Emanuele Dalla Torre
Erez Berg & Ehud Altman

HIDDEN ORDER
in ultracold atoms with dipolar interaction

KITP, February 23th, 2007

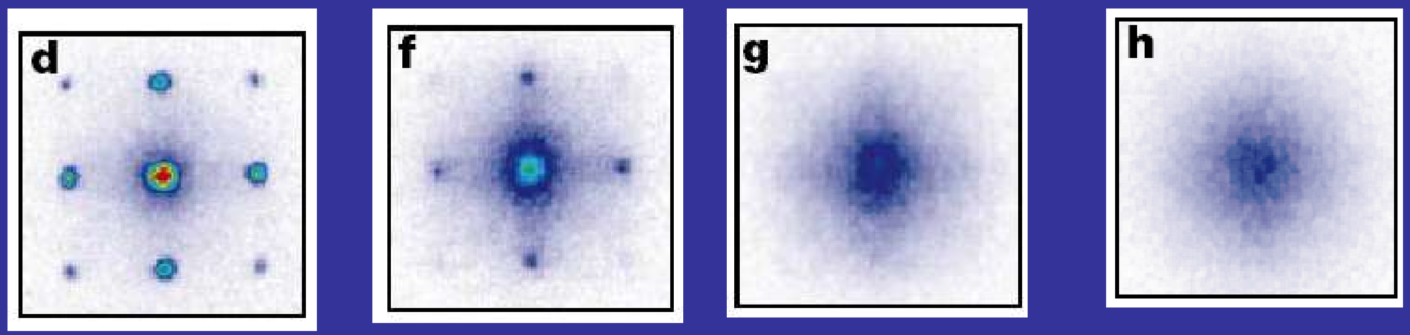
Physics faculty
Condensed Matter department

מכון ויצמן למדע
WEIZMANN INSTITUTE OF SCIENCE

Starting point

- Superfluid - Mott Insulating phase transition

lattice depth



- New physics of strong interactions, quantum fluctations

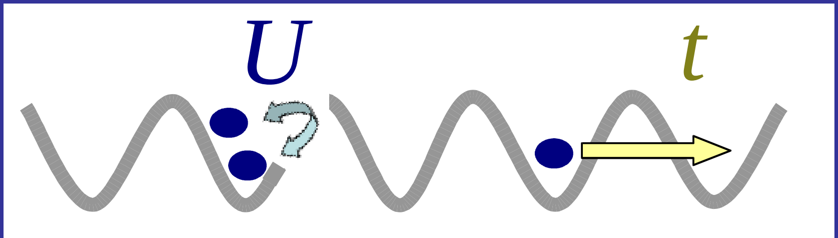
Simple phases: described by local operator

How to obtain non trivial phases?

Greiner et al. Nature (2001)

Ultra-cold Atoms in Optical Lattices

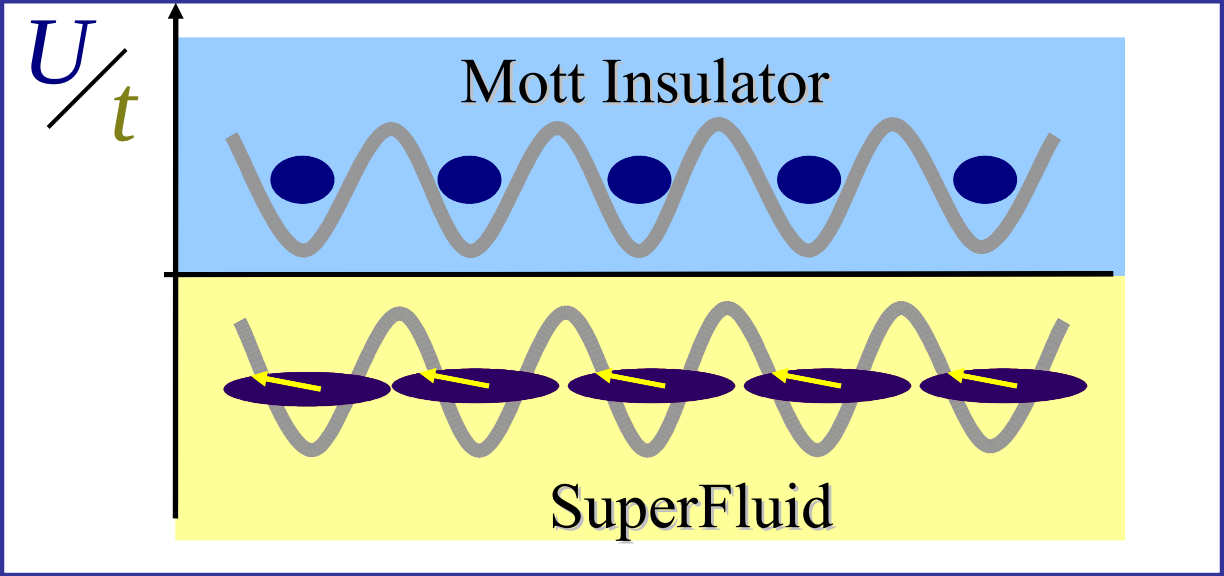
- 1D Bose- Hubbard Model



$$H = -t\sum_i (b_i^\dagger b_{i+1} + H.c.) + \frac{U}{2}\sum_i n_i(n_i - 1)$$

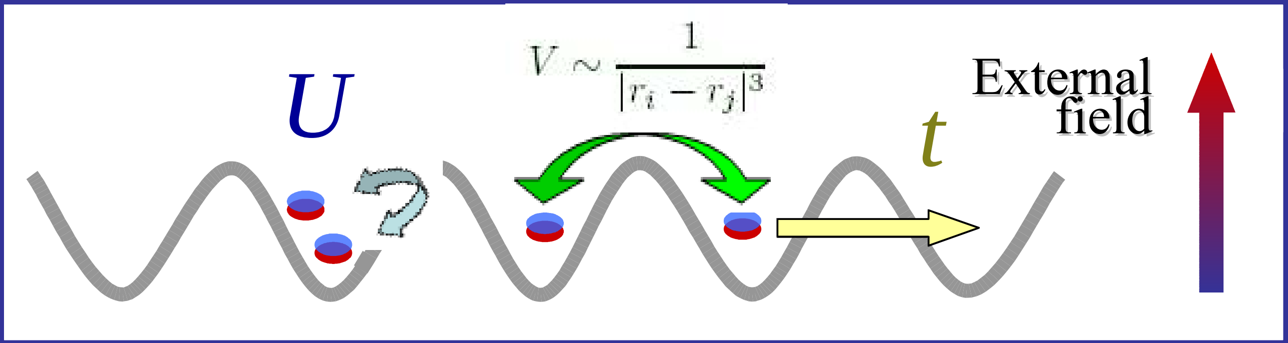
- Integer number of bosons per site

→ Two mean field phases:



Non Local Dipolar interaction

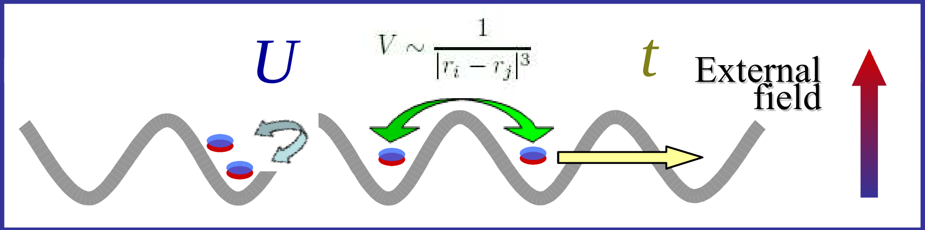
- Dipolar atoms



$$H = -t \sum_i (b_i^\dagger b_{i+1} + H.c.) + \frac{U}{2} \sum_i n_i(n_i - 1) + \sum_{i,r>0} \frac{V}{r^3} n_i n_{i+r}$$

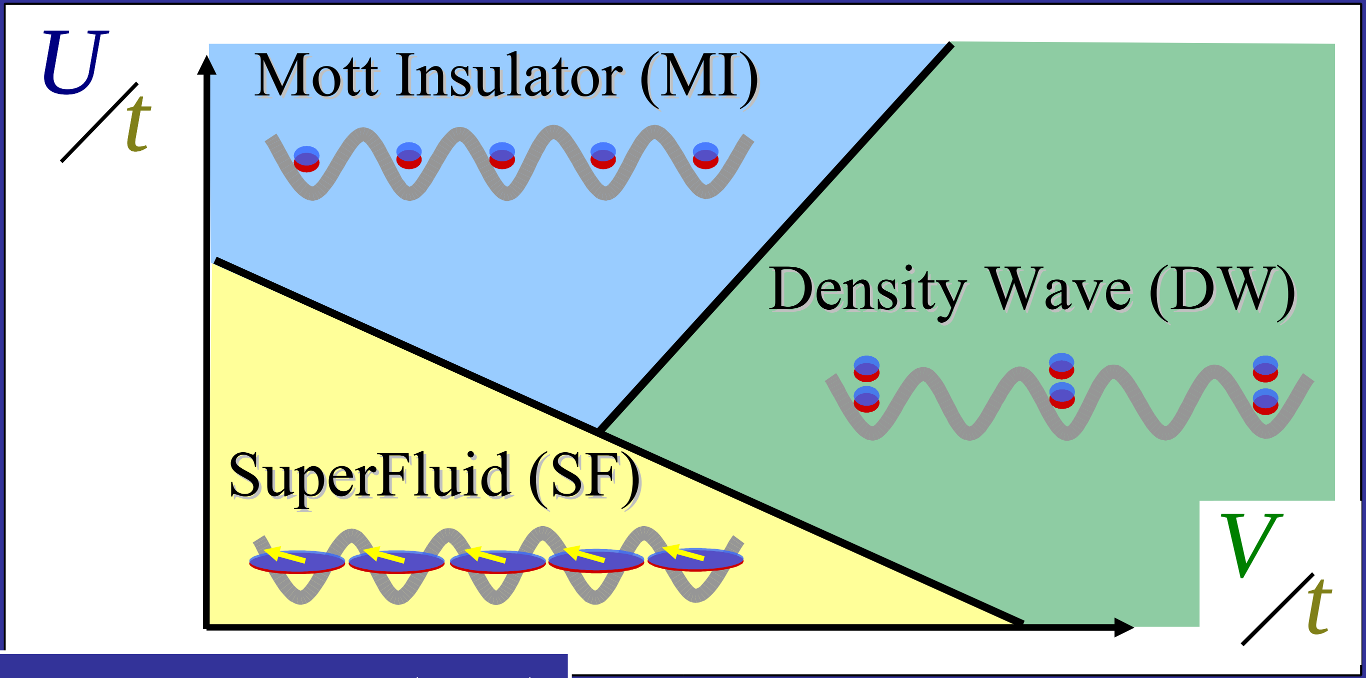
Dipolar Interaction

- Dipolar atoms



$$H = -t\sum_i (b_i^\dagger b_{i+1} + H.c.) + \frac{U}{2}\sum_i n_i(n_i - 1) + \sum_{i,r>0} \frac{V}{r^3} n_i n_{i+r}$$

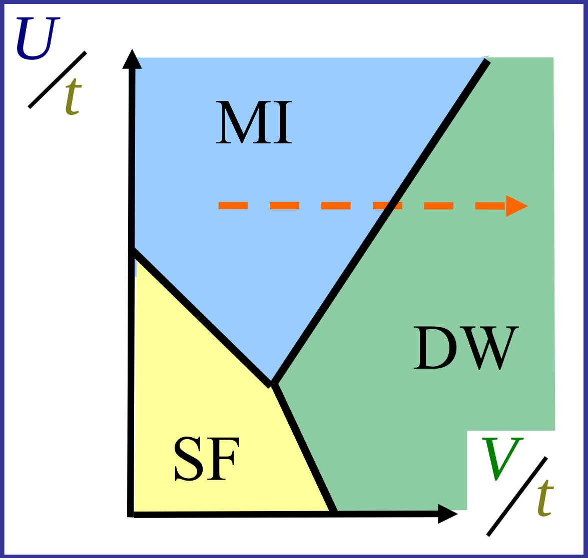
- Three Mean Field phases



Goral, Santos, Lewinstein, PRL (2002)

Numerical Methods

- 1^{st} order transition between MI and DW

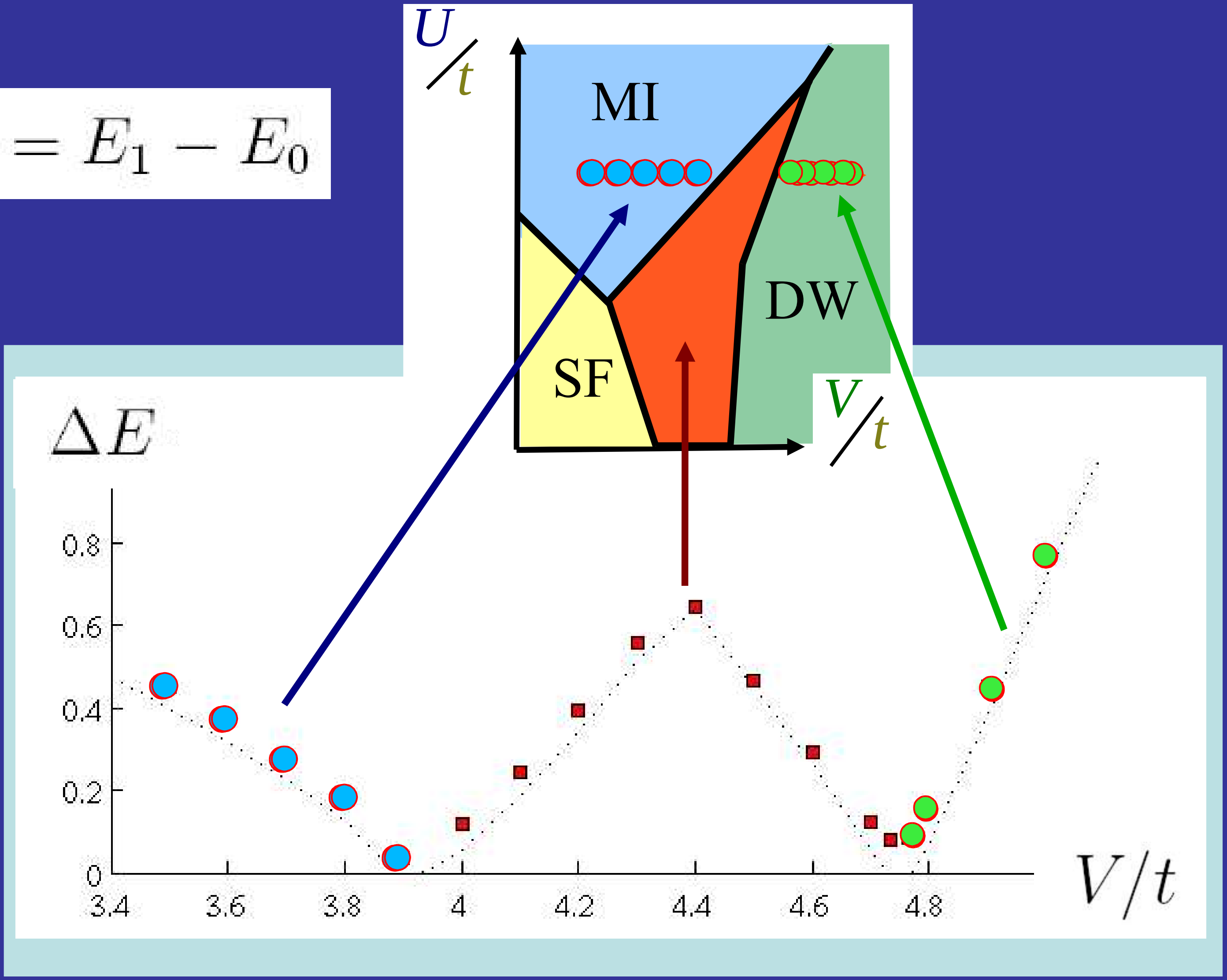


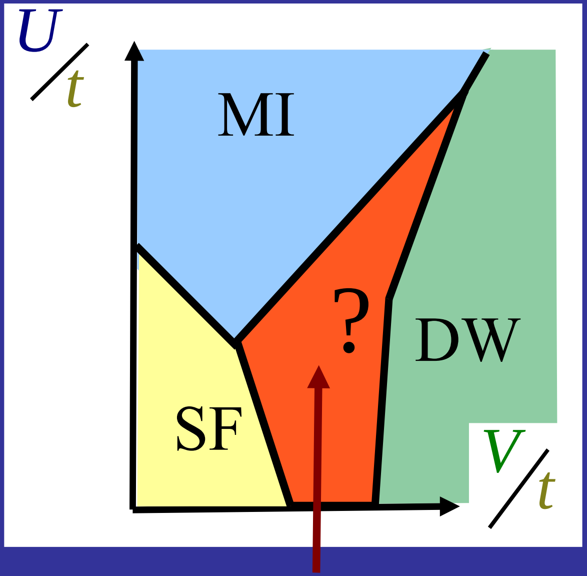
- DMRG – Density Matrix Renormalization Group : numerical method to find ground state + low excitations
- One dimensional chain, L = 256

MORE

Excitation Gap

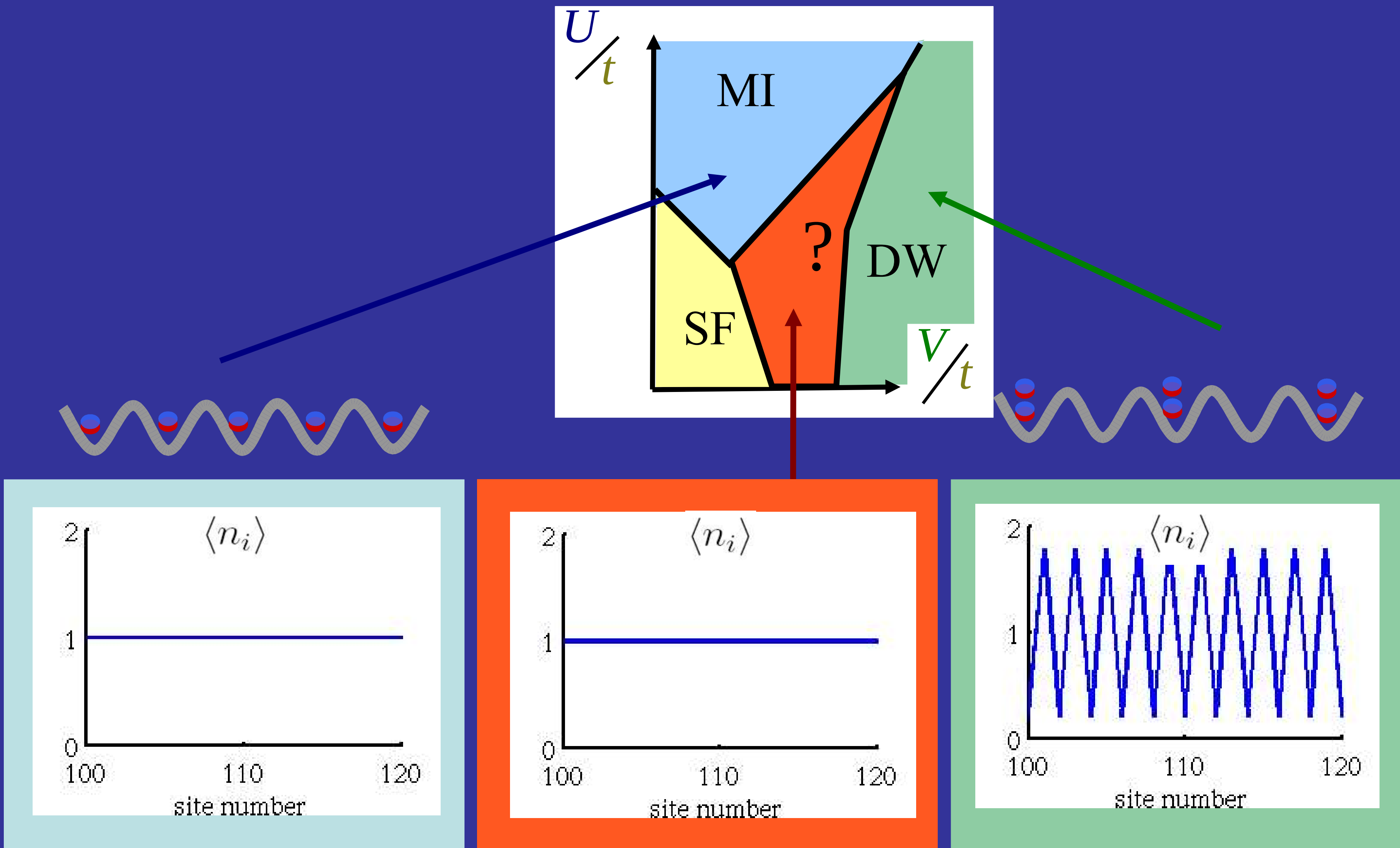
$$\Delta E = E_1 - E_0$$





How can we characterize the new phase?

Local Occupation



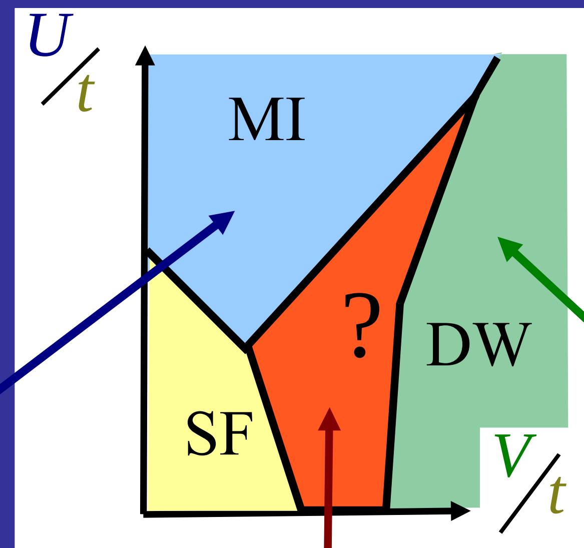
Correlation Function

DW: (blue line)

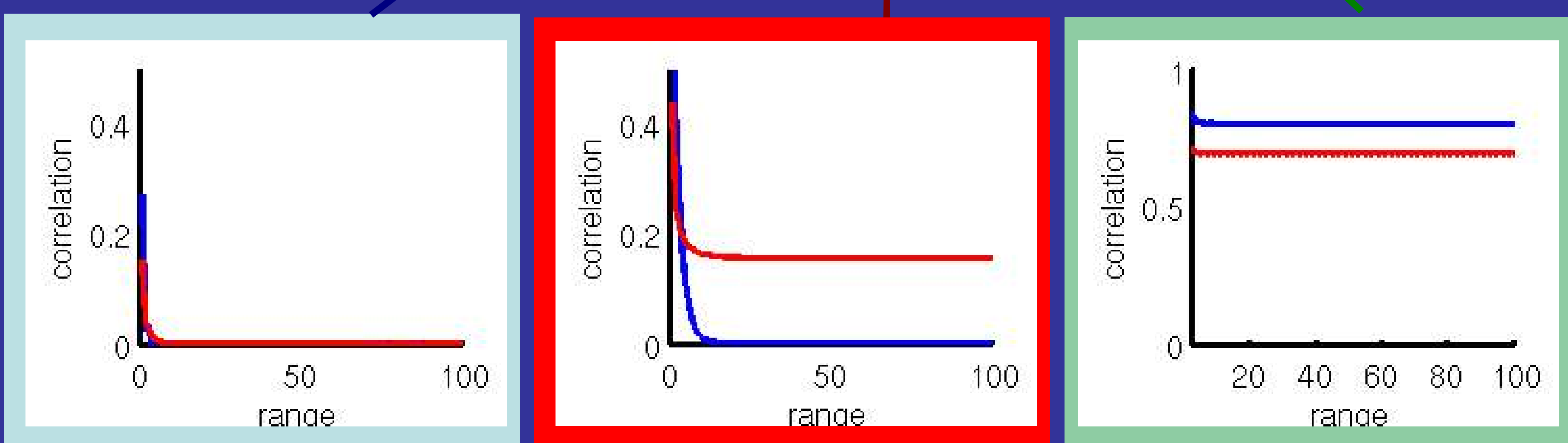
$$(-1)^{|i-j|}\langle\delta n_i\delta n_i\rangle$$

String: (red line)

$$\langle\delta n_i e^{i\pi\sum_{i<k<j}\delta n_k}\delta n_j\rangle$$

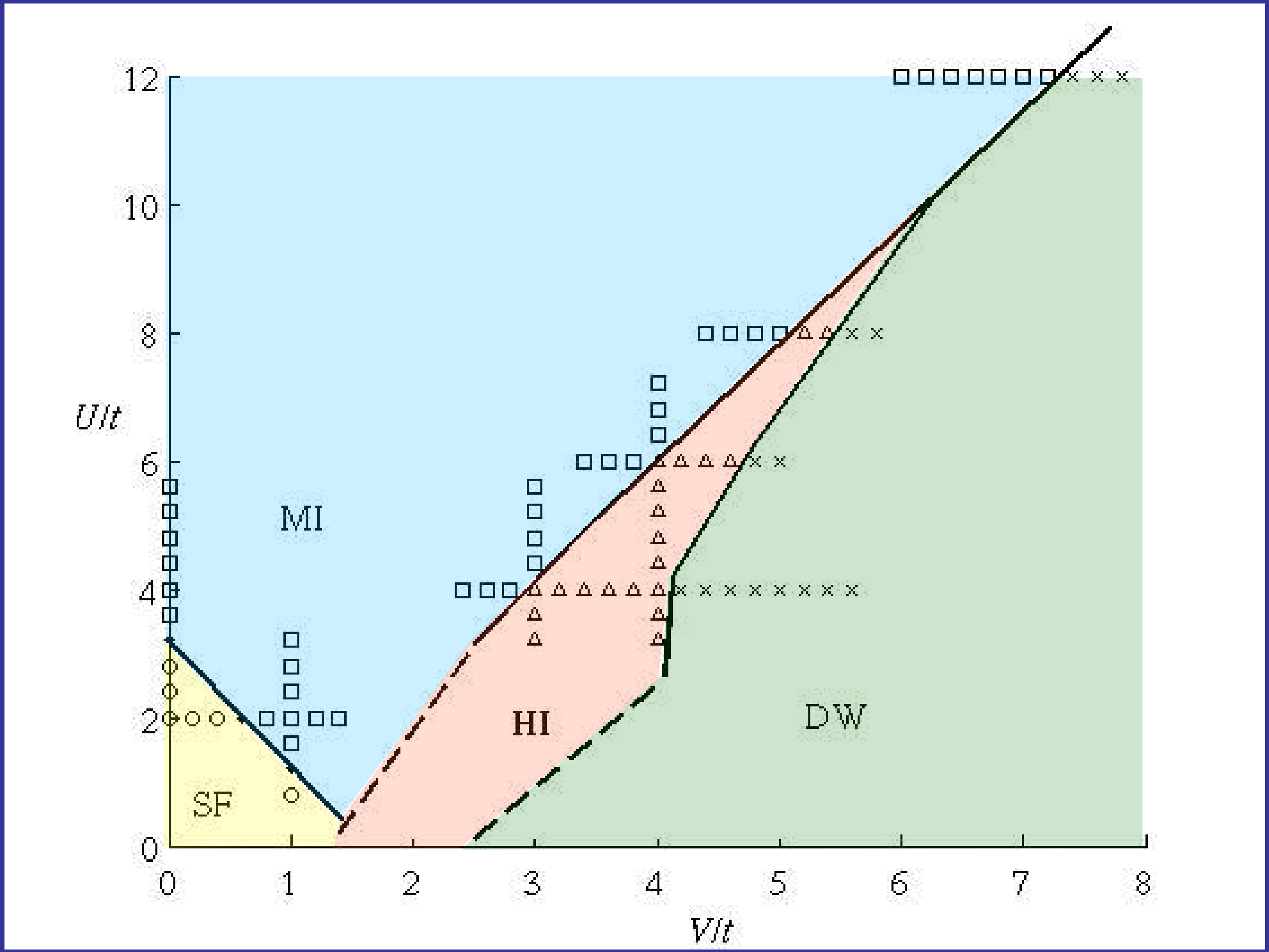


$$\delta n_i = n_i - \langle n\rangle$$



Phase Diagram

- The new phase: Haldane Insulator
- Analogous to Haldane phase for Spin-1 Chains



Spin One Analogy

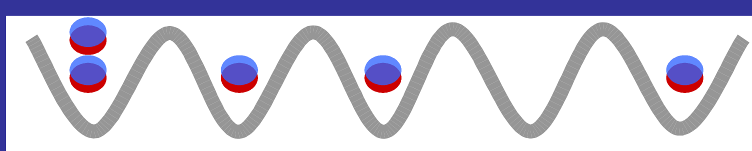
- Truncate the number of bosons to

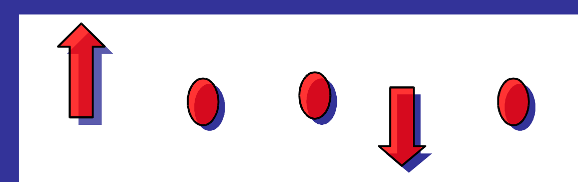
$$n = 0, 1, 2$$

$$\delta n \equiv n - \langle n \rangle = -1, 0, +1$$

- Analogous Spin operators:

$$\begin{array}{ccc} \delta n & \rightarrow & S^z \\ b^\dagger & \rightarrow & S^+ \\ b & \rightarrow & S^- \end{array}$$



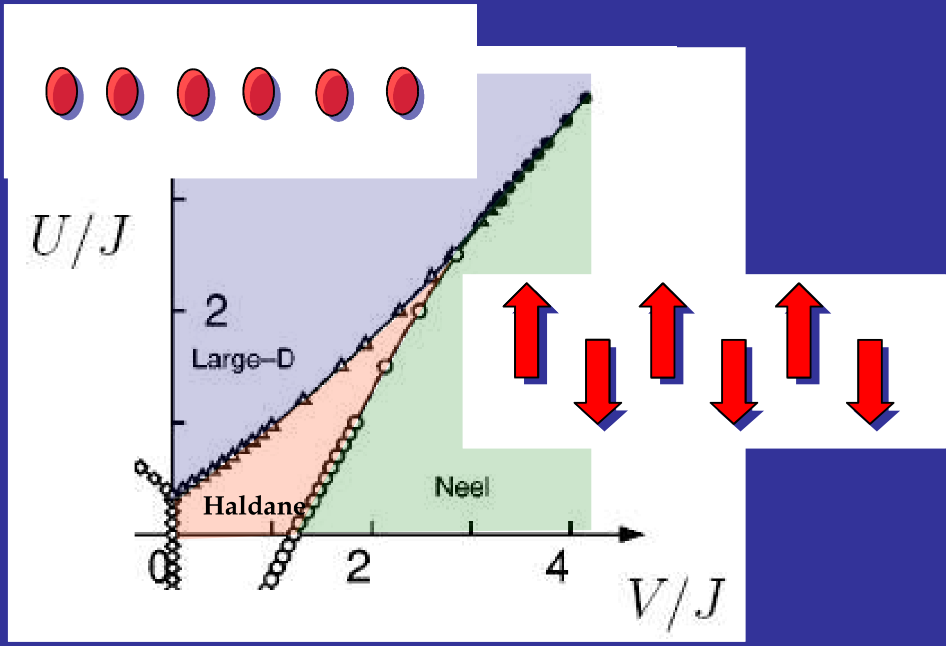


- Analogous Hamiltonian

$$H = \sum_j S_j^+ S_{j+1}^- + V \sum_j S_j^z S_{j+1}^z + U \sum_j \left(S_j^z\right)^2$$

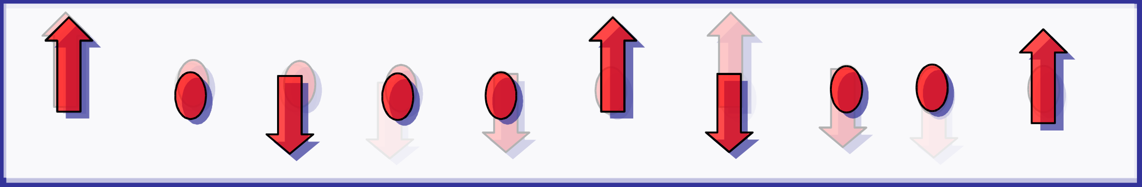
Spin One: Haldane Phase

- Phase Diagram:



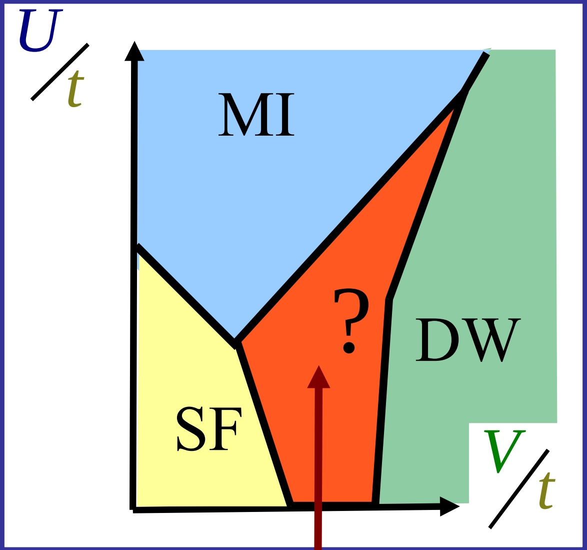
- Haldane Phase:

  String Ordered Ground State



$$R_{i,j}^{String} = \langle S_i^z \, (-1)^{\sum_{k=i}^{j} S_k^z} \, S_i^z \rangle$$

Haldane, PLA (1983)
den Nijs, Rommelse, PRB (1987)
Chen, Hida, Sanctuary, PRB (2003)



Can we (experimentally) probe the new phase?

Experiments?

- Relevant parameters:
  - Elastic scattering length $a_s \approx 30\mathring{A}$
  - Dipole effective scattering length $a_d = -\frac{md^2}{4\pi\hbar^2}$
- Needed condition $U \sim V \quad \Rightarrow \quad |a_d| \sim 30 a_s$

Heteroatomic Molecule:

(big dipole) $a_d \approx -1000\mathring{A}$

Chromium: $a_d \approx -6\mathring{A}$

+ Feschbach Resonance

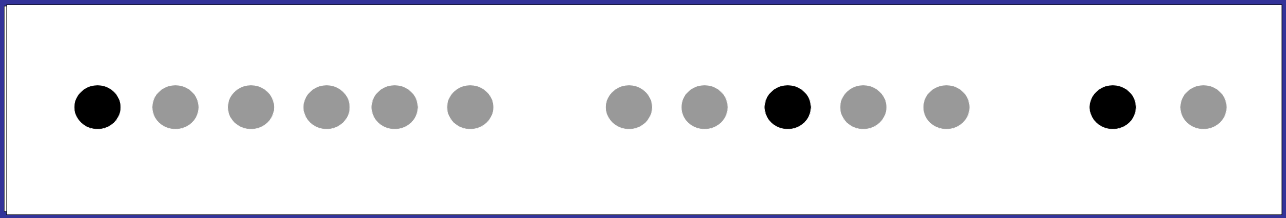
(reduce scattering length)

A. Probing the String order

- "String" = highly non local : $\langle \delta n_i e^{i\pi \sum_{k=i}^{j} \delta n_k} \delta n_j \rangle$

→ Need high resolution " in situ" imaging

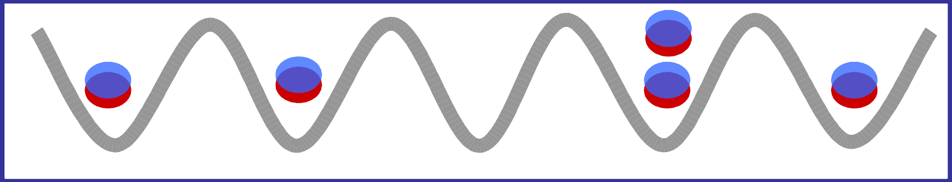
- Algorithm: take a picture and count the bosons



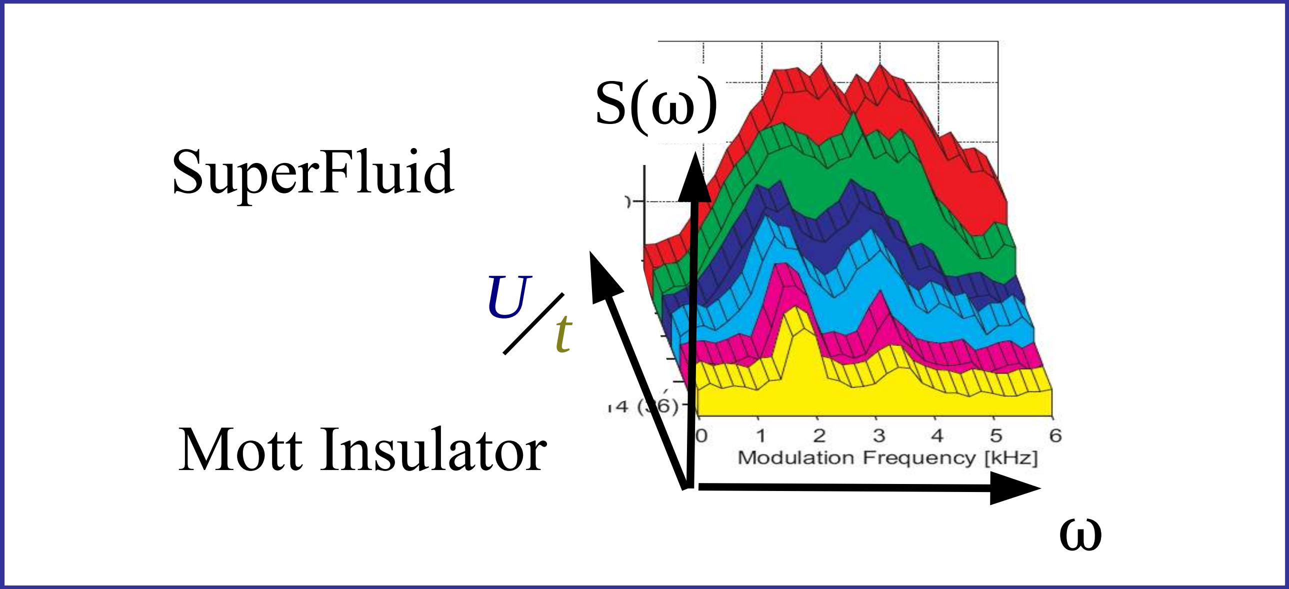
○ = 0 bosons ● (grey) = 1 boson ● = 2 bosons

B. Absorption spectrum signature

- Parametrical Excitation: Lattice Modulation



- Succesfully used to probe MI-SF phase transition



Stoferle et al., PRL (2004)

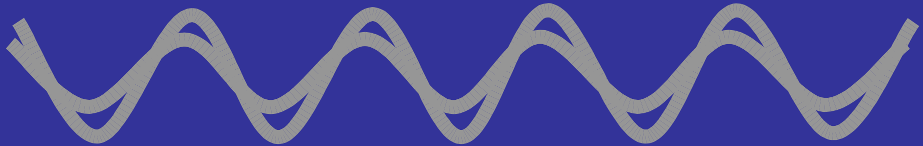
Parametric excitations

$$\hat{H} = \hat{H}_0 + \hat{V}_k(t)$$

- Lattice modulation:

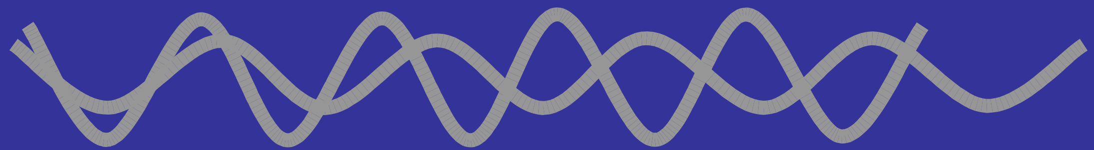
  couples to the tunneling

  (not to the density)



$$\hat{V}_{k=0}(t) = e^{i\omega t} \sum_i b_i^\dagger b_{i+1} + H.c.$$

- Bragg spectroscopy: couples (also) to the local occupation



$$\hat{V}_k(t) = e^{i\omega t} \sum_i e^{ikx_i} \, \delta n_i$$

$$S(k,\omega) = \sum_\alpha | \langle \psi_0 | \hat{V}_k | \psi_\alpha \rangle |^2 \, \delta(\omega - E_\alpha + E_0)$$

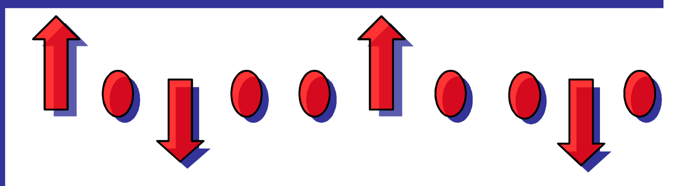
We need the elementary excitations of the system!

Non-Local Mean Field

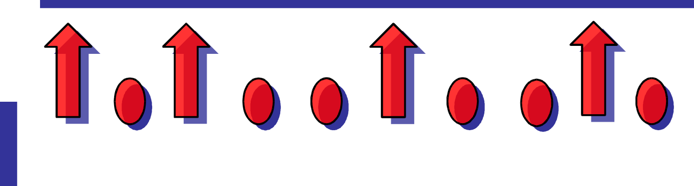
of the analogous spin model

- Non Local Transform: alternatively flips sites with $S_z = \pm 1$

$$U = \prod_{j<k} \exp(i\pi S_j^z S_k^x)$$



String order

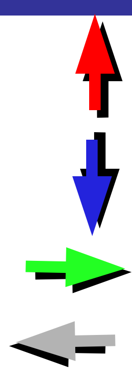


Ferromagnetic Order

$$\tilde{H} = UHU = \sum_j -S_j^x S_{j+1}^x + S_j^y \exp(i\pi S_j^z + i\pi S_{j+1}^x) S_{j+1}^y - V \sum_j S_j^z S_{j+1}^z + \frac{U}{2} \sum_i (S_j^z)^2$$

- Mean Field Ground State:

  4-times degenerate

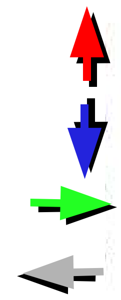


$$= \alpha|S_z = 0\rangle + \beta|S_z = 1\rangle$$
$$= \alpha|S_z = 0\rangle - \beta|S_z = 1\rangle$$
$$= \alpha|S_z = 0\rangle + \beta|S_z = -1\rangle$$
$$= \alpha|S_z = 0\rangle - \beta|S_z = -1\rangle$$

Kennedy & Tasaki, PRL (1991)

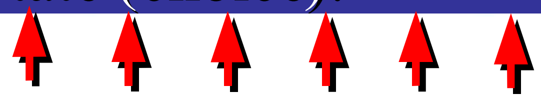
Oshikawa, Phys.Scr.T (1992)

Elementary Excitations

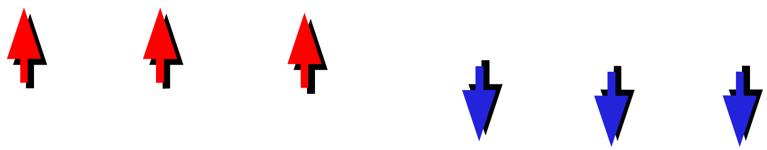


$$= \alpha|S_z=0\rangle + \beta|S_z=1\rangle$$
$$= \alpha|S_z=0\rangle - \beta|S_z=1\rangle$$
$$= \alpha|S_z=0\rangle + \beta|S_z=-1\rangle$$
$$= \alpha|S_z=0\rangle - \beta|S_z=-1\rangle$$

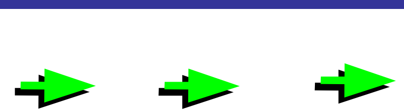
- Ground State (choice):

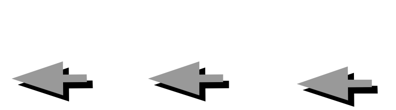
$|\Psi_{GS}\rangle =$ 

- Neutral Kink:

$|\Psi^z\rangle_k =$  $\sim \sum_i e^{ikx_i} S_i^z |\Psi_{GS}\rangle$

- Charged Kinks:

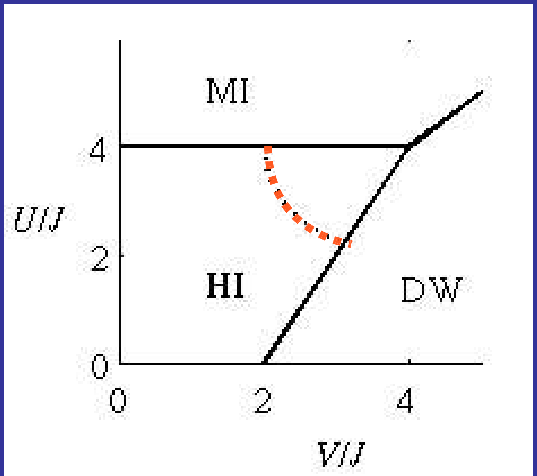
$|\Psi^+\rangle_k =$  $\sim \sum_i e^{ikx_i} S_i^+ |\Psi_{GS}\rangle$

$|\Psi^-\rangle_k =$  $\sim \sum_i e^{ikx_i} S_i^- |\Psi_{GS}\rangle$

Arovas,Auerbach,Haldane (1988)
Fath, Solyom, J.Phys (1993)

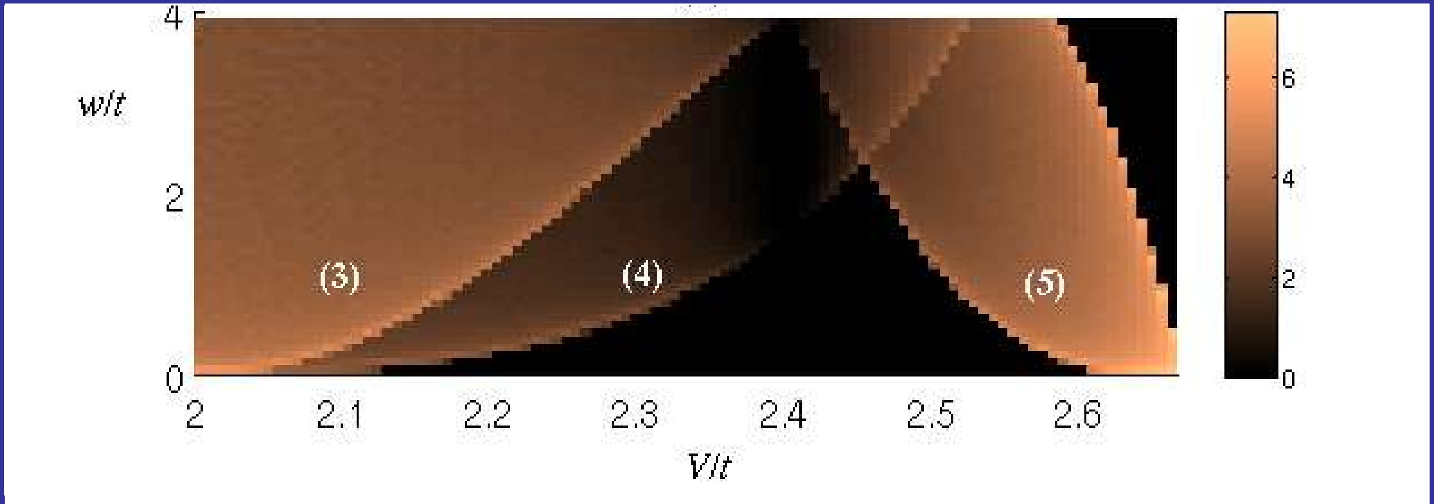
Absorption Spectrum

- Mean field phase diagram



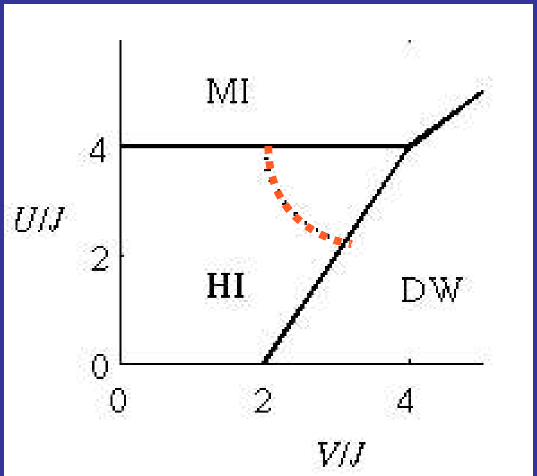
- Absorption Spectrum

Lattice modulation: $$\hat{V}_{k=0}(t) = e^{i\omega t} \sum_i b_i^\dagger b_{i+1} + H.c.$$



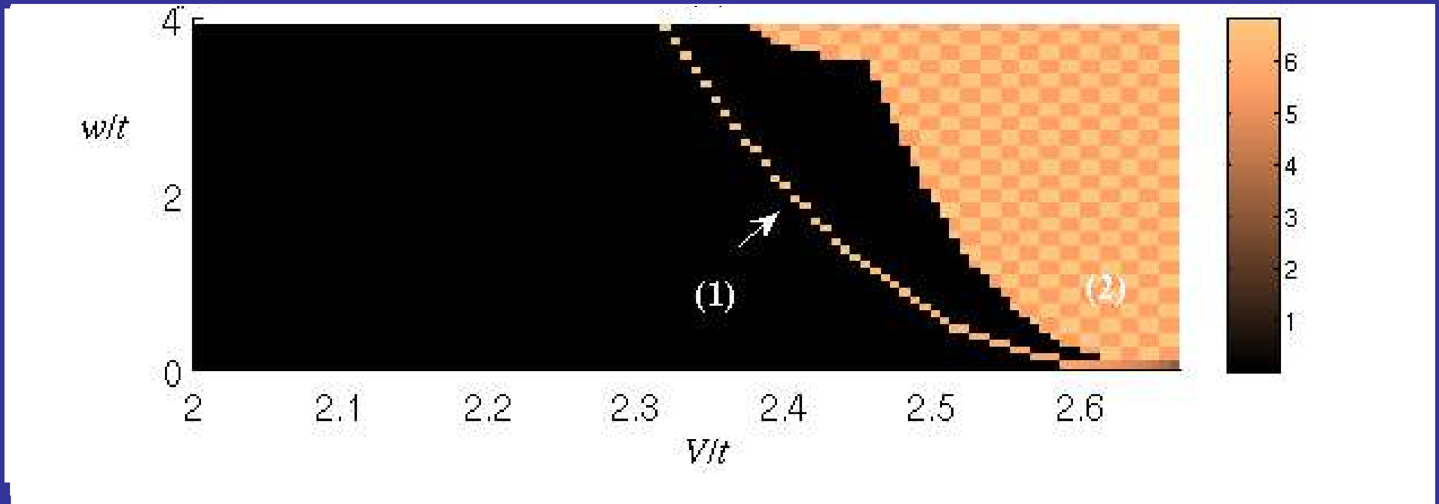
Absorption Spectrum

- Mean field phase diagram



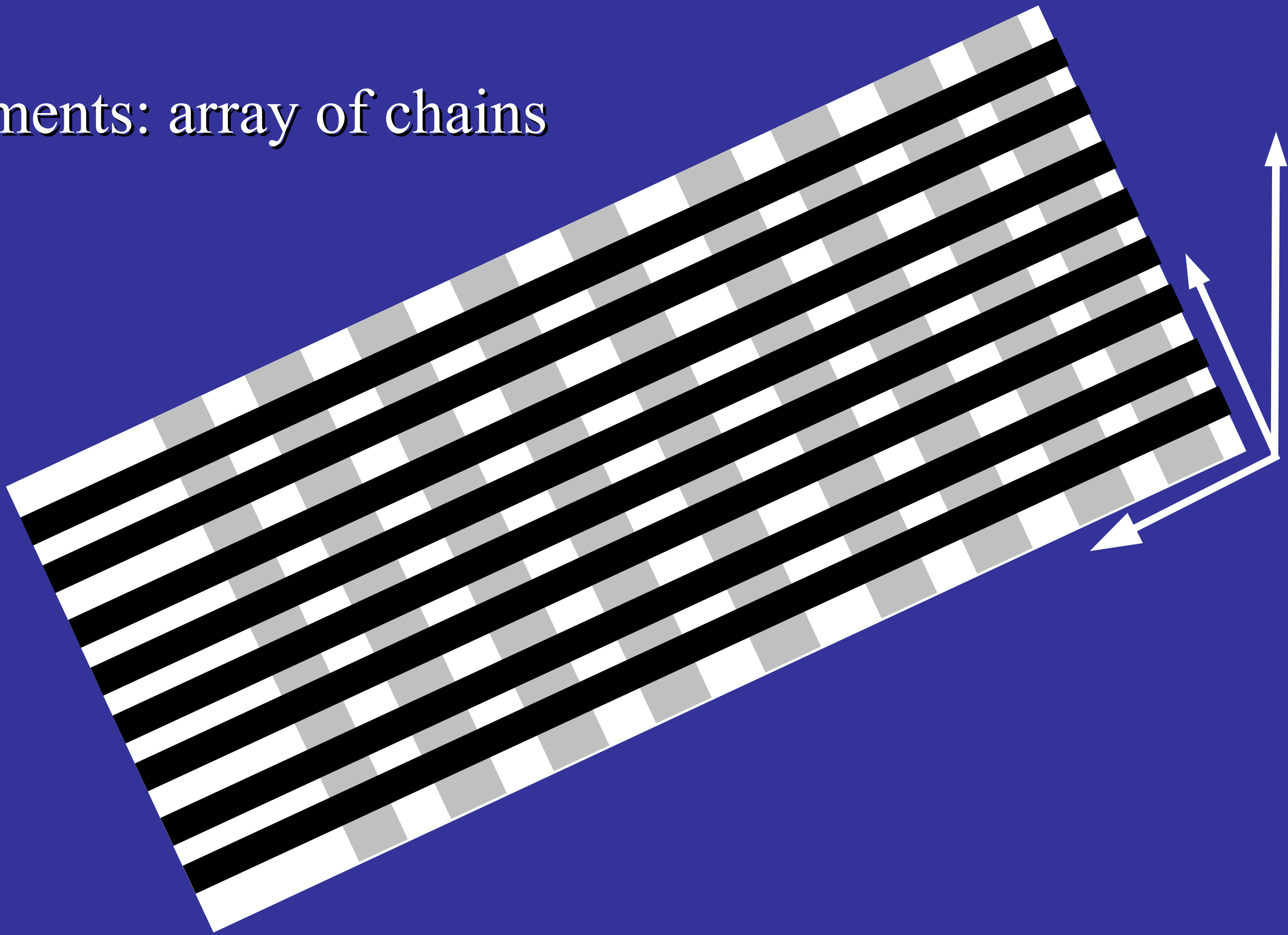
- Absorption Spectrum

Bragg Spectroscopy: $$\hat{V}_k(t) = e^{i\omega t} \sum_i e^{ikx_i}\ \delta n_i$$ (double wave-length)



2D systems

- Real experiments: array of chains

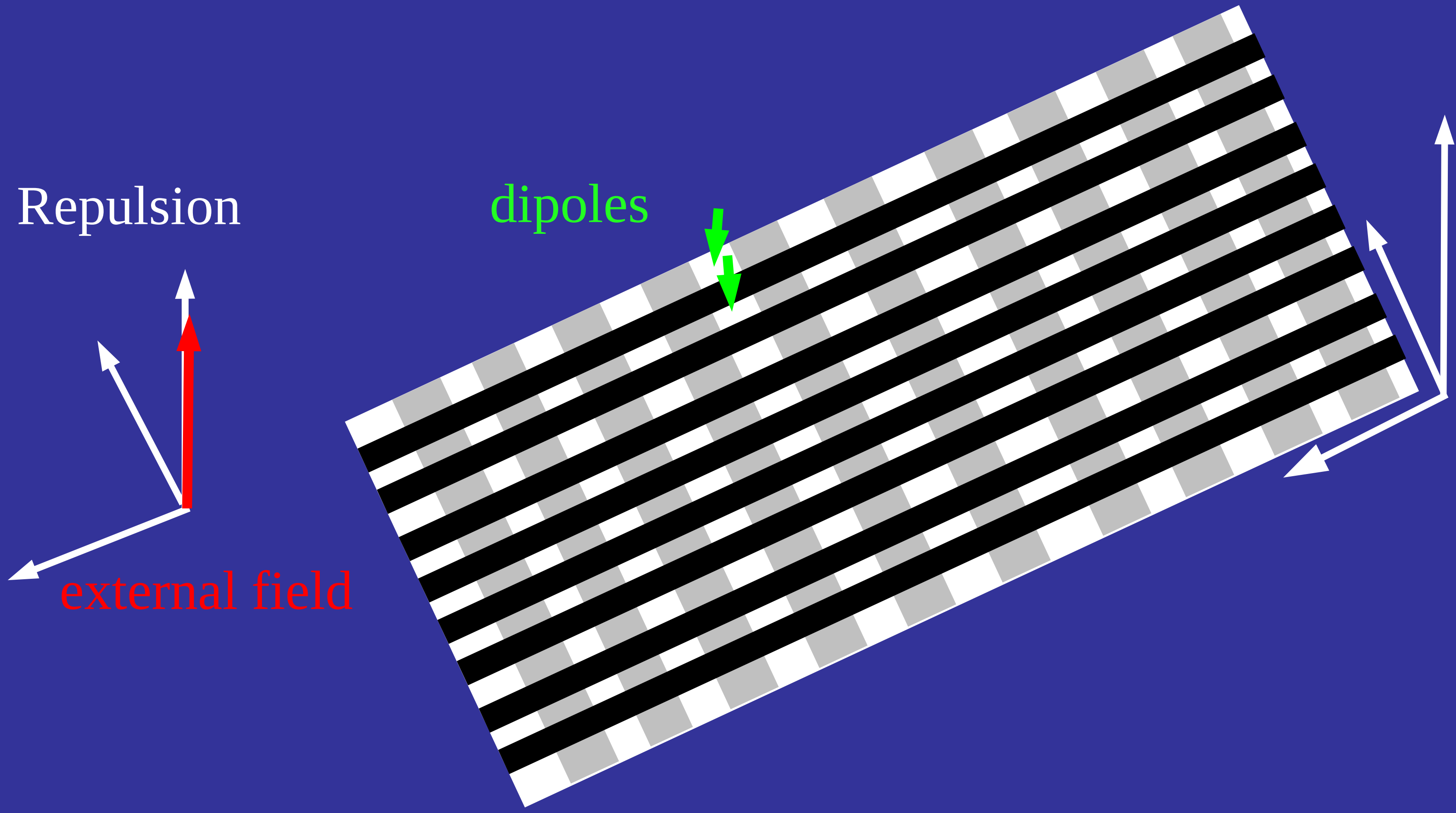


Interchain:

$t_\perp$ - tunneling ~ transverse laser intensity

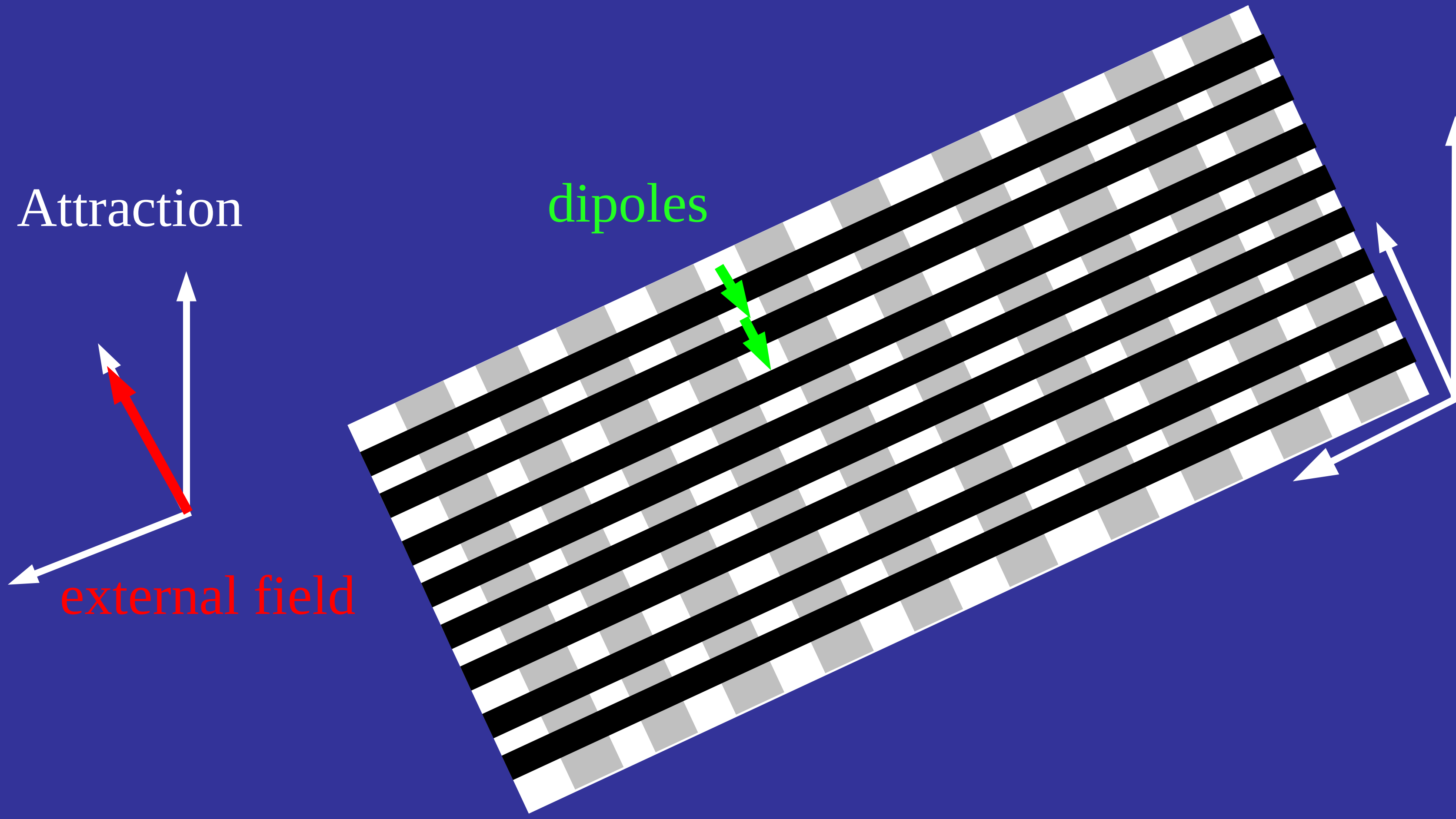
$V_\perp$ - dipolar interaction ~ direction of the external field

2D systems



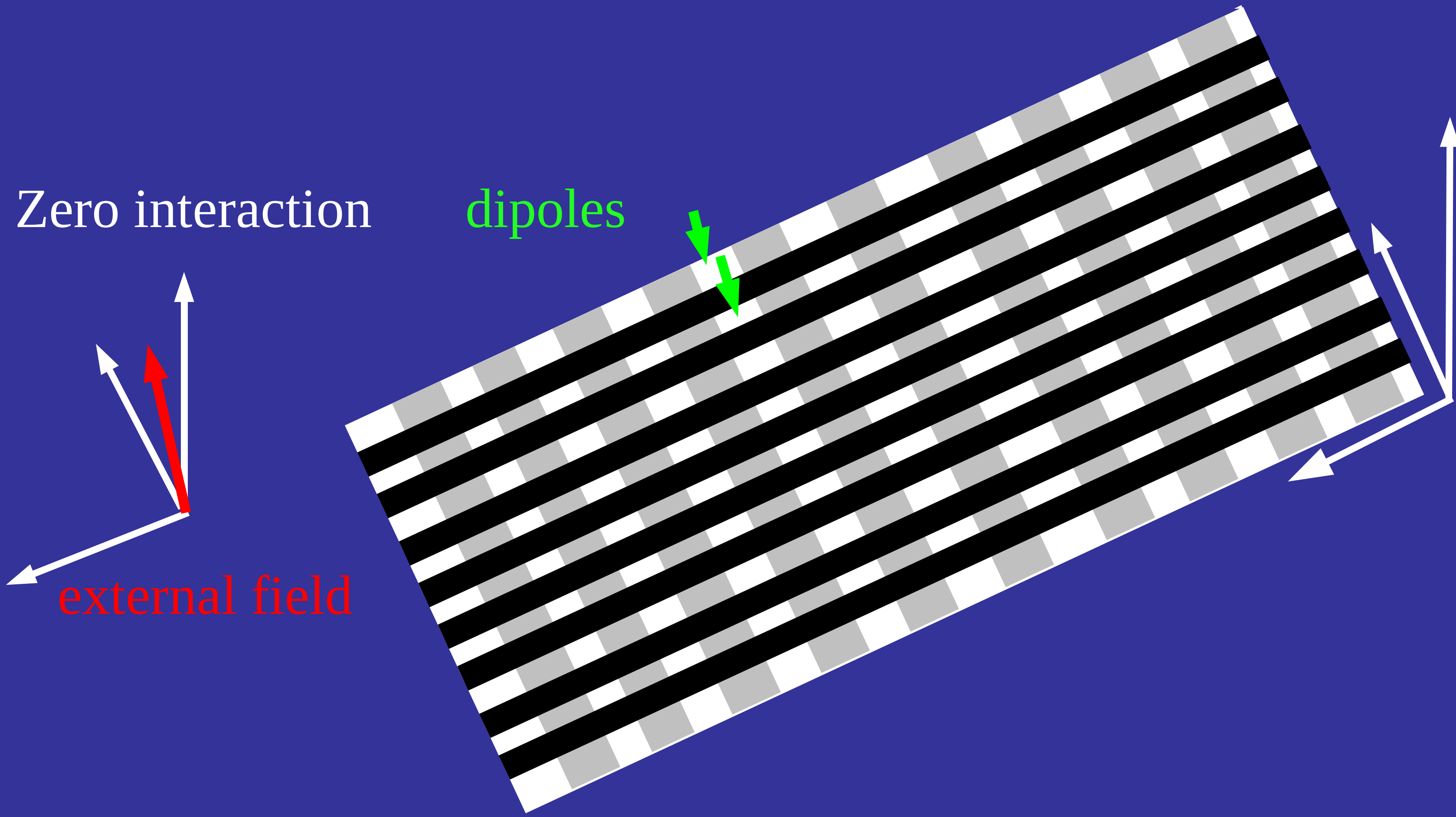
$V_\perp$ - dipolar interaction ~ direction of the external field

2D systems



$V_\perp$ - dipolar interaction ~ direction of the external field

2D systems



$V_\perp$ - dipolar interaction ~ direction of the external field

Interchain Coupling

$$H = H_0 + V_\perp n_{i,1} n_{i,2} + t_\perp \left(b^\dagger_{1,i} b_{2,i} + b^\dagger_{2,i} b_{1,i} \right)$$

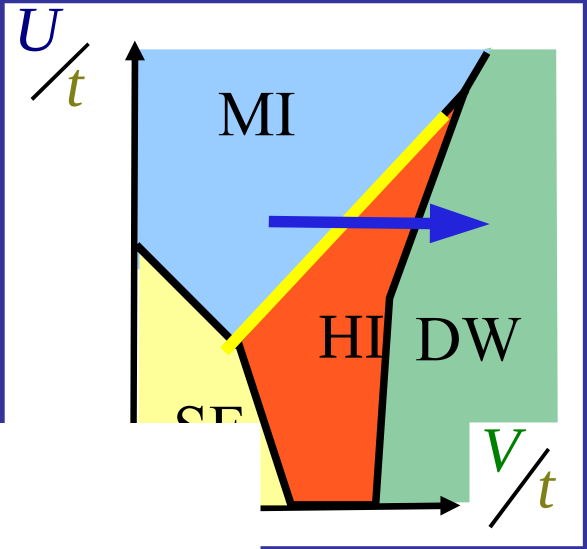
- HI – MI transition =

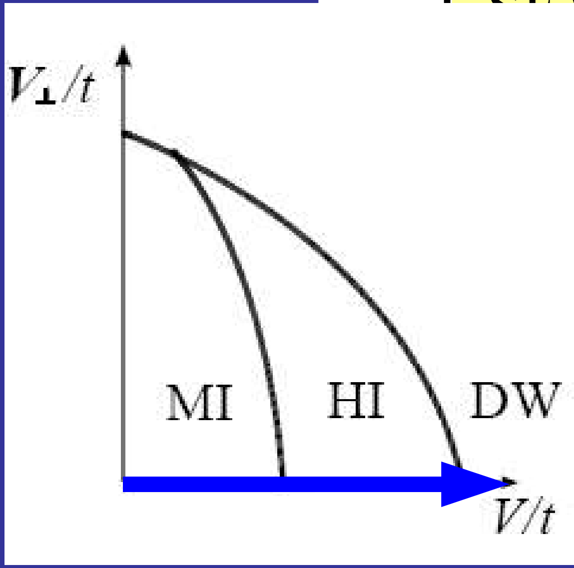
Luttinger Liquid critical line

- $V_\perp$ marginal parameter

$$K \to K' > K$$

shifts the transition





Scultz, PRB (1986)

Interchain Coupling

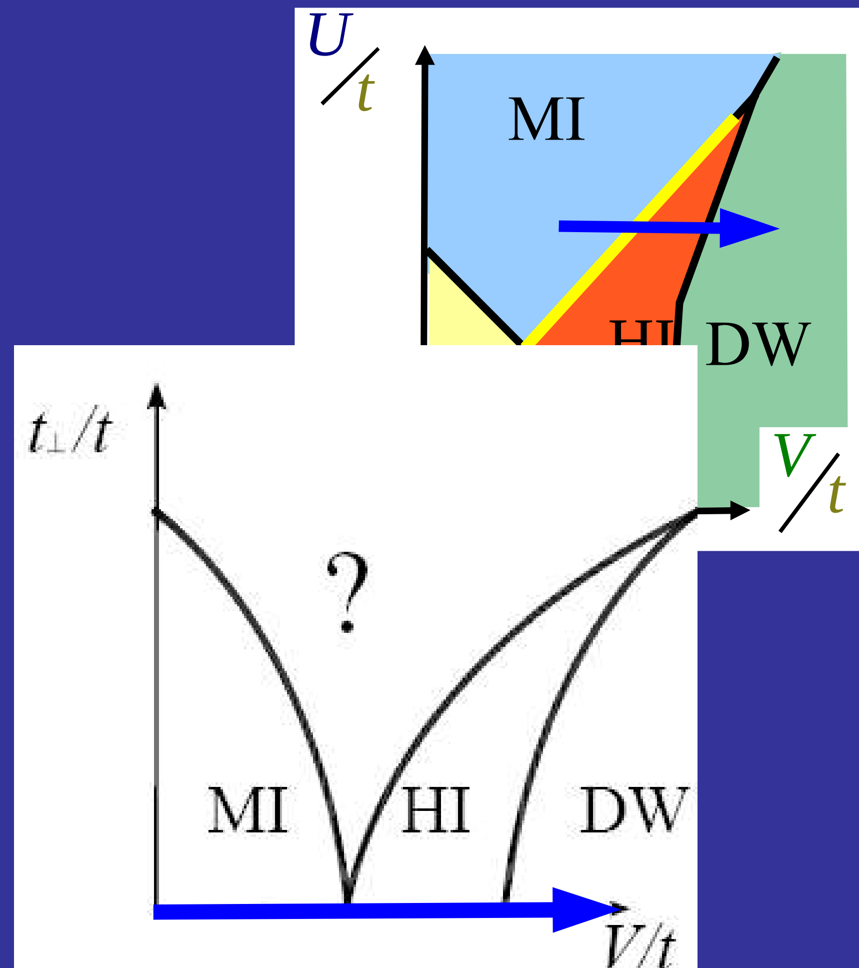
$$H = H_0 + V_\perp n_{i,1} n_{i,2} + t_\perp \left(b_{1,i}^\dagger b_{2,i} + b_{2,i}^\dagger b_{1,i} \right)$$

- HI – MI transition =

Luttinger Liquid critical line

- $t_\perp$ relevant parameter

opens a new phase?



Scultz, PRB (1986)

Conclusions

- Phase diagram of bosons with $1/r^3$ repulsion:
  a new insulating phase with string order.
- Proposed realization and detection
  with ultra cold dipolar bosons in optical lattice.
- Distinct neutral collective mode
- Open questions: 2D generalizations ?

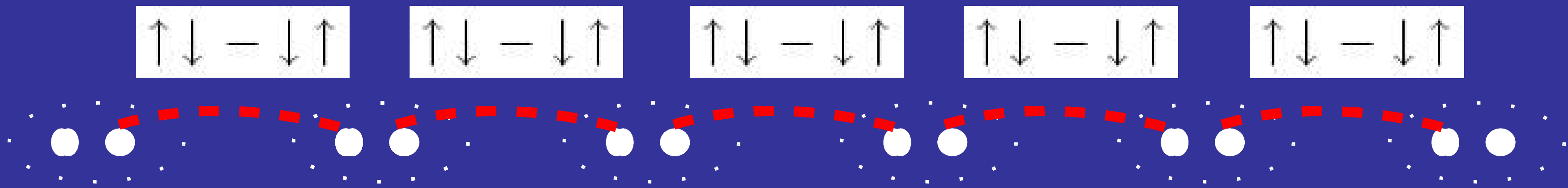
Thanks for the attention!

Phys.Rev.Lett, **97,** 260401 **(**31.12.2006)

EXTRAS

"AKLT" State for Spin-1 chain

adiabatically connected to the ground state of the Haldane phase



1. Split each "Spin-1" into two "Spin-1/2"

2. Connect neighbouring spins: Valence Bond Solid

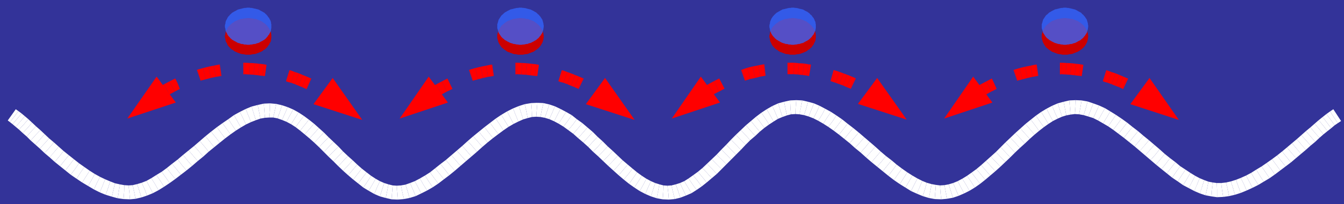
3. Project on "Spin-1" at each site ⟶ AKLT State

4. Parent Hamiltonian

$$H = \sum_i \vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3}\left(\vec{S}_i \cdot \vec{S}_{i+1}\right)^2$$

“AKLT” State for Bosons

adiabatically connected to the ground state of the Haldane Insulator



$$|\,\Psi_{GS}\,\rangle = \prod_j (b_j^\dagger + b_{j+1}^\dagger)\,|\,\Theta\,\rangle$$

- Parent Hamiltonian (after truncation)

$$H = \sum_i -\left(b_j^\dagger b_{j+1} + \mathrm{H.c.}\right) + n_j^2 + n_j n_{j+1} + \left(b_j^\dagger b_j^\dagger b_{j+1} b_{j+1} + \mathrm{H.c.}\right)$$

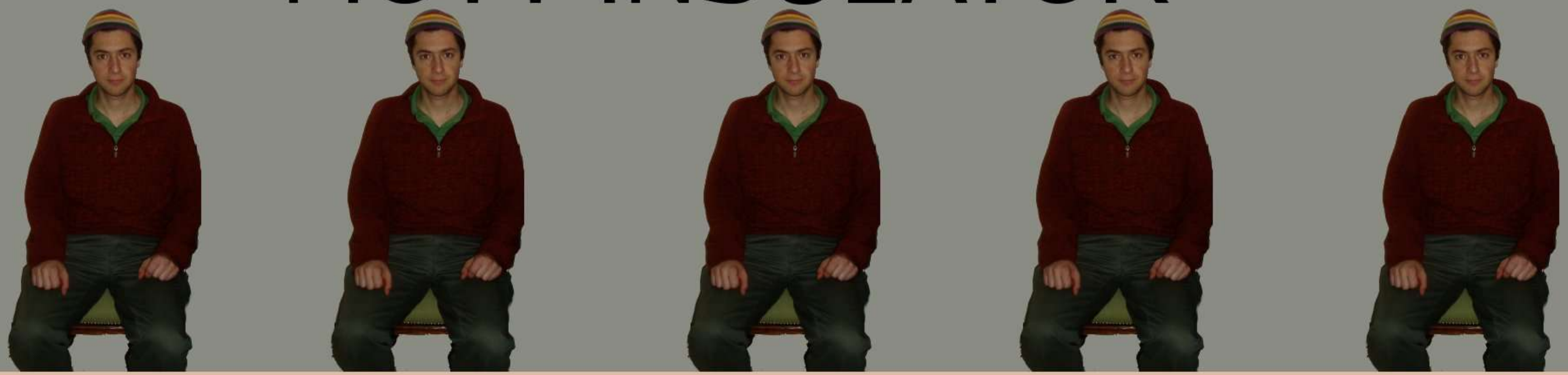
?

“AKLT” State

“...In a Mott Insulator there is an exact number of particles per site, while in a Haldane Insulator the bosons sit on the bond between two neighboring sites (quantum superposition of the sites)...”

CHAIR LATTICE

MOTT INSULATOR



HALDANE INSULATOR