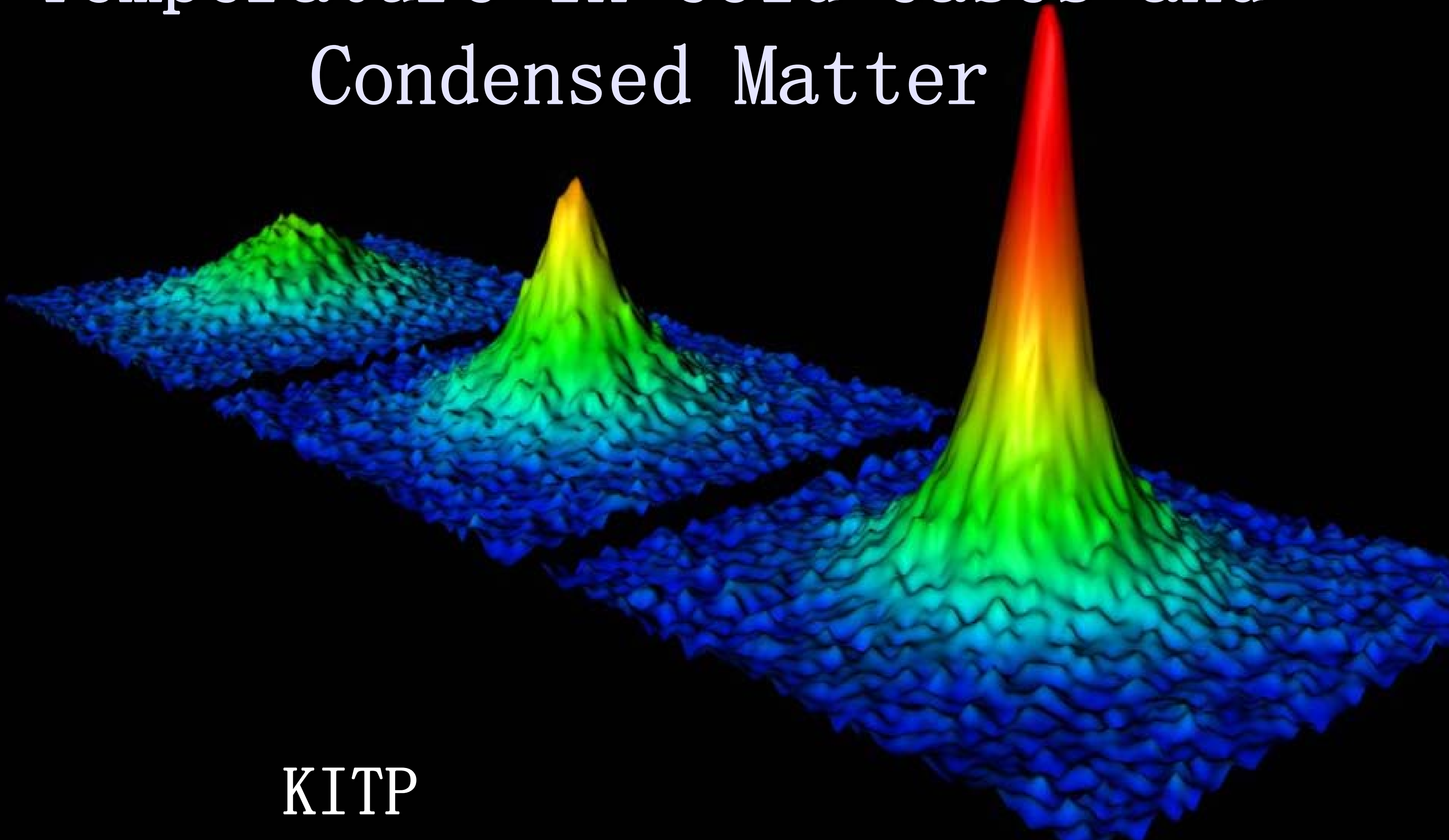
BCS-BEC Crossover at Finite Temperature in Cold Gases and Condensed Matter



KITP

May 2007

Cold Atom Collaborators: Qijin Chen

J. Stajic (U Chicago; LANL)
Yan He (U. Chicago)
ChihChun Chien (U. Chicago)

J.E. Thomas, J. Kinast, A. Turlapov (Duke)

D.S. Jin, C. Regal and M. Greiner (JILA)

M. Holland, M. Chiofalo, J. Milstein (JILA)

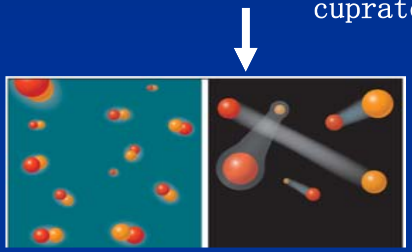
Outline of Talk

- Philosophy, motivation, requirements on theory.
- *First* generation experiments: The discovery phase in cold gases.
- *Second* generation experiments: Evidence for a pseudogap in cold gases.
- Theoretical Formalism with and without population imbalance.
- *Third* generation experiments: Polarized gases.
- *Fourth* generation experiments: Lattices (optical as well as high Tc).

Philosophy and Motivation

Motivation

- High Tc -- A. Leggett: "The small size of the cuprate pairs puts us in the intermediate regime of the so-called BCS-BEC crossover" (2006).



- To understand cold Fermi gas expts; opportunity to arrive at counterpart to Gross Pitaevskii theory.

- Opportunity to generalize the paradigm of all condensed matter theories: BCS theory.

- Novel form of fermionic superfluidity: pairing without condensation.

Philosophy

- Use BCS-Leggett wavefunction for T=0

$$\Psi_0 = \Pi_{\mathbf{k}}(u_{\mathbf{k}} + v_{\mathbf{k}} a^{\dagger}_{\mathbf{k}} a^{\dagger}_{-\mathbf{k}})|0\rangle$$

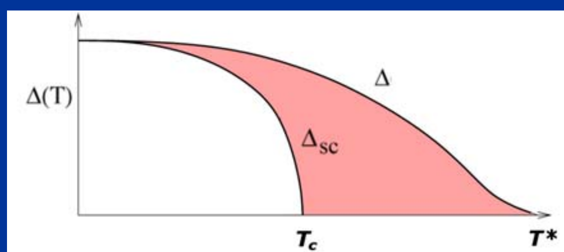
- Why? Wavefunction is basis for
  - Bogoliubov de Gennes theory (T=0).
  - T =0 Gross Pitaevskii Theory in BEC regime.
  - Unequal population theories.
  - Has simplicity and physical accessibility.

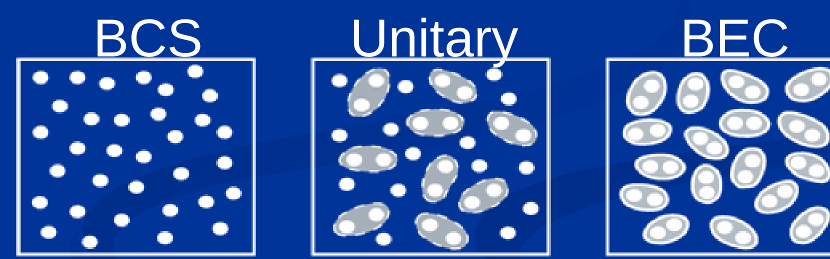
Essential Criteria for Successful Finite T Crossover Theory

- Must include pseudogap effects

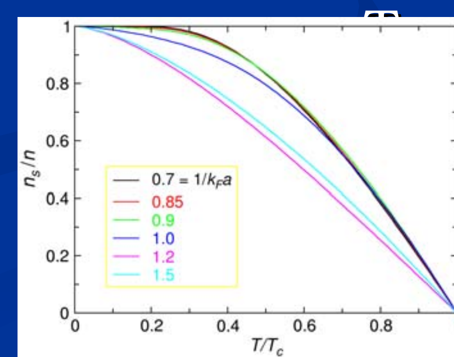
$$T^* \neq T_c \qquad \Delta(T) \neq \Delta_{sc}(T)$$

Character of Excitations





- Superfluid density must be well behaved for all T from



Comparison with Other Finite T Theoretical Approaches

- Nozieres, Schmitt-Rink (NSR) includes pairing fluctuations only in number equation. No inclusion of pseudogap in gap equation. They take

$$T^* \neq T_c \qquad \Delta(T) = \Delta_{sc}(T)$$

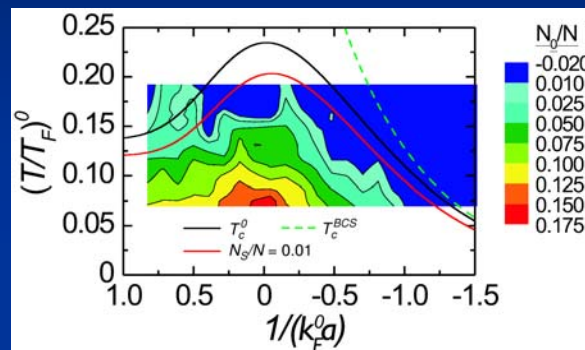
- Finite T NSR not designed to yield Leggett ground state.

- No other theory finds proper superfluid density over entire range of T. Report first order transitions (Zwerger-Haussmann), or double valued functions (Griffin) or breakdown of theory below Tc (Strinati).

First Generation Experiments

Evidence for superfludity at Unitarity

JILA Unitary Phase Diagram involves two sweeps (2004)



BEC

Unitary limit

BCS

→ ←

ΔB

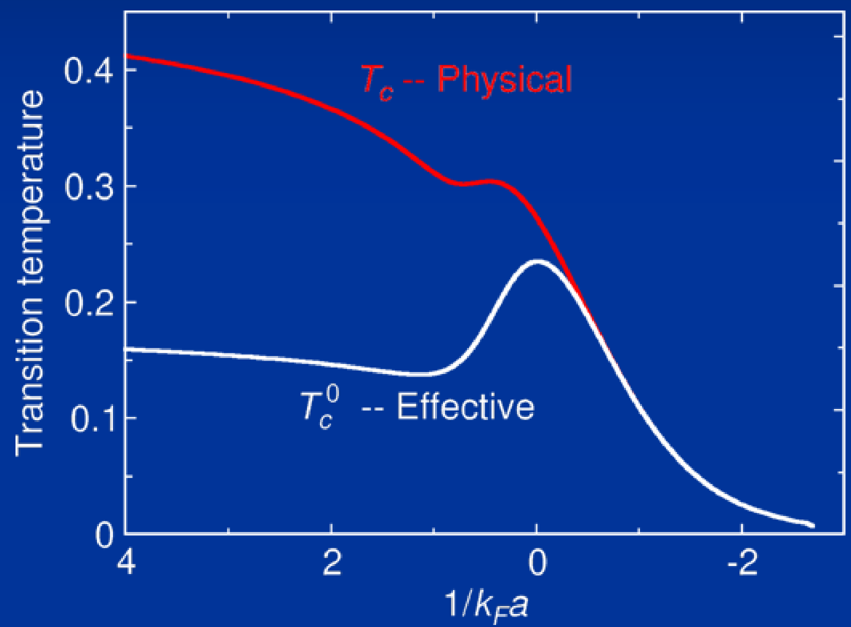
Quick sweep

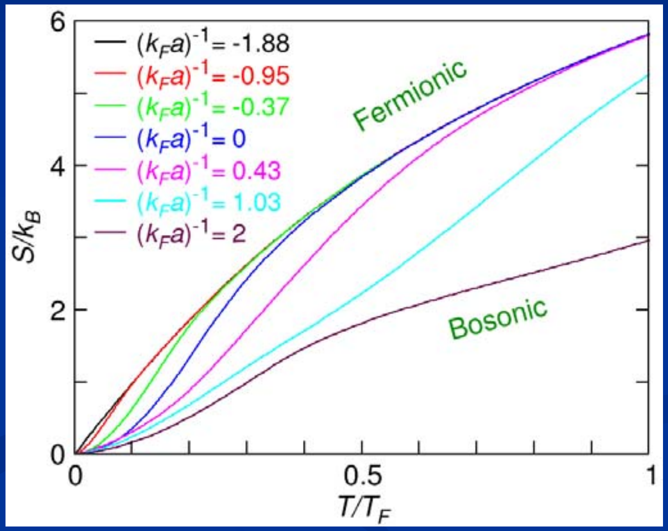
(Measures condensate)

molecules

Adiabatic sweep (sets temperature)

Theory of Adiabatic Sweep Thermometry

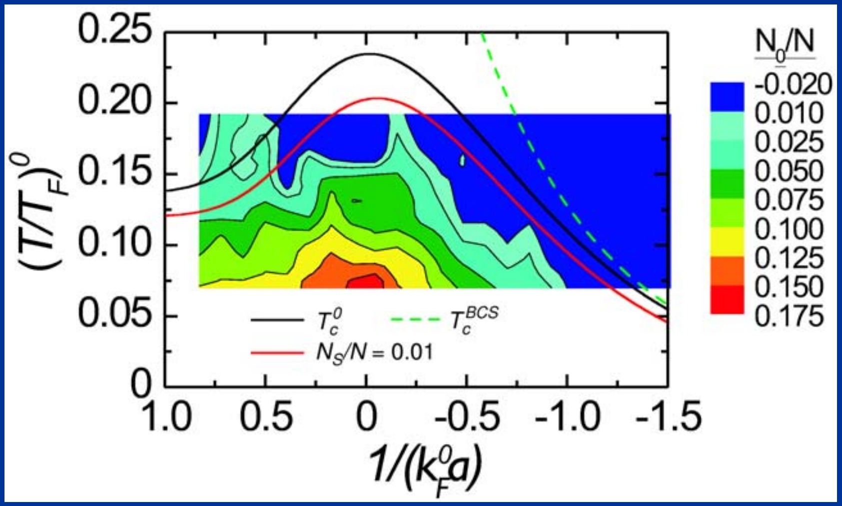




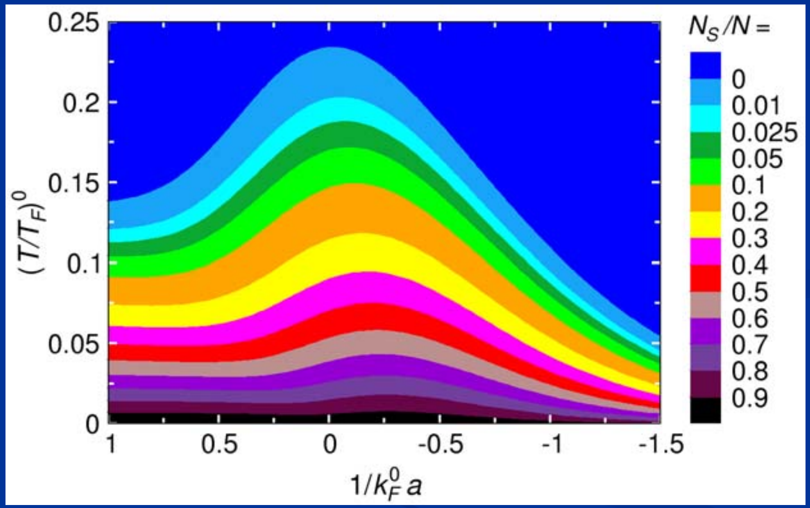
- Adiabatic cooling leads to lower effective temperature.
- *Tc* curve changes shape when projected onto temperatures measured in the noninteracting limit.

Comparison of Theoretical and Experimental Phase Diagram

Jin et al



Theory

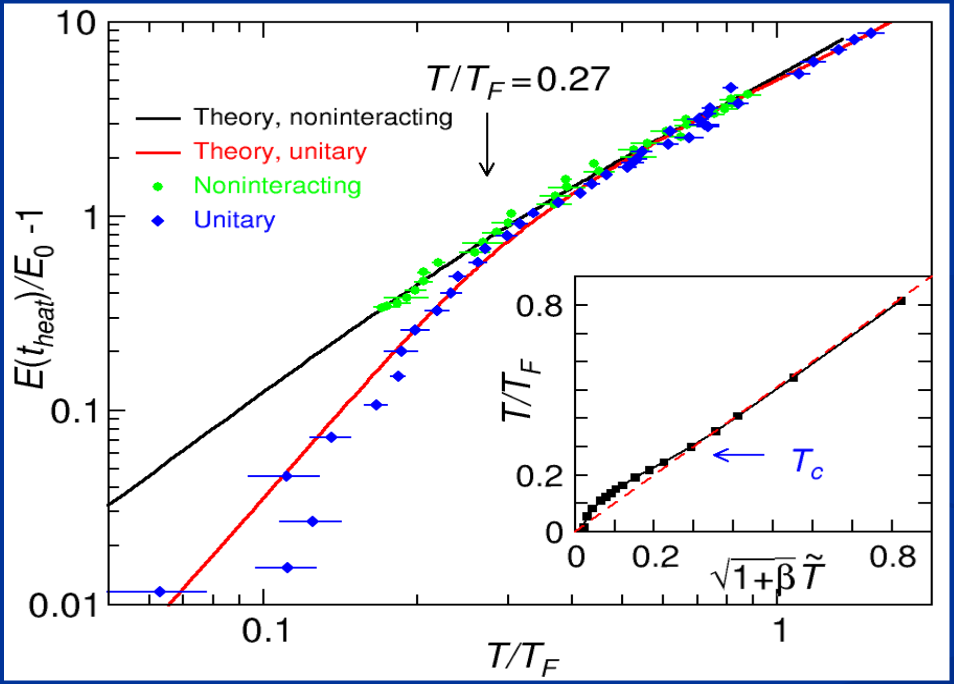


Equilibrium phase diagram

Our collaboration with JILA group.

PRA 73, 041601(R) (2006)

Thermodynamical Evidence for Phase Transition near unitarity Profiles used to Calibrate T.



Science 307, 1296

Our collaboration with Duke Group: John Thomas, Joe Kinast, Andrey Turlapov--- Feb 2005

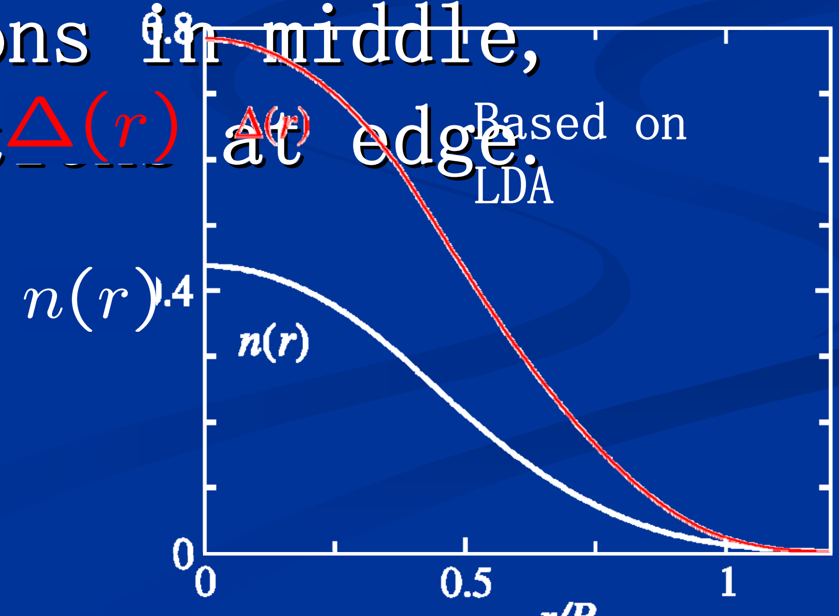
Second Generation Experiments

Evidence for a pseudogap

(i.e., pairing without condensation)

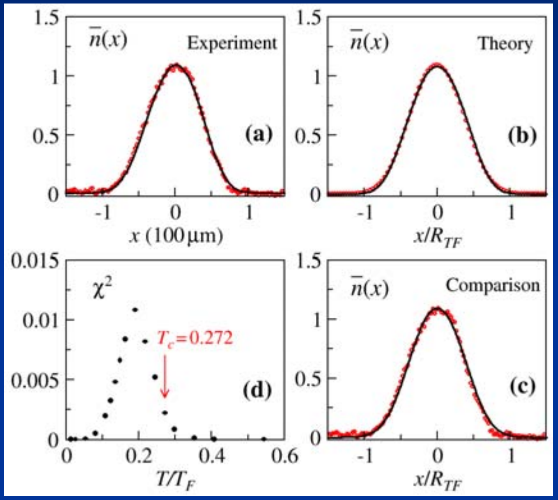
Backdground: what goes on inside a trap?

- Particle density peaked at trap center.
- Gap decreases from center to edge: bosonic excitations in middle, fermionic excitations at edge.

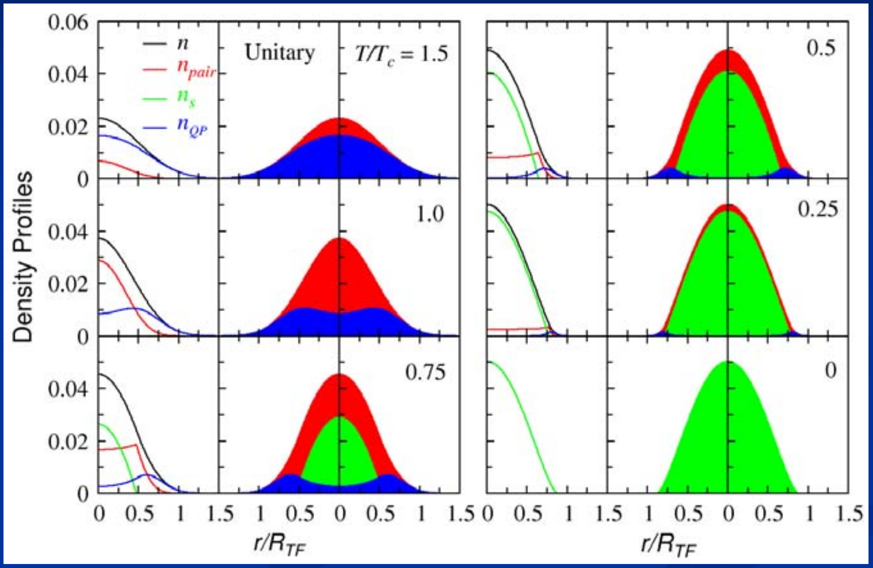


Based on LDA

Density Profiles and Pseudogap Effects as Condensate (Pair) Excitations



PRL 95, 260405 (2005)



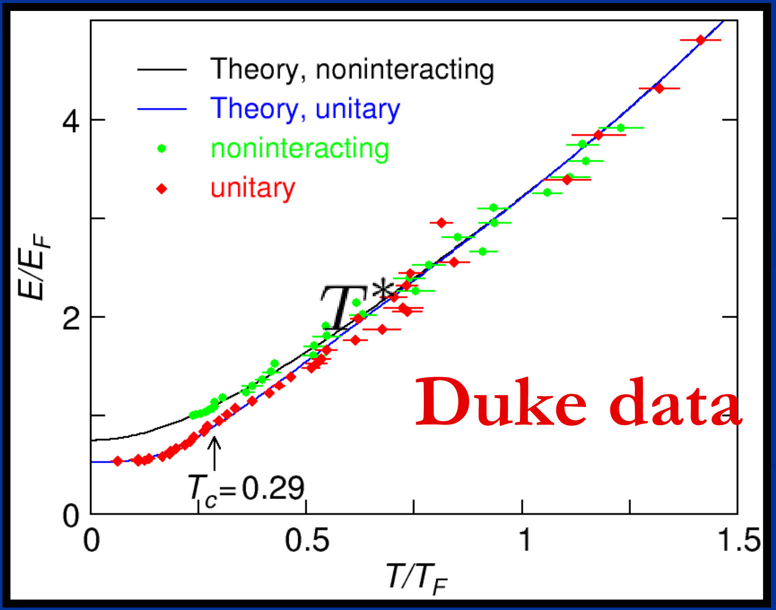
Condensate Noncondensed pairs Fermions

At unitarity, pair excitations smooth out profiles—making it hard to tell if system is normal or superfluid.

Thermodynamics and Pseudogap effects

Energy vs T

Theory and expt.



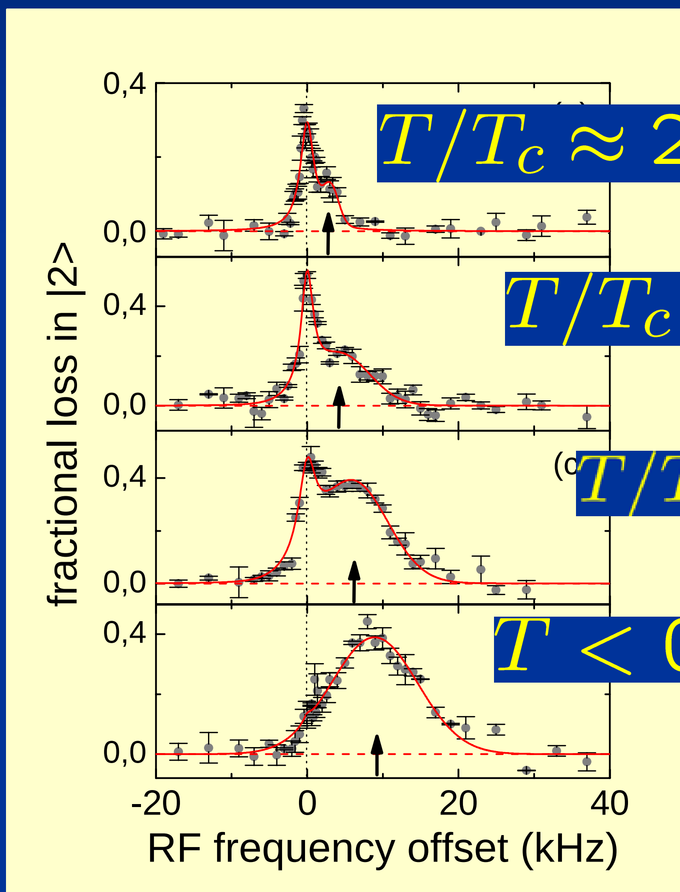
In data, T* appears as temperature where 2 curves meet

Science 307, 1296 (2005)

RF Spectroscopy and Pseudogap Effects

C. Chin et al, Science 305, 1128 (2004).

Temperatures inferred from R. Grimm updates

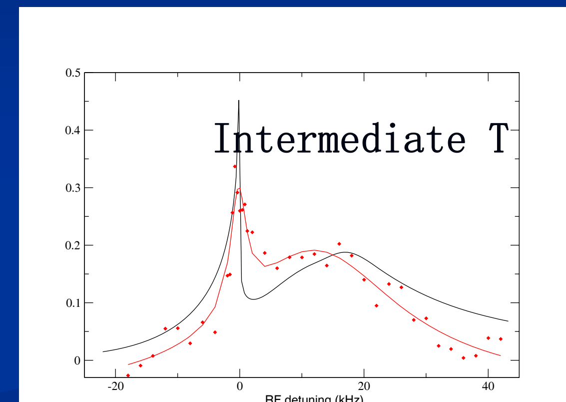


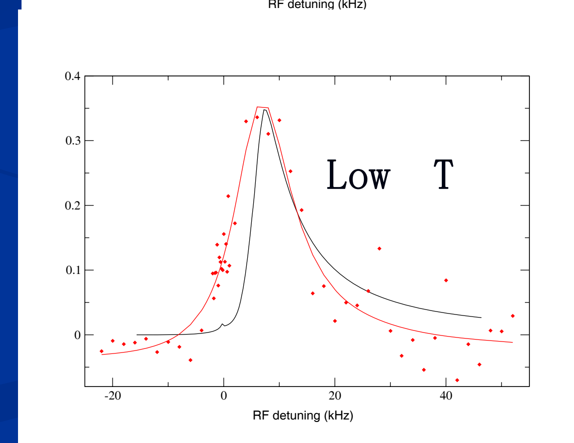
$T/T_c \approx 2 \approx T^*/T_c$

$T/T_c \approx 1.5$

$T/T_c \approx 1$

$T < 0.03T_c$





pseudogap is evident as $T > T_c$ *shoulder!*

Theoretical Formalism

BCS-BEC Crossover at Finite Temperatures

General approach to address finite temperature in BCS-BEC Crossover: *T*-matrix scheme

- Treat pair propagators (t) and particles (G) self-consistently. No higher order correlations.
- Important: this means that inter-boson interactions only treated at mean field level.
- Solve coupled equations for two propagators: G and t.

Three Possible T-matrix approaches

$pair\ \ propagator$

$$t(Q) = \frac{U}{1 + U\chi(Q)} \propto \frac{1}{\Omega - \Omega_q + \mu_{pair} + i\Gamma_q}$$

NSR pair susceptibility: $\chi = \Sigma G_0 G_0$

Present work: $\chi = \Sigma G G_o$

Associated with BCS-Leggett ground state

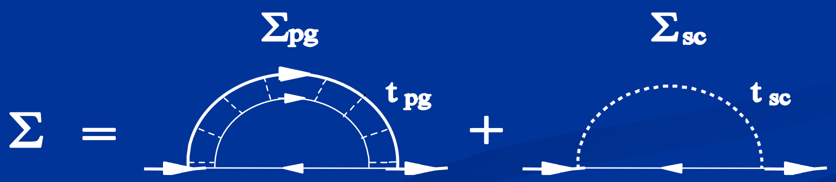
Haussmann takes $\chi = \Sigma G G$

Diagrammatic Formalism Based on BCS-Leggett Ground State

- *T*-matrix $t_{pg}(Q) =$ 

  propagator for non-condensed pairs (pg)

- Fermion self-energy:



Note:

$$1 + U\chi(Q) = 0 = \mu_{pair}$$

$$(T \le T_c)$$

$$\Sigma = -\Delta^2 G_0$$

$$\Delta^2 = \Delta_{pg}^2 + \Delta_{sc}^2$$

Self-consistent Equations Below Tc

- Gap equation: $\longleftrightarrow$ BEC condition $\mu_{pair} = 0$

$$1 + U\chi(0) = 0$$

$$1 + U \sum \frac{1 - 2f(E_k)}{2E_k} = 0 \qquad E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta^2}$$

- Pseudogap equation: Pair density (Boson number)

$$\Delta_{pg}^2 = - \sum_{Q \neq 0} t(Q)$$

$$\Delta^2 = \Delta_{pg}^2 + \Delta_{sc}^2$$

- Number equation(s)

Summary

Composite bosons

Ideal Point bosons

- Pair chemical potential:

$$\mu_{pair} = 0, \quad T \le T_c$$

$$\mu_B = 0, \quad T \le T_c$$

Leads to BCS gap equation for $\Delta(T)$.

- Total ``number" of pairs

$$\Delta^2(T) = \Delta_{sc}^2 + \Delta_{pg}^2$$

$$N = N_0 + N_T$$

- Noncondensed pairs:

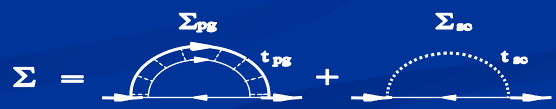
$$\Delta_{pg}^2 = \sum \mathrm{Im}\, t(Q) \; \propto \; \sum_{\mathbf{q}\neq \mathbf{0}} b(\Omega_q)$$

$$N_T = \sum_{\mathbf{q}\neq \mathbf{0}} b(\epsilon_{\mathbf{q}})$$

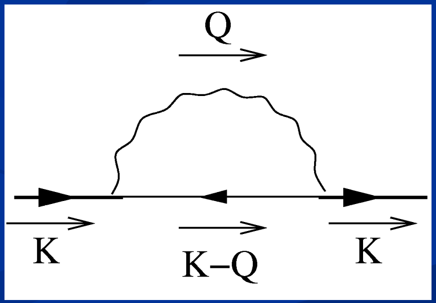
Superfluid density vanishes at same Tc as found from "gap equations"

Because of Ward Identity, pg effects do not lead to Meissner effect

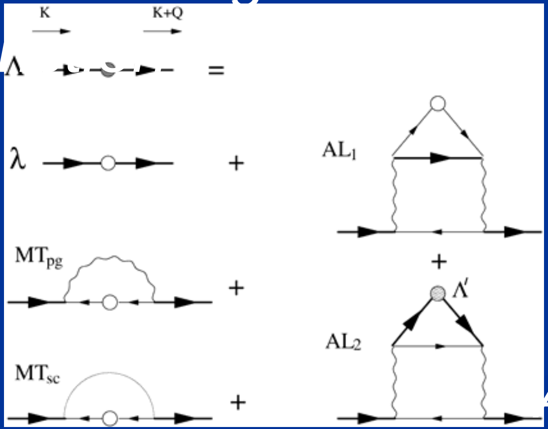
$$n_s = \frac{\Delta_{sc}^2}{\Delta^2} \quad n_s^{BCS}(\Delta)$$



$$\Sigma_{pg} =$$



Electromagnetic Vertex fu

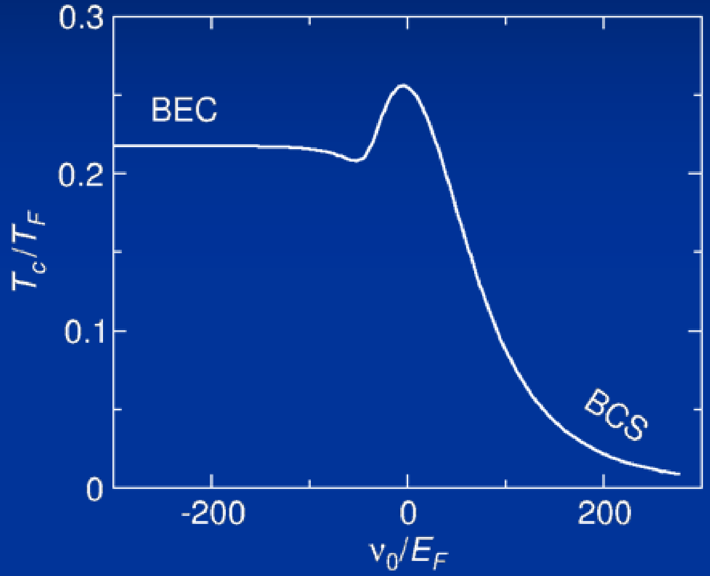


MT= Maki-Thompson

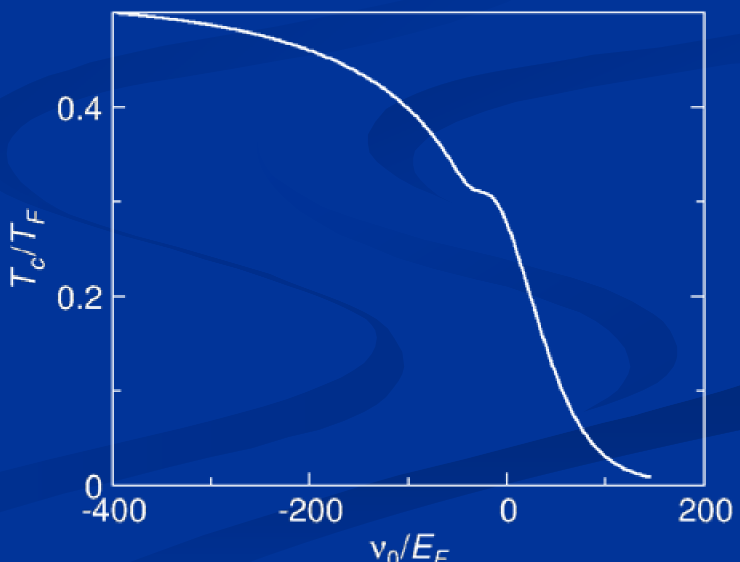
AL= Aslamazov-Larkin

Critical Temperatures

Homogeneous case:



Trap Case within LDA:



$$\mu \longrightarrow \mu - V^{trap}(r)$$

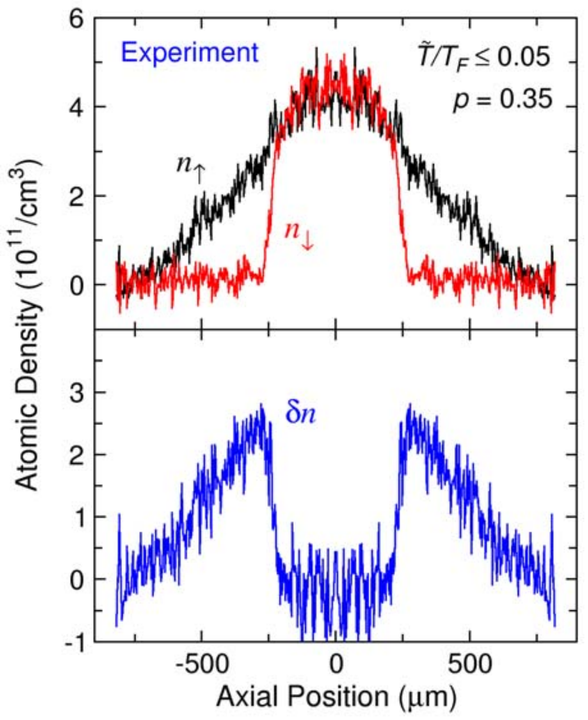
Third Generation Experiments

Polarized Gases

Three Ways to accommodate polarization

- Phase Separation
- Breached Pair (Sarma) State
- FFLO (in principle)

Rice data



Real space phase separation:

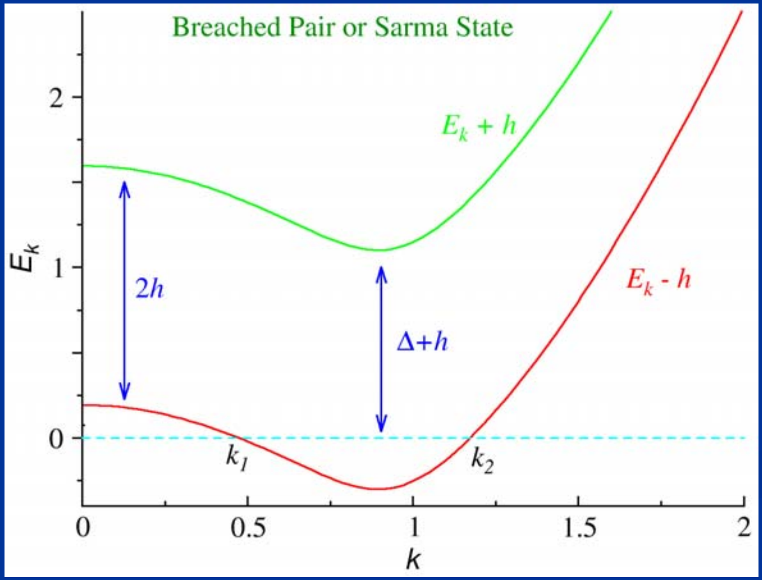
Superfluid core followed by polarized normal fluid

Breached Pair or Sarma State

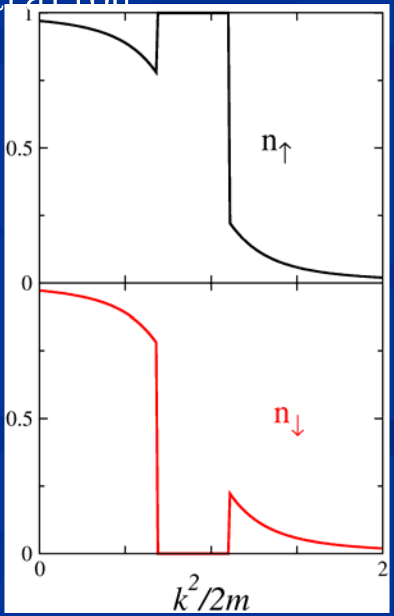
$$G_{\uparrow,\downarrow}(K) = \frac{u_{\mathbf{k}}^2}{i\omega_n \pm h - E_{\mathbf{k}}} + \frac{v_{\mathbf{k}}^2}{i\omega_n \mp h + E_{\mathbf{k}}}$$

$$E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta^2}, \quad \mu = \frac{1}{2}(\mu_\uparrow + \mu_\downarrow),\ h = \frac{1}{2}(\mu_\uparrow - \mu_\downarrow)$$

Gapless excitation spectrum

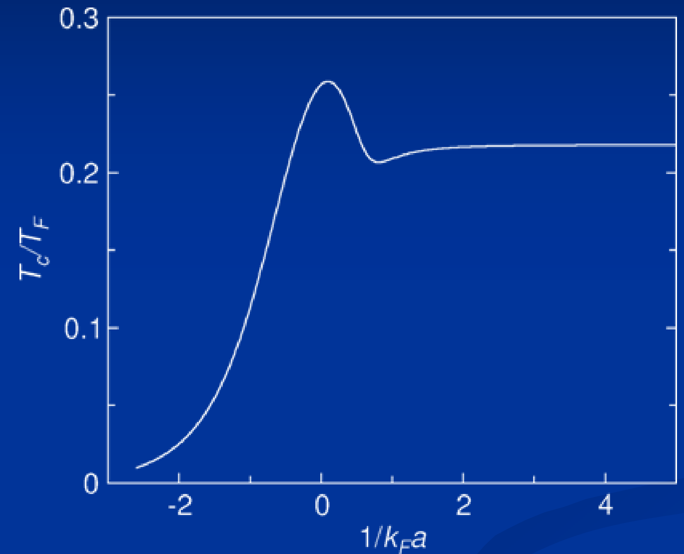


k-space "phase separation"



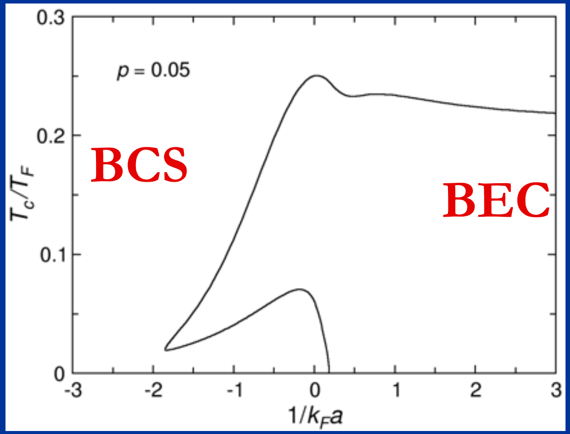
Sarma-Tc in homogeneous system: Unpolarized and Polarized Case

Away from BEC,
Sarma only stable
at finite T.

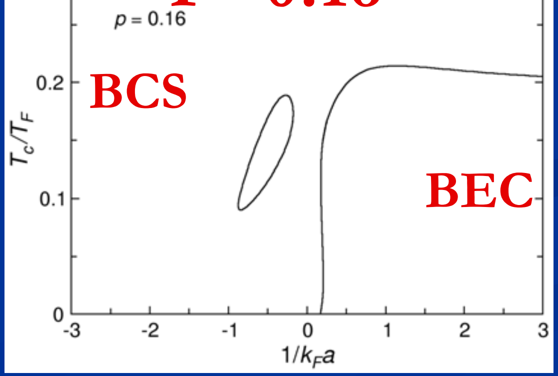


PRL 97, 090402

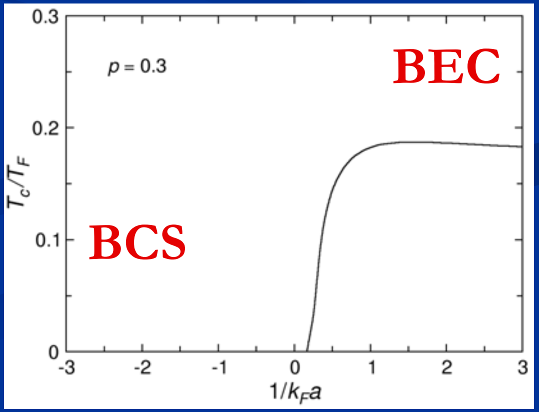
P=0.05



P=0.16



P=0.3

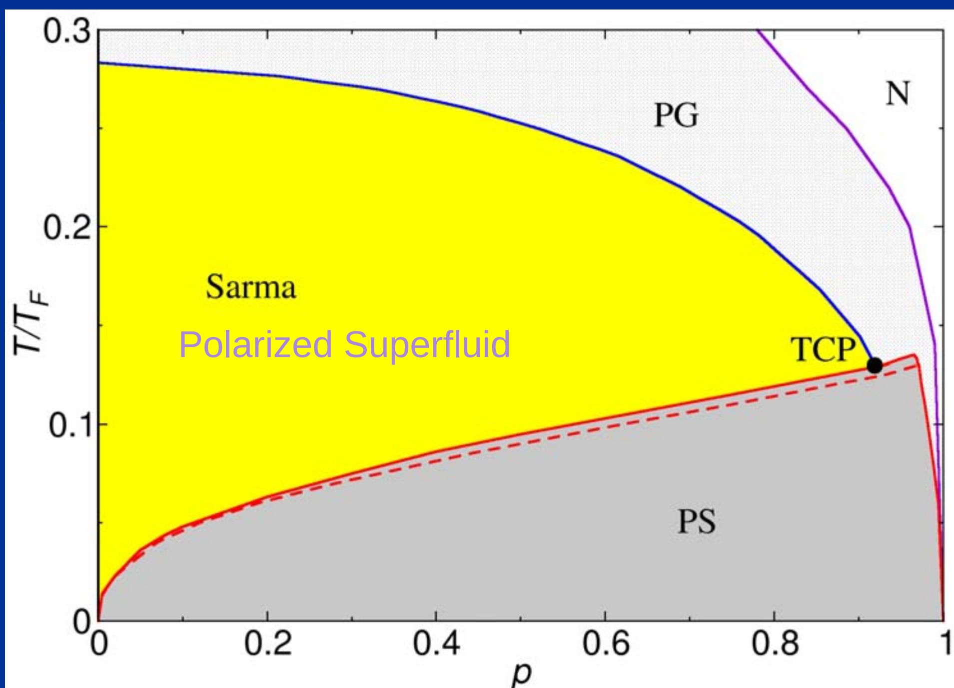


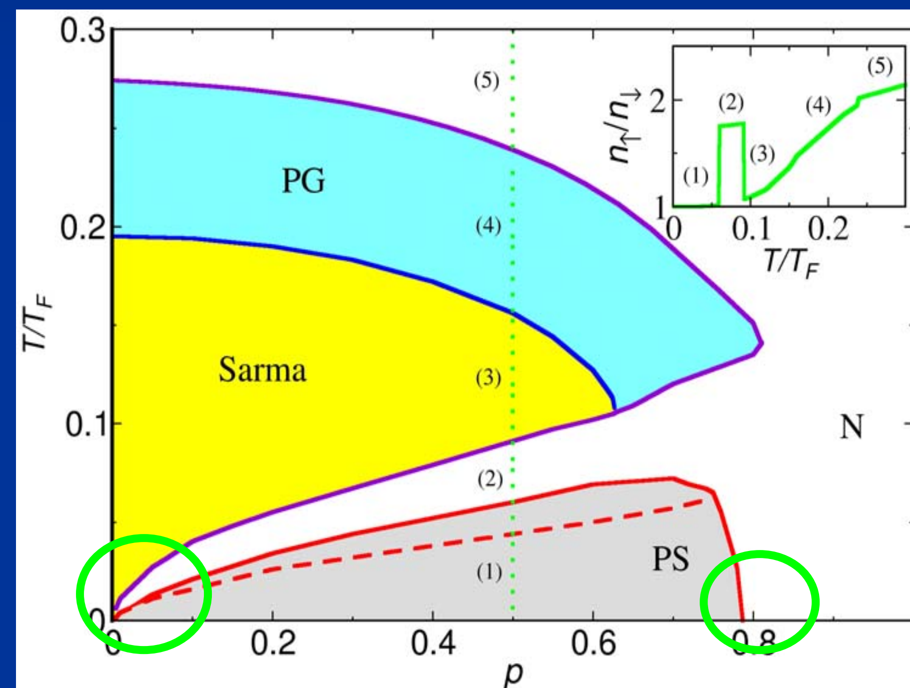
Trap Effects: Population imbalance phase diagrams

Unitary: $1/k_F a = 0$

BCS: $1/k_F a = -0.5$

BEC:



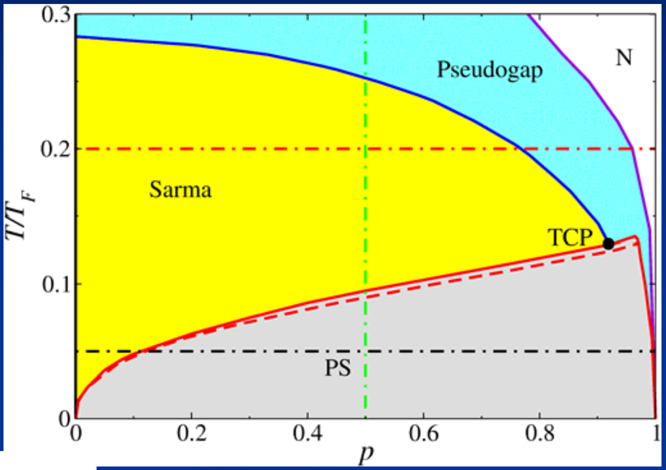


BCS case: Phase separation and Sarma states retreat from each other leaving possibility of new intervening state. FFLO, or normal,… ?

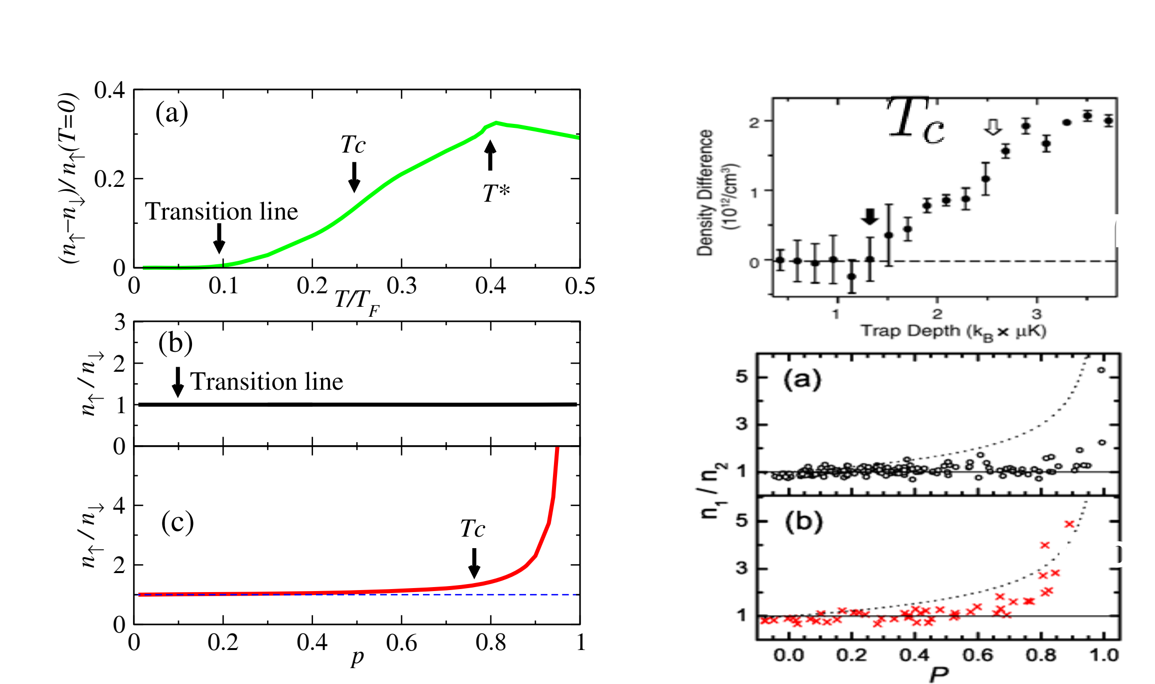
Solid lines separate different phases.

Comparing Theory and Experiment in Polarized Gases

- Unitary phase diagram:



-



T- sweeps in T

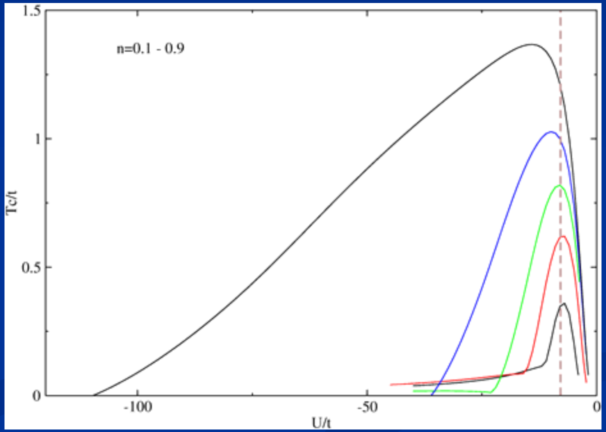
The

p- sweeps in p

Fourth Generation Experiments: Optical Lattice Effects and High Tc

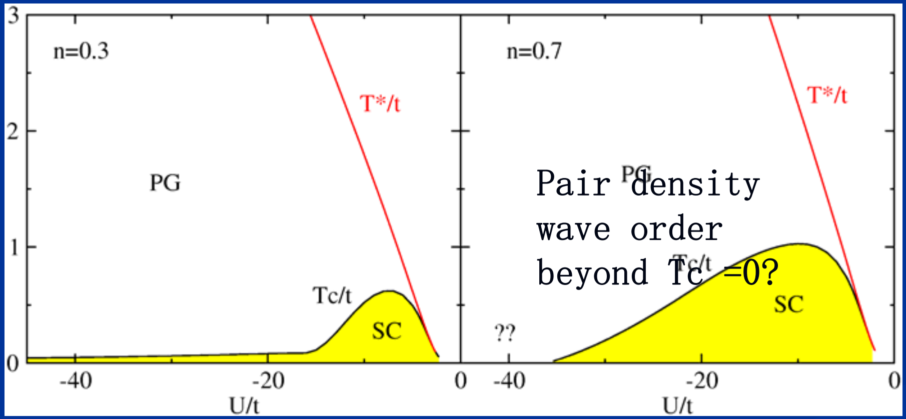
Predictions for Optical Lattices: Attractive Hubbard Model (BCS-Leggett wave function with well behaved superfluid density)

- Behavior of Tc for variable fermion density:



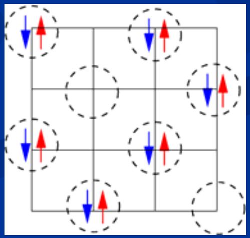
Mott-Like Effect: Pairs localize for moderate but non-integer n. Due to Pauli repulsion which inhibits pair hopping.

- Phase Diagrams:



Pair density wave order beyond Tc =0?

Shows T=0 (density induced) pseudogap phase.

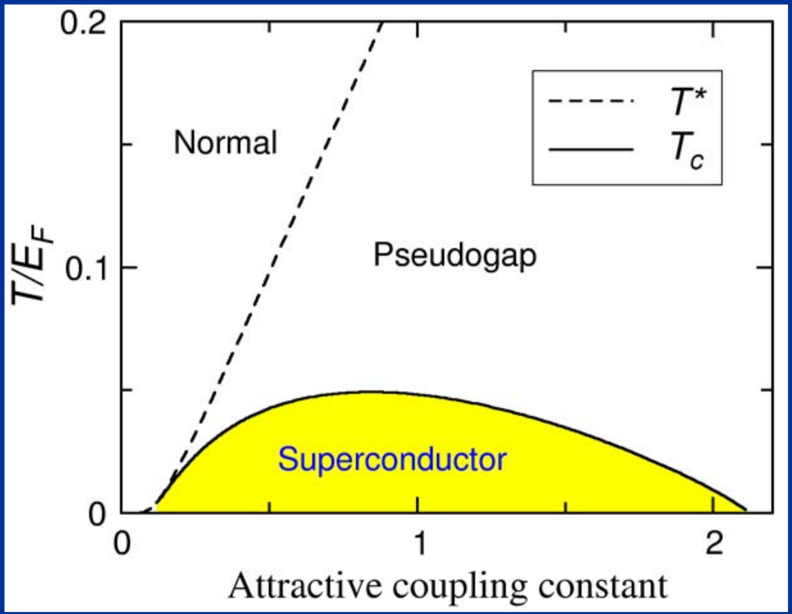


$$T_c = 0, \quad T^* \neq 0$$

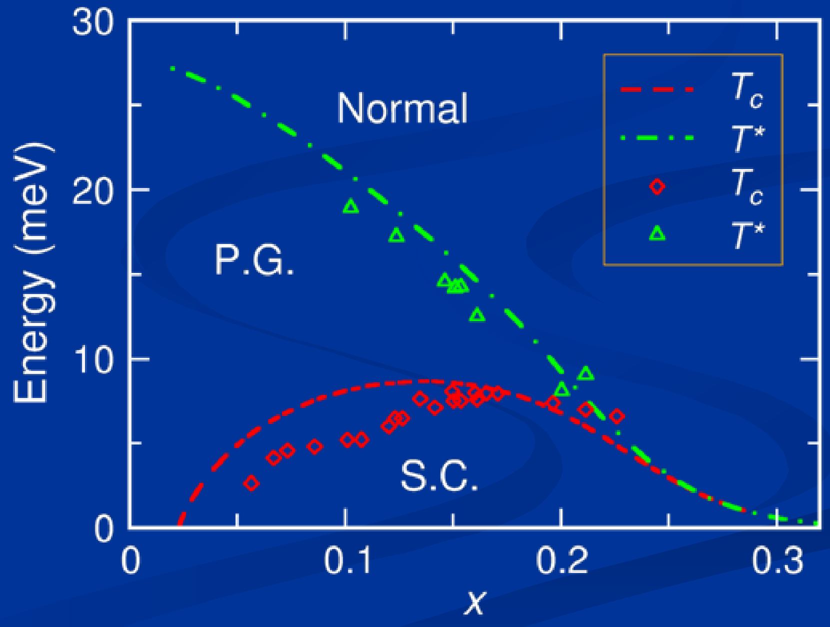
Applying Crossover Theory to d-wave Lattice case

.

Theory



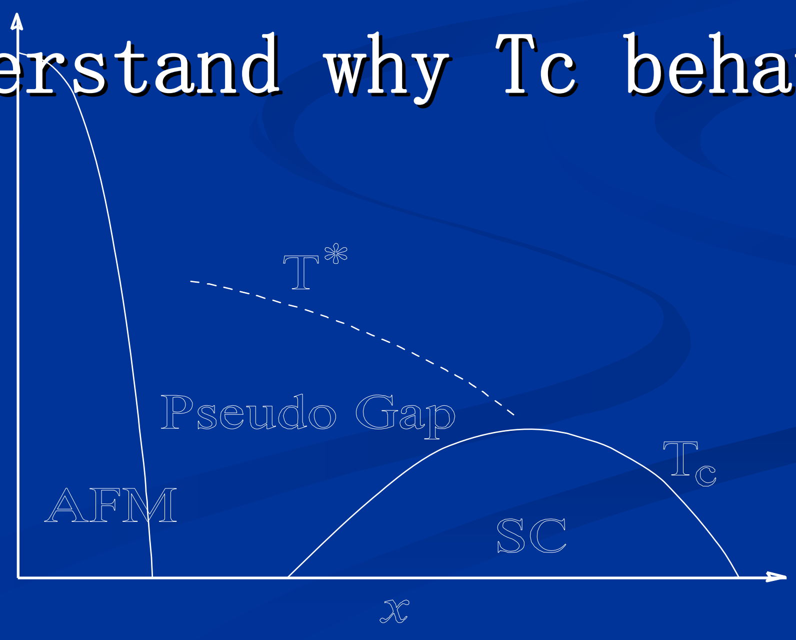
Fit T(x)*



Pairs localize and Tc vanishes well before BEC.

Where is Mott Physics?

- Attractive interaction derives from "Mott physics" since pairing interaction gets stronger with underdoping, as seen by T*.
- We can now understand why Tc behaves oppositely.

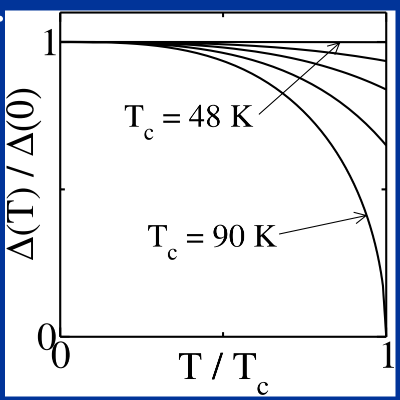


Trying To Understand the Superfluid Density in Cuprates

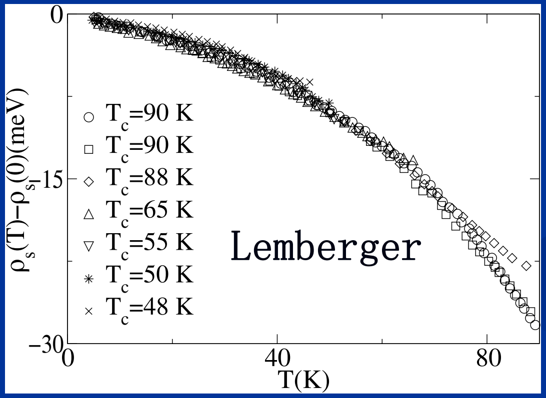
- **Paradox: superfluid density does not directly reflect the (x,T) behavior of the fermionic gap.**

Temp dependence of the gap.

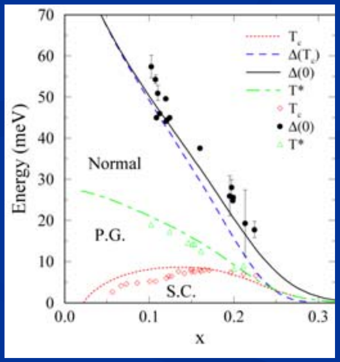
a)



b)



Doping dependence of the fermionic gap.

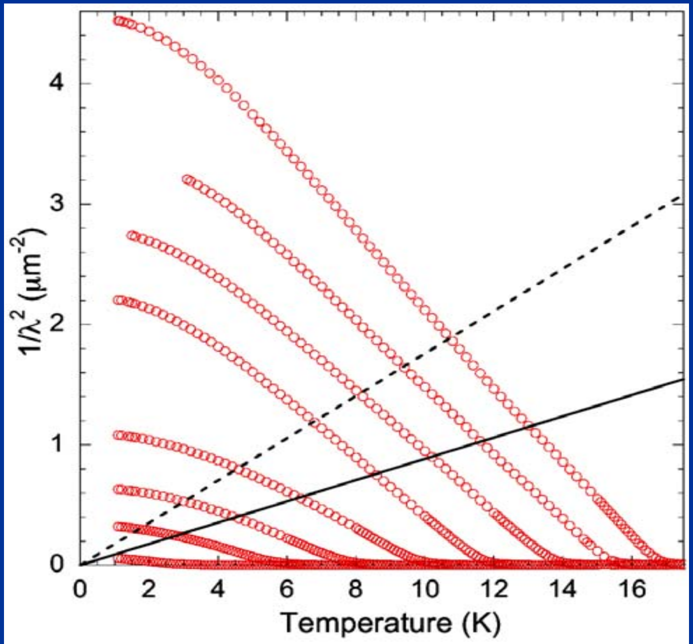


- **Then what excitations drive superfluid density to zero?**

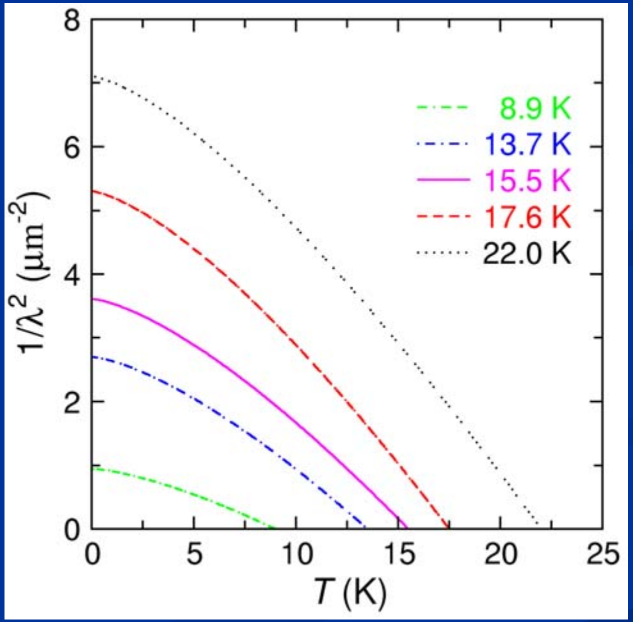
Possible Evidence for pair excitations in underdoped cuprates

Understanding universality in Penetration depth vs temperature.

UBC experiments



Crossover Theory



Conclusions

- Cold gases present opportunity to explore bigger-than-BCS theory (ie., rewrite the texts).

- Possibly relevant to high Tc.

- Future: More “exotic” phenomena in cold gases:

  1. With optical lattices can test attractive/repulsive Hubbard models.
  2. Mott physics vs “small pair physics” in high Tc needs to be sorted out.

Review References

Phys. Reports 412, 1 (2005)

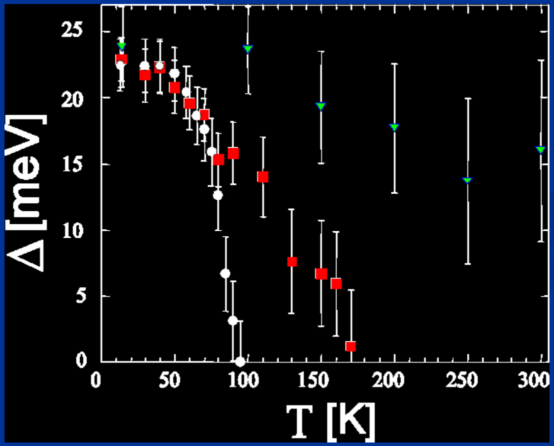
And

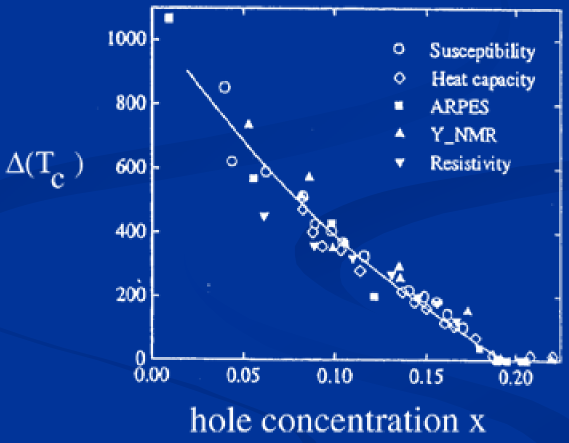
Varenna Summer School
Cond-mat/ 0605039

Also (Former Soviet J) Low Temp. Phys. 32, 406 (2006)

Rationale for Applying BCS-BEC Crossover to high Tc

- Pairs are small.
- "Pseudogap" (normal state gap) very prominent.





- Tc is high.
- Quasi 2 dimensional.