Topology and Statistics in Superconductors : Non-Abelian Matter and Topological Quantum Computation

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References

- S. Das Sarma, C. Nayak, S. Tewari, Phys. Rev. B (Rapid Comm.) (2006)
- S. Tewari, S. Das Sarma, C. Nayak, C. W. Zhang, P. Zoller, Phys. Rev. Lett. (2007)
- M. Oshikawa, Y. B. Kim, K. Shtengel, C. Nayak, S. Tewari, Annals of Physics (2006)
- S. Tewari, S. Das Sarma, D. H. Lee, arxiv-cond-mat / 0609556
- S. Tewari, C. W. Zhang, S. Das Sarma, D. H. Lee, C. Nayak, in preparation
- Charles Day, Physics Today (Search and Discovery), Dec., 2006

Overview

Motivation and background

Non-locality, degeneracy, non-Abelian statistics

Vortices in 2D p-wave SC : Majorana fermions

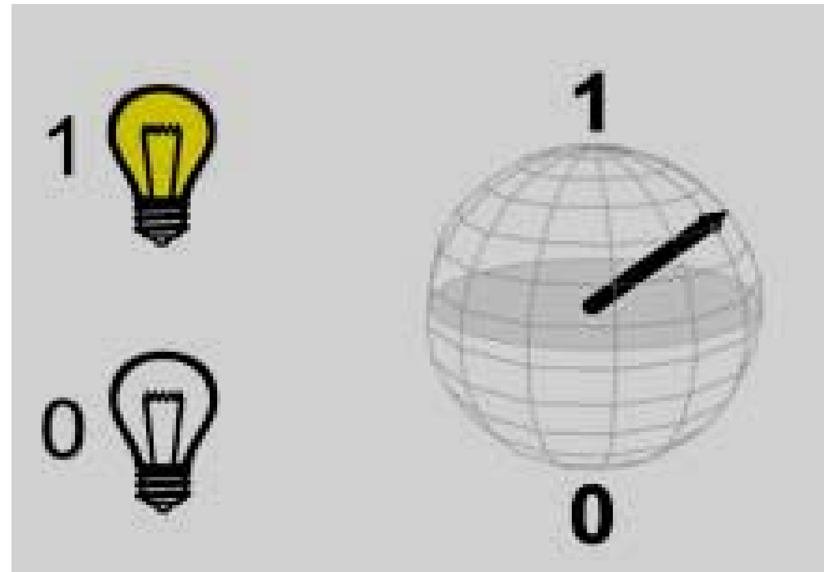
Non-locality, degeneracy, non-Abelian statistics

Applications and experiments

Contrast Classical and Quantum Bits

Classical bit has two states :

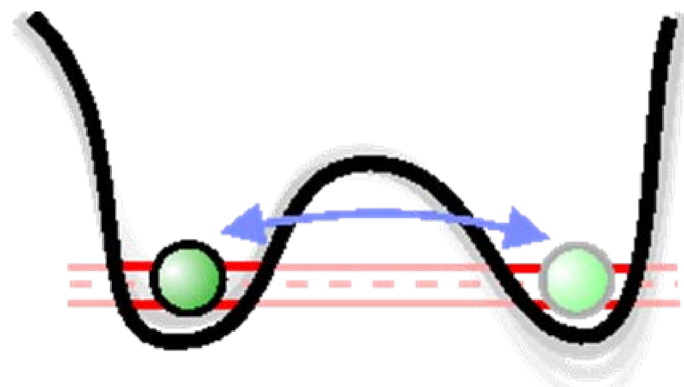
$|0>$ and $|1>$



Quantum bit is described by the state :

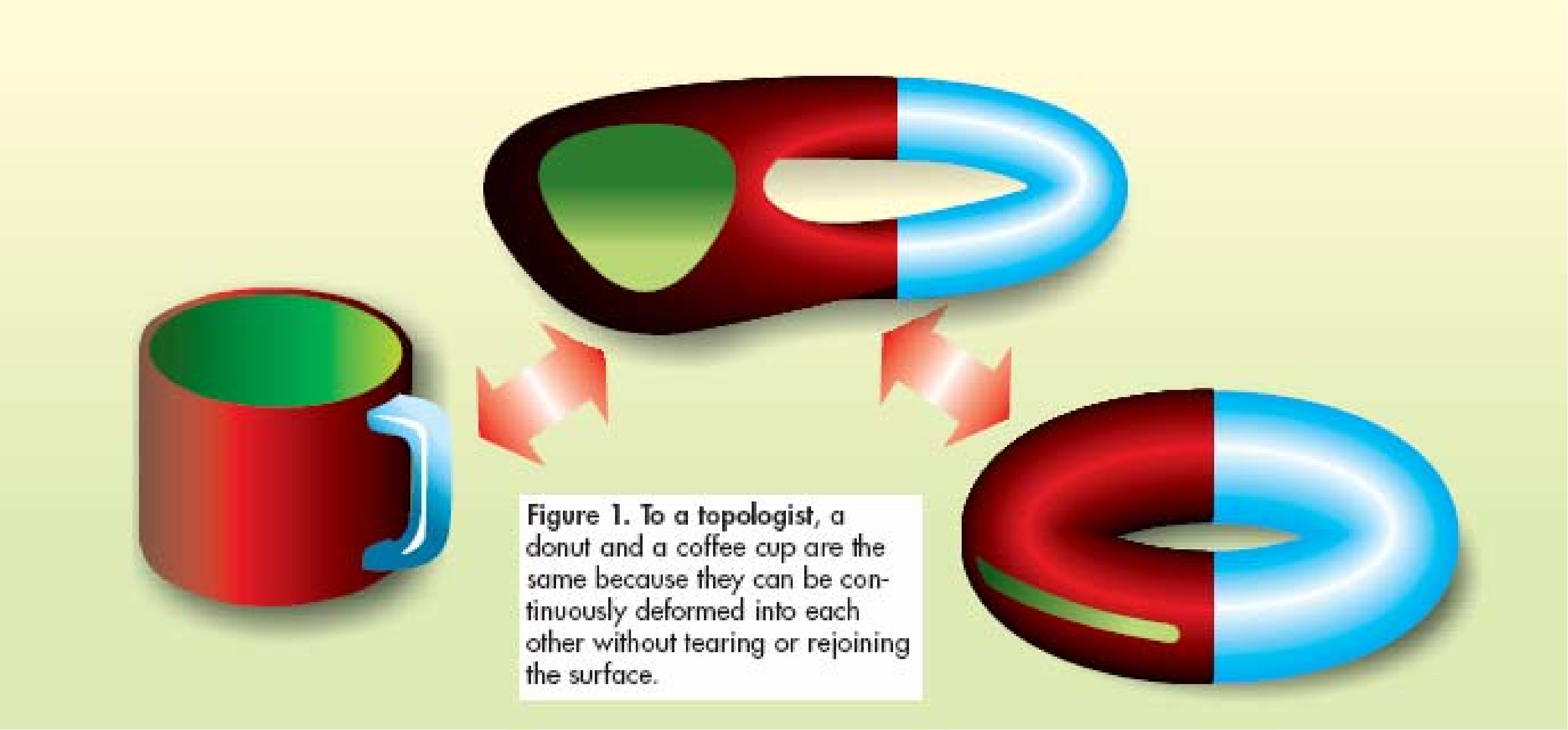
$$|\psi>=\alpha_0|0>+\alpha_1|1>$$

Environment decoheres the Quantum state



TQC: Look for a quantum state sensitive only to topology

Topology : A Global Property

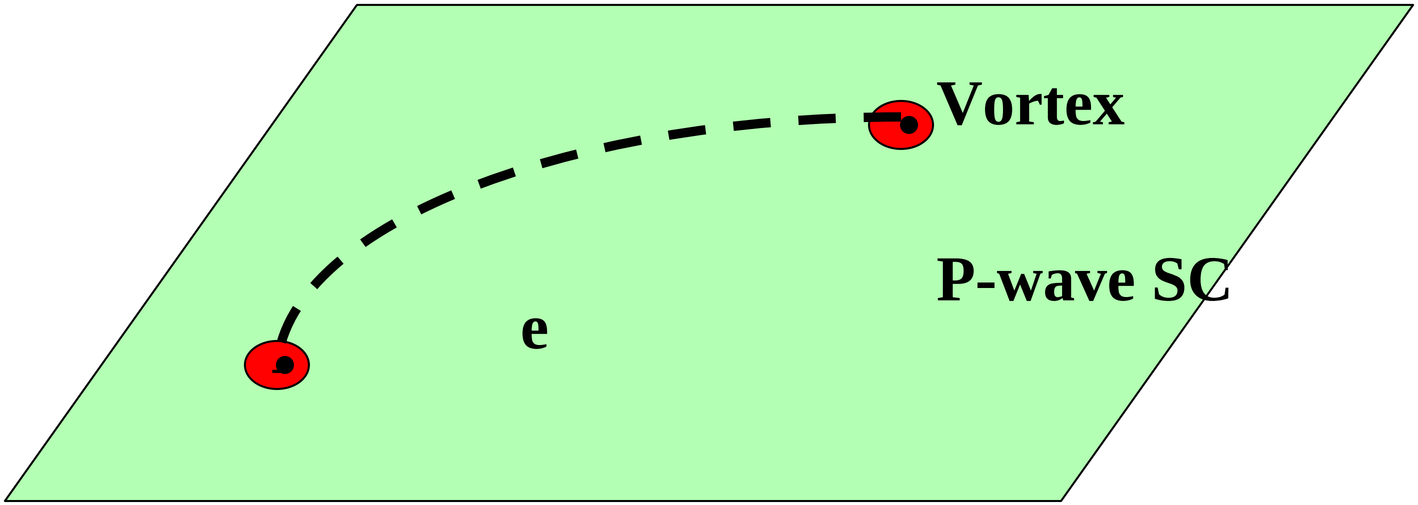


Look for a many-body quantum state sensitive only to the topology

Quasiparticles in $(\nu = 5/2)$ FQH system

Quasiparticles in vortex state of 2D p-wave superconductor

Non-Local Occupation of an Electron

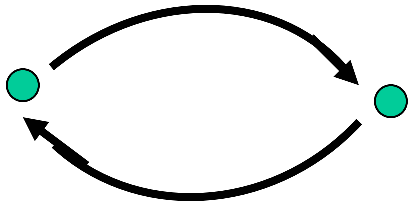


It takes a *pair* of quantum states to accommodate an electron!

Non-locality

Statistics

What happens to a many-particle wavefunction under exchange of identical particles



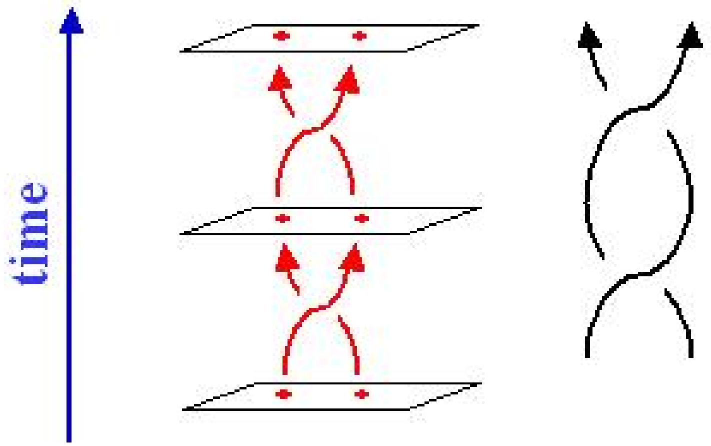
Naive Expectation

Exchanging twice should be identity

Bosons : $\psi(r_1, r_2, r_i) = \psi(r_2, r_1, r_i)$

Fermions : $\psi(r_1, r_2, r_i) = -\psi(r_2, r_1, r_i)$

In 2+1 Dimensions: Two Exchanges $\neq$ Identity

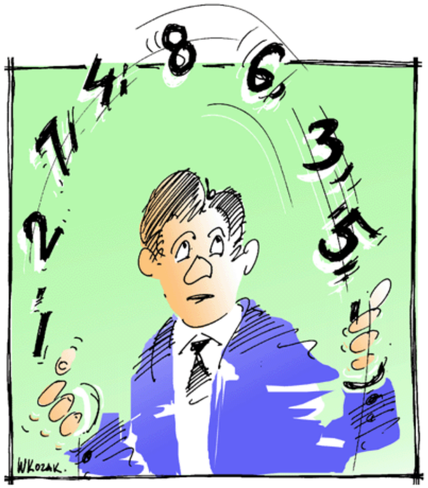


In 3+1 Dimensions: Two Exchanges = Identity

No Knots in World Lines in 3+1 D !

Slide taken from S. Simon's talk, KITP, 2004

Statistics



Bosons : $\psi(r_1, r_2, r_i) = \psi(r_2, r_1, r_i)$

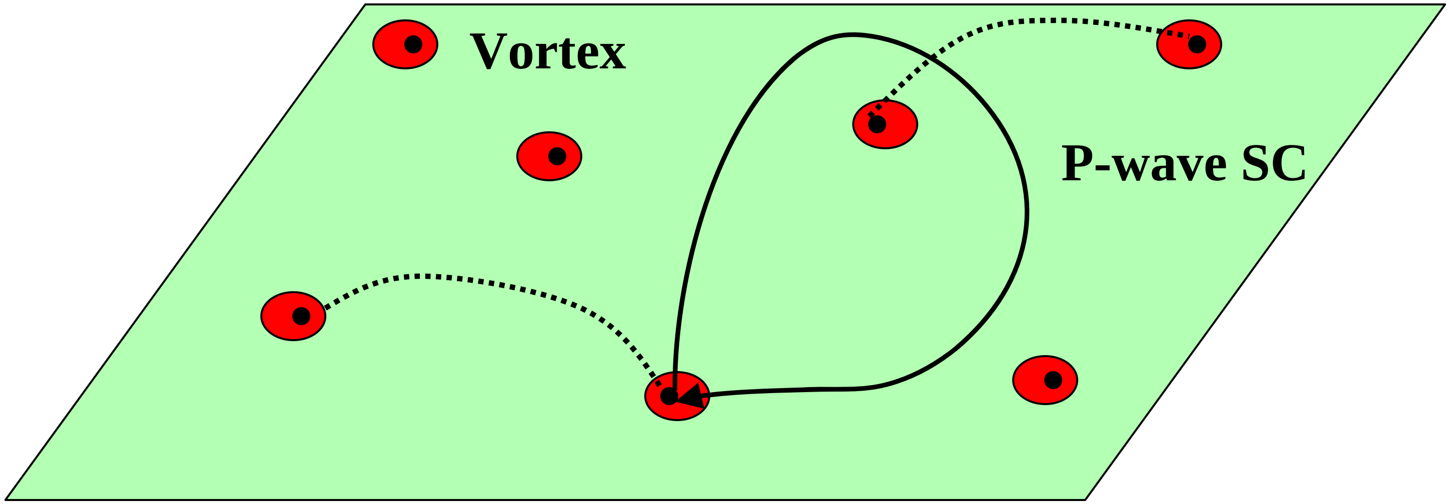
Fermions : $\psi(r_1, r_2, r_i) = -\psi(r_2, r_1, r_i)$

Anyons (2D) : $\psi(r_1, r_2, r_i) = e^{i\theta}\psi(r_2, r_1, r_i)$

Non-Abelian Anyons(2D): $\boxed{\psi_j(r_1, r_2, r_i) = M_{jk}\psi_k(r_2, r_1, r_i)}$

Statistics can be non-Abelian!

Non-Abelian Statistics



Degenerate set of ground states

$$\psi_j(r_1, r_2, r_i) = M_{jk}\psi_k(r_2, r_1, r_i)$$

Non-Locality + Non-Abelian Statistics

Many-particle quantum state is manipulable only by topology

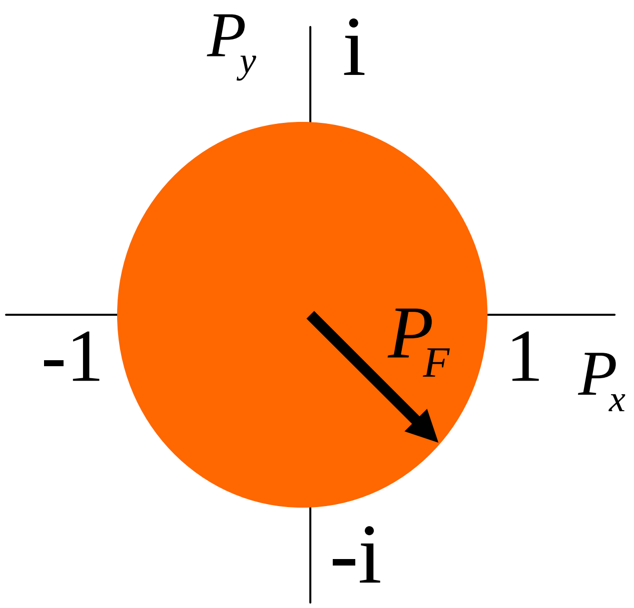
2D $(p_x + ip_y)$ Superconductor

Consider spinless
(spin-polarized) system

Order parameter :

$$\Delta(P) = \Delta_0 (P_x + iP_y)$$
$$= \Delta_0 \,|\, P \,|\, e^{i\theta_P}$$

Fermi Surface



Time reversal symmetry is broken

Superconductor forms domains that break time-reversal symmetry

Two interference experiments—one using the Kerr effect, the other using the Josephson effect—confirm strontium ruthenate's exotic pairing.

When a superconductor's temperature drops below its critical value, some of the most loosely bound electrons assemble into a single, Bose–Einstein ground state. Locked together, the electrons flow through the lattice with unimpeded ease.

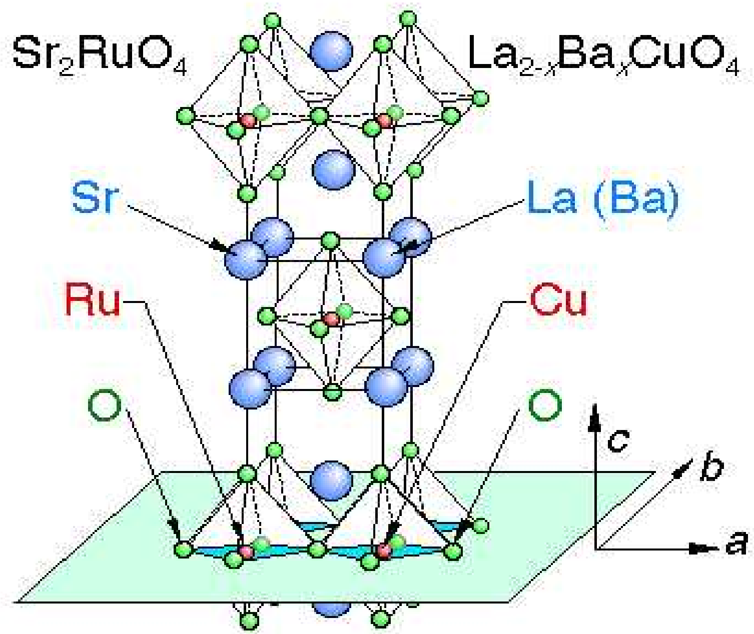
To reach that remarkable state, electrons, being fermions, must pair up to form bosons whose total spin S is an integer. Pairs of spin-½ electrons have two choices of S: 0 or 1, antiparallel or parallel. Because a pair of identical fermions must have an antisymmetric wavefunction, fixing S also constrains the pair's total orbital angular momentum L: If $S = 0$, L must be an even integer; if $S = 1$, L must be an odd integer.

How electrons follow those rules and actually pair up depends on the symmetry of the lattice and on what fluctuations polarize and nudge the electrons together. In ordinary, Bardeen-Cooper-Schrieffer superconductors, lattice vibrations mediate the pairing and S and L are both zero. By analogy with atomic orbitals, the pairing is known as *s*-wave.

When L is nonzero, the paired electrons, like electrons in single atoms, can orbit each other in more than one configuration. No one has yet identified the mediating fluctuations in high-T_c cuprates, but experiments have established that the pairing is a cloverleaf-shaped variety of *d*-wave ($S = 0$; $L = 2$).

Yoshiteru Maeno of Kyoto University discovered strontium ruthenate's superconducting state in 1994. Strontium ruthenate (Sr_2RuO_4) has the same lattice structure as lanthanum cuprate (La_2CuO_4), the parent compound of the first family of high-T_c superconductors. Based on the resemblance, one might expect the ruthenate's superconductivity to occur in the RuO_4 planes and its pairing to be *d*-wave.

The superconductivity turned out to be two-dimensional, but the preponderance of evidence soon favored *p*-wave pairing ($S = 1$; $L = 1$). Indeed,



Strontium Ruthenate: Quasi-2D lattice structure

Confirming $(p_x + ip_y)$ Pairing

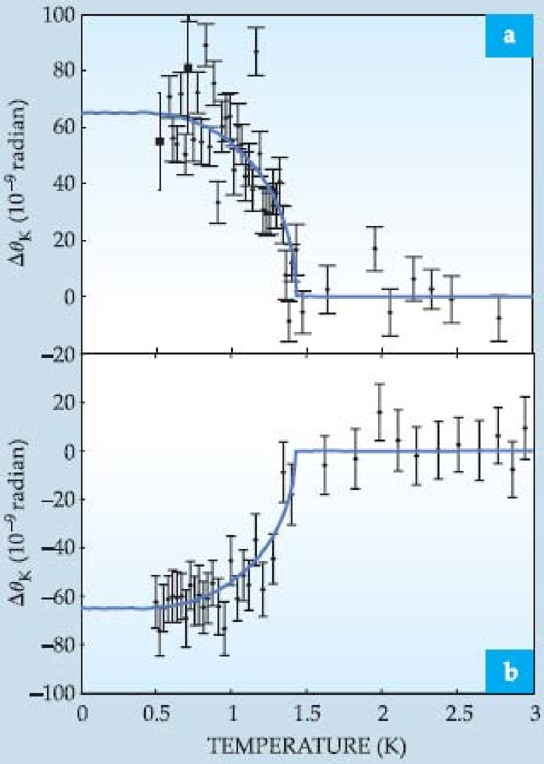


Figure 2. Electron pairs that break time-reversal symmetry in superconducting strontium ruthenate shift the phase of circularly polarized light by the Kerr angle $\Delta\theta_K$. They also form domains that can be oriented by an applied magnetic field. Here, the measured rotation is plotted against temperature above and below T_c. In (**a**), the applied field was +93 oersteds; in (**b**), −47 Oe. The curves represent a simple model. (Adapted from ref. 2.)

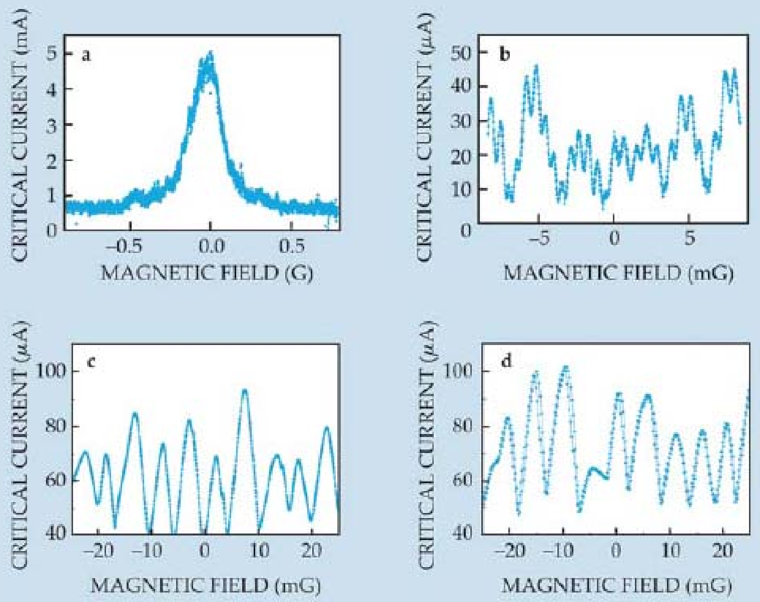


Figure 3. The current through a Josephson junction is ordinarily strongest at zero magnetic field. That's the case in (**a**), where either one domain or several domains of the same phase crossed the junction. But in (**b**), (**c**), and (**d**), multiple domains of different phase crossed the junction, each making an out-of-phase contribution to the signal. (Adapted from ref. 3.)

Shifts of circularly polarized light due to TRS breaking.
J. Xia et al., Phys. Rev. Lett. (2006)

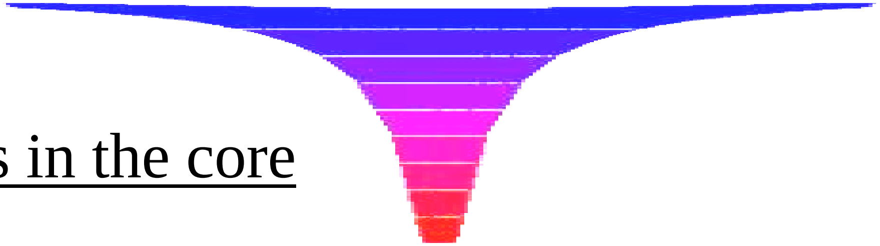
Josephson junction signatures of TRS breaking.
F. Kidwingira et al., Science (in press)

Vortices in 2D $(p_x + ip_y)$ Superconductor

Order parameter phase rotates by $2\pi n$ around the core

Order parameter amplitude suppressed at the core

Low energy normal bound states in the core



Bound State Spectrum

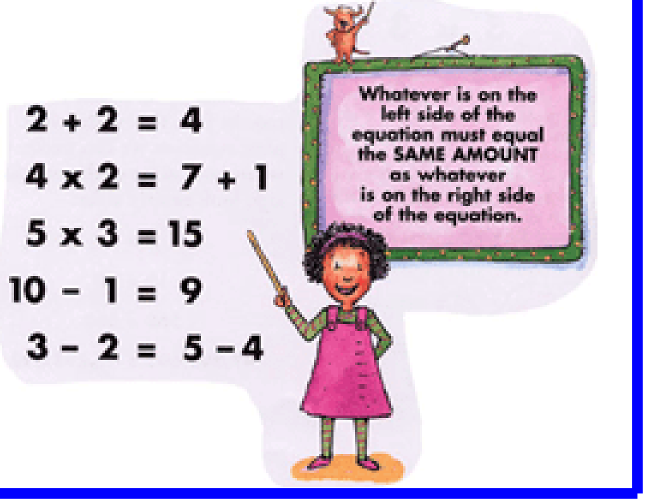
S-wave superconductor

$$E_n = (n + 1/2)\omega_0, \quad \omega_0 = \Delta_0^2 / E_F$$

($P_x + iP_y$) superconductor

$$E_n = n\omega_0, \quad \omega_0 = \Delta_0^2 / E_F$$

$n = 0,$ **zero energy states are the unusual quantum states**



BdG Equations for the ZES

M. Stone, S. B. Chung, PRB (2006)

S. T., S. D. Sarma et al., Phys. Rev. Lett. (2007)

$$H(r,r') = (-\frac{\partial^2}{2m} - \mu)\delta(r-r')\sigma_z + \frac{\Delta(r,r')}{2}\sigma^+ - \frac{\Delta^*(r,r')}{2}\sigma^-$$

$$\int_{r'} H(r,r')\psi_n(r') = E_n \psi_n(r)$$

Assume $\Delta = 0$ inside the vortex core, finite outside

Consistent solution for $E_n = 0$

Is there a theorem that guarantees it?

Index Theorem for the ZES

S. T., S. D. Sarma, D. H. Lee, cond-mat/0609556

Question

Where does such a theorem exist?

Answer

In 1D Dirac theory for electrons

In 1976, R. Jackiw and C. Rebbi prove that in 1D Dirac field theory, there is always a ZES if there is a mass domain wall.

R. Jackiw, C. Rebbi, Phys. Rev. D (1976)

1D Dirac Theory for Electrons

$$H = \int [-iv_F \psi^\dagger \sigma_z \partial_x \psi + m(x)\psi^\dagger \sigma_x \psi]$$

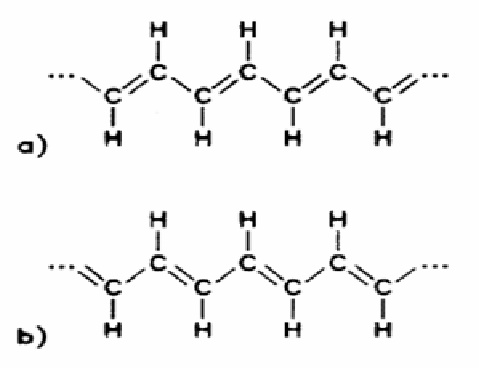
$$m(x) = sign(x)\,|\,m(x)\,|$$

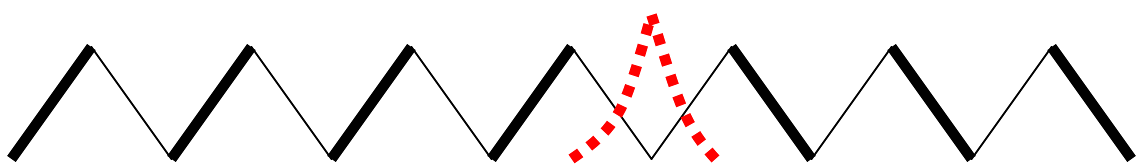
$$\psi_0(x) = e^{-\frac{1}{v_F}\int_0^x m(y)dy}\,\psi_0(0)$$

ZES is located on the mass domain wall, a topological excitation

The existence of this ZES is confirmed by experiments on polyacetylene

The Story of Polyacetylene





W. P. Su, J. R. Schrieffer, A. J. Heeger, Phys. Rev. Lett. (1979)

$$H = \int [-iv_F \psi^\dagger \sigma_z \partial_x \psi + m(x)\psi^\dagger \sigma_x \psi]$$

$$m(x) = sign(x)\,|\,m(x)\,|$$

$$\psi_0(x) = e^{-\frac{1}{v_F}\int_0^x m(y)dy}\,\psi_0(0)$$

Mapping 2D P-Wave SC on 1D

$$H = K + H_P$$

$$K = \sum \xi_k c_k^\dagger c_k$$

$$H_P = \Delta_0 \sum_k (k_x + ik_y) c_k^\dagger c_{-k}^\dagger + h.c.$$

$$c_k = \frac{1}{\sqrt{2\pi k}} \sum_{m=-\infty}^{\infty} c_{m,k} e^{im\theta_k}$$

Professor McGuire was fast regretting becoming the first man to successfully create a mini black hole in the laboratory.

2D P-Wave SC with a Vortex

$$K=\frac{1}{(2\pi)^2}\sum_m\int_{-\Lambda}^{\Lambda}dq(v_F q)c^{\dagger}_{m,q}c_{m,q}$$

$$H_P=-\Delta_0\sum_m\int dkdpu_m(k,p)c^{\dagger}_{2-m,k}c^{\dagger}_{m,p}+h.c.$$

$$\boxed{H_1=\frac{1}{(2\pi)^2}\sum_m\int_{-\Lambda}^{\Lambda}dq(v_F q)c^{\dagger}_{1,q}c_{1,q}+\Delta_0\int_{-\Lambda}^{\Lambda}dqdq'A(q-q')c^{\dagger}_{1,q}c^{\dagger}_{1,q'}+h.c.}$$

$$H_1=\int dx[-iv_F\chi^{\dagger}\sigma_z\partial_x\chi+m(x)\chi\dagger\sigma_x\chi] \qquad \chi^{\dagger}(x)=(c^{\dagger}(x),c(-x))$$

$$m(-x)=-m(x)$$

2D P-Wave SC with a Vortex

$$H_1 = \int dx[-iv_F \chi^\dagger \sigma_z \partial_x \chi + m(x)\chi^\dagger \sigma_x \chi]$$

$$\chi^\dagger(x) = (c^\dagger(x), c(-x))$$

$$m(-x) = -m(x)$$

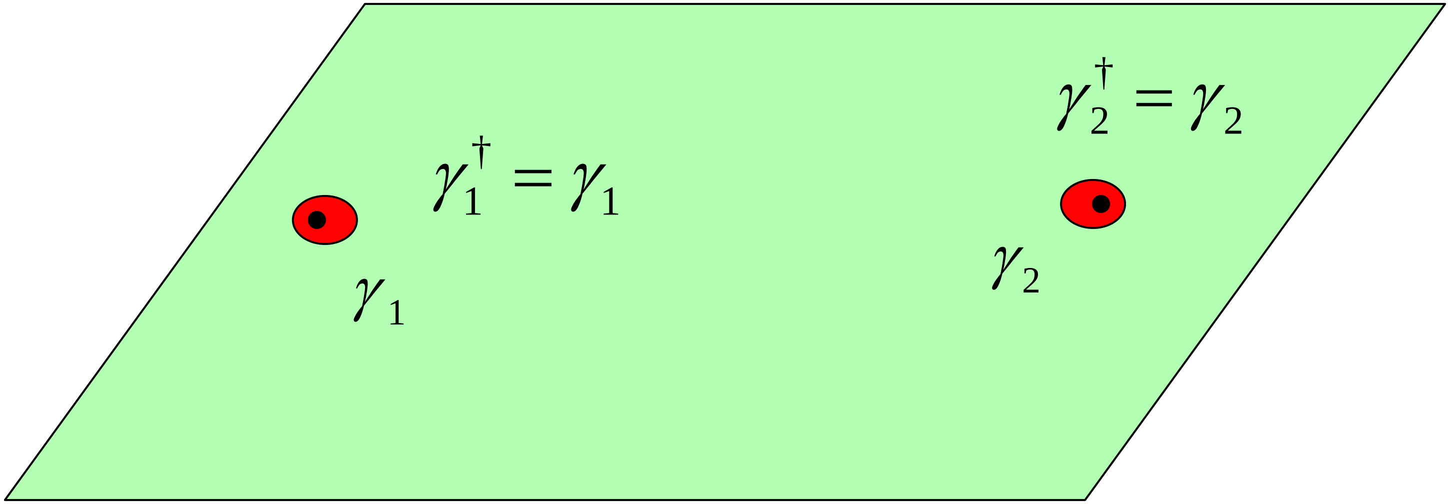
Zero-energy state!

$$\gamma^\dagger = \frac{1}{N}\int dx e^{-\frac{1}{v_F}\int_0^x m(y)dy}(c_1(x) - ic_1^\dagger(-x))$$

$$\gamma^\dagger = \gamma$$

Majorana fermion mode!

Majorana Modes : Non-Locality



$$\{\gamma_i,\ \gamma_j\} = 2\delta_{ij}$$

A single γ mode cannot accommodate an electron!

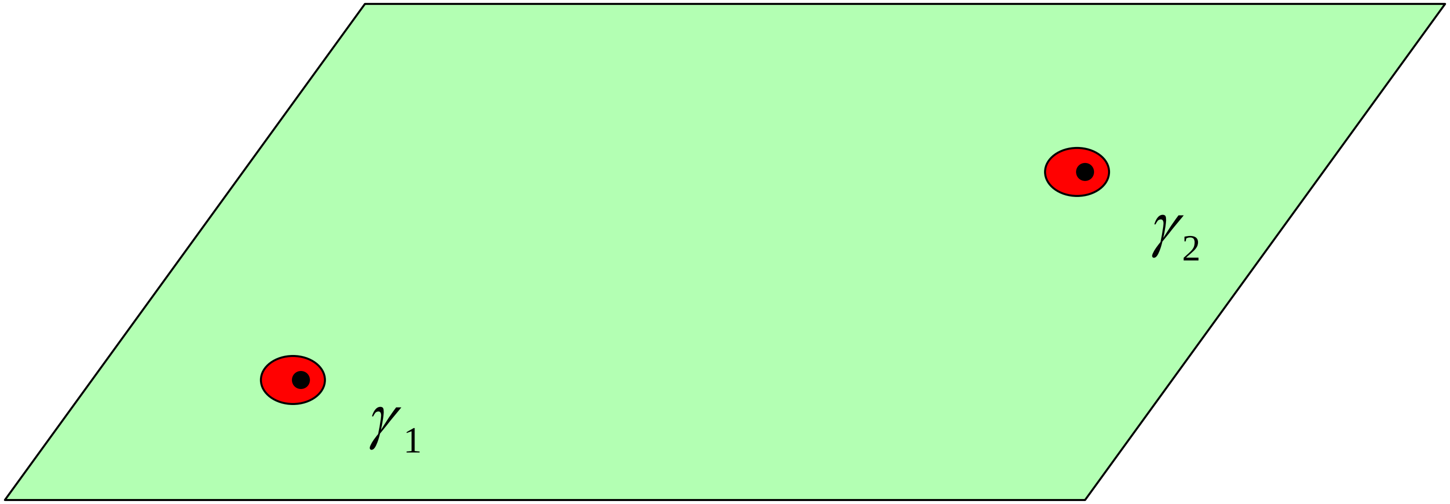
Construct $c = \gamma_1 + i\gamma_2$

$$c^\dagger = \gamma_1 - i\gamma_2$$

c's can be occupied by electrons

Non-local occupation

Majorana Modes : GS Degeneracy I



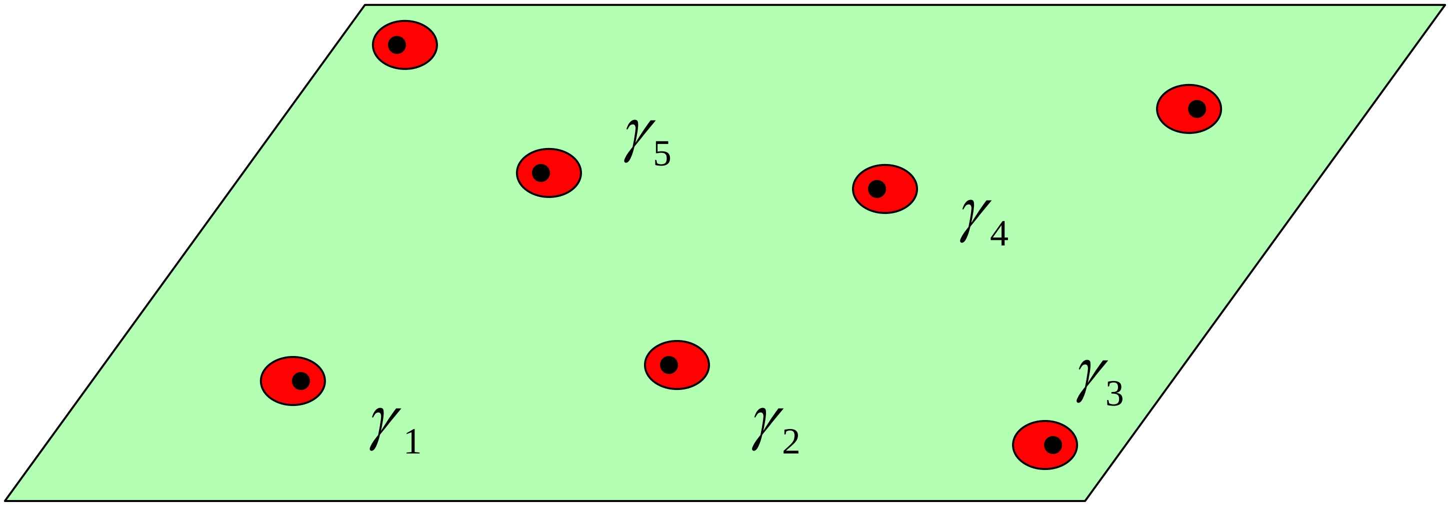
A pair of vortices support an electronic mode **at zero energy**

This mode can be unoccupied ($|0>$), or occupied ($|1>$)

Two states of a qubit

The states are degenerate and completely non-local

Majorana Modes : GS Degeneracy II



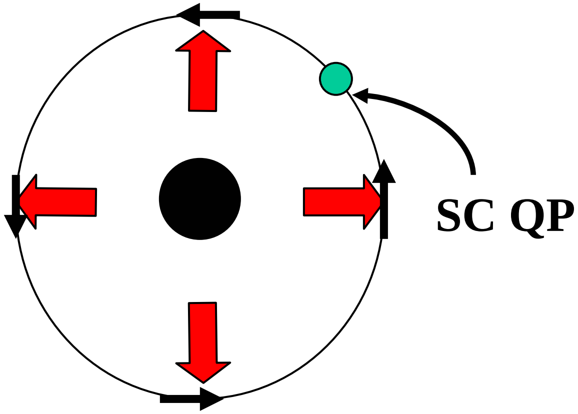
Consider 2n vortices / Majorana fermions => n electronic modes

They can be occupied / unoccupied by a SC QP

2^n -fold degenerate ground states protected by a gap $\omega_0 = \Delta_0^2 / E_F$

Any state in the GS manifold is a linear combination of them

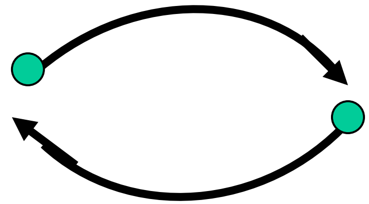
Majorana Modes : Non-Abelian Statistics



Ivanov, PRL (2001)
S. T., S. D. Sarma et al., PRL (2007)
Gurarie, Radzihovsky, Ann. Phys. (2007)

$$a^\dagger a^\dagger \mapsto 2\pi$$

$$ua^\dagger + va \mapsto \pi$$



$$\gamma_1 \to \gamma_2$$

$$\gamma_2 \to -\gamma_1$$

$$T = \frac{1}{\sqrt{2}}(1+\gamma_2\gamma_1) = \exp(\frac{\pi}{4}\gamma_2\gamma_1) = \exp(i\frac{\pi}{4}(2c^\dagger c - 1))$$

$$c = \frac{1}{2}(\gamma_1 + i\gamma_2)$$

In the basis $|0>,\ c^\dagger|0>$ $(2^1-$ fold degenerate)

$$T = \exp(-i\frac{\pi}{4}\sigma_z) \quad (2^1 \times 2^1 - \text{dimensional matrix})$$

Application to TQC

Non-local qubit + non-Abelian statistics (unitary operators)

Construct gates by braiding one vortex around another

Noiseless quantum computation

Topological ground state degeneracy

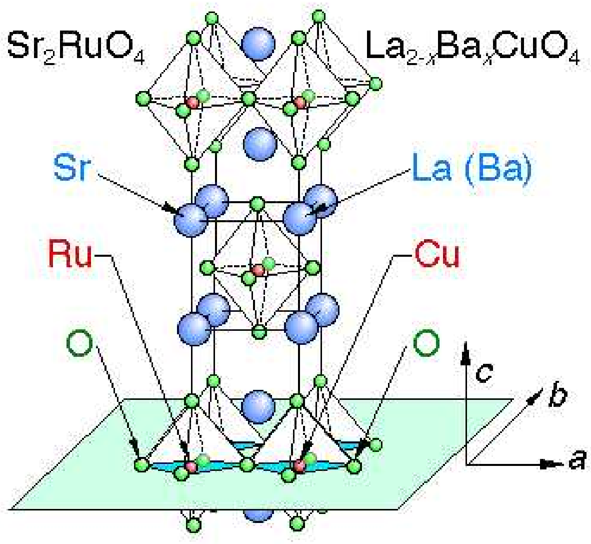
M. Oshikawa, Y. B. Kim, C. Nayak, K. Shtengel, S. Tewari
Annals of Physics (2006)

On a torus, the ground state degeneracy of (Abelian) S-wave SC is 4

The same degeneracy for non-Abelian $p_x + ip_y$ SC is 3

Half-Quantum Vortices in Strontium Ruthenate

S. Das Sarma, C. Nayak, S. Tewari
Phys. Rev. B (R) (2006)



Quasi-2D Ru-O planes

Experiments confirm a **spinful** $p_x + ip_y$ SC

Ordinary vortices do not carry Majorana fermions !

More exotic, <u>half-quantum vortices</u> needed

HQV's are effectively ordinary vortices in only one spin sector
A. J. Leggett, S. Yip, Helium 3 (1989)

Method proposed to create a dilute gas of HQV's

Experiments

2D, spinless $p_x + ip_y$ superfluidity possible in cold-atom optical trap

Gurarie, Radzihovsky et al., PRL (2005)
Cheng et al., PRL (2005)

VOLUME 90, NUMBER 5 PHYSICAL REVIEW LETTERS week ending 7 FEBRUARY 2003

Tuning p-Wave Interactions in an Ultracold Fermi Gas of Atoms

C. A. Regal, C. Ticknor, J. L. Bohn, and D. S. Jin*
JILA, National Institute of Standards and Technology, and Department of Physics, University of Colorado, Boulder, Colorado 80309-0440
(Received 29 August 2002; published 5 February 2003)

We have measured a p-wave Feshbach resonance in a single-component, ultracold Fermi gas of ^{40}K atoms. We have used this resonance to enhance the normally suppressed p-wave collision cross section to values larger than the background s-wave cross section between ^{40}K atoms in different spin states. In addition to the modification of two-body elastic processes, the resonance dramatically enhances three-body inelastic collisional loss.

DOI: 10.1103/PhysRevLett.90.053201 PACS numbers: 34.50.–s, 05.30.Fk, 32.80.Pj

The ultralow temperature regime accessible in atomic physics is characterized by collision energies so low that centrifugal forces can prevent scattering of atoms with nonzero relative angular momentum. In this case, the atoms collide predominantly via s-partial waves ($l = 0$). Even this restricted regime has demonstrated remarkably rich physics in atomic Bose-Einstein condensates [1]. However, the situation is dramatically different for ultracold fermionic atoms. For identical fermions the Pauli exclusion principle forbids s-wave collisions. This means the dominant interaction is via p-wave collisions ($l = 1$). These collisions are suppressed by centrifugal effects as described by the Wigner threshold law [2,3], which demands that the elastic scattering cross section diminishes with temperature as $\sigma \propto T^2$. For this reason, evaporative cooling of fermions to ultralow temperatures can be achieved only in a mixture of different spin states of

near this p-wave resonance, including loss at the nearby s-wave resonance.

The experiments reported here employ previously developed techniques for cooling and spin state manipulation of ^{40}K. Atoms in the $|9/2, 9/2\rangle$ and $|9/2, 7/2\rangle$ states are first held in a magnetic trap and cooled by forced evaporation [4]. The gas is then polarized in the $|9/2, 9/2\rangle$ state and loaded into a far-off resonance optical dipole trap. Adiabatic rapid passage is then used to obtain the desired spin composition [17]. First, the $|9/2, 9/2\rangle$ gas is completely transferred to the $|9/2, -9/2\rangle$ state with a 10 ms rf frequency sweep across all ten spin states at a field of ~30 G. To create a pure $|9/2, -7/2\rangle$ gas, we then move to a higher field, ~80 G, and sweep the magnetic field while applying rf at a fixed frequency to drive the $|9/2, -9/2\rangle$ to $|9/2, -7/2\rangle$ transition.

Majorana Fermions in Cold Atom Optical Trap

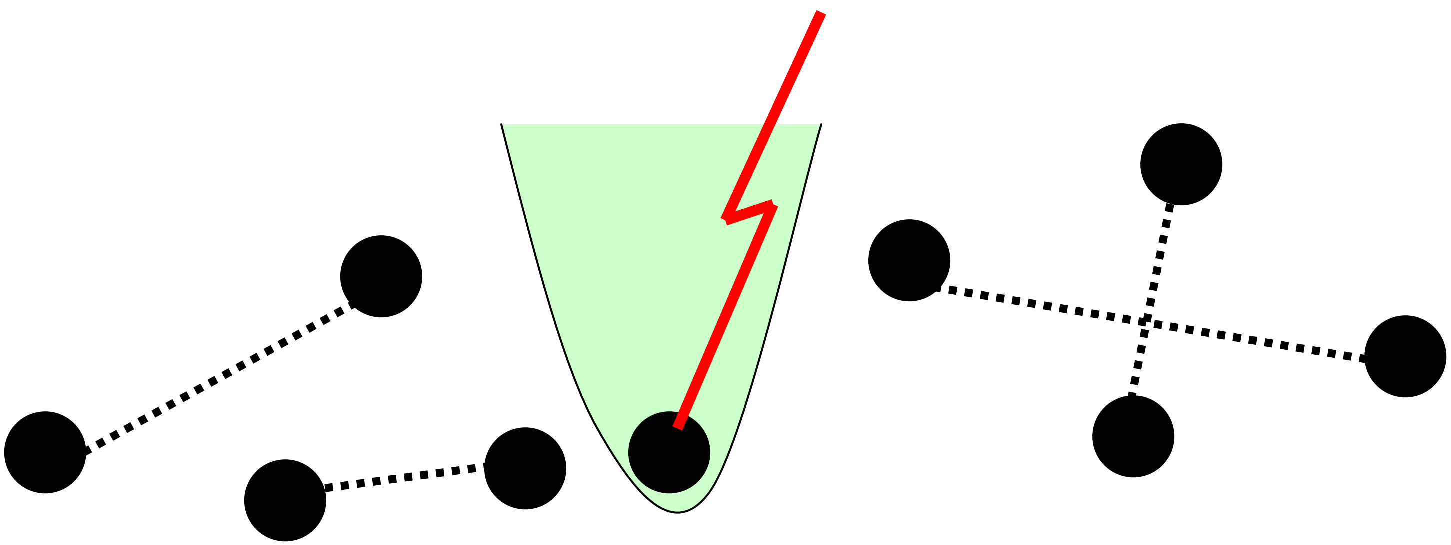
S. Tewari et al., Phys. Rev. Lett. (2007)

2D, $p_x + ip_y$ superfluidity in optical trap

Dilute gas of vortices by rotation

Majorana vortices braided to test non-Abelian statistics

Majorana fermions are chargeless, spinless : Probe?



Use the internal energy levels of an atom for detection/read-out

STM Signature of Majorana Fermions

S. T., C. Zhang, et al., in preparation

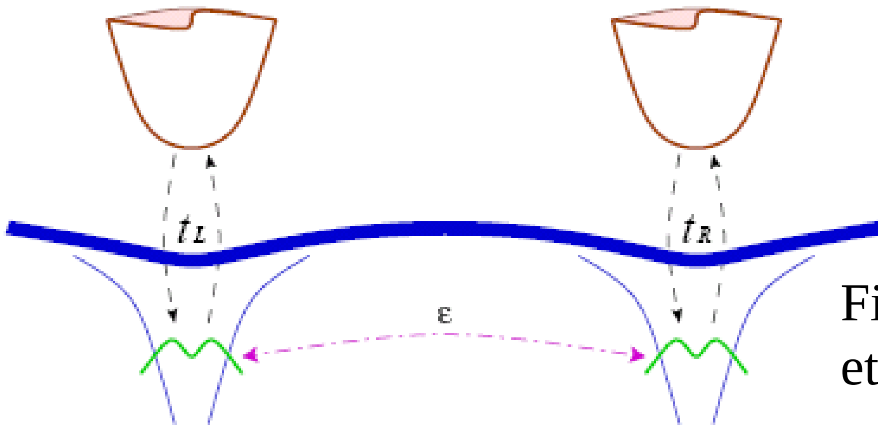


Figure from Bolech et. al. cond-mat/0607779

Inject an electron with $t_L << \omega_0, \Delta_0$

It can only go to the state $c^\dagger = \gamma_L + i\gamma_R$

Electron amplitude appears also near R and can be extracted by t_R

The process has a natural time scale $T = 1/\omega_0$, maintaining causality

Summary and Conclusion

2D spinless $p_x + ip_y$ SC / SF supports Majorana fermions

BdG equations

Index Theorem

Non-local accommodation of a quasiparticle : Qubit

Non-Abelian Statistics : Unitary gates

Topological ground state degeneracy

Experiments : HQV in strontium ruthenate, Cold atoms, STM

Thank You!