Turning back time in an optical lattice

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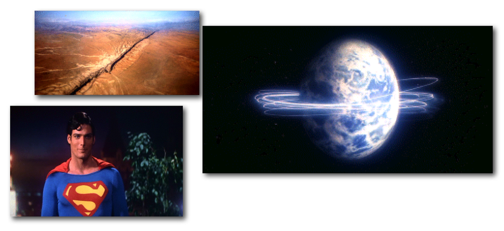
quant-ph/0609202

Turning back time



Evolution operator in quantum mechanics:
$|\psi(t)\rangle = \exp(-iHt)|\psi(0)\rangle$

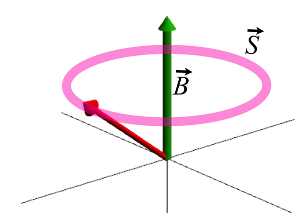
Its inverse, the time reversal operator
$|\psi(0)\rangle = \exp(+iHt)|\psi(t)\rangle$

Is symmetric with respect to change in the sign of t and H,

Negative time $\equiv$ Sign change of Hamiltonian

Routinely done! $H \rightarrow -H$

Sign change of $H \equiv$ echoes



Example: spin echo (NMR)

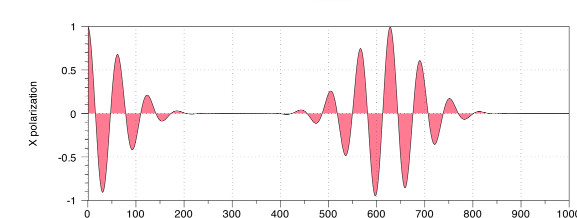
$H = -\mu\, B\, S_Z$

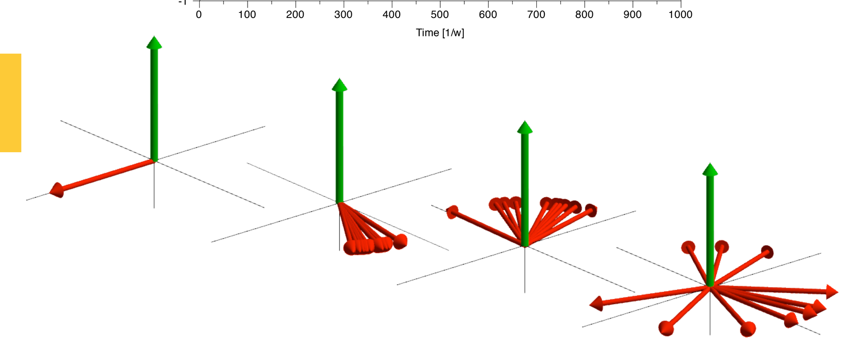
Spin precesses around $\vec{B}$

A 180 degree rotation around X axis (π_x pulse) changes S_Z into $-S_Z$, effectively changing sign of H

Spin echo reverses the decay of polarization due to spin dephasing from field inhomogeneity

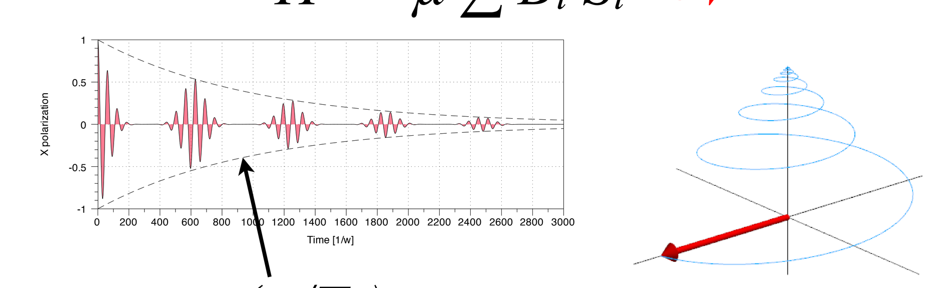
$H = -\mu \sum B_i\, S_i^Z$





The total Hamiltonian is the Zeeman part plus all other degrees of freedom (e.g. spin-spin interactions)

$H = -\mu \sum B_i\, S_i^Z + V$

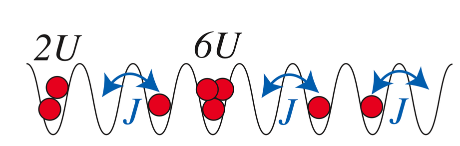


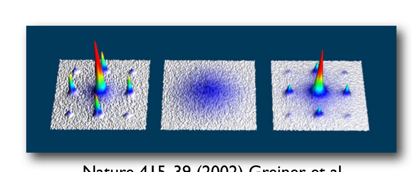
$\exp(-t/T_1)$

The echo measures the strength of V with respect to the part of the Hamiltonian that we can reverse

Echo in optical lattice

- Why?
  - Time reversal is fun
  - Applications: echoes provide pillars for manipulation/characterization techniques.
- In this case:
  - Sensing of small forces
  - Detection of quantum phase transition properties
  - Measure fidelity of quantum simulation





Nature 415, 39 (2002) Greiner et al

$H = -J \sum_{\langle i,j\rangle} a_i^\dagger a_j + a_j^\dagger a_i + U \sum_i n_i(n_i - 1)$

Optical lattice in Bose-Hubbard regime

The optical lattice is used to "simulate" the Bose-Hubbard Hamiltonian because it is very difficult to do it otherwise

This is an interesting system because it presents a quantum phase transition:

$J >> U$ Superfluid

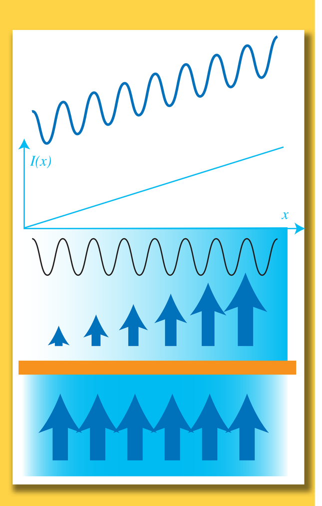
$U >> J$ Insulator

The experiment

$H = -J \sum_{\langle i,j\rangle} a_i^\dagger a_j + a_j^\dagger a_i + U \sum_i n_i(n_i - 1)$

Changing sign of J: a π phase gate

Apply linear ramp potential of slope F for a short time τ



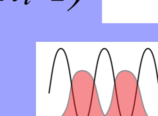
1) $e^{iF\tau x} a_i^\dagger a_{i+1}\, e^{-iF\tau x} \Rightarrow e^{iF\tau} a_i^\dagger a_{i+1}$

2) a momentum kick,
$U_F(\tau)|n\rangle = e^{inF\tau}|n\rangle$, $U_F(\tau)|k\rangle = |k+F\tau\rangle$.
$E(k) = -2J\cos(kna) \Rightarrow E(k+\pi) = -E(k)$

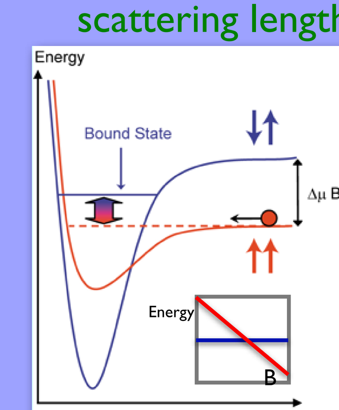
$F\tau = \pi \equiv J \Rightarrow -J$

$H = -J \sum_{\langle i,j\rangle} a_i^\dagger a_j + a_j^\dagger a_i + U \sum_i n_i(n_i - 1)$

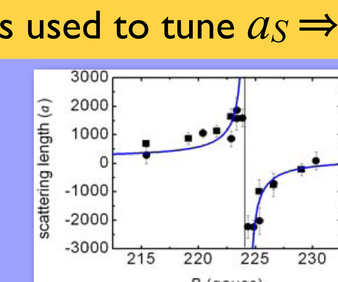
$U \propto a_S \int |\psi_w(r)|^2\, d^3r$

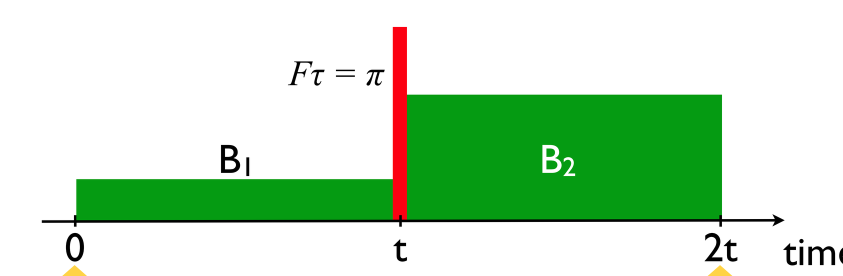


scattering length — Wannier function for site



A Feshbach resonance is used to tune $a_S \Rightarrow -a_S$





Prepare $|\psi_0\rangle$ — Measure projection on $|\psi_0\rangle$

Loschmidt echo:
$f(t) = |\langle\psi_0| e^{-i(-H_{BH}+V)t}\, e^{-i(+H_{BH}+V)t} |\psi_0\rangle|^2 \cong \exp(-aV^2t^2)$

V : Environment, known/unknown external fields

Measures fidelity of the quantum simulation

Measure small forces at small scales, Sensing devices?

Properties of QPT: critical point, exponents, etc,

Experimental feasibility

imprecision

$U \Rightarrow -U$ Limited by homogeneity of field inside trap. Variances ~ 10mG are possible

Feshbach resonances ~ 10-1000 G, widths ~ 1G

$\delta U/U \leq 10^{-3}$ $f(t) \sim \exp(-J^2\, \delta U^2 t^4)$

$J \Rightarrow -J$ Limited by laser fluctuations and diffraction limit
Continuous mask instead of sharp step
Lattices with controllable site-spacing

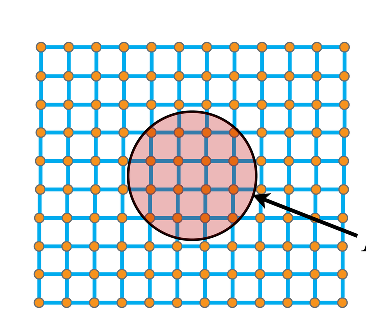
$\tau < \hbar/U$ (Interaction dynamics) $\tau < \hbar/J$ (Lattice dynamics) $\tau < 2md^2/\pi^2\hbar$ (In-site dynamics)

$\tau < 1\mu s$

initial/final states

Experimentally challenging: only few selected states can be done. For complete basis need *single site addressing*.

- Prepare $|\psi_0\rangle = |n_1, n_2, \ldots, n_M\rangle$ (a Fock state)
- Evolve forward/backward (echo)
- Freeze site dynamics by sudden ramp of laser intensity
- Measure probability $p(t)$ of finding n_1 particles in site 1, n_2 in site 2, and so on ($p(t)$ is $f(t)$)



Without *SSA*: Measure average fidelity of subspace

$|f(t) - \langle f_B(t)\rangle| < \frac{\dim \mathcal{H}_B}{\dim \mathcal{H}_T}$ (for long t)

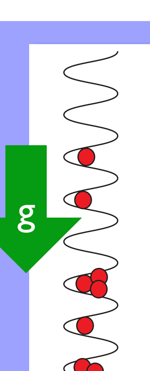
But average needs to be good...

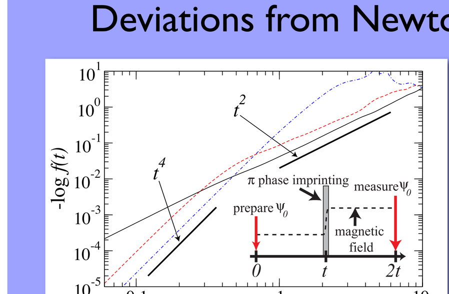
Applications

Sensing devices

Measure small forces at small scales: precision not critical - probe size is ~$5\mu m$, and close to surface

Potential advantages to measure:
- Surface properties
- Casimir effect
- Deviations from Newton





$f(t) \cong 1 - (2g\lambda mJ)^2 t^4/\hbar^4$

$g\lambda m/\hbar \cong 6700$ hz

For 20% error in U and J:

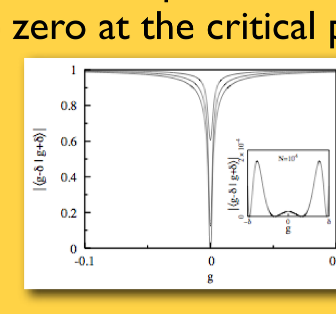
$\frac{\delta f_g}{\delta f_J} \sim 180$ $\frac{\delta f_g}{\delta f_U} \sim 30$

QPT properties

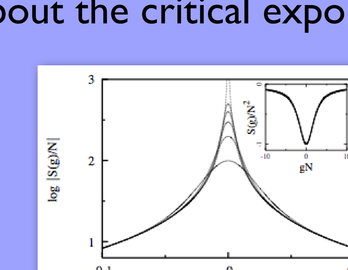
Fidelity and QPTs: critical point, exponents, etc,

QPT: a drastic change in properties of the ground state when a parameter of H is varied

Observation: overlap of two close ground states is ~ 1 if in the same phase, and goes to zero at the critical point



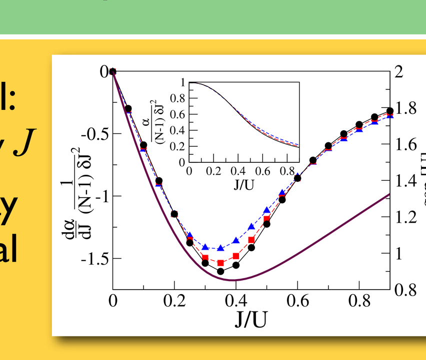
(Scaled) derivatives of the overlap contain information about the critical exponents



The same information is present in the time dependence of fidelity $f(t) \cong \exp(-aV^2t^2)$ (short times)

The Bose-Hubbard model:
Fix a perturbation δJ and vary J

From the short time decay rate one can obtain critical point and exponents



From long time decay, critical point is better obtained (but needs macroscopic numbers)

Conclusions

- Loschmidt (dynamics) echo in optical lattice is experimentally possible
- Incentive to implement it (applications)
  - Decoherence
  - Quantum simulations
  - Sensing devices
- Theoretical analysis using large (MPS, PEPS) numerical simulations can help understand quantum phase transitions.