Spinor condensates in two dimensions: Topological defects and the superfluid transition

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2D superfluid transition in *S*=1 spinor condensates

Mukerjee, Xu and Moore, Phys. Rev. Lett., 97, 120406 (2006) and cond-mat/0605102

Coarsening dynamics of *S*=1 spinor condensates

(time permitting)

Mukerjee, Xu and Moore, arXiv:0704.3440

Why spinor condensates?

- Possibility of interesting magnetic ordering: Broken symmetry more than just $U(1)$

- Interesting phase transitions and topological defects

- Realized experimentally in the last 10 years

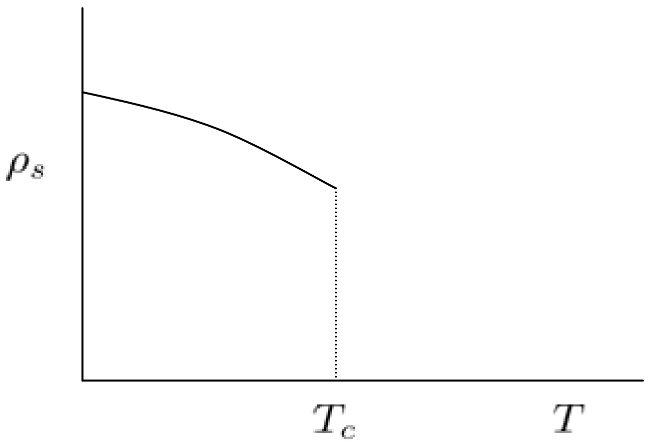
Why 2D?

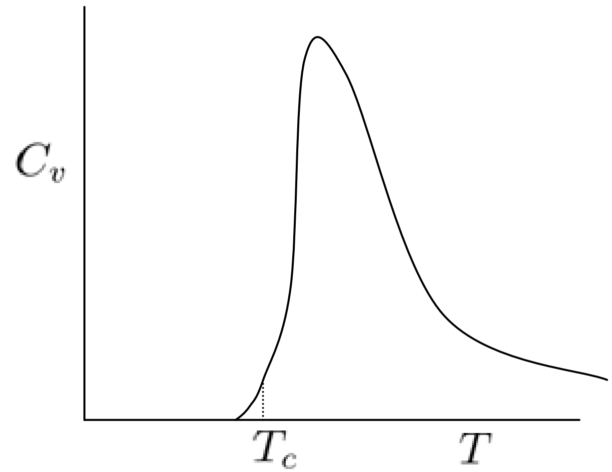
- Unconventional phases and phase transitions

- Topology and defects play an important role

- Realized experimentally in the last 2 years

2D superfluid (Kosterlitz-Thouless) transition for spinless bosons

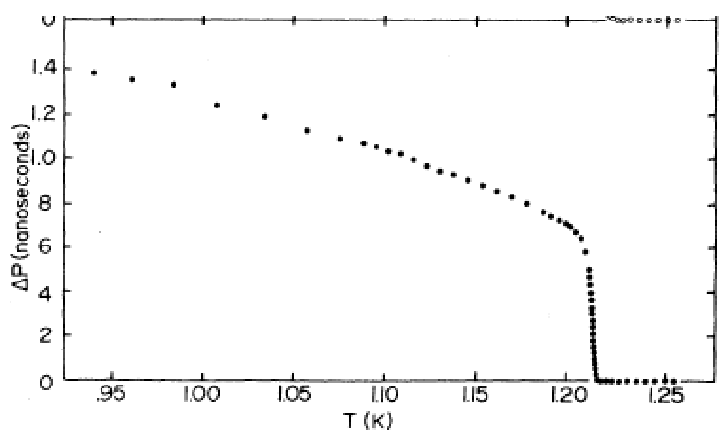




$$\frac{\rho_s(T_c^-)}{T_c} = \frac{2m^2}{\pi\hbar^2 k_B}$$

Universal jump

C_v has weak essential singularity at T_c



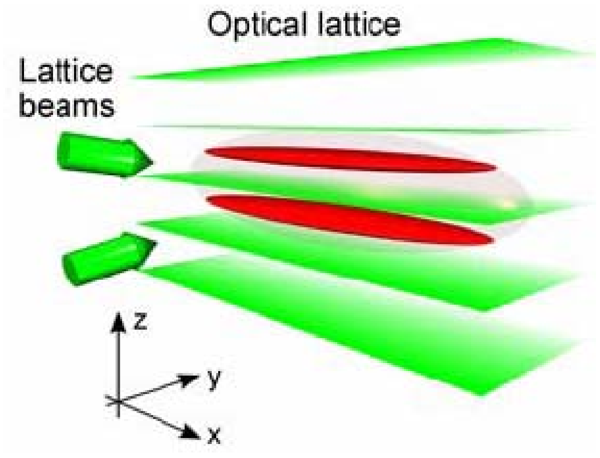
Experimental, Bishop and Reppy
PRL 40, 1727 (1978)

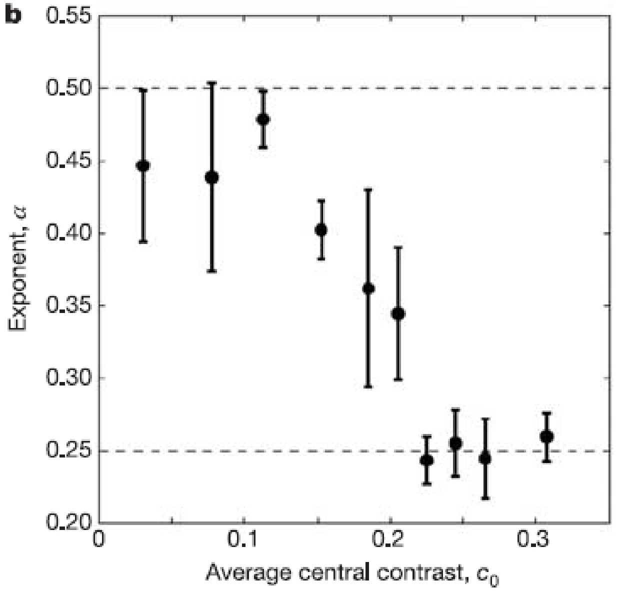
Thin helium film: torsional oscillator

KT transition in atomic BEC in a magnetic trap (^{87}Rb)

Hadzibabic et. al, Nature 441, 1118 (2006)

Interference measurement





c_0 like temperature. High c_0, low temperature.

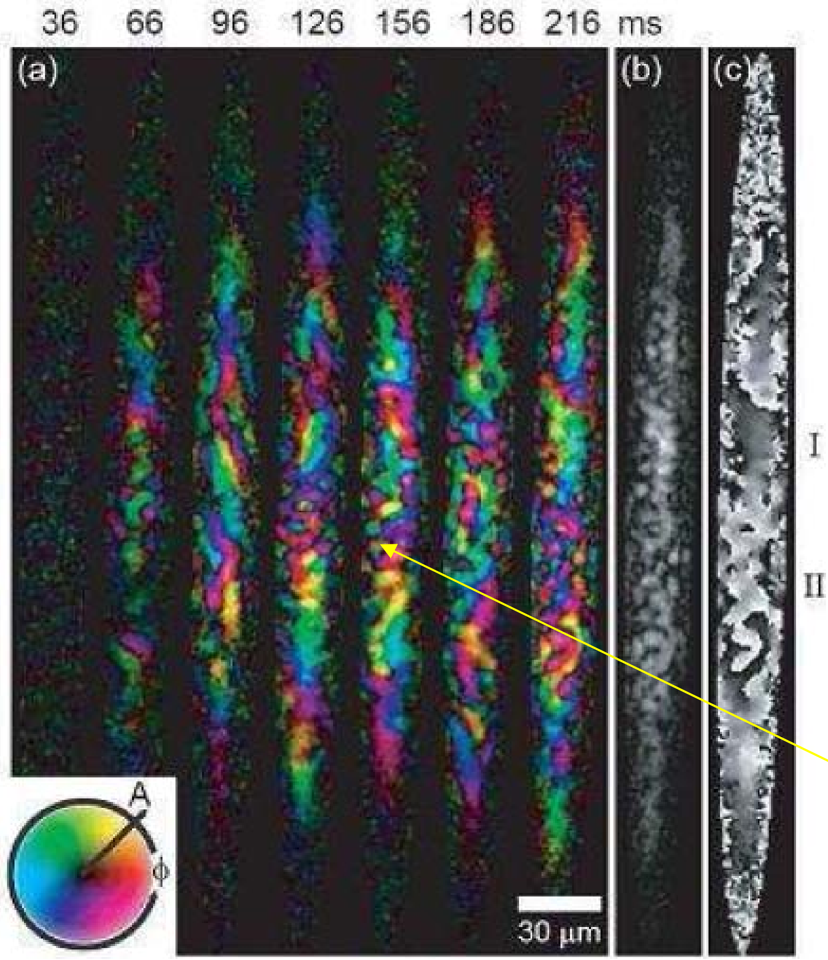
$$\left\langle C^2(L_x)\right\rangle \approx \frac{1}{L_x}\int_0^{L_x} dx\,\left[g_1(x,0)\right]^2 \propto \left(\frac{1}{L_x}\right)^{2\alpha}$$

$\alpha = 0.5$, short ranged correlations

Stiffness jump corresponds to $\alpha = 0.25 \rightarrow 0.5$

2D geometry in an optical trap (^{87}Rb)

Sadler et. al, Nature 443, 312 (2006)



Spin healing length $> r_y$

Effectively 2D for the spins

Imaging of magnetization locally

Effective two-body interaction in an optical trap

$$V_{12} = \delta(|\mathbf{r}_1 - \mathbf{r}_2|) f(\vec{S}_1, \vec{S}_2)$$

T. –L. Ho, PRL 81, 742 (1998)
A. J. Leggett, RMP 73, 307 (2001)

Only s wave scattering: $k_B T / E_c << 1$

Diluteness

E_c: Energy of bound state

$$f(\vec{S}_1, \vec{S}_2) = g_0 P_0 + g_2 P_2$$

for S=1 atoms

Only even spin projections allowed

$g_0 > 0, g_2 > 0$ for ^{23}Na and ^{87}Rb

$$V_{12} = \delta(|\mathbf{r}_1 - \mathbf{r}_2|)(g_0 P_0 + g_2 P_2)$$
$$= \delta(|\mathbf{r}_1 - \mathbf{r}_2|)(c_0 + c_2 \vec{S}_1 . \vec{S}_2)$$

$c_0 = \frac{g_0 + 2g_2}{3}$ $\quad c_2 > 0$ Antiferromagnetic or Polar (^{23}Na)

$c_2 = \frac{g_2 - g_0}{3}$ $\quad c_2 < 0$ Ferromagnetic (^{87}Rb)

Many body Hamiltonian

$$H = \int d\mathbf{r}(\frac{\hbar^2}{2M}\nabla\psi_a^\dagger\cdot\nabla\psi_a + U(\mathbf{r})\psi_a^\dagger\psi_a + c_0\psi_a^\dagger\psi_b^\dagger\psi_b\psi_a + c_2\psi_a^\dagger\psi_{a'}^\dagger\vec{S}_{ab}\cdot\vec{S}_{a'b'}\psi_{b'}\psi_b)$$

$$a, b, a', b' = \{+1, 0, -1\}$$

$U(\mathbf{r}) = 0$ (Uniform state)

Mean field ground state, $q = 0$ BEC: $\langle\psi_a^\dagger\rangle = \psi_a^*$

Stable against spin fragmentation for $c_2 > 0$ when $N \rightarrow \infty$

Yip and Ho, PRL 84, 4031 (2000)

$$\langle H \rangle = E = n^2(c_0 + c_2 \langle \vec{S} \rangle^2)$$

$$\psi_a = \sqrt{n}\chi_a; \langle \vec{S} \rangle = X^\dagger \vec{S} X; X^\dagger X = 1$$

$$X = \begin{pmatrix} \chi_{+1} \\ \chi_0 \\ \chi_{-1} \end{pmatrix}$$

spinor

E is invariant under gauge transformations and spin rotations

$$E \text{ is invariant under } X \to e^{i\theta} R X$$

Gauge ($U(1)$) Spin ($SO(3)$)

$SO(3)$ because of integer spin

$$R = e^{-i\alpha S_x} e^{-i\beta S_y} e^{-i\gamma S_z}$$

α, β, γ: Euler angles

Ground state manifold

Ferromagnetic, $c_2 < 0$

Minimum energy: $\langle \vec{S} \rangle^2 = 1$

$$X = e^{i\theta} e^{-i\alpha S_x} e^{-i\beta S_y} e^{-i\gamma S_z} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$X = e^{i(\theta-\gamma)} \begin{pmatrix} e^{-i\alpha} \cos^2 \beta/2 \\ \sqrt{2} \cos \beta/2 \sin \beta/2 \\ e^{i\alpha} \cos^2 \beta/2 \end{pmatrix}$$

Parameterized by Euler angles α, β and $\gamma - \theta$

Ground state manifold is isomorphic to $SO(3)$

Ground state manifold

Polar, $c_2 > 0$

Minimum energy: $\langle \vec{S} \rangle^2 = 0$

$$X = e^{i\theta} e^{-i\alpha S_x} e^{-i\beta S_y} e^{-i\gamma S_z} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$X = e^{i\theta} \begin{pmatrix} 1/\sqrt{2}(e^{-i\alpha} \sin\beta) \\ \cos\beta \\ 1/\sqrt{2}(e^{i\alpha} \sin\beta) \end{pmatrix}$$

Parameterized by $(\theta, \hat{\mathbf{n}})$ Zhou, PRL 87, 080401(2001)

Invariant under $\theta \to \theta + \pi$ and $\hat{\mathbf{n}} \to -\hat{\mathbf{n}}$

Ground state manifold is isomorphic to $\frac{U(1)\times S^2}{Z_2}$

Connection to other condensed matter systems

Incommensurate spin density waves in Mott insulators

Non-collinear state: $SO(3)$ (like ferromagnetic spinor)

Collinear state: $\frac{U(1)\times S^2}{Z_2}$ (like polar spinor)

Zhang, Sachdev and Demler PRB 66, 094501 (2002)

$SO(3)$ also in

Dipole locked A phase of ^{3}He

Classical triangular lattice Heisenberg antiferromagnet

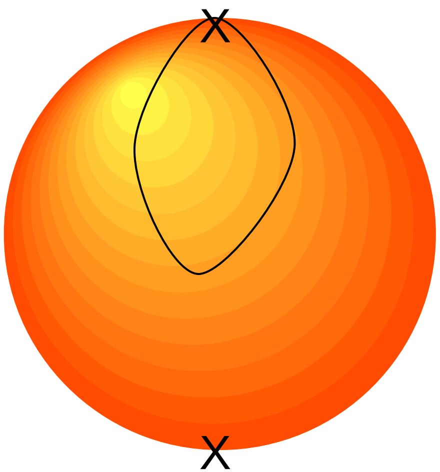
Topological defects

- Topological defects can be mapped onto non-contractable closed loops on manifolds
- Defects form a group called the first homotopy or fundamental group of the manifold (denoted as $\pi_1(M)$)
- Concatenation of defects is the group multiplication operation
- Fundamental group need not be Abelian

Topological defects (Homotopy group)

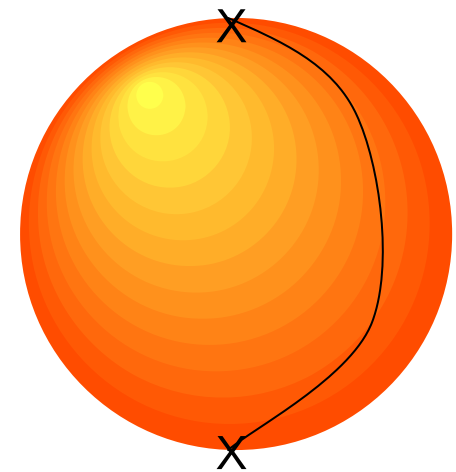
$$SO(3) = S^3/Z_2$$

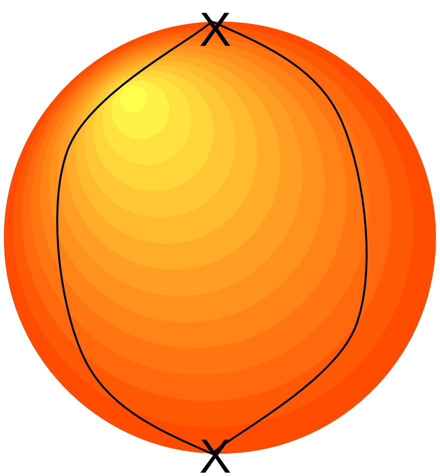
3-sphere with diametrically opposite points identified



contractable

non-contractable



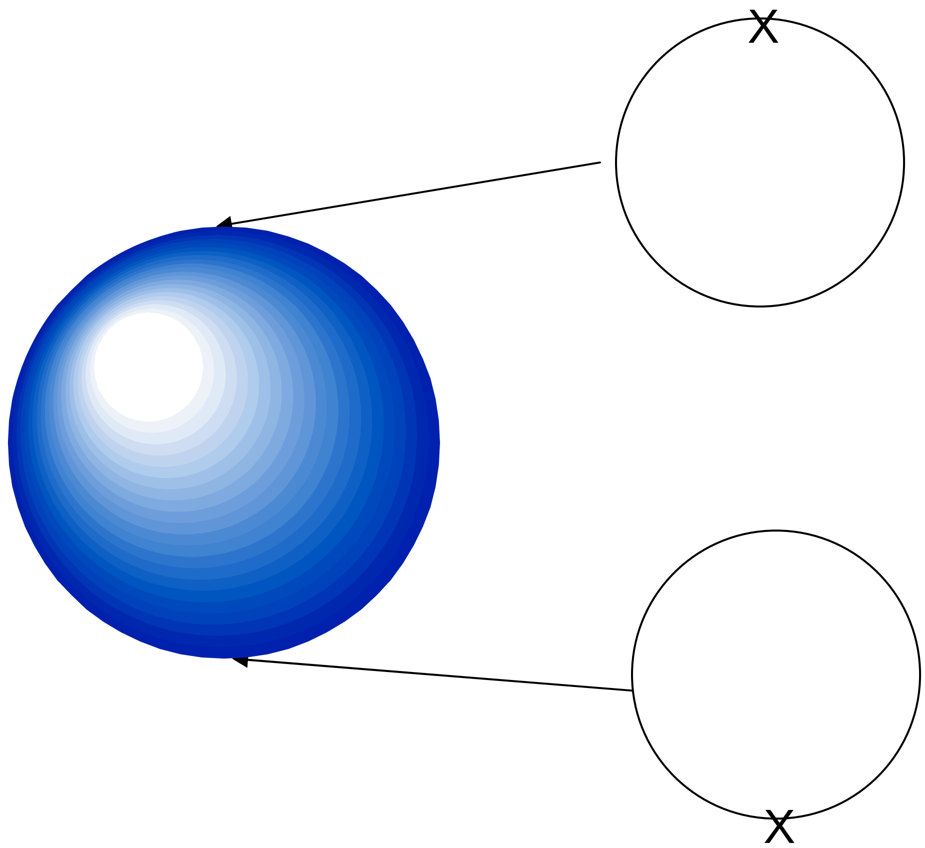


concatenation of non-contractable loops is contractable

$$\pi_1(SO(3)) = Z_2 = \{e, g\}$$

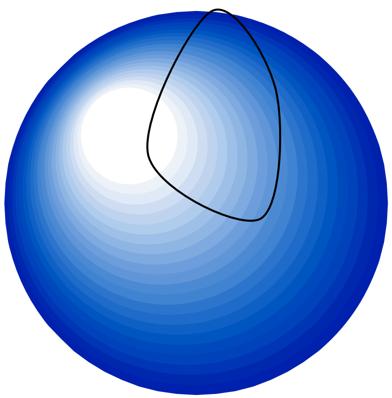
Topological defects

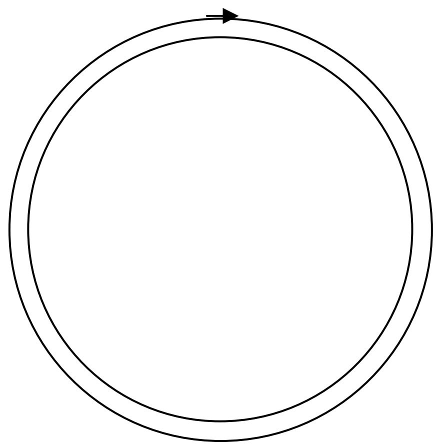
$\frac{U(1)\times S^2}{Z_2}$: Hollow sphere and circle



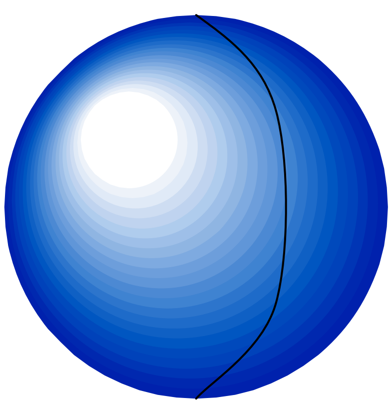
Topological defects (Homotopy theory)

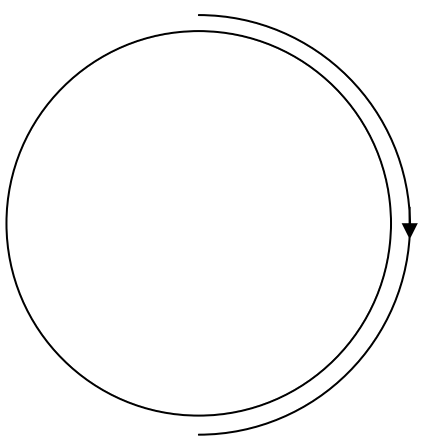
Two types of non-contractable loops





$$A = \{n, e\}$$





$$B = \{n + 1/2, g\}$$

Topological defects (Homotopy theory)

$$A = \{n, e\} \qquad B = \{n + 1/2, g\}$$

Concatenation of defects

$$AA \to A$$

$$BB \to A$$

$$AB, BA \to B$$

$$\{e, g\} \text{ label redundant}$$

$$\left[\pi_1\left(\frac{U(1)\times S^2}{Z_2}\right) = Z\right] \neq \left[\pi_1\left(U(1) \times \frac{S^2}{Z_2}\right) = Z \times Z_2\right]$$

Ferromagnetic condensate has Z_2 defects

No stable vortices, only skyrmions

No point defects exist in 3D ($\pi_2(SO(3)) = 0$)

Polar condensate has vortices

Fundamental vortex combines π phase rotation and magnetic inversion

3D point defects (monopoles) exist: $\pi_2\left(\frac{U(1)\times S^2}{Z_2}\right) = Z$

2D finite temperature

- Mermin-Wagner: No long-range order
- Quasi-long-range order possible (like for spinless bosons)
- Low T behavior describable in terms of Non-linear Sigma Model
- Perturbative Renormalization Group (RG) flows can be calculated for the stiffness of the order parameter.
- Perturbative RG flows depend only on the local structure of the order parameter manifold

$$\frac{dT}{dl} = -(d-2)T + (n-2)K_d\Lambda^{d-2}T^2 + \ldots$$

d dimensions for $O(N)$ model

In 2D for $n > 2$ flow is away from $T = 0$

No order at finite T

$n = 2$ ($U(1)$), marginal to all orders in 2D

Line of critical points

Finite T Kosterlitz-Thouless (KT) transition mediated by topological defects

Non-linear sigma model (lowest order)

$SO(3) \cong S^3/Z_2$, locally like $S^3 (n > 2)$

Ferromagnetic condensate disordered at finite T in 2D

$\frac{U(1)\times S^2}{Z_2}$ locally like $U(1) \times S^2$

Polar state: $U(1)$ critical, S^2 disordered at finite T in 2D

$$E = \frac{\Gamma}{2}(\nabla\theta)^2 + \frac{\Gamma}{2}(\nabla\hat{\mathbf{n}})^2$$

→ Flows to zero (to lowest order)

Higher order coupling?

Polar condensate

Fundamental vortex has 1/2 flux quantum

KT transition mediated by half vortices

Stiffness jump 4 times regular KT jump

Algebraic order in 2θ instead of θ (2D nematic)

Order parameter: $\psi_0^2 - 2\psi_{+1}\psi_{-1}$

Low T phase: paired singlets of bosons

Paired state formed by thermal fluctuations

Check with Monte-Carlo simulations

Free energy functional

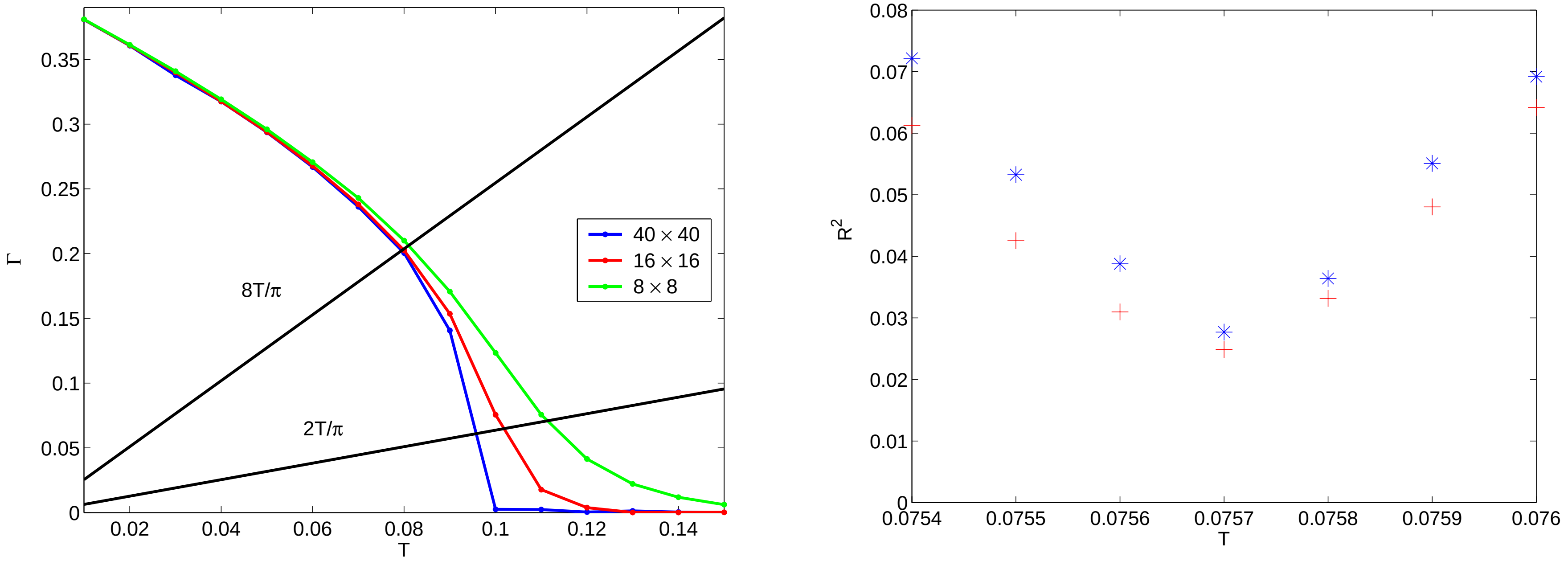
$$F = \int d\mathbf{r}(\frac{\hbar^2}{2M}\nabla\psi_a^*\cdot\nabla\psi_a + a(T)\psi_a^*\psi_a + b_0\psi_a^*\psi_b^*\psi_b\psi_a + b_2\psi_a^*\psi_{a'}^*\vec{S}_{ab}\cdot\vec{S}_{a'b'}\psi_{b'}\psi_b)$$

Phase stiffness

$$\Gamma = \left(\left\langle\frac{\partial^2 F}{\partial\phi^2}\right\rangle - \frac{1}{k_B T}\left\langle\left(\frac{\partial F}{\partial\phi}\right)^2\right\rangle\right)\Bigg|_{\phi=0}$$

ϕ is a twist in θ

2D finite temperature



$$\Gamma(N, T_c) = \Gamma_\infty \left[1 + \frac{1}{2}\frac{1}{\ln N + C}\right]$$

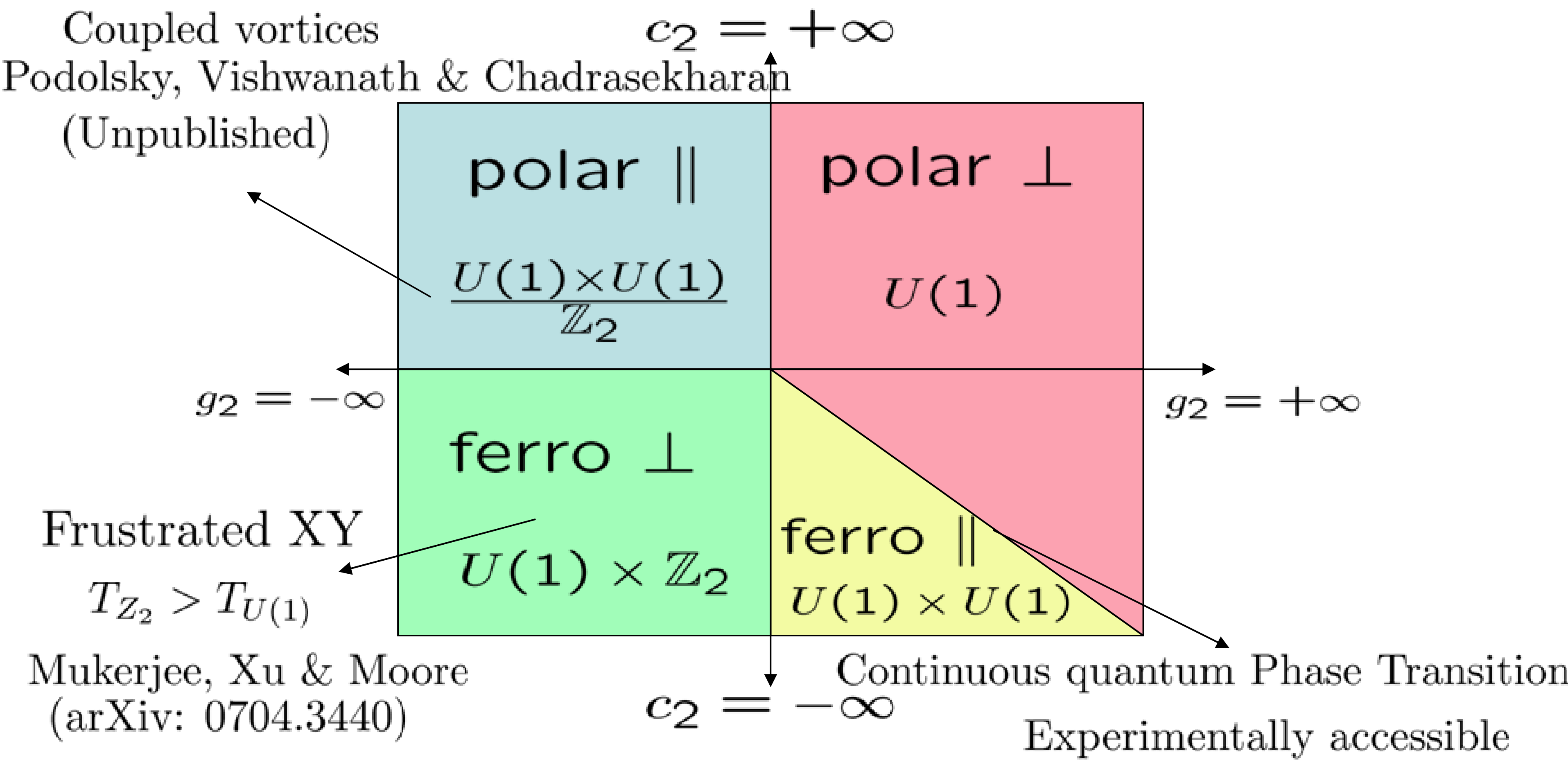
$$\Gamma_\infty = 8T_c/\pi$$

Also in two-color lattice QCD in 2+1 dimensions

Chandrasekharan, PRL 97, 182001 (2006)

Quadratic Zeeman field

$$E = n^2(c_0 + c_2\langle\vec{S}\rangle^2 + g_2\langle S_z^2\rangle)$$



Conclusions and summary

- S=1 condensates have interesting topological defects
- Ferromagnetic condensate remains disordered in 2D at finite temperature
- Mean field polar condensate is unstable to forming a paired superfluid at finite temperature, with a KT transition mediated by half vortices

Phase-ordering kinetics of spinor condensates

- Experiments allow direct observation of magnetic domain formation after initial quench.
- What is the correct dynamical universality class? Are quantum effects important at experimental time scales?
- Previous models: deterministic dynamics of either isolated condensate (Gross-Pitaevskii (GP)) or with some quantum fluctuations (quasiparticles).
- What happens because of normal cloud interactions on long time scales? What is the crossover time scale?
- What effect do the various conservation laws have on the dynamics?

Pu et al., PRA 60, 1463 (1999), Saito and Ueda, PRA 72, 023610 (2005)

Lamacraft, PRL 98, 160404 (2007)

Finite temperature dynamical models

Hohenberg and Halperin, RMP 49, 435 (1977)

Model A: No conservation laws, as is. Condensate in contact with heat bath. Does not know about the “normal cloud”.

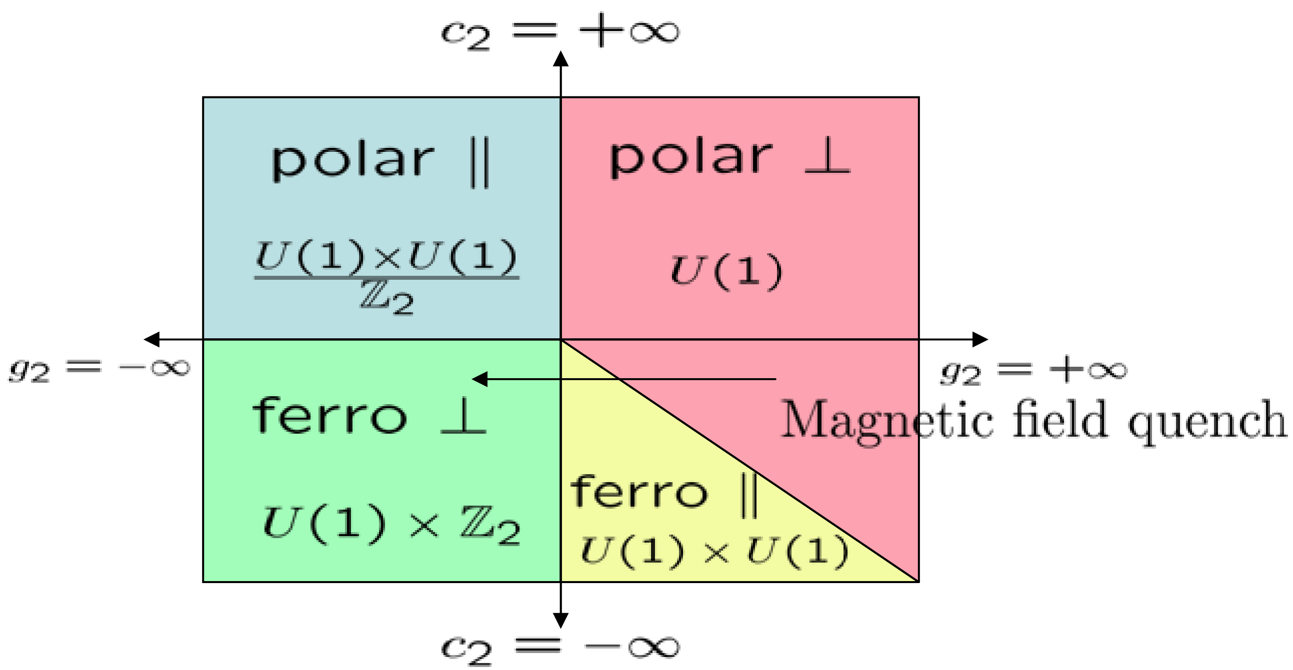
Model F: Condensate coupled to the “normal cloud”. Conserved “mass-energy” second sound mode that couples to condensate fields. Magnetization of the condensate not conserved, as is.

Magnetization conservation can be externally imposed in both models.

Different dynamic universality classes

Compare to zero temperature GP which conserves everything

Dynamic modes with model F parameters contain those of GP



Investigate Z_2 domain formation in 2D.

Domain size $L(t) \propto t^{1/z}$ for large t

Numerically simulate the different models

Compare z for different models

Results

$z = 2$ for models A and F without magnetization conservation

$z = 3$ for both with conserved magnetization $(M = 0)$

$z = 3$ for GP $(M = 0)$

Magnetization conservation important to dynamics of magnetic domain formation

Ising model, high T quench Huse, PRB 34, 7845 (1986)

Second sound mode makes no difference to magnetic ordering

But it does affect superfluid ordering

Spinless bosons $(T < T_c$ quench).
Model F = GP $(z = 1)$
Model A $(z = 2)$

Damle, Majumdar and Sachdev
PRA **54**, 5037 (1996)

Questions to be addressed

- What are the relevant time scales for the different conservation laws?
- What happens in the other parts of the phase diagram?
- What is the role of topological defects?
- Other kinds of order (pairing)?
- Contact between quantum and classical dynamics.
- Higher spin systems

THANKS FOR YOUR ATTENTION