Quantum fluctuations in FFLO superconductors

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Outline

- FFLO SC phase transition
- Fluctuation propagator
- Fluctuation corrections to susceptibility and quasiparticle decay rate
- Discussion

Superconductor in magnetic field

Magnetic field suppresses SC by acting on:

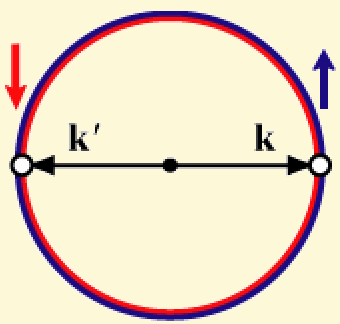
(a) charge (orbital motion of electrons)

condensation energy competes with kinetic energy

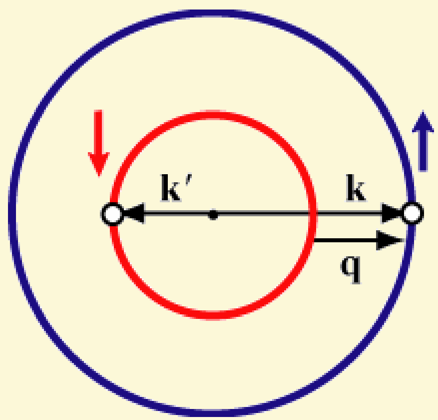
(b) spin (paramagnetic, or Pauli, mechanism)

condensation energy competes with polarization energy (Clogston-Chandrasekhar limit)

→ Non-uniform FFLO SC state with $q \neq 0$ and $H_C > H_{CC}$.



$\mathbf{q}=\mathbf{k}+\mathbf{k}'=0$



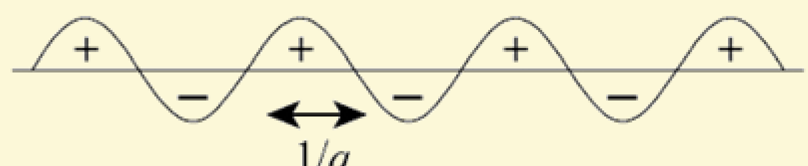
$\mathbf{q}=\mathbf{k}+\mathbf{k}'\neq 0$

$$\Delta(\mathbf{q}_c,\mathbf{r}) = \Delta_0 e^{i\mathbf{q}_c\mathbf{r}}$$

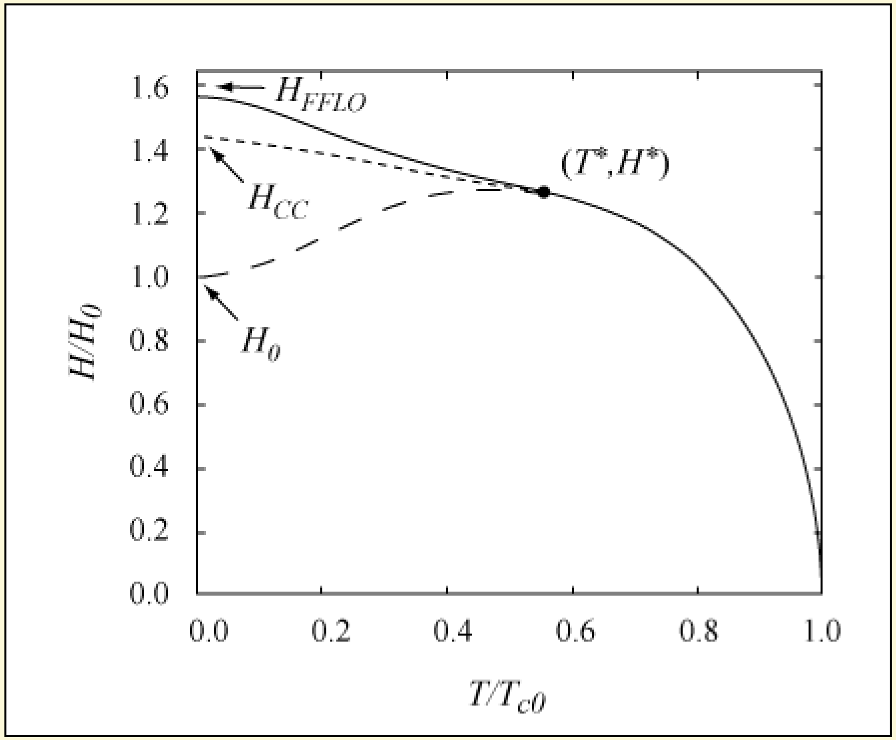
Fulde, Ferrell, '64

$$\Delta(\mathbf{q}_c,\mathbf{r}) = \Delta_0 \cos \mathbf{q}_c\mathbf{r}$$

Larkin, Ovchinnikov, '64



FFLO superconductivity



Generic phase diagram of FFLO superconductor

Relative importance of orbital and spin pair-breaking (Maki parameter) $\alpha_M = \sqrt{2} H_{c2} / H_{CC}$

Paramagnetic limit: $\alpha_M = \infty$

Gruenberg, Gunter '66: FFLO at $\alpha_M \geq 1.8$

Most "classical" SC's: $\alpha_M \leq 1$

<u>Reduce orbital effects</u>

Layered SC, parallel or tilted magnetic field (Bulaevskii '73, Shimahara, Rainer '97) thin films (Fulde '73), surface SC (Barzykin, Gorkov '02)

<u>Get rid of impurities</u>

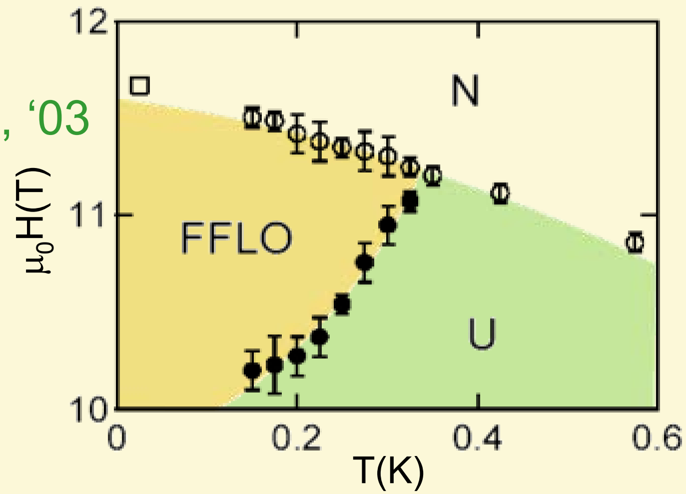
Aslamazov '68, Bulaevskii, Guseinov '76

Experimental situation

Clean, paramagnetically limited materials: heavy fermions, organic SC’s

$CeCoIn_5$

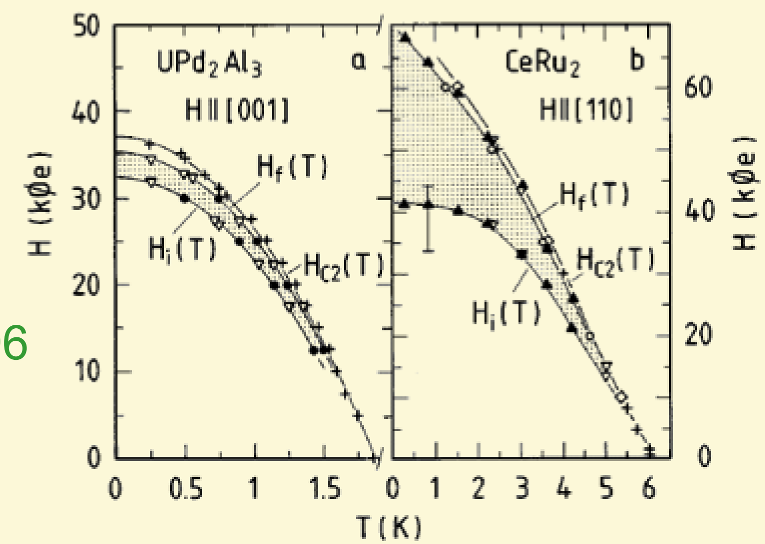
Radovan et. al, ‘03



UPd_2Al_3

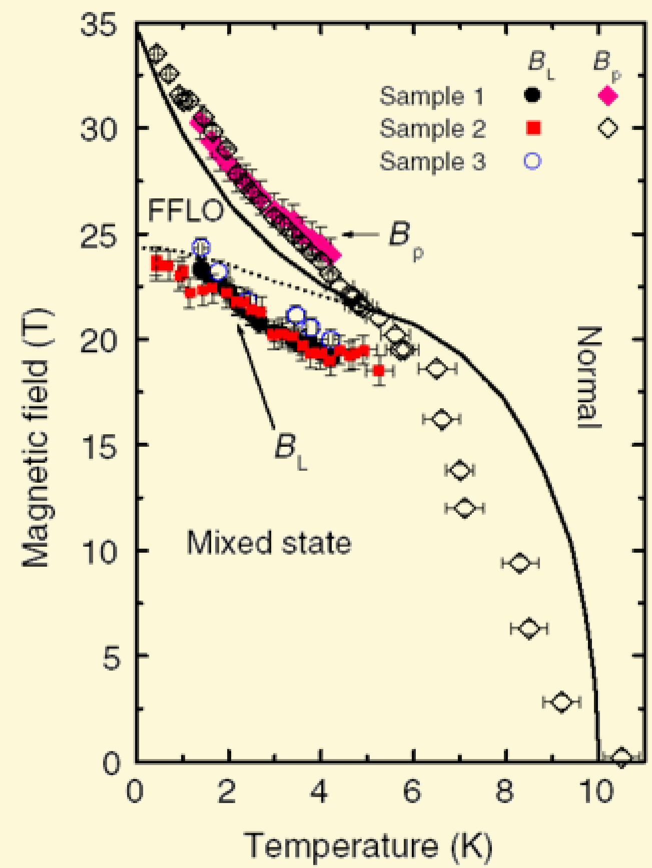
$CeRu_2$

Modler et. al, ‘96



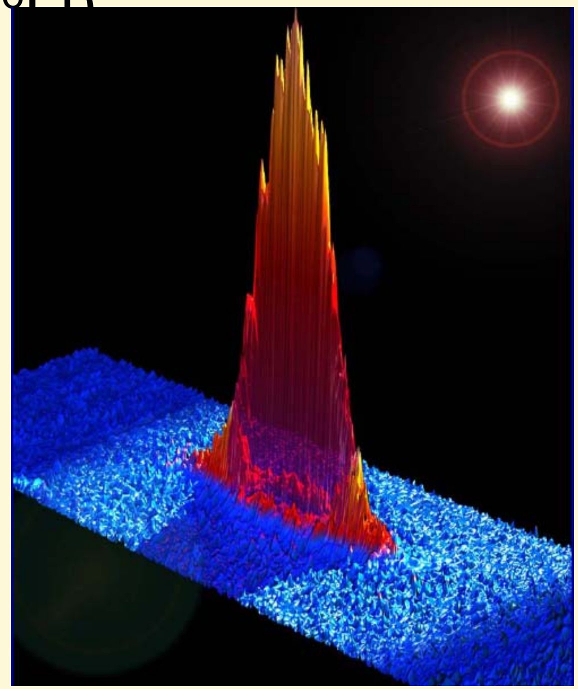
κ-$(BEDT\text{-}TTF)_2$-$Cu(NCS)_2$

Singleton et. al, ‘00



Fermi gas with population imbalance

no impurities, orbital effects, interaction could be adjusted (^{40}K, ^{6}Li)



Combescot '01

Mizushima et. al '05

Sedrakian et. al '05

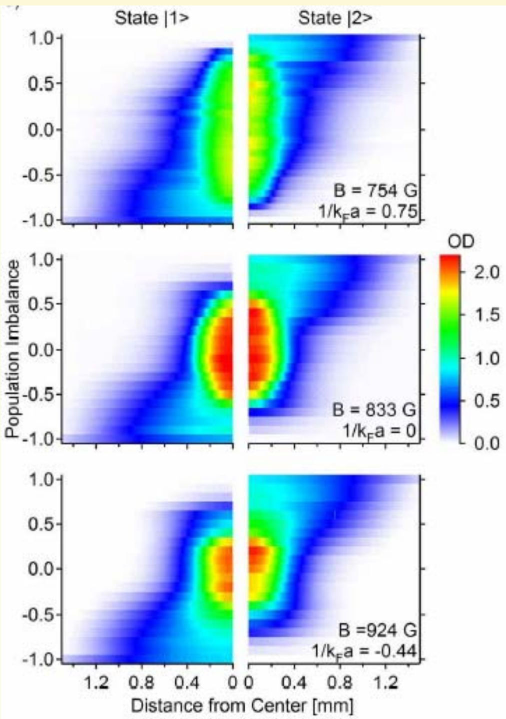
Sheehy, Radzihovsky '06

Kinnunen, Jensen, Torma '06

Samokhin, Marienko '06

Partridge, Kamar, Liao, Hulet '06

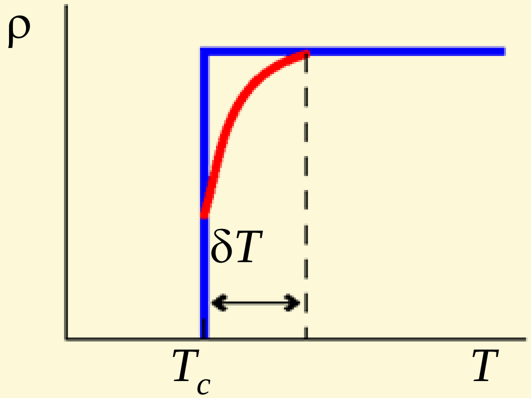
Zwierlein, Schirotzek, Schunck, Ketterle '06



Population imbalance is equivalent to Zeeman splitting

→ FFLO (and other nonuniform phases) with $q \neq 0$?

Superconducting fluctuations



Aslamazov, Larkin '68, Maki '68, Thomson '70

Corrections to the normal-state conductivity due to Cooper pairs existing at $T \geq T_C$ at H=0.

Free energy (corrections to the specific heat):

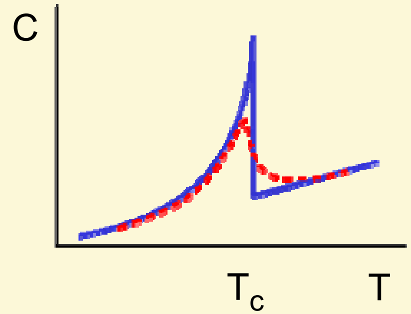
$$F[\Psi(r)] = F_N + \int dV \left\{ a|\Psi(\mathbf{r})|^2 + \frac{b}{2}|\Psi(\mathbf{r})|^4 + \frac{1}{4m}|\nabla\Psi(\mathbf{r})|^2 \right\}$$

$$F[\Psi_{\mathbf{k}}] = F_N + \sum_{\mathbf{k}} [a + \frac{\mathbf{k}^2}{4m}]|\Psi_{\mathbf{k}}|^2 = \alpha T_c \sum_{\mathbf{k}} \left(\epsilon + \xi^2 \mathbf{k}^2\right) |\Psi_{\mathbf{k}}|^2.$$

$$Z = \prod_{\mathbf{k}} \int d^2\Psi_{\mathbf{k}} \exp\left\{ -\alpha(\epsilon + \frac{\mathbf{k}^2}{4m\alpha T_c})|\Psi_{\mathbf{k}}|^2 \right\}$$

$$F(\epsilon > 0) = -T \ln Z = -T \sum_{\mathbf{k}} \ln \frac{\pi}{\alpha(\epsilon + \frac{\mathbf{k}^2}{4m\alpha T_c})}$$

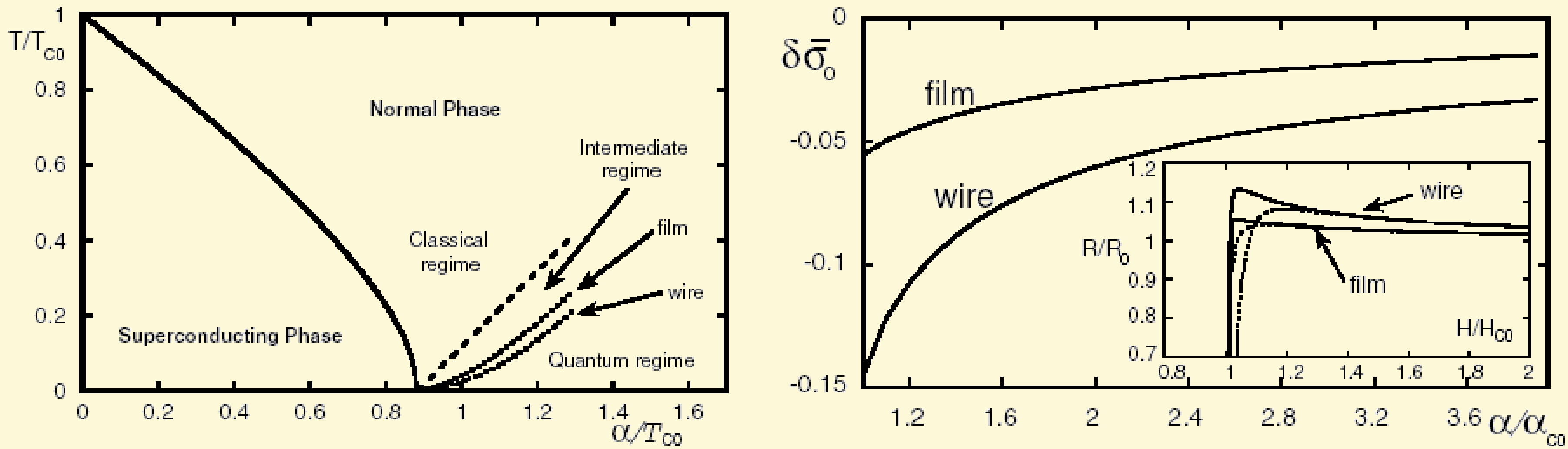
$$\delta C_+ = -\frac{1}{VT_c}\left(\frac{\partial^2 F}{\partial \epsilon^2}\right) = \frac{1}{V}\sum_{\mathbf{k}} \frac{1}{(\epsilon + \frac{\mathbf{k}^2}{4m\alpha T_c})^2}$$



$$\delta C_+ \propto \frac{1}{\sqrt{\varepsilon}}$$

Larkin, Varlamov, *Theory of Fluctuations in Superconductors* (Oxford '05)

Fluctuations conductivity near SC QPT



Superconductor at T=0: controllable QCP, allows for systematic quantitative study

$$\ln(T_c/T_{c0}) = \psi(1/2) - \psi(1/2 + \alpha/2\pi T_c)$$

Ramazashvili, Coleman '97

Mineev, Sigrist '01

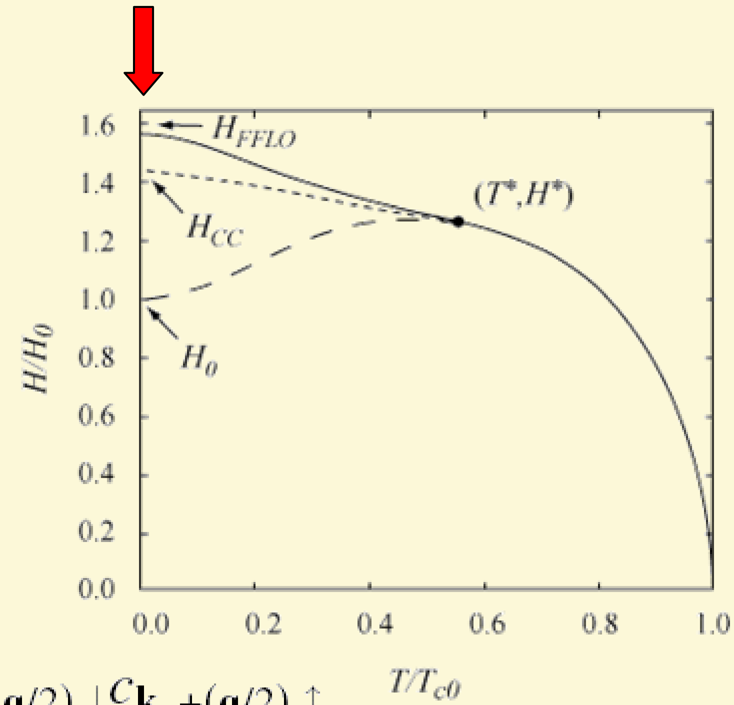
Galitski, Larkin '01, Galitski, Das Sarma '03

Lopatin, Shah, Vinokur '05

Fluctuations near FFLO state?

Since the phase volume of fluctuations is greater than near uniform BCS transition (the wave vectors of fluctuating modes are close to q_c, i.e. sphere in isotropic 3D), the fluctuation effects will be considerably increased. (Brazovskii '75)

Consider: clean spin-singlet SC in external magnetic field (enters through Zeeman splitting) at T=0



Generalized BCS Hamiltonian:

$$H = \sum_{\mathbf{k}} (\xi_{\mathbf{k}}\delta_{\alpha\beta} - h\sigma_{3,\alpha\beta}) c^{\dagger}_{\mathbf{k}\alpha} c_{\mathbf{k}\beta}$$

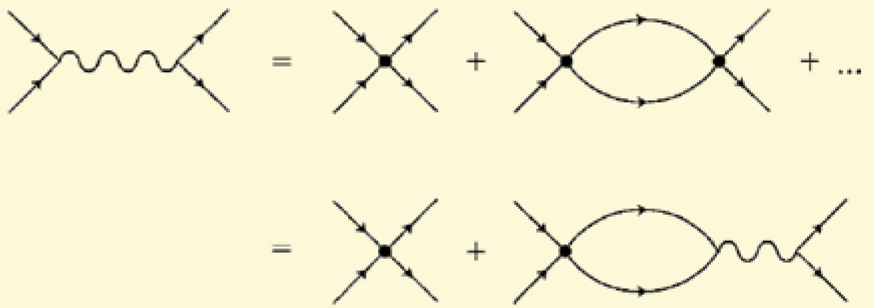
$$+ \sum_{\mathbf{q},\mathbf{k}_{1,2}} V_{\mathbf{k}_1\mathbf{k}_2}(\mathbf{q}) c^{\dagger}_{\mathbf{k}_1+(\mathbf{q}/2),\uparrow} c^{\dagger}_{-\mathbf{k}_1+(\mathbf{q}/2),\downarrow} c_{-\mathbf{k}_2+(\mathbf{q}/2),\downarrow} c_{\mathbf{k}_2+(\mathbf{q}/2),\uparrow}$$

$$h = \mu_B H \qquad V_{\mathbf{k}_1\mathbf{k}_2}(\mathbf{q}) = -\lambda(\mathbf{q})\phi_{\mathbf{k}_1}\phi_{\mathbf{k}_2}$$

Fluctuation propagator

Order parameter dynamics is described by fluctuation propagator

$$\mathcal{L}(\mathbf{q},\nu_m) = \frac{1}{\lambda^{-1} - \mathcal{C}(\mathbf{q},\nu_m)}$$



Calculating the diagrams, one obtains:

$$\frac{1}{N_F}\mathcal{L}^{-1}(\mathbf{q},\nu_m) = \ln\frac{T}{T_{c0}} - \Psi\left(\frac{1}{2}\right) + \left\langle \phi_{\mathbf{k}}^2 \,\mathrm{Re}\,\Psi\left(\frac{1}{2} + \frac{iW_{\mathbf{k}} + |\nu_m|}{4\pi T}\right)\right\rangle$$

$$W_{\mathbf{k}} = \xi_{\mathbf{k}+(\mathbf{q}/2)} - \xi_{\mathbf{k}-(\mathbf{q}/2)} - 2h = \mathbf{v}_{\mathbf{k}}\mathbf{q} - 2h + O(\mathbf{q}^3) \qquad \mathbf{v}_{\mathbf{k}} = \nabla_{\mathbf{k}}\xi_{\mathbf{k}}$$

The solution $T(\mathrm{q},h)$ of the equation $\mathcal{L}^{-1}(\mathbf{q},0)=0$ determines the critical temperature of the SC state with the wave vector **q** in a given field h. Setting $q=T=0$, one restores the second-order phase transition into uniform SC state at $h_0=0.88T_{c0}$. (T_{c0} is zero-field critical temperature).

In general, in clean isotropic SC at $T<T^*\approx 0.56\, T_{c0}$, the maximum of critical field is at $q_c\neq 0$.

Near $h_c(T)$ at $T\to 0$, we will try to find power expansion of fluctuation propagator near $q=q_c$ and $\nu_m=0$.

Fluctuation propagator, isotropic 3D

3D parabolic band: $\xi_{\mathbf{k}} = \mathbf{k}^2/2m - \mu$

$$\frac{1}{N_F}\mathcal{L}^{-1}(\mathbf{q},\nu_m) = A(\mathbf{q},h) + \widetilde{A}(\mathbf{q},h)\frac{|\nu_m|}{2h}$$

$$A(\mathbf{q},h) = \ln\frac{h}{h_0} + F(\mathbf{Q})$$

$$F(\mathbf{Q}) = \frac{1}{2}\ln|x^2-1| + \frac{1}{2x}\ln\left|\frac{x+1}{x-1}\right| - 1$$

$$x = v_F Q = v_F q/2h$$

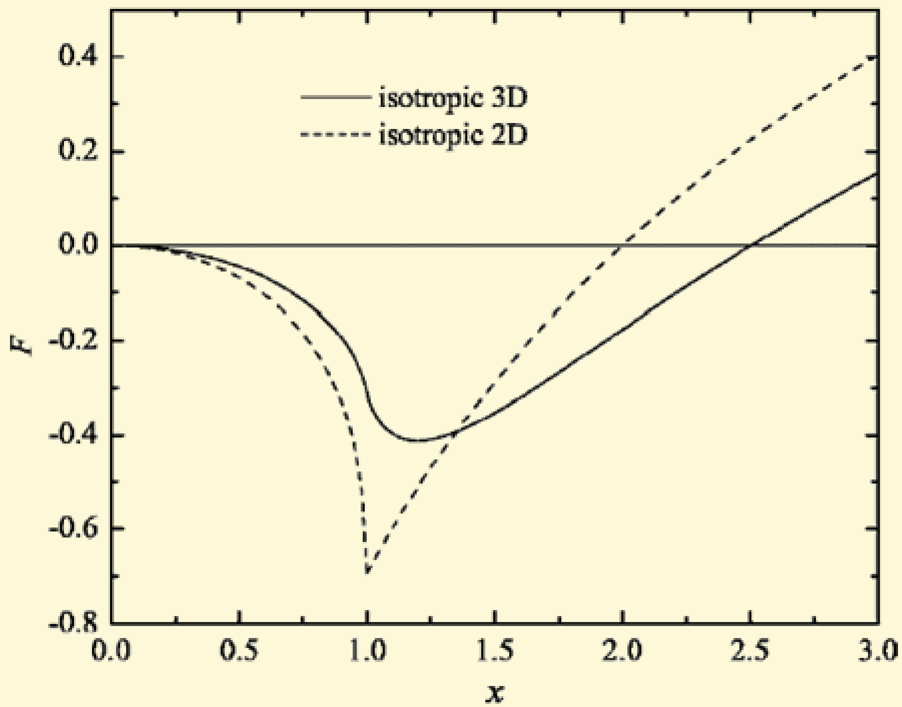
$$x = x_c \simeq 1.20$$

$$h_c \simeq 1.51 h_0 \simeq 0.75\Delta_0$$

$$q_c = 2x_c h_c/v_F \simeq 0.51\xi_0^{-1}$$

$$\xi_0 = v_F/2\pi T_{c0}$$

$$\widetilde{A}(\mathbf{q},h) = \frac{\pi}{2x}\theta(x-1)$$



$$\mathcal{L}(\mathbf{q},\nu_m) = \frac{1}{N_F}\frac{1}{\epsilon + \gamma|\nu_m| + K(|\mathbf{q}| - q_c)^2}$$

$$K \simeq 0.28 v_F^2/h_c^2$$

$$\gamma \simeq 0.65/h_c$$

$$\epsilon = \frac{h-h_c}{h_c}$$

Does not depend on direction of *q*!

Fluctuation propagator, generic band

Suppose the infinite degeneracy of FFLO state is lifted by general band structure (crystal symmetry) or by gap anisotropy.

This will make minima of $A(q,h)$ well separated, and the fluctuation modes with different q can be treated independently.

$$\mathcal{L}_a(\mathbf{q}, \nu_m) = \frac{1}{N_F} \frac{1}{\epsilon + \gamma|\nu_m| + K(\mathbf{q} - \mathbf{q}_c^{(a)})^2}$$

is fluctuation propagator near ath minimum.

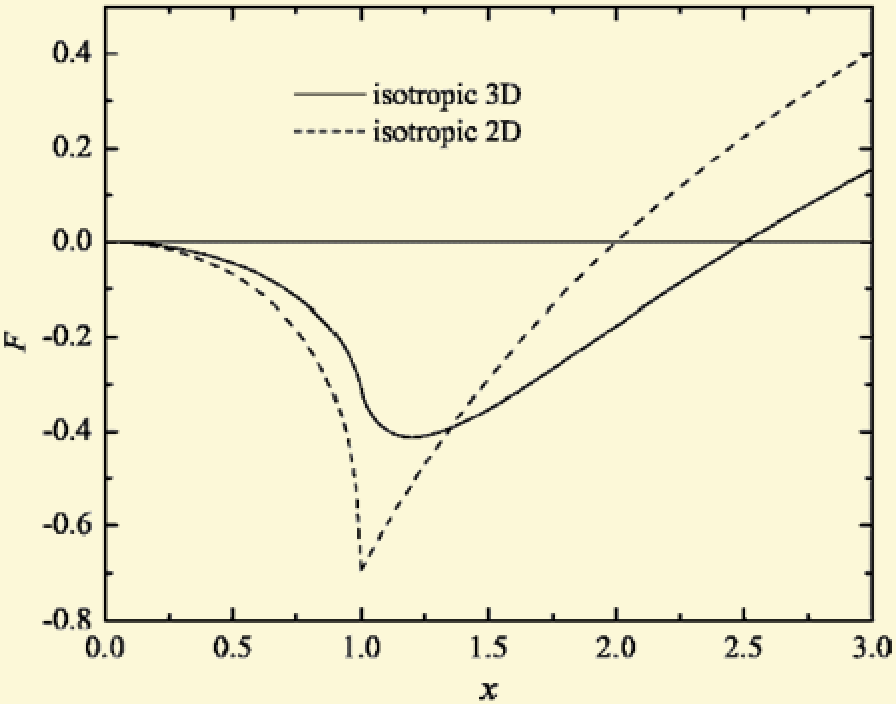
Fluctuation propagator, isotropic 2D

In clean isotropic 2D case with parabolic band dispersion, expansion in powers of frequency fails.

$$F(\mathbf{Q}) = \mathrm{Re}\ln\frac{1+\sqrt{1-x^2}}{2}$$

$$\widetilde{A}(\mathbf{q},h) = \frac{1}{\sqrt{x^2-1}}\theta(x-1)$$

$$x = v_F Q = v_F q/2h$$



$F(\mathbf{Q})$ has nonanalytical minimum at $x=1$

$$F(\mathbf{Q}_c) = -\ln 2$$

$$h_c = 2h_0$$

$$q_c = 4h_0/v_F$$

$\widetilde{A}(\mathbf{q},h)$ diverges at critical point, thus:

$$\frac{1}{N_F}\mathcal{L}^{-1}(\mathbf{q},\nu_m) = \ln\frac{h}{h_0} + \mathcal{F}\left(\frac{v_F|\mathbf{q}|}{2h},\frac{|\nu_m|}{2h}\right)$$

$$\mathcal{F}(x,y) = \mathrm{Re}\ln\frac{1+iy+\sqrt{(1+iy)^2-x^2}}{2}$$

(nonanalyticity persists oven in layered 2D case

Fluctuation corrections, spin susceptibility

$$\delta F = 2\sum_{\mathbf{q}} \int_0^{\nu_{\max}} \frac{d\nu}{2\pi} \ln \mathcal{L}^{-1}(\mathbf{q},\nu)$$

$$\delta\chi = -\frac{\partial^2}{\partial H^2}\delta F = -N_F^2 \frac{1}{H_c^2}\frac{\partial^2}{\partial \epsilon^2}\delta F$$

Isotropic 3D:

$$\delta\chi = \frac{N_F}{\pi\gamma H_c^2}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1}{\epsilon + K(|\mathbf{q}|-q_c)^2} \simeq \frac{N_F q_c^2}{2\pi^2\gamma H_c^2\sqrt{K}}\frac{1}{\sqrt{\epsilon}}$$

$$\frac{\delta\chi}{\chi_P} \simeq 1.21\left(\frac{\Delta_0}{\epsilon_F}\right)^2\left(\frac{H_c}{H-H_c}\right)^{1/2}$$ singular, but small

Magnetization

$$\delta M = -\frac{\partial}{\partial H}\delta F = -N_F\frac{1}{H_c}\frac{\partial}{\partial \epsilon}\delta F \propto \sqrt{\epsilon}$$ not singular

Fluctuation corrections, spin susceptibility

Generic 3D (fluctuation phase space reduced) :

$$\delta\chi = \frac{N_F}{\pi\gamma H_c^2}\sum_a \int \frac{d^3\mathbf{q}}{(2\pi)^3}\frac{1}{\epsilon + K(\mathbf{q}-\mathbf{q}_c^{(a)})^2} = N_q\frac{N_F q_{\max}}{2\pi^3 H_c^2 \gamma K}\left(1 - \frac{\pi}{2q_{\max}}\sqrt{\frac{\epsilon}{K}}\right)$$ not singular

Generic 2D:

$$\delta\chi = \frac{N_F}{\pi\gamma H_c^2}\sum_a \int \frac{d^2\mathbf{p}}{(2\pi)^2}\frac{1}{\epsilon + K\mathbf{p}^2}$$

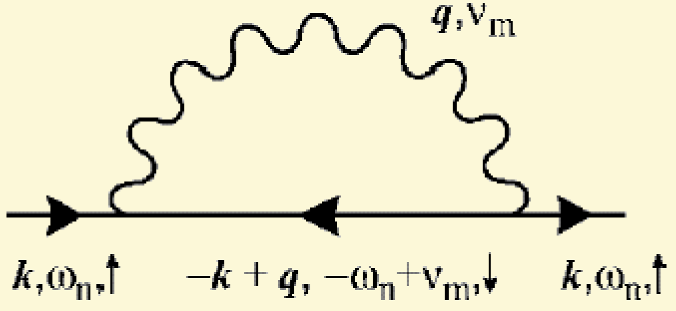
$$\frac{\delta\chi}{\chi_P} \sim \frac{\Delta_0}{\epsilon_F}\ln\frac{H_c}{H-H_c}$$ logarithmic singularity

Formally, for normal metal to uniform SC transition (q_c=0), correction to susceptibility in isotropic 3D is nonsingular; in isotropic 2D it is logarithmically divergent.

Fluctuation corrections, self energy

Self energy of spin-up fermions

$$\Sigma_{\uparrow}(\mathbf{k},\omega_n) = -T\sum_m \sum_{\mathbf{q}} \mathcal{L}(\mathbf{q},\nu_m) \times G_{\downarrow}(-\mathbf{k}+\mathbf{q},-\omega_n+\nu_m)$$



Quasiparticle decay rate at spin-up FS, T=0

$$\Gamma(\hat{\mathbf{k}},\omega) \equiv -\operatorname{Im}\Sigma^R_{\uparrow}(\hat{\mathbf{k}},\omega) = \sum_{\mathbf{q}} \operatorname{Im}\mathcal{L}^R(\mathbf{q},\omega - W_{\mathbf{k}})$$

Corrections to decay rate at QCP (ε=0), at $\omega \to 0$:

Isotropic 3D:

$$\frac{\Gamma(\omega)}{\Delta_0} \simeq 1.57\left(\frac{\Delta_0}{\epsilon_F}\right)^2\left(\frac{\omega}{\Delta_0}\right)^{1/2}$$

Generic 3D: strongly anisotropic:

$\hat{\mathbf{k}}$ for which $w_a = \mathbf{v}_{\mathbf{k}}\mathbf{q}_c^{(a)} - 2h_c \neq 0$

$$\frac{\Gamma(\hat{\mathbf{k}},\omega)}{\Delta_0} \sim \left(\frac{\Delta_0}{\epsilon_F}\right)^2\left(\frac{\omega}{\Delta_0}\right)^2$$

$\mathbf{v}_{\mathbf{k}}\mathbf{q}_c^{(a)} = 2h_c$

$$\frac{\Gamma(\hat{\mathbf{k}},\omega)}{\Delta_0} \sim \left(\frac{\Delta_0}{\epsilon_F}\right)^2 \frac{\omega}{\Delta_0}$$

Fluctuation corrections, self energy

Generic 2D: also strongly anisotropic:

$\hat{\mathbf{k}}$ for which $w_a = \mathbf{v}_\mathbf{k}\mathbf{q}_c^{(a)} - 2h_c \neq 0$

$$\frac{\Gamma(\hat{\mathbf{k}},\omega)}{\Delta_0} \sim \frac{\Delta_0}{\epsilon_F}\left(\frac{\omega}{\Delta_0}\right)^2$$

$\mathbf{v}_\mathbf{k}\mathbf{q}_c^{(a)} = 2h_c$

$$\frac{\Gamma(\hat{\mathbf{k}},\omega)}{\Delta_0} \sim \frac{\Delta_0}{\epsilon_F}\left(\frac{\omega}{\Delta_0}\right)^{1/2}$$

Formally, for normal metal to uniform SC transition (q_c=0):

Isotropic 3D:

$$\Gamma(\omega) = \frac{1}{64\pi^2}\frac{\gamma v_F}{N_F K^2 \tilde{h}_c^2}\omega^2$$

Isotropic 2D:

$$\Gamma(\omega) = \frac{1}{128\pi}\frac{\gamma v_F^2}{N_F K^2 h_c^3}\omega^2$$

Here Fermi liquid character of quasiparticle excitations is not destroyed by quantum fluctuations

Conclusions

- We studied superconducting fluctuations tear QPT (T=0) at H=Hc from the normal state to FFLO state.
- We derived the general form of fluctuation propagator at finite *q* and ν_m.
- In the absence of impurities and orbital effects, we analyzed momentum and frequancy dependence of the propagator in 3D and 2D as well as in the case of generic spectrum.
- The fluctuations are more pronounces in isotropic 3D compared to the generic situations: the susceptibility diverges at H→Hc. $\delta\chi \propto (H-H_c)^{-1/2}$
  The quasiparticle decay rate shows non-Fermi-liquid behavior at QCP: $\Gamma(\omega) \propto \omega^{1/2}$
- In generic case, the phase volume of fluctuations is reduced, resulting in non-singular spin susceptibility in 3D. Fluctuations still are strong enough to cause breakdown of Fermi-liquid at QCP, which manifests itself in highly anisotropic dependence of quasiparticle decay rate on the Fermi surface: $\Gamma(\omega) \propto \omega$ (lines in 3D) $\Gamma(\omega) \propto \omega^{1/2}$ (points in 2D)

- Impurities, order of phase transition, non-zero temperatures, 2D???