Hagedorn Inflation

talk by

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Hagedorn Inflation

with Steve Abel, Ian Kogan

IDEA:
Open strings on branes at high temperatures (close to string scale) drive inflation

n.b. NO potential required

How it works:
Negative pressure in bulk causes our brane to inflate.

Why Inflation? Puzzles of Hot Big Bang Cosmology

1) 'Smoothness' of the Universe

- homogeneity & isotropy on large scales

microwave background radiation

surface of last scattering

$t \sim 10^6$ yrs

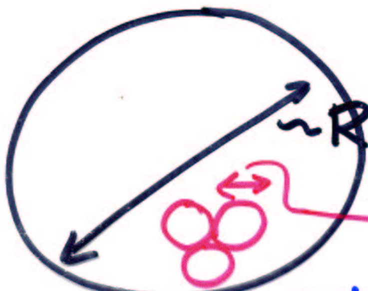
$e^- p \rightarrow H \gamma$

$T \sim 4000\,K$

$\frac{\Delta T}{T} \simeq \mathcal{O}(10^{-5})$

$ct \Rightarrow \theta \sim 1^\circ$

horizon



$\sim R_{then} \simeq \frac{10^{28}\,cm}{1000} \simeq 10^{25}\,cm$

$ct \sim 10^{20}\,cm$

Present observable universe was then $\sim 10^5$ causally distinct regions.

The Cosmological Horizon Problem

Comoving size of observable universe today is

$$L_0 = \int_{t_{dec}}^{t_0} \frac{dt}{a(t)}$$

t_0 — today

$a(t)$ — scale factor

t_{dec} — time of radiation decoupling

Must fit inside comoving size of horizon (causal region) at some early time (before decoupling)

$$L_n = \int_0^{t_n} \frac{dt}{a(t)}$$

To explain causal contact of all points of our observable universe at t_n need $L_0 < L_n$

For $a \sim t^p$ $\quad p<1$

before t_n and after t_{dec},

$$L_0 \sim \frac{t_0}{a_0} \sim \frac{1}{H_0 a_0} \qquad \left(H = \frac{\dot a}{a}\right)$$

$$L_n \sim \frac{t_n}{a_n} \sim \frac{1}{H_n a_n}$$

causality condition becomes

$$\frac{1}{a_n H_n} \geq \frac{1}{a_0 H_0} \Rightarrow \frac{1}{\dot a_n} > \frac{1}{\dot a_0}$$

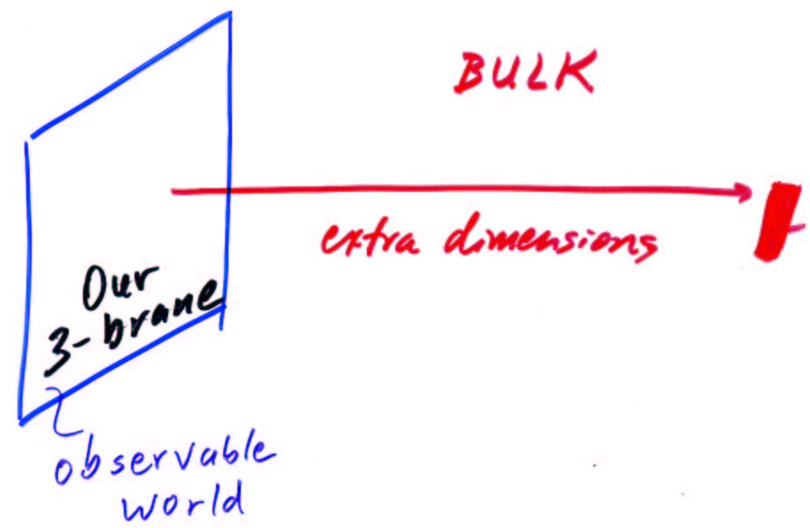
Inflation solves with

$\ddot a > 0$ somewhere between t_n + t_0

$$\frac{\ddot a}{a} = -\frac{4\pi}{3 m_{pl}^2}(\rho + 3p)$$

e.g. vacuum energy with $p = -\rho$

Braneworld Scenarios



The Hagedorn Phase

Fundamental strings have a large number of degrees of freedom.

Many oscillator modes →

Density of States

$$\boxed{\omega(\mathcal{E}) \propto e^{\beta_H \mathcal{E}}}$$

where $T_H = \frac{1}{\beta_H}$ = Hagedorn temperature

and $\mathcal{E}$ is energy and

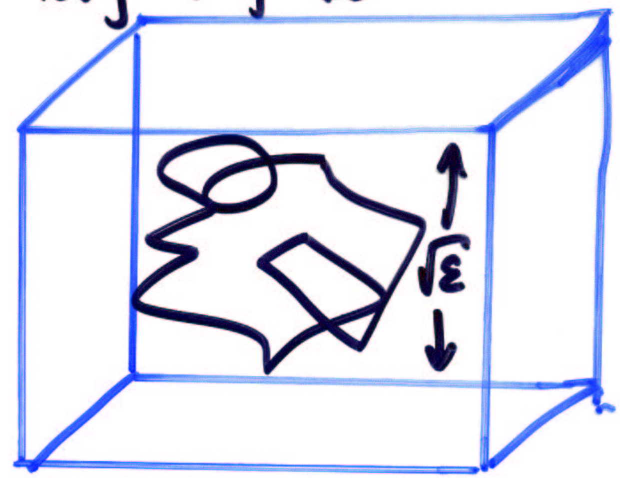
e.g. $\langle E \rangle = \int d\mathcal{E}\, \omega(\mathcal{E})\, e^{-\beta \mathcal{E}}$

physical quantities $(F, \langle E \rangle \dots)$

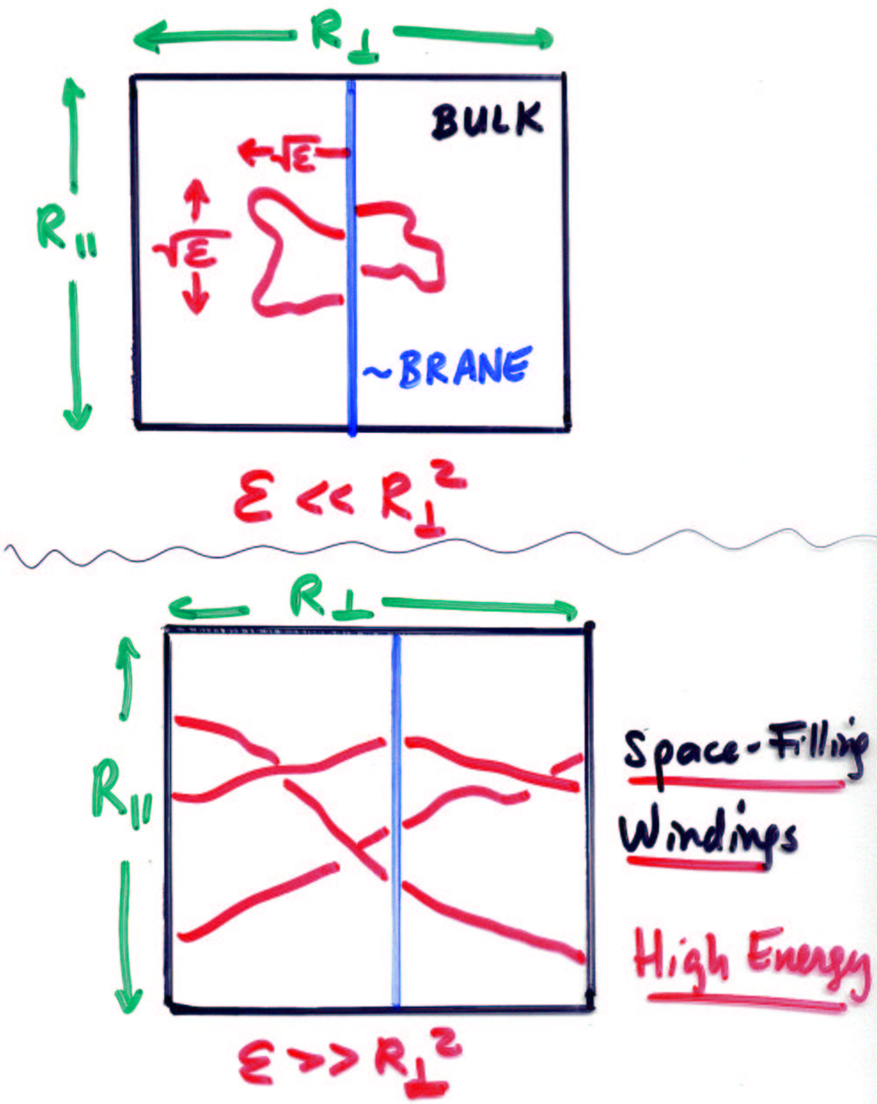
$\sim (\beta - \beta_H)^{-x}$

typically diverge at T_H.

Get 'long string' behavior



~ random walk



Density of States for Open Strings on Branes at High Energy:

$$\boxed{\omega(E)_{open} \sim \frac{V_\parallel}{V_\perp} e^{\beta_H E}}$$

obtained via

- microcanonical ensemble

$$E \to \Omega \to S = \ln \Omega \to T^{-1} = \frac{\partial S}{\partial E}$$

- random walk

→ Same answer

Abel, Barbon, Kogan, Rabinovici

So Far:

Density of states of open strings attached to branes near T_H

Next:

find partition function Z

$\Downarrow$

Bulk $T_{\mu\nu}$

Result: obtain negative pressure

Put into Einstein's equations to find inflation of our brane.

Partition Function

$$\log Z \sim \int d\mathcal{E}\, w(\mathcal{E})\, e^{-\beta \mathcal{E}}$$

$$\sim \frac{2 V_{\parallel}^2 \beta_H^2}{V_\perp (\beta^2 - \beta_H^2)} + \text{nonsingular cutoff terms}$$

Bulk Energy-Momentum Tensor

$$\langle T^{\mu\nu} \rangle = \left\langle \frac{\delta S}{\delta g^{\mu\nu}} \right\rangle$$

$$\sim \frac{\delta \log Z}{\delta g^{\mu\nu}}$$

forces $T_{05} = 0$

Assume: small changes in metric correspond to small changes in volumes

eg $$\frac{\delta Z}{\delta g^{55}} = \int dx' \frac{\delta Z}{\delta V_\perp(x')} \underbrace{\frac{\delta V_\perp(x')}{\delta g_{55}}}_{\frac{1}{2\sqrt{g_{55}}\sqrt{B}}}$$

for a single extra dim:

Results for Bulk Energy-Momentum

$$T^{\mu\nu} = \begin{pmatrix} E/V_{\parallel}V_{\perp} & & & & \\ & p & & & \\ & & p & & \\ & & & p & \\ & & & & -p \end{pmatrix}$$

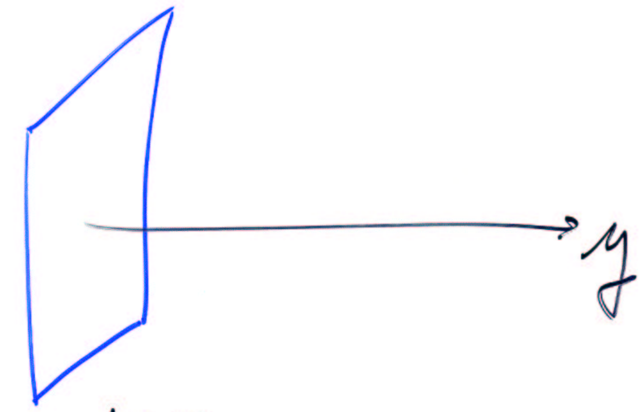
← local energy density of strings

$$p \sim \frac{1}{V_{\perp}^{3/2}}\,\rho^{1/2}$$

$$(\rho \equiv E/V_{\parallel})$$

Negative pressure in Bulk

Consider 5 dimensional universe:



brane at $y=0$

$$ds^2 = -n^2 dt^2 + a(t,y)^2 d\vec{x}^2 + b(t,y)^2 dy^2$$

scale factor of spatial 3-planes

scale factor of extra dimensions

Cosmological Equations in D=5

$$ds^2 = -n^2 dt^2 + a(t,y)^2 d\vec{x}^2 + b(t,y)^2 dy^2$$

scale factor of spatial 3-planes

scale factor of extra dimension

Only one transverse dimension "y" that supports winding modes.

For simplicity, impose Z_2 symmetry under $y \to -y$.

Take 3-brane of observable universe to be at $y=0$.

Equations:

5-D bulk Einstein's eqns.

+

Israel conditions for boundary

$$3\left[\frac{a'}{a}\right]_0 = -\frac{8\pi}{M_5^3} b_0 \rho_{br}$$

$$3\left[\frac{n'}{n}\right]_0 = \frac{8\pi}{M_5^3} b_0 (2\rho_{br} + 3p_{br})$$

5-D bulk Einstein eqns:

$$G_{00} = 3\left\{\frac{\dot a}{a}\left(\frac{\dot a}{a} + \frac{\dot b}{b}\right) - \frac{n^2}{b^2}\left[\frac{a''}{a} + \frac{a'}{a}\left(\frac{a'}{a} - \frac{b'}{b}\right)\right]\right\} = \hat{\kappa}^2 T_{00}$$

$$G_{ii} = \frac{a^2}{b^2}\left\{\left(\frac{a'}{a} + 2\frac{n'}{n}\right)\frac{a'}{a} - \frac{b'}{b}\left(\frac{n'}{n} + 2\frac{a'}{a}\right) + 2\frac{a''}{a} + \frac{n''}{n}\right\}$$

$$+ \frac{a^2}{n^2}\left\{\frac{\dot a}{a}\left(-\frac{\dot a}{a} + 2\frac{\dot n}{n}\right) - 2\frac{\ddot a}{a} + \frac{\dot b}{b}\left(-2\frac{\dot a}{a} + \frac{\dot n}{n}\right) - \frac{\ddot b}{b}\right\} = \hat{\kappa}^2 T_{ii}$$

$$G_{05} = 3\left(\frac{n'}{n}\frac{\dot a}{a} + \frac{a'}{a}\frac{\dot b}{b} - \frac{\dot a'}{a}\right) = \hat{\kappa}^2 T_{05}$$

$$G_{55} = 3\left\{\frac{a'}{a}\left(\frac{a'}{a} + \frac{n'}{n}\right) - \frac{b^2}{n^2}\left[\frac{\dot a}{a}\left(\frac{\dot a}{a} - \frac{\dot n}{n}\right) + \frac{\ddot a}{a}\right]\right\} = \hat{\kappa}^2 T_{55}$$

where $\hat{\kappa}^2 = 8\pi G = \frac{8\pi}{M_5^3}$

prime $\to \frac{\partial}{\partial y}$ and dot $\to \frac{\partial}{\partial t}$

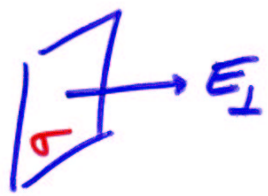
Use $T_{\mu\nu}$ appropriate to Hagedorn regime.

Use negative bulk pressure.

Israel Conditions for Boundary

similar to electrostatics

Charged Plate

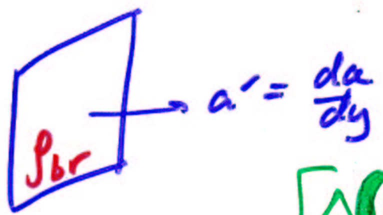


Relates change to electric field on either side.

$$\Delta E_\perp = 4\pi\sigma$$

σ = charge/area

in braneworlds



$$\Delta a' \propto \rho_{br}$$

$$\left[\Delta (K_{MN} - K h_{MN}) = t_{MN}\right]$$

Relates energy-momentum on our brane to its extrinsic curvature, i.e. to the way it's embedded in the bulk.

Results: Behavior of Scale Factors a(t) and b(t) at y=0

(y=0: on the brane)

Case: $\Lambda_{br} = \Lambda_{bulk} = \dot{b} = 0$

Look at 55 equation:

1) in High energy limit of windings in all transverse dimensions, $T_5{}^5 \sim \sqrt{\frac{E}{V_{11}}} \sim a^{-p/2}$

$\Rightarrow a \sim t^{4/p}$, p = # large parallel dimensions

n.b. true for any number of extra dimensions

For our observable universe, $p = 3 \Rightarrow$

$$\boxed{a \sim t^{4/3}}$$ **SUPERLUMINAL EXPANSION**

Amusing result: inflation requires $p \leq 3$ regardless of total # dimensions

2) For 2 dimensions w/ no windings:

Exponential expansion

n.b. Results for a(t) assume $E \approx S$ = constant.

Def'n: $\gamma = \frac{d_0}{2} - 1$

where d_0 = # dimensions transverse to brane with NO windings

Standard high-energy case:
$d_0 = 0 \Rightarrow \gamma = -1$

Results for "p" large parallel dimensions

Regime	$\rho = E/V_{\parallel}$	$-P_{bulk}$	$\boxed{a(t)/a(0)}$
$d_0 = 0$ (High Energy)	$\frac{1}{V_\perp}(\beta - \beta_H)^{-2}$	$\rho^{1/2}$	$t^{4/p}$
$d_0 = 1$	$(\beta-\beta_H)^{-3/2}$	$\rho^{1/3}$	$t^{6/p}$
$d_0 = 2$	$(\beta-\beta_H)^{-1}$	$\log \rho$	$\exp(-Ct^2 + Dt)$
$d_0 = 3$	$(\beta-\beta_H)^{-1/2}$	$1/\rho$	$t^{-2/p}$
$d_0 = 4$	$-\log(\beta-\beta_H)$	$e^{-\rho}$	constant

Explanation for 3 large dimensions?

In high energy regime, superluminal expansion requires brane with ($p \leq 3$). Our brane has $p=3$.

Case: 'nett' cosmological const = 0

$$= -\frac{\hat{\kappa}^2}{12} \Lambda_{br}^2 + \Lambda_{bulk} = 0$$

Take $\dot{b} \neq 0$.

Family of power law solutions

$$a_0(t) \sim A t^q, \quad q = \frac{\gamma - 1}{2\gamma}\left(\frac{4}{3} - r\right)$$

$$b_0(t) \sim B t^r$$

For high energy $\gamma = -1$, find superluminal $q > 1$ for $r < 1/3$
e.g. $r < 0$ (shrinking bulk)

We also find a family of hyperbolic solutions.

$$\frac{a_0(t)}{a_0(0)} = \left(\frac{\sinh 2C(t+t_1)}{\sinh 2Ct_1}\right)^{1/2}$$

Can be exponentially expanding.
Which, if any, of these solutions is appropriate depends on initial value for $\dot{b}_0$.

Sustaining Inflation

We find: inflation begins.

We know: once T drops out of Hagedorn regime, inflation ends

We cannot calculate in between!

Once inflation begins, our calculations break down [our interpretation of $\delta/\delta g \to \delta/\delta V$ fails]

We can speculate

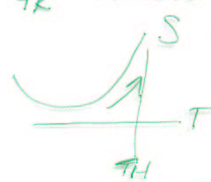
1) Turok: strings in dS (de Sitter) sustain inflation. Our work leads to this setup. Present work provides explanation for how universe enters de Sitter.

2) Bath of branes + bulk strings (eg. branes smashing into each other) keep system hot ⇐ TO DO

3) Generalized 2nd law

$$S = S_{matter} + \underset{\text{de Sitter}}{S_{horizon}} = S_{matter} + \frac{1}{4K^2} A_{horizon}$$

$\delta S > 0$ drives $T \to T_H$



Conclusion

Hagedorn inflation:

open strings on branes near T_H

⇓

negative pressure in bulk

⇓

drives our brane to inflate