Quantum Criticality and Fermi Surfaces

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KITP, April 12, 2007

- Brief overview of heavy fermion quantum criticality
- Fermi surfaces of Kondo lattice
  - Kondo effect in the presence of AF order
  - Global phase diagram

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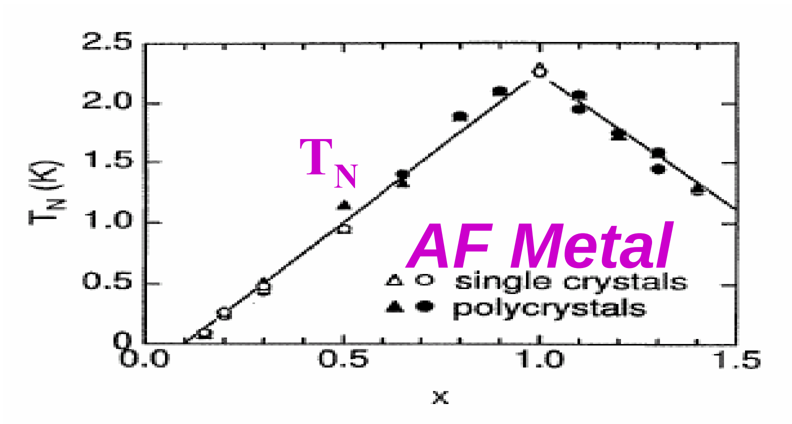
Heavy fermions near a magnetic QCP:

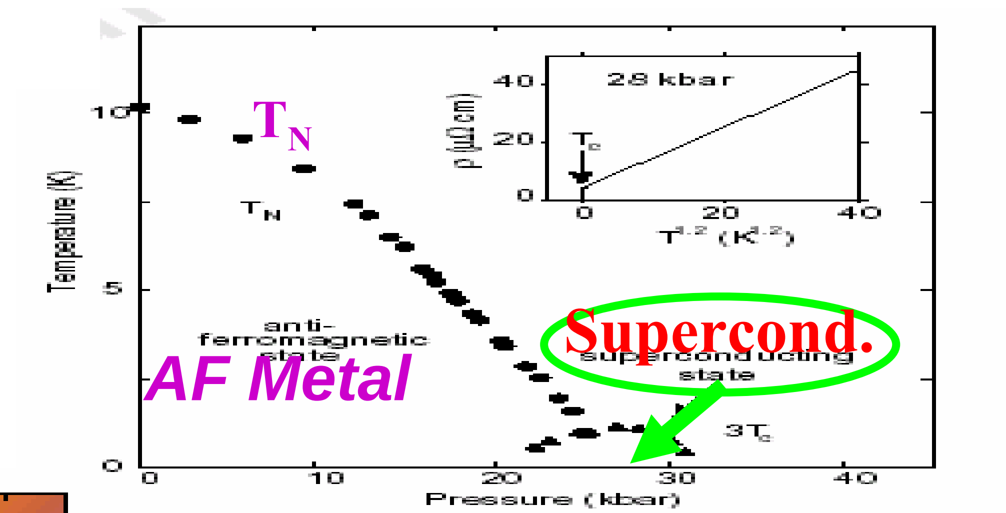
$CeCu_{6-x}Au_x$

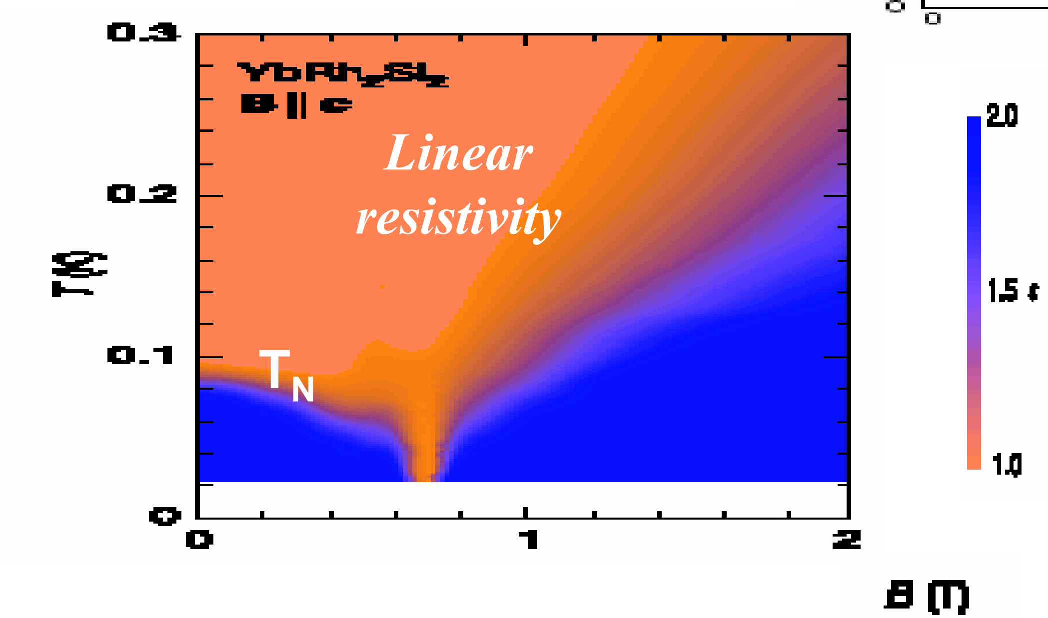
H. v. Löhneysen et al, PRL 1994



$CePd_2Si_2$

N. Mathur et al, Nature 1998





$YbRh_2Si_2$

J. Custers et al, Nature 2003

T=0 spin-density-wave transition

$$m(x,\tau)$$

order parameter fluctuations in space and (imaginary) time

$$d_{eff} = d + z > 4,$$

Gaussian

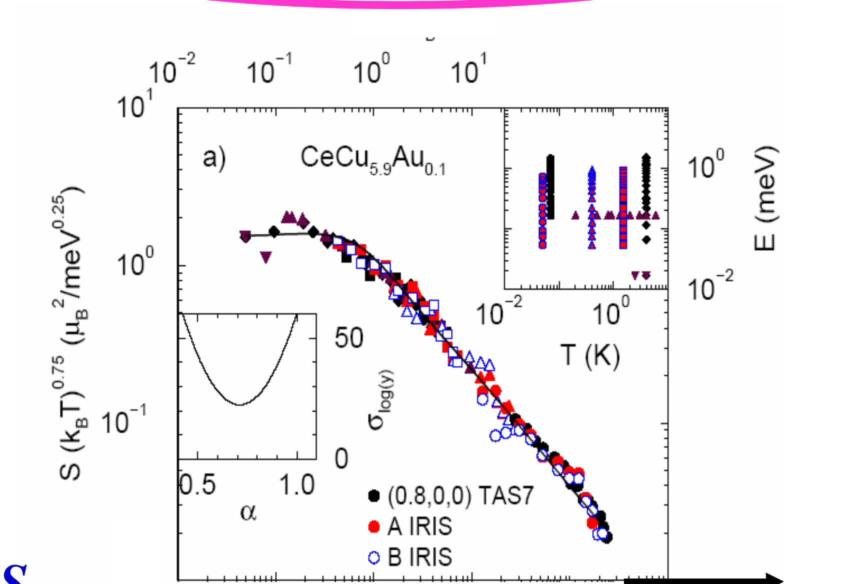
no $\frac{\omega}{T}$ *scaling*

MF exponent

Dynamical and Static Susceptibilities in $CeCu_{5.9}Au_{0.1}$

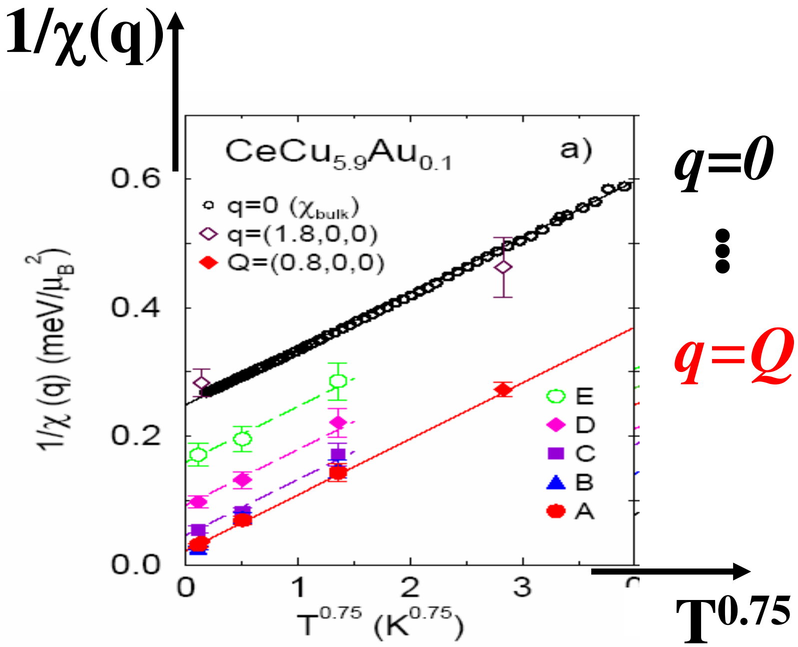
- ω/T scaling
- Fractional exponent $\alpha=0.75$

- $\alpha=0.75$ `everywhere' in **q**.



INS
@ q=Q

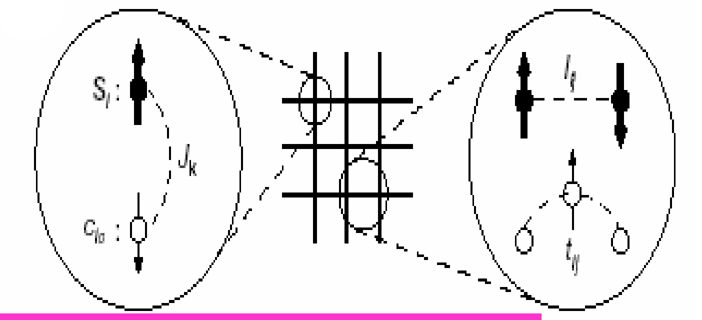
ω/T



INS and M/H

A. Schröder et al., Nature '00; PRL '98;
O. Stockert et al., PRL '98; M. Aronson et al., PRL '95

• Kondo lattices:



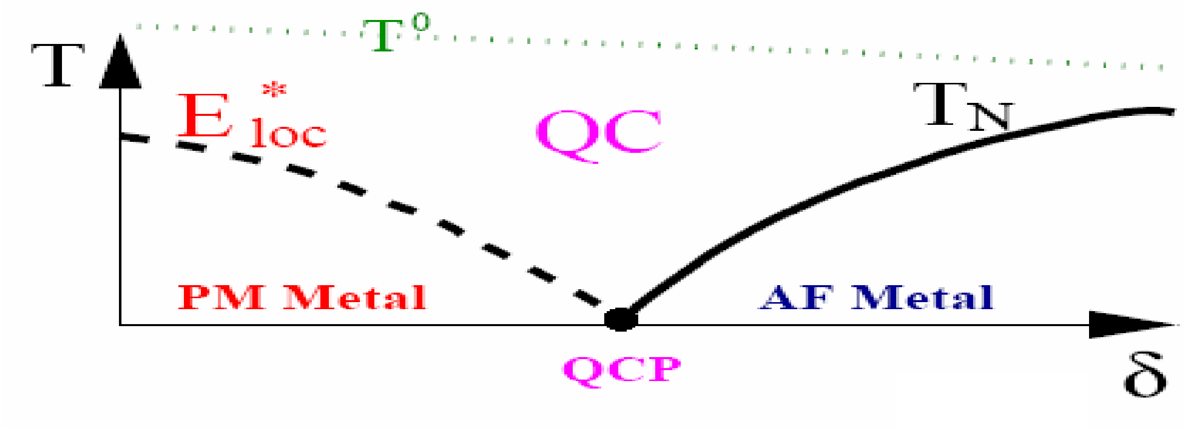
$$\mathcal{H} = \sum_{ij,a} I^a_{ij}\, S^a_i\, S^a_j + \sum_{ij,\sigma} t_{ij}\, \mathbf{c}^\dagger_{i\sigma}\, \mathbf{c}_{j\sigma} + \sum_{i,a} J^a_K\, S^a_i\, s^a_{c,i}$$

Historical development of heavy Fermi liquid:

Single-impurity: Anderson, Wilson, Nozières, Andrei, Wiegmann, Coleman, Read & Newns, …

Lattice: Varma, Doniach, Auerbach & Levin, Millis & Lee, Rice & Ueda, …

Local Quantum Critical Point

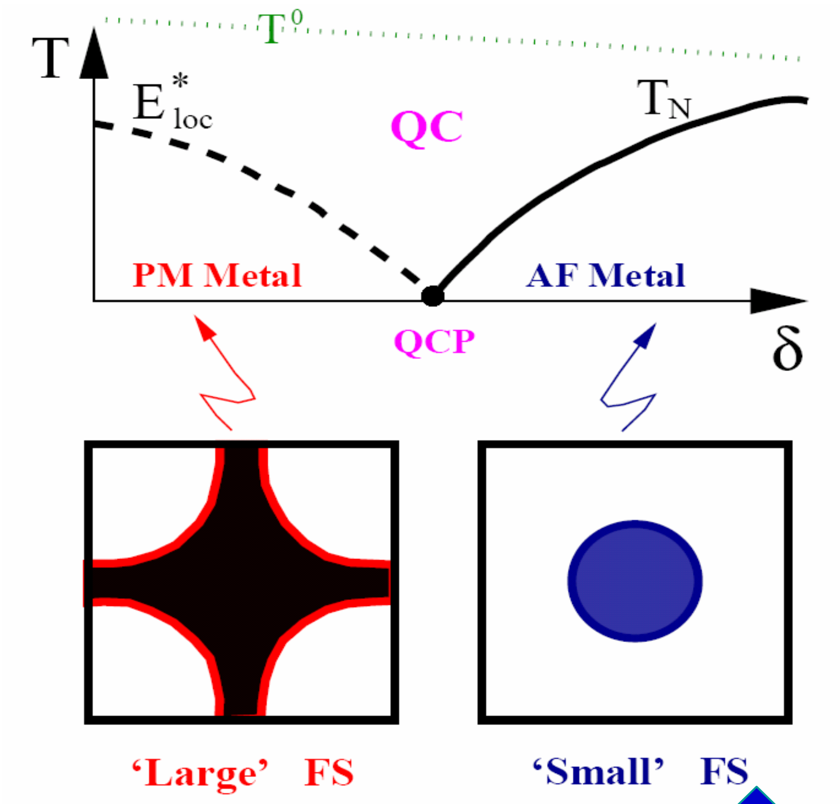


- **Destruction of Kondo screening:**
  energy scale E_{loc}* → 0 marks an electronic slowing down at the magnetic QCP

- **Spin damping: anomalous exponent and ω/T scaling**

- **Fermi surface jumps from "large" to "small"**

QS, S. Rabello, K. Ingersent, & J. L. Smith, Nature 413, 804 (2001); Phys. Rev. B68, 115103 (2003)



Fermi surface inside the antiferromagnetic part of the Kondo lattice phase diagram

Some Further Experiments

- **Fermi surface jump:**
  - Hall coeff.: a rapid crossover,
    → a jump at zero temperature, YbRh2Si2
    [Paschen et al, Nature 432, 881 (2004)]
  - dHvA: Fermi surface jump w/ divergent mass, CeRhIn5
    [Shishido et al, JPSJ 74, 1103 (2005); Park et al]
- **Multiple energy scales:**
  - thermodynamics (magnetostriction & magnetization) and transport, in YbRh2Si2
    [Gegenwart et al, Science 315, 969 (2007)]

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Kondo lattice

$$\mathcal{H} = \sum_{ij,a} I^a_{ij} S^a_i S^a_j + \sum_{ij,\sigma} t_{ij} \mathbf{c}^\dagger_{i\sigma} \mathbf{c}_{j\sigma} + \sum_{i,a} J^a_K S^a_i s^a_{c,i}$$

$$J_K << I_{rkky} << W$$

J_K=0 as the reference point of expansion

Local Moments:
Quantum non-Linear Sigma Model

Heisenberg model + coherent spin path integral ➡ QNLσM

$$\mathcal{A} = \mathcal{A}_{\mathrm{NL\sigma M}} + \mathcal{A}_{\mathrm{Berry}} + \mathcal{A}_K + \mathcal{A}_c$$

Haldane (1983)
Affleck (1985)
:

$$\mathcal{A}_{\mathrm{NL\sigma M}} = \frac{c}{2g}\int d\tau d^d x \left[(\nabla \vec{n}(\tau,\vec{x}))^2 + \frac{1}{c^2}\left(\frac{\partial \vec{n}(\tau,\vec{x})}{\partial \tau}\right)^2 \right]$$

Chakravarty, Halperin, Nelson
PRB 39, 2344 (1989)

$$\mathcal{A}_{\mathrm{Berry}} = i\sum_{\vec{x}} \int_0^1 du \int_0^\beta d\tau\, \vec{n}_{\vec{x}}(\tau,u) \cdot \left(\frac{\partial \vec{n}_{\vec{x}}(\tau,u)}{\partial u} \times \frac{\partial \vec{n}_{\vec{x}}(\tau,u)}{\partial \tau} \right)$$

Not important inside ordered phase

$$\mathcal{A}_c = \int d\omega d^d k\, \psi(\omega,\vec{k})(i\omega - \xi_{\vec{k}})\psi(\omega,\vec{k}) + \int \psi^4$$

Conduction electrons

$$\mathcal{A}_K = \lambda \int d\tau d^d x \left[\underbrace{\vec{s}_c(\tau,\vec{x})}_{\psi^\dagger \vec{\tau} \psi} \cdot \underbrace{\left(\vec{n}(\tau,\vec{x}) \times \frac{\partial \vec{n}(\tau,\vec{x})}{\partial \tau} \right)}_{\vec{\phi}} \right]$$

Effective fermi-magnon coupling

N.B.: $\vec{s}_c(\tau,\vec{x}) \cdot \vec{n}(\tau,\vec{x})$ coupling irrelevant (see next slide)

S. Yamamoto & QS, Cond-mat/0610001

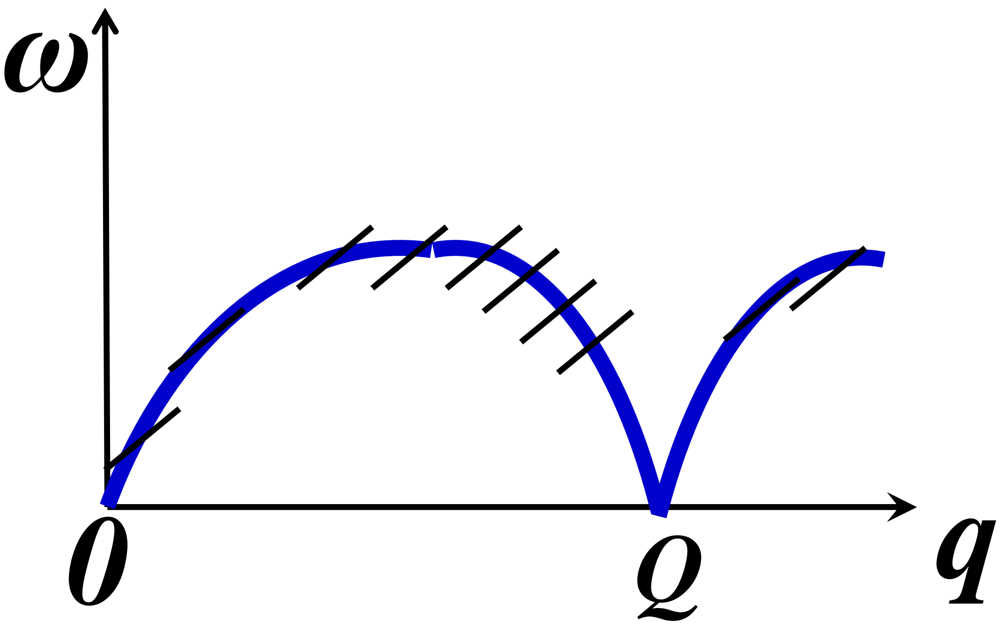
Kondo lattice

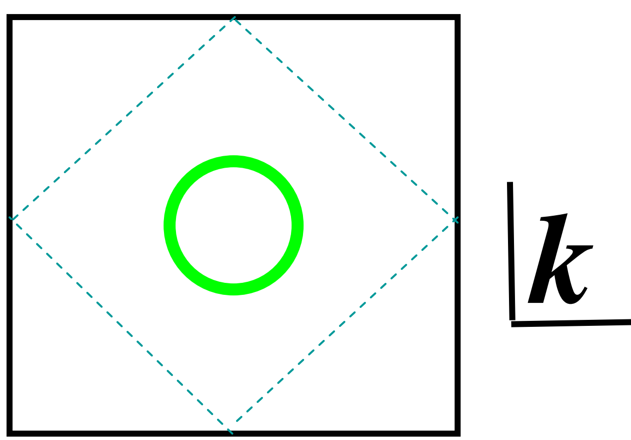
$$\mathcal{H} = \sum_{ij,a} I_{ij}^a S_i^a S_j^a + \sum_{ij,\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_{i,a} J_K^a S_i^a s_{c,i}^a$$

At J_K=0:

Local-moment antiferromagnetism

Conduction electrons





Effective Kondo coupling w/ magnons

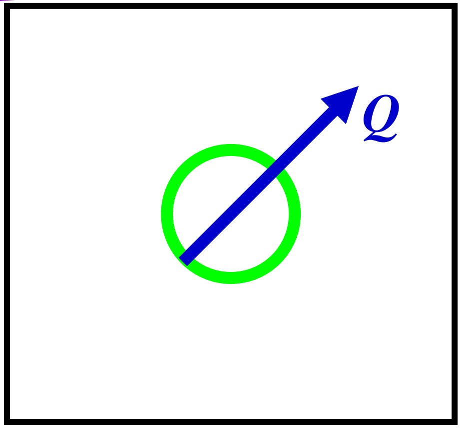
$$\mathcal{A}_K = \lambda \int d\tau d^d x \left[\vec{s}_c(\tau,\vec{x}) \cdot \left(\vec{n}(\tau,\vec{x}) \times \frac{\partial \vec{n}(\tau,\vec{x})}{\partial \tau} \right) \right]$$

with AF order

$$S(x,\tau) \approx m + n\, e^{i Q \bullet x}$$

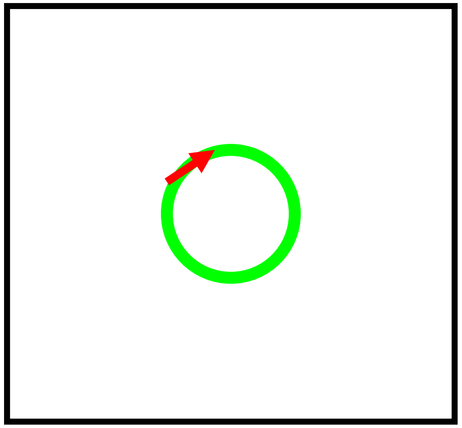
$q \approx Q$

Large momentum transfer scattering NOT allowed kinematically.



$q \approx 0$

Forward scattering ALLOWED. RELEVENT ???



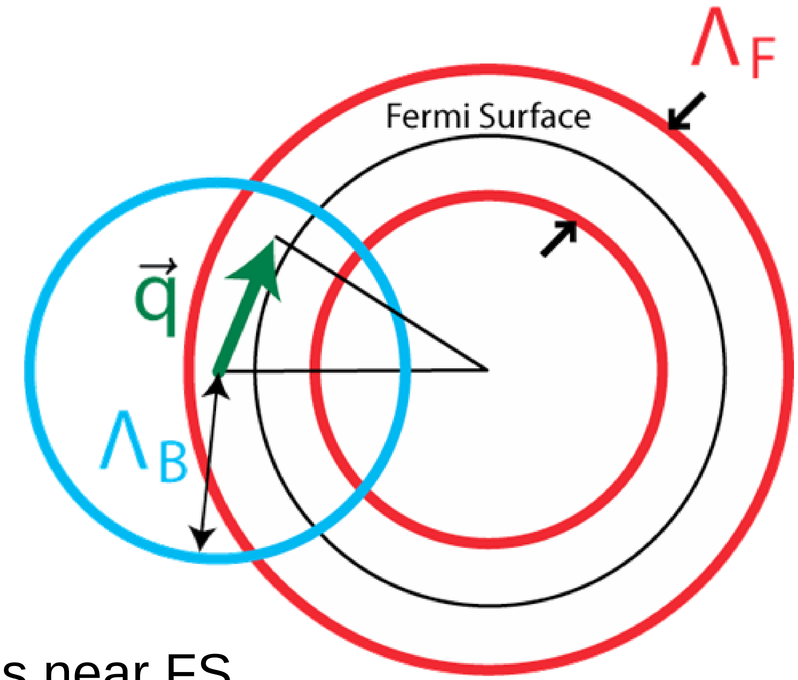
Combined bosonic/fermionic* RG

Tree-level scaling (*ferminoc RG: Shankar, RMP '94)

$$\mathcal{A}_K = \lambda \int \frac{d\omega}{e^l}\frac{d\varepsilon}{e^l}\frac{dk_\perp}{e^l}\frac{d^{d-1}k_\parallel}{e^0}\frac{dq_\perp}{e^l}\frac{d^{d-1}q_\parallel}{e^{(d-1)l}}\frac{\psi^\dagger_\sigma(\omega+\varepsilon,\vec{k}+\vec{q})}{e^{-3l/2}}\frac{\psi_{\sigma'}(\omega,\vec{k})}{e^{-3l/2}}\tau^a_{\sigma\sigma'}\frac{\phi^a(\varepsilon,\vec{q})}{e^{-dl}}$$

$$\frac{d\lambda}{dl} = 0 + O(\lambda^2)$$

marginal!

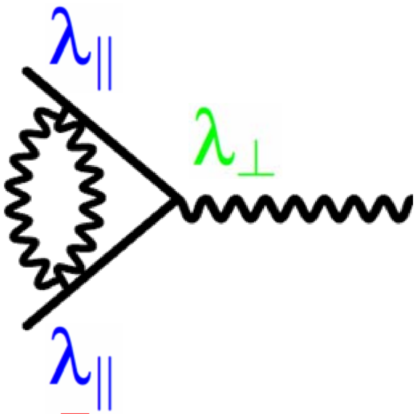


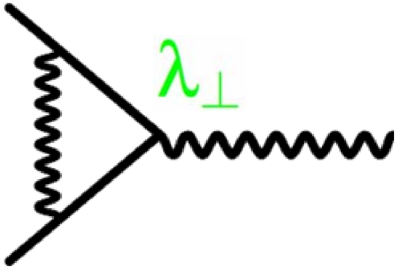
$[\vec{\phi}(\varepsilon,\vec{q})] = -d$ ← From NLσM

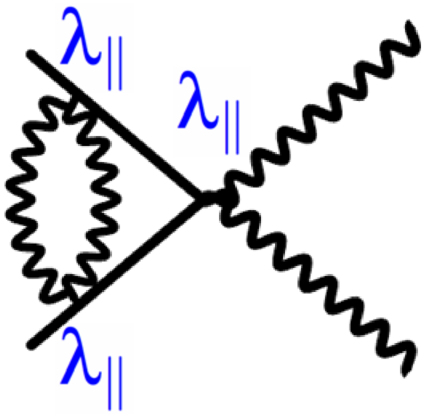
$[\psi(\omega,\vec{k})] = -3/2$ ← From electrons near FS

note: remember $\vec{q} \approx 0$ for forward scattering

RG at 1-loop & beyond:

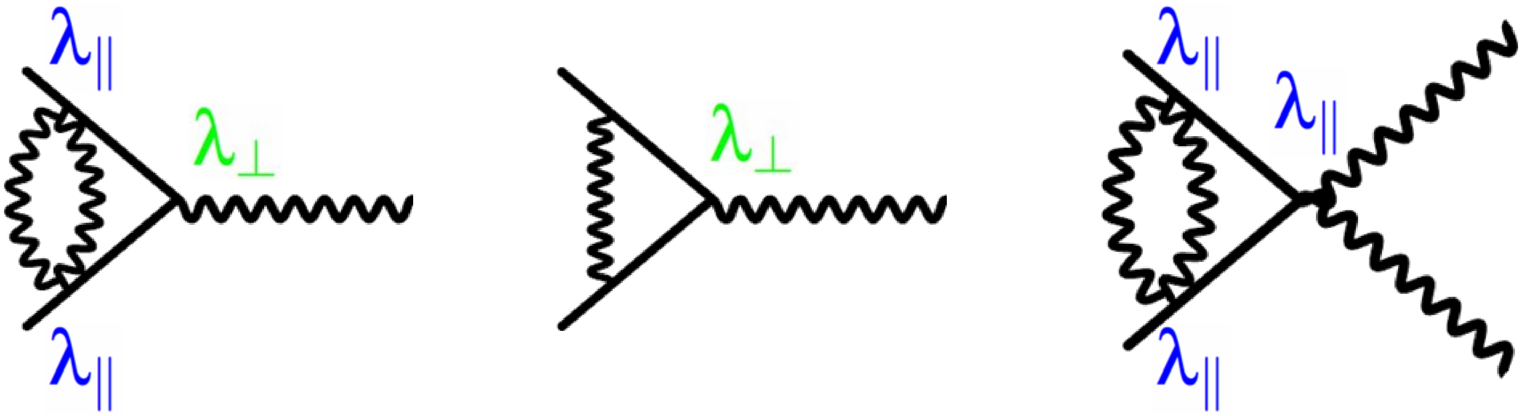






Marginal even at 1-Loop and beyond.

RG at 1-loop & beyond:



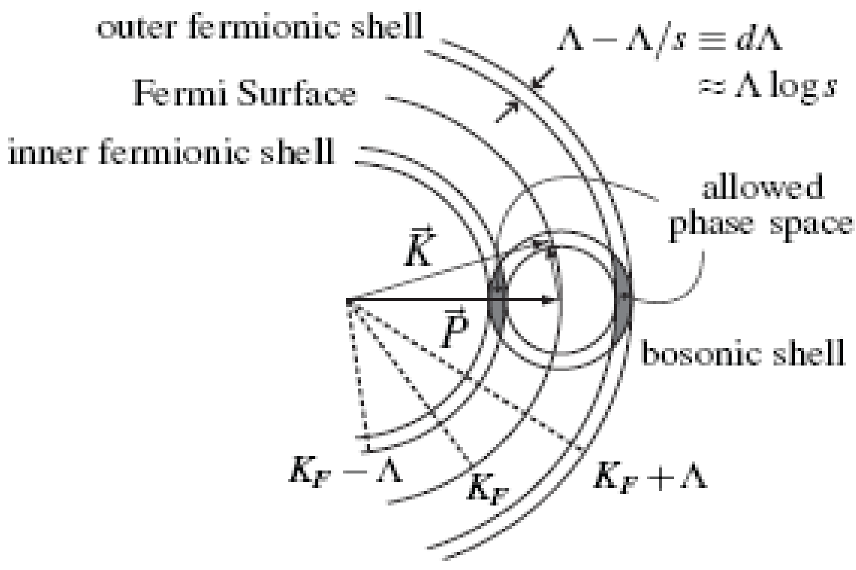
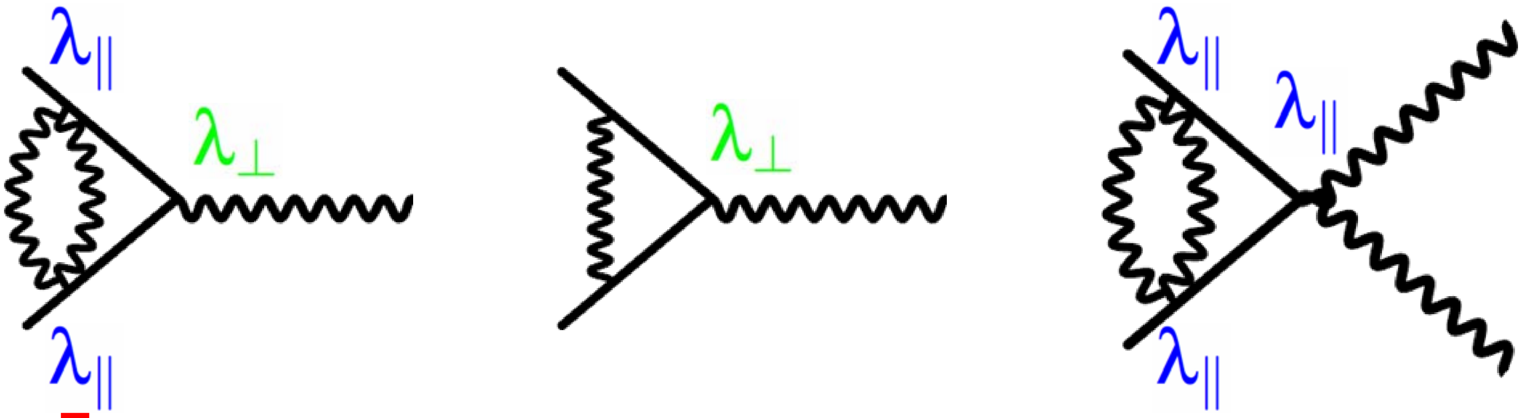


FIG. 4: Kinematics for the momentum-shell RG. Only the shaded region is integrated over.

RG at 1-loop & beyond:



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AF phase w/ “small” Fermi surface!

S. Yamamoto & QS, Cond-mat/0610001

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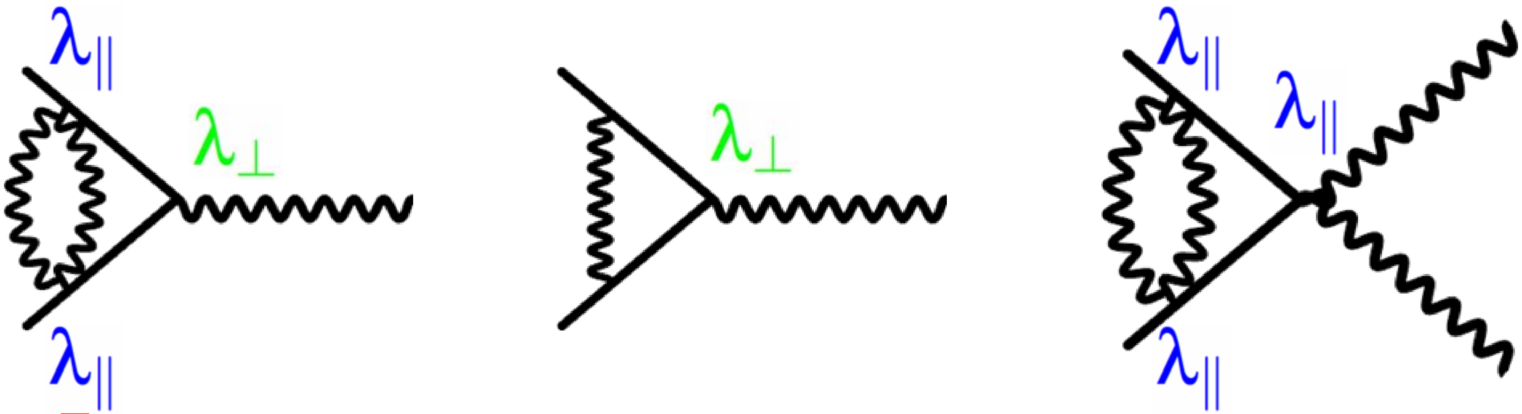

Marginal even at 1-Loop and beyond.

Large N:

$$\Sigma(\vec{k}, \omega) = $$

$$\sim \quad \omega^d$$

No pole in self energy.
Fermi surface remains "small".

AF phase w/ "small"
Fermi surface!

S. Yamamoto & QS,
Cond-mat/0610001

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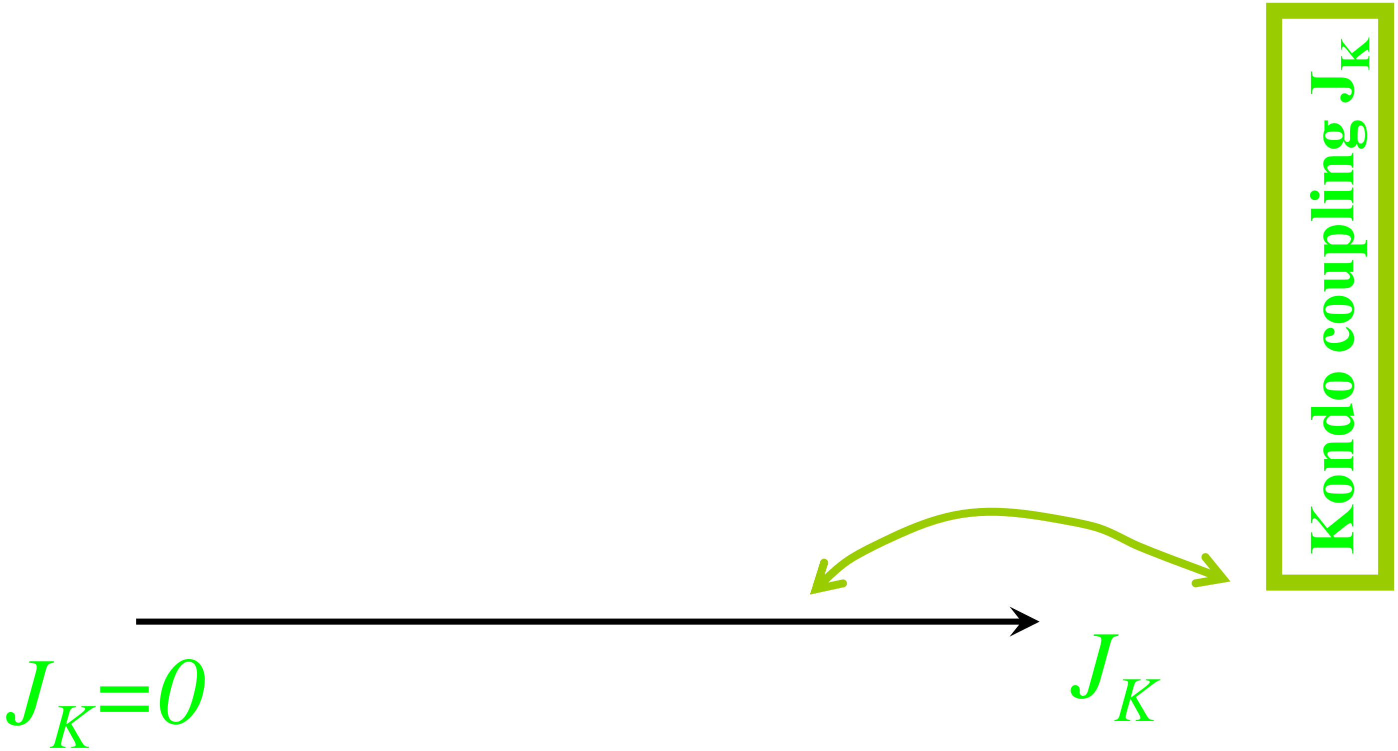
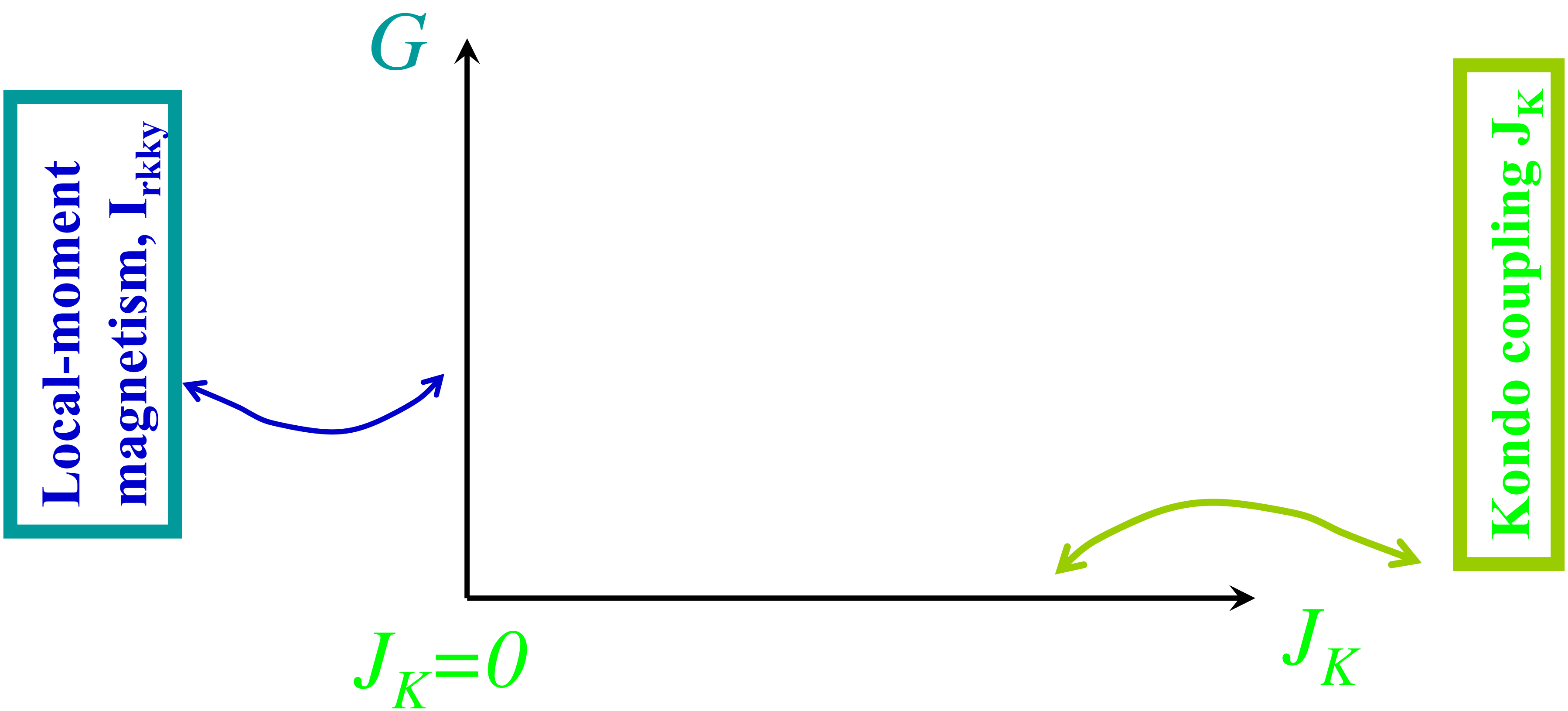


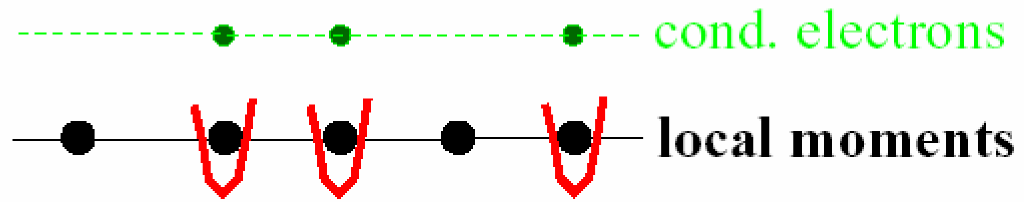
Kondo lattice

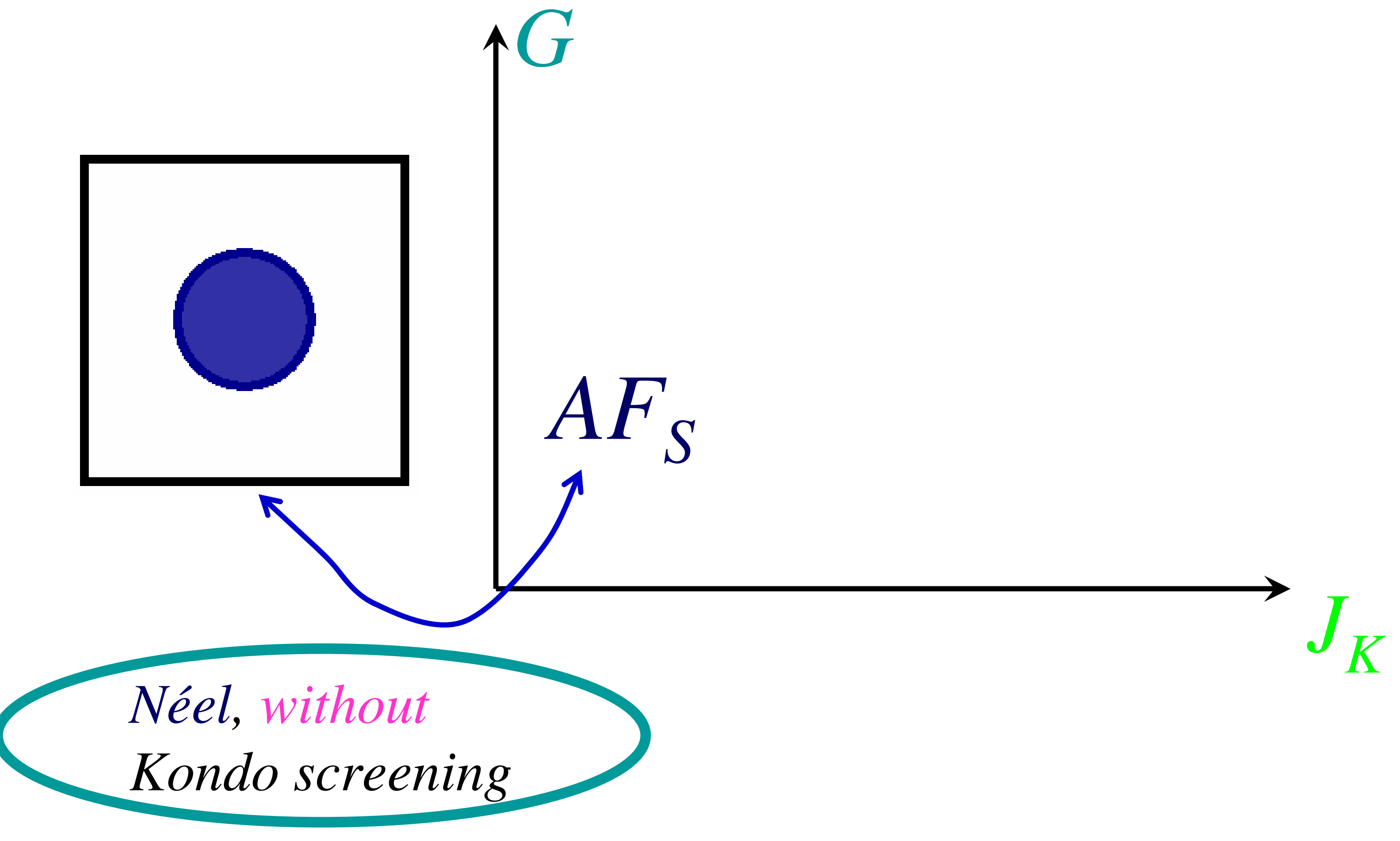
$$\mathcal{H} = \sum_{ij,a} I^a_{ij}\, S^a_i\, S^a_j + \sum_{ij,\sigma} t_{ij}\, \mathbf{c}^\dagger_{i\sigma}\, \mathbf{c}_{j\sigma} + \sum_{i,a} J^a_K\, S^a_i\, s^a_{c,i}$$

G ~ frustration, reduced dimensionality, etc.

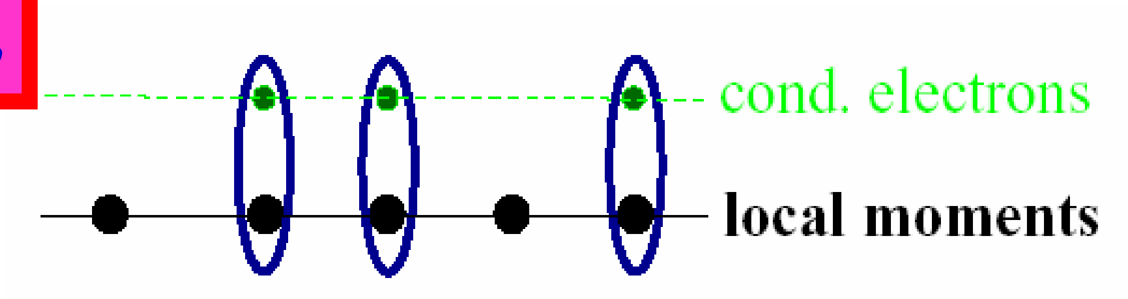


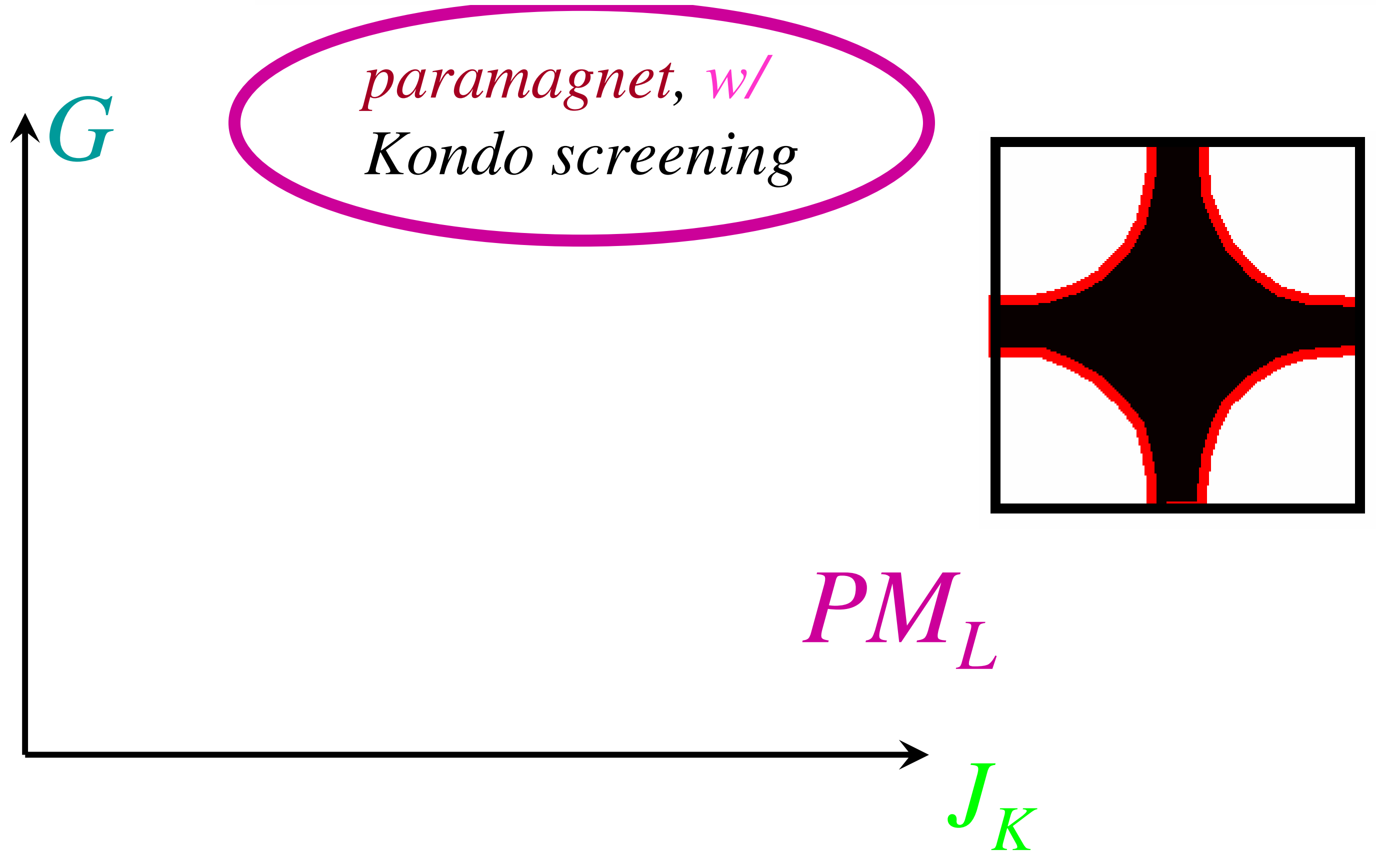
$J_K \ll I_{rkky} \ll W$





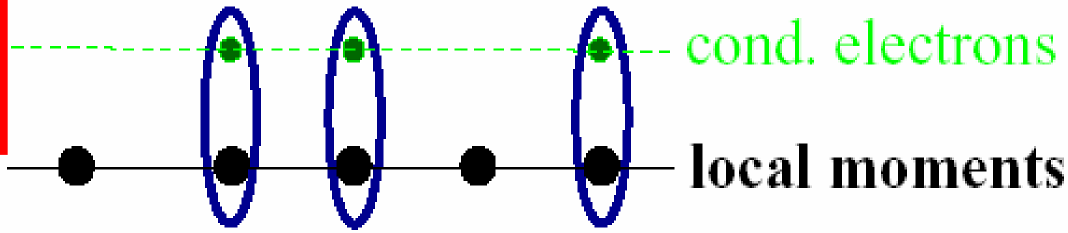
$J_K >> W >> I_{rkky}$





Cf. A. C. Hewson, The Kondo Problem to Heavy Fermions
(Cambridge Univ. Press, 1993)

$J_K >> W >> I_{rkky}$



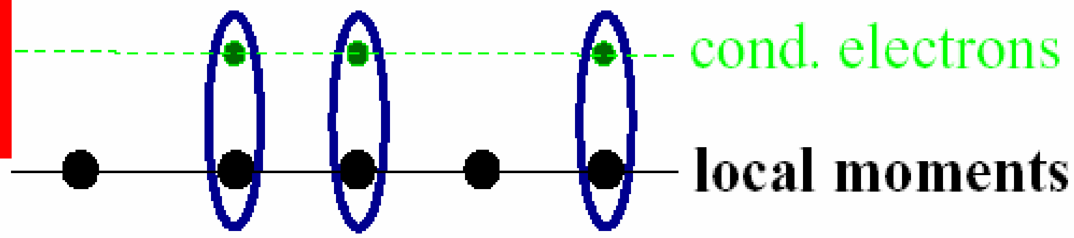
- xN_{site} **tightly bound local singlets**

$$|s>_i = \frac{1}{\sqrt{2}}\left(|\uparrow>_f|\downarrow>_c - |\downarrow>_f|\uparrow>_c\right)_i$$

(cf. If x were =1, Kondo insulator)

- $(1\text{-}x)N_{site}$ **lone moments:**

$J_K >> W >> I_{rkky}$



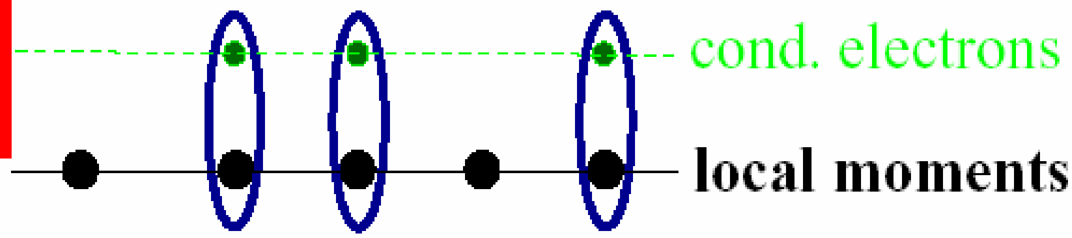
- ***xN_{site}* tightly bound local singlets**

$$|s>_i = \frac{1}{\sqrt{2}}\left(|\uparrow>_f |\downarrow>_c - |\downarrow>_f |\uparrow>_c\right)_i$$

(cf. If x were =1, Kondo insulator)

- ***(1-x)N_{site}* lone moments:**
  - **projection:** $$|\text{lone moment}>_{i,\sigma} = \left(-\sqrt{2}\sigma\right) c_{i,\bar{\sigma}} |s>_i$$
  - **(1-x)N_{site} holes with U=∞**

$J_K >> W >> I_{rkky}$



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- $(1\text{-}x)N_{site}$ **lone moments:**
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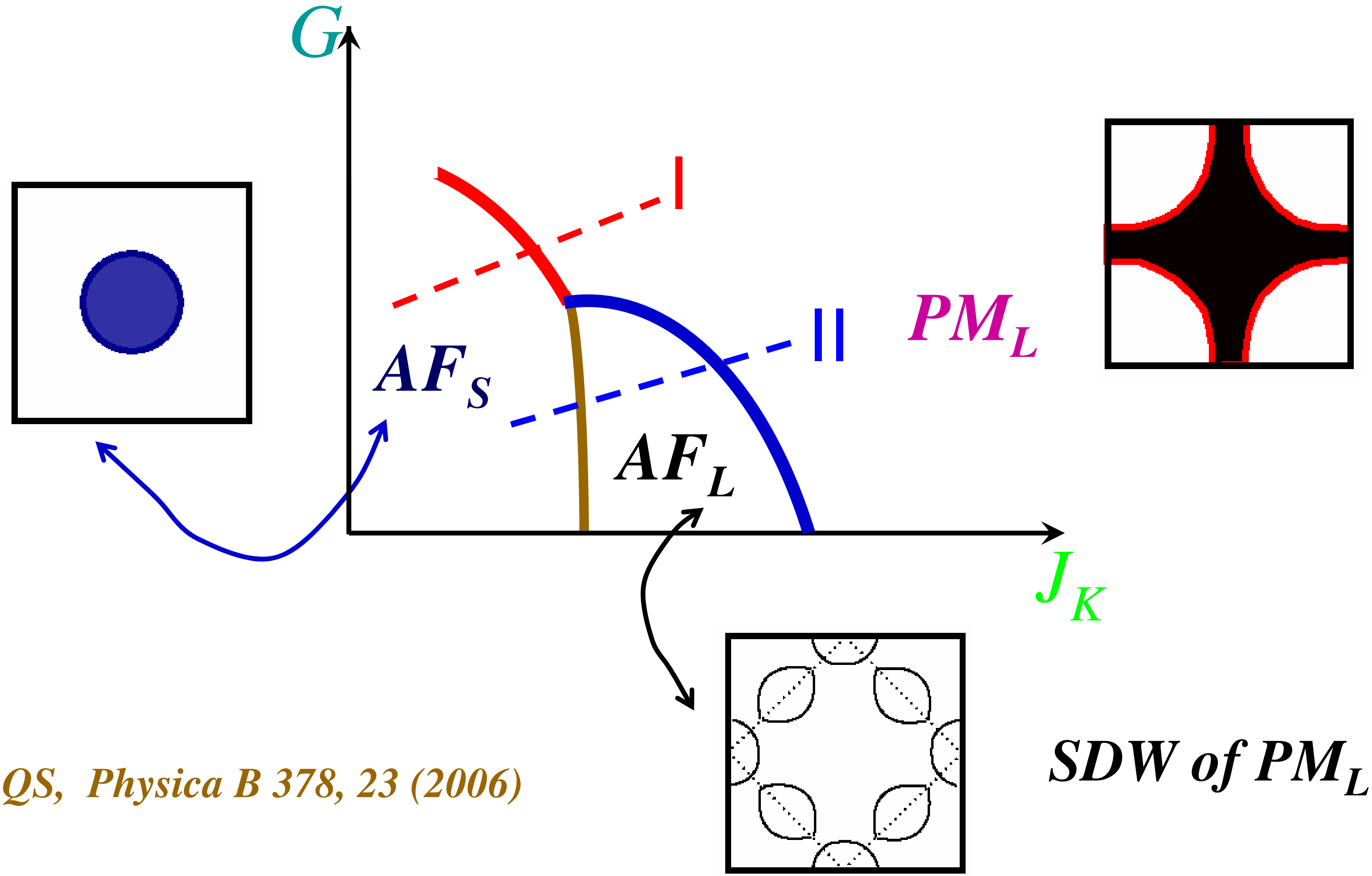
  - **$(1\text{-}x)N_{site}$ holes with $U=\infty$**

- **Luttinger's theorem:**

(1-x) holes/site in the Fermi surface

⟺ **(1+x) electrons/site ---- Large Fermi surface!**

Global phase diagram



SDW of PM_L

QS, Physica B 378, 23 (2006)

Type II transition

Hertz-Moriya-Millis fixed point for T=0 SDW transition

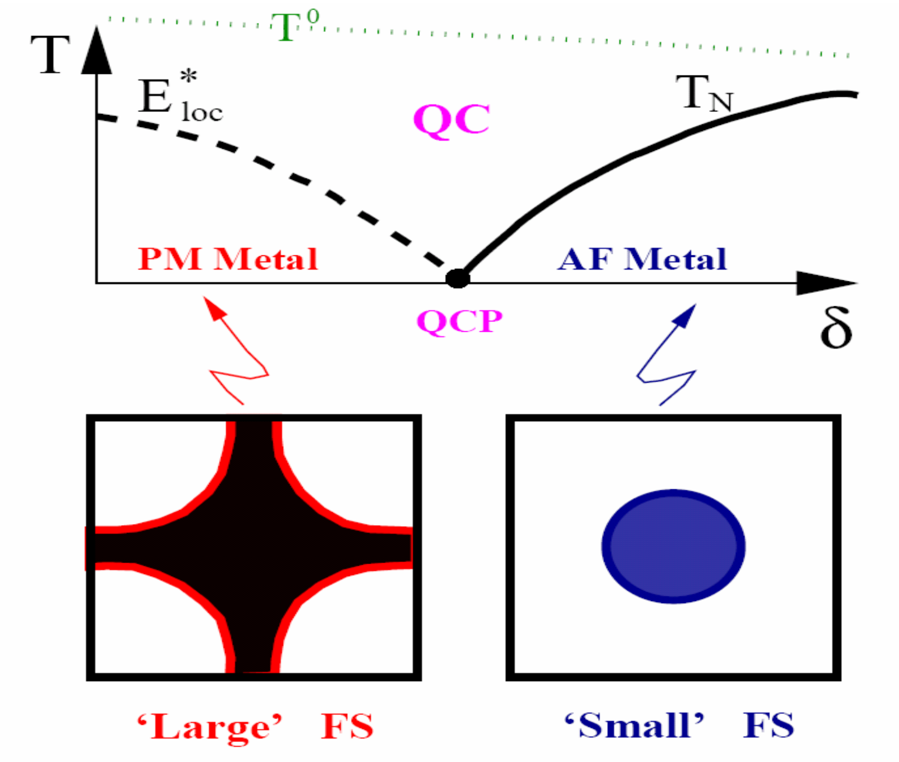
Type I transition

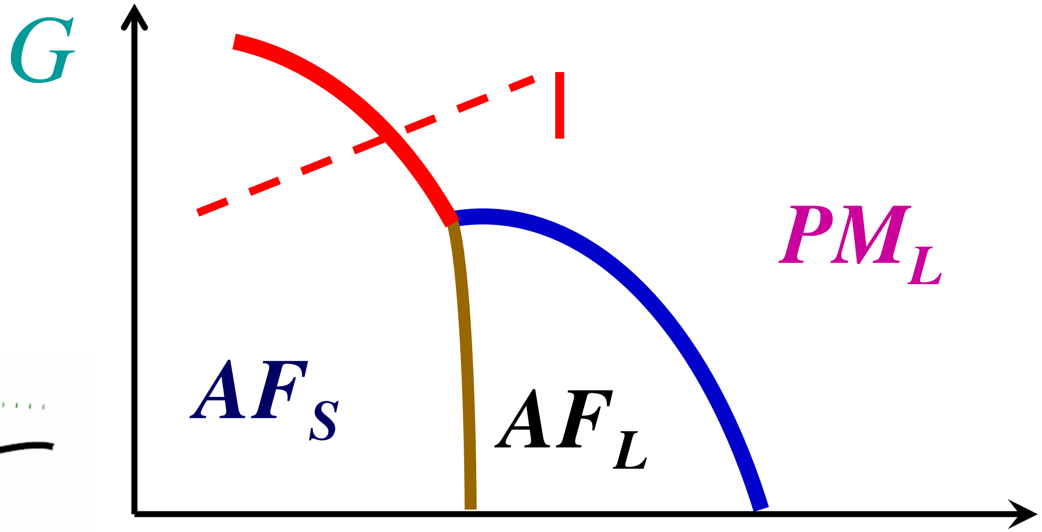
Second order if

$$\begin{cases} z_L \longrightarrow 0 \quad \text{as} \quad \delta \longrightarrow \delta_c^+ \\ z_S \longrightarrow 0 \quad \text{as} \quad \delta \longrightarrow \delta_c^- \end{cases}$$

Destruction of Kondo screening where magnetism sets in

Type I quantum transition (cont'd)





SUMMARY

- **Quantum criticality**
  - **heavy fermion metals as prototype**
  - non-Fermi liquid behavior
  - beyond the order parameter fluctuation picture

- **Antiferromagnetic metal with "small" Fermi Surface**
  [An asymptotically exact solution in d>1 dimensions]

- **Global phase diagram**

- **Evidence for AF_S and PM_L phases, and direct transition between them: dHvA and Hall effect**