RF-spectroscopy and imbalanced Fermi gases

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Contents

- Pairing gap and RF-spectroscopy
  - Basics
  - Innsbruck 2004 experiment
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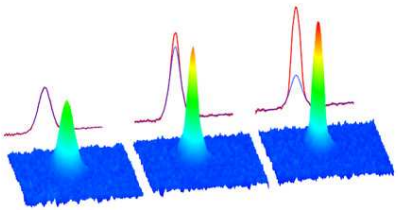
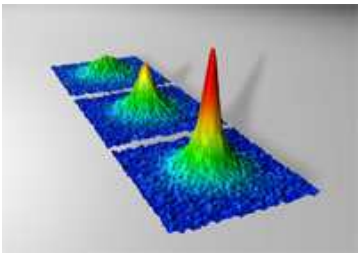
    - How the spectral peak actually depends on the gap (absence of Pauli blocking, trapping effects, Hartree fields)
- Strongly interacting Fermi gases with density imbalance
  - FFLO-type features in a trapped gas
  - FFLO state in an optical lattice
- New MIT experiments (pairing gap in an imbalanced density Fermi gas)
  - Discussion

Strongly interacting Fermi gases experiments 2003-2005

BEC of molecules (dimers of two Fermions) 2003-2004
Grimm, Jin, Ketterle, Salomon

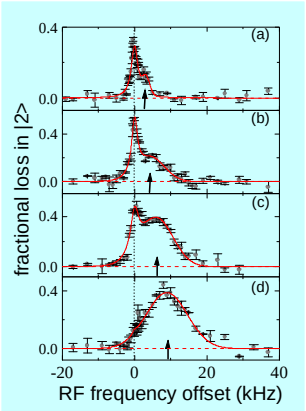
Fermion pairs near the Feshbach Resonance 2004
Jin, Ketterle





Density profile throughout the crossover
Grimm 2004

Collective modes
Thomas 2004,
Grimm 2004

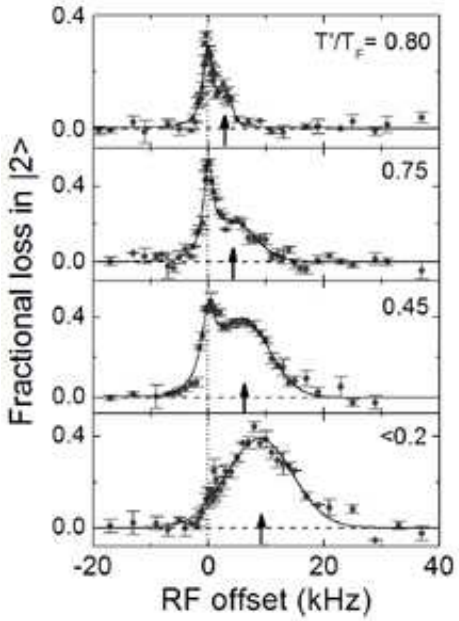


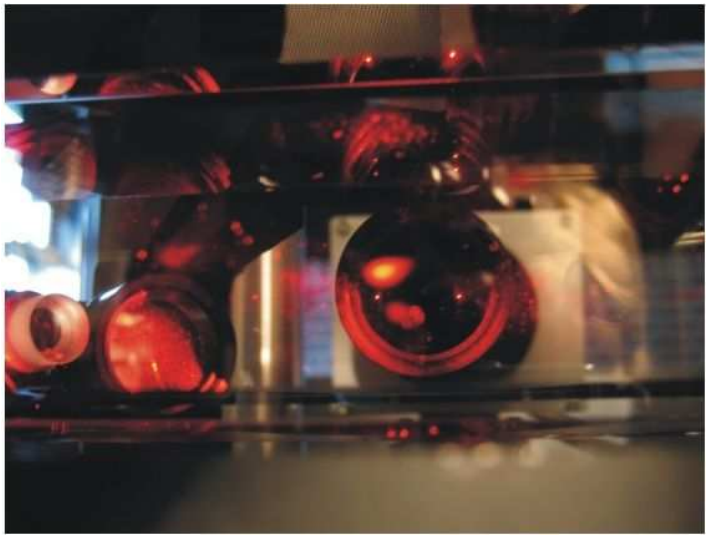
Heat capasity
Thomas 2005

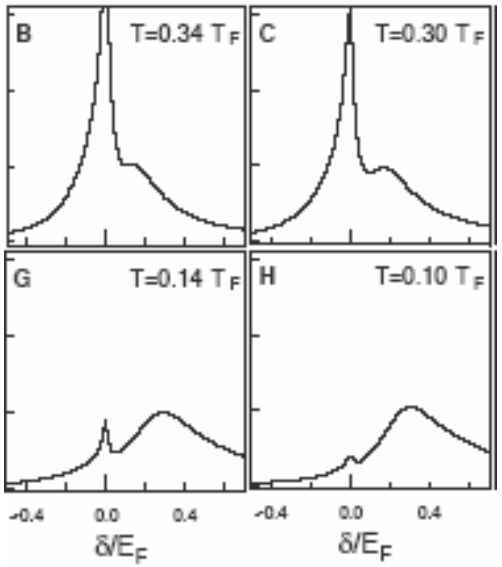
Vortices
Ketterle 2005

Pairing gap *Grimm* 2004

The pairing gap in strongly interacting Fermi gases





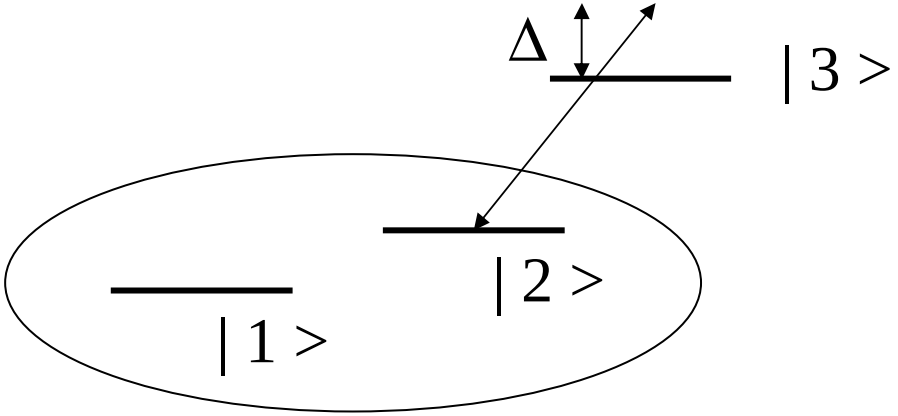


C. Chin, M. Bartenstein, A. Altmayer, S. Riedl, S. Jochim, J.H. Denschlag, and R. Grimm, Science 305, 1128, 2004

J. Kinnunen, M. Rodriguez, and P. Törmä, Science 305, 1131, 2004

Spectroscopy of the pairing gap

- Driving a transition between a paired and an unpaired state



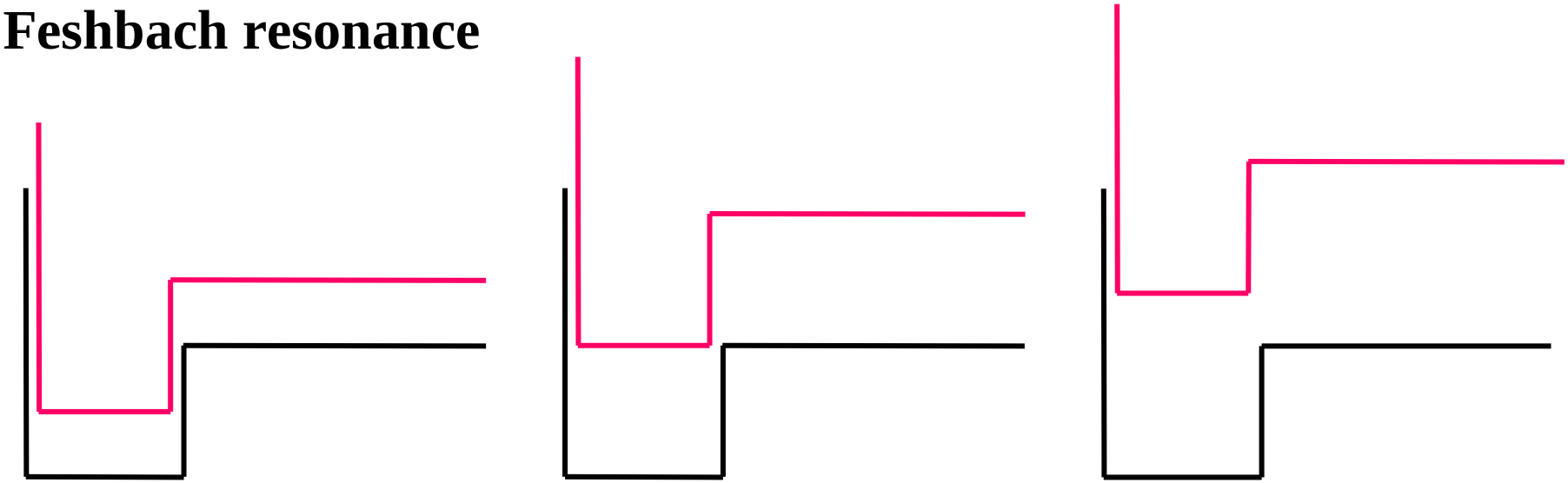
Probing the superfluid excitation gap

- P. Törmä and P. Zoller, PRL 85, 487 (2000)

RF-spectroscopy of mean field effects

- C. Regal and D. Jin, PRL 90, 230404 (2003)
- S. Gupta, Z. Hadzibabic, M.W. Zwierlein, C.A. Stan, K. Dieckmann, C.H. Schunck, E.G.M. van Kempen, B.J. Verhaar, W. Ketterle, Science 300, 1723 (2003)

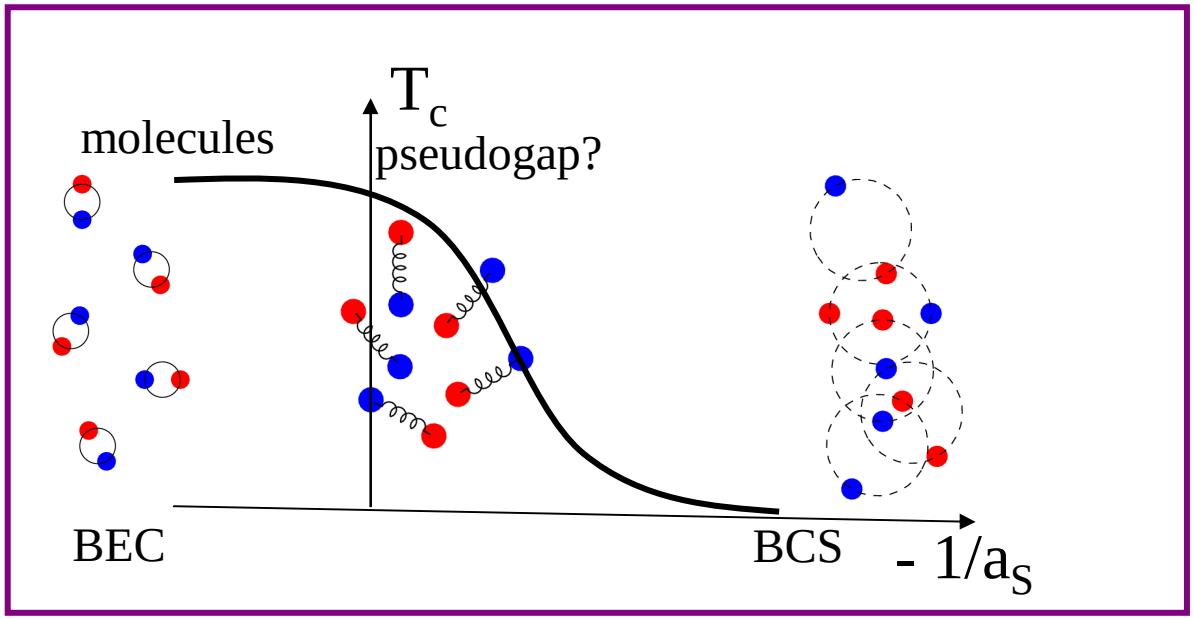
Feshbach resonance



$a_S > 0$
bound molecules

$E_{int} \sim E_F$

$a_S < 0$



Equilibrium state: Resonance superfluidity with a pseudogap

$$H = \sum_{k,\sigma} \epsilon_k^\sigma c_k^{\sigma\dagger} c_k^\sigma + \sum_q \left(E_q^0 + \nu\right) b_q^\dagger b_q + \sum_{q,k,k'} U(k,k') c_{q/2+k}^{g\dagger} c_{q/2-k}^{g'\dagger} c_{q/2-k'}^{g'} c_{q/2+k'}^{g}$$
$$+ \sum_{q,k} \left[g(k) b_q^\dagger c_{q/2-k}^{g} c_{q/2+k}^{g'} + H.c. \right]$$

M. Holland, S.J.J.M.F. Kokkelmans, M.L. Chiofalo, R. Walser, PRL **87**, 120406 (2001)
c.f. E. Timmermans, K. Furuya, P.W. Milonni, A.K. Kerman, Phys.Lett.A 63, 130402 (2002)
Y. Ohashi and A. Griffin, PRL 89, 130402 (2002)

$$\Delta^2 = \tilde{\Delta}_{\rm sc}^2 + \Delta_{\rm pg}^2$$

$$\Delta_{\rm pg}^2 = -\sum_{Q\neq 0} t_{\rm pg}(Q)$$

$$g_{\rm eff}^{-1}(0) \sum_k \frac{1 - 2n_{\rm F}(E_k)}{2E_k} \phi_k^2 = 0, \quad T \leq T_{\rm c}$$

$$N = N_{\rm f} + 2N_{\rm b}^0 + 2N_{\rm b}$$

$$\Sigma(K) = \frac{\tilde{\Delta}_{\rm sc}^2 \phi_k^2}{i\omega + \epsilon_k} + \frac{\Delta_{\rm pg}^2 \phi_k^2}{i\omega + \epsilon_k + i\gamma}$$

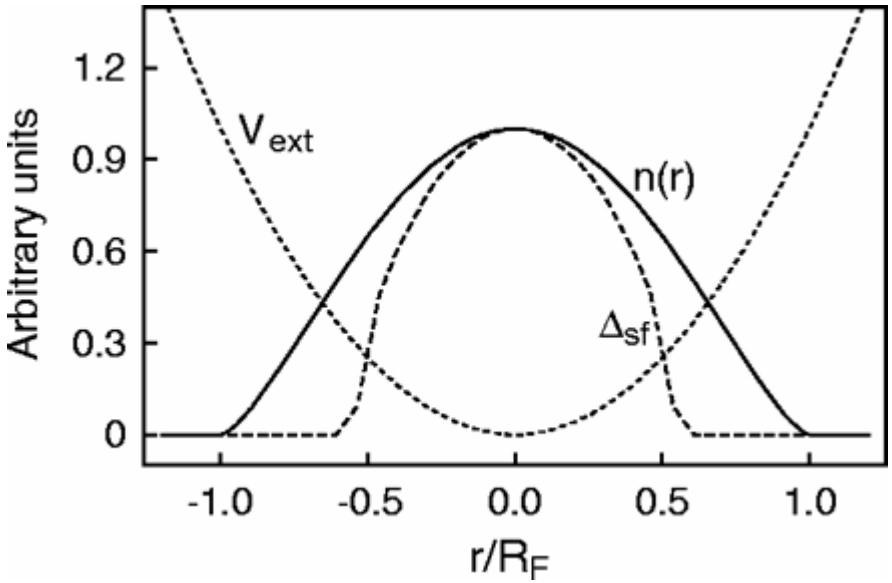
J. Stajic, J.N. Milstein, Q. Chen, M.L. Chiofalo, M.J. Holland, K. Levin, PRA 69, 063610 (2004)
Q.J. Chen, I. Kosztin, B. Janko, and K. Levin, PRL 81, 4708 (1998)
c.f. A. Perali, P. Pieri, L. Pisani, G.C. Strinati, PRL 92, 220404 (2004)

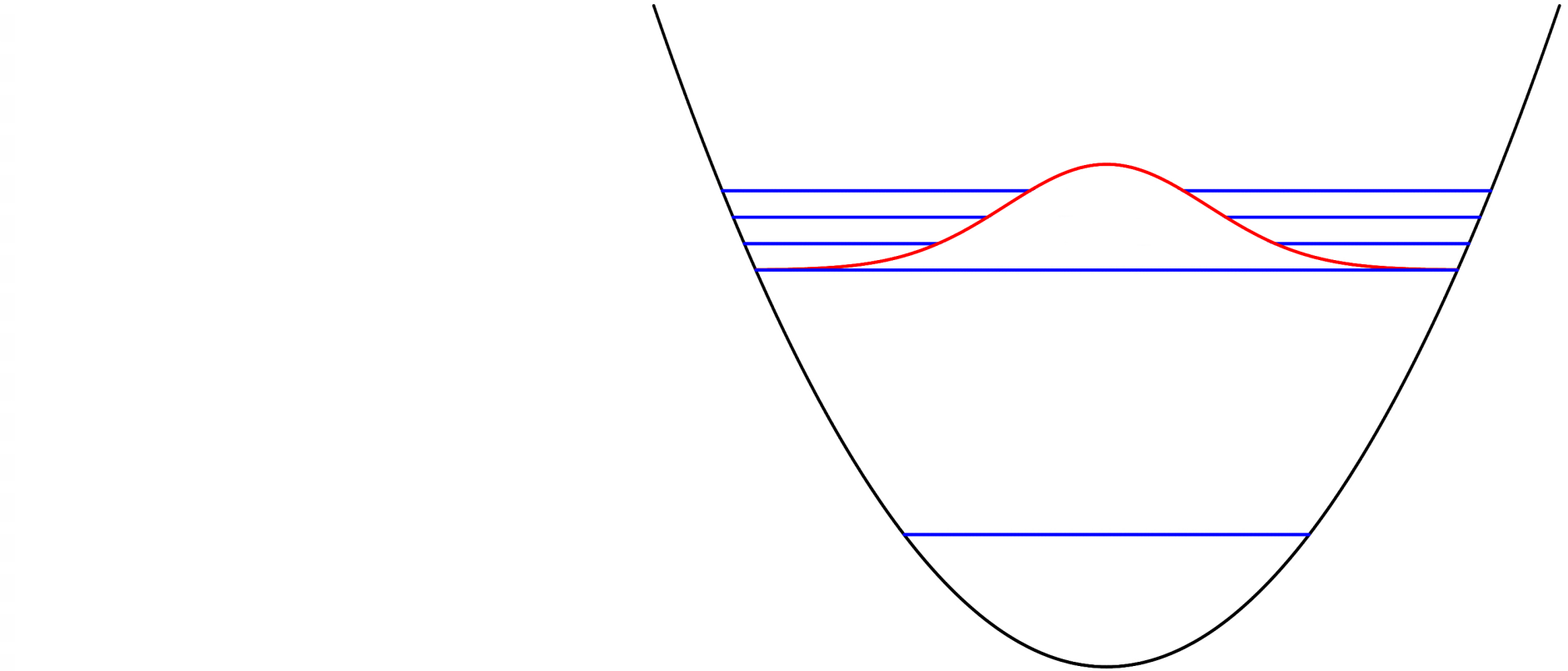
The spectrum is calculated using second-order perturbation theory

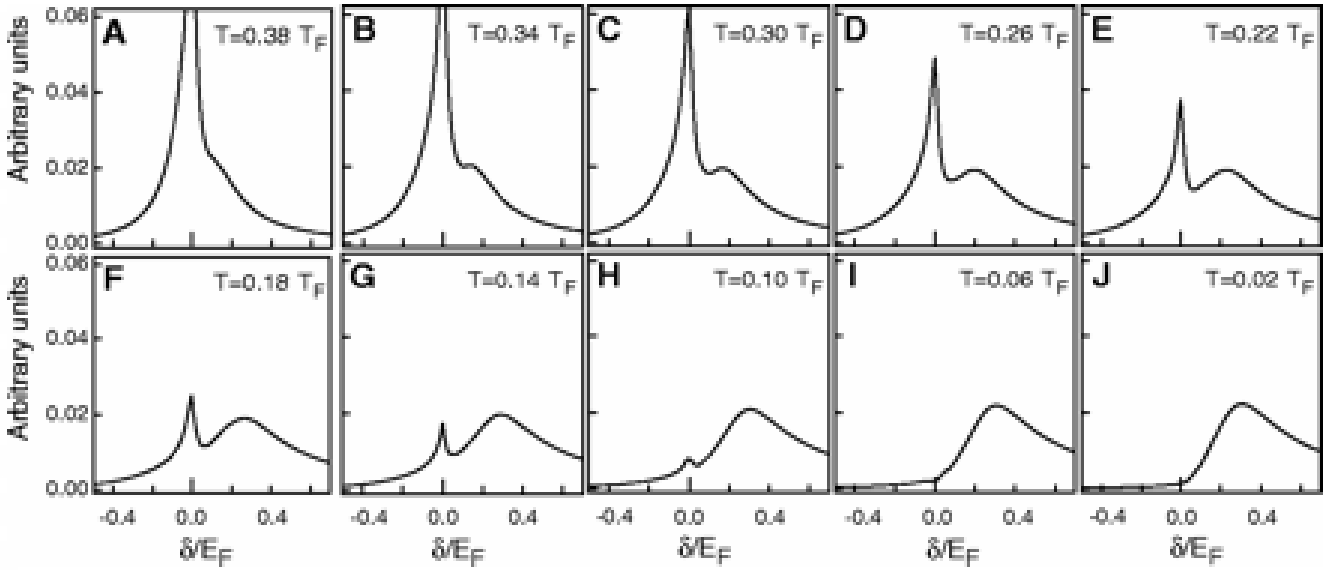
$$X(i\omega) = \sum_k |M_{kk}|^2 \int \frac{dz}{2\pi i} n_{\mathrm{F}}(z) G_{\mathrm{e}}(k,z) G_{\mathrm{g}}(k, z - i\omega)$$

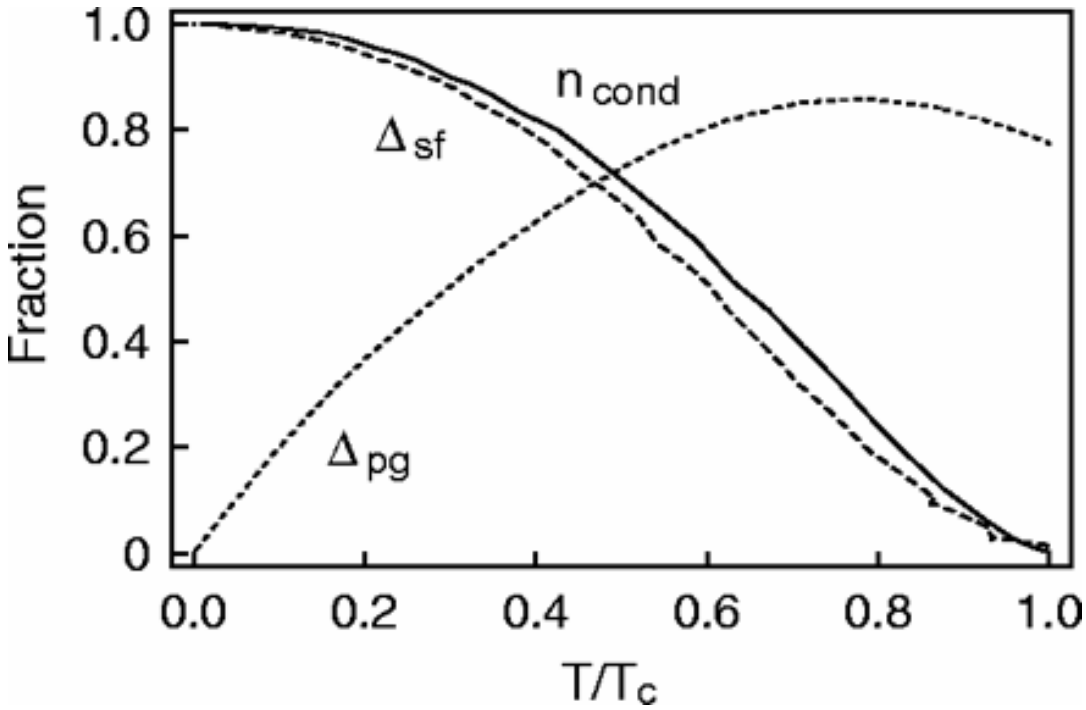
and the local density approximation using Thomas-Fermi density distribution

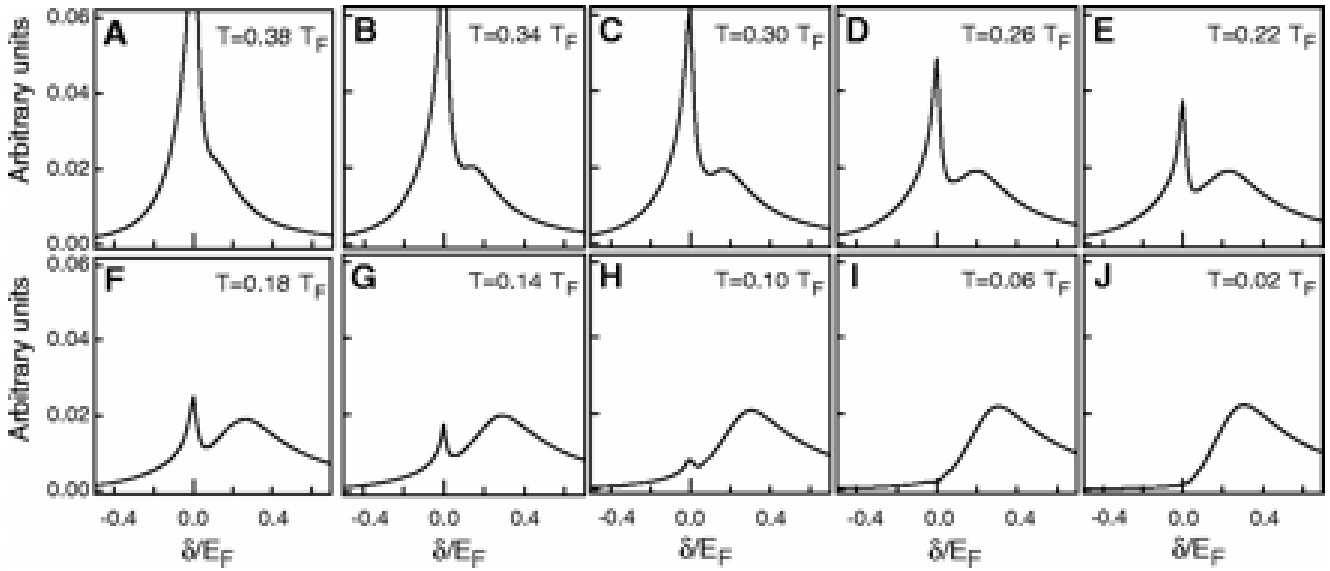
$$n(r) = n_0 \left[1 - \left(\frac{r}{r_{\mathrm{TF}}} \right)^2 \right]^{3/2}$$

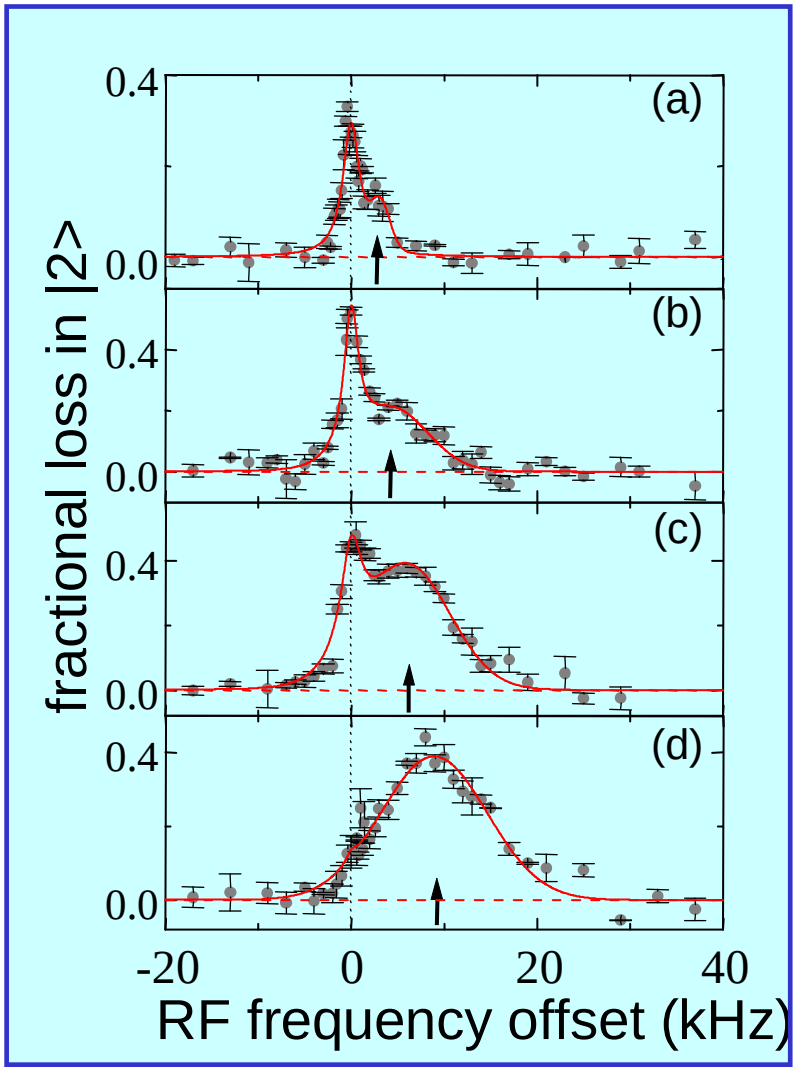




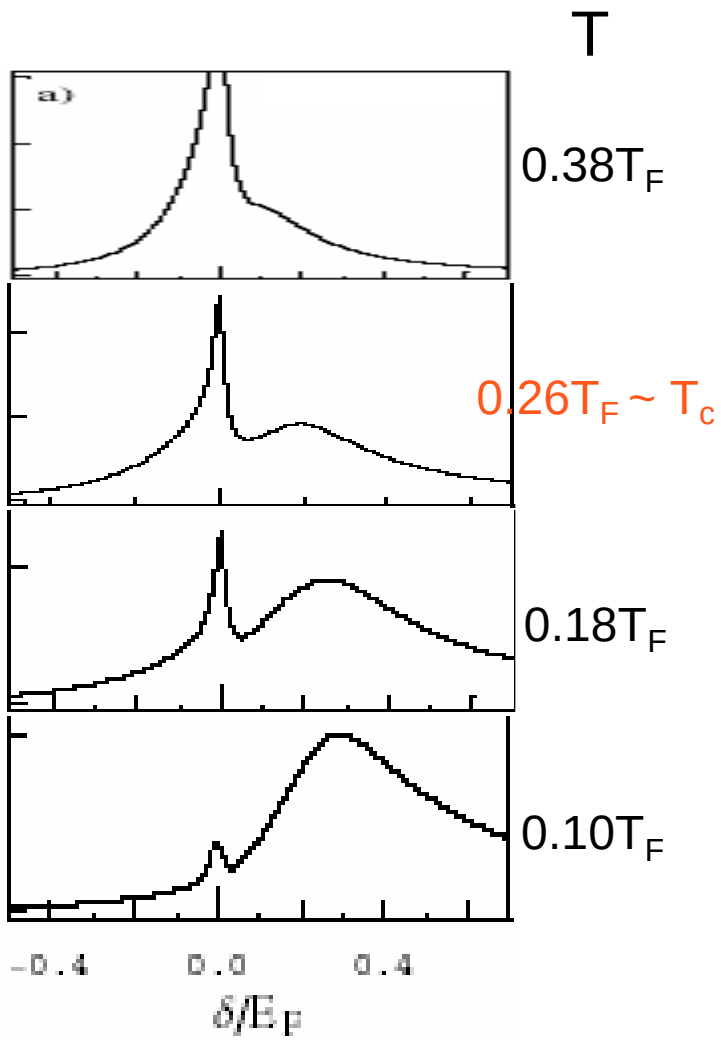

A
T=0.38 T_F
B
T=0.34 T_F
C
T=0.30 T_F
D
T=0.26 T_F
E
T=0.22 T_F
F
T=0.18 T_F
G
T=0.14 T_F
H
T=0.10 T_F
I
T=0.06 T_F
J
T=0.02 T_F
Arbitrary units
0.06
0.04
0.02
0.00
-0.4
0.0
0.4
δ/E_F


1.0
0.8
0.6
0.4
0.2
0
Fraction
n cond
Δ sf
Δ pg
0.0
0.2
0.4
0.6
0.8
1.0
T/Tc


A T=0.38 TF
B T=0.34 TF
C T=0.30 TF
D T=0.26 TF
E T=0.22 TF
F T=0.18 TF
G T=0.14 TF
H T=0.10 TF
I T=0.06 TF
J T=0.02 TF
Arbitrary units
0.06
0.04
0.02
0.00
-0.4
0.0
0.4
δ/EF

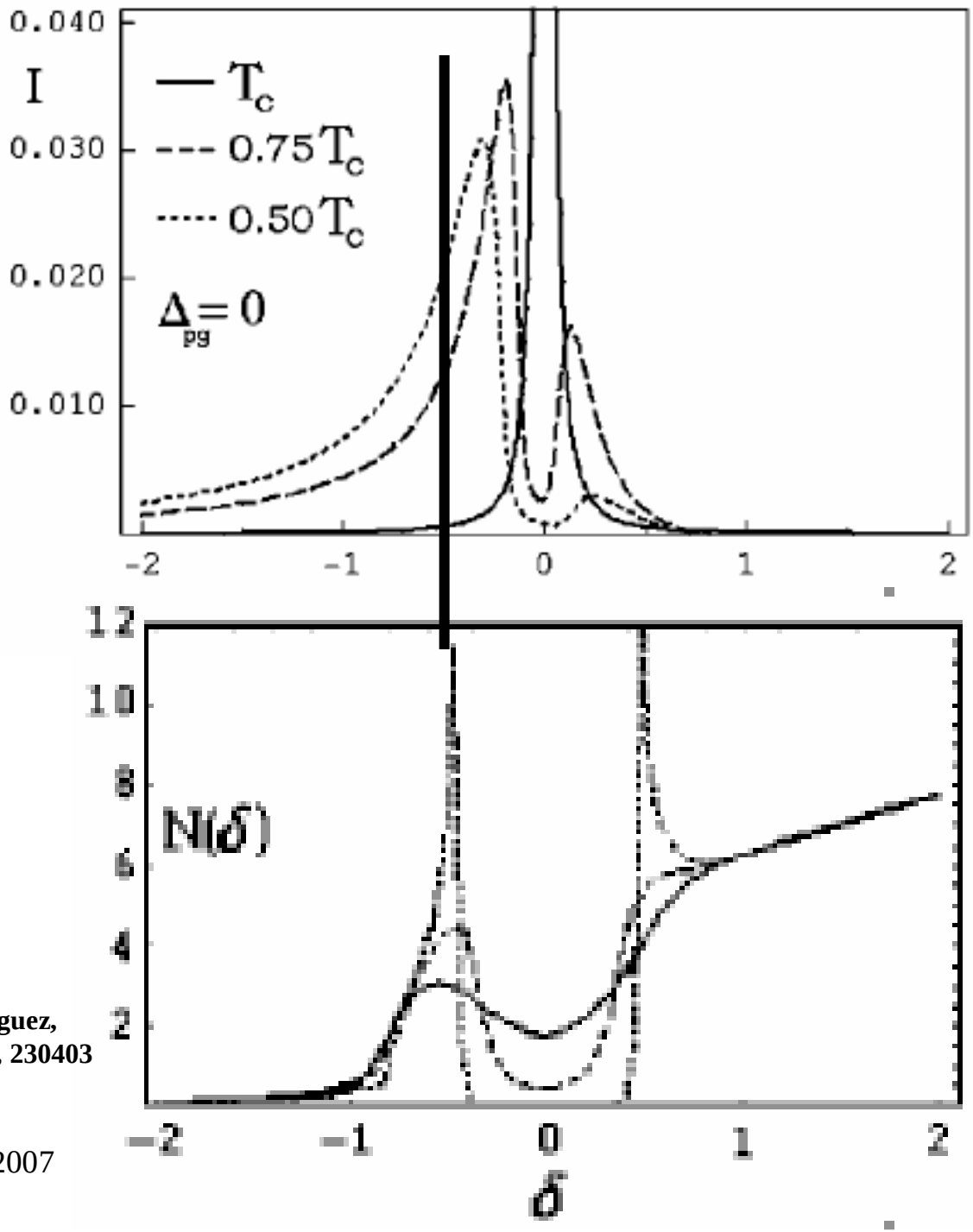


C. Chin, M. Bartenstein, A. Altmayer, S. Riedl, S. Jochim, J.H. Denschlag, and R. Grimm, Science 305, 1128, 2004



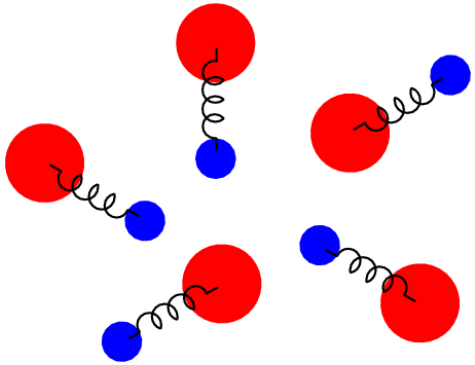
J. Kinnunen, M. Rodriguez, and P. Törmä, Science 305, 1131, 2004

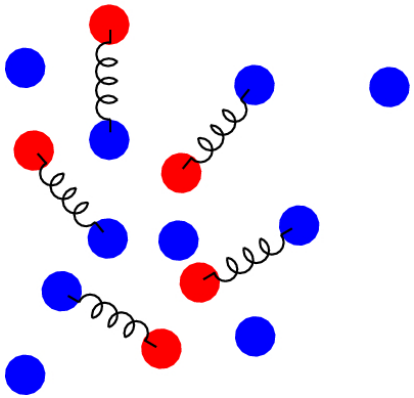
Beyond linear response rf-spectroscopy, J. Kinnunen and P. Törmä, PRL 96, 070402 (2006)



J. Kinnunen, M. Rodriguez, and P. Torma, PRL 92, 230403 (2004)

Polarized Fermi gases





Polarization

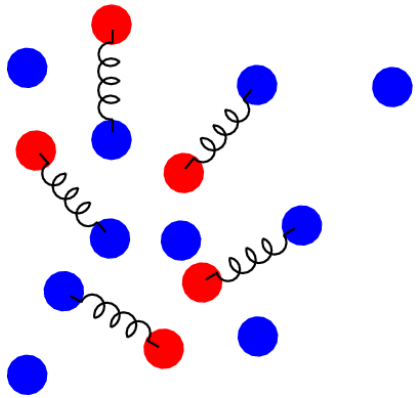
$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

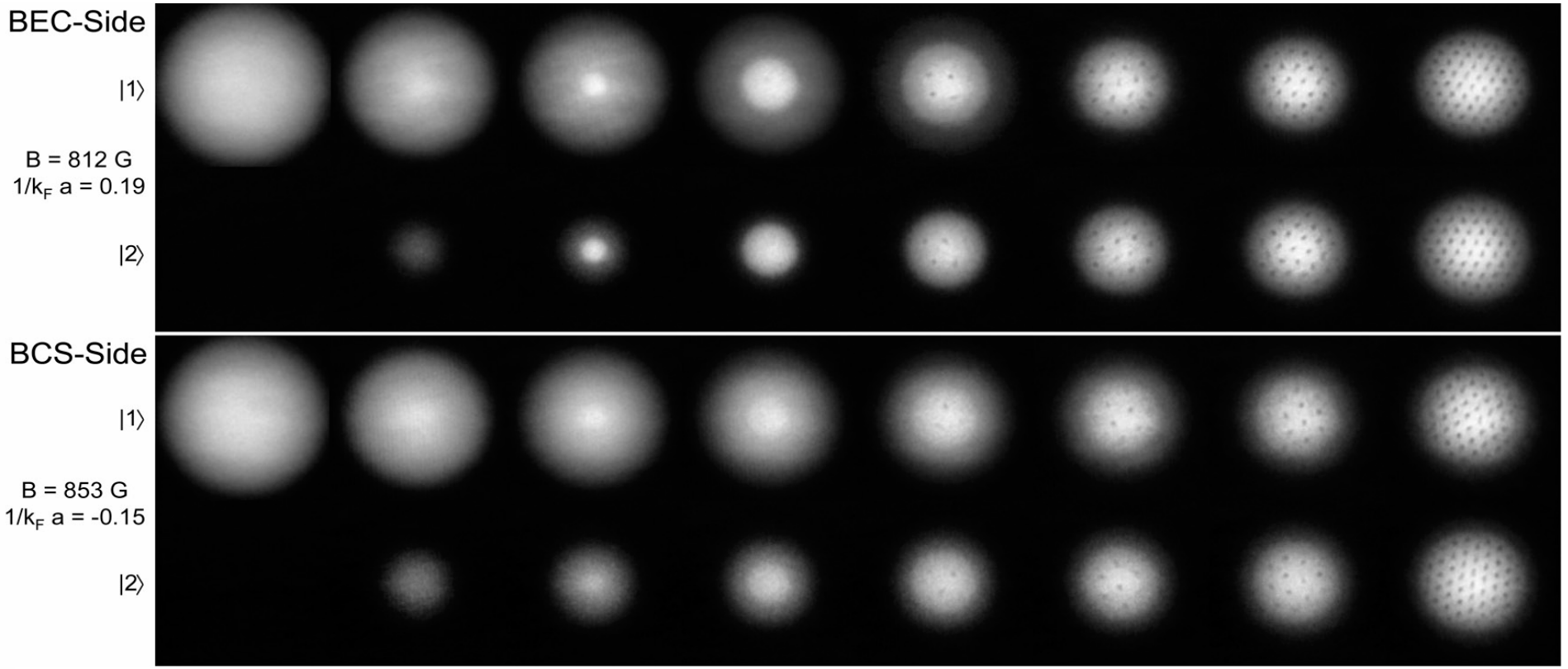
Pairing between particles with unequal mass or unequal total number

Polarized (density-imbalanced) Fermi gases in traps

- Zwierlein, Schirotzek, Schunck, Ketterle, Science 311, 492 (2006)
- Partridge, Li, Kamar, Liao, Hulet, Science 311, 503 (2006)
- Zwierlein, Schunk, Schirotzek, Ketterle, Nature 442, 54 (2006)
- Shin, Zwierlein, Schunck, Schirotzek, Ketterle, PRL 97, 030401 (2006)
- Partridge, Li, Liao, Hulet, Haque, Stoof, PRL 97, 190407 (2006)

Polarization $P = \dfrac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$





P=1 **P=0**

Ketterle 2005

- Bogoliubov-deGennes (BdG) equations in a spherical harmonic trap
- Imbalanced densities
- ~ 20 000 atoms, strong interactions, T ~ 0

$$H = \sum_\sigma \int d\mathbf{r}\, \Psi_\sigma^\dagger(\mathbf{r}) \left[-\frac{\nabla^2}{2m} + V_{\mathrm{trap}}(r) - \mu_\sigma \right] \Psi_\sigma(\mathbf{r}) - U \int d\mathbf{r}\, \Psi_\uparrow^\dagger(\mathbf{r}) \Psi_\downarrow^\dagger(\mathbf{r}) \Psi_\downarrow(\mathbf{r}) \Psi_\uparrow(\mathbf{r})$$

$$\Psi_\sigma(\mathbf{r}) = \sum_{nlm} \psi_{nlm}(\mathbf{r}) c_{nlm\sigma} = \sum_{nlm} R_{nl}(r) Y_{lm}(\hat{r}) c_{nlm\sigma}$$

$$R_{nl}(r) = \sqrt{2}\,(m\omega_0)^{3/4} \sqrt{\frac{n!}{(n+1+1/2)!}} e^{-\frac{\overline{r}^2}{2}} \overline{r}^l L_n^{l+1/2}\left(\overline{r}^2\right)$$

$$H = \sum_{nlm\sigma} \xi_{nl\sigma} c_{nlm\sigma}^\dagger c_{nlm\sigma} - \frac{U}{2} \sum_{nn'lm\sigma} J_{nn'\sigma}^l c_{nlm\sigma}^\dagger c_{n'lm\sigma} - \sum_{nn'lm} F_{nn'}^l \left[c_{nlm\uparrow}^\dagger c_{n'l-m\downarrow}^\dagger + h.c. \right]$$

$$\Delta(r) = U \sum_{nn'l} \frac{2l+1}{4\pi} R_{nl}(r) R_{n'l}(r) \langle c_{nl0\downarrow} c_{n'l0\uparrow} \rangle$$

$$n_\sigma(r) = \sum_{nn'l} \frac{2l+1}{4\pi} R_{nl}(r) R_{n'l}(r) \langle c_{nl0\sigma}^\dagger c_{n'l0\sigma} \rangle$$

J. Kinnunen, L.M. Jensen, and P. Törmä, Phys. Rev. Lett. 96, 110403 (2006)

L.M. Jensen, J. Kinnunen, and P. Törmä, cond-mat/0604424

c.f other recent theory work on imbalanced atom gases
Sheehy, Radzihovsky, Pieri, Strinati, Yi, Duan, Haque, Stoof, Chevy, De Silva, Muller, Bedaque, Caldas, Rupak, Gubbels, Romans, Chien, Chen, He, Levin, Sachdev, Yang, Imambekov, Bolech, Lukin, Demler, Machida, Mizushima, Ichioka, Martikainen, Lobo, Recati, Giorgini, Stringari, Ho, Zhai, Bulgac, Forbes, Schwenk, Castorina, Grasso, Oertel, Urban, Zappala

+ many others

- For comparison: LDA in a trap
- Imbalanced densities
- Strong interactions, T ~ 0
- Breached pair (BP)/Sarma state allowed $\delta\mu > \Delta(r)$

$$|\Psi_{BP}\rangle = \prod_{E_{\mathbf{k}\sigma}<0} c^{+}_{\mathbf{k}\sigma} \prod_{E_{\mathbf{k}\uparrow},E_{\mathbf{k}\downarrow}>0} (u_{\mathbf{k}} + v_{\mathbf{k}} c^{+}_{\mathbf{k}\uparrow} c^{+}_{-\mathbf{k}\downarrow}) |\Psi_0\rangle$$

$$\mu = (\mu_\uparrow + \mu_\downarrow)/2 \qquad \delta\mu = (\mu_\uparrow - \mu_\downarrow)/2$$

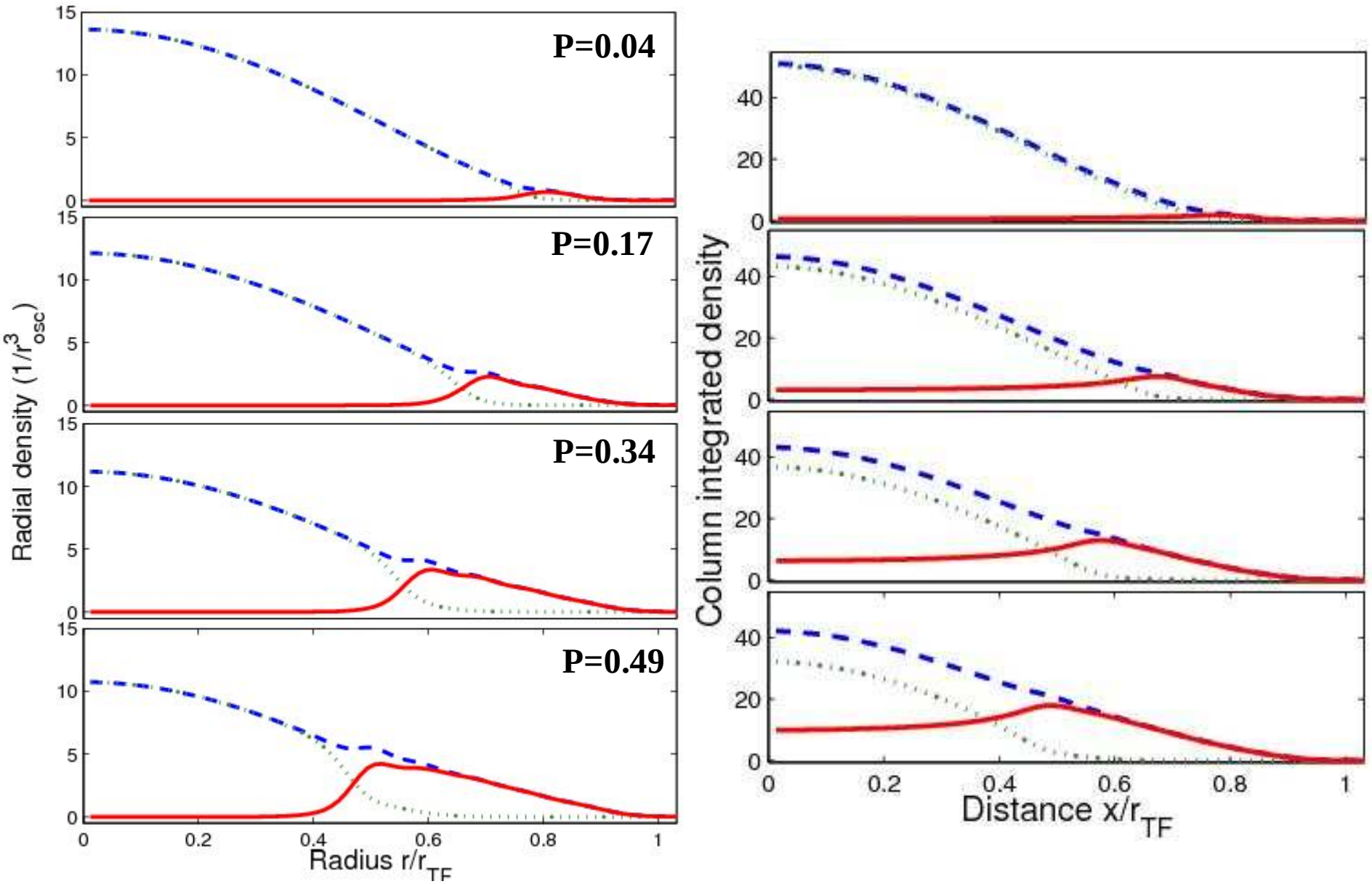
$$\xi_{\mathbf{k}} = \varepsilon_{\mathbf{k}} - \mu \qquad E_{\mathbf{k}\sigma} = \delta\mu + \sigma E_{\mathbf{k}}$$

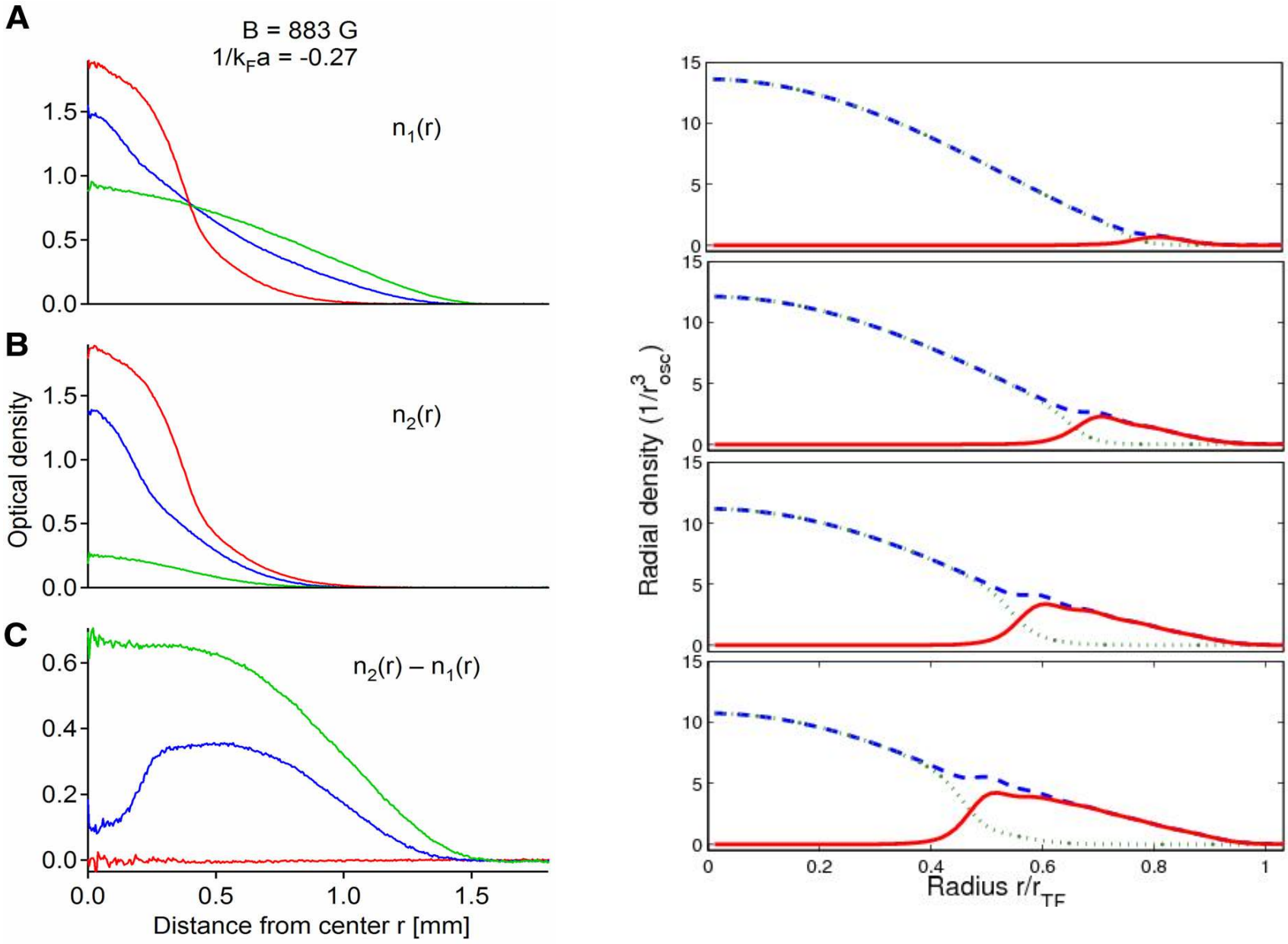
We solve the system by fixing the polarization P and the minority component density, and using δμ and μ as variables.

$$P = 4\pi \int dr r^2 \delta n(r) \qquad \frac{1-P}{2} = 4\pi \int dr r^2 n_\downarrow(r)$$

L.M. Jensen, J. Kinnunen, and P. Törmä, cond-mat/0604424

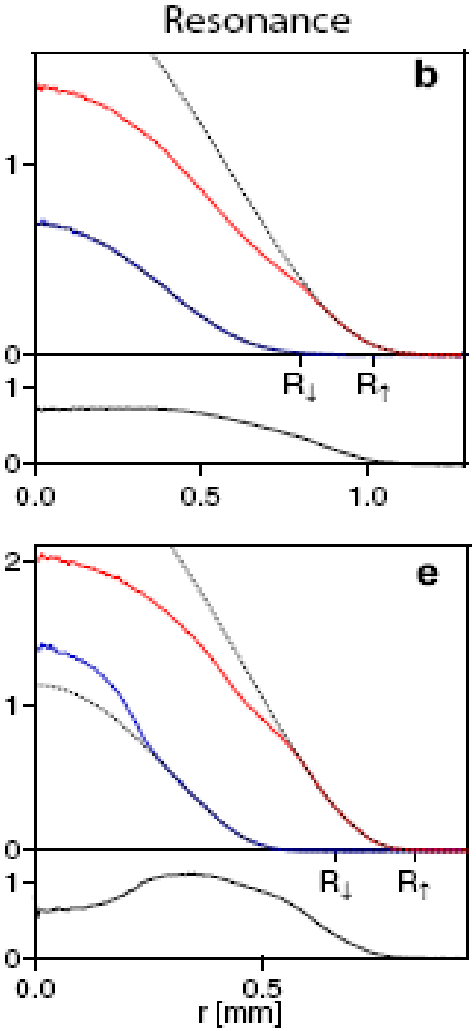
BdG


P=0.04
P=0.17
P=0.34
P=0.49
Radial density (1/r_osc^3)
Radius r/r_TF
Column integrated density
Distance x/r_TF



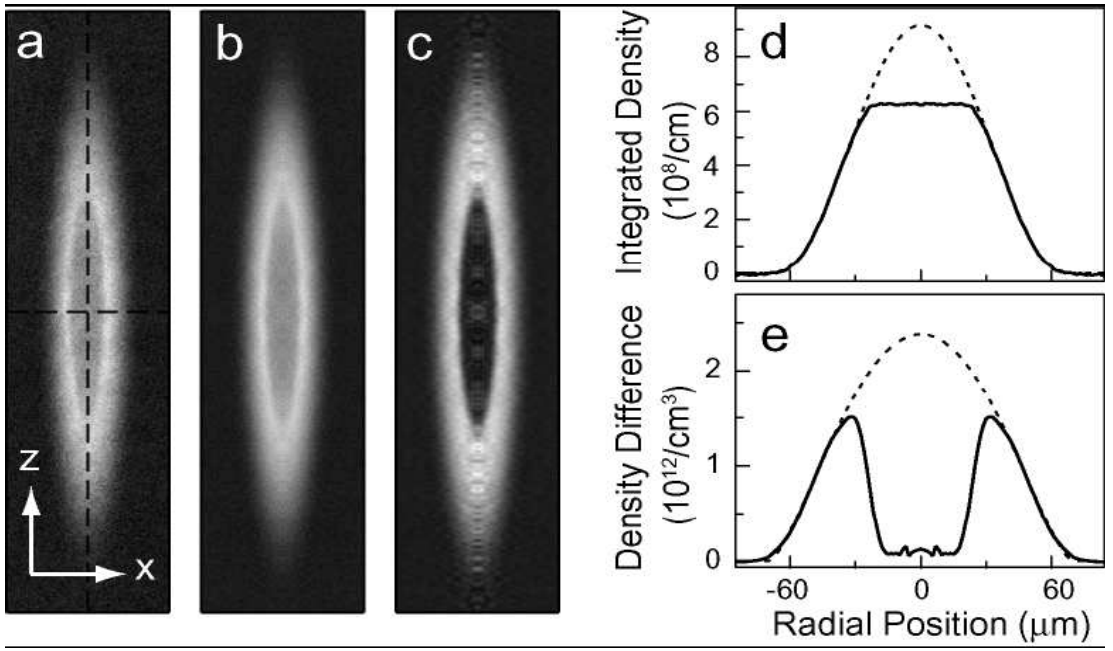
Zwierlein, Schirotzek, Schunck, Ketterle, Science 311, 492 (2006)

Condensation seen in the density profiles



Zwierlein, Schunck, Schirotzek, Ketterle 2006

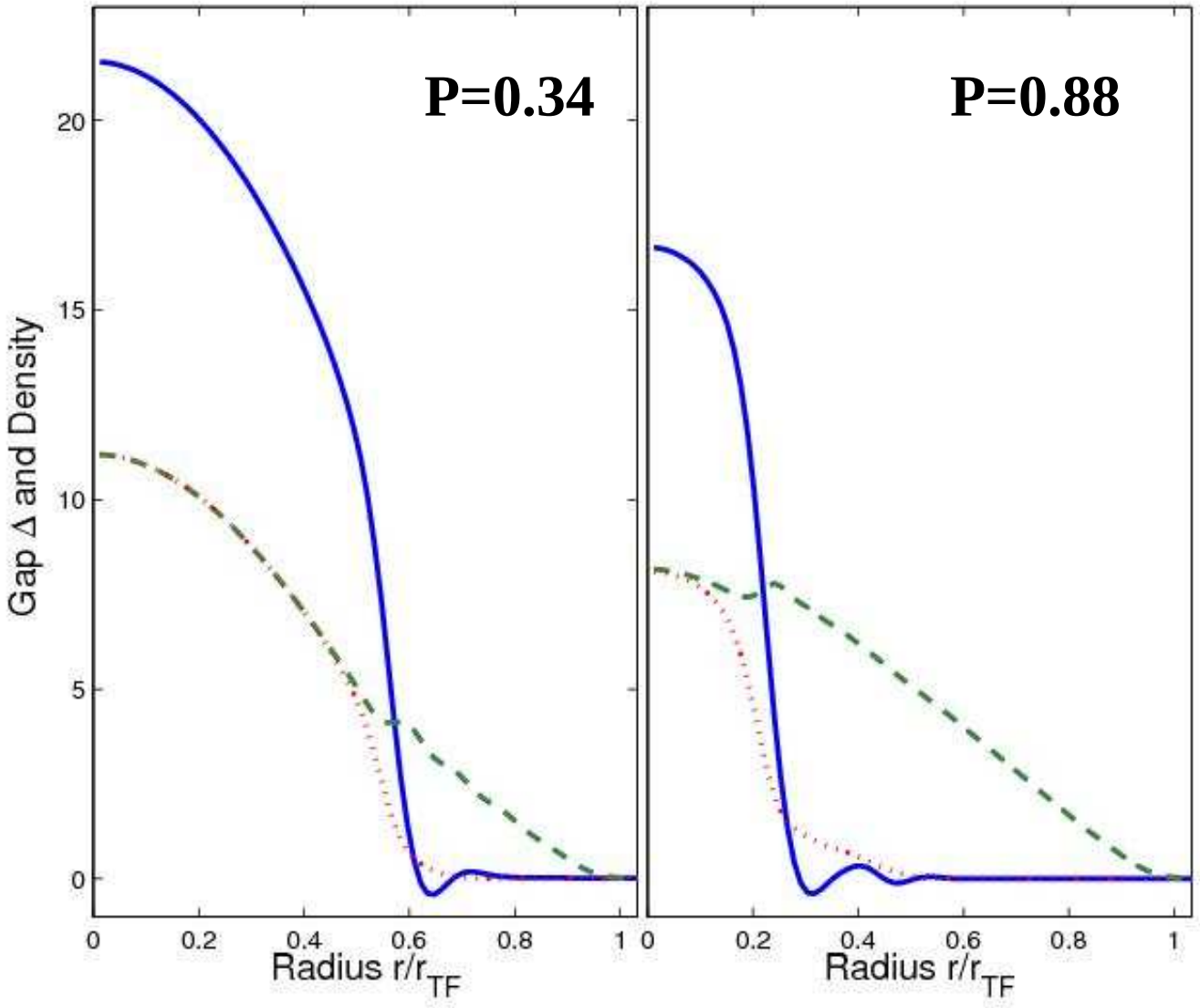
3D reconstruction



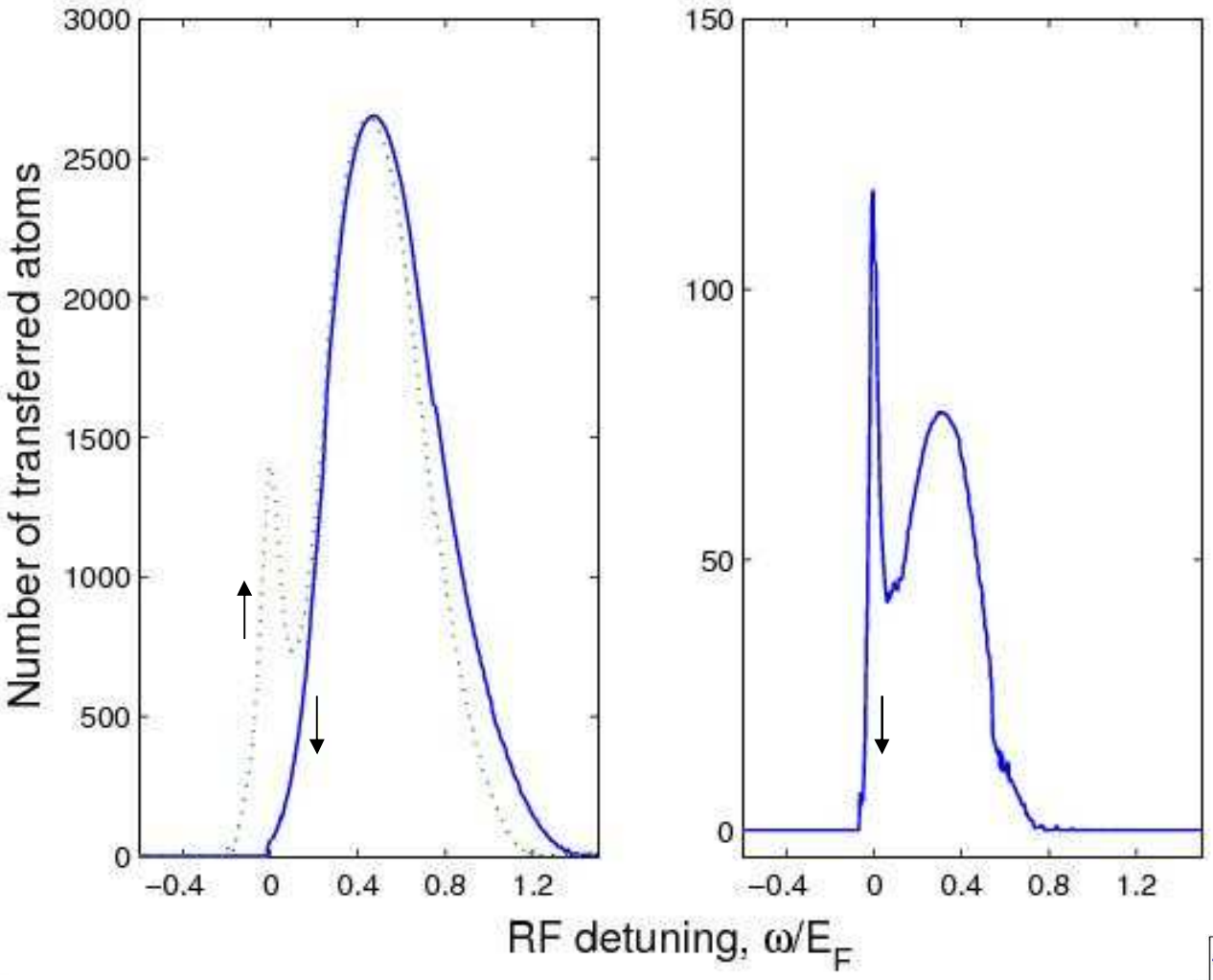
Shin, Zwierlein, Schunck, Schirotzek, Ketterle 2006

The shell structure established by the experiments and many theoretical calculations

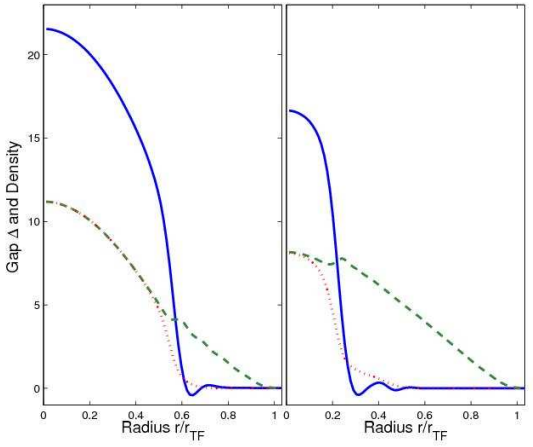
FFLO features



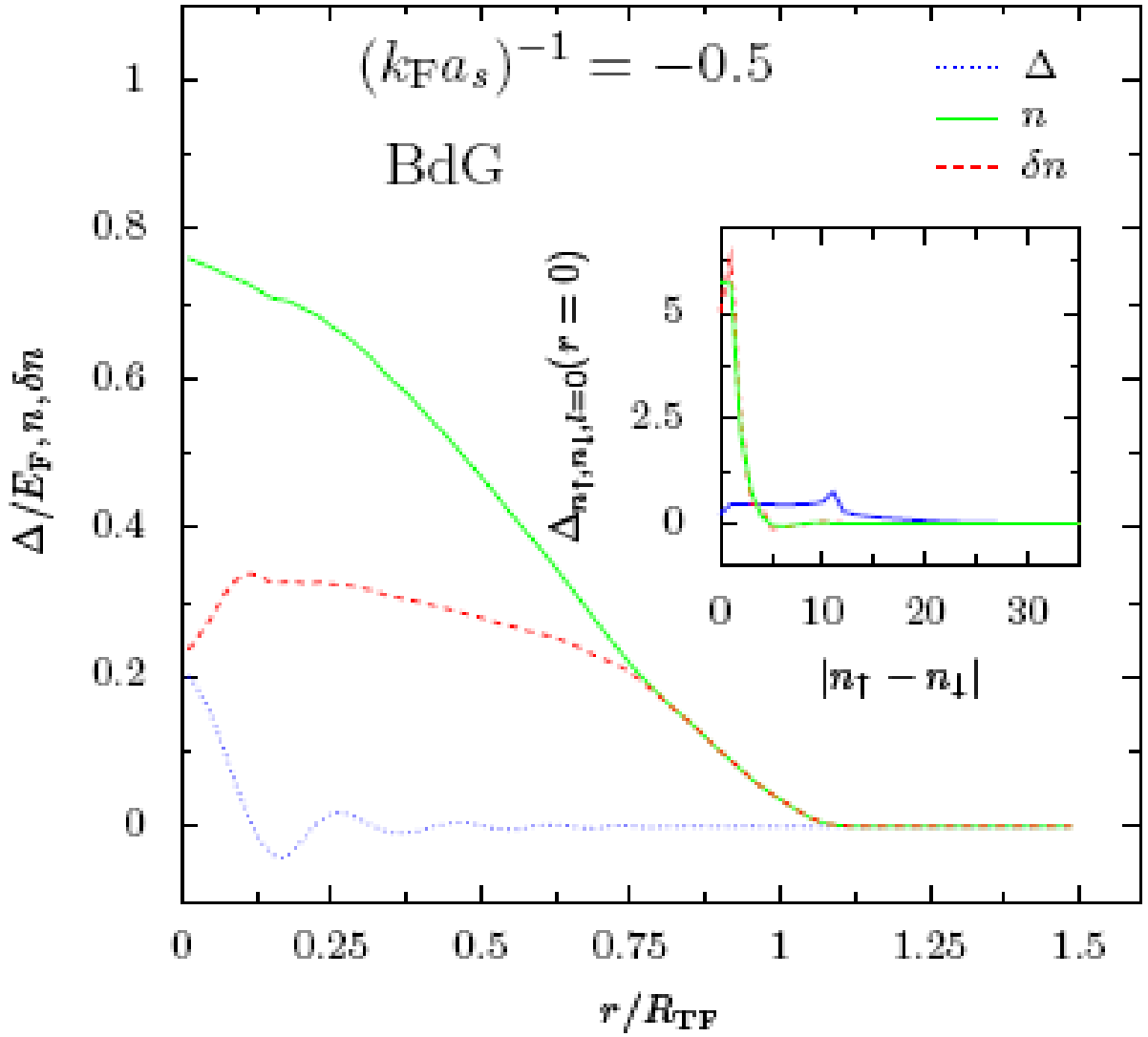
Ours: strong interactions (unitarity), c.f. Castorina et al. PRA 2005 and Mizushima et al. PRL 2005 BSC-limit, different geometry



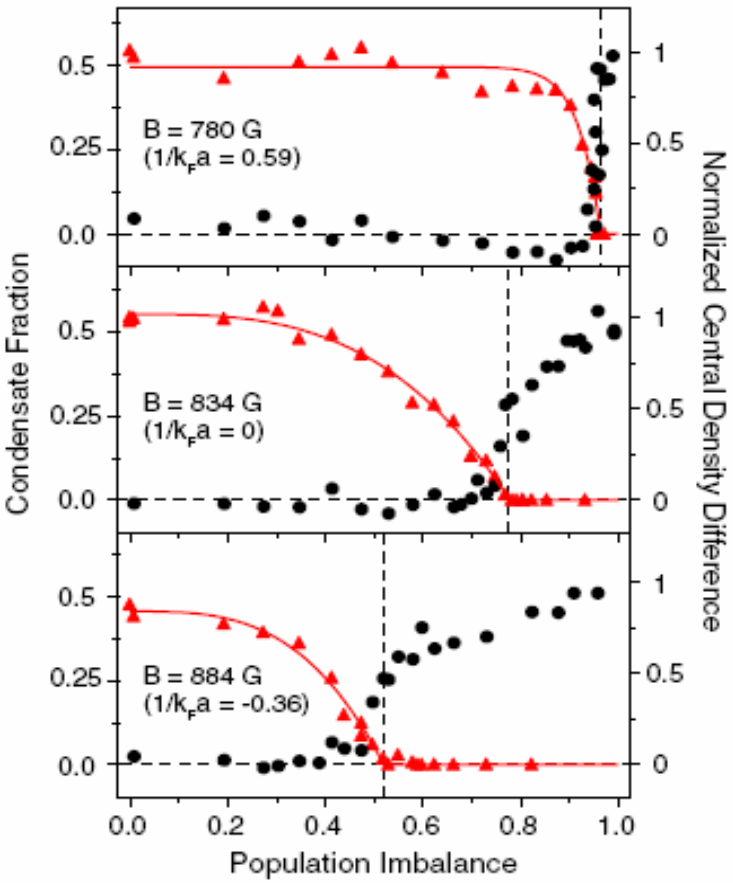
Observation
by RF-spectroscopy



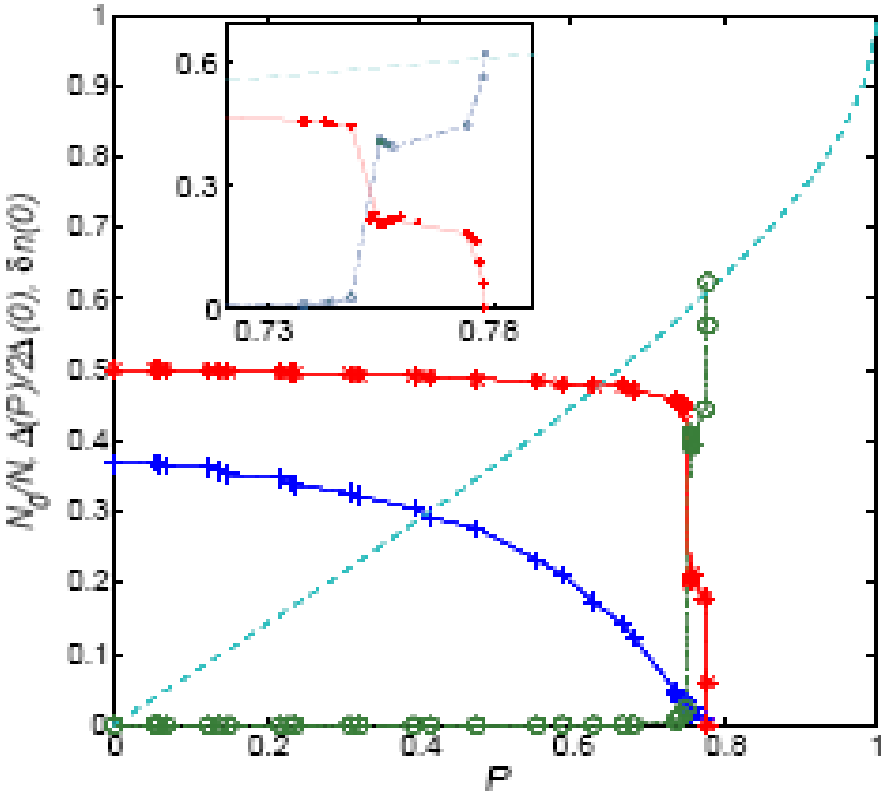
Is the ”FFLO” an edge effect (c.f. superconductor-ferromagnet interfaces)?



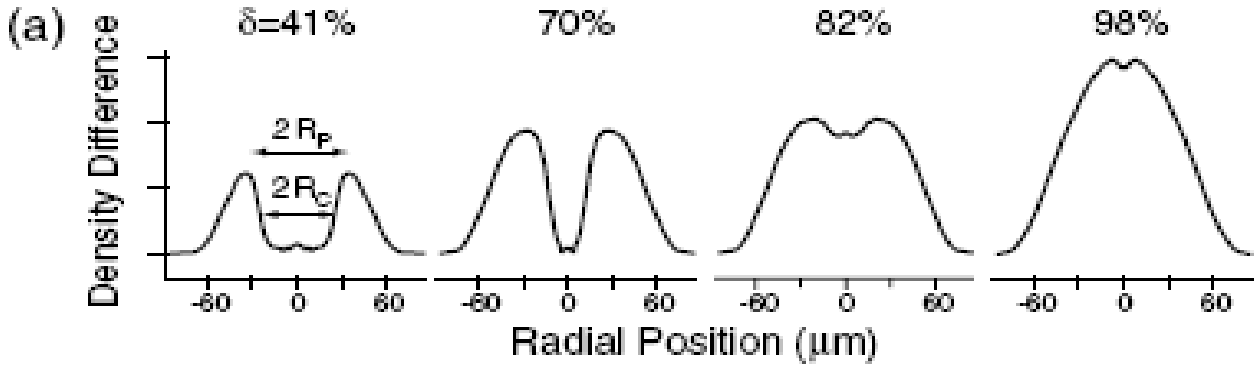
Central density difference
Condensate fraction



Condensate fraction
Central density difference
Central gap

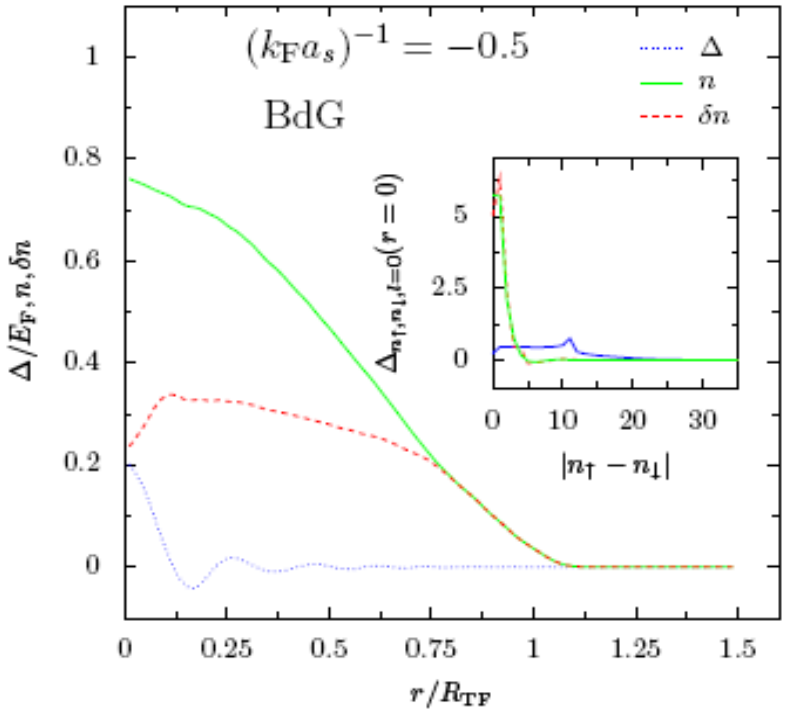


Shin, Zwierlein, Schunck, Schirotzek, Ketterle 2006



Shin, Zwierlein, Schunck, Schirotzek, Ketterle 2006

Central ”dip” at high P: Signature of FFLO (T=0), BP or pseudogap



Imbalanced gas in a 3D optical lattice

- BCS+BP in a lattice (Hubbard Hamiltonian), FFLO-type ansatz by Q, comparison to phase separation (BCS+normal)

$$\Delta e^{iQr}$$

$$E_{k,Q} = \left(\frac{\xi_{k+Q} - \xi_{k-Q}}{2} \pm \sqrt{\frac{\xi_{k+Q} + \xi_{k-Q}}{2} + \Delta^2} \right)$$

$$n = (n_\uparrow + n_\downarrow)/2$$

$$P = (N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow)$$

Minimize

$$F = \Omega + \mu_\uparrow n_\uparrow + \mu_\downarrow n_\downarrow$$

Phase separation

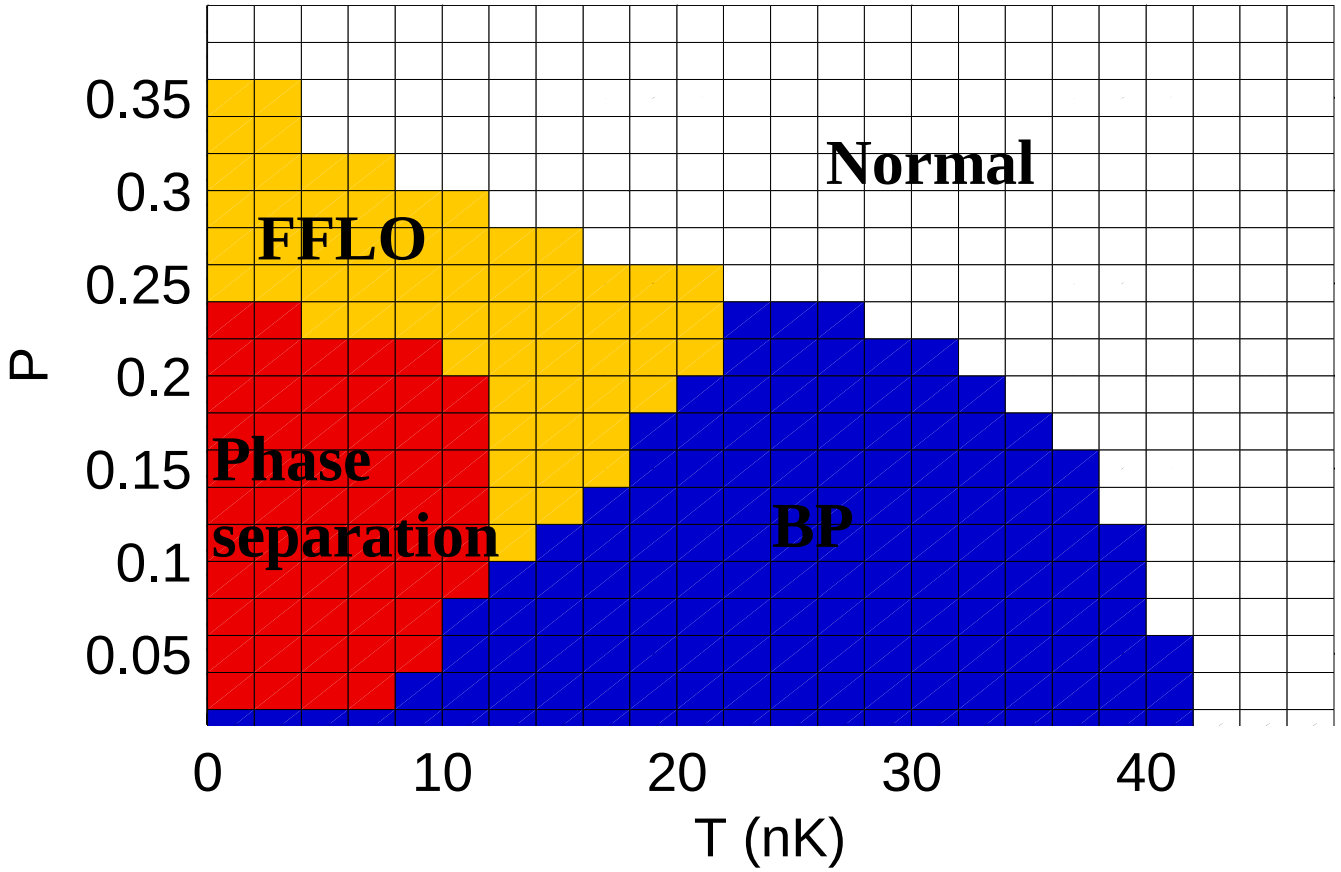
$$x, (1-x)$$

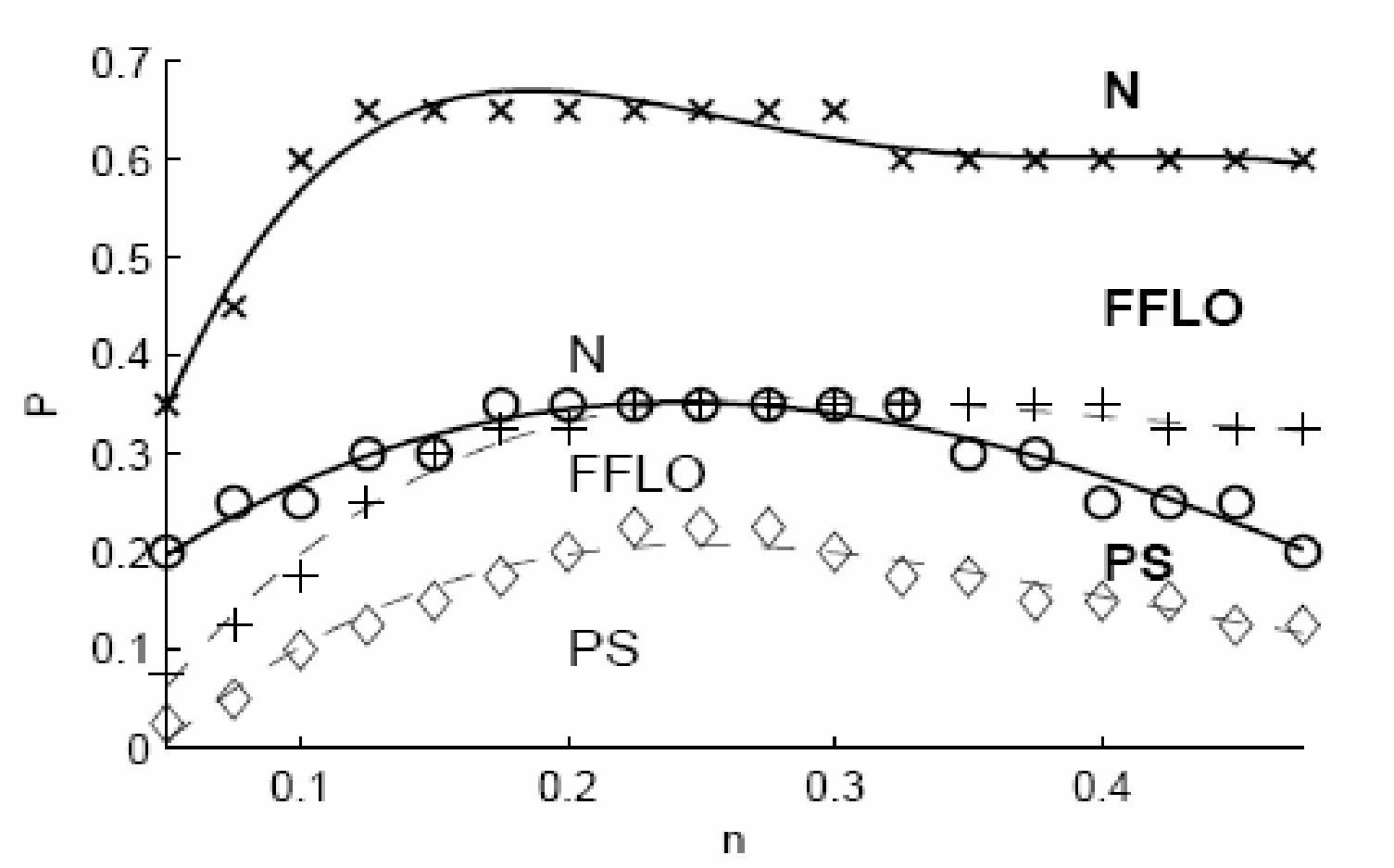
$$x : 2N_{BCS}$$

$$(1-x) : N_\uparrow - N_{BCS}, N_\downarrow - N_{BCS}$$

T. Koponen, T. Paananen, J.-P. Martikainen, and P. Törmä, cond-mat/0701484 (2007)

T. Koponen, J. Kinnunen, J.-P. Martikainen, L.M. Jensen, and P. Törmä, New J. Phys. 8, 179 (2006)

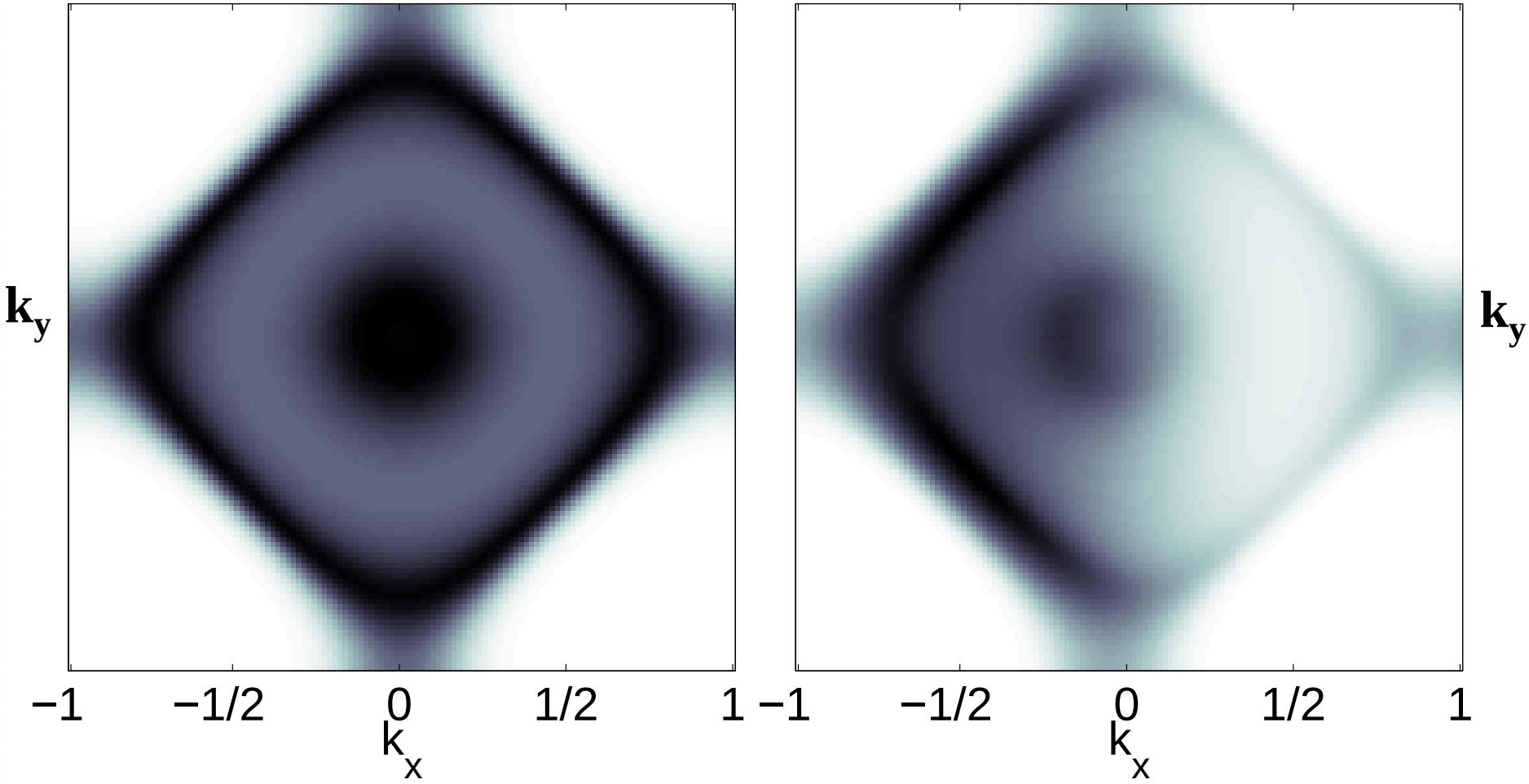

0.35
0.3
0.25
0.2
0.15
0.1
0.05
p
Normal
FFLO
Phase
separation
BP
0
10
20
30
40
T (nK)



T=0

Observation from density distributions

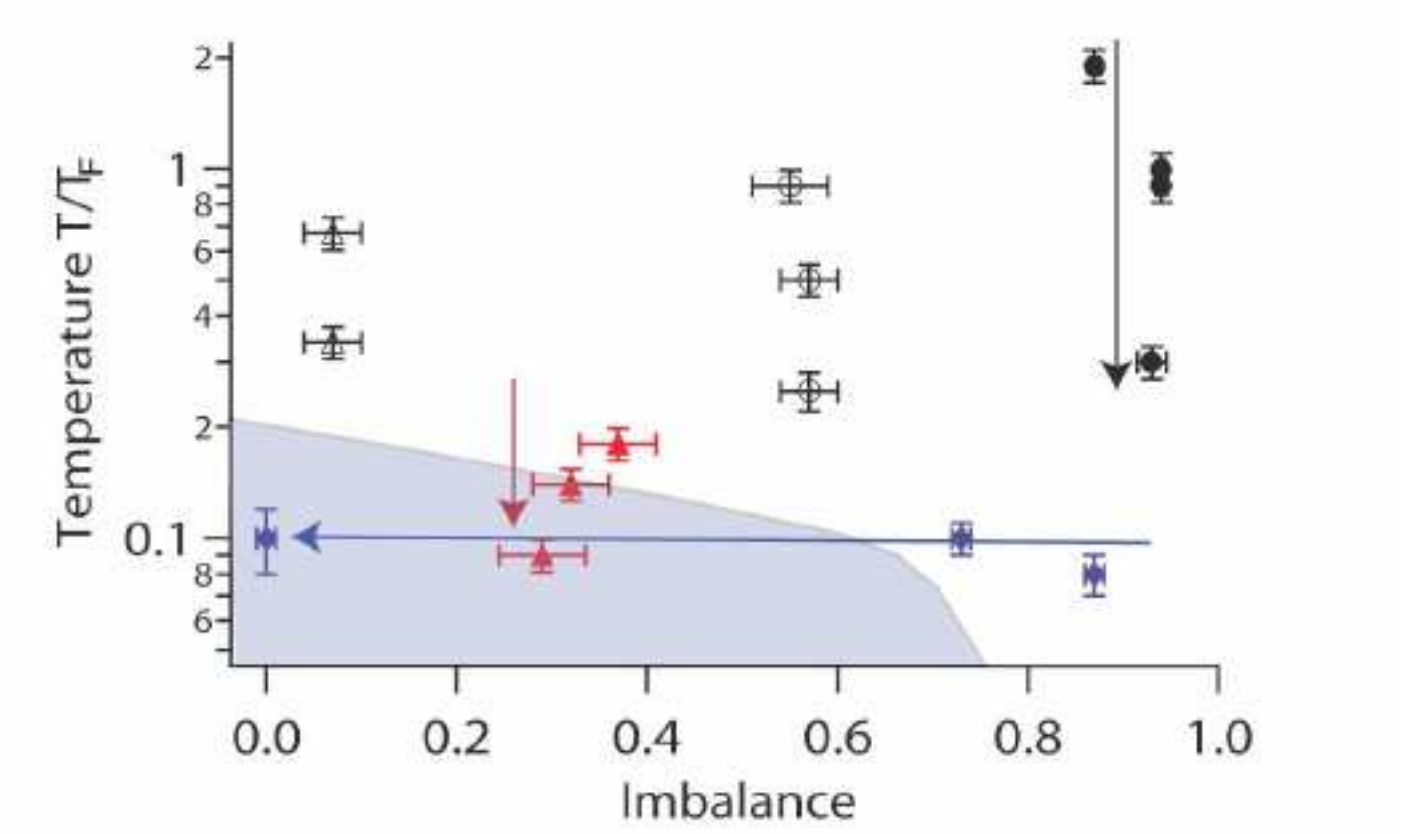
$$n_{k\uparrow} - n_{k\downarrow}$$

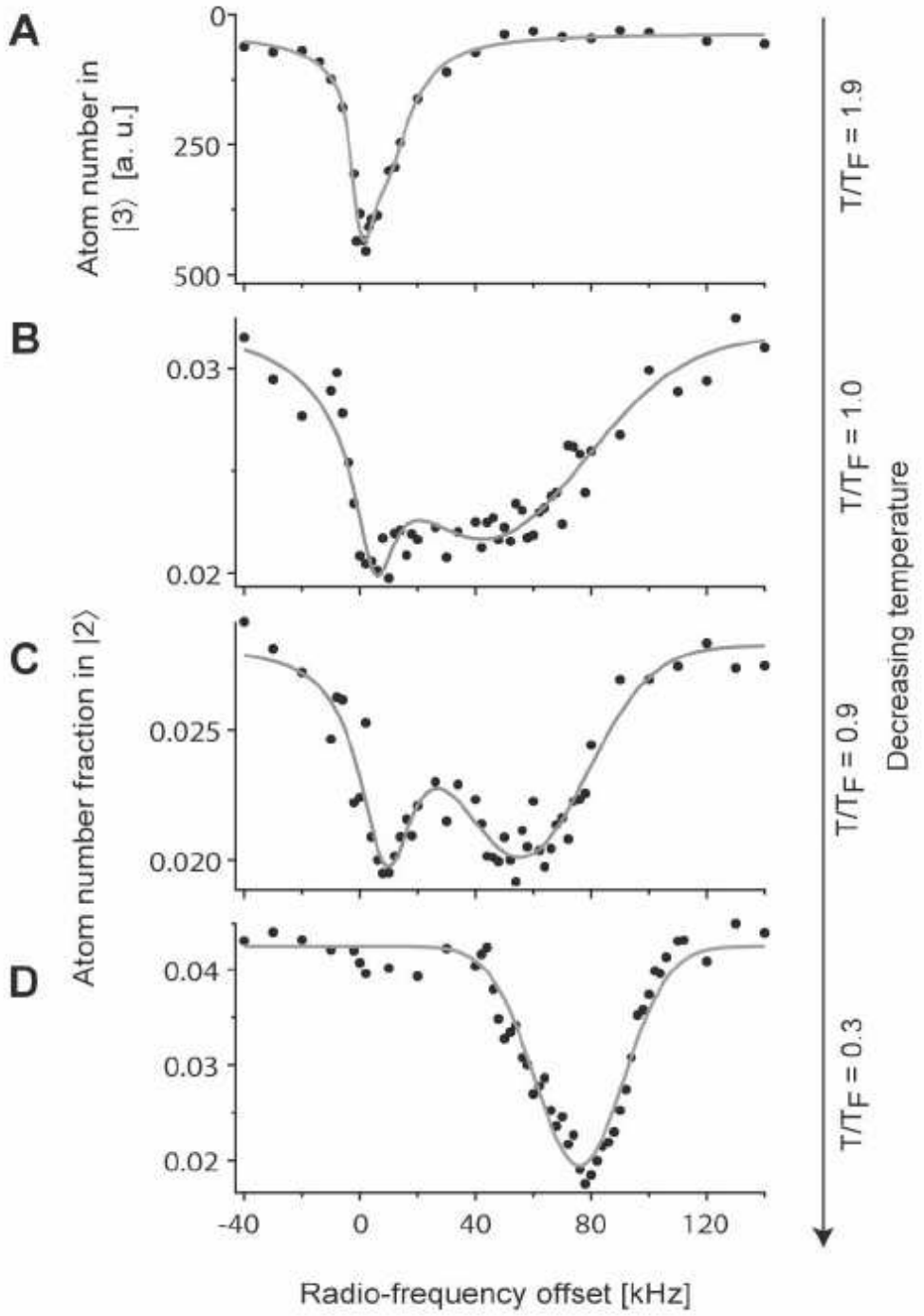


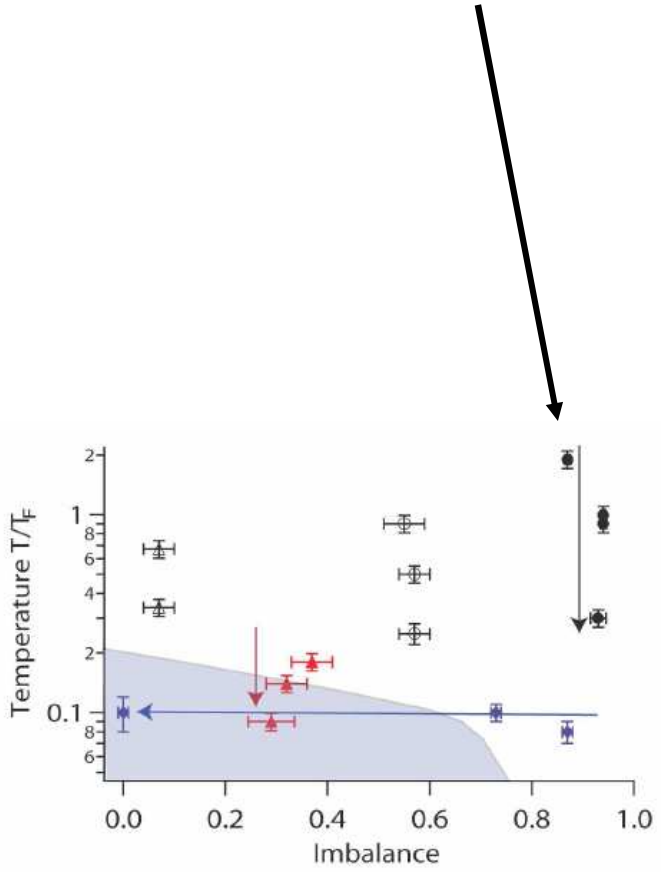
BP, T = 20 nK

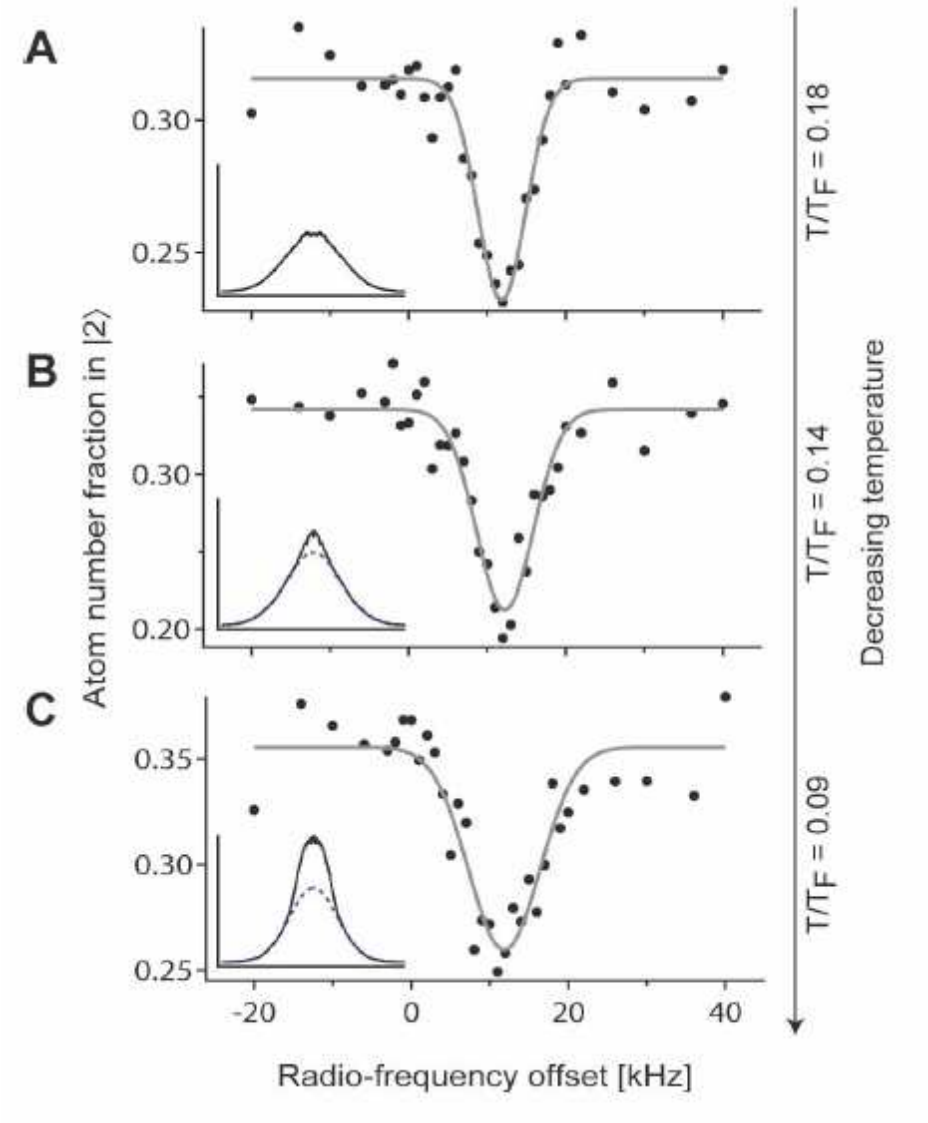
FFLO, T = 15 nK

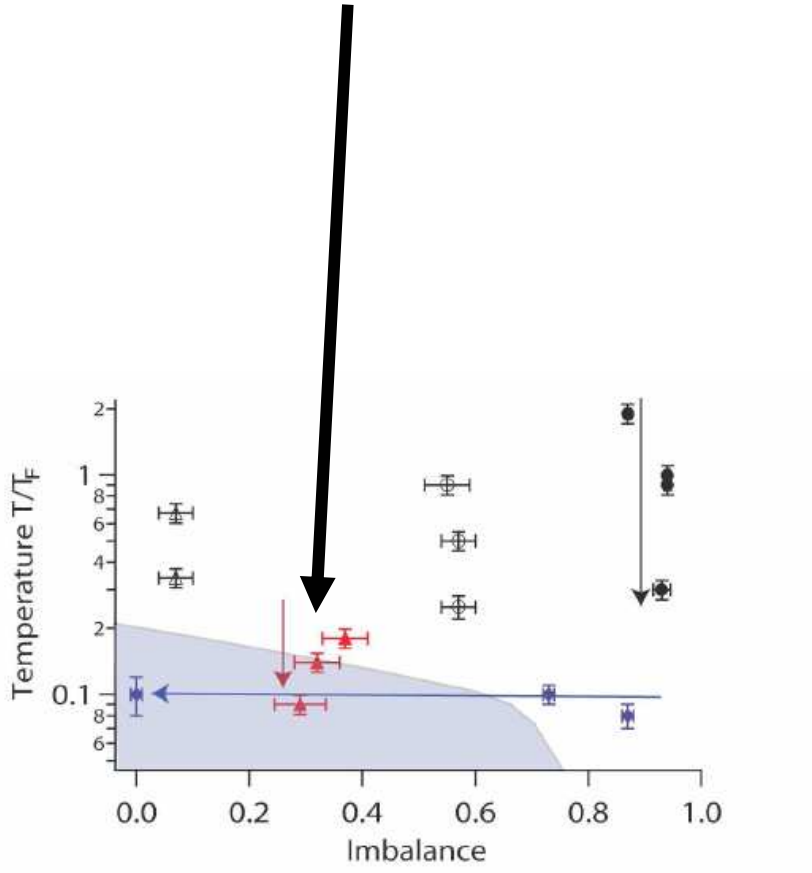
Schunck, Shin, Schirotzek, Zwierlein, Ketterle 2007

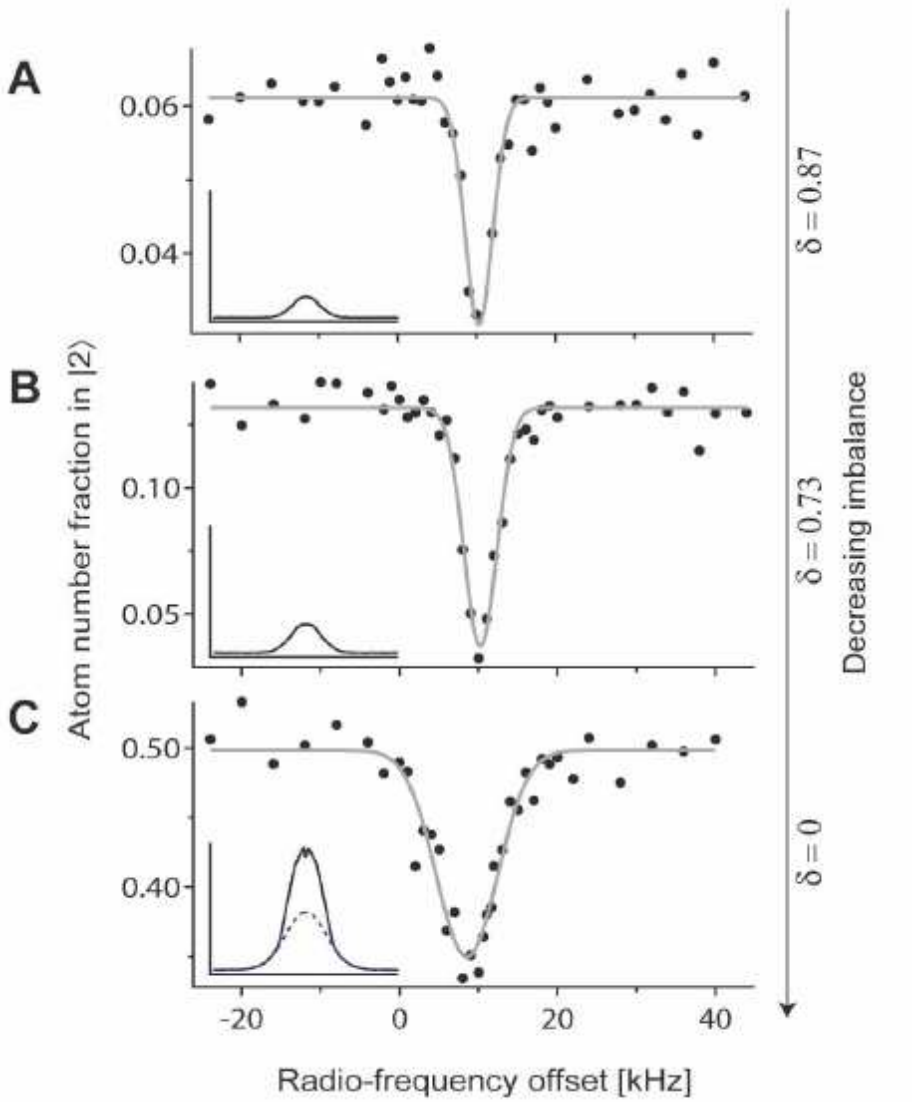


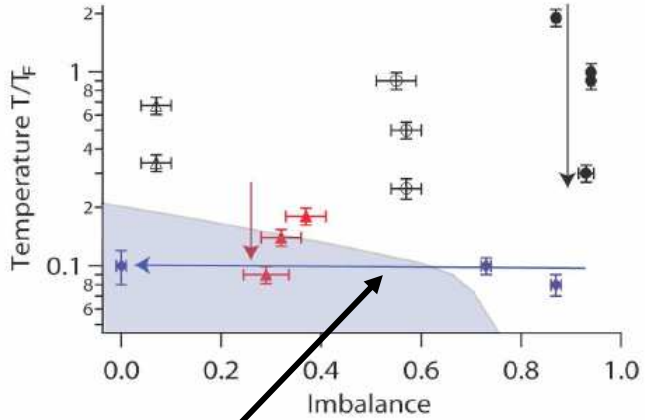

A
B
C
D
Atom number in |3⟩ [a. u.]
Atom number fraction in |2⟩
0
250
500
0.03
0.02
0.025
0.020
0.04
0.03
0.02
-40
0
40
80
120
Radio-frequency offset [kHz]
T/TF = 1.9
T/TF = 1.0
T/TF = 0.9
T/TF = 0.3
Decreasing temperature

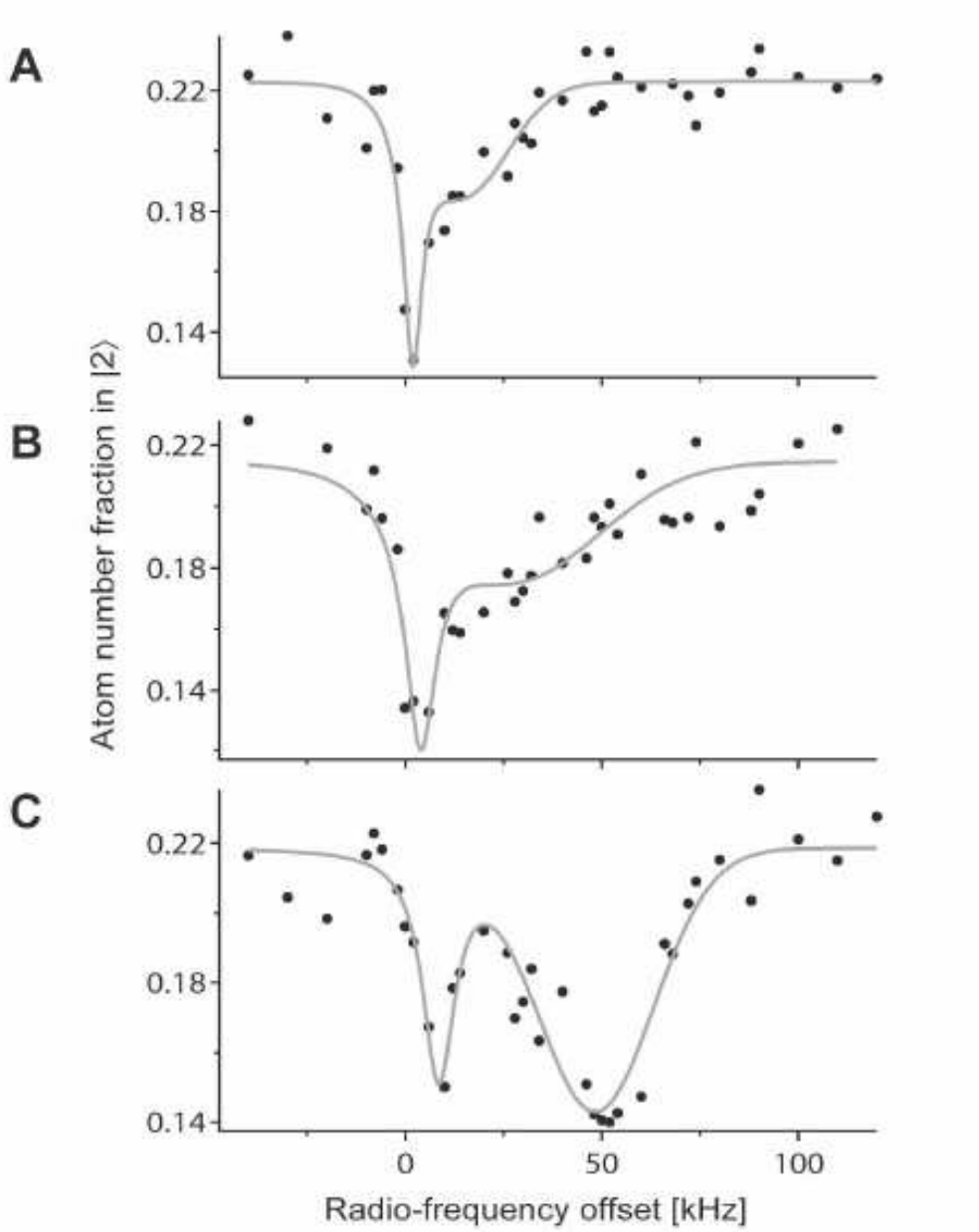

Temperature T/TF
1
0.1
0.0
0.2
0.4
0.6
0.8
1.0
Imbalance

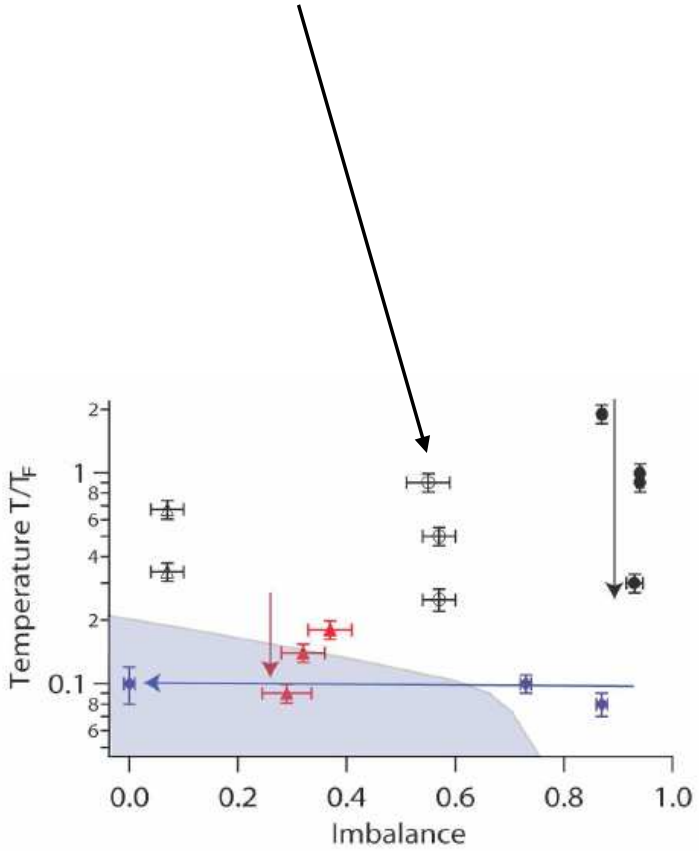

A
B
C
0.30
0.25
0.30
0.20
0.35
0.30
0.25
-20
0
20
40
Atom number fraction in |2⟩
Radio-frequency offset [kHz]
T/T_F = 0.18
T/T_F = 0.14
T/T_F = 0.09
Decreasing temperature

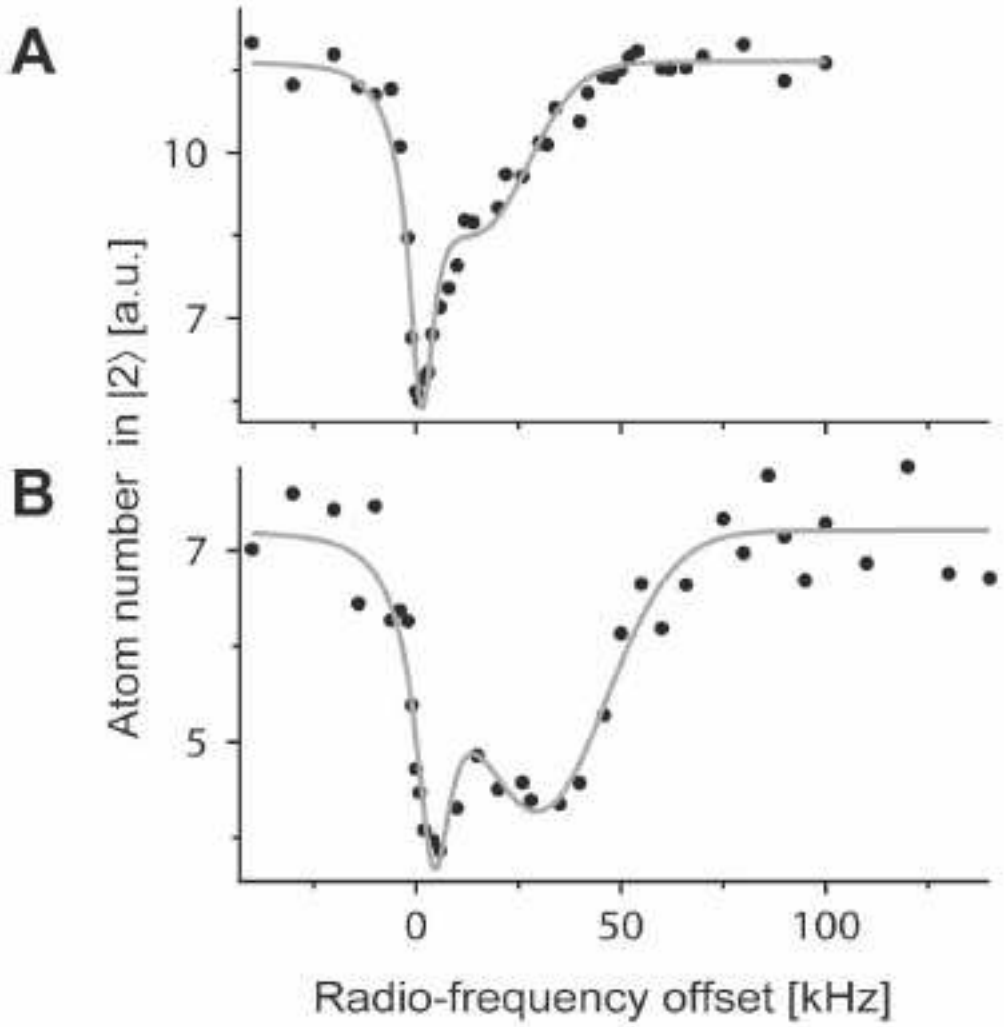

Temperature T/T_F
1
0.1
0.0
0.2
0.4
0.6
0.8
1.0
Imbalance

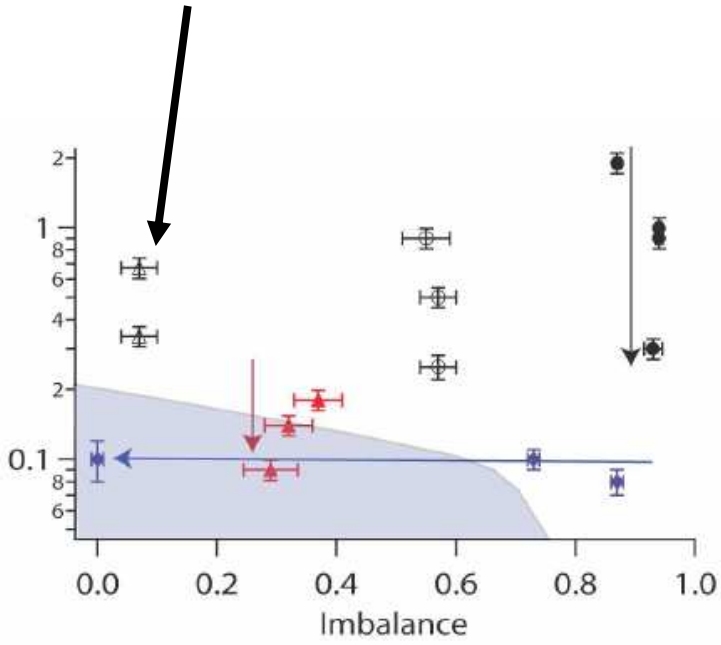

A
B
C
0.06
0.04
0.10
0.05
0.50
0.40
Atom number fraction in |2⟩
δ = 0.87
δ = 0.73
δ = 0
Decreasing imbalance
-20
0
20
40
Radio-frequency offset [kHz]

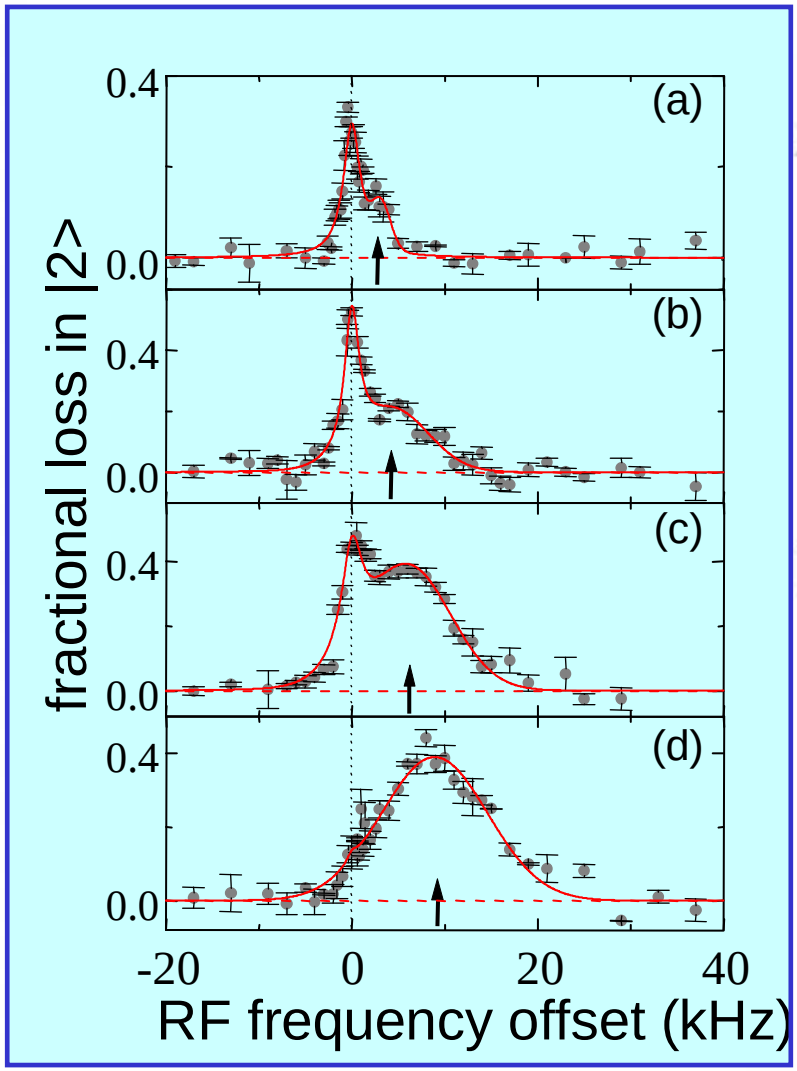

Temperature T/T_F
2
1
0.1
0.0
0.2
0.4
0.6
0.8
1.0
Imbalance

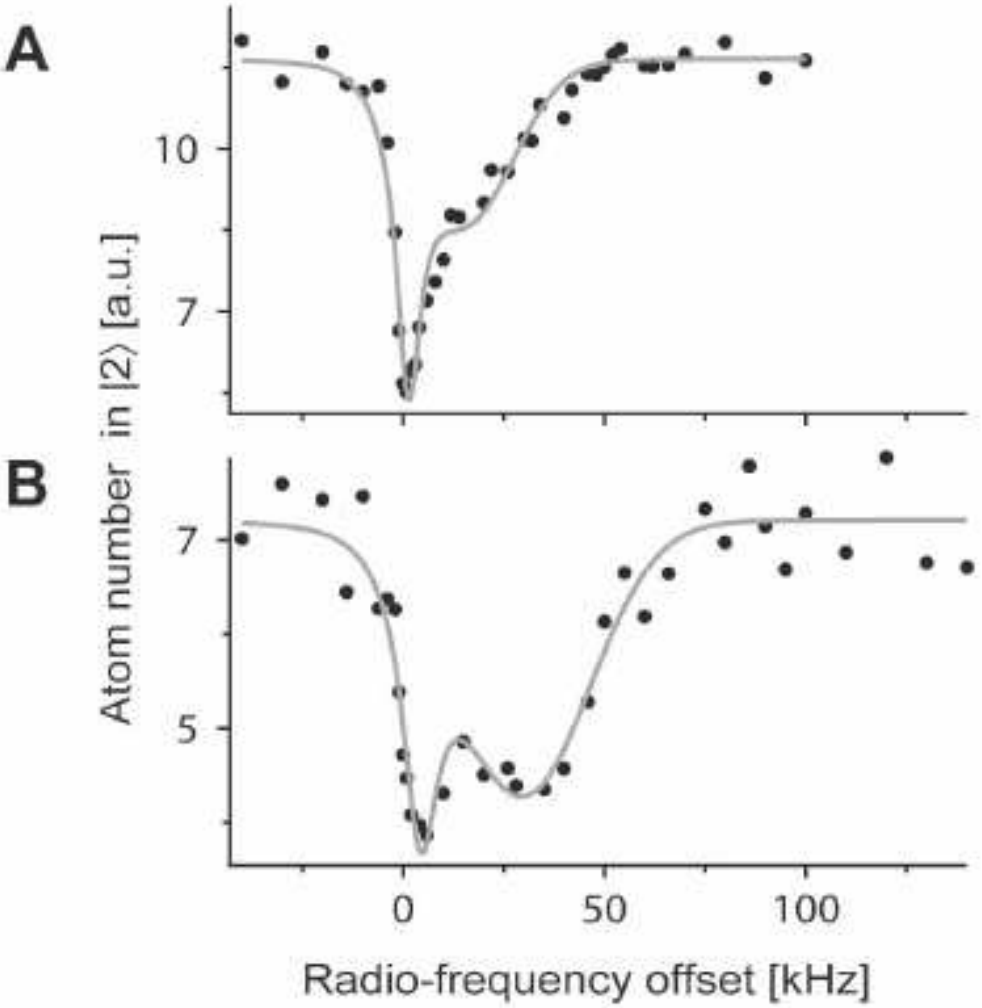

A
B
C
0.22
0.18
0.14
Atom number fraction in |2⟩
0
50
100
Radio-frequency offset [kHz]

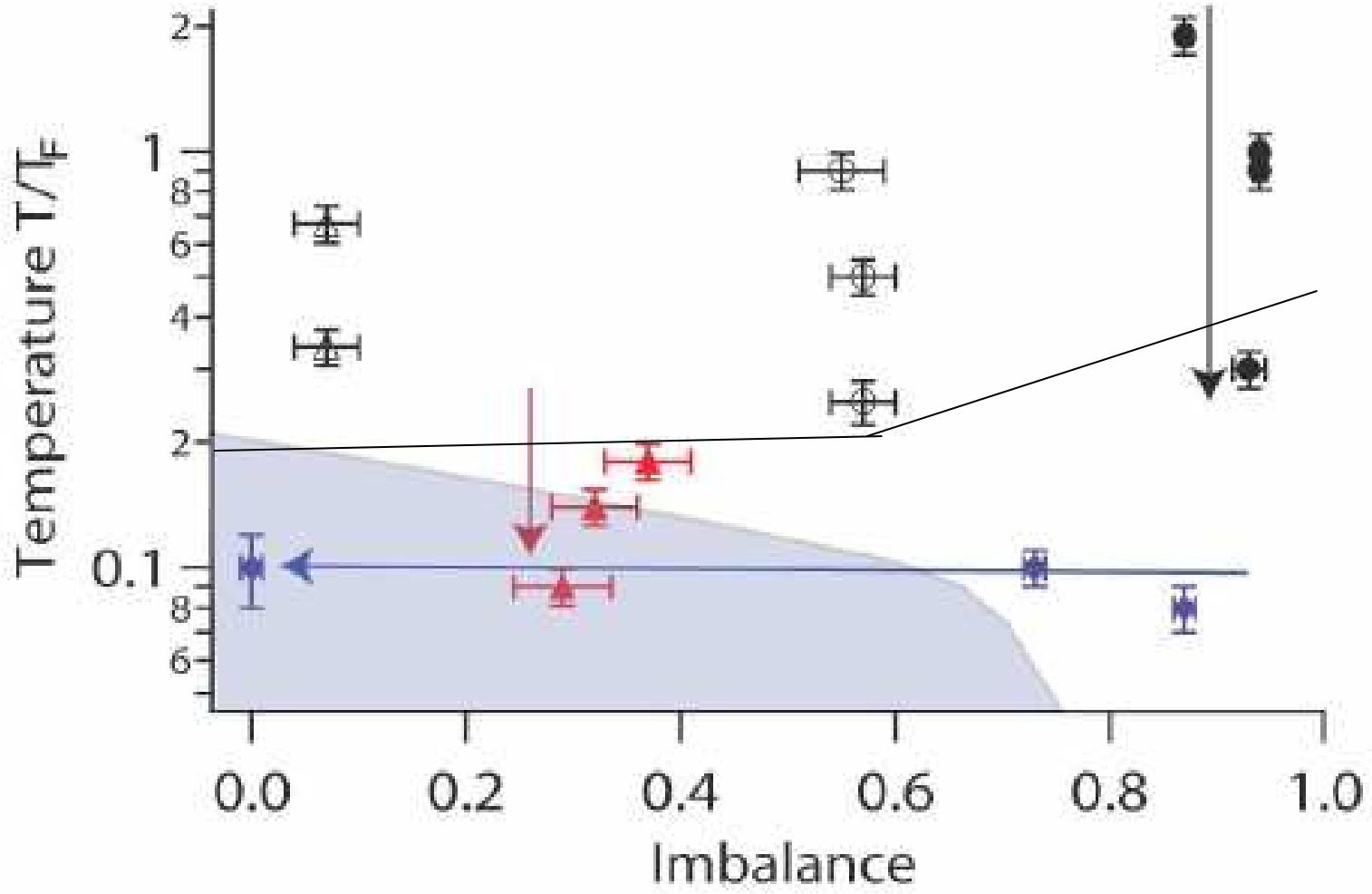

Temperature T/T_F
1
0.1
0.0
0.2
0.4
0.6
0.8
1.0
Imbalance


A
B
10
7
7
5
0
50
100
Atom number in |2⟩ [a.u.]
Radio-frequency offset [kHz]


2
1
8
6
4
2
0.1
8
6
0.0
0.2
0.4
0.6
0.8
1.0
Imbalance


fractional loss in |2>
0.4
0.0
(a)
(b)
(c)
(d)
-20
0
20
40
RF frequency offset (kHz)


A
B
Atom number in |2⟩ [a.u.]
10
7
7
5
0
50
100
Radio-frequency offset [kHz]


Temperature T/T_F
1
8
6
4
2
0.1
8
6
0.0
0.2
0.4
0.6
0.8
1.0
Imbalance

Summary

- RF-spectroscopy of the pairing gap
- Imbalanced density Fermi gas
  - BdG: FFLO-type state in a trap
  - Phase diagrams (PS, FFLO, BP) in optical lattices: FFLO stabilized in a large parameter window
- Pairing gap in imbalanced gases (new experiments from MIT)