KITP Santa Barbara, 24 May 2007

Exploring New Phases in Polarised Fermi Gases

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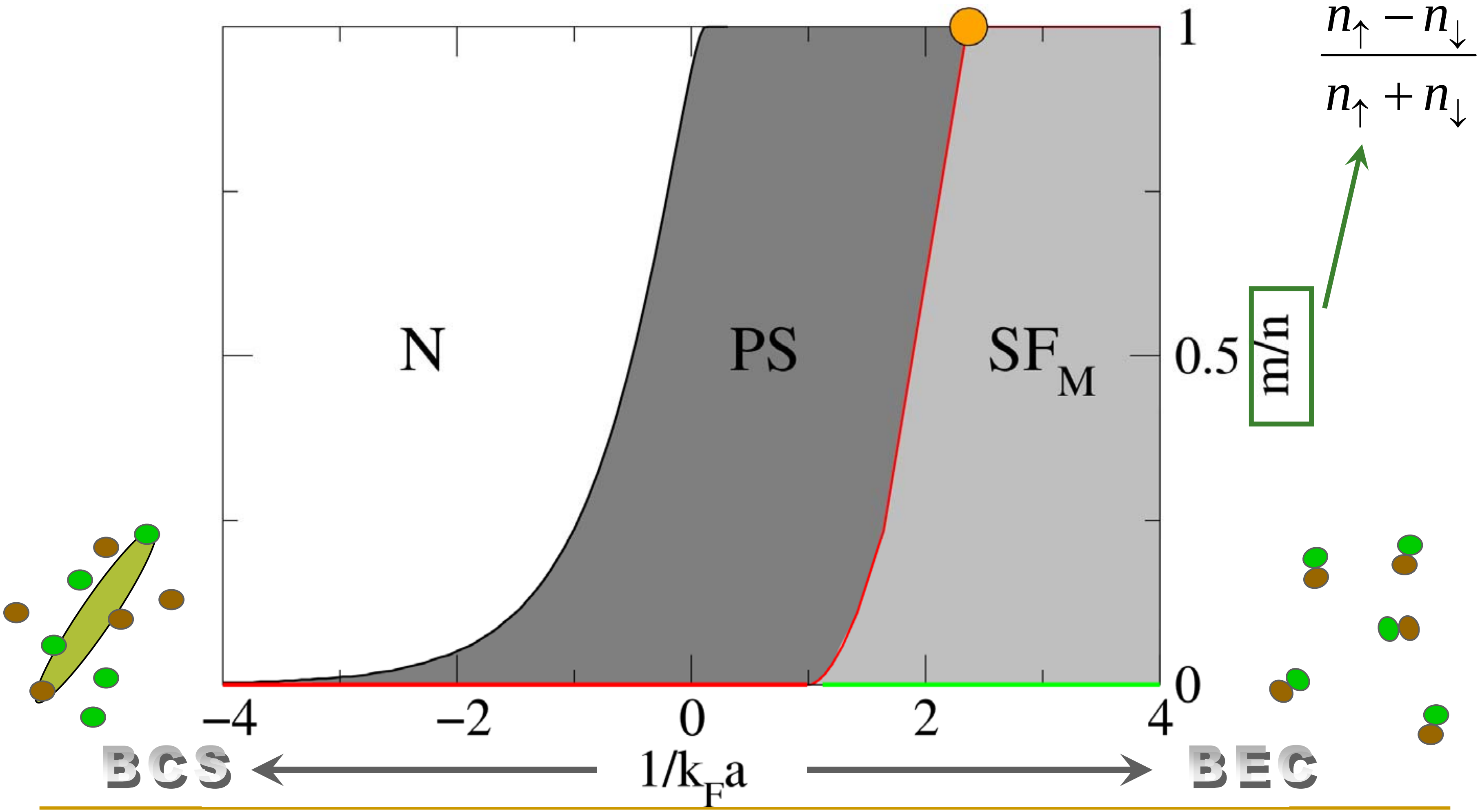
Spin-imbalanced Fermi gases

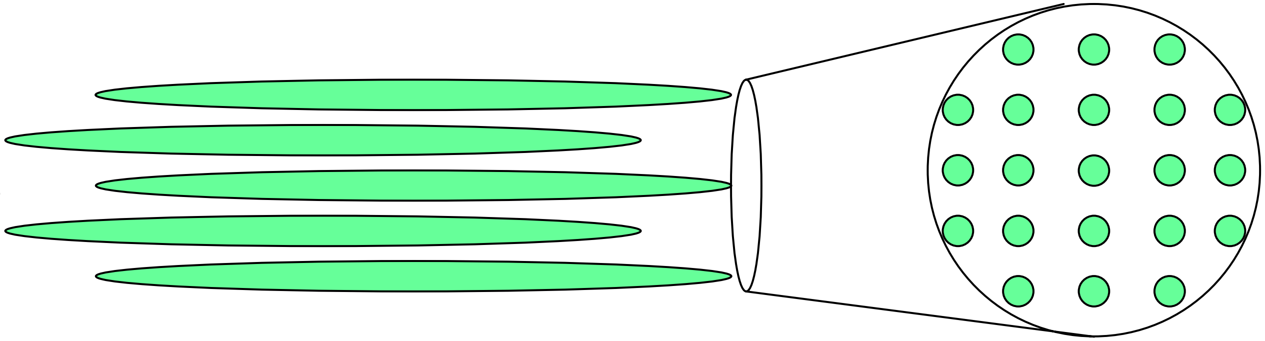
Recall: equal masses, zero T



See F. M. Marchetti's talk

Outline

- Fermi gases with unequal masses
  - BCS-BEC crossover
  - Evolution of tricritical point
  - Existence of different ‘breached pair’ states?
  - Trapped gases at finite temperature
- Quasi-1D
  - FFLO states



- Conclusion

Model for unequal masses

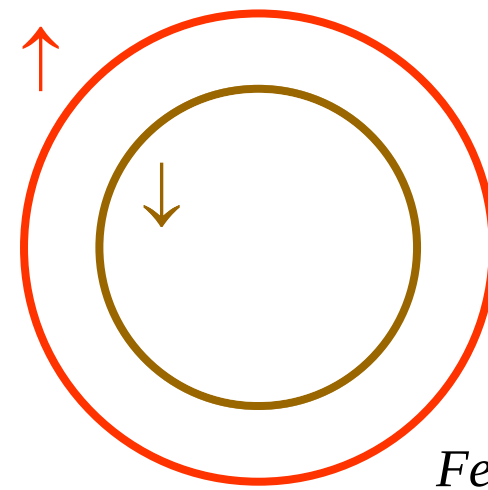
$$\hat{H} - \mu_\uparrow N_\uparrow - \mu_\downarrow N_\downarrow = \sum_{\mathbf{k}\sigma} \left(\frac{\mathrm{k}^2}{2m_\sigma} - \mu_\sigma \right) a^+_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma}$$

$$+ U \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} a^+_{\mathbf{k}\uparrow} a^+_{\mathbf{k}'\downarrow} a_{\mathbf{k}'-\mathbf{q}\downarrow} a_{\mathbf{k}+\mathbf{q}\uparrow}$$

$$\mu_\uparrow \equiv \mu + h$$
$$\mu_\downarrow \equiv \mu - h$$

- Define mass ratio $r = \dfrac{m_\downarrow}{m_\uparrow}$
- Assume $k_{F\uparrow} \geq k_{F\downarrow}$

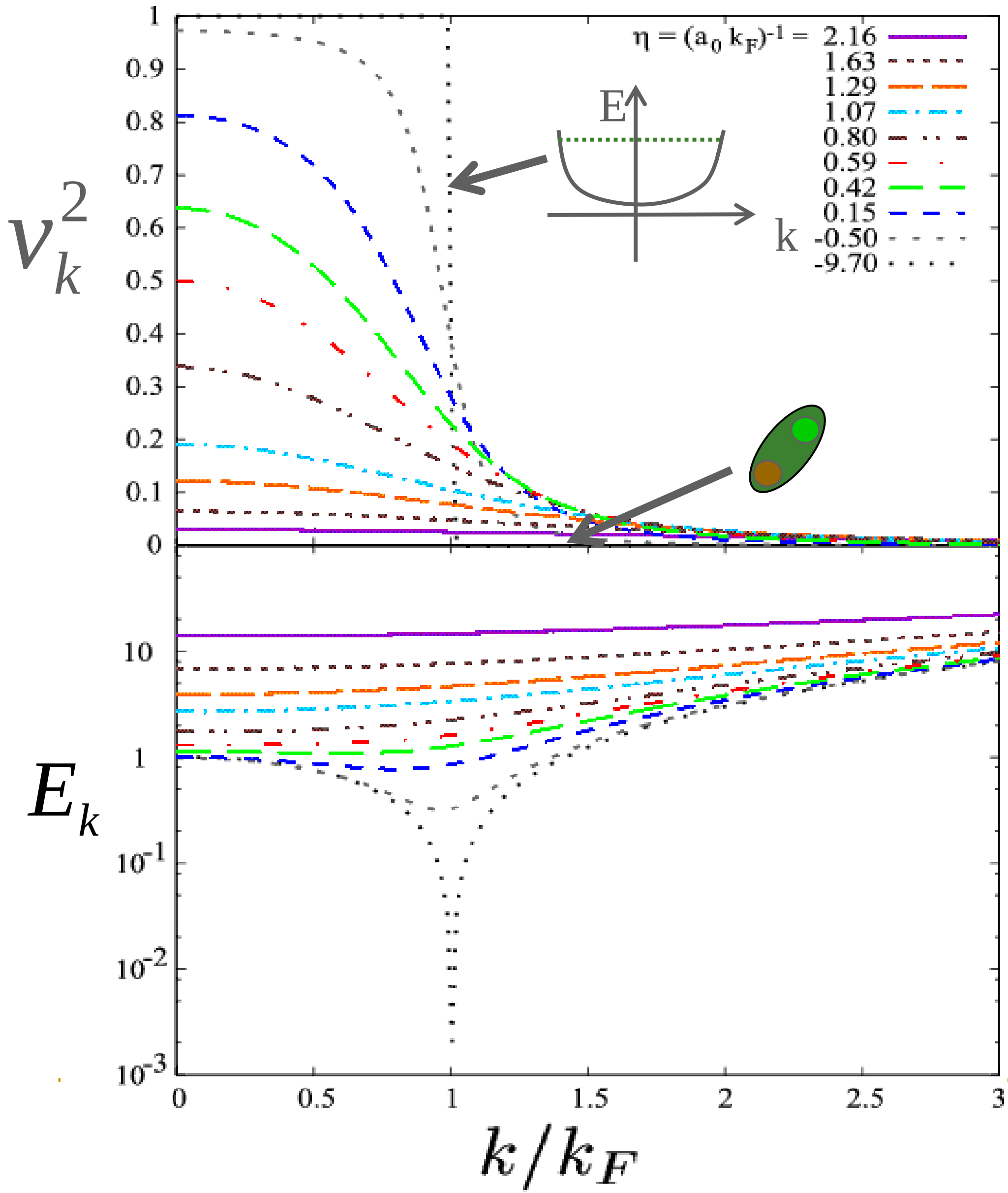
  → r > 1 corresponds to the majority species being heavier
- Minimise mean-field free energy



Fermi surfaces

Related work: Liu and Wilczek, PRL 2003; Iskin et al., PRL 2006; Wu et al., PRB 2006; Lin et al, PRA 2006.

Mean-field BCS-BEC crossover (r = 1)



Wave function

$$|\Phi\rangle = \prod_k \left(u_k + v_k c^+_{k\uparrow} c^+_{-k\downarrow}\right)|0\rangle$$

Same for unequal masses

Quasiparticle excitations

$$E_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta^2}$$

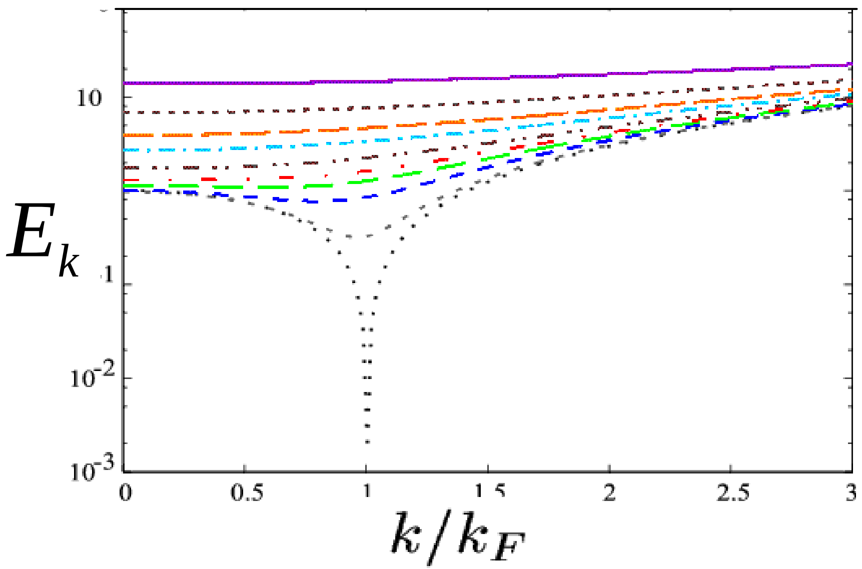
Parish et al. PRB 71, 064513 (2005)

Quasiparticle spectrum

- Equal masses

$$E_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta^2}$$

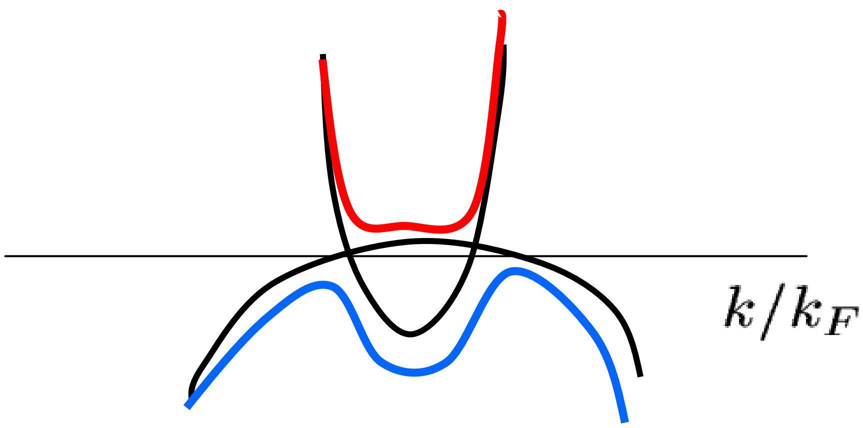
- Minimum at nonzero *k* for $\mu > 0$
- Minimum at k = 0 for $\mu < 0$



- Unequal masses

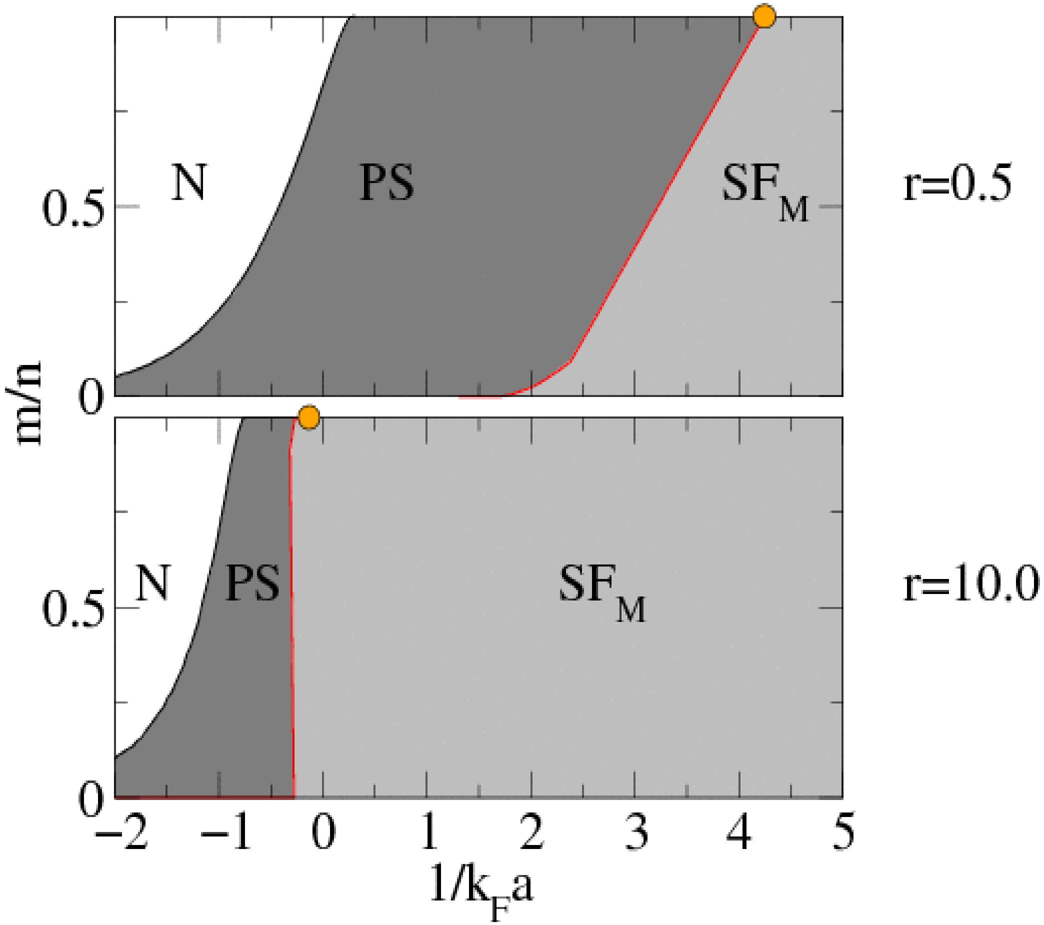
- Minimum at nonzero k for *one* branch when:

$$\left|\frac{\mu}{\Delta}\right| < \frac{|r-1|}{2\sqrt{r}}$$

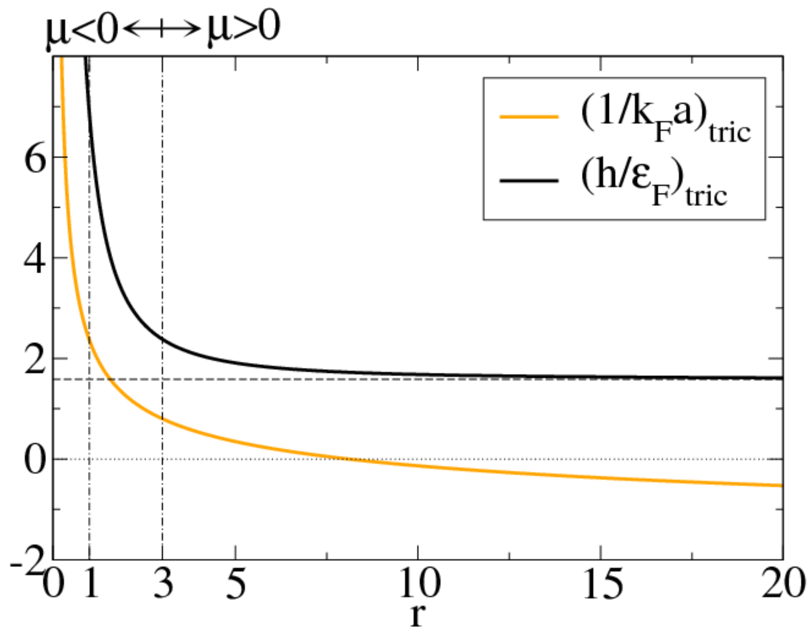


$r = m_\downarrow / m_\uparrow$

Zero temperature phase diagram



- Evolution of tricritical point



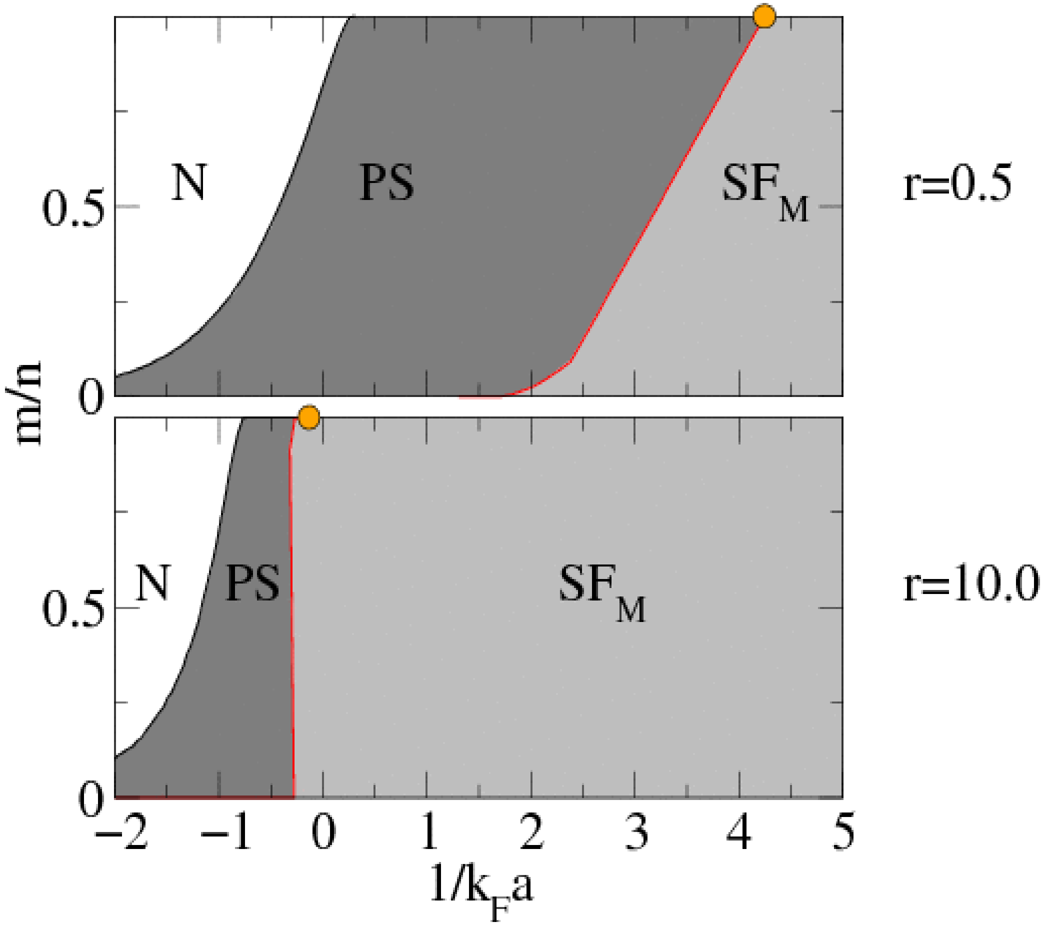
As $r \to 0$

$$(1/k_F a)_{tric} \to \infty$$

Region of PS expands

Parish et al., PRL 98, 160402 (2007)

Zero temperature phase diagram



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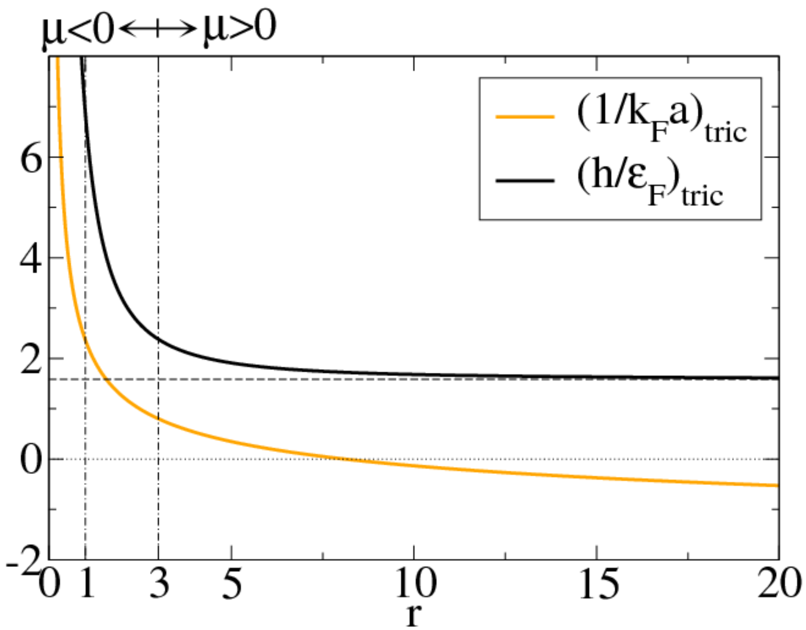


As $r \to \infty$

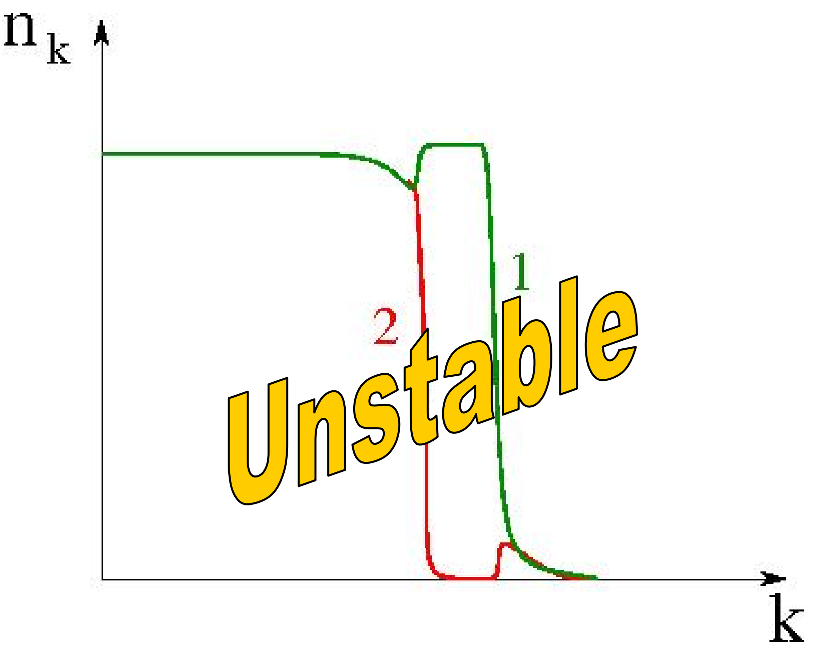
$$(1/k_F a)_{tric} \to -\infty$$

Region of SF_M expands

Parish et al., PRL 98, 160402 (2007)

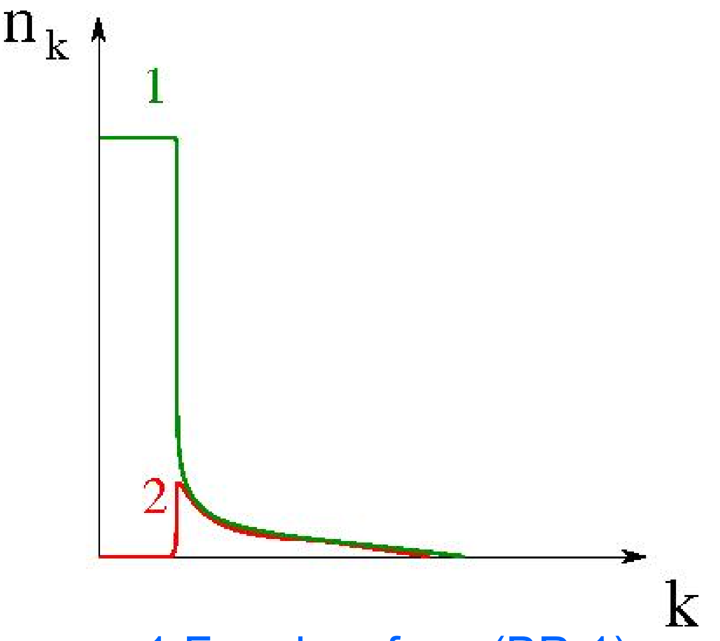
Breached pair states

- Magnetised superfluids that are *homogeneous* in real space
- Two types:



2 Fermi surfaces (BP-2)

Liu and Wilczek, PRL 2003



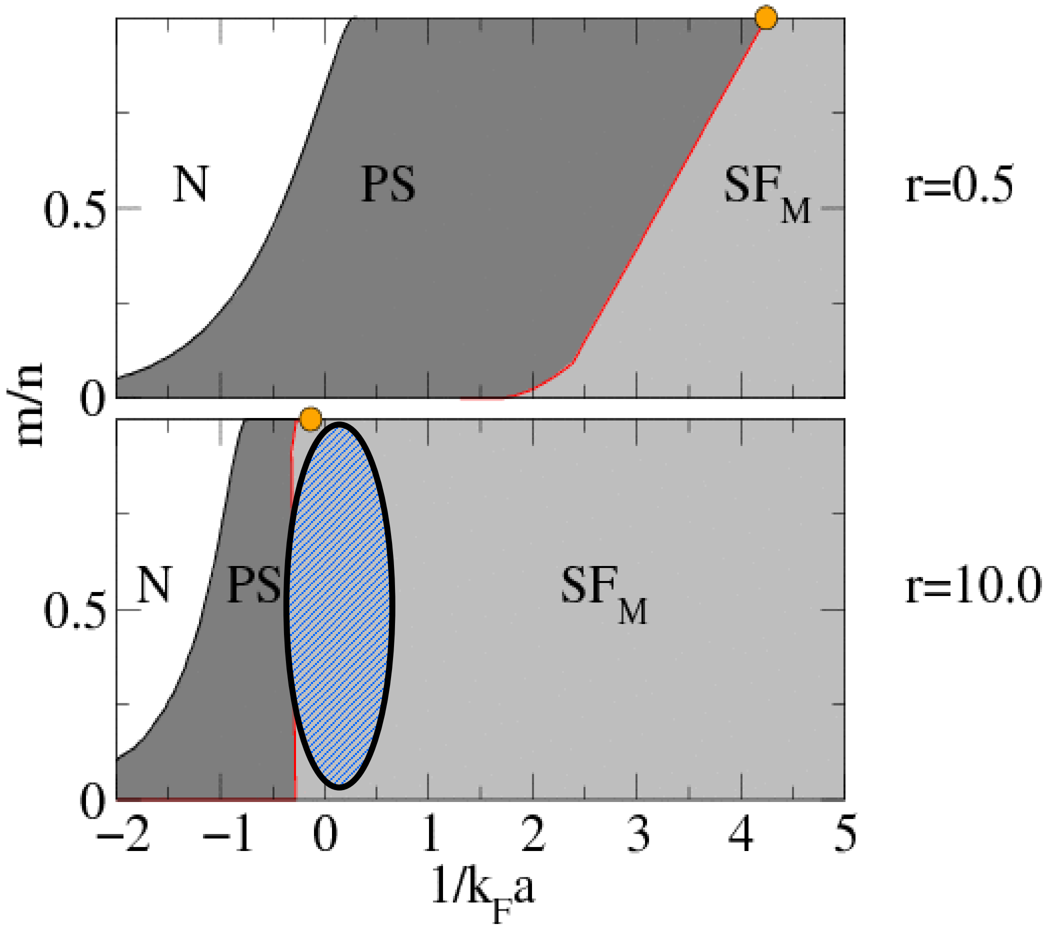
1 Fermi surface (BP-1)

Exists in BEC limit of the equal mass phase diagram

Is BP-2 ever stable?

Breached pair states

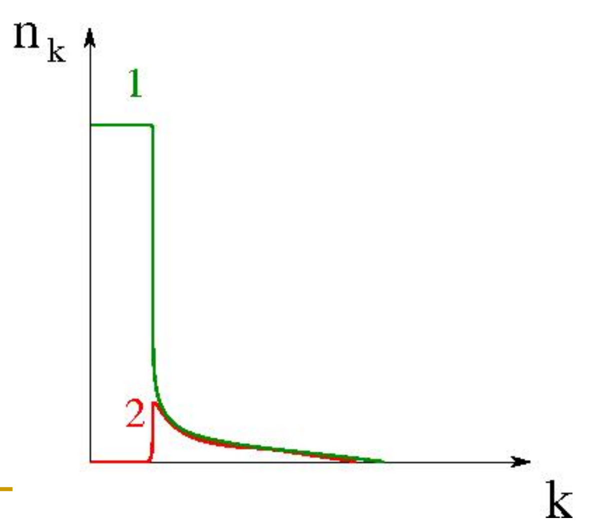
$r = m_{\downarrow} / m_{\uparrow}$



- BP-2 requires $\mu > 0$

Consider r >> 1

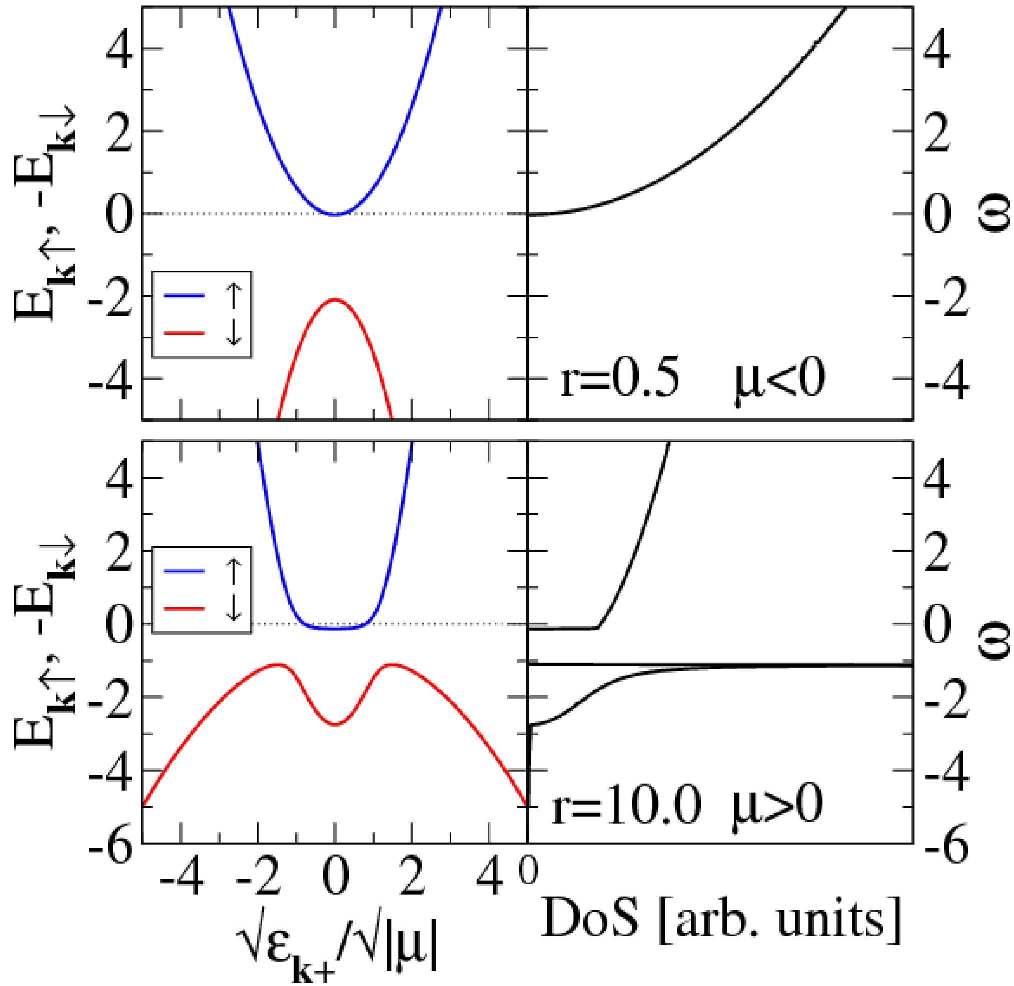
Here, we only find BP-1



Is BP-2 ever stable? NO, but…

Density of states

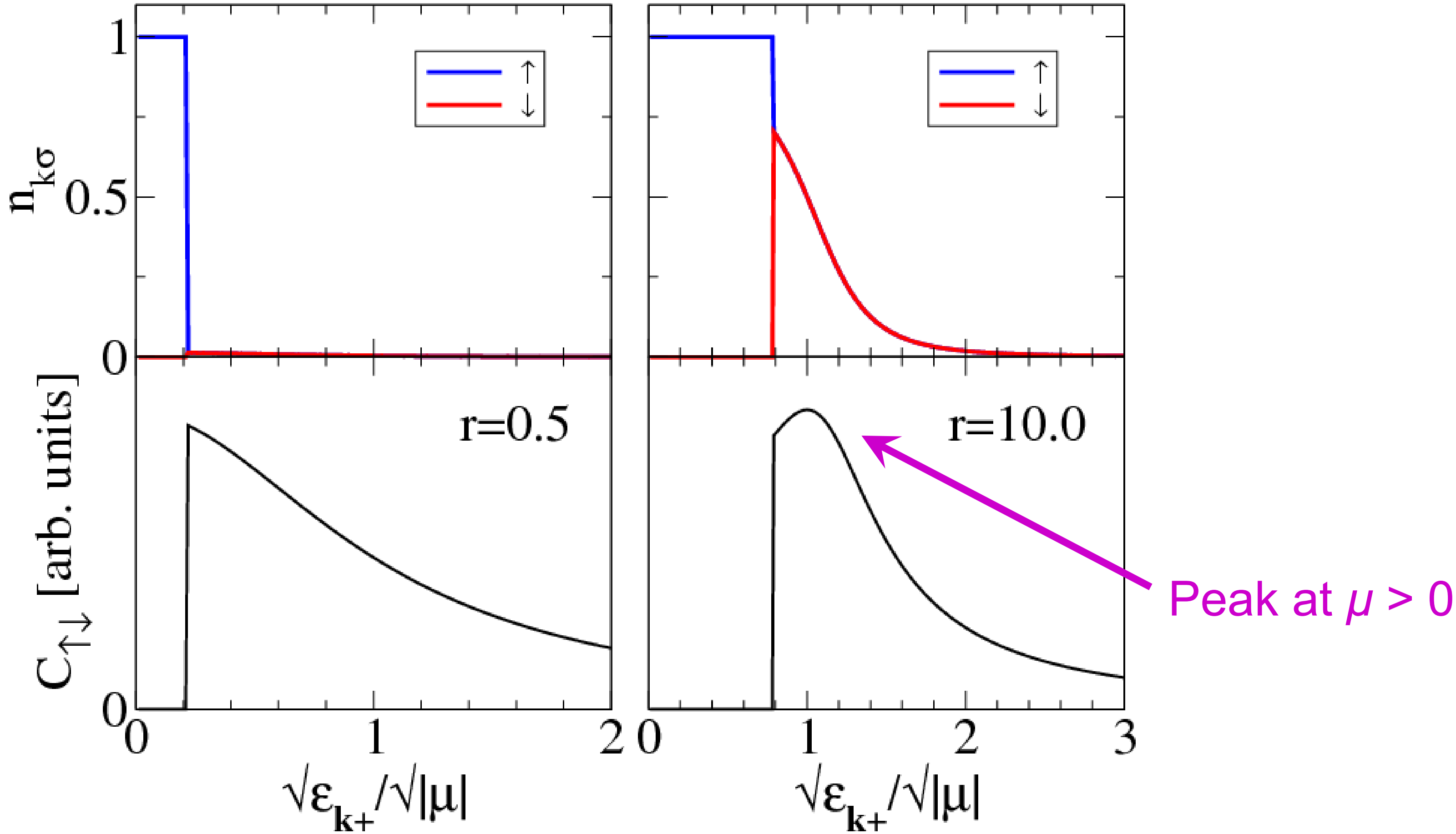
- BP-1 can have two different types of quasi-particle spectra and DoS
- Singularity in DoS only occurs for r > 1



Parish et al., PRL 98, 160402 (2007)

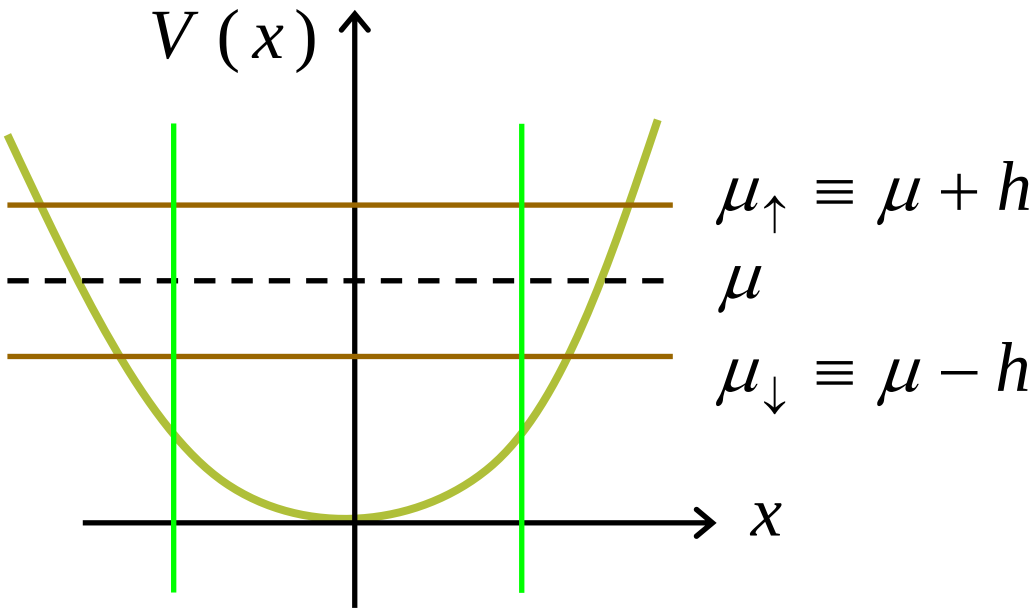
Pair correlations

$$C_{\uparrow\downarrow}(k_1, k_2) = \langle n_{k_1\uparrow} n_{k_2\downarrow} \rangle - \langle n_{k_1\uparrow} \rangle \langle n_{k_2\downarrow} \rangle$$



Parish et al., PRL 98, 160402 (2007)

Trapped Fermi gases



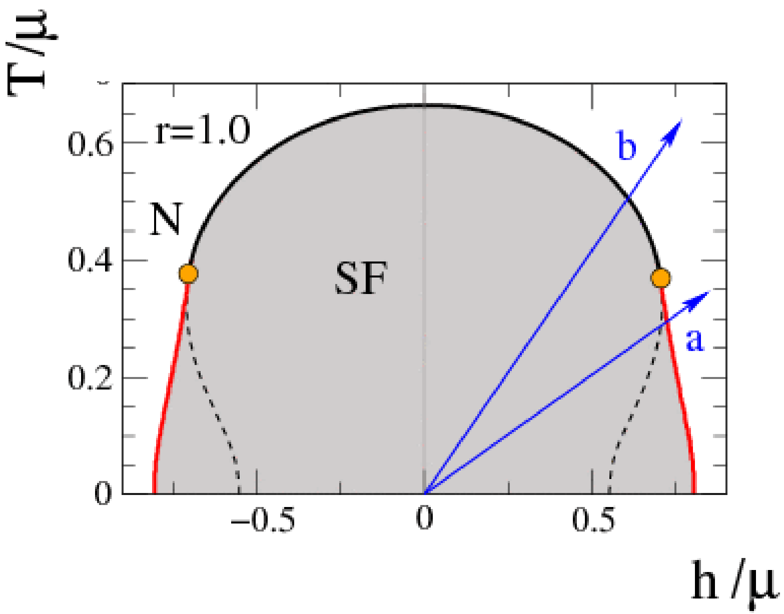
Trap cuts across uniform phase diagram

- Local density approximation $\mu_{eff} = \mu - V(x)$

  $\rightarrow$ valid when trap slowly varying with respect to all other length scales
- *h* is constant across trap, but h/μ_{eff} varies

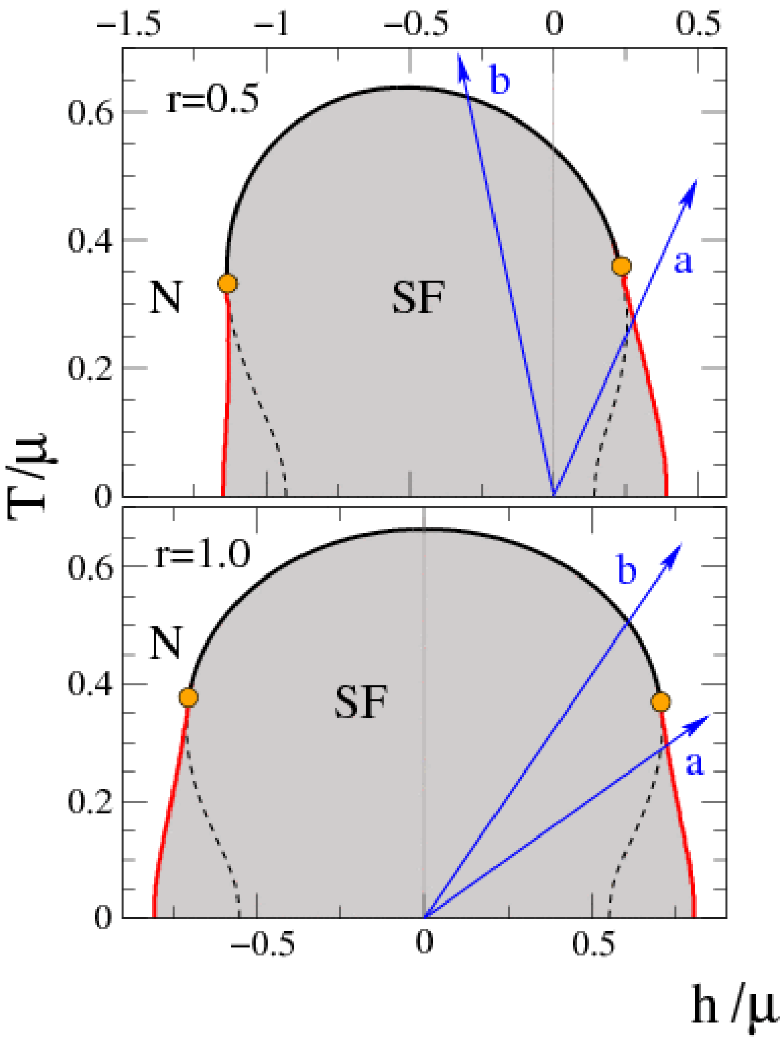
Trapped gases at zero T: Lin et al, PRA 2006.

Trapped gases at finite temperature



Unitarity limit $r = m_\downarrow / m_\uparrow$ *Parish et al., PRL 98, 160402 (2007)*

Trapped gases at finite temperature



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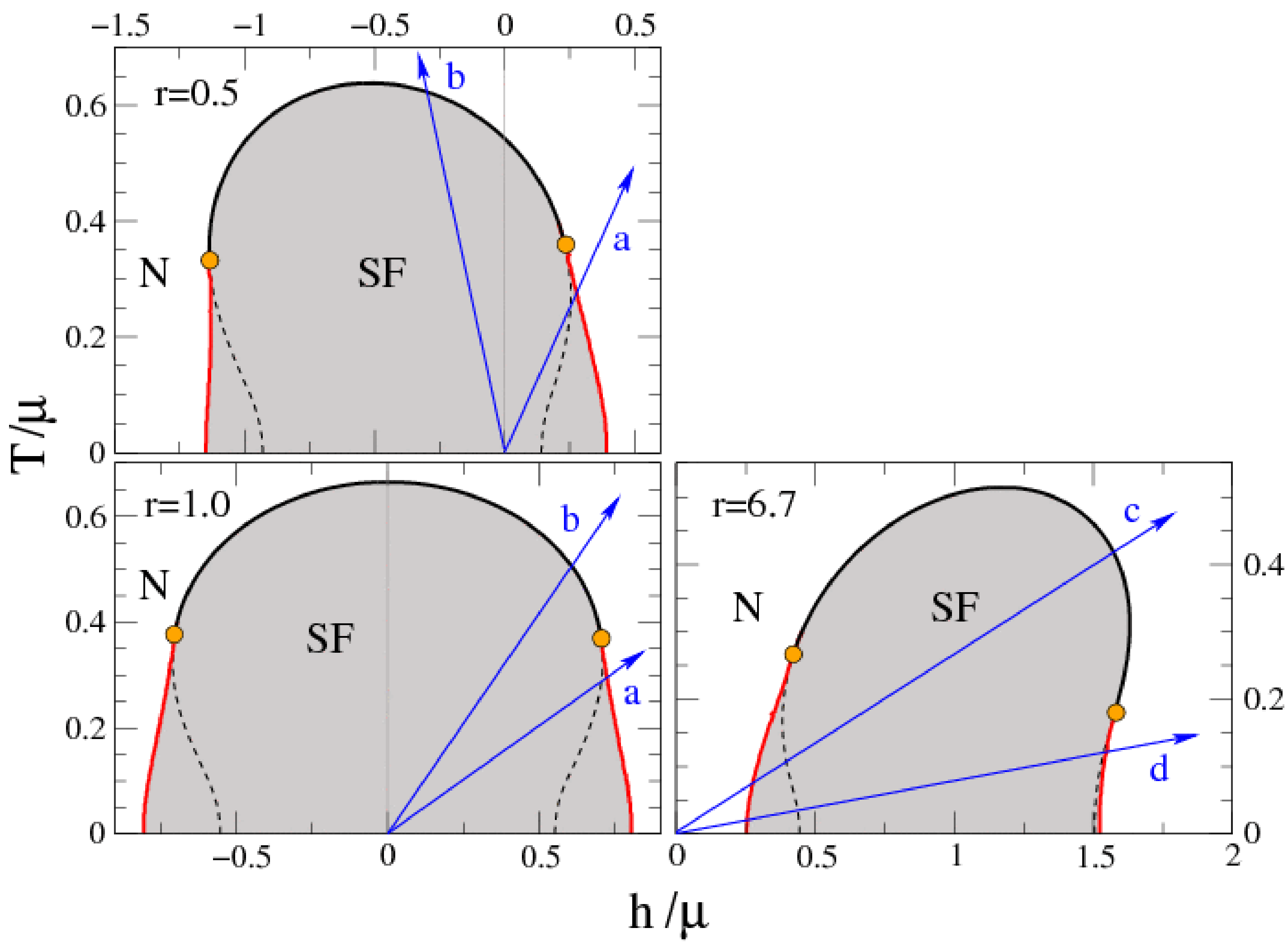
Trapped gases at finite temperature



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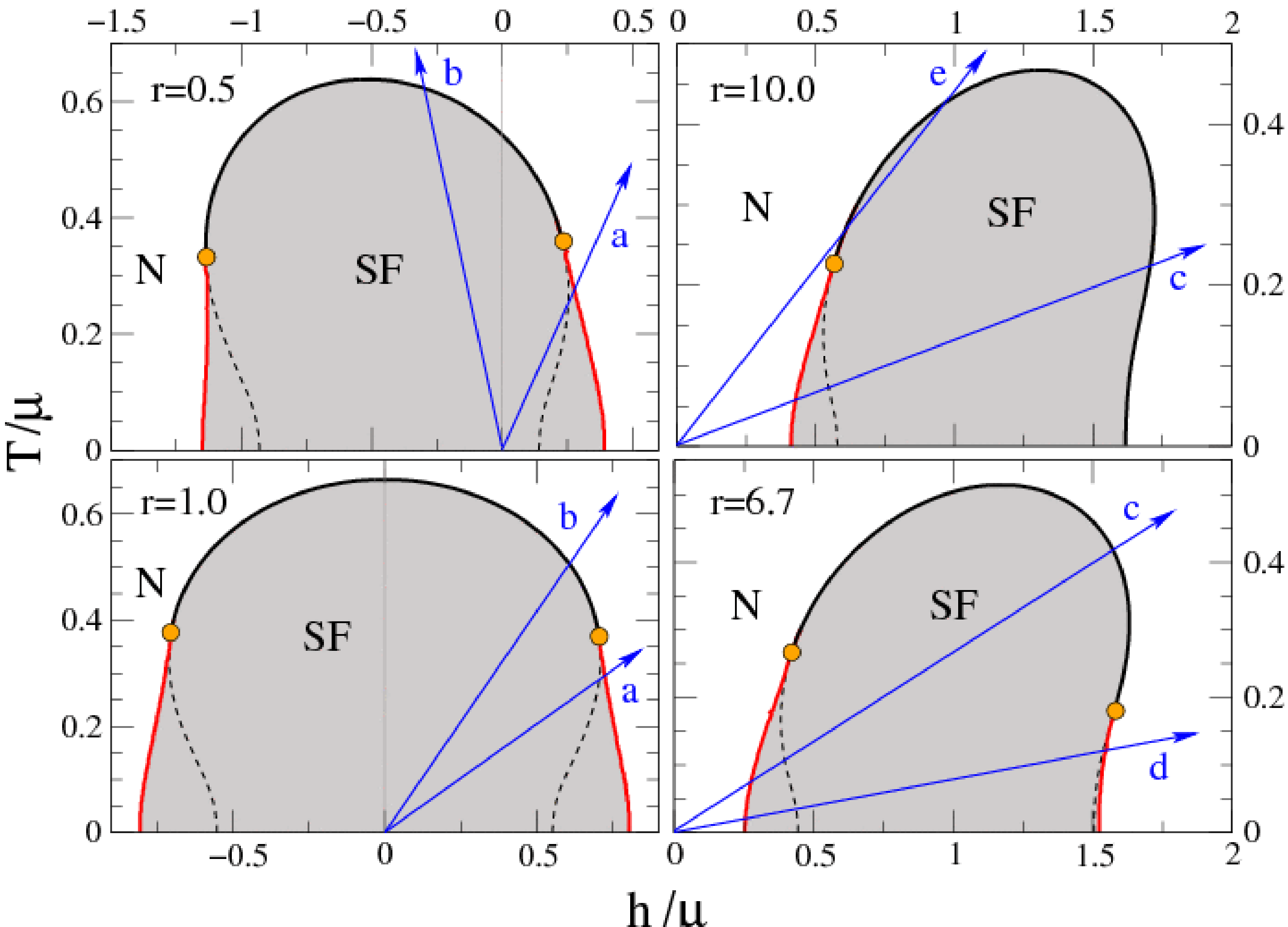

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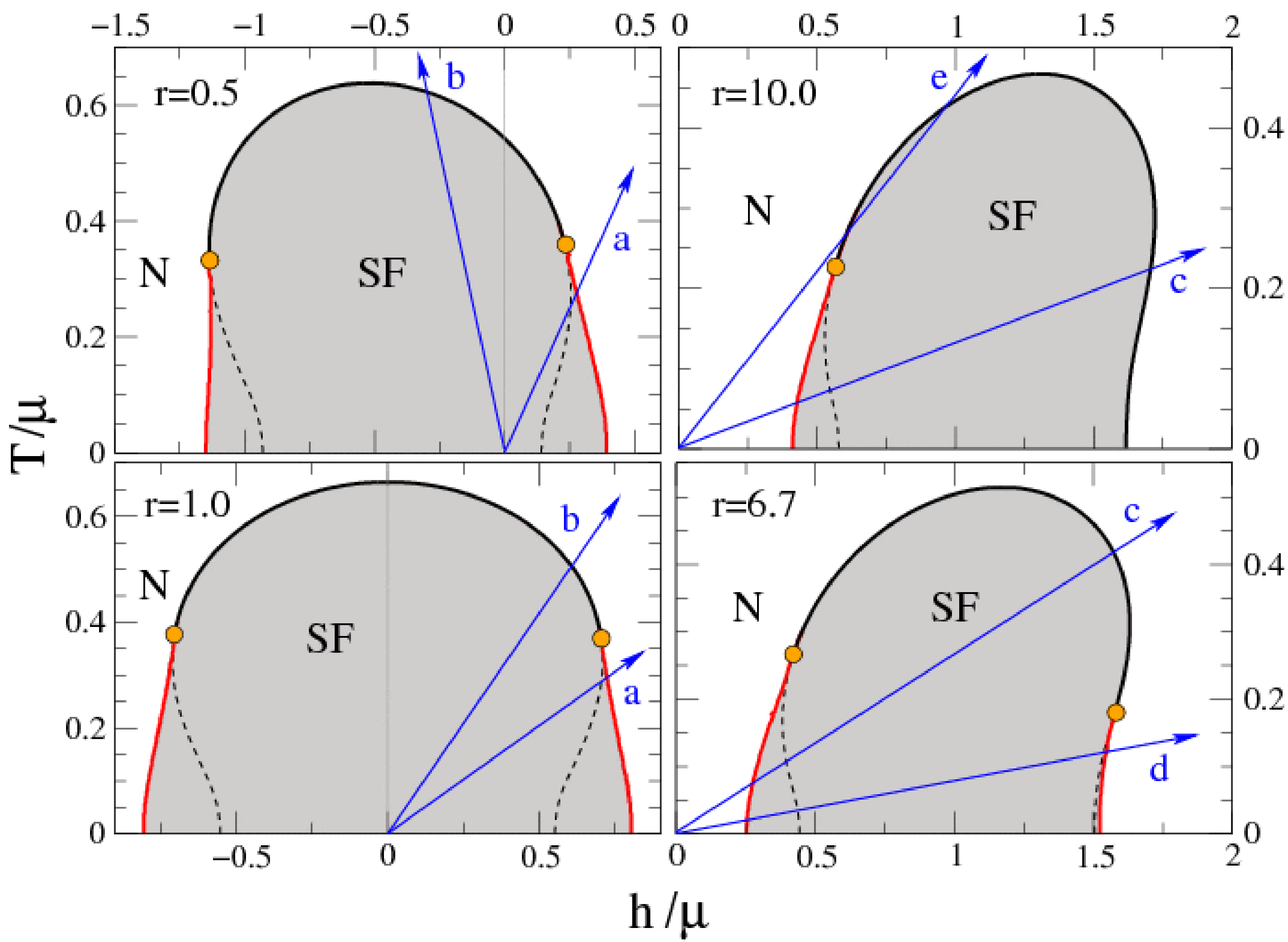
Trapped gases at finite temperature



"Superfluid shells" for r > 3.95

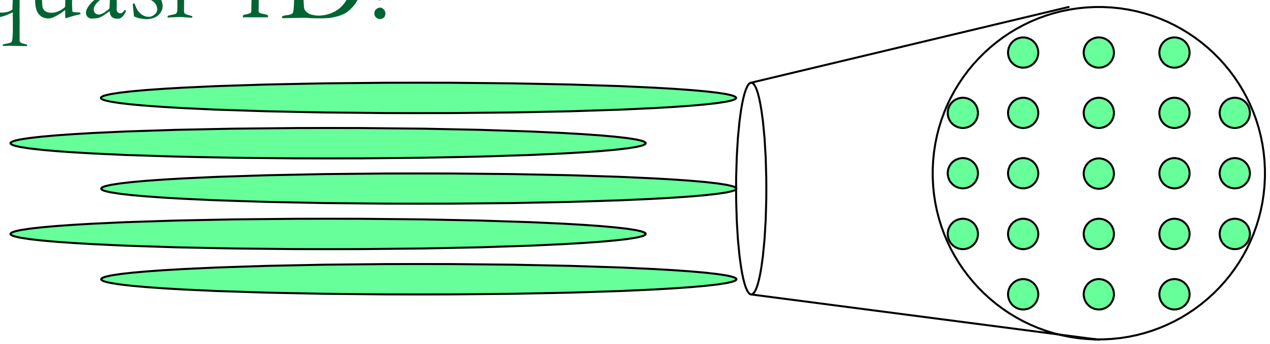
Parish et al., PRL 98, 160402 (2007)

Conclusion

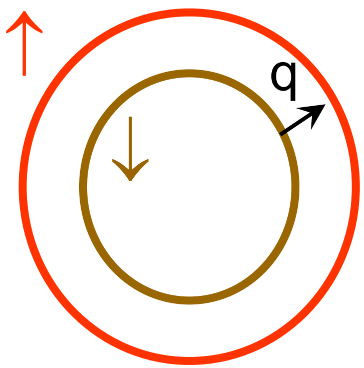
- The zero temperature tricritical point smoothly evolves from the BEC to BCS limits with increasing r
- The interior gap state or BP-2 state is never stable for s-wave interactions
- However, differences in the breached pair states show up in the DoS and pair correlations
- Trapped gases at finite temperature exhibit superfluid shells for $r > 3.95$ at unitarity

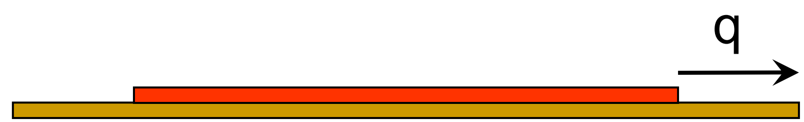
NEXT: Quasi-1D polarised Fermi gases

Why quasi-1D?



- Experimentally realisable
- It expands the region of FFLO in the phase diagram





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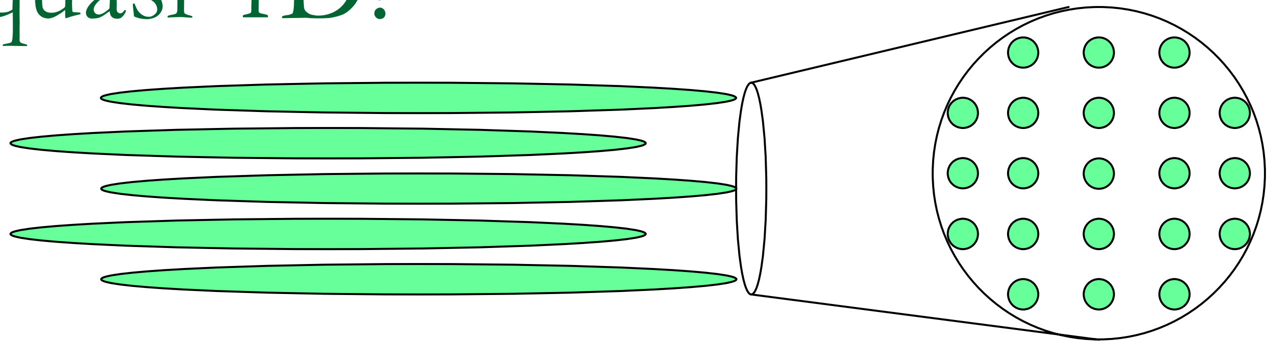

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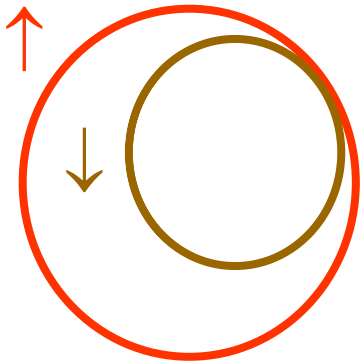
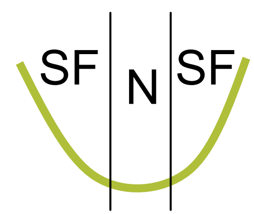
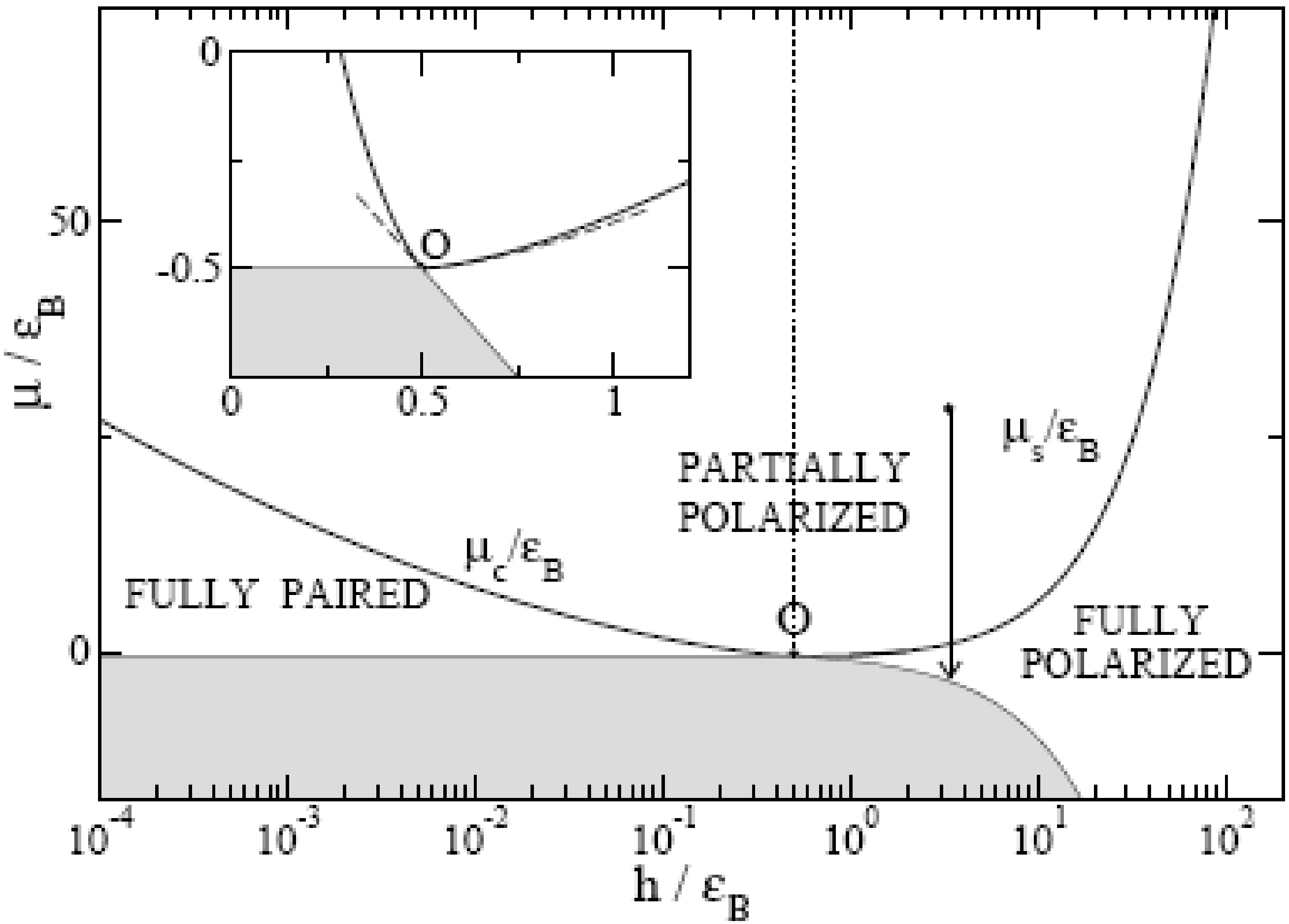




- Possibility of phase inversion in a trap
  - E.g. BCS limit: $\Delta \sim \varepsilon_F \exp(-1/UN(\varepsilon_F))$, $\quad N(\varepsilon_F) \propto 1/\sqrt{\varepsilon_F}$

Δ can increase with decreasing density

Spin-imbalanced Fermi gases in 1D



<u>Bethe Ansatz calculation</u>: Orso, PRL 2007; Hu, Liu & Drummond, PRL 2007

Quasi-1D model

$$\mu_{\uparrow} \equiv \mu + h$$
$$\mu_{\downarrow} \equiv \mu - h$$

- Add hopping *t* between 1D tubes

$$\hat{H} = \sum_{\mathbf{k}\sigma} (\varepsilon_{\mathbf{k}} - \mu_{\sigma}) a^{+}_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma} + g_{1D} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} a^{+}_{\mathbf{k}\uparrow} a^{+}_{\mathbf{k}'\downarrow} a_{\mathbf{k}'-\mathbf{q}\downarrow} a_{\mathbf{k}+\mathbf{q}\uparrow}$$

$$\varepsilon_{\mathbf{k}} = \frac{k_z^2}{2m} - t(\cos(k_x) + \cos(k_y) - 2)$$

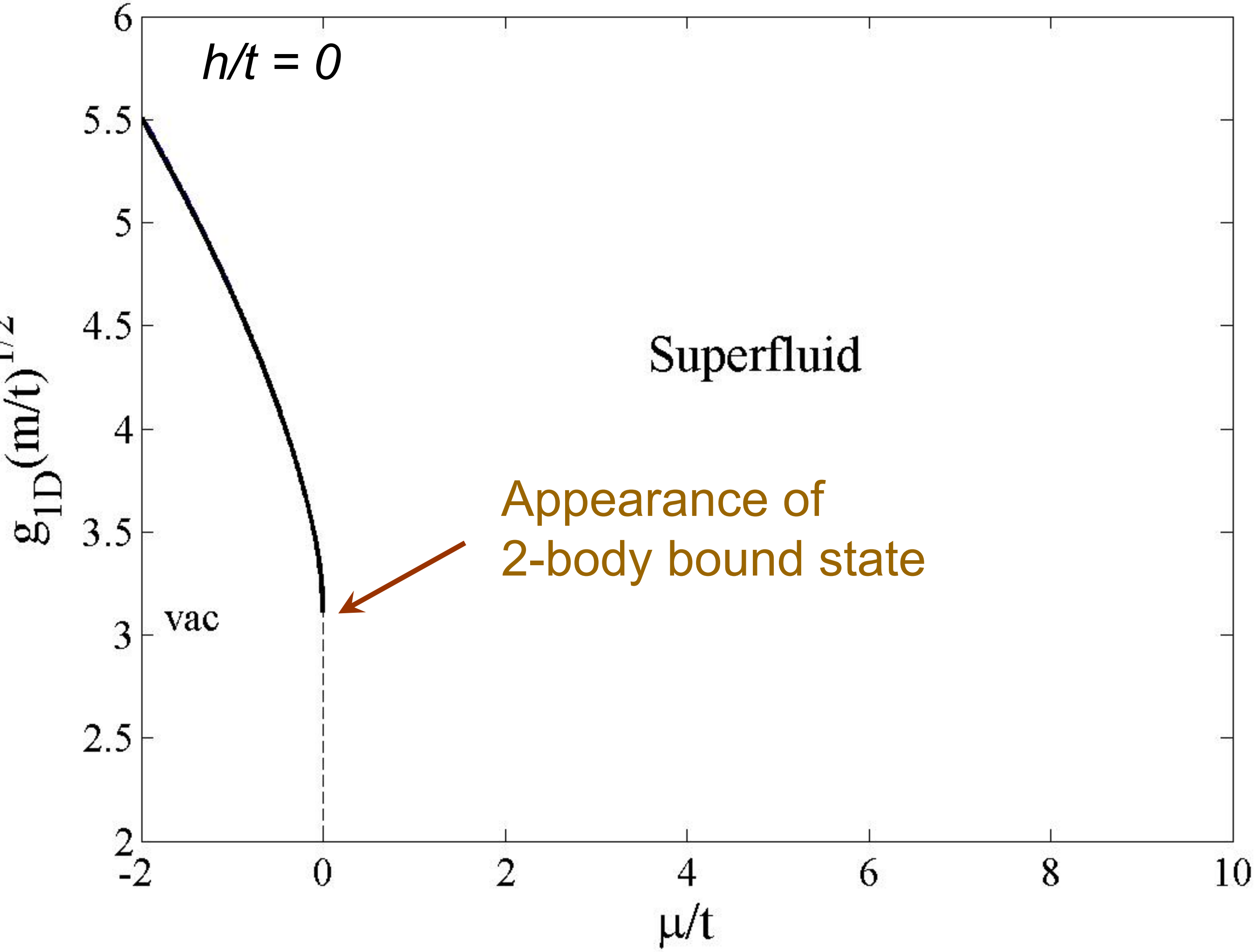
$$g_{1D} = \frac{2a_{3D}}{ma_{\perp}^2} \frac{1}{1 - Aa_{3D}/a_{\perp}}$$

Bergeman et al. PRL 2003

- Density-driven crossover from 3D to 1D
  - 3D limit *h*, $\mu \ll t$
  - 1D limit $\mu \gg t$
- Mean-field approach

3 dimensionless parameters: h/t, μ/t, $g_{1D}(m/t)^{1/2}$

Phase diagram



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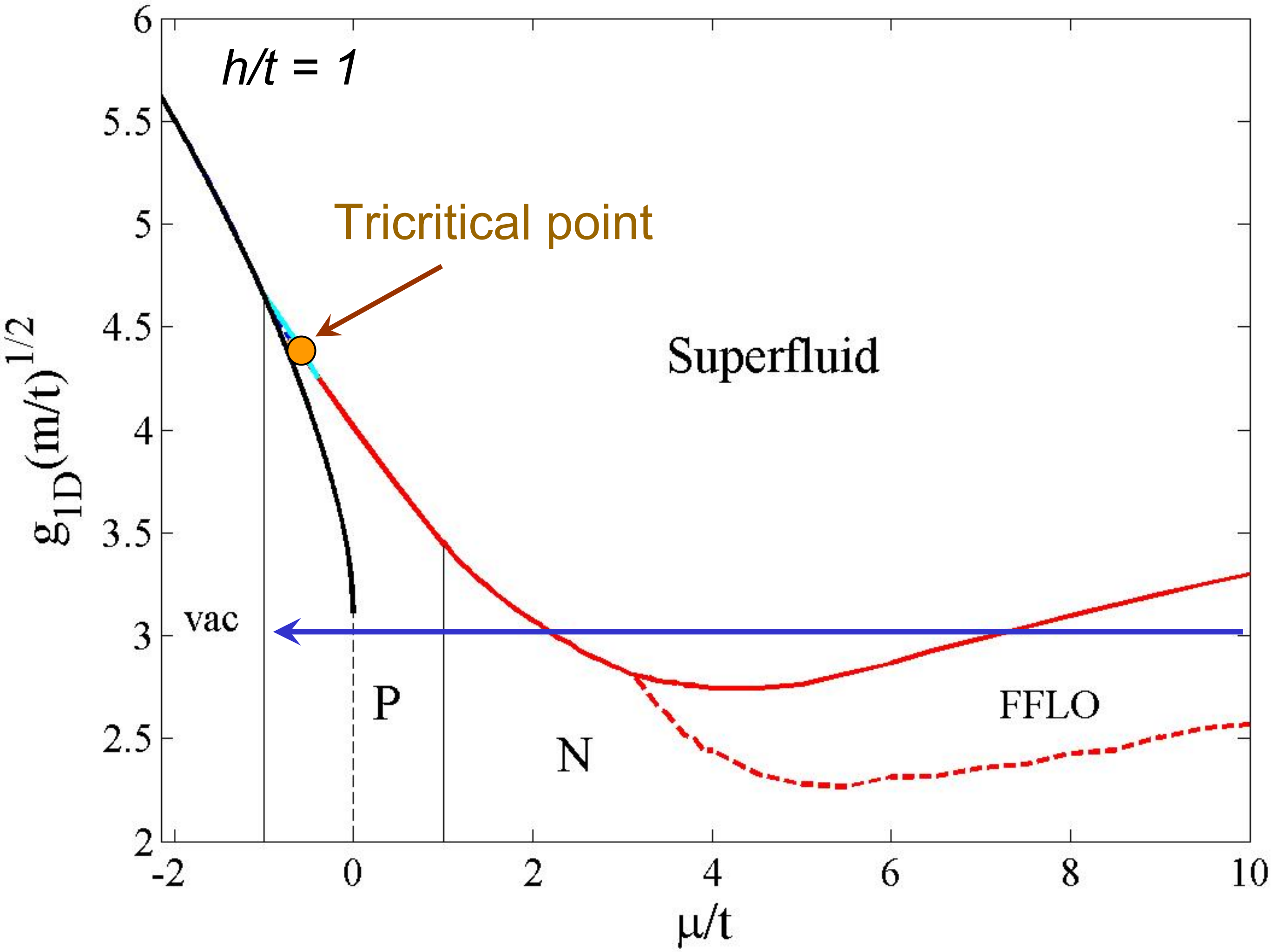

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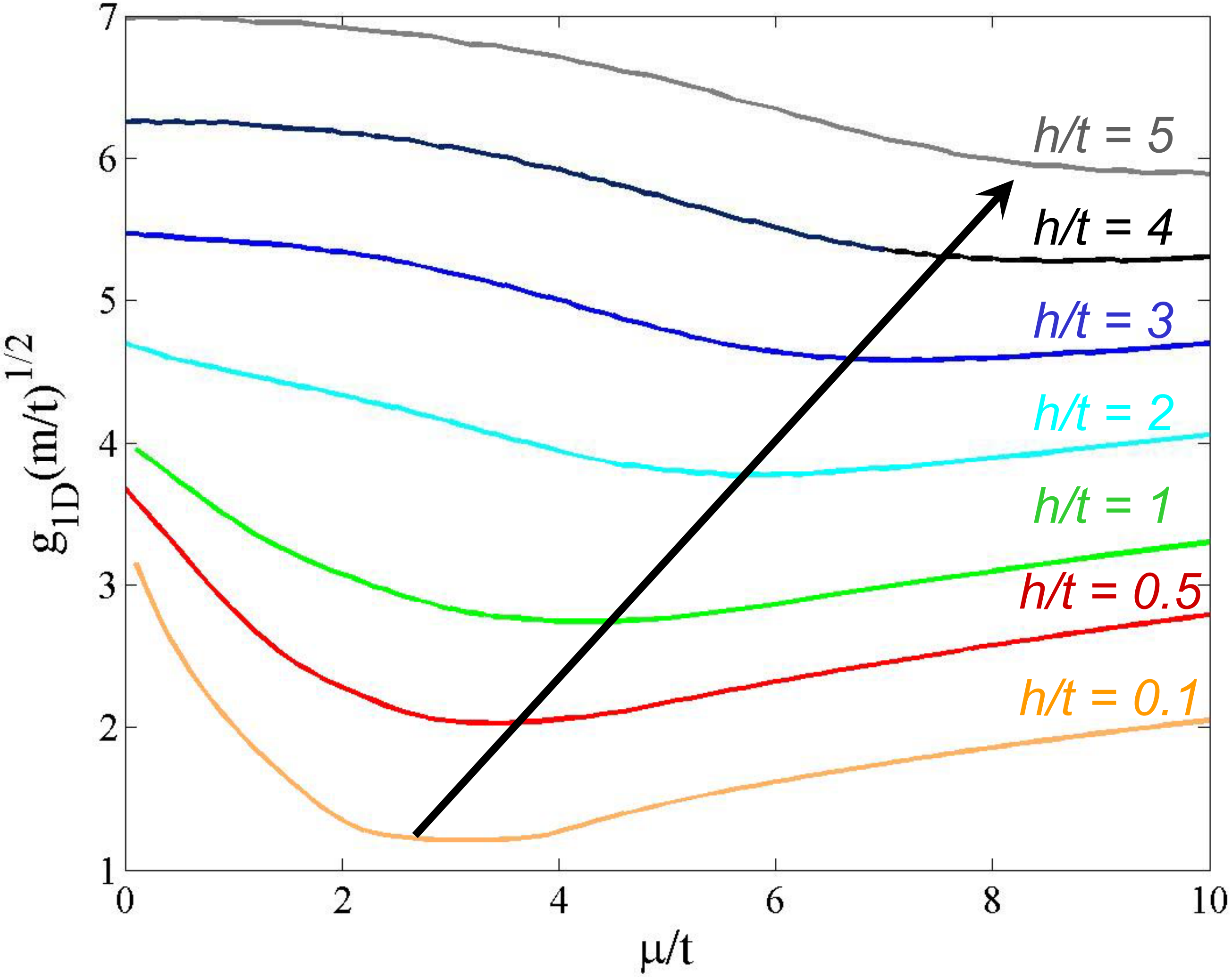
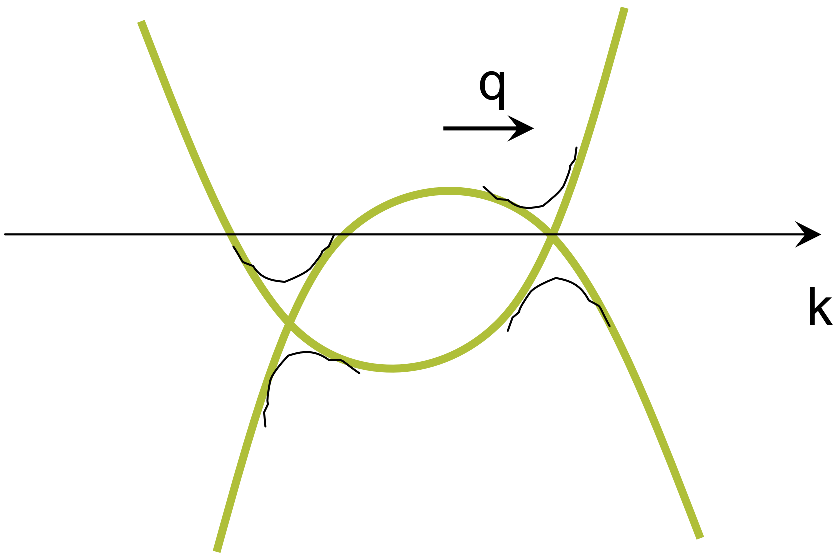
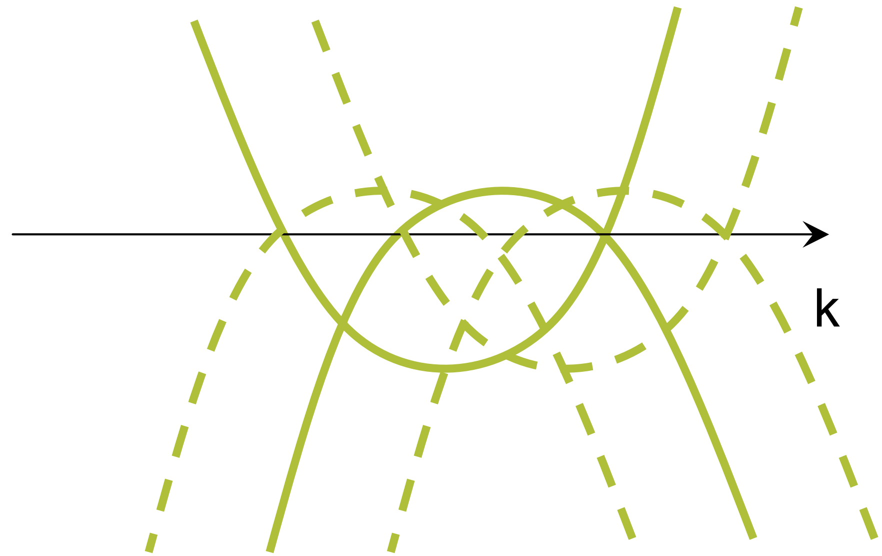


FFLO phases

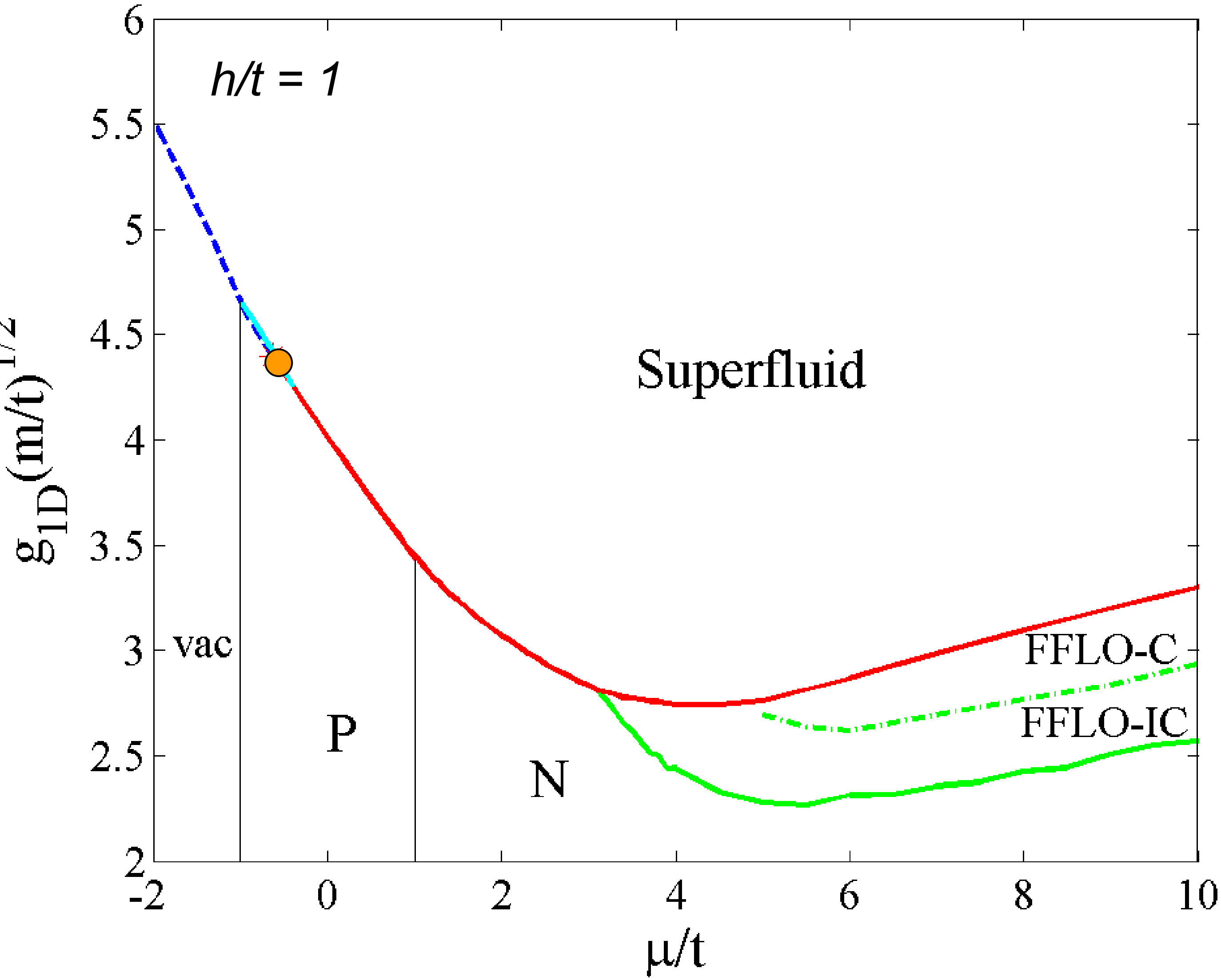
- FF state $\Delta(r) = \Delta e^{iqr}$
- LO state $\Delta(r) = \Delta \cos(qr)$





- Two types of FFLO in quasi-1D
  - Commensurate – gapped
  - Incommensurate - gapless

FFLO phases



h/t = 1
Superfluid
vac
P
N
FFLO-C
FFLO-IC
$g_{1D}(m/t)^{1/2}$
μ/t
6
5.5
5
4.5
4
3.5
3
2.5
2
-2
0
2
4
6
8
10

Conclusion

- Quasi-1D system has an enhanced region of FFLO in the phase diagram
- It exhibits a rich collection of phases in the trapped gas
- Two types of FFLO states – commensurate and incommensurate

- Open question – when are the FFLO modulations in each tube phase-locked?

Acknowledgements

- Unequal masses
  - F. M. Marchetti, Oxford
  - A. Lamacraft, Oxford
  - B. D. Simons, Cambridge

- Quasi-1D
  - D. Huse, Princeton
  - E. Mueller, Cornell