Physics of Vacuum Energy: Gravitation of the Casimir Effect

(astro-ph/0209312)

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"Cosmological Constant"

"Quintessence"

"Vacuum Energy"

?

Wrong Question?

What's 70% of the Universe made of? We don't know! Yet...

"On the Nature of the Universe" Lucretius

(c.100 - c.55 BC)


TITI
LVCRETII
CARI
DE
RERVM NATVRA
LIBRI SEX,
Ad postremam OBERTI GIFANII Ic.
emendationem accuratissimè restituti.
Quæ præterea in hoc opere sint præstita, paginâ
post dedicationem indicabit.
LVGDVNI BATAVORVM,
EX OFFICINA PLANTINIANA,
Apud Franciscum Raphelengium,
cIↃ. IↃ. XCV.

Matter and Space:
Six Primary Propositions

Nothing is ever created out of nothing.

Nothing is ever annihilated.

Matter exists in the form of invisible particles (atoms).

Besides matter, the Universe contains empty space (vacuity).

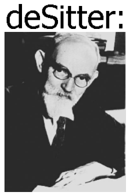
The Universe consists of matter, vacuity, and of nothing else.

Atoms are indestructible.

Gravity and Inertia:

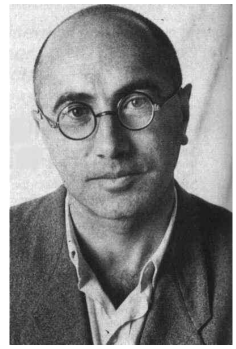
Einstein & deSitter's First Forays into Cosmology

deSitter:



Spacetime filled by "world-matter" (Λ) responsible for inertia; Stars and nebulae, like massive particles in the fixed background, responsible for gravitation (1917)

Zero-Point Energy and the Cosmological Constant?


Zeldovich

1968: "The genie (Λ) has been let out of the bottle"

"A new field of activity arises, namely the determination of Λ"

Showed that the vacuum of scalar particles

$$\rho = \tfrac{1}{2}\hbar c \int \frac{d^3k}{(2\pi)^3}\sqrt{k^2+(mc/\hbar)^2}$$

gives a uniform and constant regularized stress-energy

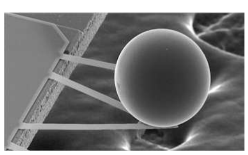
$p=-\rho$

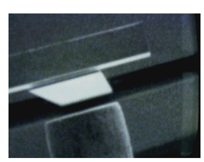
A potential source of Λ !

Q: What are the physical laws of vacuum energy?

Q: Where can we find a handful of vacuum energy to analyze in the laboratory?

Lamoreaux, 1997


Mohideen & Roy, 1998


Bressi et al, 2002

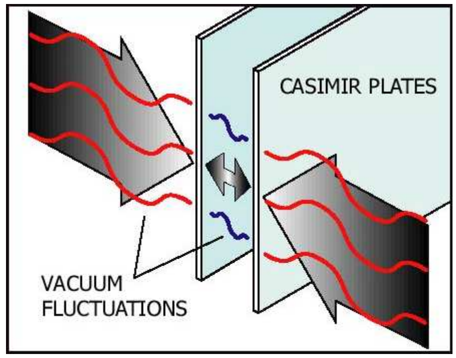
"Inertia of Casimir Energy" Jaekel & Reynaud 1993

"Quantum Systems in Weak Gravity" Papini 2001

"Test of Equivalence Principle with a Freely Falling Quantum Object" Onofrio & Viola 1997

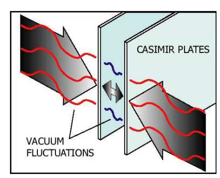
Casimir Effect: *a physical manifestation of zero-point energy*

H. Casimir



…made of the same "stuff" as Λ?

Sketch of Basic Calculation



$E = \hbar\omega(n + \tfrac{1}{2})$ — oscillator energy

$$\rho = \tfrac{1}{2}\hbar c \int \frac{d^3k}{(2\pi)^3} |k|$$

field vacuum energy density

$$\rho = \tfrac{1}{2a}\hbar c \int \frac{d^2k_\times}{(2\pi)^2} \sum_{n=1}^{\infty} \sqrt{k_\times^2 + (n\pi/a)^2}$$

apply boundary conditions

$$\rho = \rho_{\text{w/ plates}} - \rho_{\text{w/o plates}}$$

evaluate influence of boundary

$$p = -\frac{\partial}{\partial a}\left(a \times \left[\rho_{\text{w/ plates}} - \rho_{\text{w/o plates}}\right]\right)$$

determine force/area

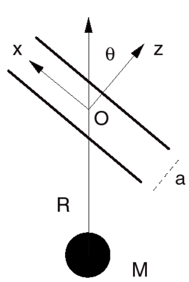
$$T_\mu^{\ \nu} = \frac{\hbar c \pi^2}{720 a^4}\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}$$

stress-energy tensor: uniform across plates

Sticky geckos use Van der Waals forces

Q: How do virtual quanta interact with gravity?

Calculation: Analyze the parallel-plate Casimir effect in a gravitational field



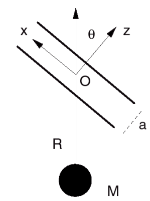
Replace E&M by two scalar fields

$\hat{n}\times\vec{E}=0$ (Dirichlet Φ)

$\hat{n}\cdot\vec{B}=0$ (Neumann Φ)

And so evaluate $\langle T_{\mu}^{\nu}\rangle$

Calculation: Some details...



obtain stress-energy tensor

$$\langle T_{ab} \rangle \underset{x \to x'}{=} \left[\partial_a \partial_b - \tfrac{1}{2} g_{ab} \partial^c \partial_c \right] \Big|_x \langle \Phi(x) \Phi(x') \rangle$$

using Feynman propagator

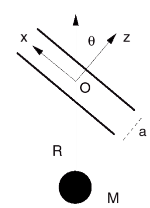
$$\langle \Phi(x) \Phi(x') \rangle = -i\hbar \mathrm{G}(x,x')$$

satisfying bc's and weak-field wave equation

$$g^{ab} \partial_a \partial_b \Big|_x \mathrm{G}(x,x') = -\delta\,(x,x')/\sqrt{-g}$$

Brown & Maclay, 1969; Milton, 2001

Calculation: Some details...



use small parameter $\varepsilon = GM/R$

$$ds^2 = -(1 - \tfrac{2GM}{r})dt^2 + (1 + \tfrac{2GM}{r})d\vec{x}^2$$

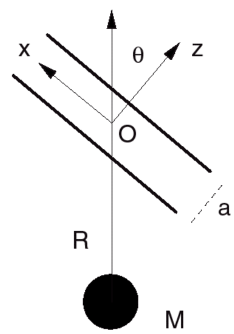
make perturbative expansion

$$\mathrm{G}(x,x') = \mathrm{G}^{(0)}(x,x') + \varepsilon\ \mathrm{G}^{(1)}(x,x') + O(\varepsilon^2)$$

obtain first-order correction

$$\mathrm{G}^{(1)}(x,x'') = -4\,\varepsilon \int d^4x'\ \mathrm{G}^{(0)}(x,x')\ \frac{R}{r'} \left(\partial_{t'}\right)^2 \mathrm{G}^{(0)}(x',x'')$$

Determine influence on vacuum energy and pressure:



$$F/A = -\frac{\hbar c \pi^2}{240 a_p^4} \times \left\{ 1 + \cdots + \frac{GM}{R^3} k a_p^2 (1 + 3\cos 2\theta) + \cdots \right\}$$

$$E/A = -\frac{\hbar c \pi^2}{720 a_p^3} \times \left\{ 1 + \cdots + \frac{GM}{R^3} k' a_p^2 (1 + 3\cos 2\theta) \right\}$$

$$R_{tztz} = -\frac{1}{2}\frac{GM}{R^3}(1 + 3\cos 2\theta)$$

Lesson: zero-point energy is not immutable

Vacuum energy and pressure respond to variations in the gravitational field!

Details: Stress-Energy tensor is conserved, but...

Contact divergences

$$L = \tfrac{1}{2} g^{ab} \partial_a \Phi \partial_b \Phi$$

$$\tilde{L} = -\tfrac{1}{2} g^{ab} \Phi \partial_a \partial_b \Phi$$

remove boundary terms
Deutsch & Candelas, 1979
Kennedy et al, 1980

Anomalous trace

minimally-coupled fields, not conformally-coupled

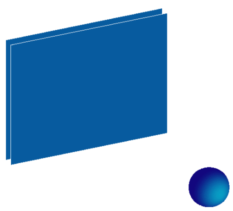
Off-diagonal stress

orientation dependence

If we could compute other finite quantities due to vacuum energy, we would expect a similar distortion due to gravitating sources (corrections like $\sim GMa^2/R^3$)

Cosmological Constant? (If we could compute something finite…)

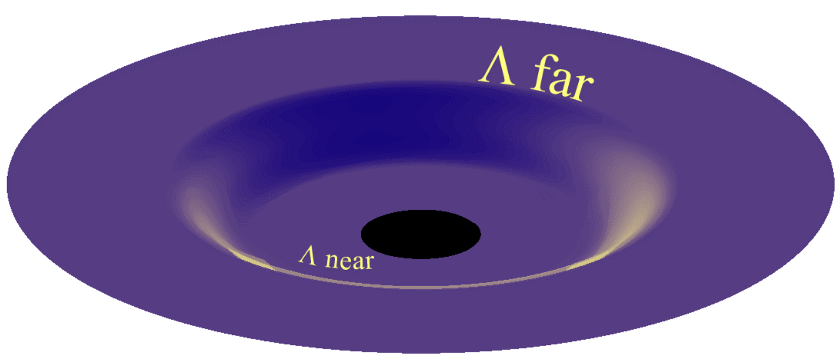
Not So Constant?



Recall earlier lessons:

Inflation is not just a deSitter epoch

Quintessence is not just a time-evolving Λ



…this "constant" may vary in space

Zero-Point Shenanigans:

"Weight of Vacuum" $w = \hat{r}^a f_a = E \times GM/R^2$

Since $E<0$, can we make negative mass?

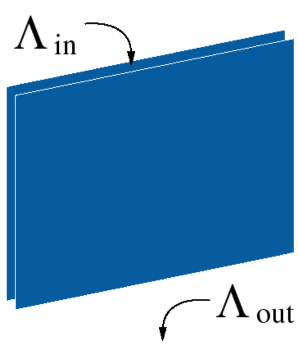
Stress required to keep plates fixed offsets vacuum pressure

Active gravitational mass and inertial mass of isolated system are the same (Misner & Putnam 1959)

Beware ZPE perpetual motion!
(Call the Patent Office!)

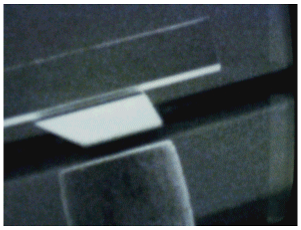
Casimir Effect and a different sort of Vacuum Energy

$$\rho = -\frac{\hbar c \pi^2}{720 a^4}\left[1 + O\left(a^2 \Lambda\right)\right]$$



First Measurement of the Parallel-Plate Casimir Effect
Bressi et al, 2002

gap ~ 3 µm



$$\rho = -\hbar c \pi^2 / 720 a^4 \quad \sim -6\times10^{-26}\, g/cm^3$$

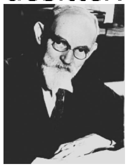
$$\rho_{crit} \sim 8\times10^{-30}\, g/cm^3$$

Idle speculation: $F/A = -\hbar c \pi^2 / 240 a^4 + \Lambda_{em}$?

Gravity and Inertia:

Einstein & deSitter's First Forays into Cosmology

deSitter:



Spacetime filled by "world-matter" (Λ) responsible for inertia; Stars and nebulae, like massive particles in the fixed background, responsible for gravitation (1917)

Einstein:

Sun and stars may be condensed "world-matter", and the world-matter itself may very well be inhomogeneous on small scales, approaching a constant density on sufficiently large scales (1917)

Some Sources:

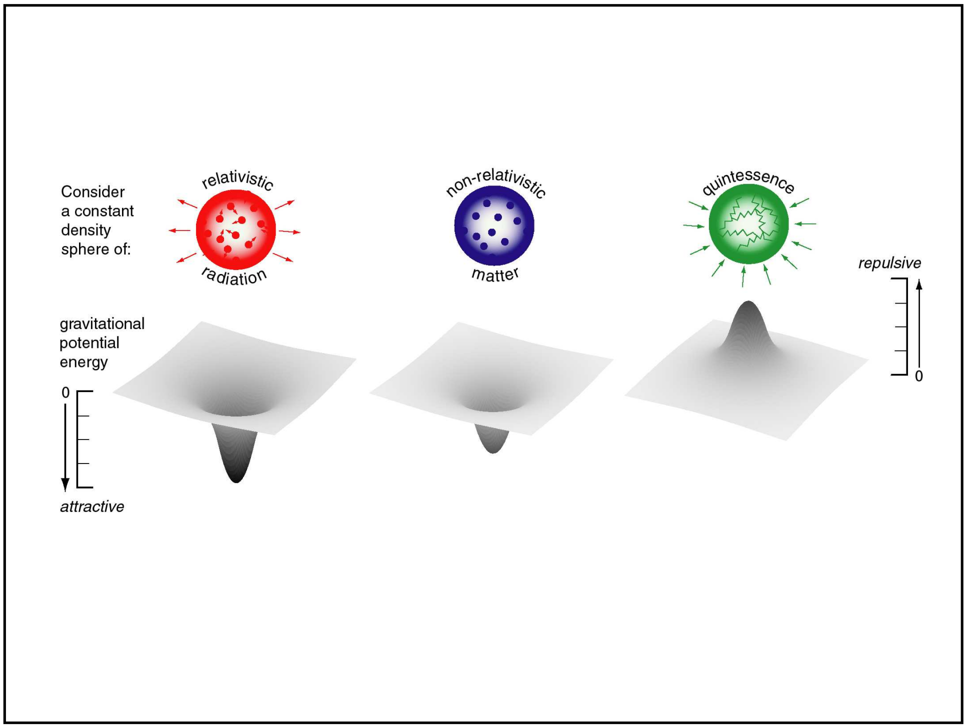
A. Lambrecht, Physics World, September 2002

Bordag, Mohideen, Mostepanenko, Phys. Rept. 353, 1 (2001)

K. Milton, "The Casimir Effect" (2001)

Bressi et al, Phys. Rev. Lett. 88, 041804 (2002)
Mohideen & Roy, Phys. Rev. Lett. 81, 4549 (1998)
S. Lamoreaux, Phys. Rev. Lett. 78, 5 (1997)

Sticky geckos use Van der Waals forces



Consider
a constant
density
sphere of:
relativistic
radiation
non-relativistic
matter
quintessence
gravitational
potential
energy
0
attractive
repulsive
0