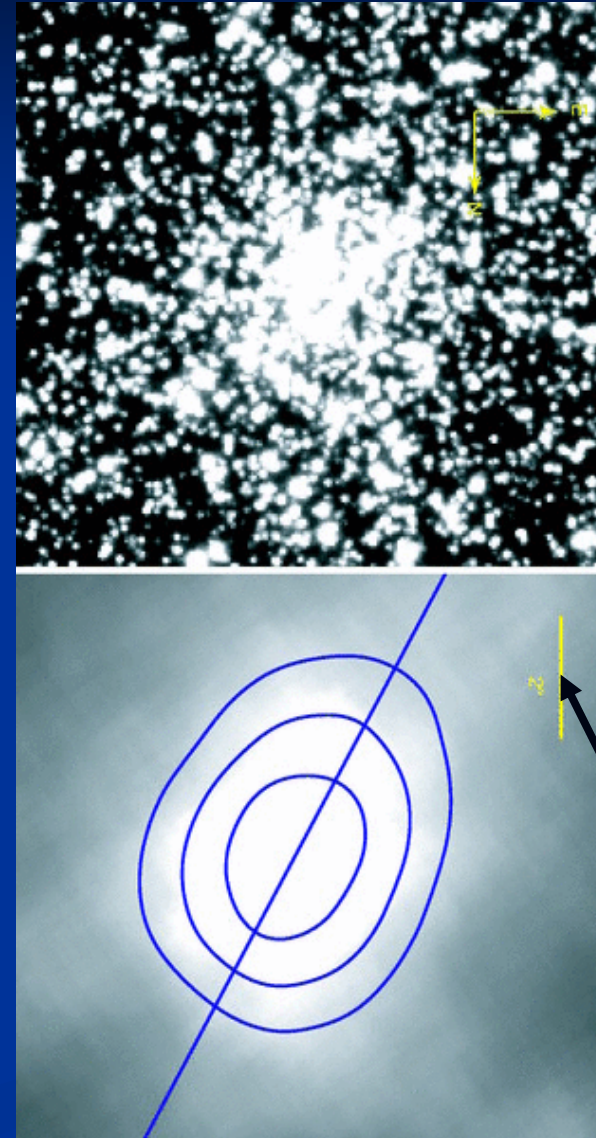
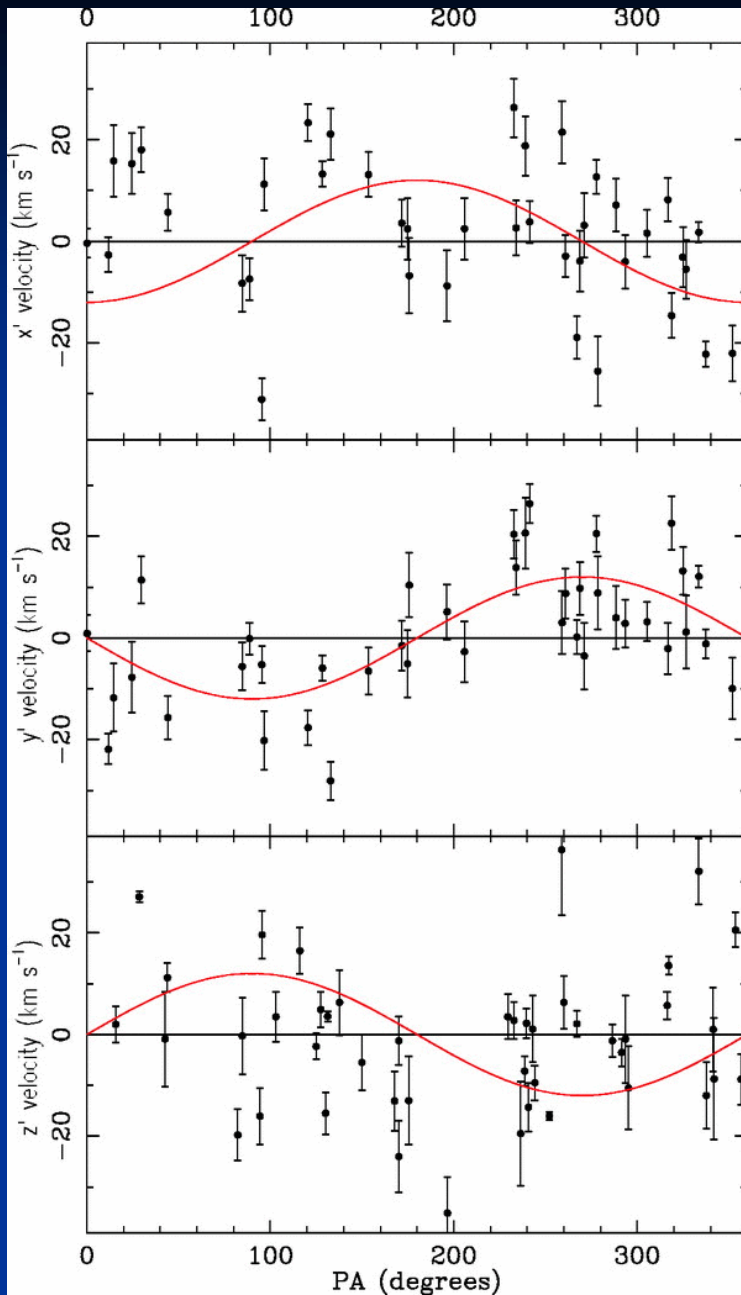
vert.

l.o.s.



1 arcmin  $\sim$  3 pc

# Decoupled core?



$2'' = 0.1 \text{ pc}$