Strongly interacting Fermi gases

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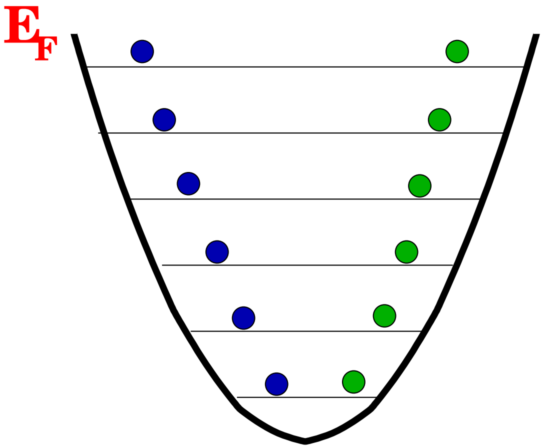
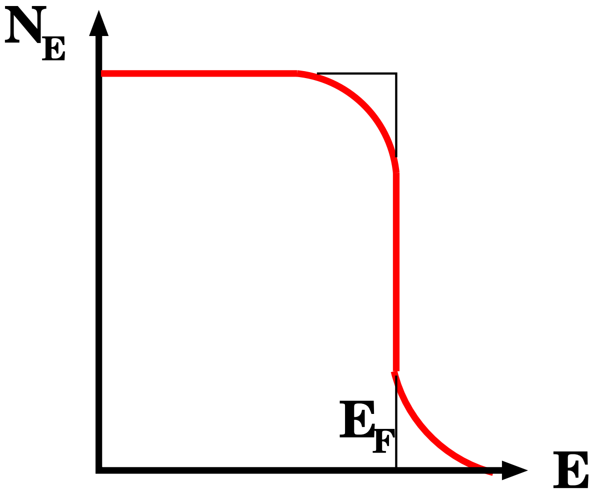
Outline

- Introduction.
- BCS-BEC crossover and strongly interacting regime
- Molecular BEC regime
- Molecules in Fermi-Fermi mixtures
- Crystalline phase and quantum transitions
- Conclusions

Collaborations: D.S. Petrov, C. Salomon (ENS), G.Astrakharchik (Barcelona)

Two-component trapped Fermi gas





$$E_F = \frac{\hbar^2 k_F^2}{2m}; \quad k_F = (3\pi^2 n)^{1/3}; \quad E_F \sim N^{1/3}\hbar\omega$$

Weakly interacting gas $n|a|^3 \ll 1;\ k_F|a| \ll 1$

$a < 0$ →Interspecies attraction →Cooper pairing at low T

$\vec{\mathbf{k}}$ ● ● $-\vec{\mathbf{k}}$

Superfluid BCS transition → $T_c \sim E_F \exp\{-\pi/2k_F|a|\}$

$T_c \ll 0.1 E_F$ for ordinary a Very hard to reach

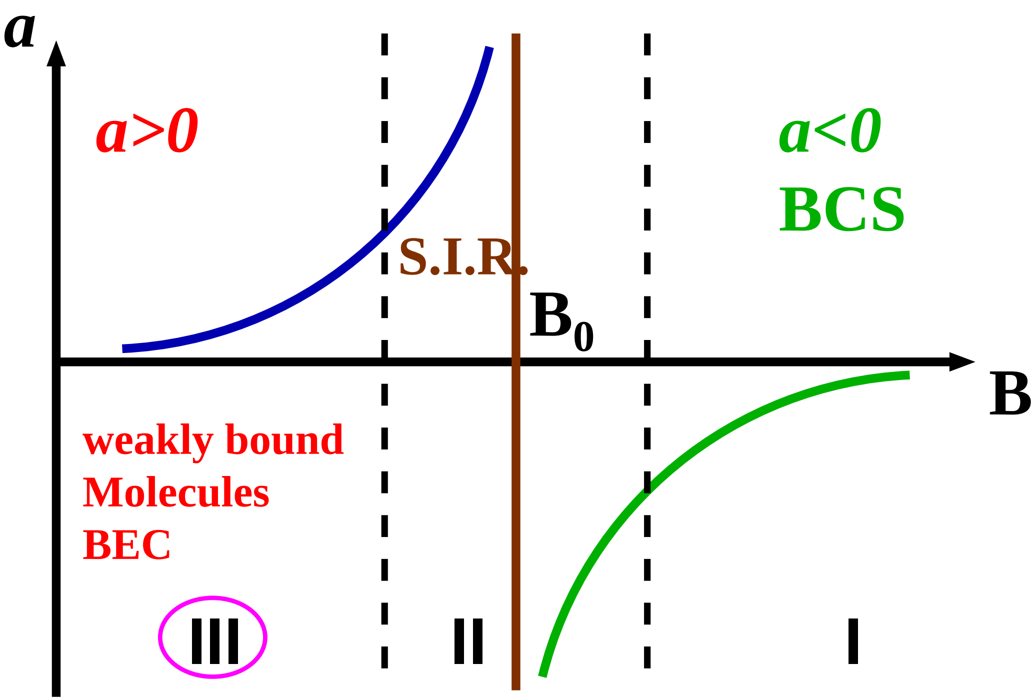
Experiments ^{40}**K** ^{6}**Li**

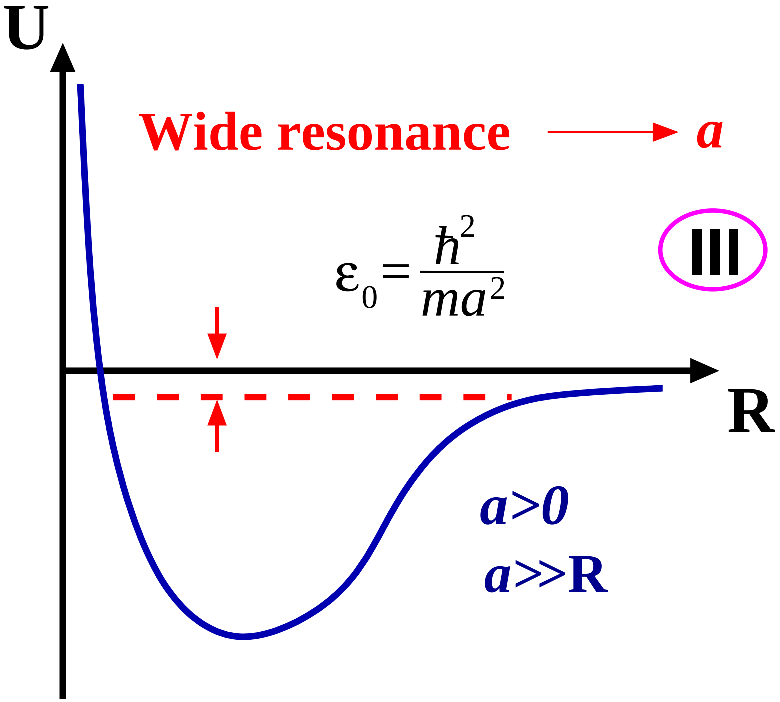
Dilute limit $nR_e^3 \ll 1$ Ultracold limit $\Lambda_T \gg R_e$

Quantum degeneracy $\longrightarrow$ JILA 1998 ^{40}K

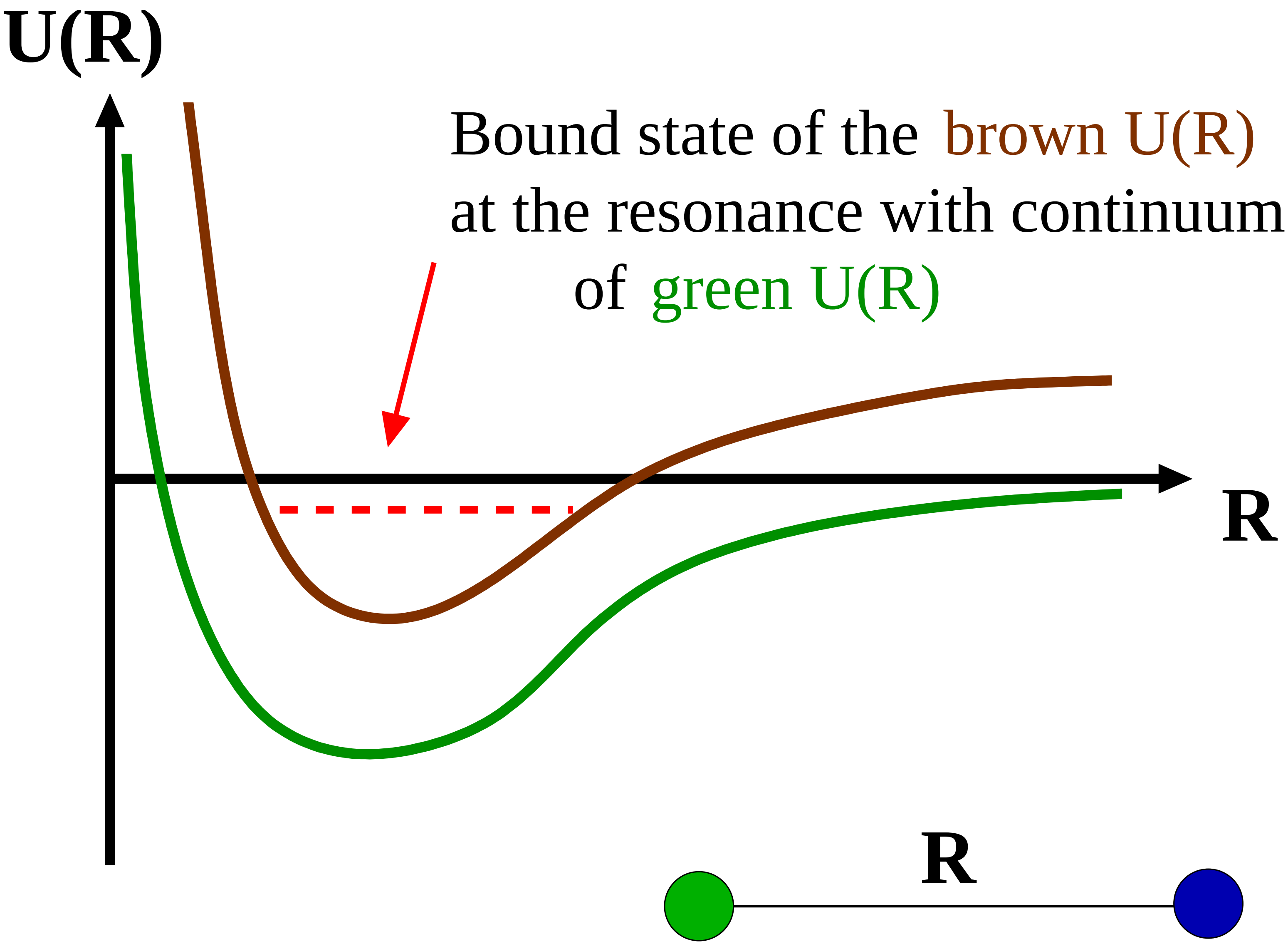
At present $n \sim 10^{13} - 10^{14}\mathrm{cm}^{-3}; \quad T \sim 1\mu\mathrm{K}$

JILA, LENS Innsbruck,MIT,ENS,Rice,Duke, ETH, Hamburg, Tuebingen, Toronto





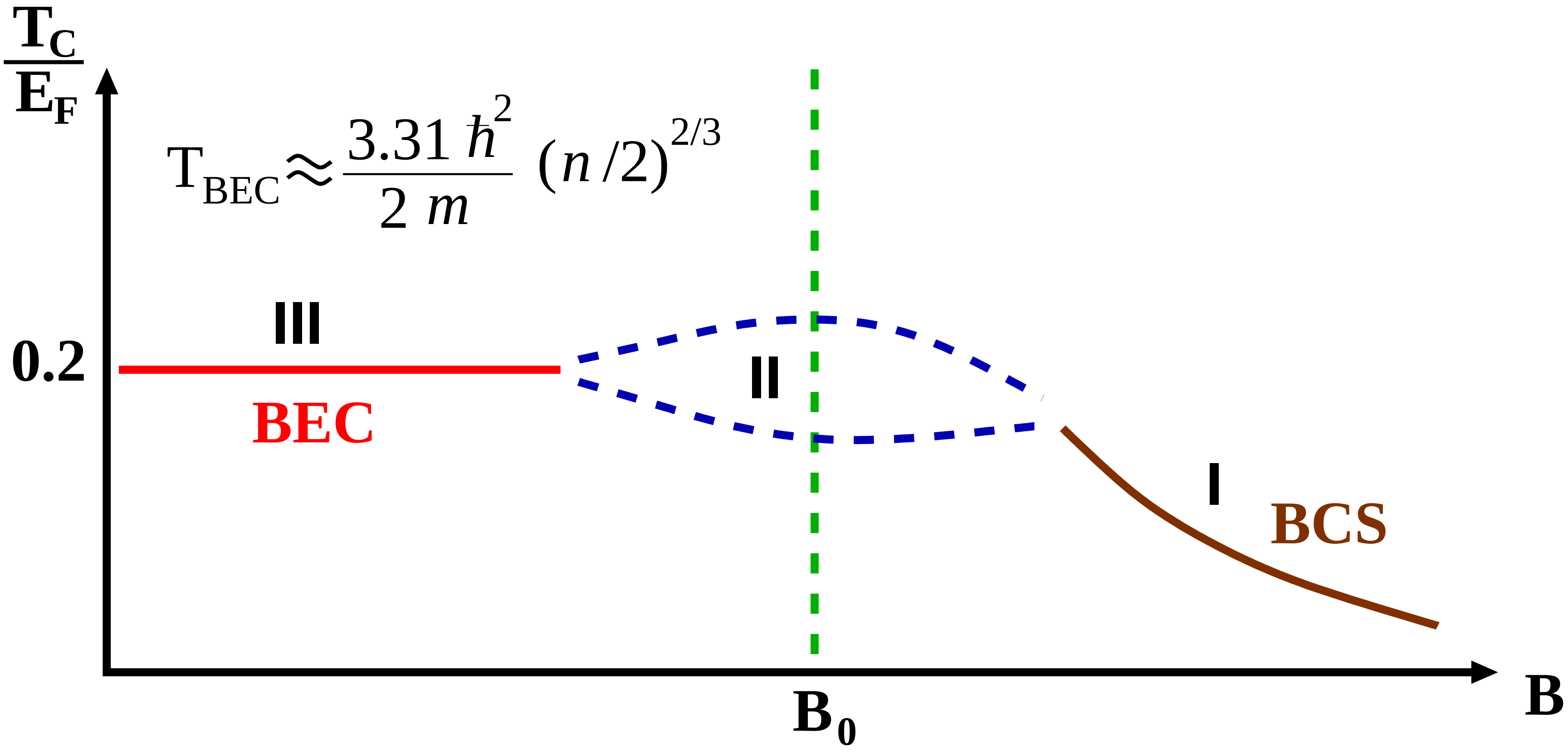
Feshbach resonance



Bound state of the brown U(R)
at the resonance with continuum
of green U(R)

Superfluid regimes

- **I** $k_F|a| \ll 1 \;\rightarrow$ **BCS**
- **II** $k_F|a| > 1 \;\rightarrow$ **Strongly interacting regime**
- **III** $na^3 \ll 1 \;\rightarrow$ **Gas of bosonic molecules**
  - $a \gg R_e \;\rightarrow$ BEC of weakly bound molecules



BCS-BEC crossover: Leggett, Nozieres-Schmitt-Rink

Strongly interacting regime

$T = 0 \quad k_F|a| \gg 1 \quad \rightarrow$ Only one distance scale $n^{-1/3}$

Only one energy scale $E_F \sim \hbar^2 n^{2/3}/m$

Universal thermodynamis (J. Ho)

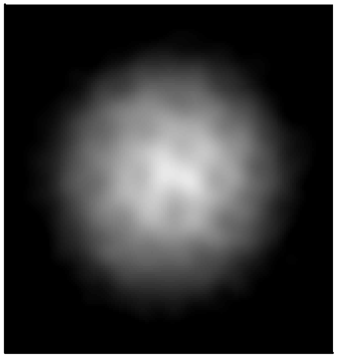
Monte Carlo studies $\rightarrow \ \mu \approx 0.4 E_F$
(Carlson et al, Giorgini/Astracharchik, etc.)
$T_c = 0.15 E_F$ UMASS-ETH

Theory $\rightarrow$ Nature of superfluid pairing, Transition temperature, Excitations

Experiments (JILA, MIT,Innsbruck, Duke,ENS)
Vortices (MIT)

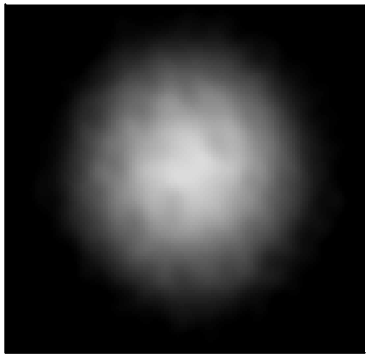
Vortex lattices

MIT, Zwierlein et al., Science 05

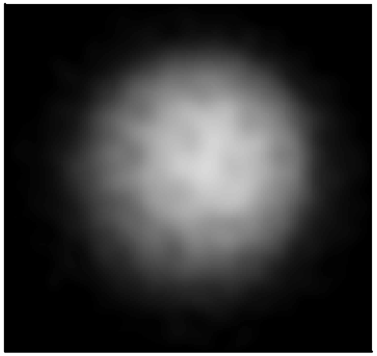


B_f = 835 G
1 / k_F a = 0

B_f = 843 G
1 / k_F a = -0.13



B_f = 854 G
1 / k_F a = -0.27

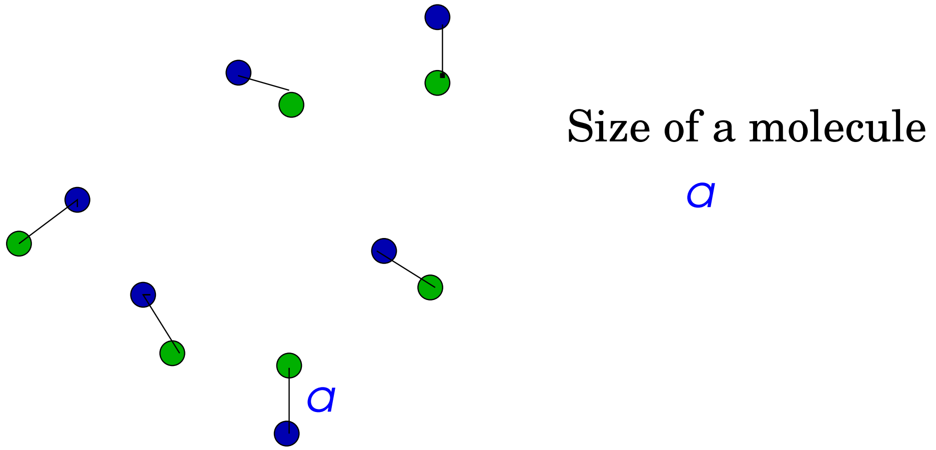


B_f = 864 G
1 / k_F a = -0.39

Direct proof of superfluidity !

Gas of bosonic molecules (dimers)

Region III $(a > 0) \Rightarrow$ gas of weakly bound bosonic molecules



$a \ll n^{-1/3}$ or $na^3 \ll 1 \quad \Rightarrow$ weakly interacting Bose gas

Interaction energy $E_{int} = \frac{N(N-1)}{2}\varepsilon_{int}$

$$\varepsilon_{int} = \frac{g}{V}; \qquad g = ?$$

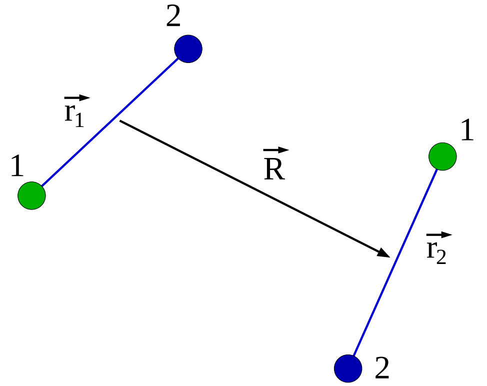
$g < 0 \;\Rightarrow$ collapse of a Bose-Einstein condensate

$g > 0 \;\Rightarrow$ stable BEC

Weakly interacting gas of bosonic dimers

Elastic interaction BEC stability "Old answer" $\rightarrow 2a$

4-body problem **Exact solution for** $a \gg R_e$ (Petrov et al 2003)



$\Psi \quad \rightarrow$ 9 variables

Zero-range approximation

$$\Psi_{r_1 \rightarrow 0} \;\rightarrow f(\vec{r}_2, \vec{R})(1/4\pi r_1 - 1/4\pi a)$$

Integral equation for f $k \rightarrow 0$ s-wave scattering; 3 variables

$$R \rightarrow \infty \quad \Psi = \phi_0(r_1)\phi_0(r_2)(1 - a_{dd}/R); \;\; \phi_0(r) = \frac{1}{\sqrt{2\pi a}}\exp(-r/a)$$

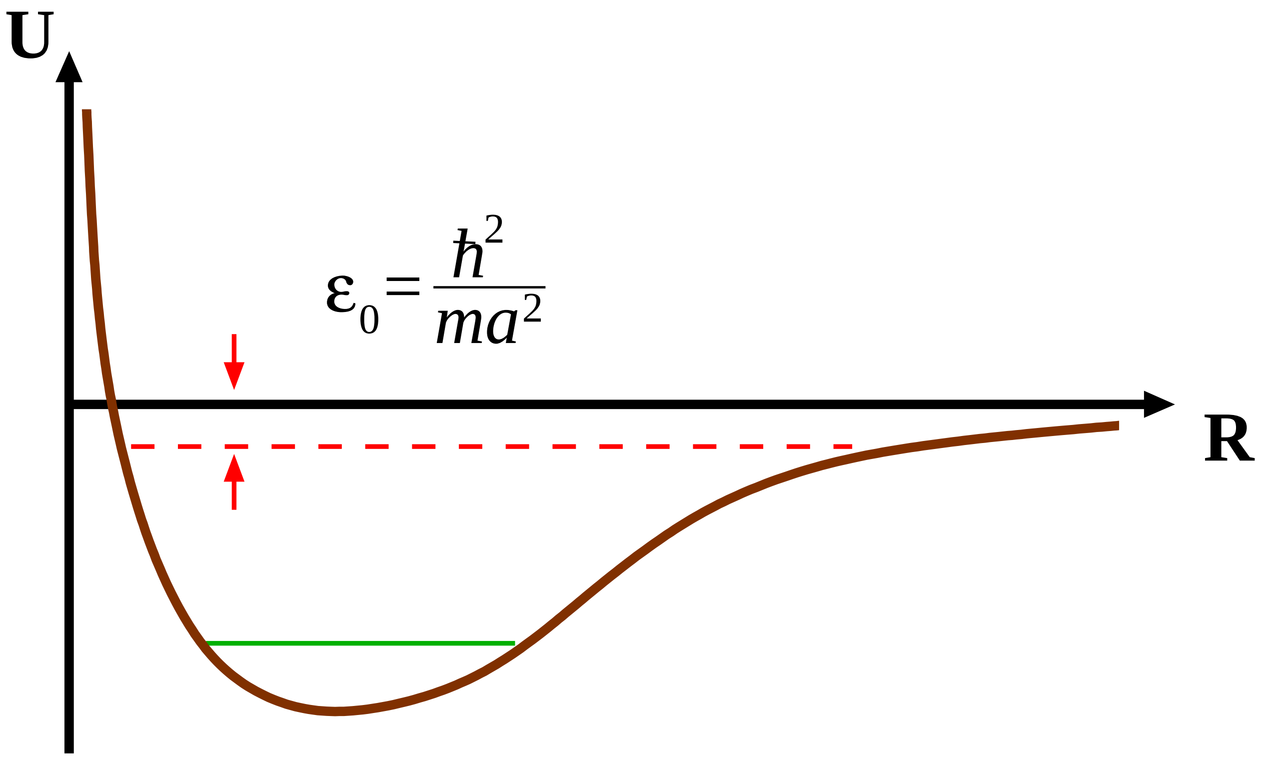
$$a_{dd} = 0.6a$$

Monte Carlo (Giorgini/Astracharchik, 2004)

Diagrammatic approach (M.Kagan et al,2005; Gurarie et al,2006)

Weakly bound dimers

Weakly bound dimers $\rightarrow$ The highest rovibrational state of the diatomic molecule



Collisional relaxation to deep bound states

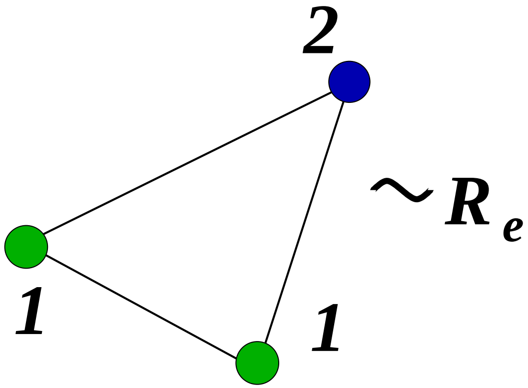
(~ 1ms for Rb_2 at $n \sim 10^{13}$cm^{-3})

Atom-dimer collisions

Weakly bound dimer $\sim a$

Size →

Deep bound state $\sim R_e$ (50 Å) $\ll a$

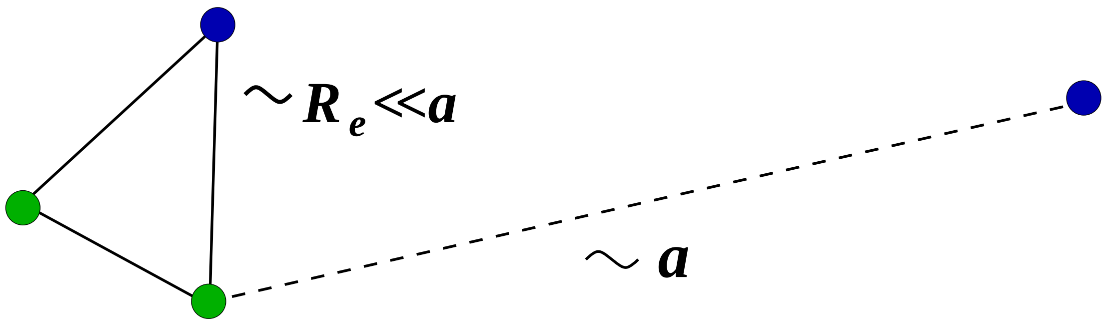


2 particles are identical fermions

Pauli principle

$$\alpha_{rel} \sim (k_{eff}R_e)^{2?} \sim (R_e/a)^{2?}$$

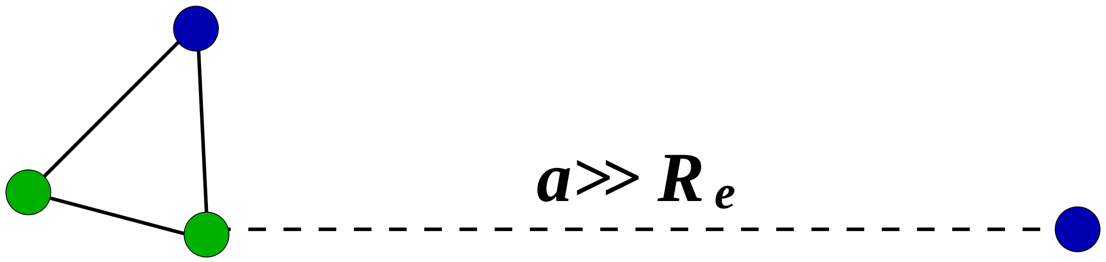
Molecule-molecule relaxation collisions



$$\alpha_{rel} = C \frac{\hbar R_e}{m} \left(\frac{R_e}{a} \right)^s ; \qquad s = 2.55$$

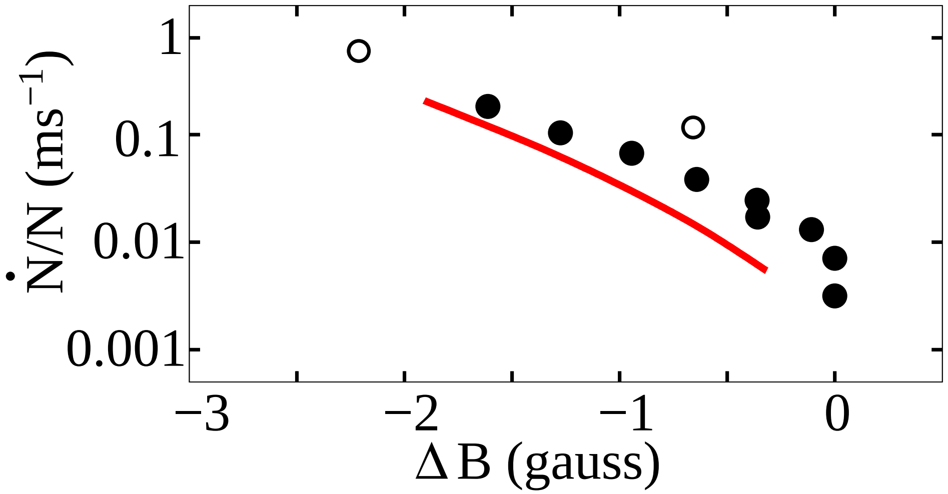
$\tau \sim (\alpha_{rel} n)^{-1} \sim$ seconds (Petrov et al 2003)

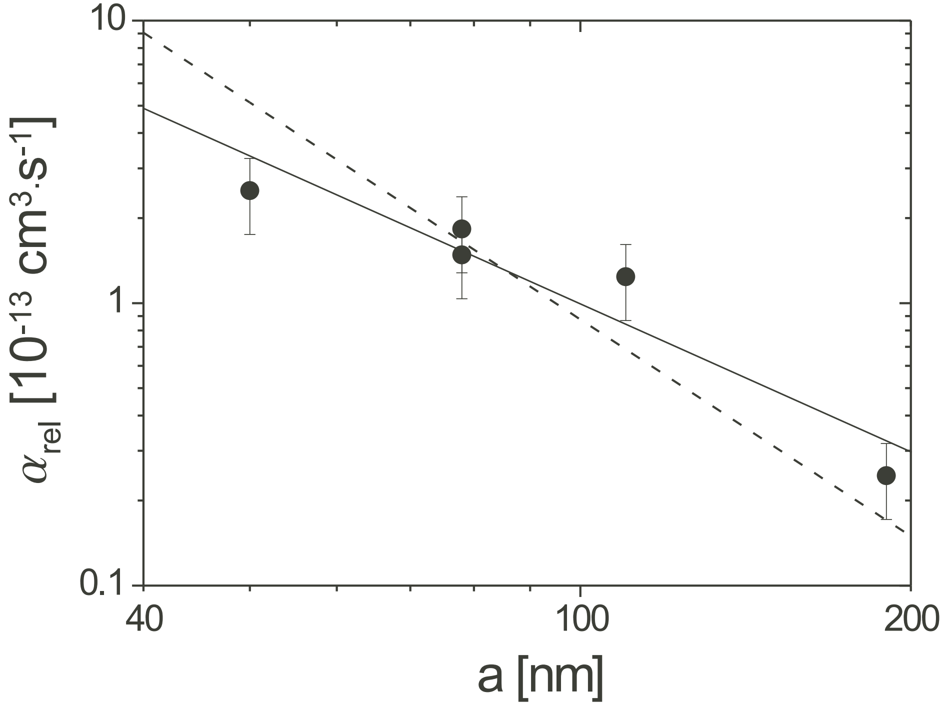
Molecules of bosonic atoms



Resonant enhancement $\alpha_{rel} \sim \hbar a/m$ $\tau < 1$ms

Suppressed collisional relaxation





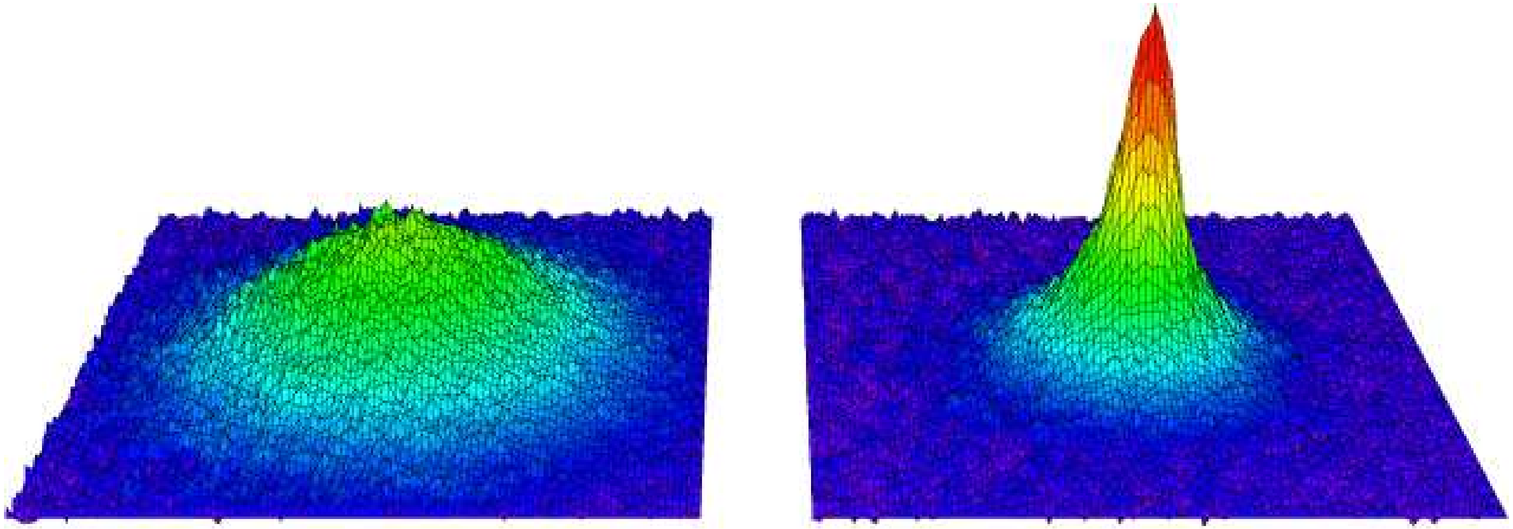
Bose-Einstein condensates of molecules

Suppressed relaxation **Fast elastic collisions** $a_{dd} = 0.6a$

$$^6\mathrm{Li}_2 \rightarrow \frac{\alpha_{rel}}{\alpha_{el}} \leq 10^{-4}$$

Efficient evaporative cooling → BEC

JILA, Innsbruck, MIT, ENS, Rice



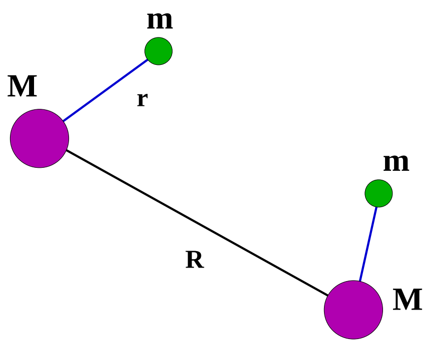
Molecules in Fermi-Fermi mixtures

$^{6}\mathrm{Li}^{40}\mathrm{K} \quad ^{6}\mathrm{Li}^{87}\mathrm{Sr}$

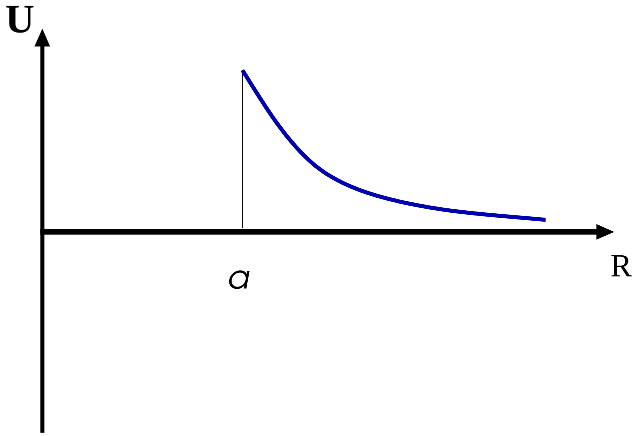
What happens with collisional stability and molecular BEC?

Molecules of heavy and light fermions Born-Oppenheimer picture

$\mathrm{R} \gg \mathrm{a}$



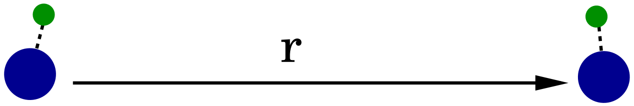
$$U(R) = 2\left(\frac{\hbar^2}{maR}\right)\exp(-2R/a)$$



$$P \sim \exp\left(-0.9\sqrt{\frac{M}{m}}\right)$$

$M >>> m \rightarrow$ Collisional stability independent of a

It is a gas?



$$U(r) = \frac{2\hbar^2}{ma^2(r/a)} \exp\left(-\frac{2r}{a}\right)$$

$$K = -\frac{\hbar^2}{2Ma^2}\Delta_{(r/a)} = -\frac{m}{M}\frac{\hbar^2}{2ma^2}\Delta_{(r/a)}$$

$$H = \sum_i K(r_i) + \frac{1}{2}\sum_{i\neq j} U(r_{ij}) \quad \Rightarrow \quad \frac{\hbar^2}{2ma^2}\sum_i \tilde{H}\left(\frac{r_i}{a}; \frac{M}{m}\right)$$

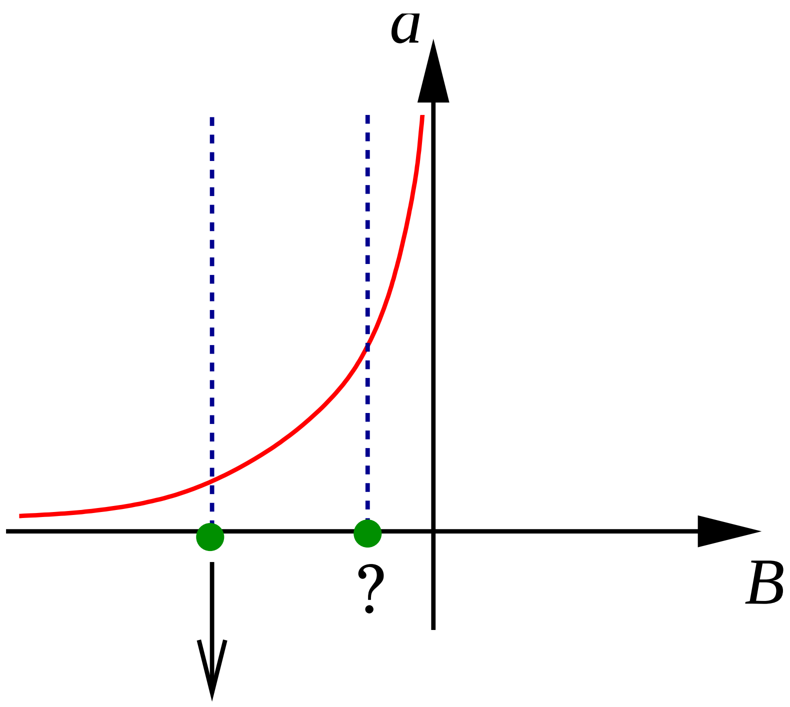
Large $\frac{M}{m}$ $\Rightarrow$ small $\frac{K}{U}$

$\rightarrow$ Wigner crystal for $\frac{M}{m} > \left(\frac{M}{m}\right)_c$

Quantum transitions

$$\frac{M}{m} > \left(\frac{M}{m}\right)_c \quad \text{and } n \text{ fixed}$$

Increase a



depends on $\frac{M}{m}$ but always $na^3 \ll 1$

first-order transition

Crystalline phase

$$\frac{M}{m} \approx 200$$

2D motion of heavy atoms $\longrightarrow$ triangular lattice

Binary approach $\rightarrow \left(\frac{a}{r_{min}}\right) \exp(-r_{min}/a) \ll 1 \quad r_{min} > 2.2a$

How to obtain the crystalline phase?

Optical lattice for heavy fermions

Small filling factor $\Rightarrow$ Increase of M/m

Formation of a superlattice

Crystalline phase

2D motion of light fermions

$$U(r) = 4[(\kappa r)K_0(\kappa r)K_1(\kappa r) - K_0^2(\kappa r)]$$

$\dfrac{\hbar^2\kappa^2}{2m}$ $\rightarrow$ binding energy of a light-heavy molecule

Binary approach $\rightarrow K_0(\kappa r_{min}) \ll 1 \quad \kappa r_{min} > 2.3$

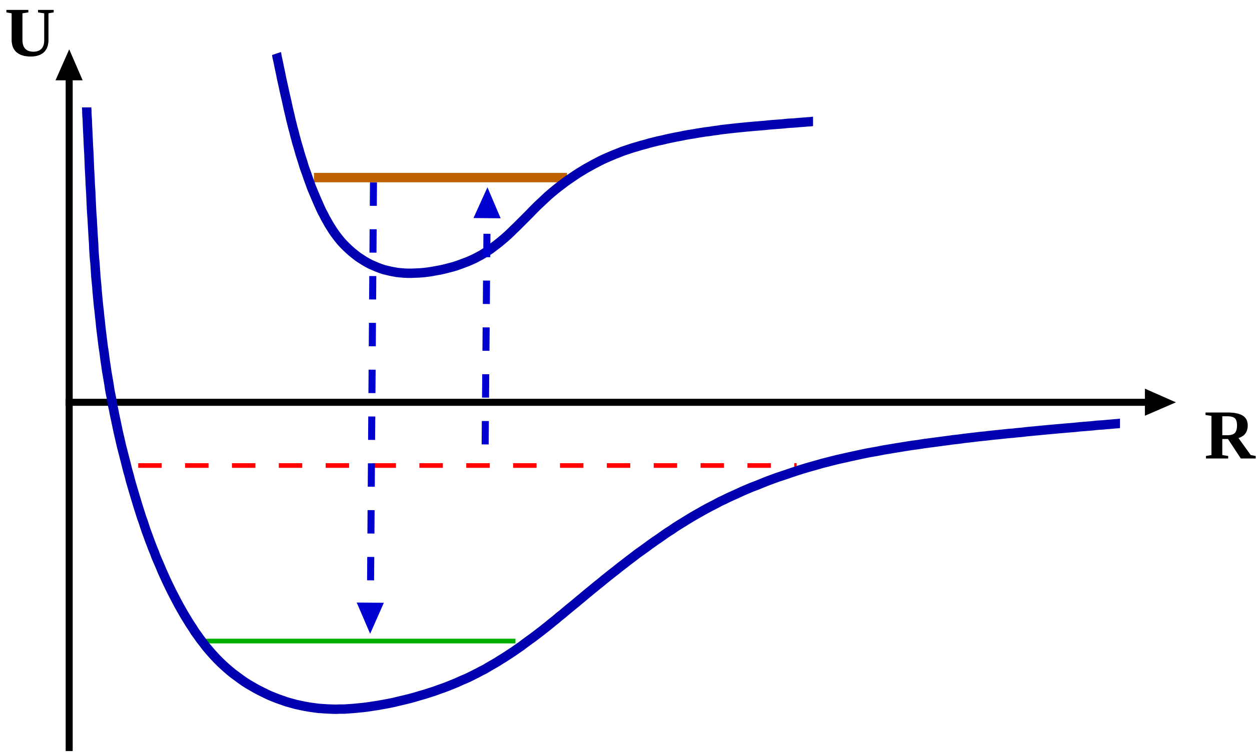
$\dfrac{M}{m} \approx 120$ Triangular lattice

Conclusions

- Remarkable physics of weakly bound molecules in cold Fermi gases
- Novel physics of molecular collisional stability in mixtures of Fermi gases
- Possibilities to create new macroscopic quantum systems

Ideas for future

Idea from Yalle studies of molecules of bosonic atoms



Replace bosons by fermions

Large n Large τ

$\rightarrow$ Dipolar gas