Fermi surface change across quantum phase transitions

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cond-mat/0609106

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Talk online at http://sachdev.physics.harvard.edu

Consider a system of bosons and fermions at non-zero density, and N particle-number (U(1)) conservation laws.

Then, for each conservation law there is a "Luttinger" theorem constraining the momentum space volume enclosed by the locus of gapless single particle excitations, *unless*:

- there is a broken translational symmetry, and there are an integer number of particles per unit cell for every conservation law;
- there is a broken U(1) symmetry due to a boson condensate – then the associated conservation law is excluded;
- the ground state has "topological order" and fractionalized excitations.

Outline

A. Bose-Fermi mixtures

Depleting the Bose-Einstein condensate in trapped ultracold atoms

B. Fermi-Fermi mixtures

Normal states with no superconductivity

C. The Kondo Lattice

The heavy Fermi liquid (FL) and the fractionalized Fermi liquid (FL)*

D. Deconfined criticality

Changes in Fermi surface topology

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Mixture of bosons *b* and fermions *f*

(*e.g.* ^{7}Li+^{6}Li, ^{23}Na+^{6}Li, ^{87}Rb+^{40}K)

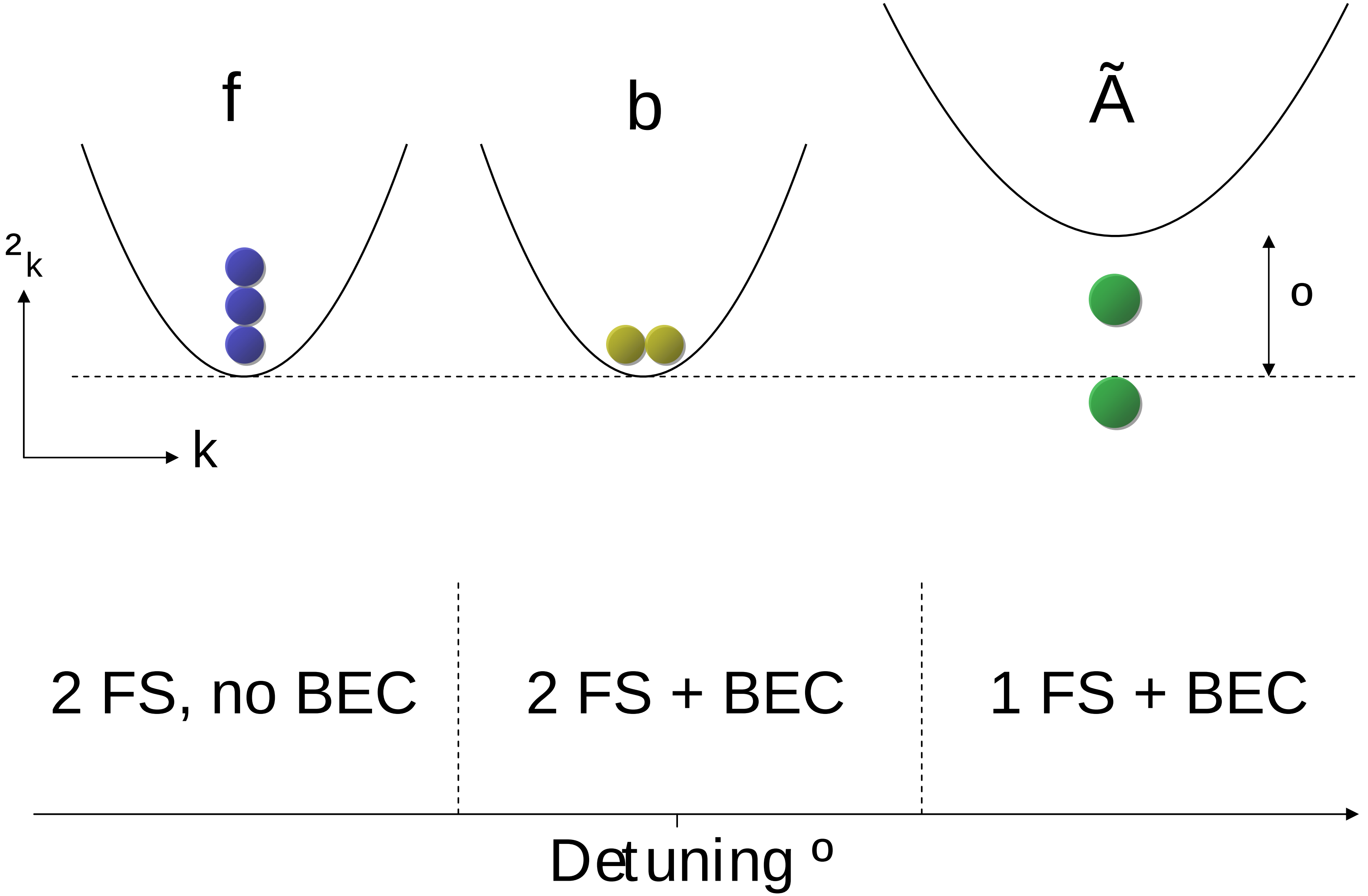
Tune to the vicinity of a Feshbach resonance associated with a molecular state ψ

Conservation laws:

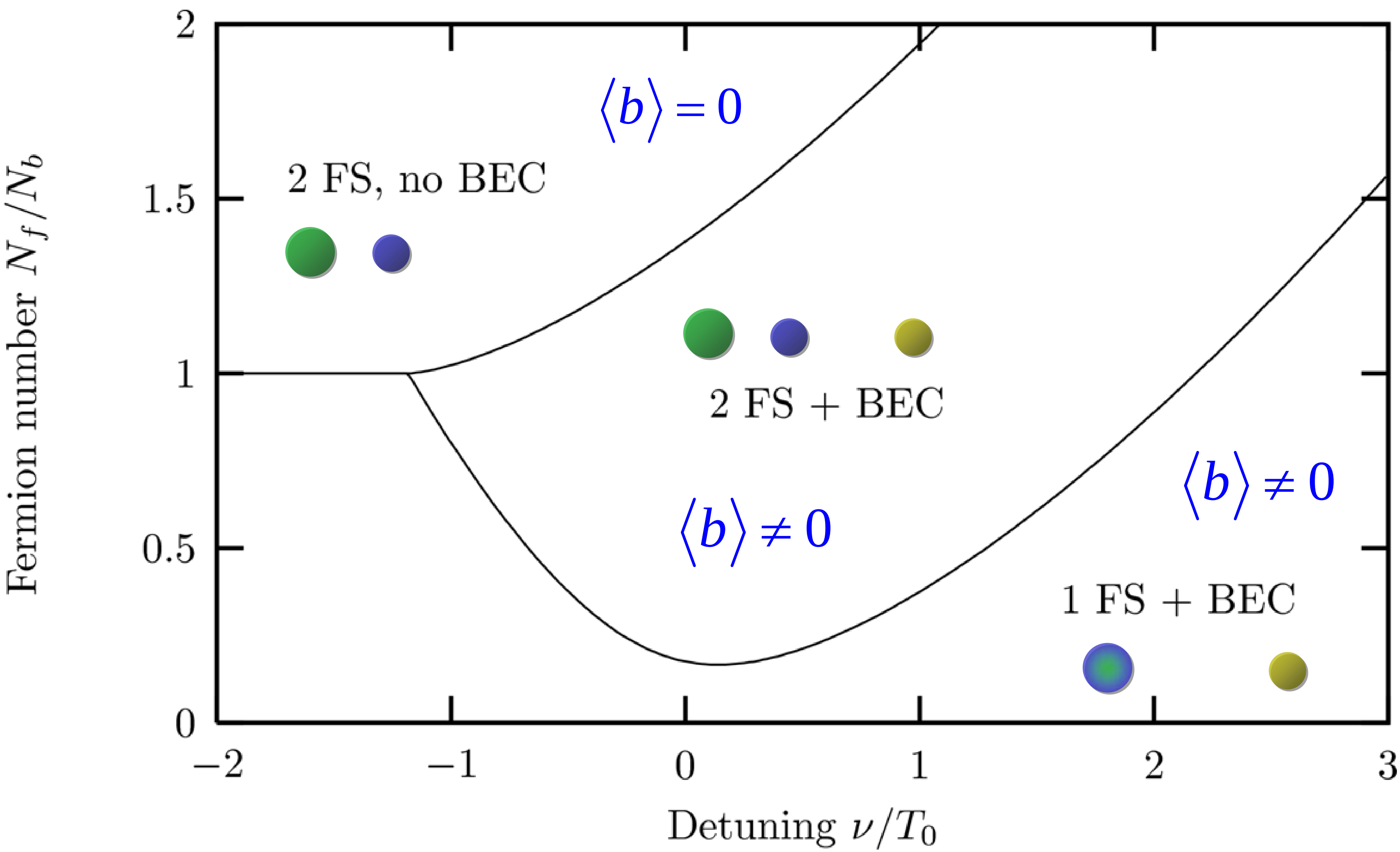
$$b^\dagger b + \psi^\dagger \psi = N_b$$

$$f^\dagger f + \psi^\dagger \psi = N_f$$

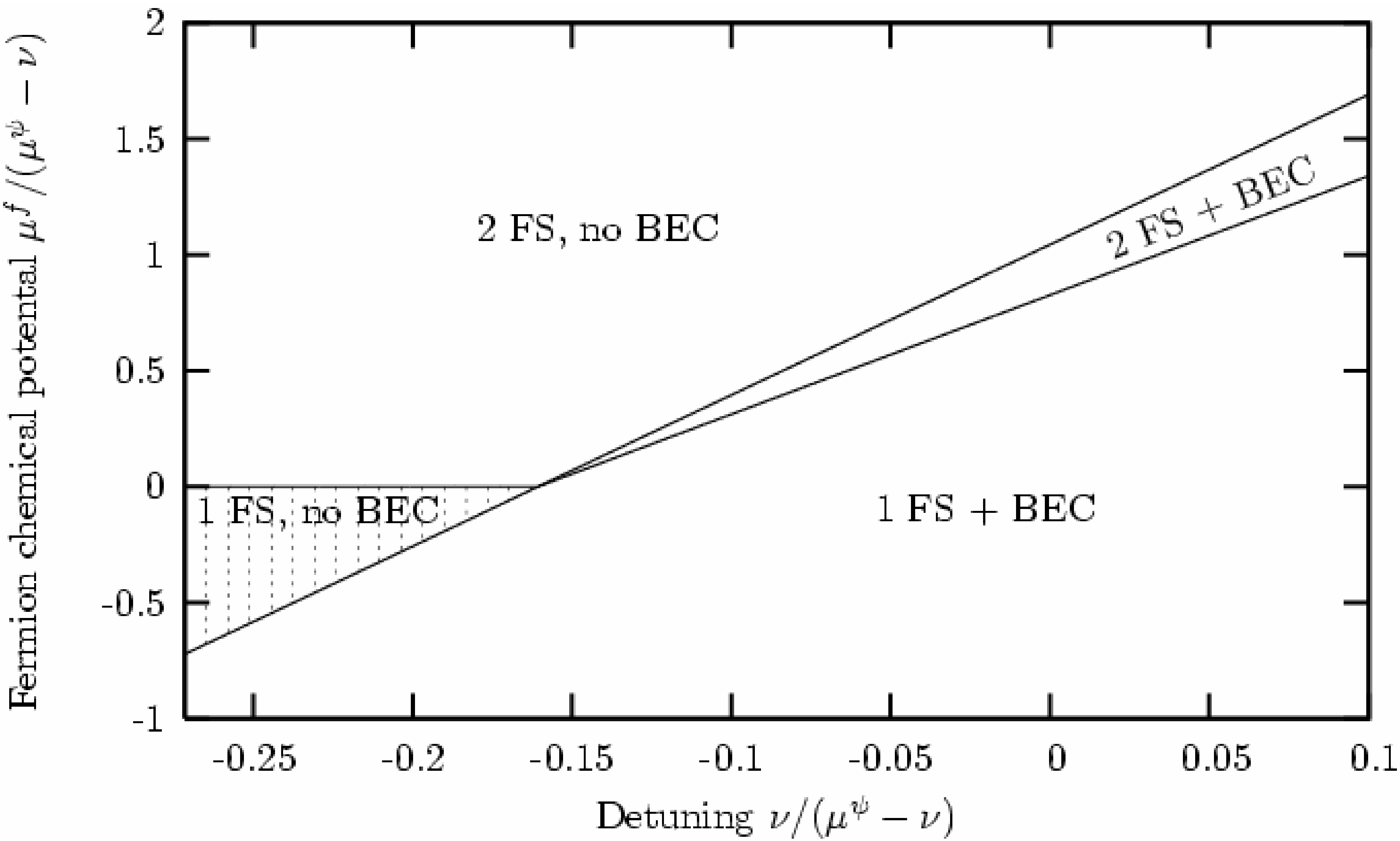
Phases



Phase diagram



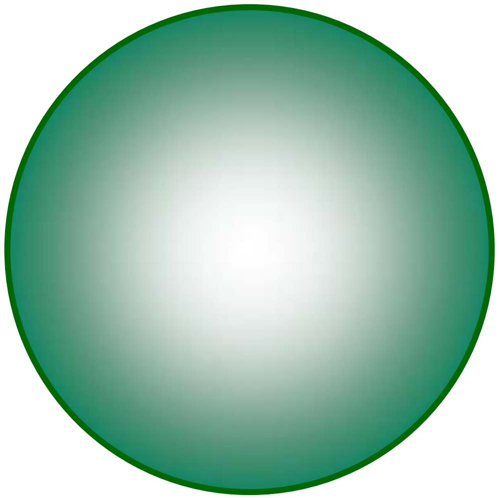
Phase diagram



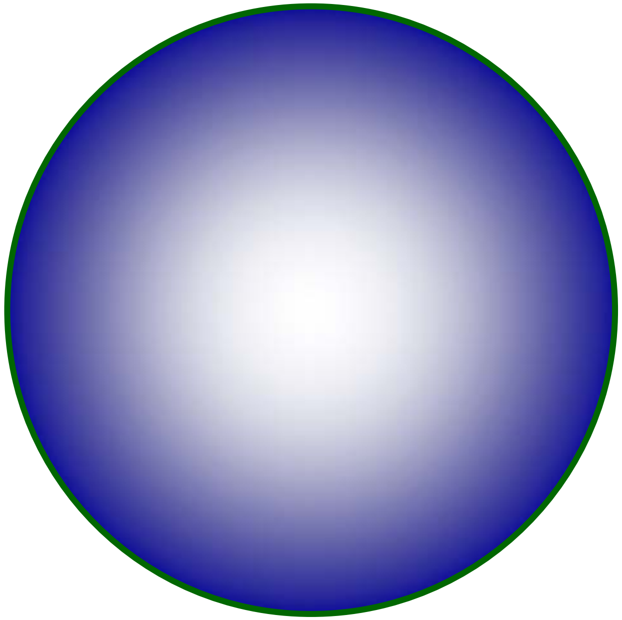
2 FS, no BEC phase

"atomic" Fermi surface

"molecular" Fermi surface



$$\langle b \rangle = 0$$

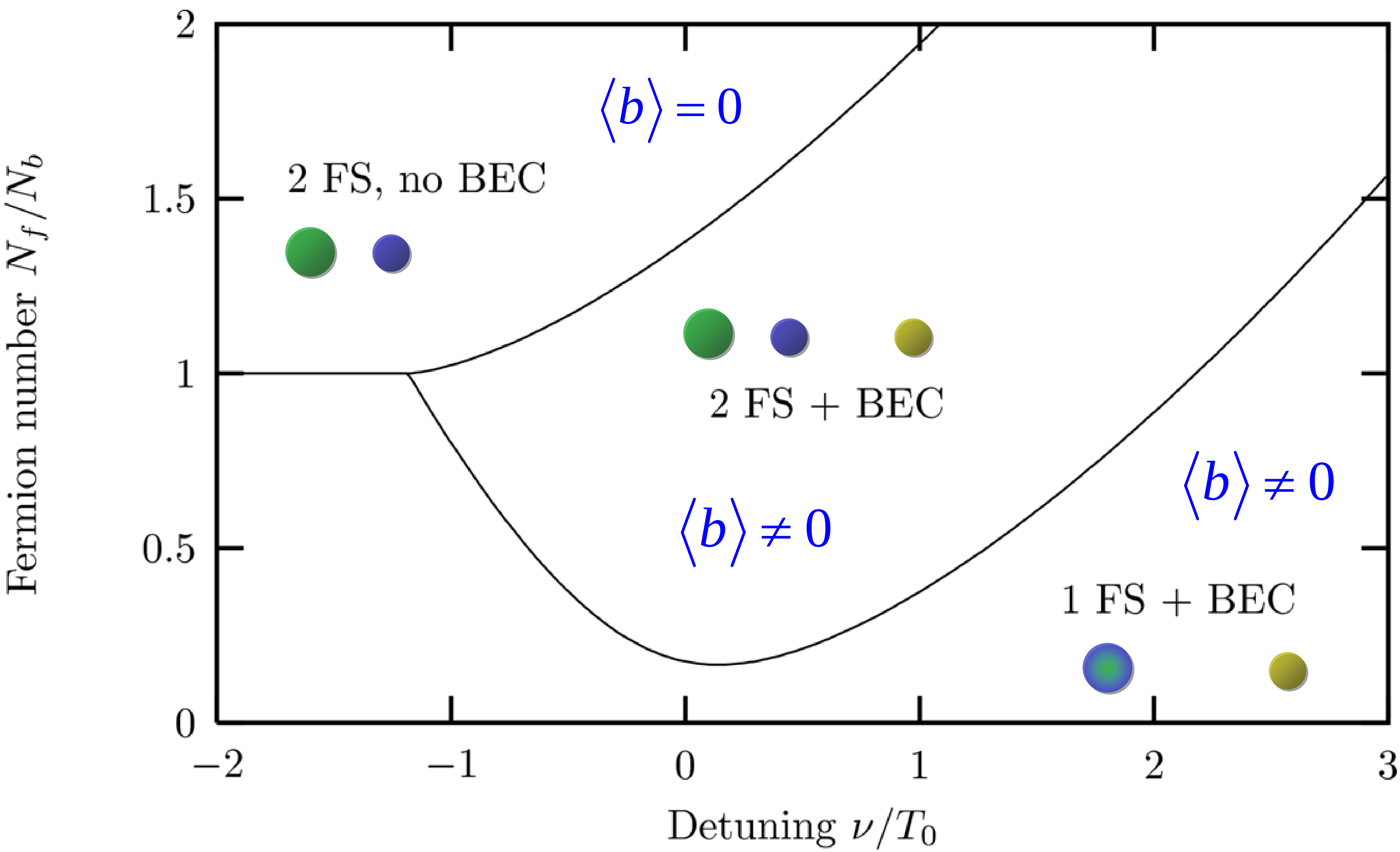


Volume $= N_b$

Volume $= N_f - N_b$

2 Luttinger theorems; volume within both Fermi surfaces is conserved

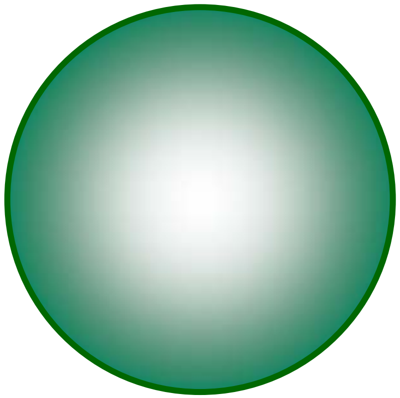
Phase diagram



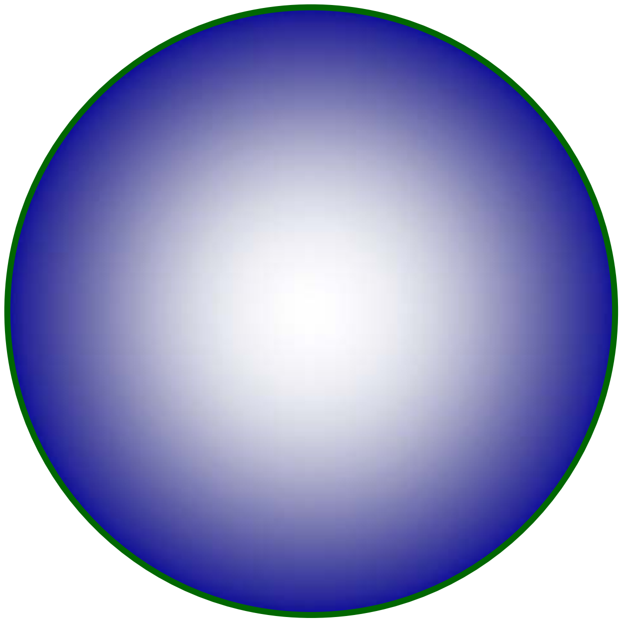
2 FS + BEC phase

"atomic" Fermi surface

"molecular" Fermi surface



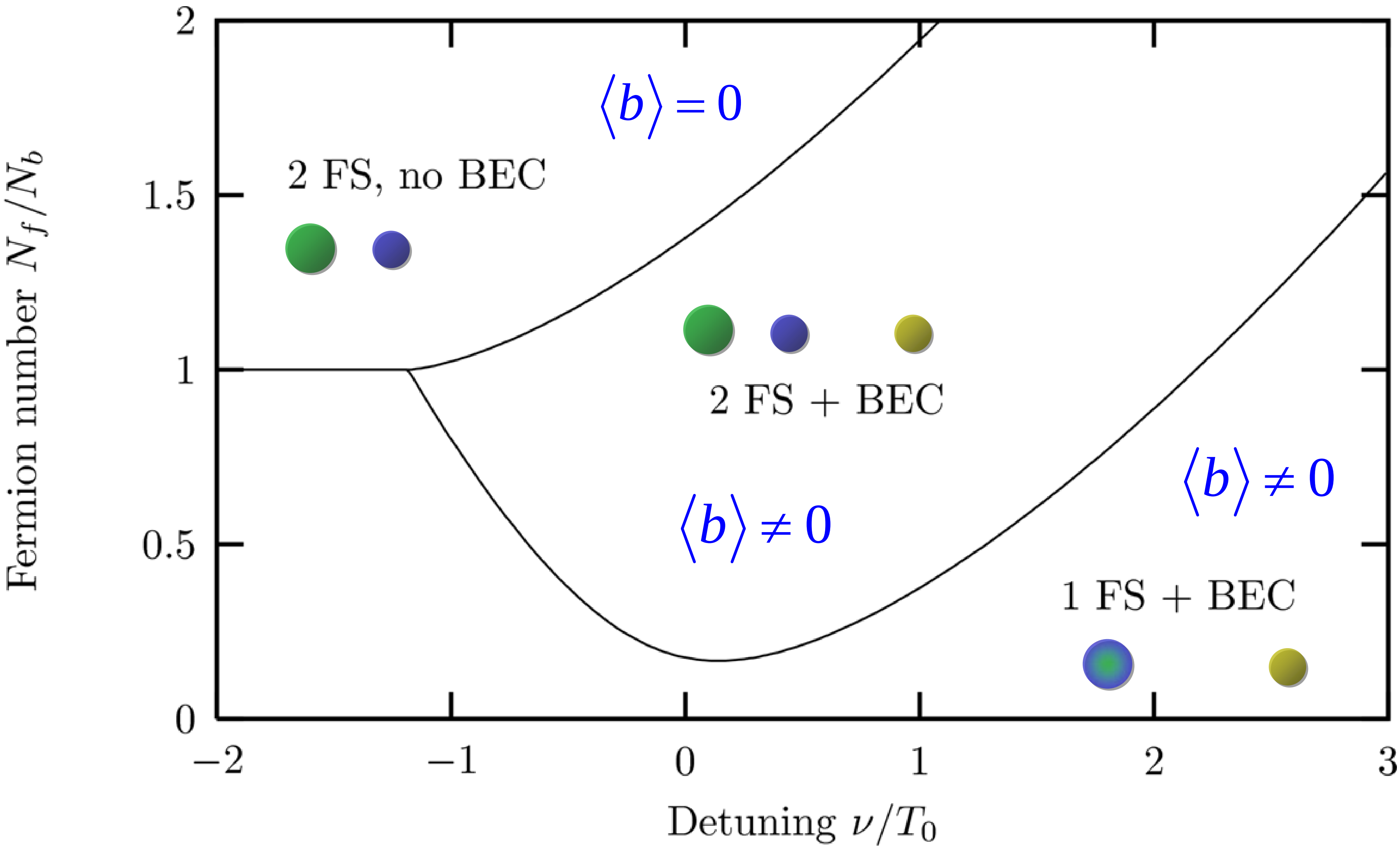
$$\langle b \rangle \neq 0$$



$$\text{Total volume} = N_f$$

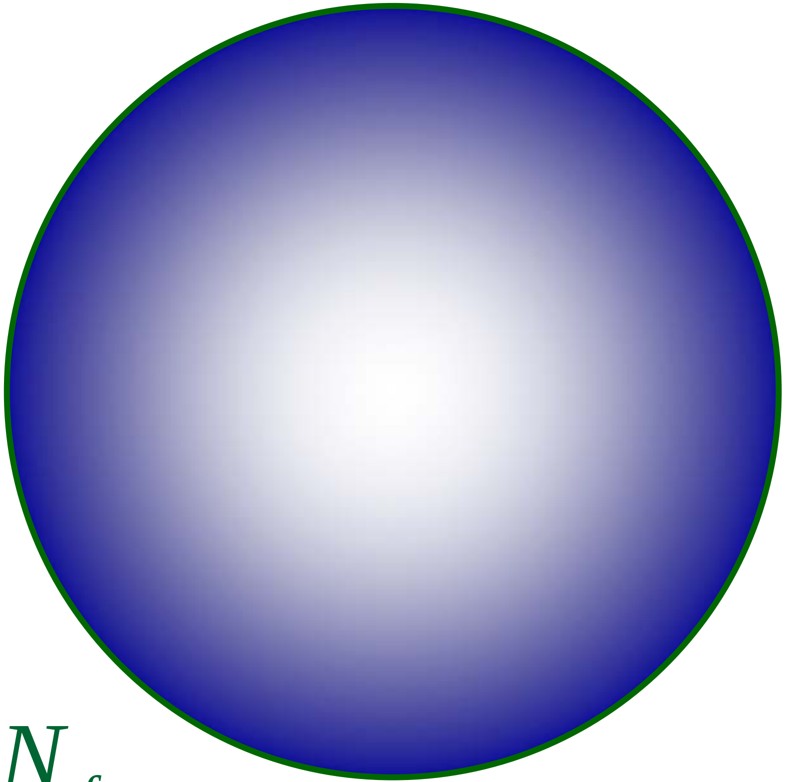
1 Luttinger theorem; only total volume within Fermi surfaces is conserved

Phase diagram



1 FS + BEC phase

"atomic" Fermi surface



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Changes in Fermi surface topology

Mixture of fermions $f_\downarrow$ and $f_\uparrow$

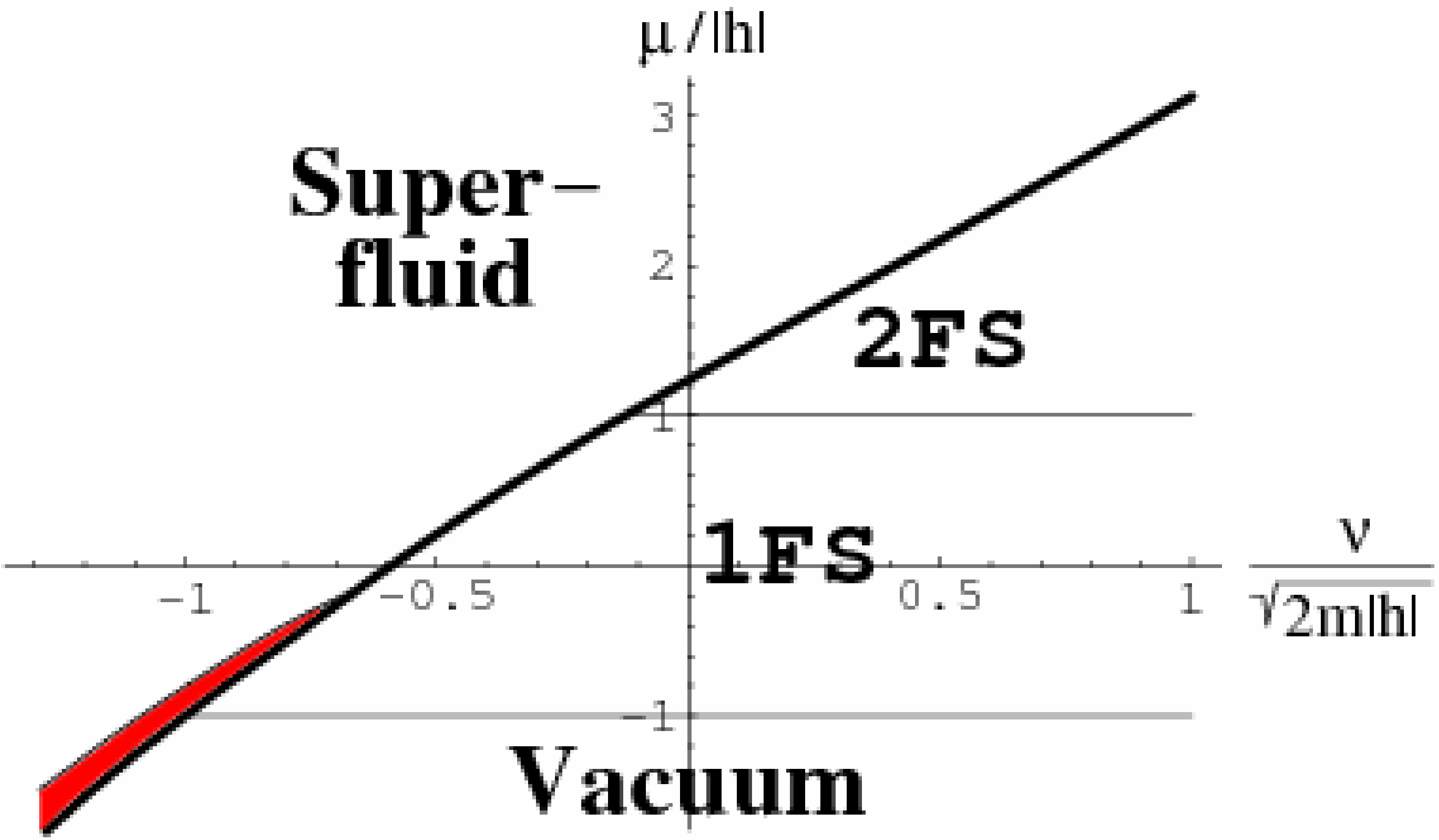
Tune to the vicinity of a Feshbach resonance
associated with a Cooper pair Δ

Conservation laws:

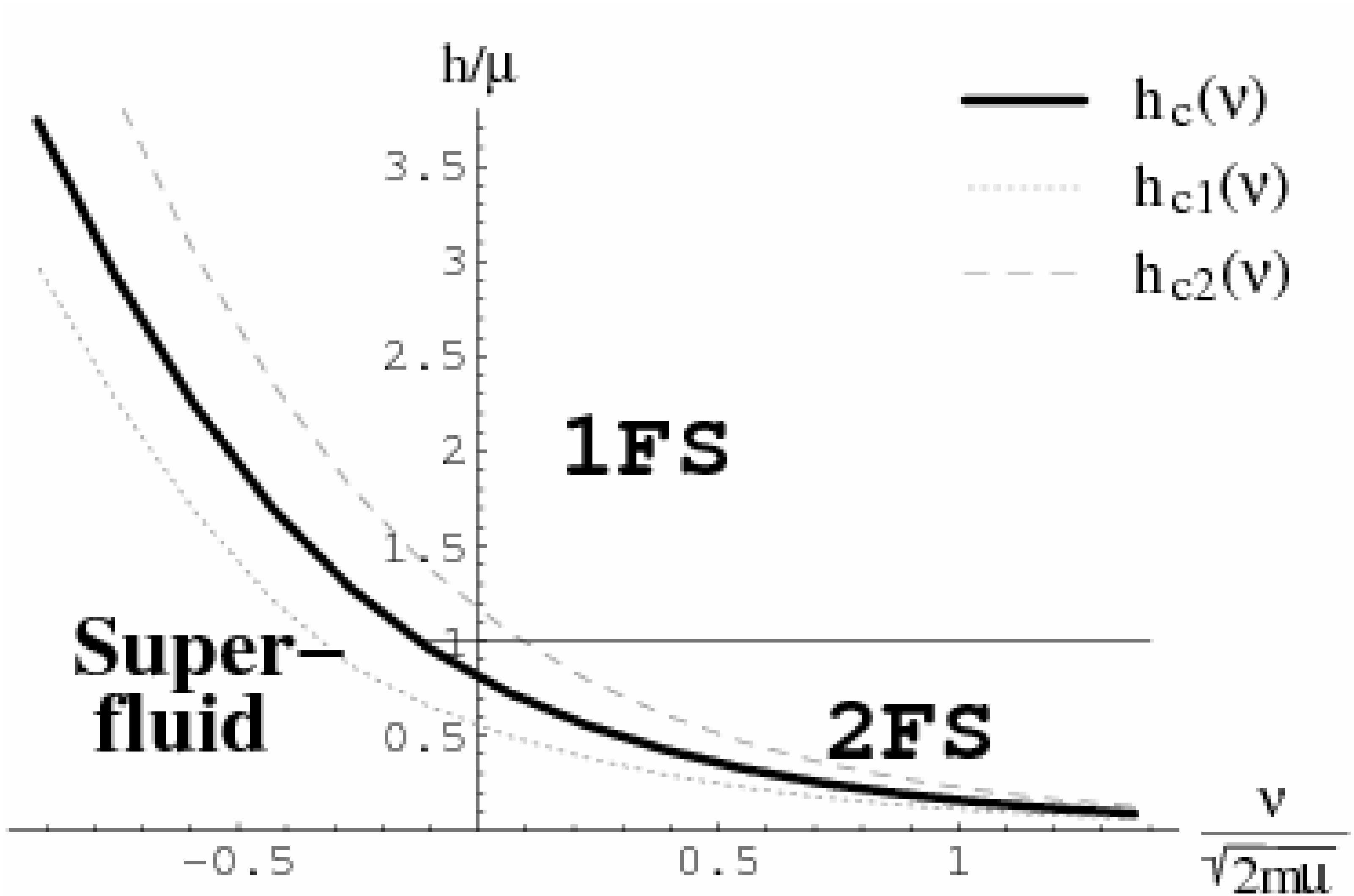
$$f_\downarrow^\dagger f_\downarrow + \Delta^\dagger \Delta = N_\downarrow$$

$$f_\uparrow^\dagger f_\uparrow + \Delta^\dagger \Delta = N_\uparrow$$

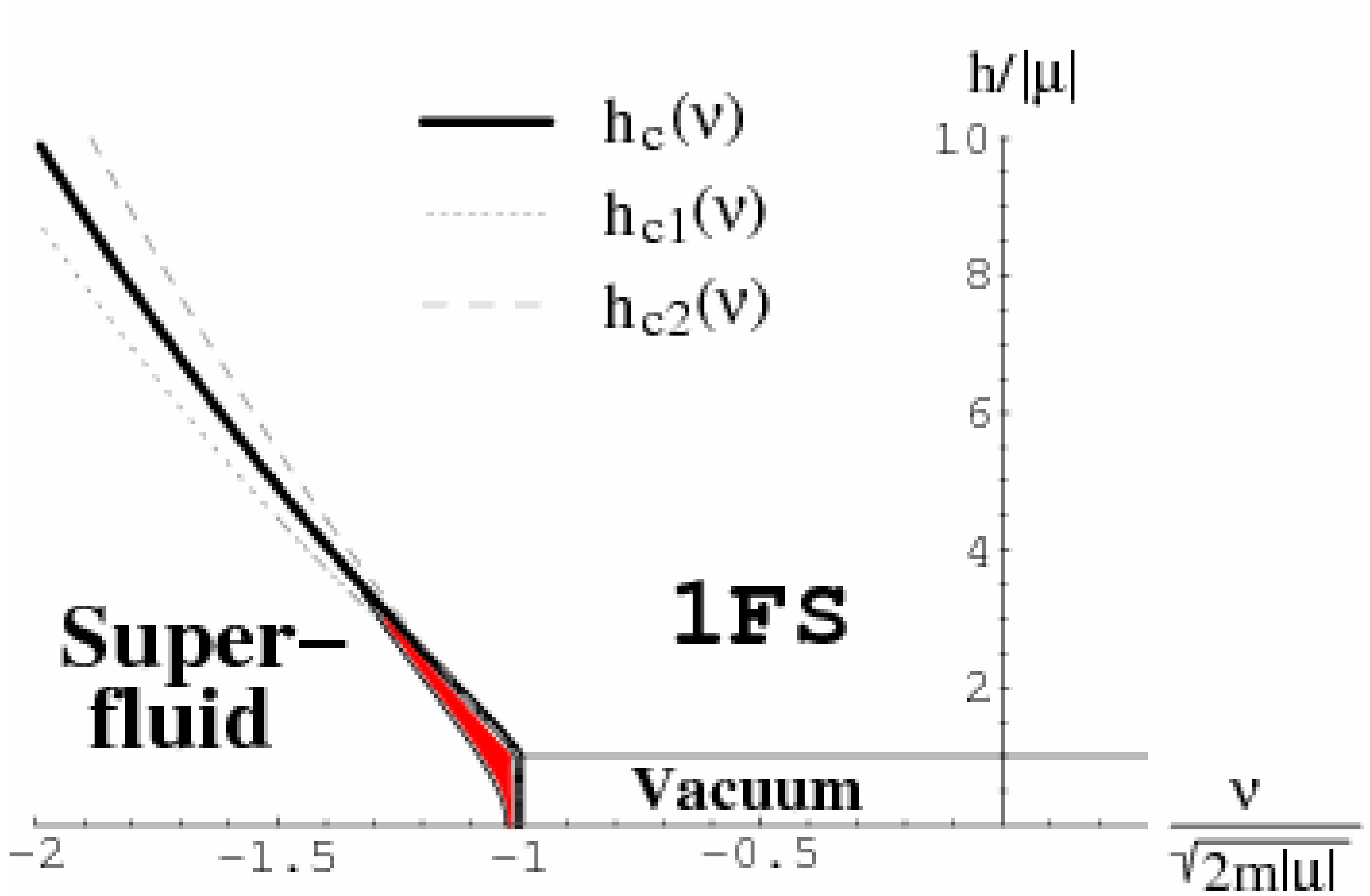
D. E. Sheehy and L. Radzihovsky, *Phys. Rev. Lett.* **96**, 060401 (2006);
M. Y. Veillette, D. E. Sheehy, and L. Radzihovsky, cond-mat/0610798.



μ chemical potential; h "magnetic" field; ν detuning



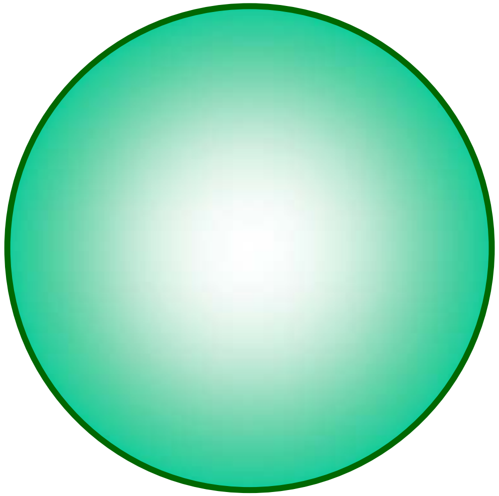
μ chemical potential; h "magnetic" field; ν detuning



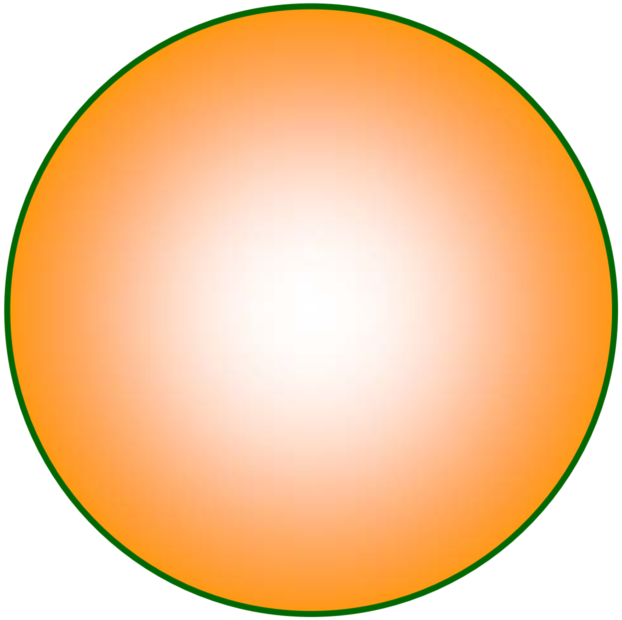
μ chemical potential; h "magnetic" field; ν detuning

2 FS, normal state

minority Fermi surface



majority Fermi surface



$$\langle \Delta \rangle = 0$$

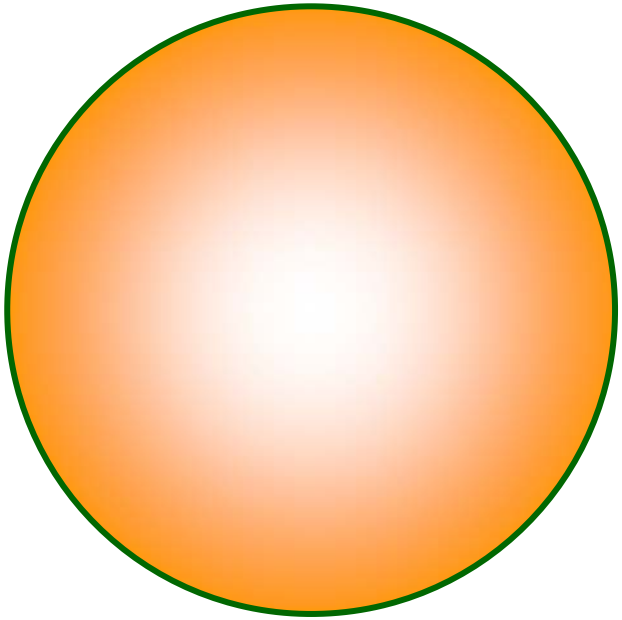
$$\text{Volume} = N_{\downarrow} \qquad\qquad \text{Volume} = N_{\uparrow}$$

2 Luttinger theorems; volume within both Fermi surfaces is conserved

1 FS, normal state

majority Fermi surface

minority Fermi surface



$$\langle \Delta \rangle = 0$$

$$N_{\downarrow} = 0 \qquad \text{Volume} = N_{\uparrow}$$

2 Luttinger theorems; volume within both Fermi surfaces is conserved

Superfluid

minority Fermi surface

majority Fermi surface

$$\langle \Delta \rangle \neq 0$$

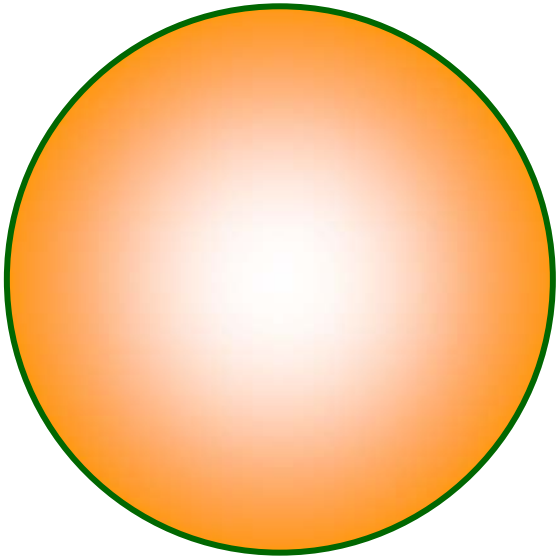
$$\text{Volume}_\uparrow - \text{Volume}_\downarrow = N_\uparrow - N_\downarrow$$

1 Luttinger theorem; difference volume within both Fermi surfaces is conserved

Magnetized Superfluid

minority Fermi surface

majority Fermi surface



$$\langle \Delta \rangle \neq 0$$

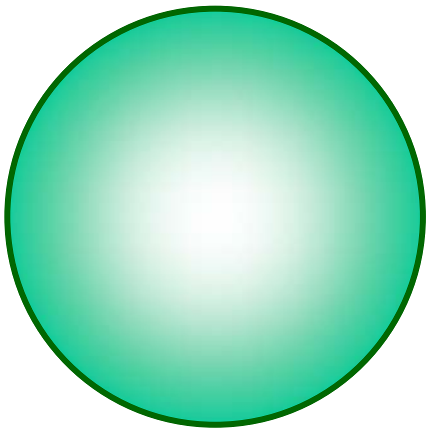
$$\text{Volume}_{\uparrow} - \text{Volume}_{\downarrow} = N_{\uparrow} - N_{\downarrow}$$

1 Luttinger theorem; difference volume within both Fermi surfaces is conserved

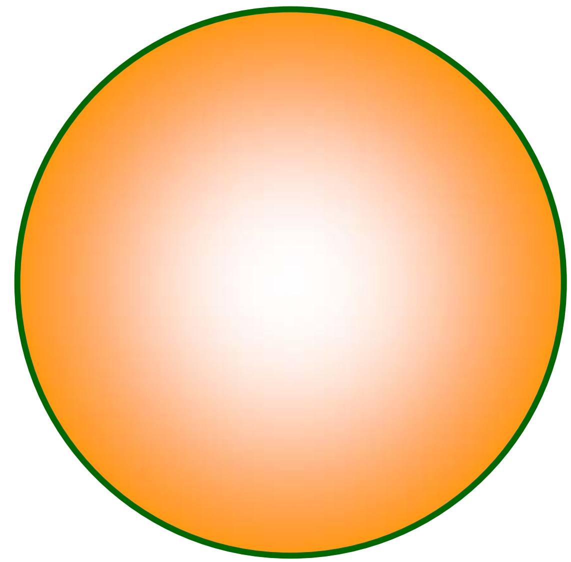
Sarma (breached pair) Superfluid

minority Fermi surface

majority Fermi surface



$$\langle \Delta \rangle \neq 0$$



$$\mathrm{Volume}_{\uparrow} - \mathrm{Volume}_{\downarrow} = N_{\uparrow} - N_{\downarrow}$$

1 Luttinger theorem; difference volume within both Fermi surfaces is conserved

Any state with a density imbalance
must have at least one Fermi surface

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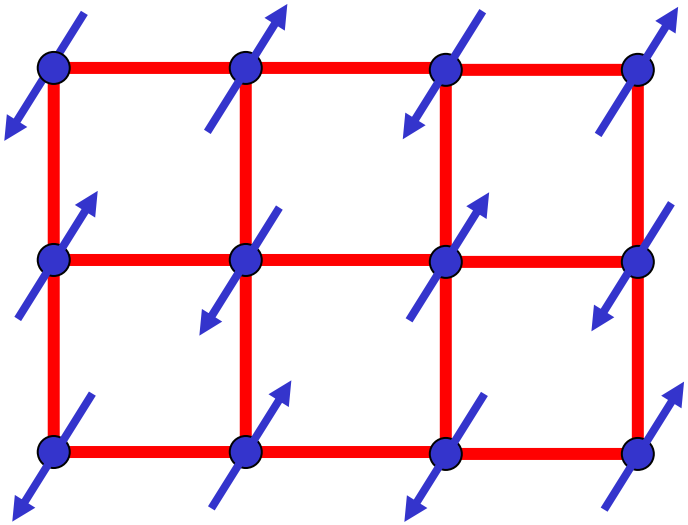
T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003).

D. Deconfined criticality

Changes in Fermi surface topology

The Kondo lattice



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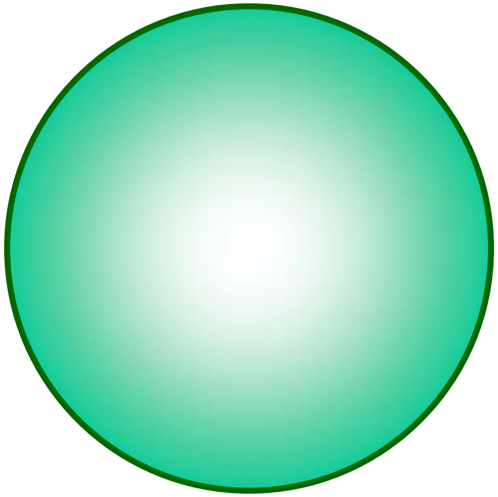


Local moments f_σ

Conduction electrons c_σ

$$H_K = \sum_{i<j} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + J_K \sum_i c^\dagger_{i\sigma} \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} + J \sum_{\langle ij \rangle} \vec{S}_{fi} \cdot \vec{S}_{fj}$$

Number of f electrons per unit cell = $n_f = 1$

Number of c electrons per unit cell = n_c

Define a bosonic field which measures the hybridization between the two bands:

$$b_i \sim \sum_{\sigma} c_{i\sigma}^{\dagger} f_{i\sigma}$$

Analogy with Bose-Fermi mixture problem:

$c_{i\sigma}$ is the analog of the "molecule" ψ

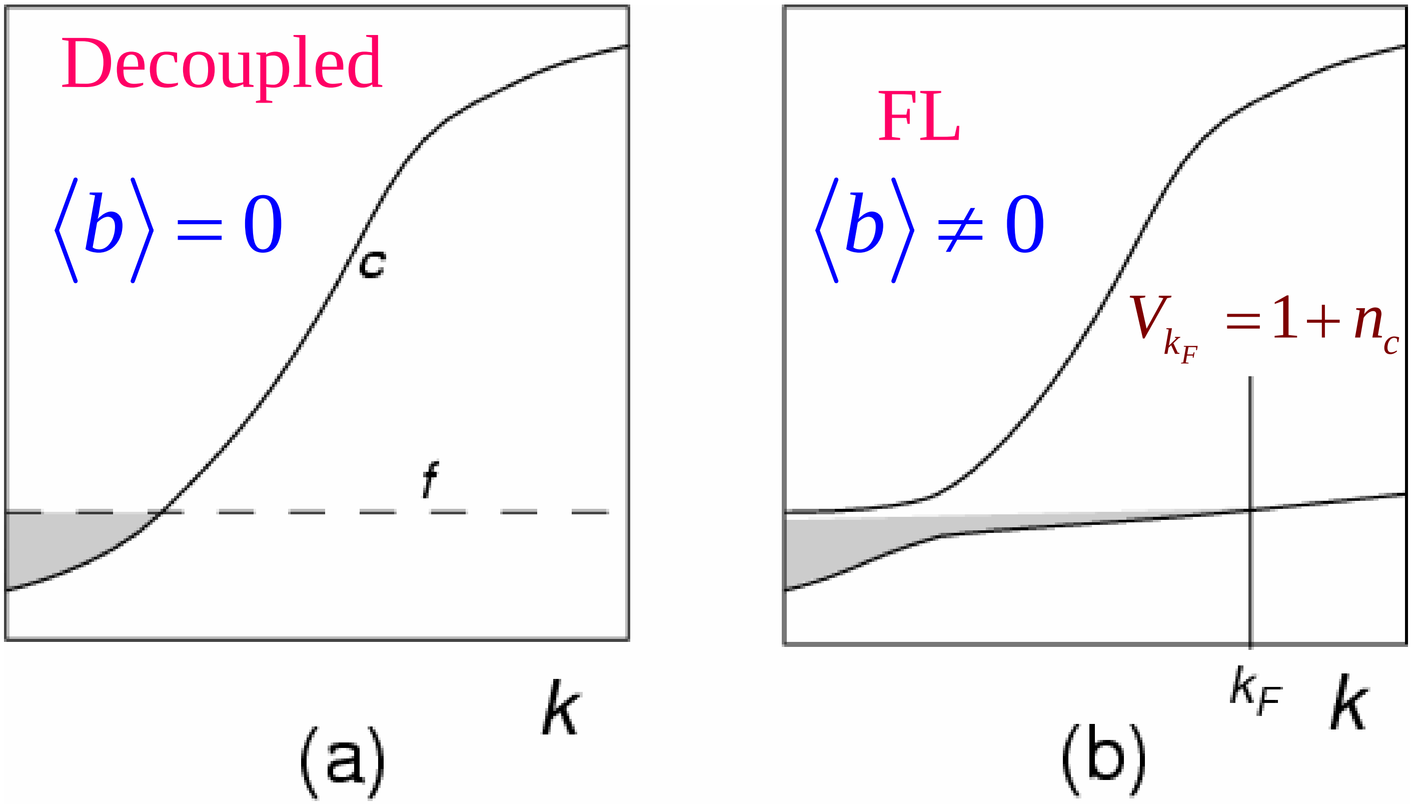
Conservation laws:

$$f_{\sigma}^{\dagger} f_{\sigma} + c_{\sigma}^{\dagger} c_{\sigma} = 1 + n_c \quad \text{(Global)}$$

$$f_{\sigma}^{\dagger} f_{\sigma} + b^{\dagger} b = 1 \quad \text{(Local)}$$

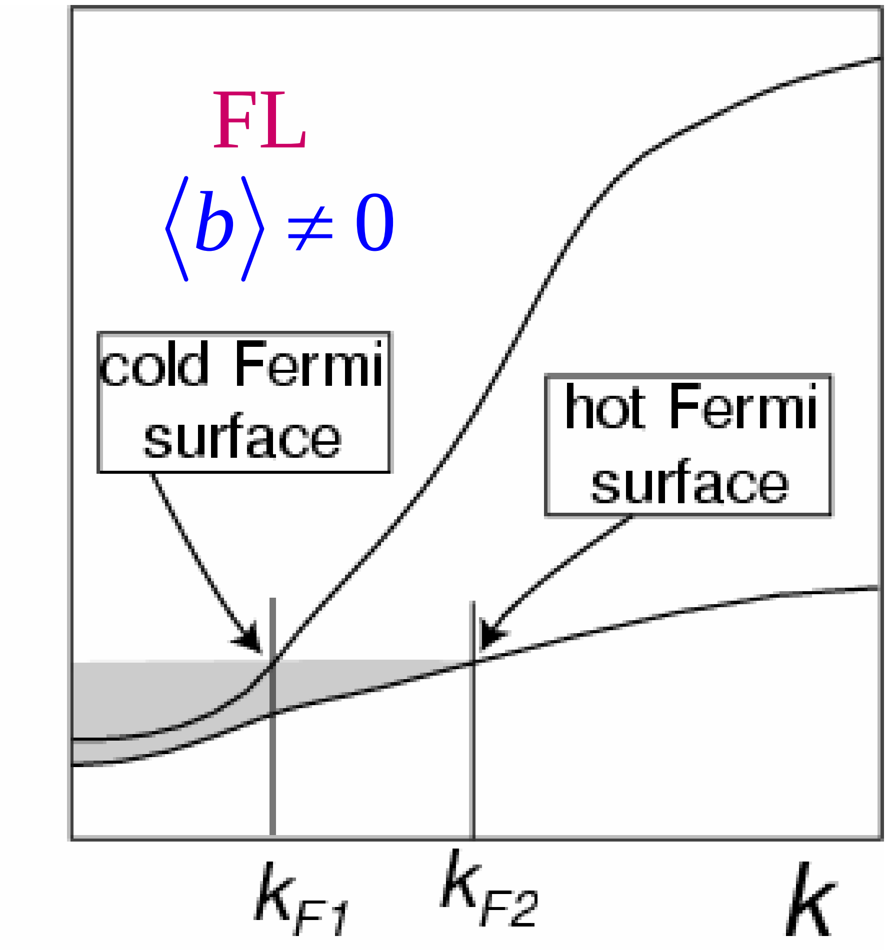
Main difference: second conservation law is *local* so there is a U(1) gauge field.

1 FS + BEC ⇔ Heavy Fermi liquid (FL) ⇔ Higgs phase



If the f band is dispersionless in the decoupled case, the ground state is always in the 1 FS FL phase.

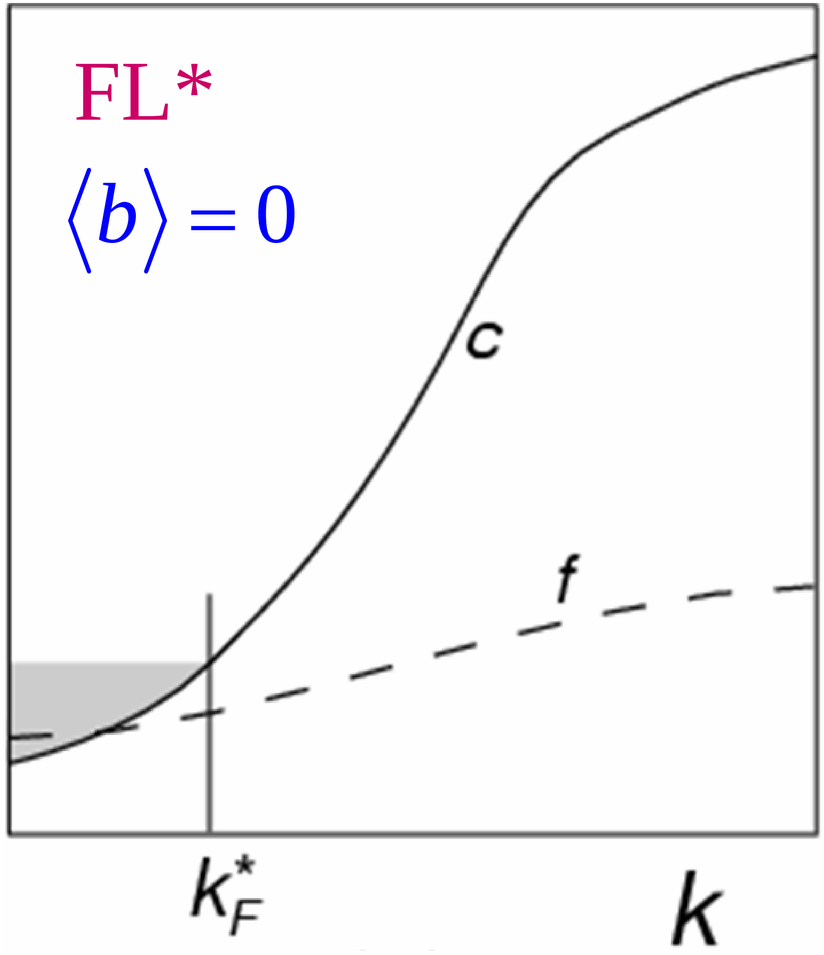
$2\ \mathrm{FS} + \mathrm{BEC} \Leftrightarrow$ Heavy Fermi liquid (FL) $\Leftrightarrow$ Higgs phase



A bare f dispersion (from the RKKY couplings) allows a 2 FS FL phase.

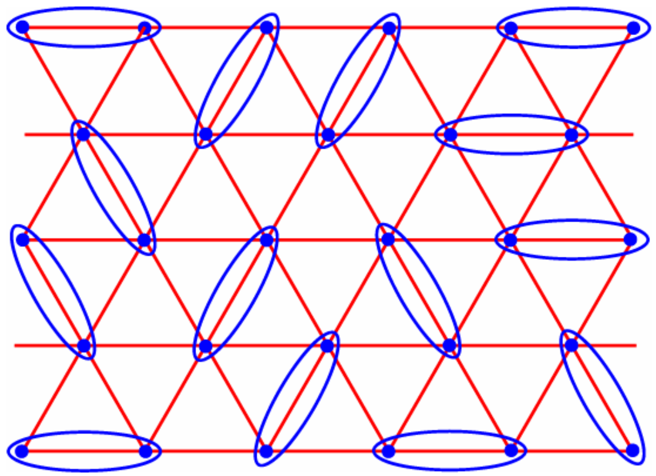
2 FS, no BEC $\Leftrightarrow$ Fractionalized Fermi liquid (FL*)

$\Leftrightarrow$ Deconfined phase

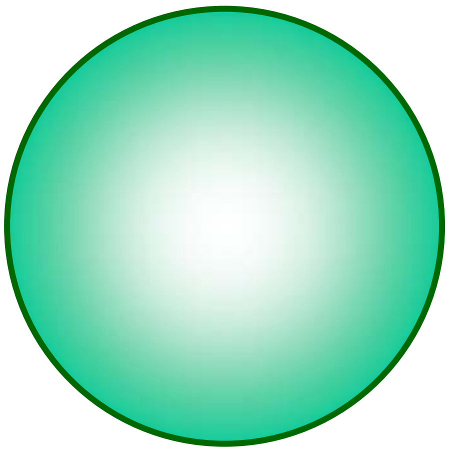


The f band "Fermi surface" realizes a spin liquid (because of the local constraint)

Another perspective on the FL* phase



Local moments f_σ



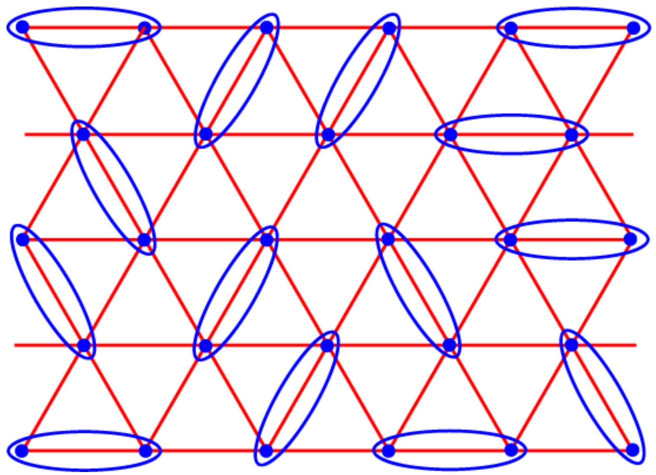
Conduction electrons c_σ

$$H = \sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i \left(J_K c_{i\sigma}^\dagger \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} \right) + \sum_{i<j} J_H (i,j) \vec{S}_{fi} \cdot \vec{S}_{fj}$$

Determine the ground state of the quantum antiferromagnet defined by J_H, and then couple to conduction electrons by J_K

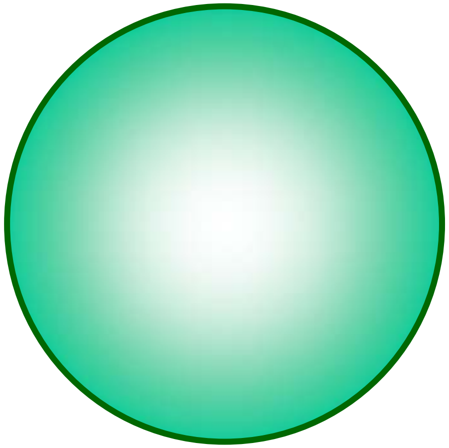
Choose J_H so that ground state of antiferromagnet is a Z_2 or U(1) spin liquid

Influence of conduction electrons



Local moments f_σ

+



Conduction electrons c_σ

At $J_K = 0$ the conduction electrons form a Fermi surface on their own with volume determined by n_c.

Perturbation theory in J_K is regular, and so this state will be stable for finite J_K.

So volume of Fermi surface is determined by **$(n_c+n_f$ -1)= n_c(mod 2),** and does not equal the Luttinger value.

The (U(1) or Z_2) FL* state

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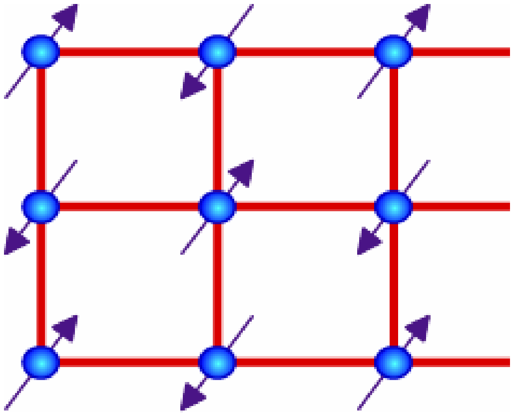
Changes in Fermi surface topology

R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, *cond-mat/0702119*.

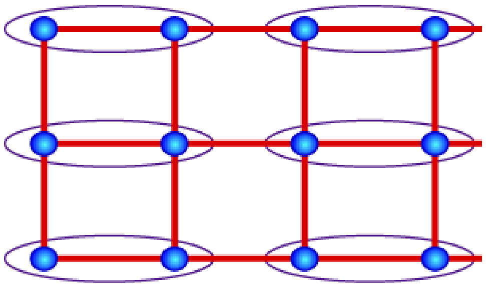
Phase diagram of S=1/2 square lattice antiferromagnet



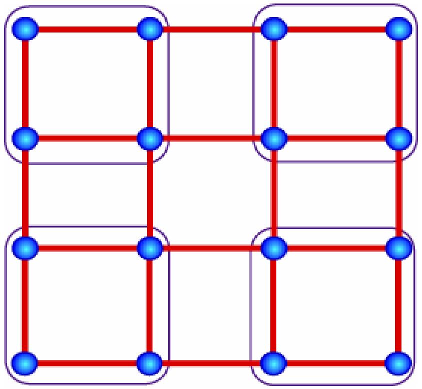
Neel order

$\vec{\varphi} \sim z^*_\alpha \vec{\sigma}_{\alpha\beta} z_\beta \neq 0$

(Higgs)



or



VBS order $\Psi_{\rm VBS} \neq 0$,

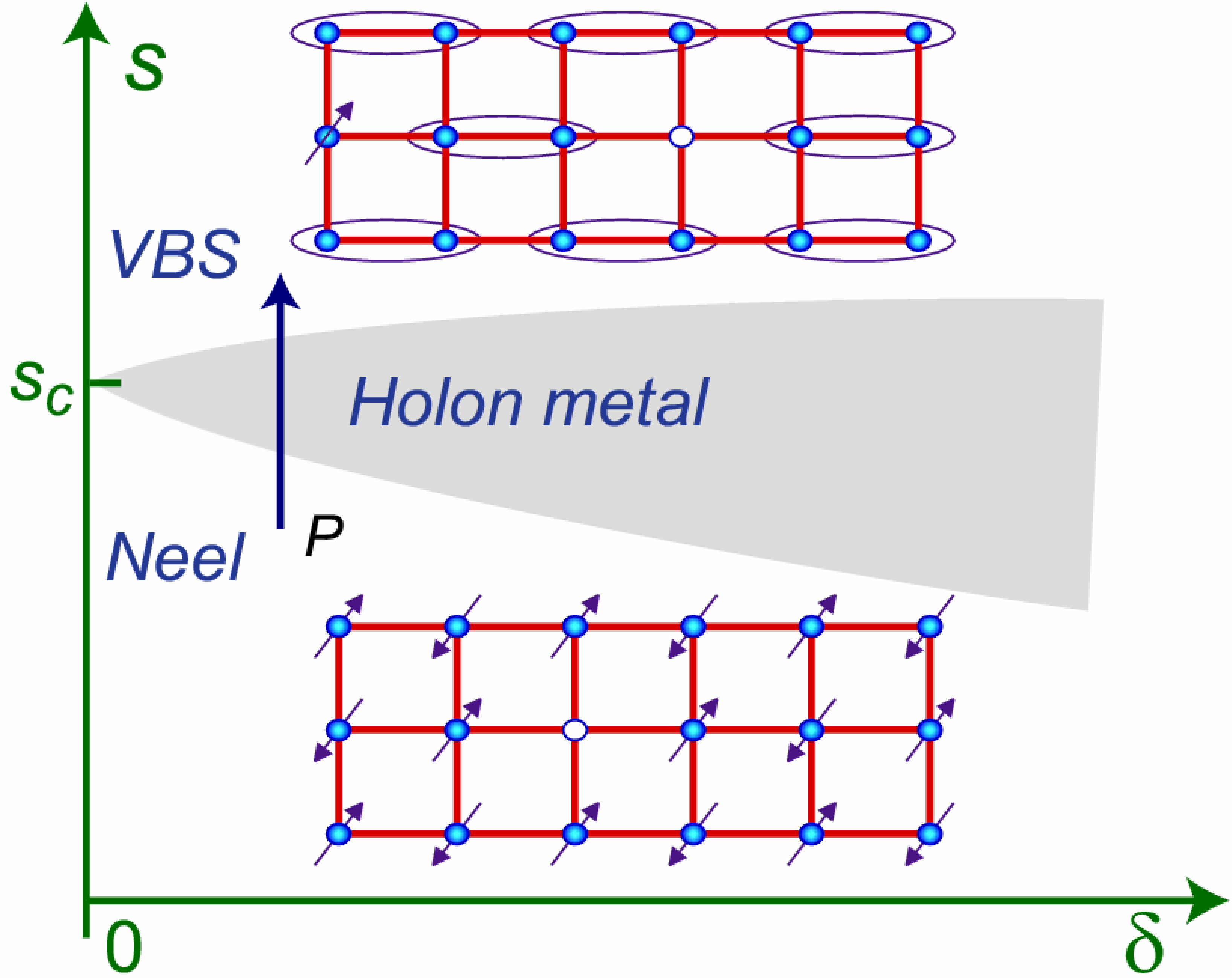
$S = 1/2$ spinons z_α confined,

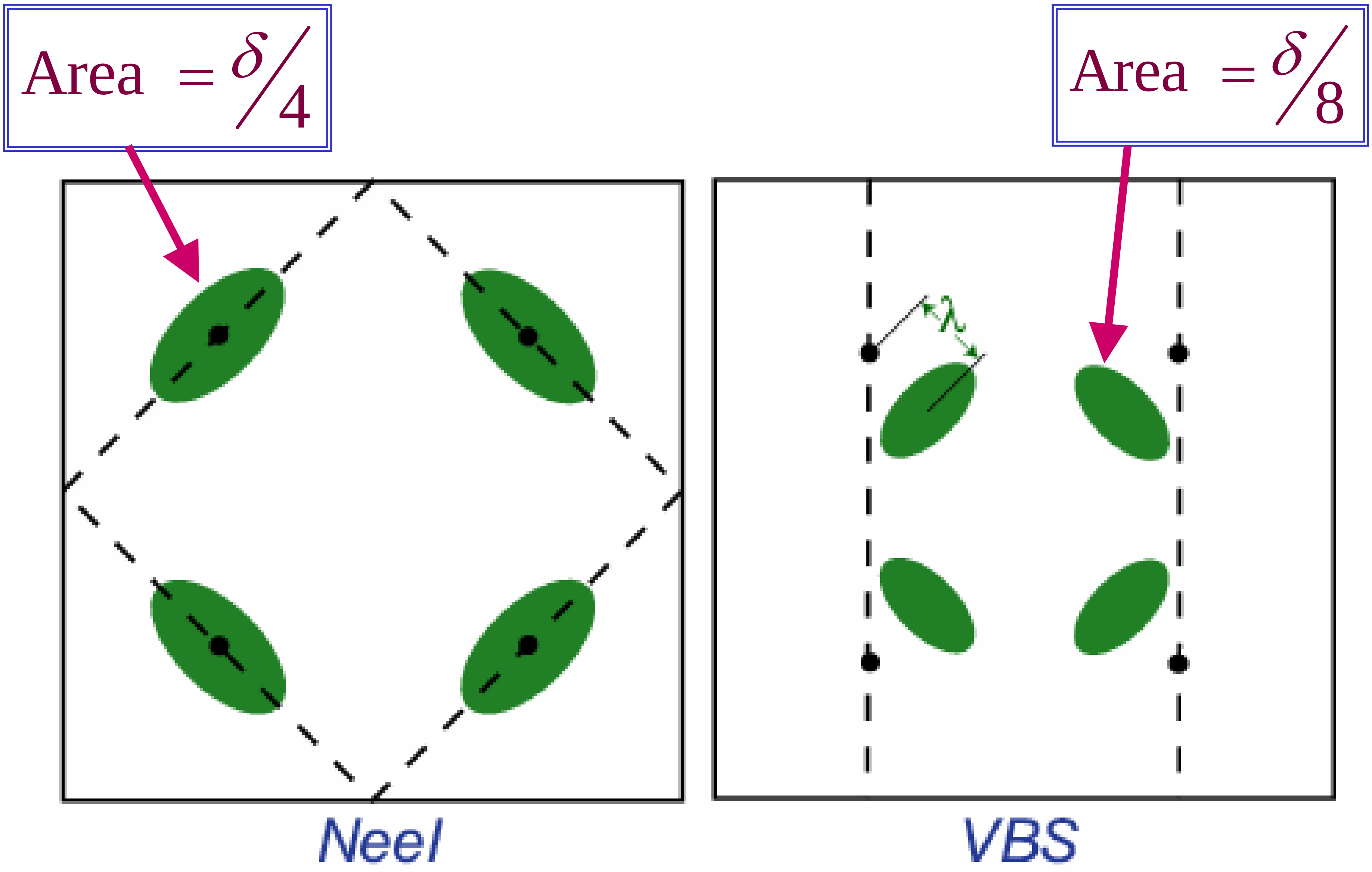
$S = 1$ triplon excitations

Deconfined critical point described by a theory of spinons

$$\mathcal{S}_{\rm critical} = \int d^2x d\tau \left[\left|(\partial_\mu - iA_\mu) z_\alpha\right|^2 + s\left|z_\alpha\right|^2 + \frac{u}{2}\left(\left|z_\alpha\right|^2\right)^2 + \frac{1}{4e^2}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 \right]$$

Landau-forbidden transition between phases which break "unrelated" symmetries


s
VBS
s_c
Holon metal
P
Neel
0
δ



Area $= \delta/4$
Area $= \delta/8$
λ
Neel
VBS