Exploring new phases of cold atomic matter with or without an optical lattice

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Two topics:

A. Unconventional vortex properties of a (gapless) breached pair superfluid [with M. Forbes, E. Gubankova, Y. B. Kim, V. M. Stojanovic, **F. Wilczek**, P. Zoller]

B. Orbital order of *p*-band bosons in the optical lattice [with S. Das Sarma, J. Moore, C. Wu]

Thank Funding Support from

- ORAU (Oakridge) 2006-2007
- **ARO** (Army research office) 2007-2010

Topic A. Breached pair superfluidity (BP)

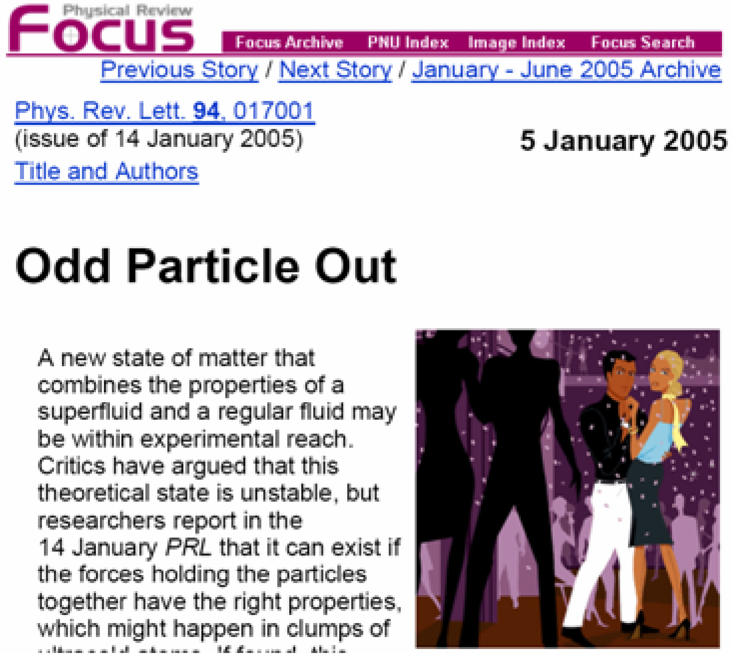
Collaborators:

M. Forbes (MIT graduate; now postdoc at UW Seattle)
E. Gubankova (MIT postdoc)
Y. B. Kim (U of Toronto)
F. Wilczek (MIT)
V. Stojanovic (Carnegie-Mellon student)
P. Zoller (Innsbruck)

publications

A. PRL **90**, 047002 (2003)
B. PRL **91**, 032001(2003)
C. PRA **70,** 033603 (2004)
D. PRL **94**, 017001 (2005)
E. cond-mat/0611295

News story:
"Odd particle out",
Phys. Rev. Focus
(January 5, 2005; story 1)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Phys. Rev. Lett. **94**, 017001
(issue of 14 January 2005)
Title and Authors

5 January 2005

Odd Particle Out

A new state of matter that combines the properties of a superfluid and a regular fluid may be within experimental reach. Critics have argued that this theoretical state is unstable, but researchers report in the 14 January *PRL* that it can exist if the forces holding the particles together have the right properties, which might happen in clumps of

Part A.1.
A brief remark on Breached Pair Superfluidity
(BP state)

Why called Breached Pair

BCS vs BP wavefunctions:

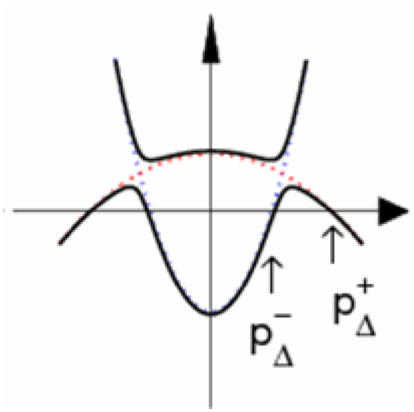
Unpaired matter??

$$
\begin{aligned}
|BCS\rangle &= \prod_{\mathbf{p}} (u_{\mathbf{p}} + v_{\mathbf{p}} \psi^{\dagger}_{\mathbf{p}\uparrow} \psi^{\dagger}_{-\mathbf{p}\downarrow})|0\rangle \\
|BP\rangle &= \prod_{\mathbf{p}\notin \mathrm{breach}} (u_{\mathbf{p}} + v_{\mathbf{p}} \psi^{\dagger}_{\mathbf{p}\uparrow} \psi^{\dagger}_{-\mathbf{p}\downarrow}) \boxed{\prod_{\mathbf{p}\in \mathrm{breach}} \psi^{\dagger}_{\mathbf{p}\downarrow}} |0\rangle
\end{aligned}
$$

where

$$
\begin{pmatrix} u_{\mathbf{p}}^2 \\ v_{\mathbf{p}}^2 \end{pmatrix} = \frac{1}{2}\left(1 \pm \frac{\epsilon_{\mathbf{p}}^{+}}{\sqrt{{\epsilon_{\mathbf{p}}^{+}}^2 + \Delta_{\mathbf{p}}^2}}\right)
$$

“breach” region: $p_{\Delta}^{-} \le |\mathbf{p}| \le p_{\Delta}^{+}$



[WVL and F. Wilczek, PRL (2003)]

Essential Difference: *Sarma vs Breached Pair*

Sarma [J. Phys. Chem. Solids (1963)]

1. Equal masses only.
2. Used Debye-like energy cutoff ω_D for interaction.
3. Did not pay attention to the new possibility of $\delta k_F > \omega_D / v_F$.
4. Correctly concluded an unstable (now known as) Sarma state.

WVL-Wilczek [PRL 2003a] (see also our [PRL 2005])

1. Re-discover the interest of mismatched Fermi surfaces.
2. First time introduced the effect of unequal masses.
3. Used momentum cutoff λ.
4. First recognized the interest of $\delta k_F > \lambda$.
5. Stable breached pair (BP) state.

My mistake: The conclusion of [Gubankova-WVL-Wilczek, PRL 2003b].

How stable?

The stability of BP criticized by:

1. Shin-Tza Wu, Sungkit Yip, PRA (2003)
2. P. F. Bedaque, H. Caldas, G. Rupak, PRL (2003); Caldas, PRA (2004)

Both are correct, but are done for a short-range delta-interaction.
[WVL-Wilczek, PRL 2003] is valid and correct.

The stability issue was clarified and examined in:

[Forbes, Gubankova, WVL, Wilczek, PRL 94, 017001 (2005)]

Stable for interaction of

- *a finite or long range;*

or

- *a momentum cutoff*

$$R^* \gtrsim k_F^{-1}$$

effective range — inter-atom distance

Effective range in real atomic gases

From D. Petrov, talk given at KITP Conference: Quantum gases

	R_e [Å]	B_0 [G]	Δ_B [G]	$\partial E_{res}/\partial B$	a_{bg} [Å]	R^* [Å]
^{6}Li	30	543.25	0.1	$2\mu_B$	32	19000
^{23}Na	45	907	1	$3.7\mu_B$	33	260
^{87}Rb	85	1007.4	0.17	$2.5\mu_B$	60	320
^{133}Cs	100	19.8	0.005	$0.55\mu_B$	160	13000

[http://online.itp.ucsb.edu/online/gases_c04/petrov/]

Gas density: $n \sim 10^{14}\mathrm{cm}^{-3} \quad \Rightarrow \quad k_F^{-1} \sim 1.0\mu\mathrm{m}$

Summary of Stability Criteria of BP

Two essential/necessary conditions (for weak coupling):

- Unequal masses
- Momentum dependent interaction: either a finite or long range or a momentum cutoff

Clarified in [Forbes, Gubankova, WVL, Wilczek, PRL 2005]

Part. A.2

Strong coupling imbalanced superfluid

Focus on the following case:

Strong interaction, wide Feshbach resonance

- There is a long list of papers before the phase-separation experiments of imbalanced fermi gases [M.. Zwierlein, W. Ketterle et al. *Science* (2006); G. Partridge, R. Hulet et al., *Science* (2006)] --- (it seems) in part stimulated by our work.
- There is even a longer list of papers after the experiments (>150?? as based on the **citation record** of [WVL and F. Wilczek PRL 2003])

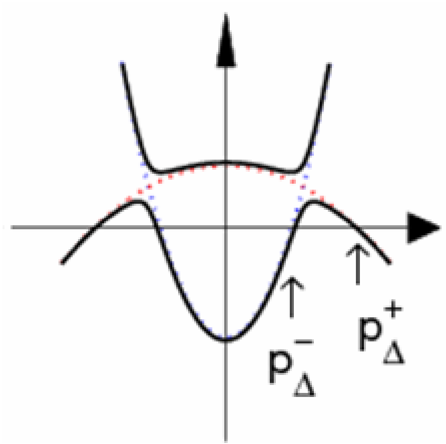
BP states of one or two fermi surfaces: BP2 vs BP1

quasiparticle spectrum

weak coupling

$$\mu(\Delta) \approx \mu(\Delta = 0) > 0$$

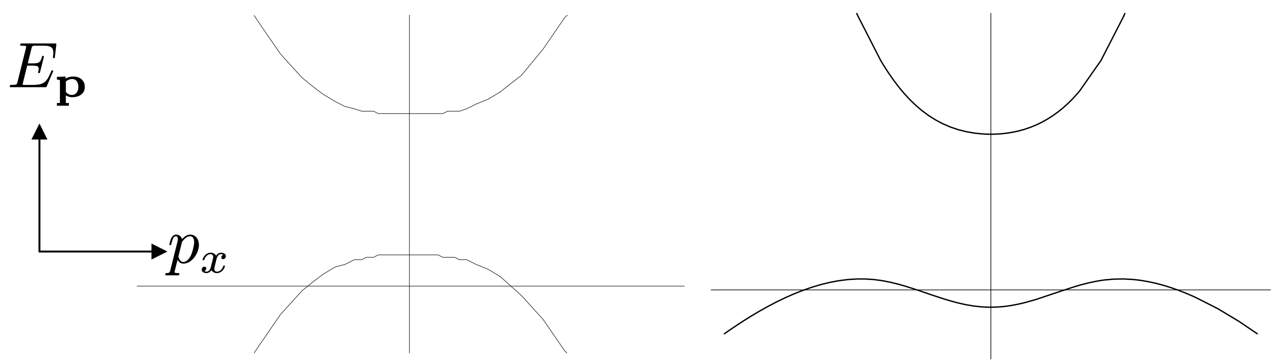
$$m_\uparrow < m_\downarrow$$



strong coupling

$$\bar{\mu}(\Delta \sim \epsilon_F) < 0$$

$$m_\uparrow = m_\downarrow \qquad\qquad m_\uparrow < m_\downarrow$$

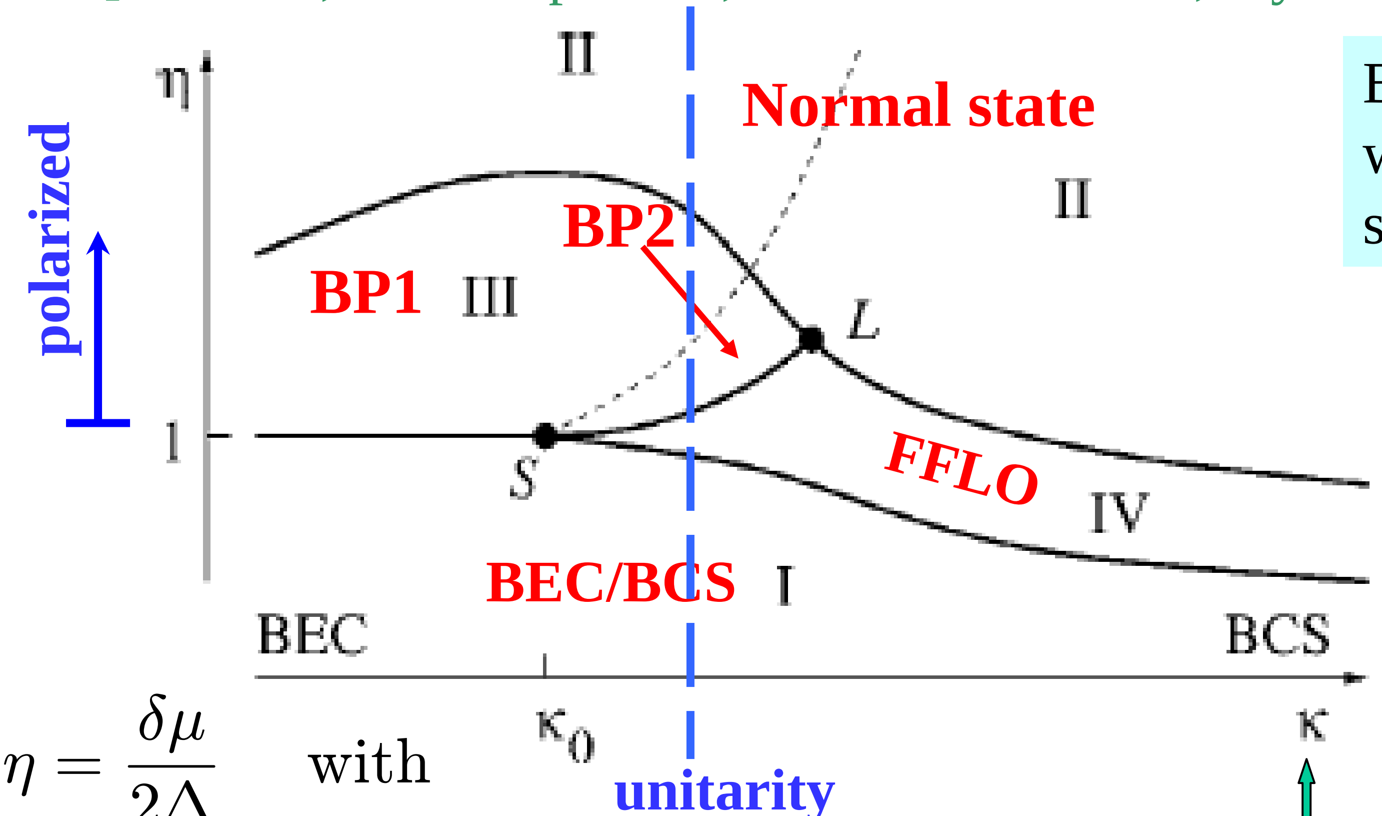


one fermi surface **two fermi surfaces**

WVL (unpublished)

Spin imbalanced Fermi gas: phase diagram

[D.T.Son, M.A.Stephanov, cond-mat/0507586,Phys. Rev. A 2006]



BP=breached pair with 1 or 2 Fermi surfaces

$\eta = \dfrac{\delta\mu}{2\Delta}$ with

$\delta\mu = \mu_\uparrow - \mu_\downarrow,$

Δ=energy gap.

$$\kappa = -\frac{1}{na^3} \propto -\left(\frac{1}{k_F a}\right)^3$$

Many other versions of phase diagram (see next slide).

BP1 (Breached pairing with 1 Fermi surface)
[same as ‘magnetized’ superfluid of Sheehy-Radzihovsky]

Found in the homogenous space by:

1. C. H. Pao, S.-T. Wu, and S. K. Yip, *Phys. Rev. B* **73** (2006) 132506.
2. D. E. Sheehy and L. Radzihovsky, *Phys. Rev. Lett.* **96** (2006) 060401.
3. D. T. Son and M. A. Stephanov, *Phys. Rev. A* **74** (2006) 013614.
4. M. Iskin and C. A. R. Sá de Melo, *Phys. Rev. Lett.* **97** (2006) 100404.
5. P. Nikolic and S. Sachdev, cond-mat/0609106.
6. Y. Nishida and D. T. Son, cond-mat/0607835.

***Correspondingly, a superfluid-normal mixture phase found in a trap*:**

1. P. Pieri and G. C. Strinati, *PRL* **96** (2006) 150404.
2. W. Yi and L. M. Duan, *Phys. Rev. A* **73** (2006) 031604.
3. T. N. De Silva and E. J. Mueller, *Phys. Rev. A* **73** (2006) 051602.
4. W. Yi and L. M. Duan, *Phys. Rev. Lett.* **97** (2006) 120401.
5. C. H. Pao and S. K. Yip, *J. Phys.: Cond. Matt.* **18** (2006) 5567.

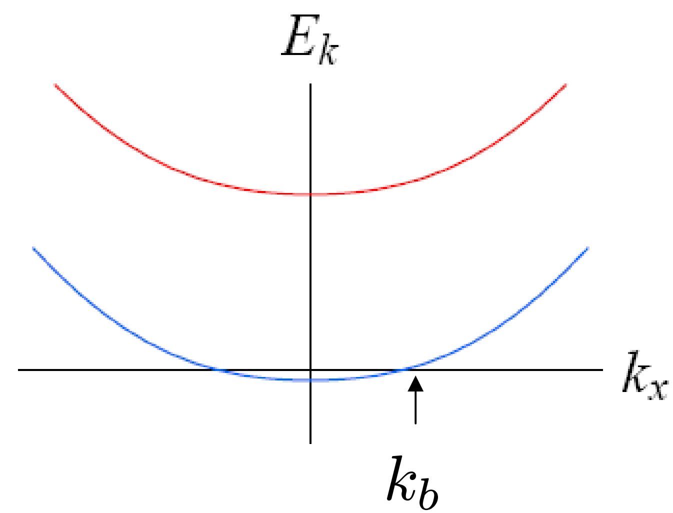
BP1 is predicted to be on the molecular (BEC) side of the Feshbach resonance, but not in the unitary regime!

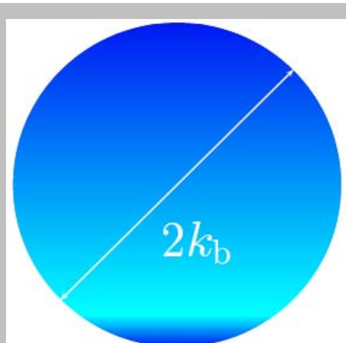
Unconventional vortex interaction
in a ‘polarized’ homogeneous gapless (BP1) superfluid.
[V. M. Stojanovic, WVL and Y. B. Kim, cond-mat/0611295]

The key is gapless fermions:

$$E_{\mathbf{k}} = \sqrt{(\frac{\mathbf{k}^2}{2m} - \bar{\mu})^2 + \Delta^2} \pm \frac{\delta}{2}$$

$$(\bar{\mu} \equiv \frac{\mu_\uparrow + \mu_\downarrow}{2} < 0 \text{ in BEC;} \quad \delta = \mu_\uparrow - \mu_\downarrow)$$





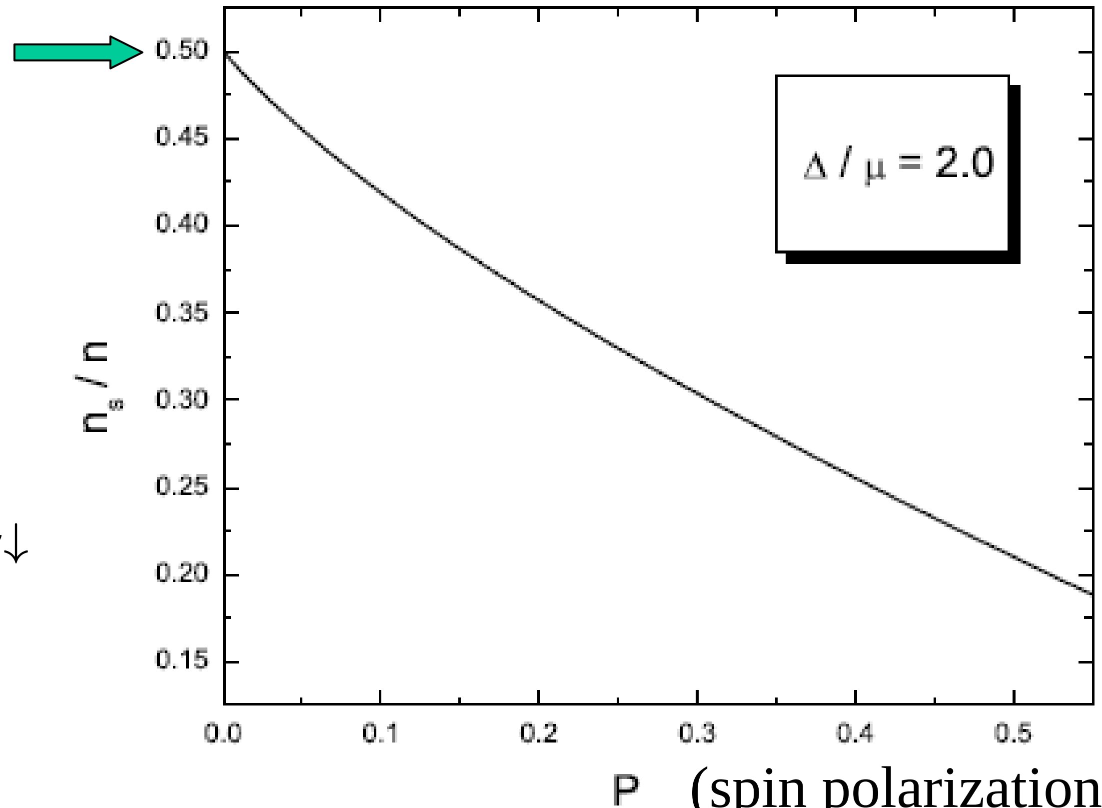
$$P = \frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow} \quad \leftrightarrow \quad k_b = \left(6\pi^2(n_\uparrow - n_\downarrow)\right)^{1/3}$$

Breached-pairing Fermi ball

Check BP1 stability: superfluid density

"0.5" corresponds to n_s equal to the total number of fermion pairs per unit volume.

Definition: $n = n_{\uparrow} + n_{\downarrow}$



(spin polarization)

[V. Stojanovic, WVL, Y.B. Kim, unpublished]

Note: Used the method of L. He, M. Jin, P. Zhuang [Phys. Rev. B. (2006)] originally derived for computing superfluid density for BP2.

Vortex-vortex interaction

Conventional superfluid (s-wave BCS-like):

$$V_{\mathrm{vortex}}(\mathbf{r}) \propto -\ln \frac{r}{\xi}$$

(strictly repulsive 2D Coulomb potential)

⇒ Vortex lattice is **triangular**.

Question arises for BP1 phase:

What is the effect (if any) of those gapless fermions around the surface the ‘breach’ Fermi ball ?

Properties and assumptions of BP1

- The ground state is ASSUMED, not derived, to be superfluid!
- One superfluid phase (Goldstone) mode --- θ
- One gapless branch of fermionic quasiparticles --- ψ

Low energy modes

- Continuous symmetries are: two global U(1) (“charge” and “spin”) + Galilei invariance

Effective field theory of the polarized fermionic superfluid

Effective Lagrangian

[extension of Son-Stephanov's to the case of arbitrary polarization]

$$\mathcal{L} = \psi^*[\partial_\tau + \varepsilon(-i\nabla)]\psi + c_1(\partial_\tau\theta)^2 + c_2(\nabla\theta)^2$$
$$+c_3\psi^*\psi\left[i\partial_\tau\theta + \frac{1}{2m_p}(\nabla\theta)^2\right] + \nabla\theta\cdot\mathbf{j} + \cdots$$

Coefficients c_1, c_2, c_3 are NOT universal but determined phenomenologically/experimentally:

$$c_1 = \frac{\partial n}{\partial \mu}$$

$$4mc_2 + c_3(n_\uparrow - n_\downarrow) = n_s$$

Unit coefficient and the form of composite objects in the [...] all dictated by Galilei invariance [Greiter, Wilczek, and Witten (1989)]

Outline of Method

Task: Study the problem of parallel vortex lines pointing to the z-direction, say, generated by rotation.

phase = "spin wave" part + singular vortex part

$$\theta = \phi + \theta_v$$

Vortex gauge field: $\mathbf{a} = -\nabla\theta_v \quad \Rightarrow \quad \nabla\theta = \nabla\phi - \mathbf{a}$

Standard relation between vortex charge density and ***a***

$$\rho(\mathbf{x}) = 2\pi \sum_\alpha \delta^{(2)}(\mathbf{x} - \mathbf{x}_\alpha), \qquad \nabla \times \mathbf{a} = -\frac{\pi\hbar}{m}\rho(\mathbf{x})\hat{\mathbf{e}}_z$$

- Integrate out gapless quasi-particle fermions.
- Integrate out the regular part of the phase field, ϕ
- Retain effective action for vortices → effective vortex interaction

Effective vortex interaction (momentum space)

$$V_{\text{eff}} = V_0 + V_{\text{ind}}$$

$$V_0 \quad \propto \quad \frac{n_s}{4m} \frac{1}{\mathbf{q}^2}$$

2D Coulomb potential

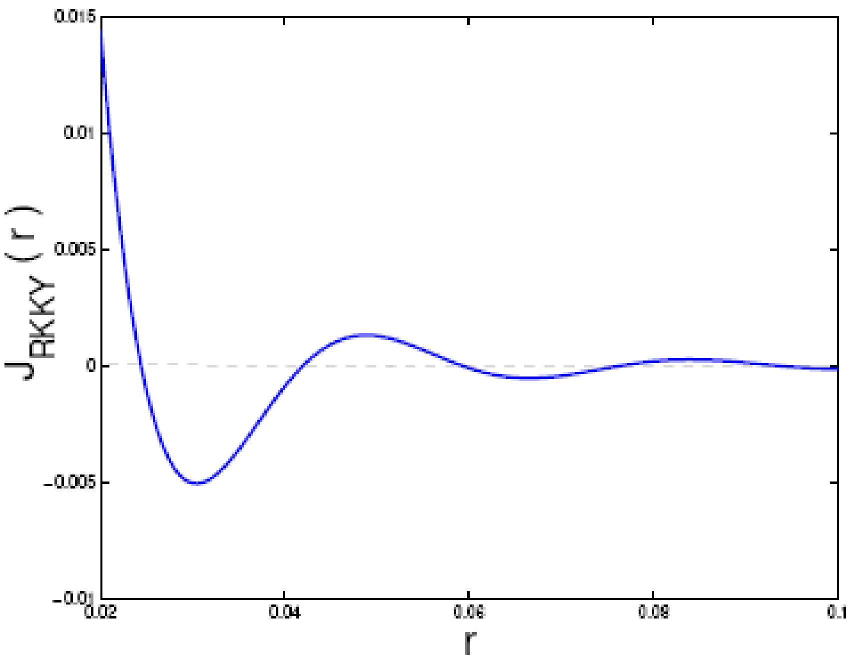
$$V_{\text{ind}} \quad \propto \quad \frac{P^0_{\mathbf{q}}}{\mathbf{q}^2}$$

fermion-induced potential

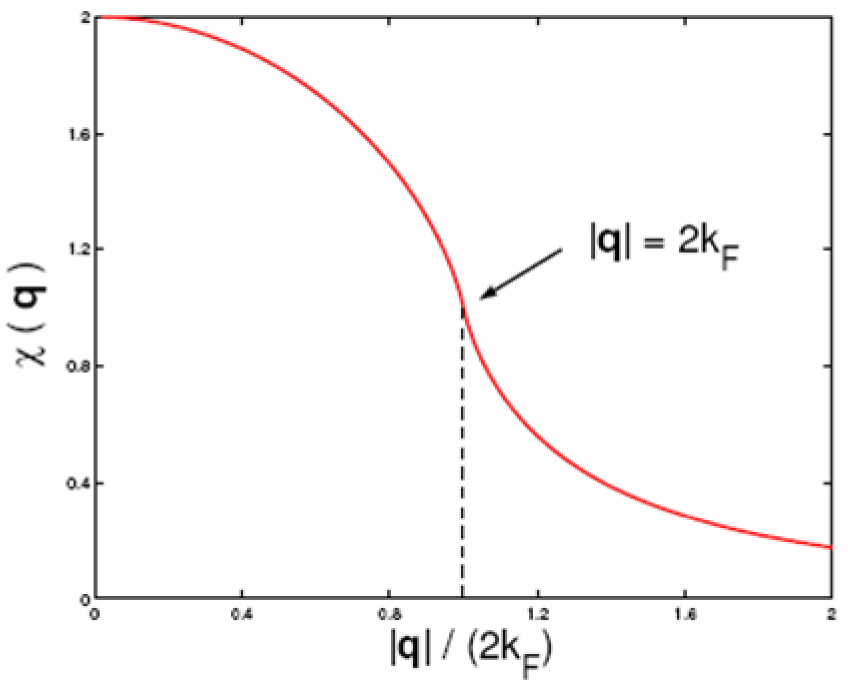
$P^0_{\mathbf{q}} \quad \longrightarrow \quad$ zero-temperature static limit of the transverse current-current correlator

Recall RKKY (Ruderman-Kittel-Kasuya-Yosida) in metals

Classic result: Indirect exchange interaction between magnetic impurities (mediated by the conduction electrons) in non-magnetic metals (1950's)

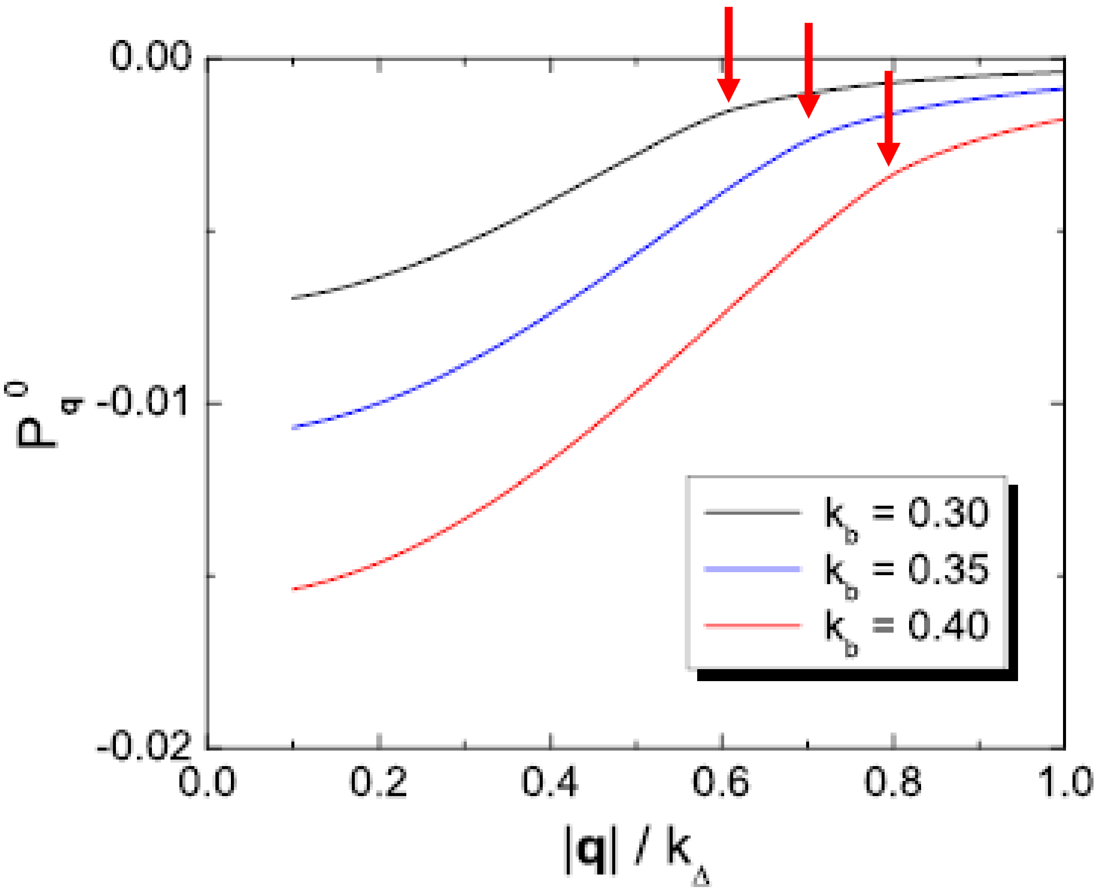


Spin-spin interaction J(r)



Spin density response function

Analogue of RKKY oscillation in BP1



Momentum units

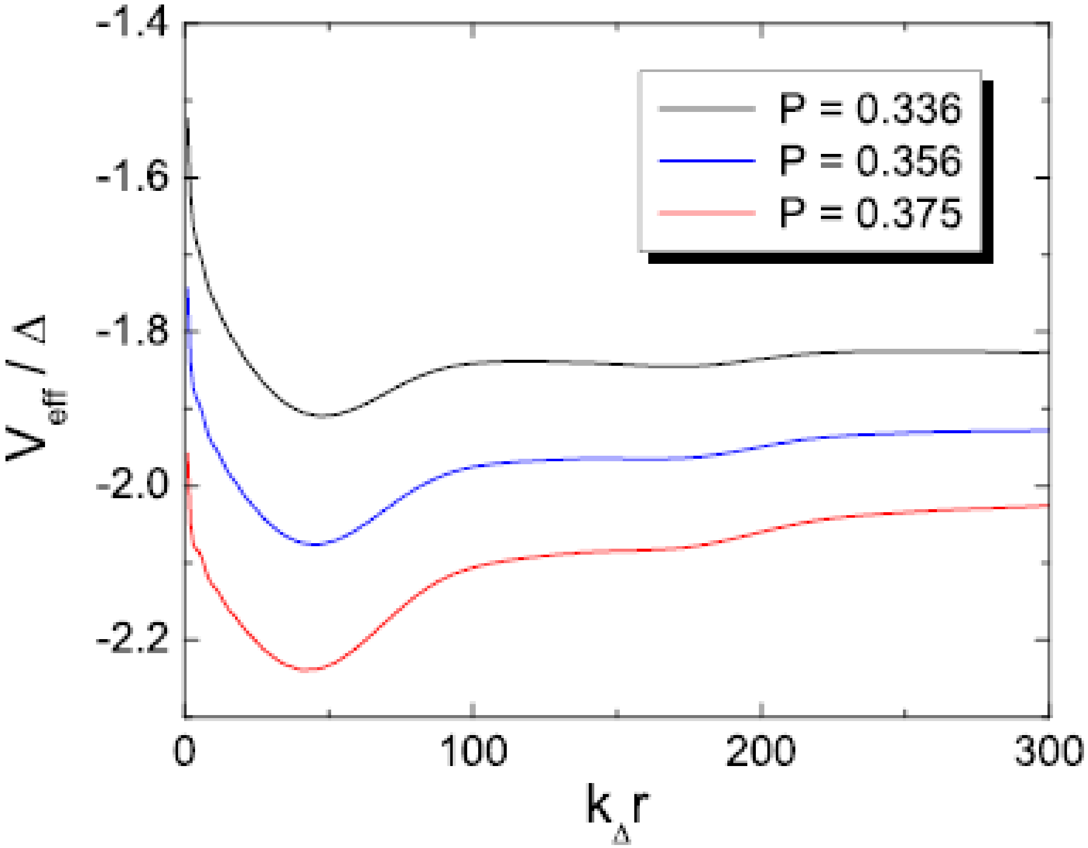
$$k_\Delta \equiv \sqrt{\frac{2m\Delta}{\hbar}}$$

The transverse current-current correlation, $P_{\mathbf{q}}$, has a knee at $|\mathbf{q}| = 2k_b$.
(Recall k_b is the gapless Fermi wavevector.)

Effective potential in real space (I)

Intermediate polarization:

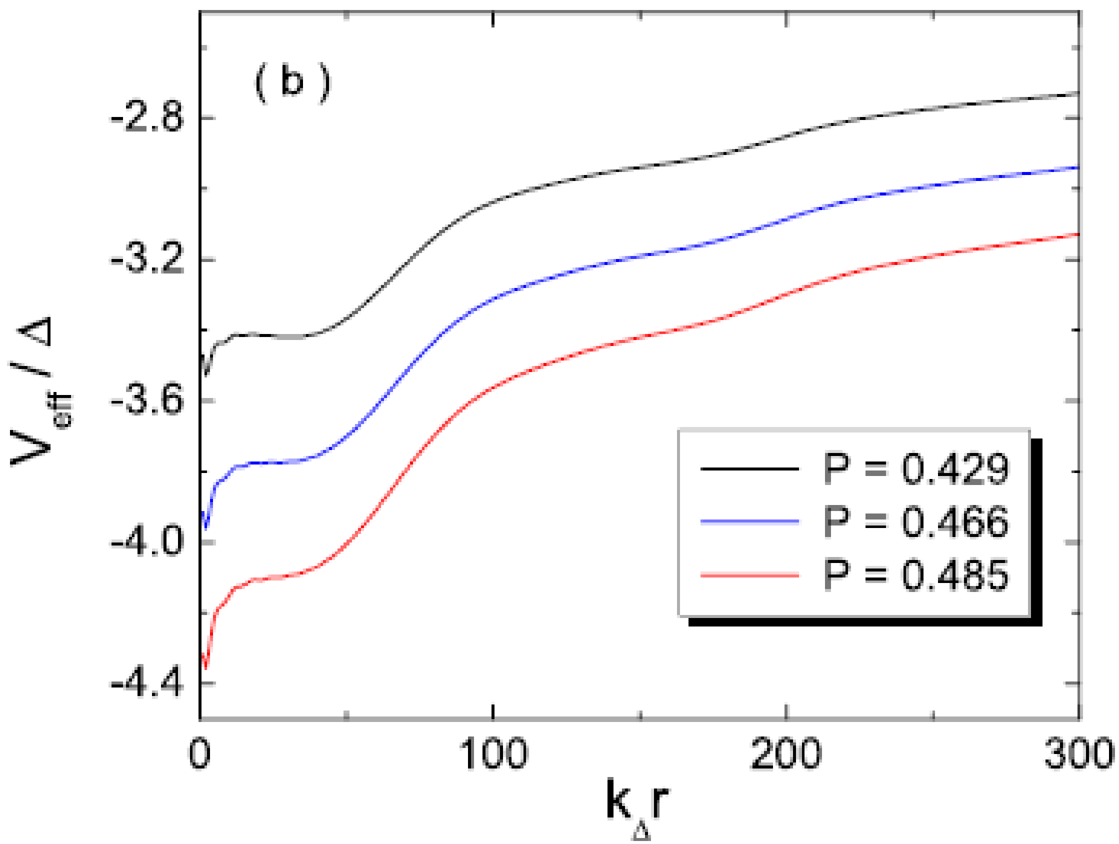
$$P_{c1} \le P \le P_{c2}$$
$$(P_{c1} \sim 0.2; \quad P_{c2} \sim 0.4)$$



New vortex
lattice structure?!

Effective potential in real space (II)

High spin polarization: $P \geq P_{c2}\,,\qquad (P_{c2} \sim 0.4)$



vortex lattice instability at short distance?!

Related recent studies

- Non-monotonic (as a function of distance) interaction between vortices in a multi-component superconductor [Babaev & Speight, PRB 72, 180502(R) (2005)]

- Nodal-quasiparticle mediated interaction between vortices in a d-wave superconductor [Nikolic & Sachdev, PRB 73, 134511 (2006)]

Summary of New Results for the first topic

New results (to the best of our knowledge) are:

- First find that vortex interaction is not strictly repulsive due to gapless fermions!
- It has RKKY-like oscillating character!
- After the Friedel oscillation (charge sector) and the RKKY (spin sector), oscillating behavior is first shown to occur in the vortex sector !

Future:

- What is the form of the resulting vortex lattice in the BP1 state ?
- Will the new form of vortex interaction change the Kosterlitz-Thouless transition in 2D? How?

Topic B.
Bosonic atoms in the ***p-orbital*** band of an optical lattice

Collaborators: Congjun Wu (KITP)
Joel Moore (UC Berkeley)
Sankar Das Sarma (U Maryland)

Our work:

- WVL and C. Wu, cond-mat/0601432, *Phys. Rev. A* (2006)
- C. Wu, WVL, J. Moore and S. Das Sarma, *Phys. Rev. Lett.* (2006)

Other related theoretical studies

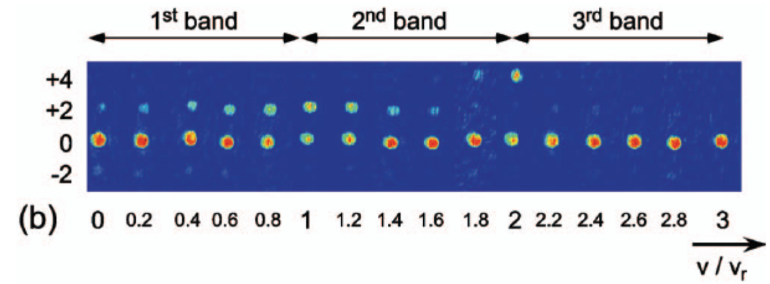
- V. W. Scarola and S. Das Sarma, *Phys. Rev. Lett*. 65, 33003 (2005).
- A. Isacsson and S. Girvin, *Phys. Rev. A* 72, 053604 (2005).
- A. B. Kuklov, *PRL* 97, 110405 (2006)
- C. Xu et al., cond-mat/0611620.
- C. Wu, D. Bergman, L. Balents, and S. Das Sarma, cond-mat/0701788.

Motivations:

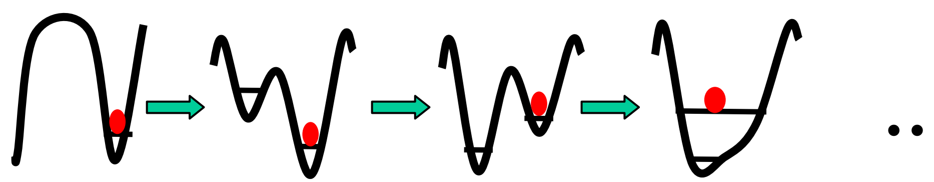
- Look beyond s-band; Beyond cold atom models of spins.
- Explore orbital degeneracy and symmetries, and **new** aspects of strong correlation [those **not** well studied in usual condensed matter systems]
- Anisotropy is not a problem, but a new feature.
- Possible quantum (cold atoms) simulation of the difficult orbital-related problems [as observed in electronic materials, e.g., transition-metal oxides]?
- New experiments on *p-band* at NIST [A. Browaeys, et al, PRA (2005); J.J. Sebby-Strabley, Porto, et al., PRA (2006)] and Mainz [T. Mueller and I. Bloch et al., Mueller thesis (2006); T. Mueller, I. Bloch, et al, arXiv:0704.2856]

Preparation of p-band bosons: three experiment groups

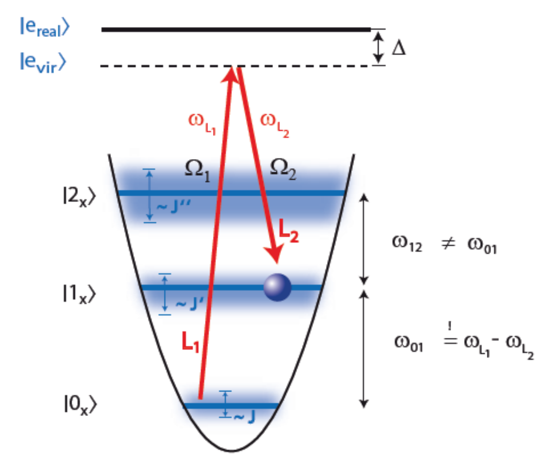
By moving lattice [A. Browaeys, W. D. Phillips, et al, PRA **72**, 053605 (2005)]



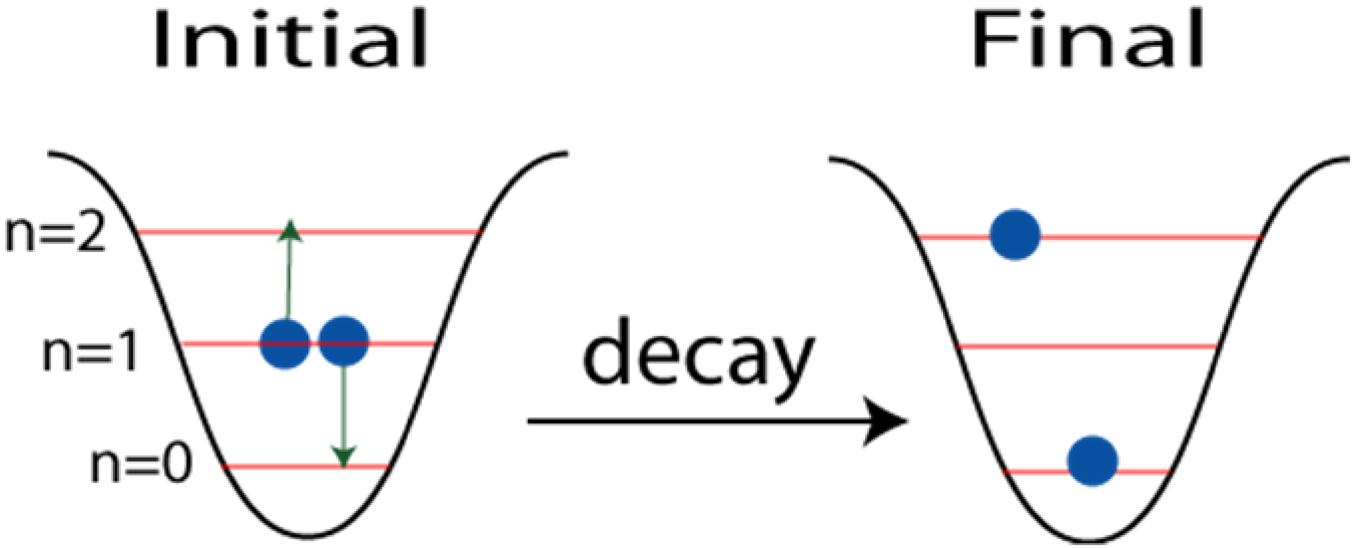
Dynamically deforming the double-well lattice [M. Anderlini, J. V. Porto, et al., J. Phys. B 39, S199 (2006)]



Pumping bosons by Raman transition [T. Mueller, I. Bloch et al., thesis of Mueller (2006); arXiv:0704.2856]



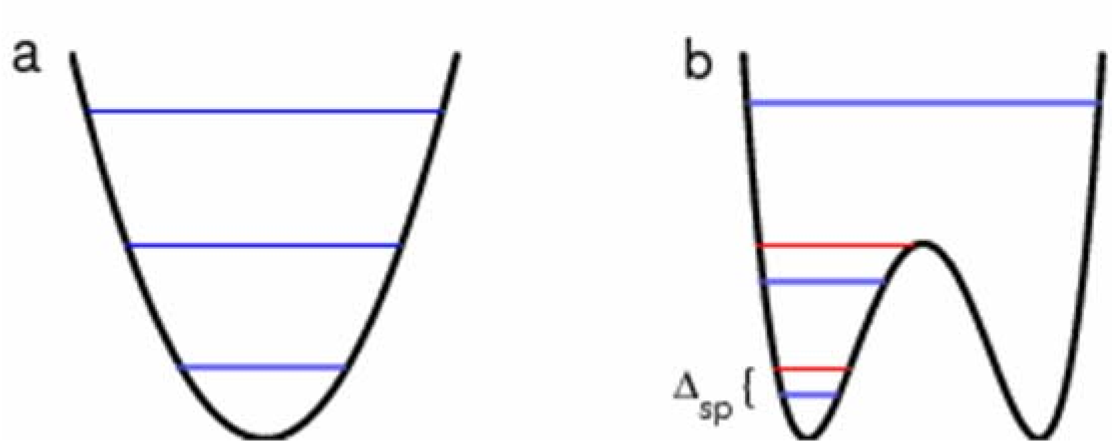
The decay problem of p-orbital bosons



The decay process where two p-bosons collide, promoting one to the 2nd excited band and bringing one down to the s-band. [Studied by Isacsson and Girvin (2005).]

"Energy-blocking" mechanism to suppress the decay

single well lattice ***double well lattice***



- Low energy motion of bosons is an effective two-band model;
- *p*-orbital bosons cannot decay to the "s" by energy conservation.

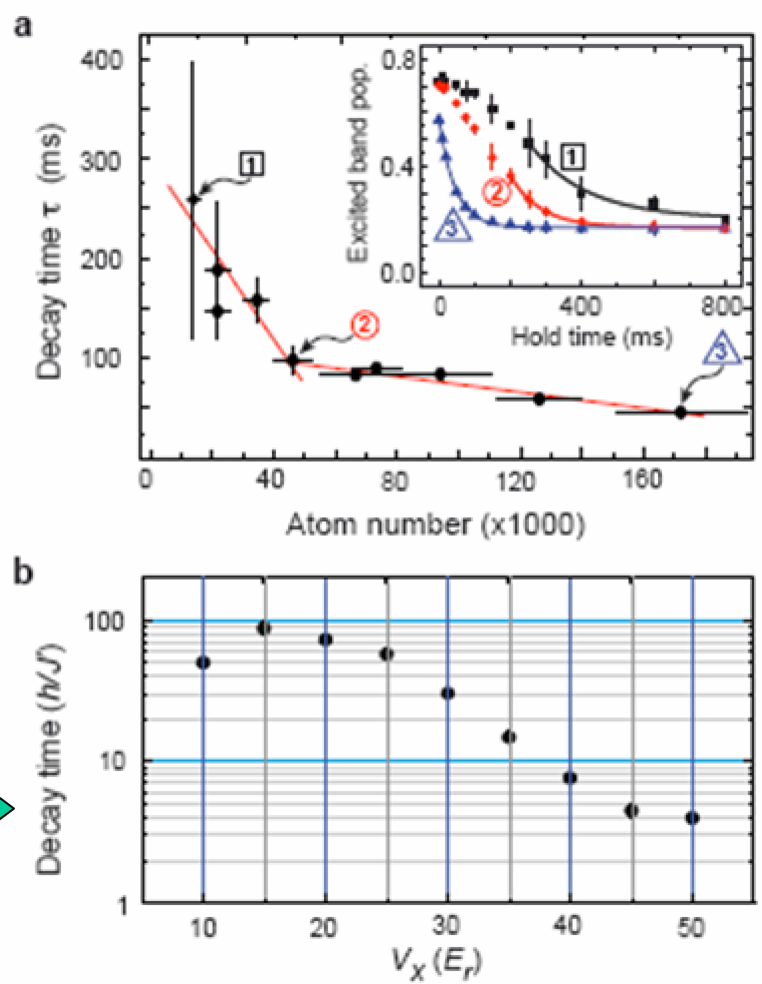
[WVL and C. Wu, PRA (2006)]

p-band decay time measured by the Mainz group

[T. Mueller, I. Bloch, et al, arXiv:0704.2856]

Decay time in (ms)
vs atom number

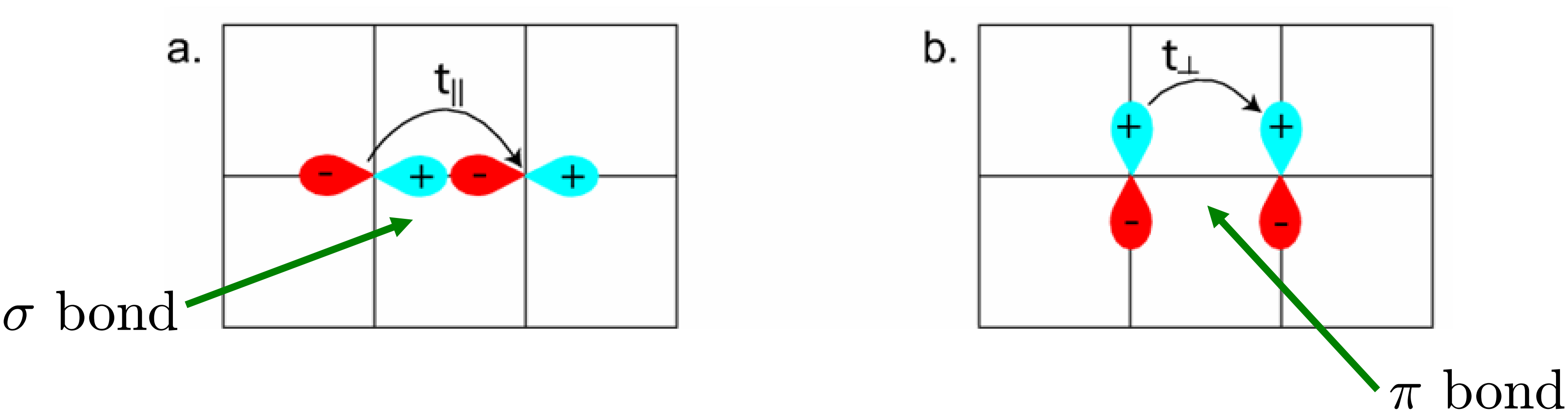
Decay time in units of
tunneling time scale
vs
lattice depth



A key to slow decay as explained by Bloch et al: anharmonicity

p-orbital Bose-Hubbard model: 3D cubic lattice

$$H = \sum_{\mathbf{r}\mu} [t_{\|}\delta_{\mu\nu} - t_{\perp}(1-\delta_{\mu\nu})] \left(b^{\dagger}_{\mu,\mathbf{r}+a\mathbf{e}_{\nu}} b_{\mu\mathbf{r}} + h.c. \right) + \frac{1}{2} U \sum_{\mathbf{r}} \left[n_{\mathbf{r}}^2 - \frac{1}{3} \mathbf{L}_{\mathbf{r}}^2 \right]$$



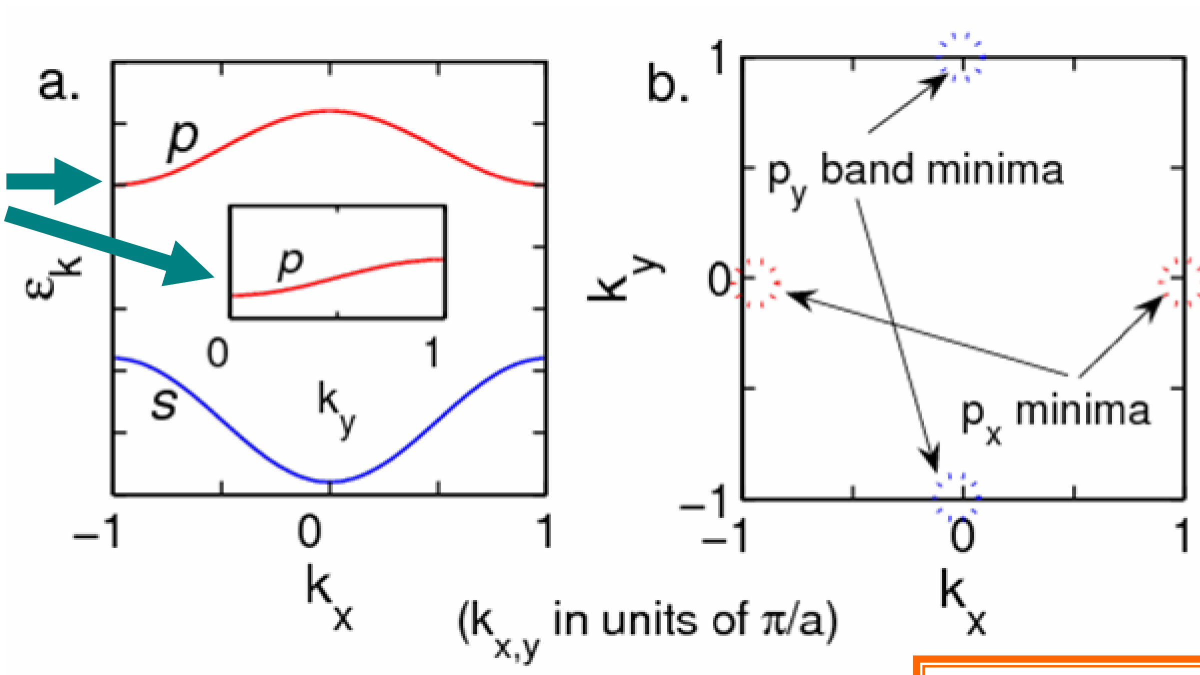
$$\mu, \nu = x, y, z \qquad \text{or} \qquad p_x, p_y, p_z$$

Density field operator $n_{\mathbf{r}} = \sum_{\mu} b^{\dagger}_{\mu\mathbf{r}} b_{\mu\mathbf{r}}$

Angular momentum operator: $L_{\mu\mathbf{r}} = -i \sum_{\nu\lambda} \epsilon_{\mu\nu\lambda} b^{\dagger}_{\nu\mathbf{r}} b_{\lambda\mathbf{r}}$

p-band minima in momentum space (k-space)

p_x band



a = lattice constant

Bose-Einstein condensation (BEC) of

p_x, p_y, and p_z orbital atoms occurs at finite momenta:

$$\mathbf{Q}_x = \left(\frac{\pi}{a}, 0, 0\right), \ \mathbf{Q}_y = \left(0, \frac{\pi}{a}, 0\right), \ \mathbf{Q}_z = \left(0, 0, \frac{\pi}{a}\right)$$

The *p*-orbital BEC (*p*-OBEC)

Parameterization of Order parameter:

$$\begin{pmatrix} \langle b_{x\mathbf{k}=\mathbf{Q}_x} \rangle \\ \langle b_{y\mathbf{k}=\mathbf{Q}_y} \rangle \\ \langle b_{z\mathbf{k}=\mathbf{Q}_z} \rangle \end{pmatrix} = \rho e^{i\varphi - i\mathbf{T}\cdot\theta} \begin{pmatrix} \cos\chi \\ i\sin\chi \\ 0 \end{pmatrix}$$

U(1) phase

SO(3) orbital

T-reversal

[**T**'s are three 3x3 matrices]

For a dilute lattice gas of U>0, the condensate is found to be:

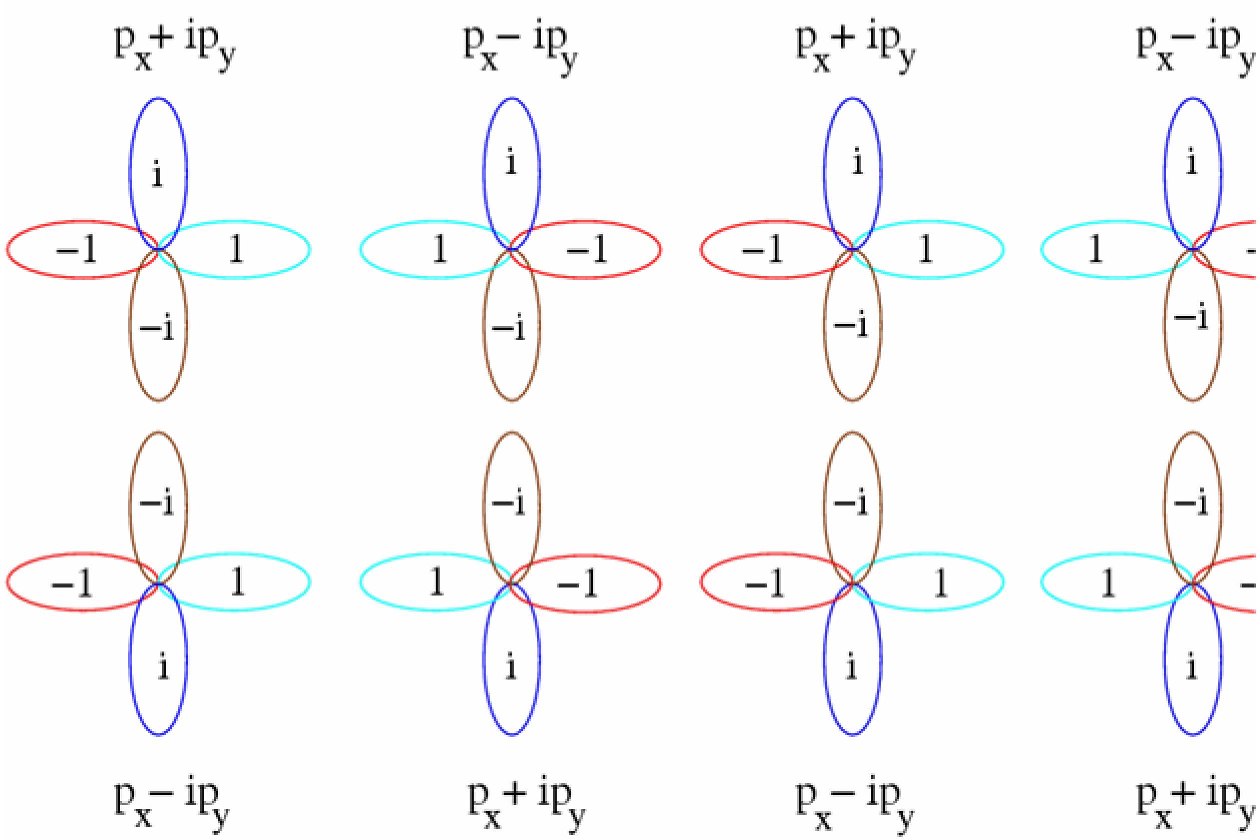
$$\begin{pmatrix} \langle b_{x\mathbf{k}=\mathbf{Q}_x} \rangle \\ \langle b_{y\mathbf{k}=\mathbf{Q}_y} \rangle \\ \langle b_{z\mathbf{k}=\mathbf{Q}_z} \rangle \end{pmatrix} = \sqrt{\frac{\text{Vol.} \times n_0^b}{2}} \begin{pmatrix} 1 \\ \pm i \\ 0 \end{pmatrix}$$

It is also an order of …

Transversely Staggered Orbital Current (TSOC)

Example:

$p_x + ip_y$ state



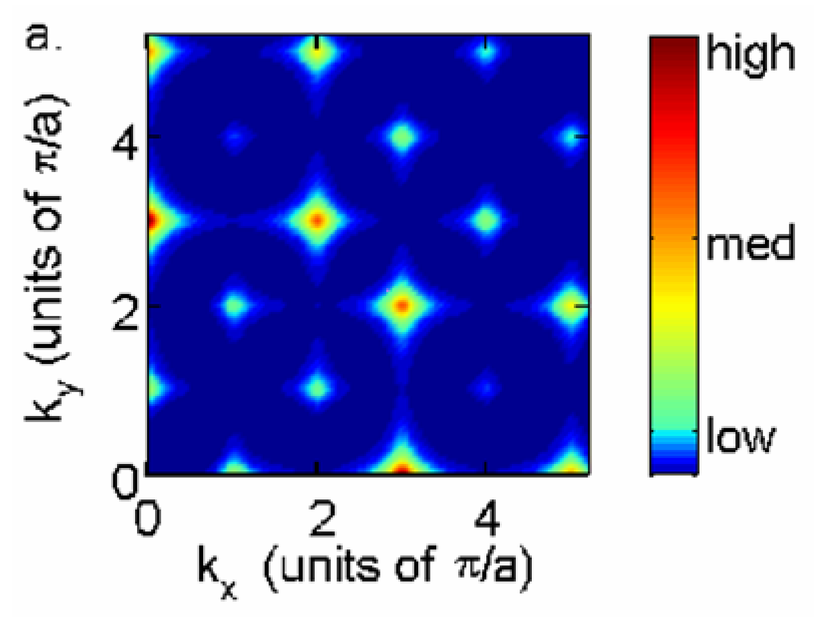
Quantitative Description of TSOC:

$$\langle L_{x\mathbf{r}} \rangle = \langle L_{y\mathbf{r}} \rangle = 0, \langle L_{z\mathbf{r}} \rangle = n_0^b (-)^{\frac{x+y}{a}} .$$

Prediction: non-zero momentum BEC of *p-orbital* atoms

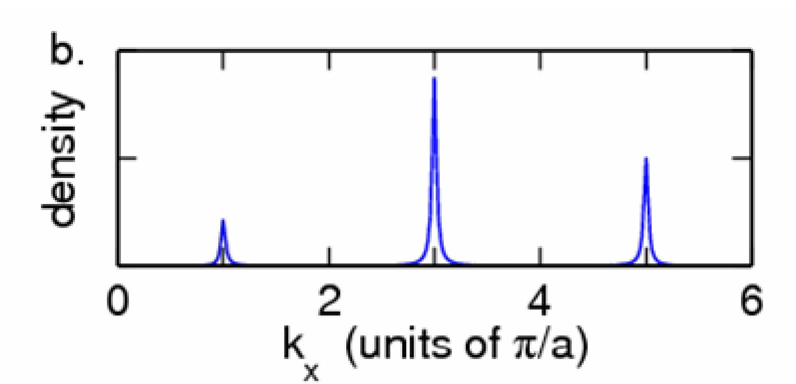
Time-of-flight (TOF) experiment

Peaks not at (0,0)!



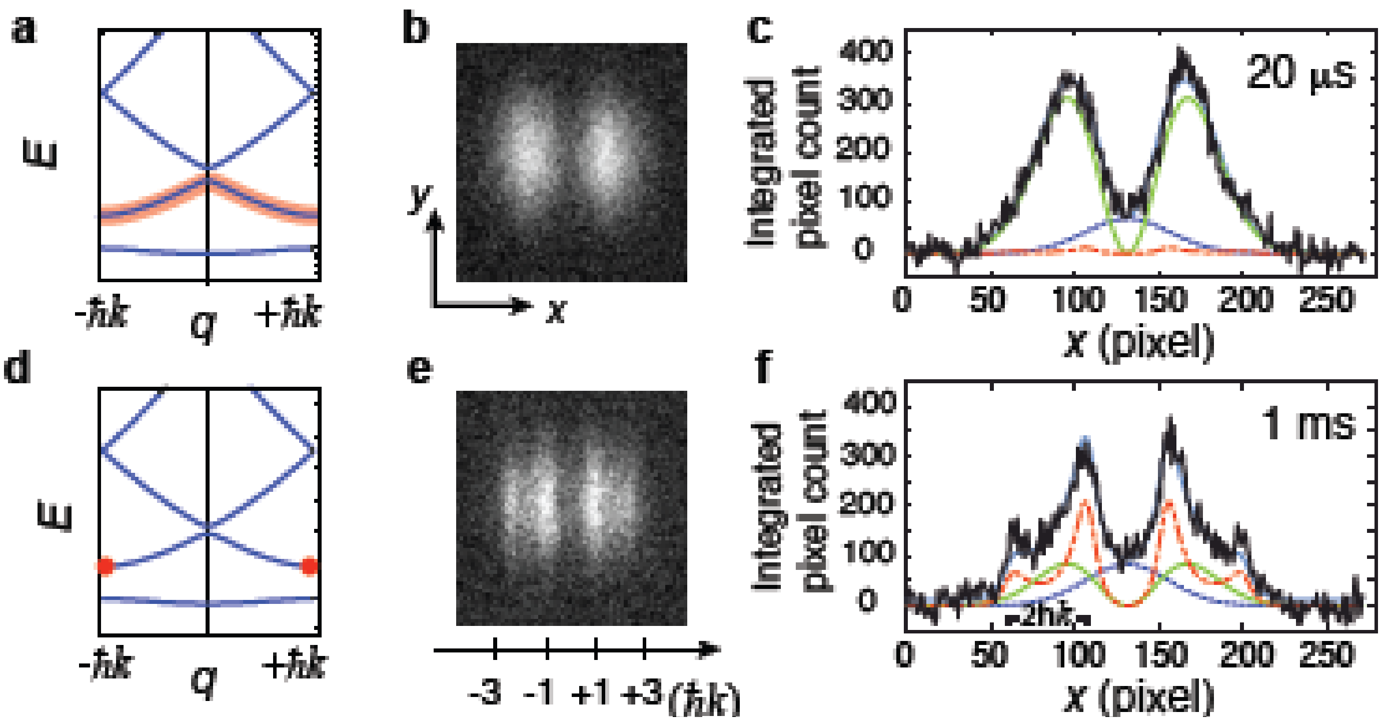
p-orbital wavefunction imposes a *non-Gaussian* profile;

The highest moves when varying the size of the *p*-wavefunction.



[Related results independently by: A. Isacsson, S. Girvin, PRA (2005); A. B. Kuklov, PRL 97, 110405 (2006)]

Experimental discovery of the Mainz group



Initial time (incoherent)

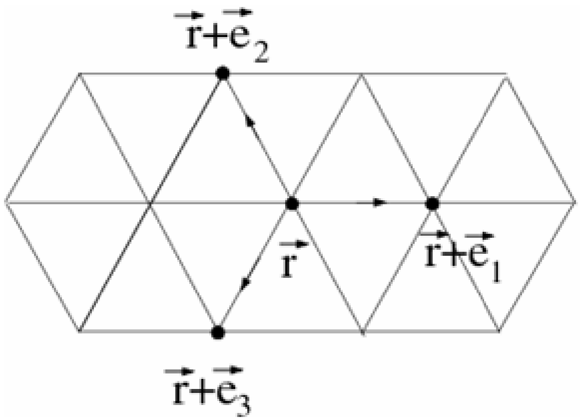
Later time (coherent)

Confirms our prediction!

p-band bosons in a triangular lattice

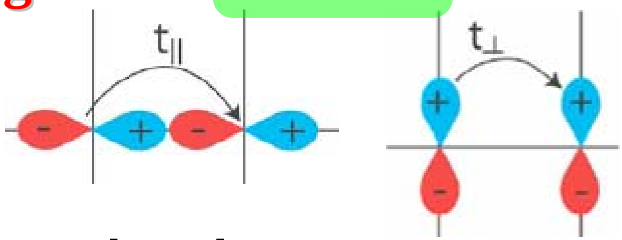
[C. Wu, WVL, J. Moore, and S. Das Sarma, *Phys. Rev. Lett.* (2006).]

$$H = t_{\parallel} \sum_{\mathbf{r}} \sum_{i=1,2,3} \left[b_i^{\dagger}(\mathbf{r}+\mathbf{e}_i) b_i(\mathbf{r}) + h.c. \right] + \tfrac{1}{2} U \sum_{\mathbf{r}} \left[n_{\mathbf{r}}^2 - \tfrac{1}{3} L_{z\mathbf{r}}^2 \right]$$

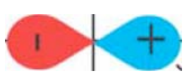
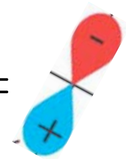


For hopping

$t_{//} >> t_{\perp}$



σ-bond **π-bond**

$|p_1\rangle =$  $|p_2\rangle =$  $|p_3\rangle =$ 

For interaction

Number density $n_{\mathbf{r}} = \sum_{\mu} b_{\mu\mathbf{r}}^{\dagger} b_{\mu\mathbf{r}}$

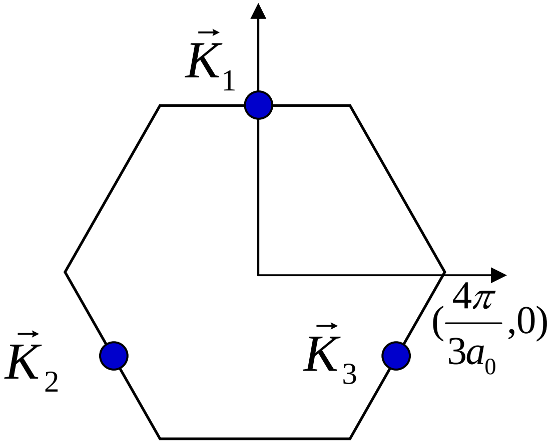
Angular momentum $L_{z\mathbf{r}} = -i \sum_{\mu\nu} \epsilon_{\mu\nu} b_{\mu\mathbf{r}}^{\dagger} b_{\nu\mathbf{r}}$

(with $\mu, \nu = x, y$ for p_x, p_y)

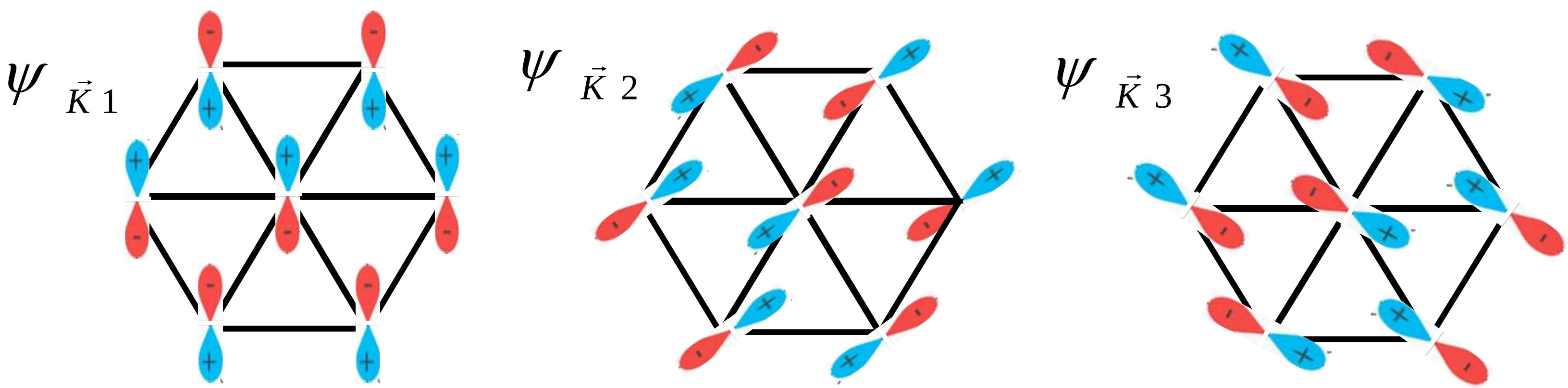
Band structure

$\vec{K}_{1,2,3}$ lowest energy states in the 1st Brillouin zone

$$
\begin{aligned}
K_1 &= (0, \frac{2\pi}{\sqrt{3}a_0}) \\
K_2 &= (\frac{\pi}{a_0}, \frac{\pi}{\sqrt{3}a_0}) \\
K_3 &= (-\frac{\pi}{a_0}, \frac{\pi}{\sqrt{3}a_0})
\end{aligned}
$$



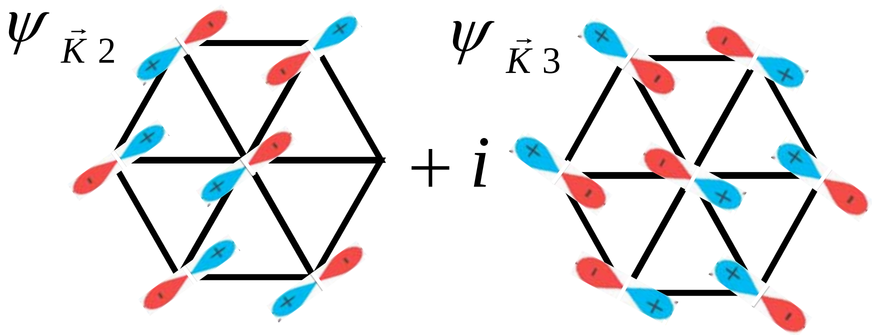
Real space configurations of the three states

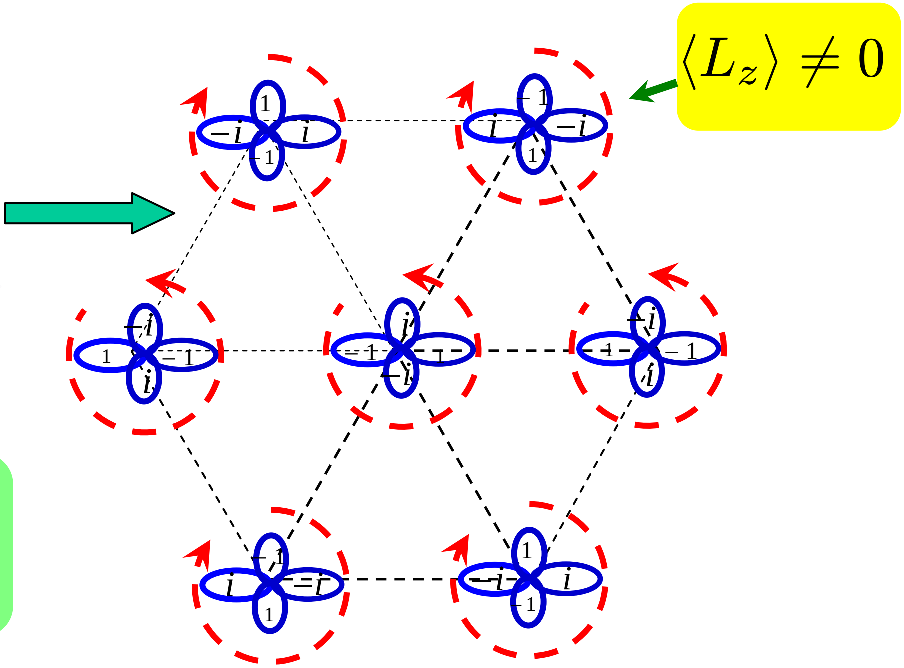


Note: For lowering kinetic energy, any superposition of the three equally works.

Stripe ordered superfluidity

- Interactions select the condensate as (weak coupling analysis)





$$\frac{1}{\sqrt{N_0!}}\left\{\frac{1}{\sqrt{2}}\left(\psi^{+}_{K_2} + i\,\psi^{+}_{K_3}\right)\right\}^{N_0}|0\rangle$$

- Time-reversal, translational, and rotational symmetries are broken.

- c.f. charge stripe orderings in electronic solids (e.g., high Tc cuprates, quantum Hall systems): long range Coulomb interactions, fermionic.

Stripe ordering found for all couplings:

[new in both AMO and condensed matter]

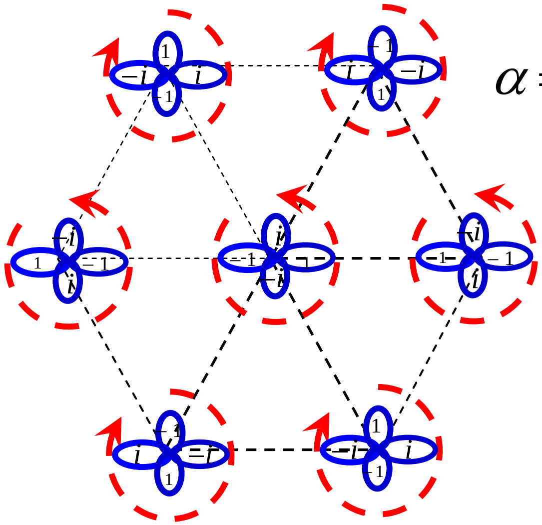
U(1) phase

Ising variable +1 or -1

- Orbital wavefunction in lattice site ***r***:

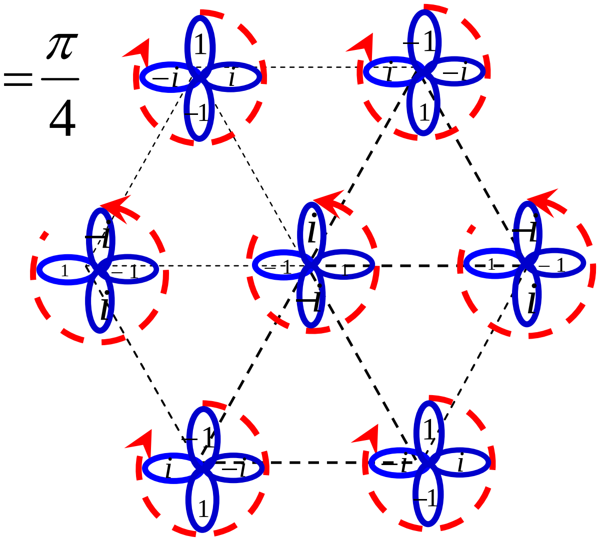
$$e^{i\phi_r}(\cos\alpha |p_x\rangle + i\sigma_r \sin\alpha |p_y\rangle)$$

mixing angle



$\alpha = \frac{\pi}{6}$

$\alpha = \frac{\pi}{4}$



weak coupling

strong coupling

- c.f. strong coupling results also apply to the $p_x + ip_y$ Josephson junction array systems (e.g. Sr_2RuO_4).

Effective gauge theory for strong coupling SF

$$H_{\text{eff}} = -\frac{1}{2} n t_{\|} \sum_{\langle \vec{r}_1, \vec{r}_2 \rangle} \cos\left\{ \phi_{\vec{r}_1} - \phi_{\vec{r}_2} - A_{\vec{r}_1, \vec{r}_2}(\sigma_{\vec{r}_1}, \sigma_{\vec{r}_2}) \right\} + \frac{1}{3} U \sum_{\vec{r}} n_{\vec{r}}^2$$

The gauge field (as an "external flux" for ϕ)

External flux in a triangular plaquette:

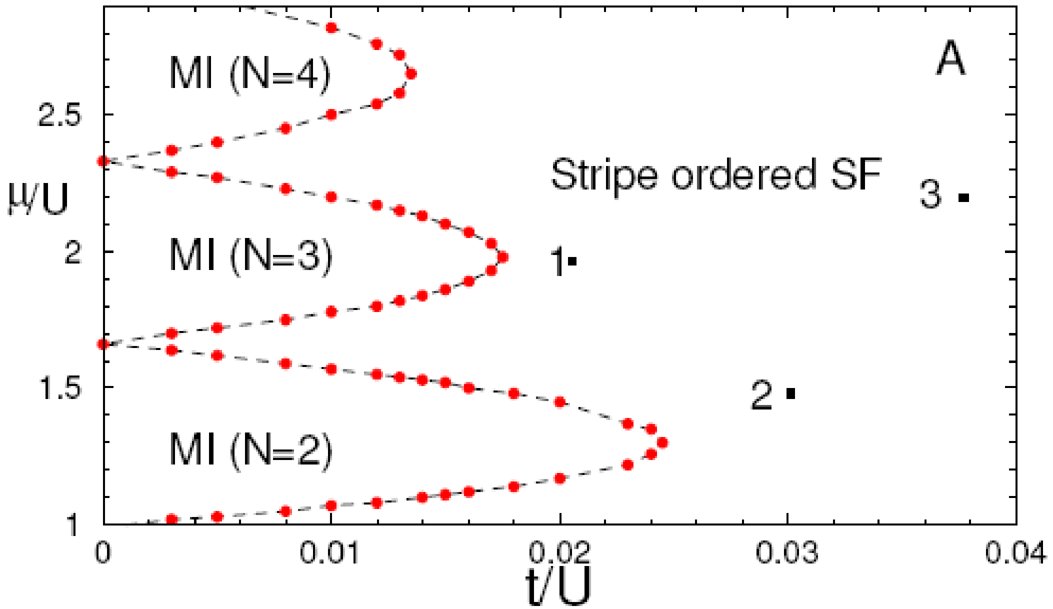
$$\Phi = \frac{1}{2\pi} \sum_{\langle r, r' \rangle} A_{r,r'} = \frac{1}{6} (\sigma_{\vec{r}_1} + \sigma_{\vec{r}_2} + \sigma_{\vec{r}_3}) \bmod 1 \quad \text{must be} \quad \pm\frac{1}{6}$$

Require minimum flux in each plaquette [as shown, e.g., by Moore and Lee (2004) for a Josephson array of superconductors].

U(1) vortex theory: Duality mapping to a lattice Coulomb gas

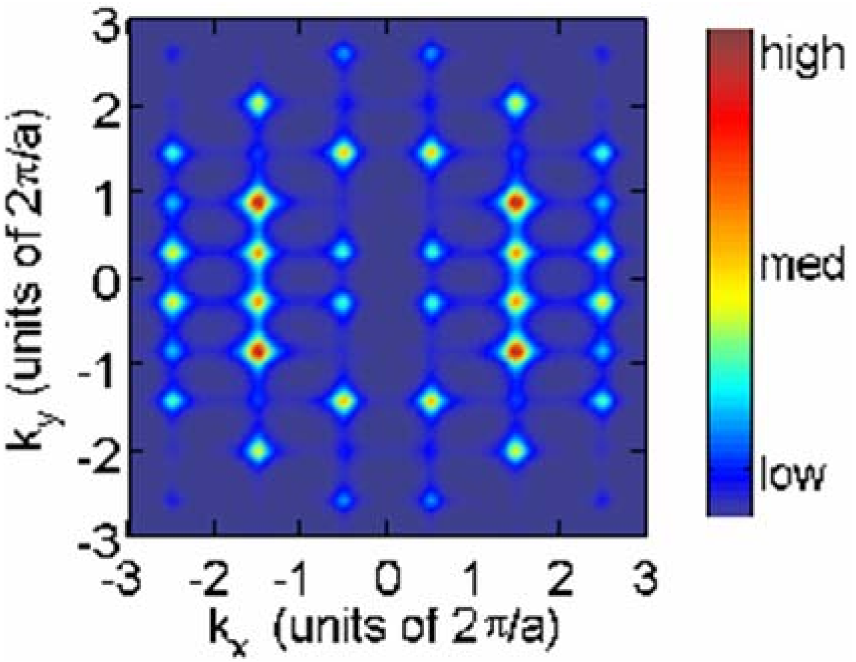
→ **Staggered fluxes (stripe order)**

Gutzwiller mean field phase diagram for the p-band bosons in a triangular lattice



Note: Stripe ordering even persists into Mott-insulating states without phase coherence!

Prediction: Time-of-flight experiment



Coherence peaks occur at **non-zero wavevectors.**

Predicted TOF density distribution for the stripe-ordered p-orbital superfluid

[C. Wu, WVL, J. Moore, and S. Das Sarma, *Phys. Rev. Lett.* (2006)]

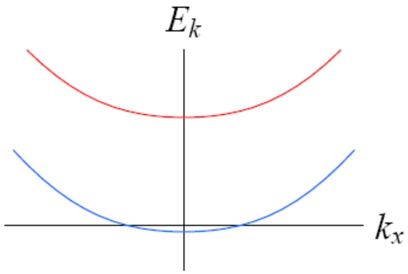
Novelty of lattice p-band Bose gases

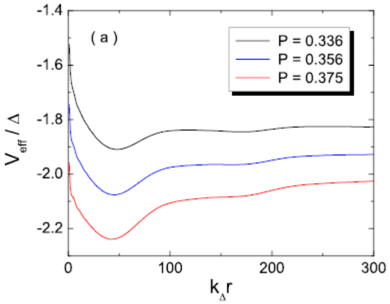
- Non-zero momentum BEC---defying the paradigm: predicted in theory, and discovered in experiments (for square lattice).
- Quantum orbital stripe order in a triangular lattice, in both the superfluid and Mott insulator phase.
- A rich set of broken symmetries: time reversal (T), orbital unitary transformation, space rotation and translation, and U(1).
- ***Different than He-3 superfluid:*** *p*-wave of the center-of-mass motion vs *p*-wave of the relative motion as in He-3.
- ***Future:*** novel excitations, topological defects, and topologically bound states?

Summary and Conclusion

Topic A: Breached Pair (without lattice)

[with Forbes, Gubankova, Kim, Stojanovic, Wilczek, Zoller]



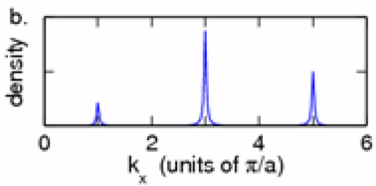


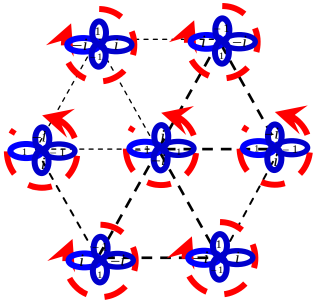
Phys Rev Focus

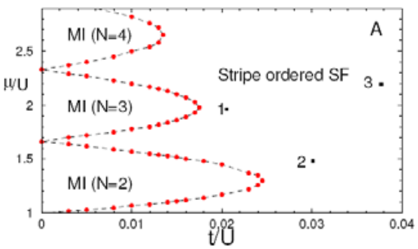
BP1 fermions

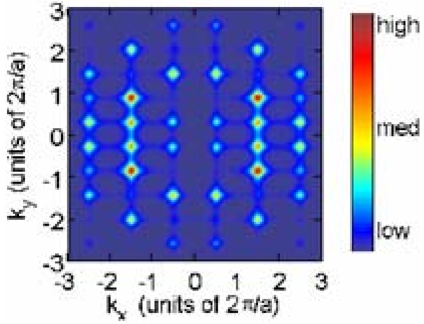
RKKY-like, attractive vortex interaction

Topic B: lattice *p*-band bosons [with Das Sarma, Moore, Wu]









$\mathbf{k} \neq 0$ BEC

stripe

phase diagram

prediction