FERMION HUBBARD MODEL AND COLD ATOMS

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Main references: Paramekanti et. al PRL 87, 217002 (2001); PRB 70, 054504 (2004)

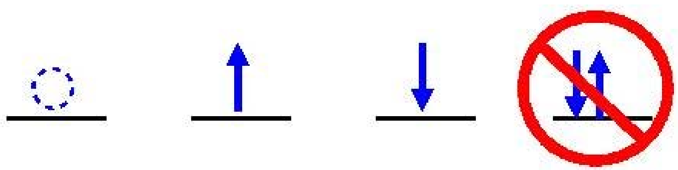
Variational Ground State Wavefunction

$$| \Psi_0 > = \exp(-iS)\ \mathbf{P}\ | dBCS >$$

| dBCS > = d-wave BCS state

variational parameters Δ and μ

$$U = \infty$$



Gutzwiller Projection:

$$P = \prod_i \left(1 - n_{i\uparrow} n_{i\downarrow}\right)$$

exp(-iS) = Unitary transformation:

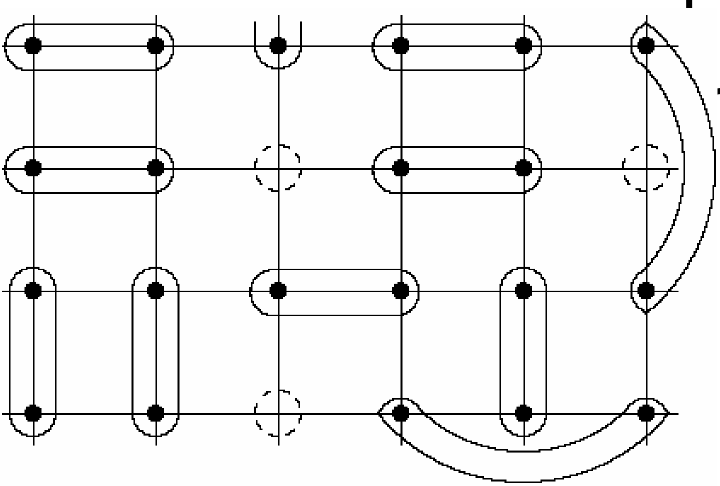
Hubbard → tJ + … brings in the scale J

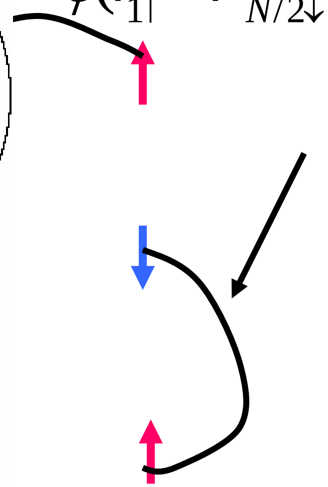
Kohn (`64); Gross, Joynt, Rice *(`87)*

Configuration of electrons

$$R = \{r_{1\uparrow}, r_{2\uparrow}, \cdots r_{N/2\uparrow}; r_{1\downarrow}, r_{2\downarrow}, \cdots r_{N/2\downarrow}\}$$

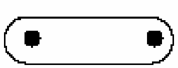
$$\mathbf{P}\,\langle R \,|\, \psi_{BCS}\rangle = \mathbf{P}\begin{vmatrix} \phi(r_{1\uparrow} - r'_{1\downarrow}) \cdots\cdots\cdots \phi(r_{1\uparrow} - r'_{1\downarrow}) \\ \phi(r_{1\uparrow} - r'_{2\downarrow}) \cdots\cdots \\ \vdots \\ \vdots \\ \phi(r_{1\uparrow} - r'_{N/2\downarrow}) \cdots\cdots\cdots \phi(r_{N/2\uparrow} - r'_{N/2\downarrow}) \end{vmatrix}$$





Projected SC
Resonating Valence
Bond (RVB) liquid

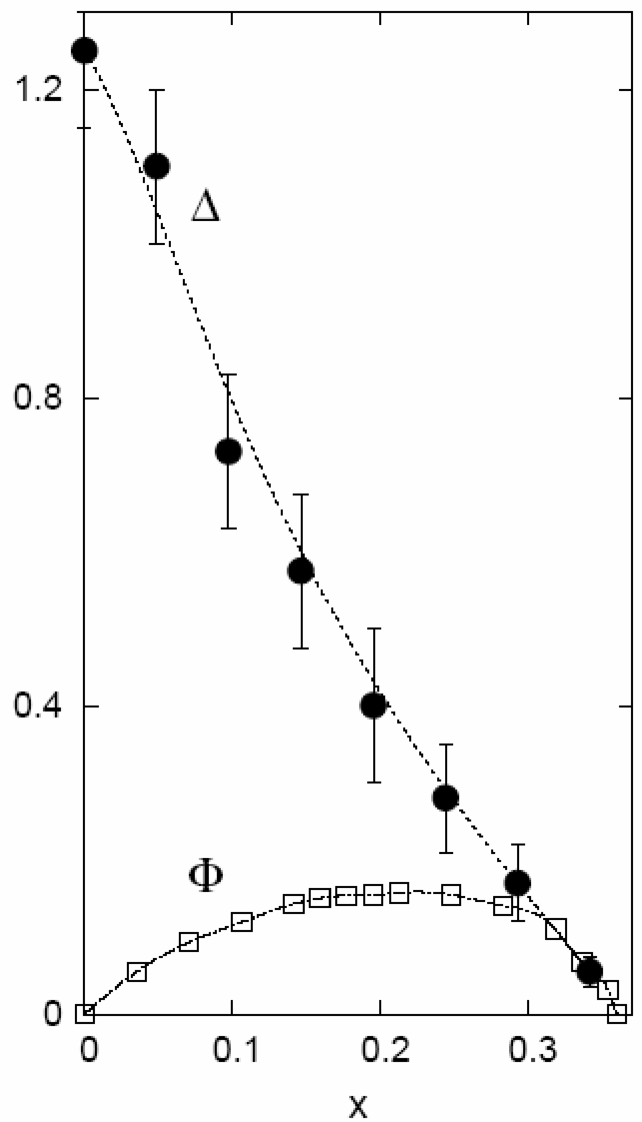
P.W. Anderson,
Science 235, 1196 (1987)

r  $r' = \frac{|\uparrow_r \downarrow_{r'}\rangle - |\downarrow_r \uparrow_{r'}\rangle}{\sqrt{2}}$ $\quad \varphi(\mathbf{r} - \mathbf{r}')$

Electrons paired into singlets

$$\varphi(\mathbf{r} - \mathbf{r}') = \sum_k \exp(i\mathbf{k} \cdot (\mathbf{r} - \mathbf{r}'))\,(v_{\mathbf{k}}/u_{\mathbf{k}})$$

Two Energy Scales



Pairing & Superconductivity

Pairing

→ **variational Δ**

SC order parameter Φ

→ **from ODLRO** $\langle c^{\dagger} c^{\dagger} c c \rangle$

Strong Coulomb U

Φ(x) ~ x as x $\rightarrow$ 0

Quantum phase fluctuations

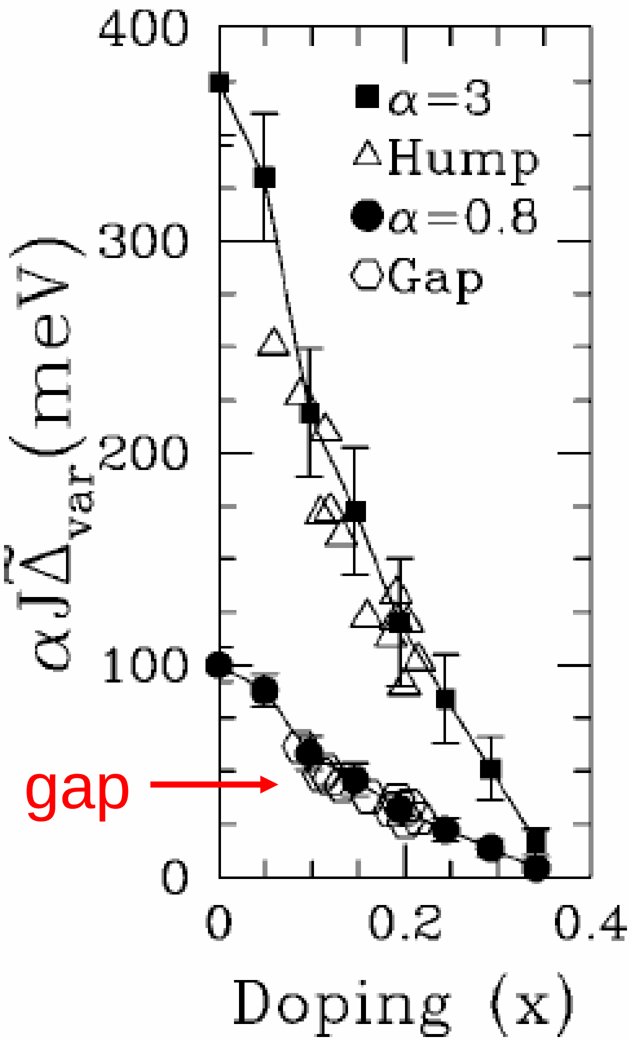
Paramekanti, Randeria & N. T.,
PRL 87, 217002 (2001); PRB 70, 054504 (2004)

Variational Energy Gap

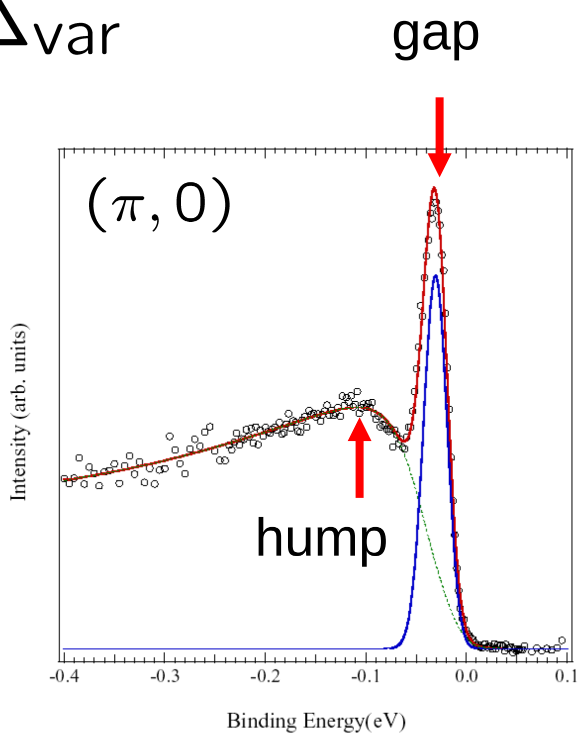
$$|0\rangle = e^{-iS} P |\text{dBCS}\rangle$$

$$|k\sigma\rangle = e^{-iS} P \gamma^{\dagger}_{k\sigma} |\text{dBCS}\rangle$$

$$E_{\text{gap}} \simeq J\tilde{\Delta}_{\text{var}}$$



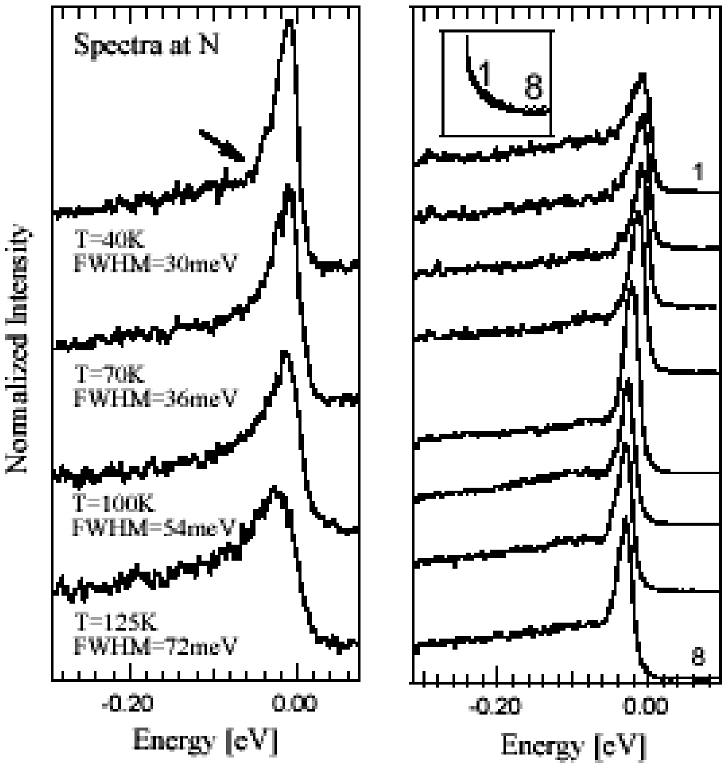
Filled symbols: theory
Open symbols: expt
[Campuzano *et al*, PRL (1999)]



ARPES: Ding et al. PRL (2001)

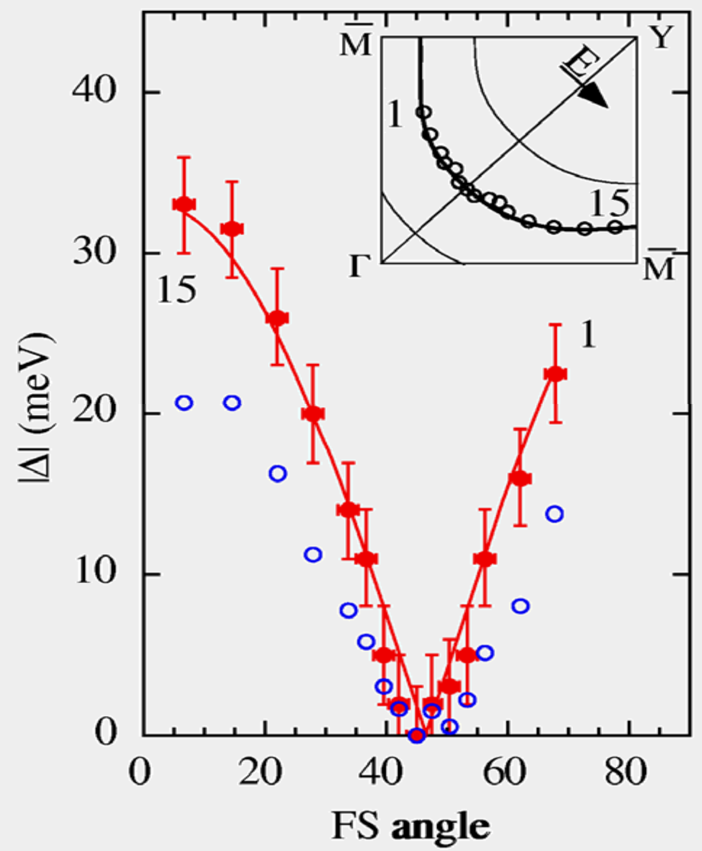
Quasiparticles in SC state

A. Kaminski et al., PRL (2000; 2001)

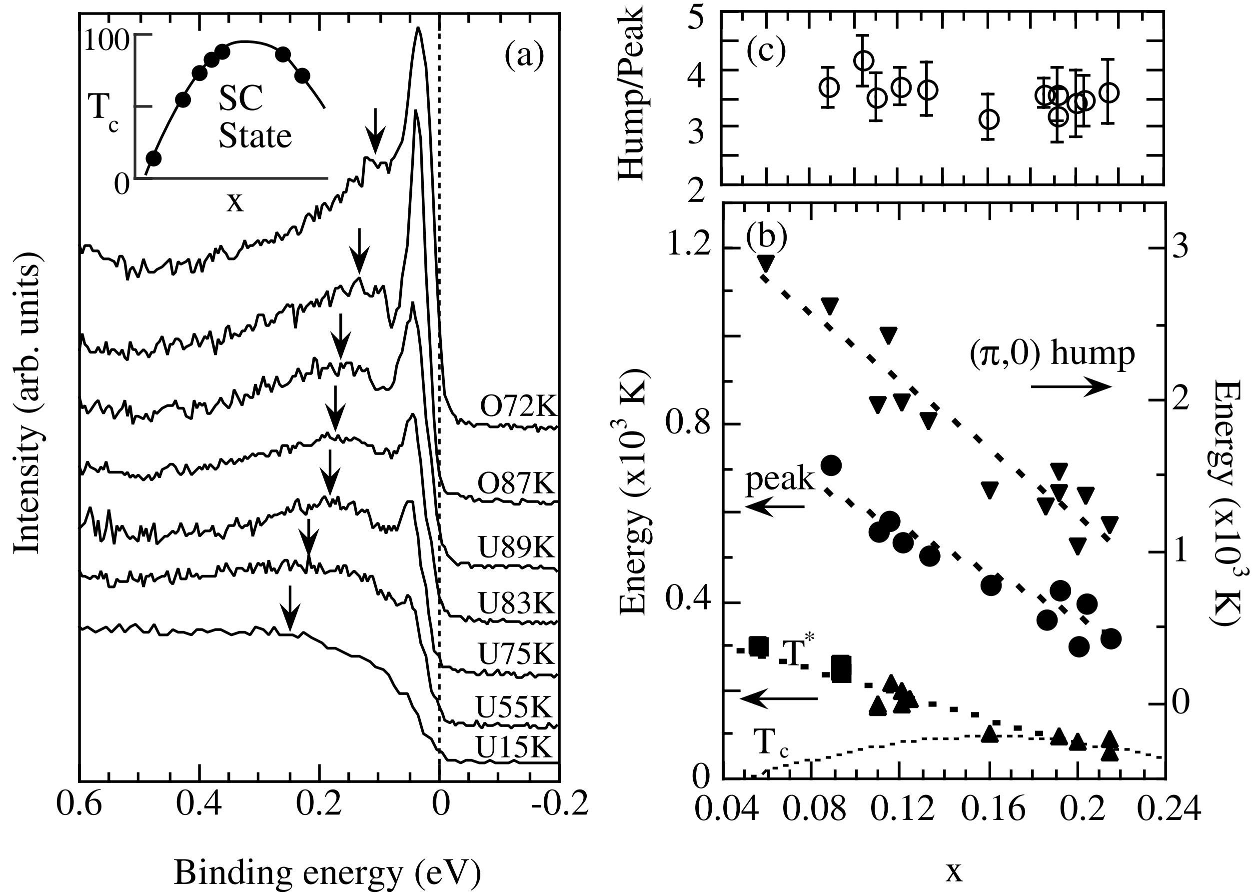


ARPES: Bi2212 SC Gap Anisotropy

H. Ding et al., PRB (1996)



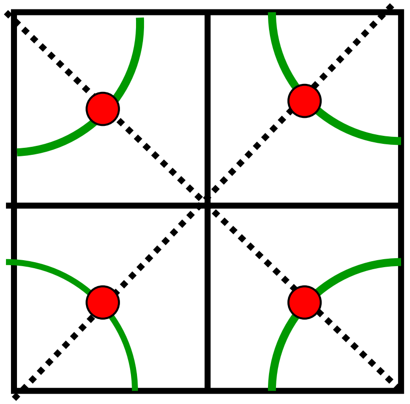
$$\Delta(\mathbf{k}) = \Delta_0 (\cos k_x - \cos k_y)$$



$$\Delta \not\propto T_c$$

Low energy excitations

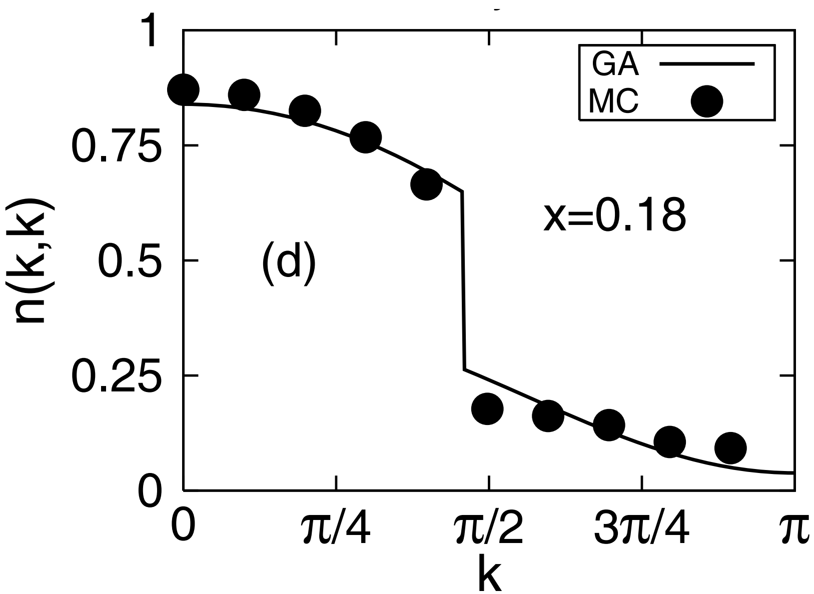
NODAL QUASIPARTICLES



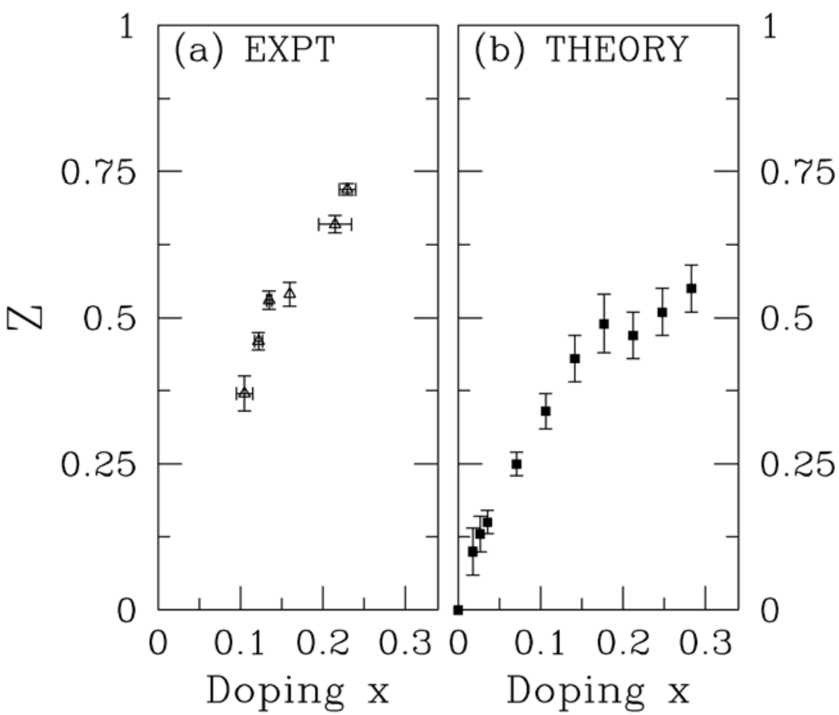
- coherent spectral weight Z(x)
- dispersion vf(x)
- current carried by a quasiparticle

technical developments relevant for many other variational calculations

Nodal Quasiparticle Z:
Loss of coherence with $x\downarrow$



Paramekanti et al, PRL (2001)

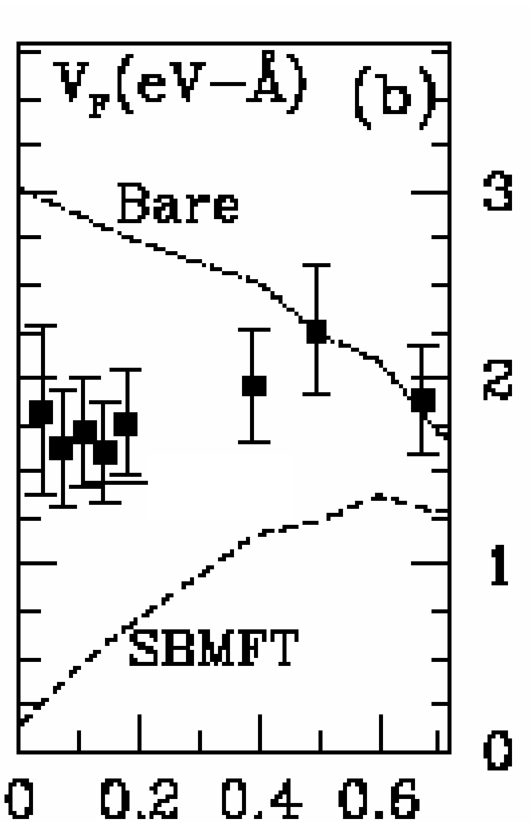


Expt: Johnson et al, PRL (2001)

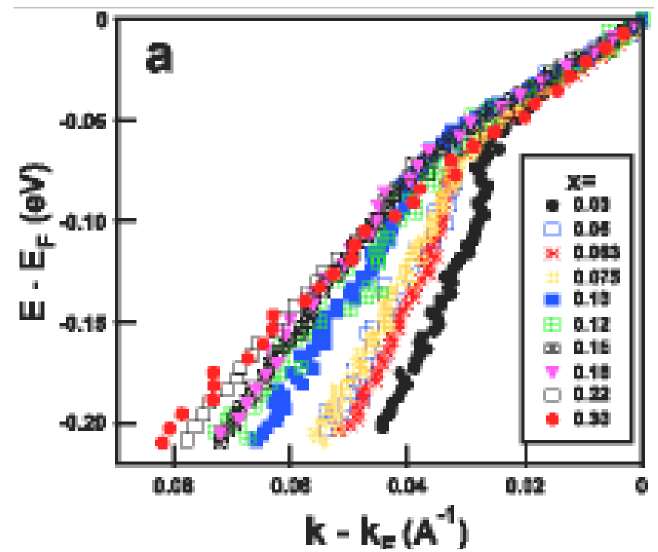
Nodal Quasiparticles:
Dispersion of low-energy excitations in the SC state

const. VF

VF indep. of x

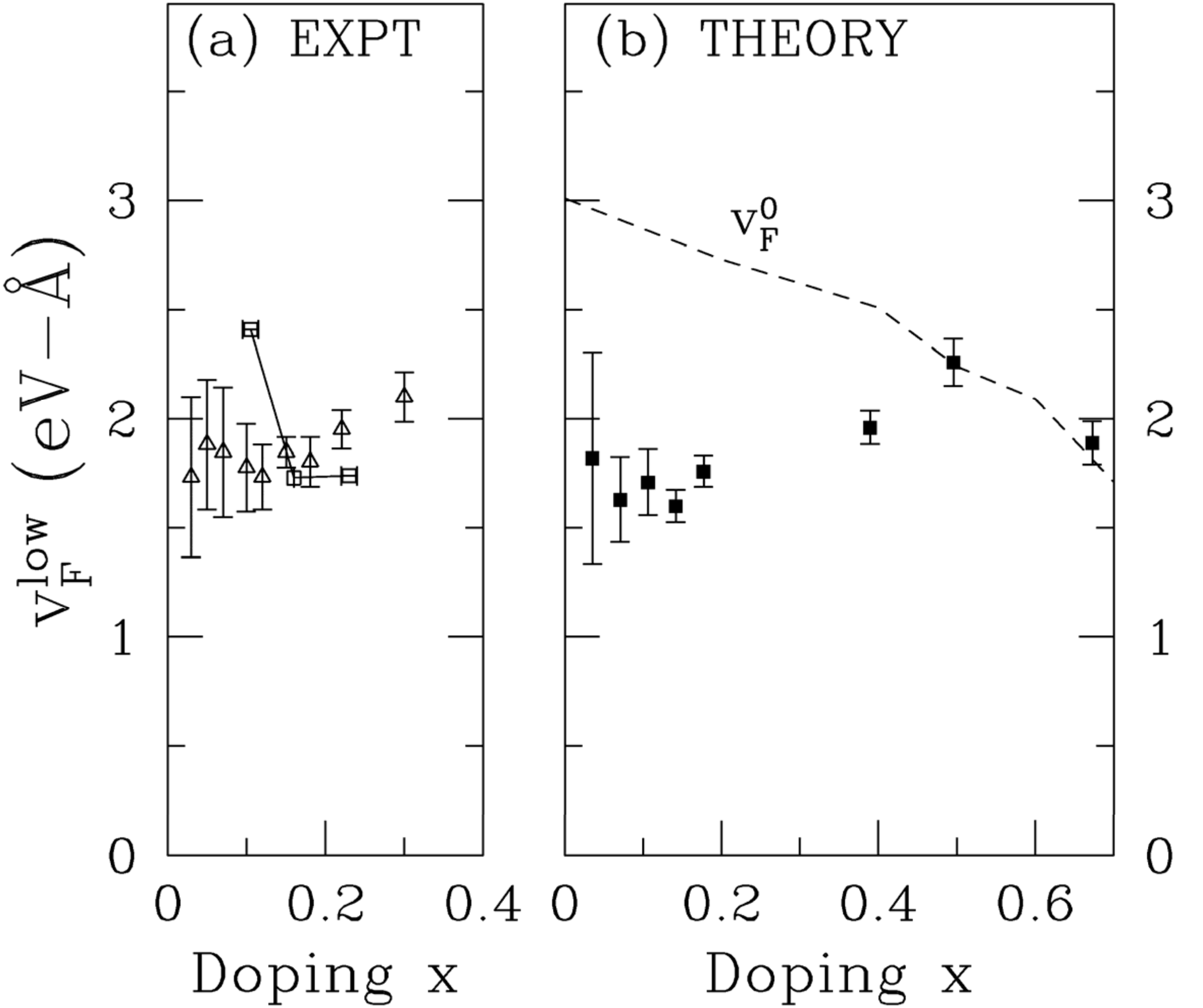


Theory: Paramekanti et al, PRL (2001)



Expt: Zhou et al, Nature (2003)

Z ~ x and VF ~ const.
→ Σ (k,ω) singularities!

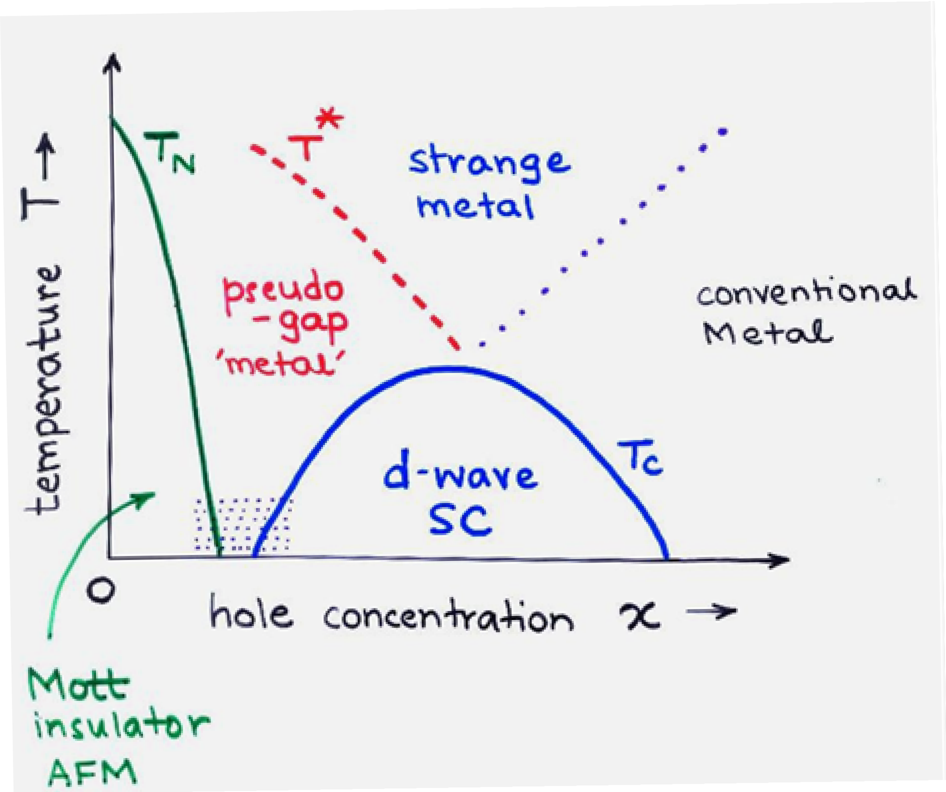


Expt: Zhou et. al
Nature **423**, 398 (2003)

Theory: Paramekanti, Randeria, NT;
PRL **87,** 217002 (2001)

Pseudogap phase

The Pseudogap regime



Loss of low-energy spectral weight for T > Tc:

"gap" above Tc in underdoped cuprates

Evidence for pairing between spins

Pseudogap above Tc

for small x

- **NMR: spin pairing**

  T > Tc

- **C(T): loss of entropy**

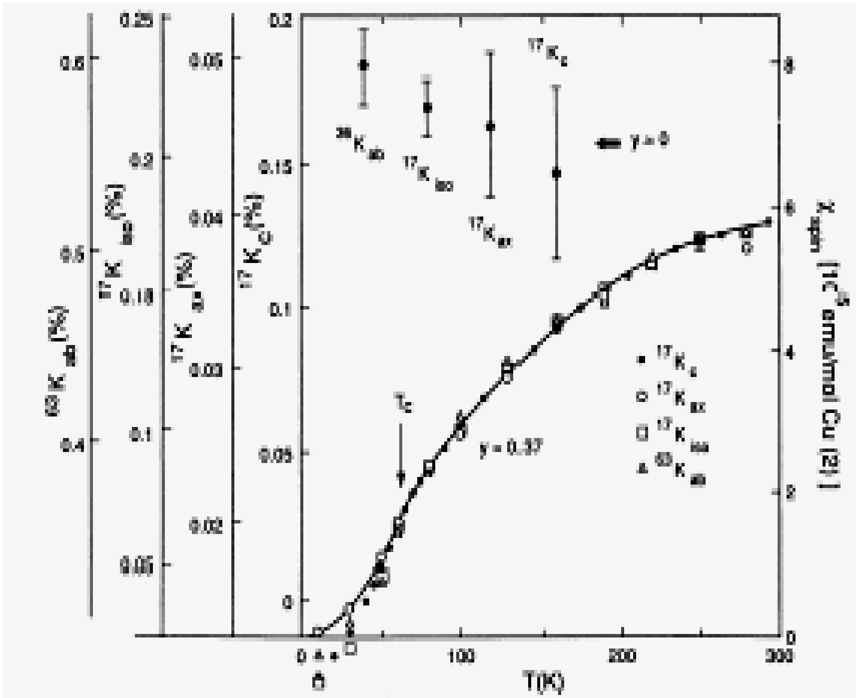
  T > Tc

- **Tunneling & Photoemision:**

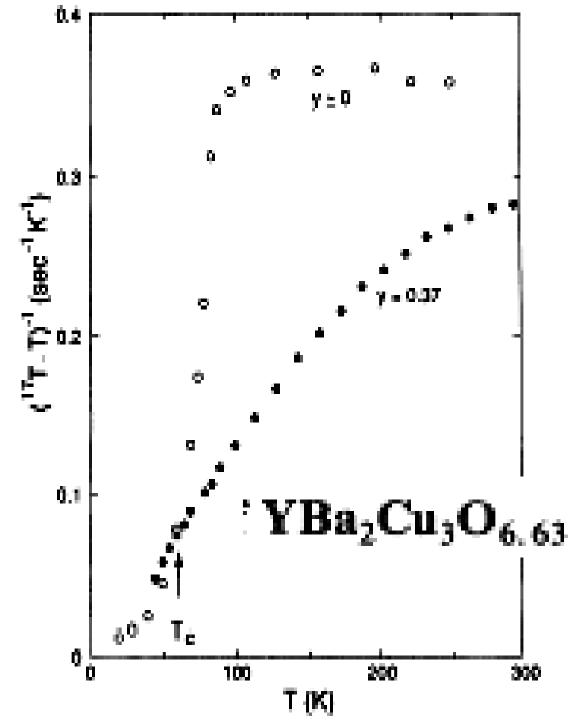
Spectral Gap visible above Tc!

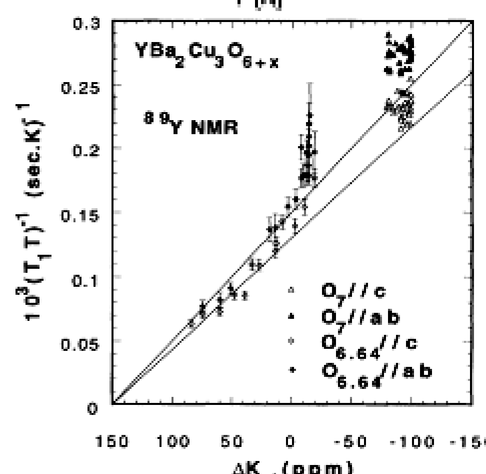
- **No quasiparticles**
- **Fermi surface destroyed**

NMR spin gap: pairing of spins above Tc



Takigawa et al (1991)



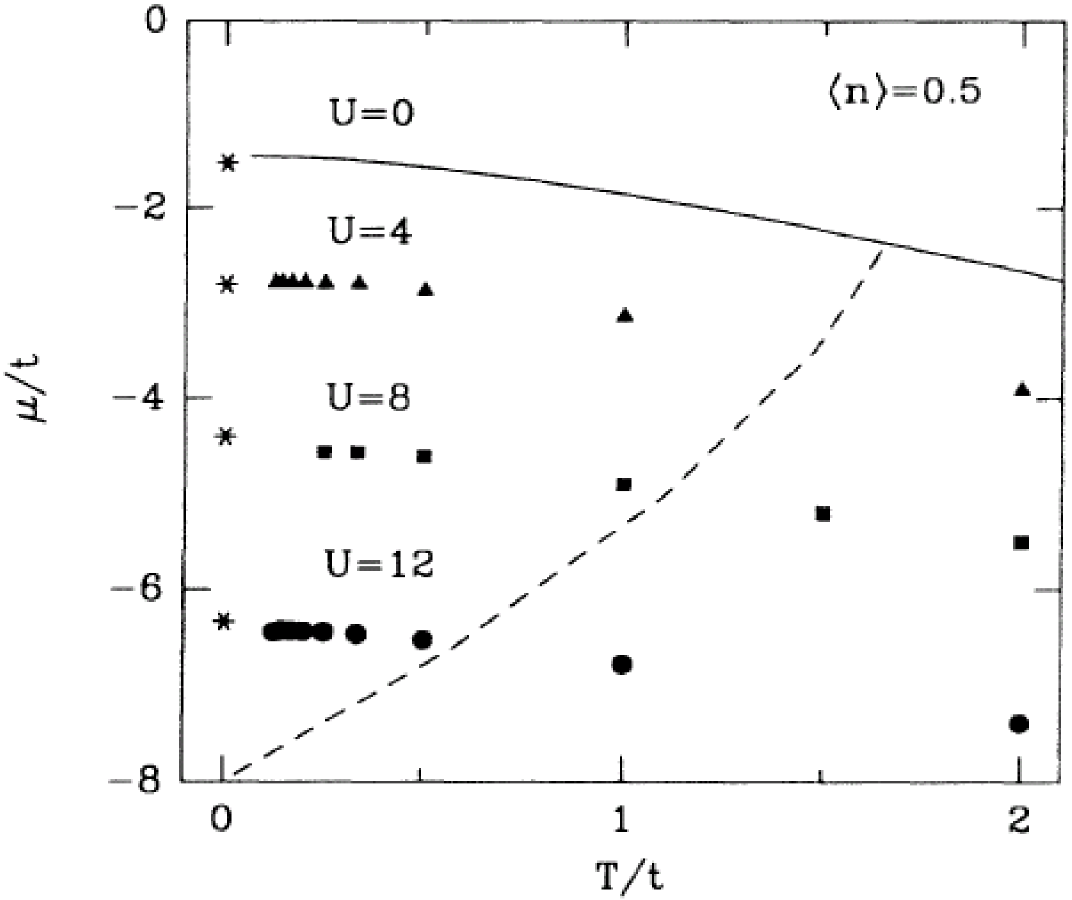


Alloul et al (1989,93)

- $d\chi/dT > 0$
- $1/T_1T \sim \chi(T)$

$$1/T_1T = \lim \Sigma_{\mathbf{q}} F(\mathbf{q}) \, \mathrm{Im}\, \chi(\mathbf{q},\omega)/\omega$$

Attractive U Hubbard model (no sign problem)

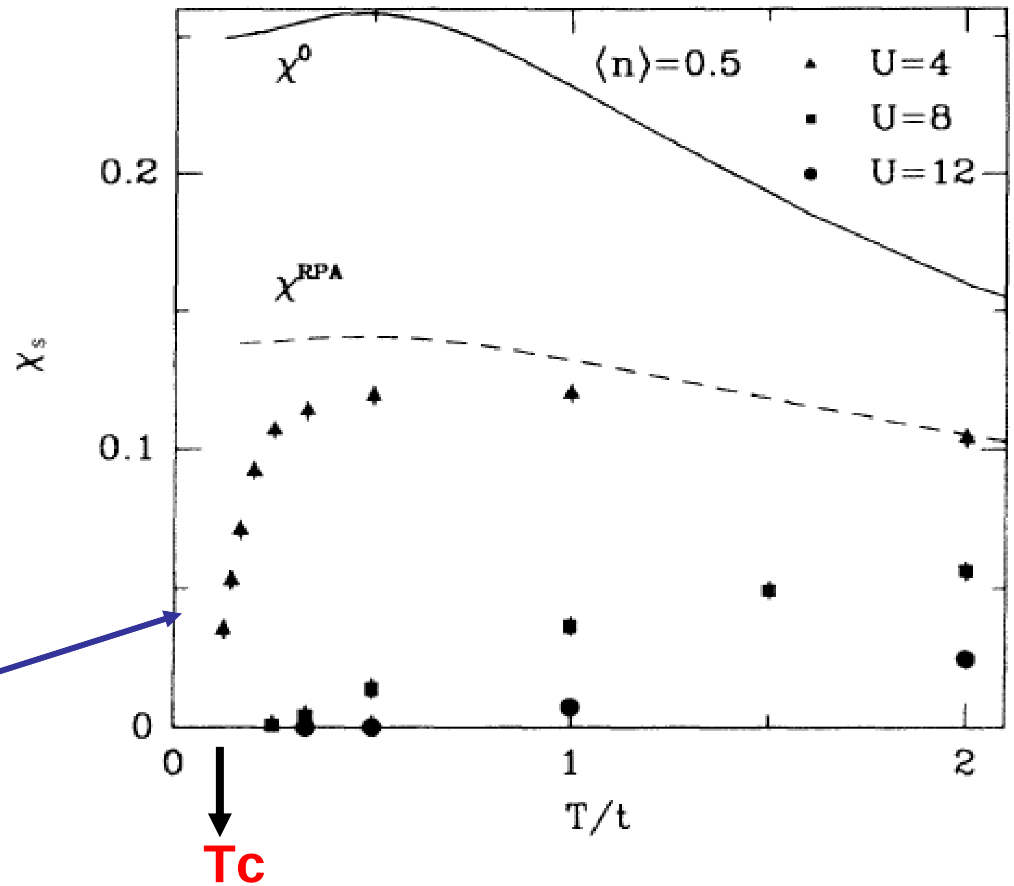


$$\mu(T,U)+4+\langle n\rangle U/2 > T$$

Attractive U Hubbard model

Static Spin susceptibility

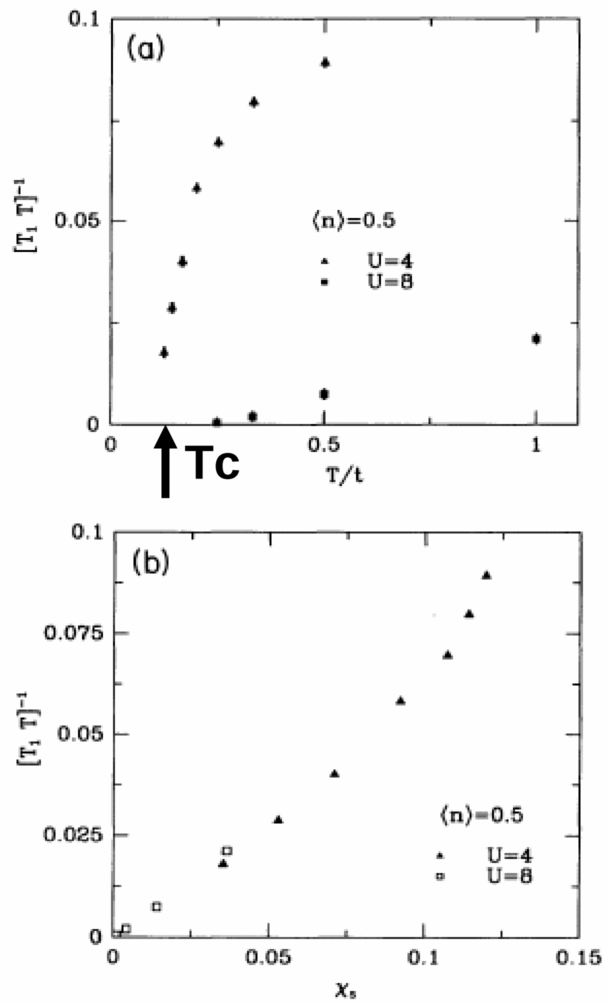
$$\chi(q=0,\omega=0)$$



Downturn indicates pairing of spins into singlets

Randeria, NT, Scalettar, Moreo; PRL 69, 2001 (1992)

Electron spin relaxation rate



$$\frac{1}{T_1 T} = \sum_q \left. \frac{\chi''(q,\omega)}{\omega} \right|_{\omega \to 0}$$

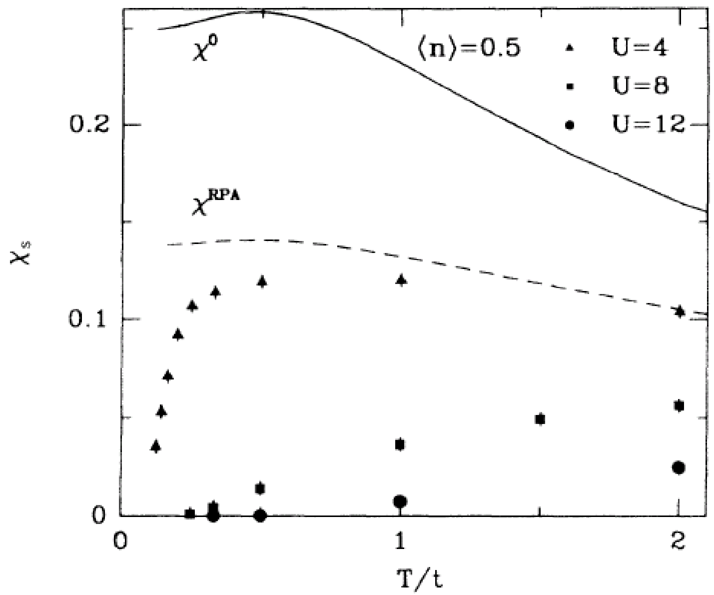
Dynamical spin susceptibility

Strong T-dependence implies a loss of weight
In the low energy spin spectrum that starts
much above Tc

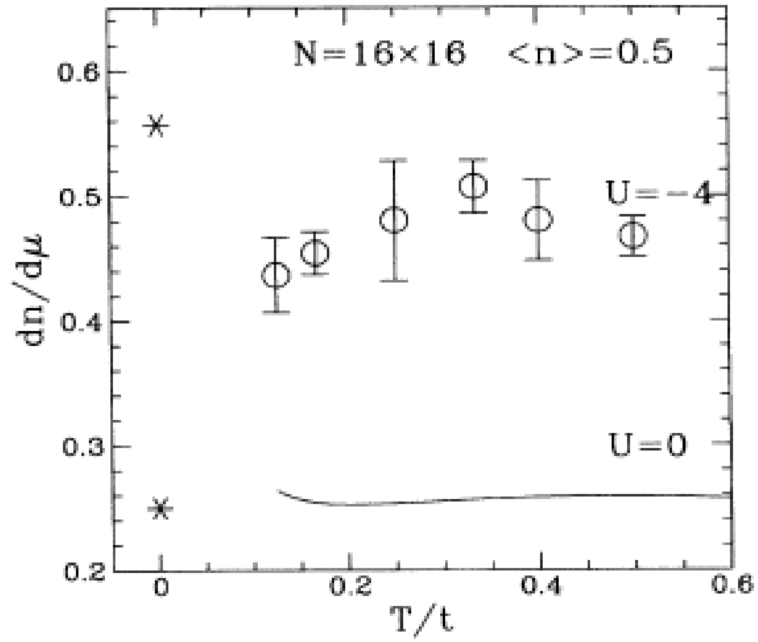
$$\frac{1}{T_1 T} \approx \chi(T)$$

Non-Korringa behavior

Spin susceptibility



compressibility



Fermi Liquid $\chi \sim N(\varepsilon_F)$ $\kappa \sim N(\varepsilon_F)$

Both are equal and independent of T

Electronic Specific Heat

C = T (dS/dT)

Loss of entropy above Tc in underdoped cuprates

J. Loram et al, PRL (1993)

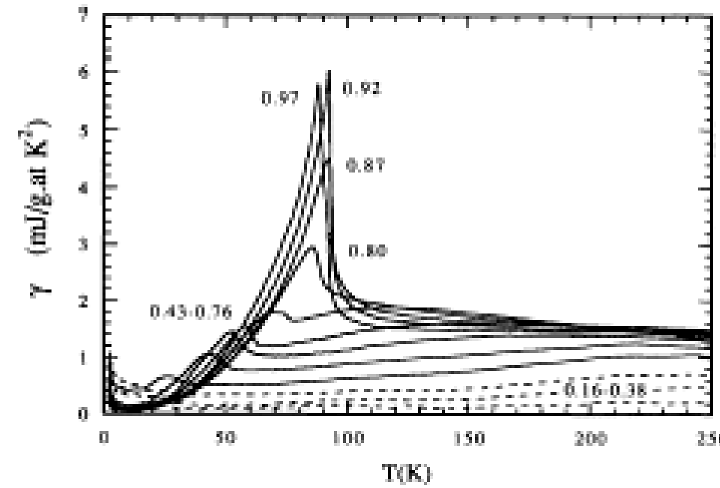


FIG. 4. Electronic specific heat coefficient $\gamma(x,T)$ vs T for $YBa_2Cu_3O_{6+x}$ relative to $YBa_2Cu_3O_6$. Values of x are 0.16, 0.29, 0.38, 0.43, 0.48, 0.57, 0.67, 0.76, 0.80, 0.87, 0.92, and 0.97.

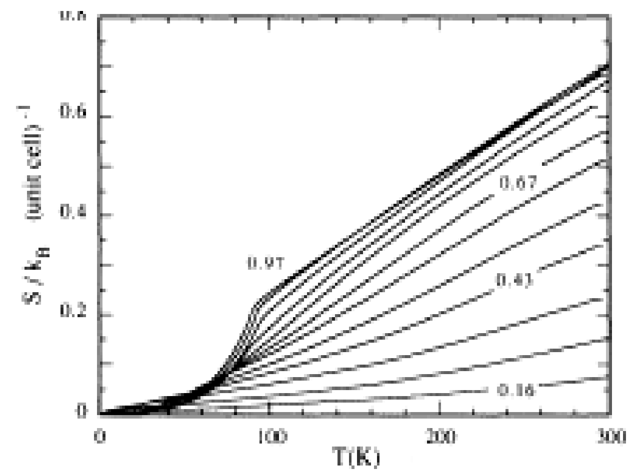


FIG. 5. Electronic entropy $S(x,T)$ for $YBa_2Cu_3O_{6+x}$. Values of x as in Fig. 4.

Pseudogap from other probes

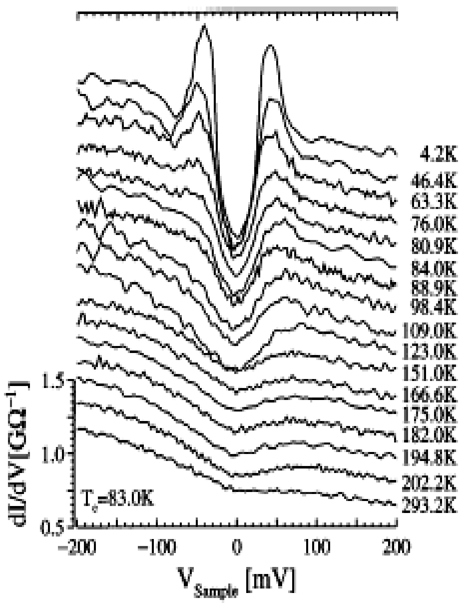
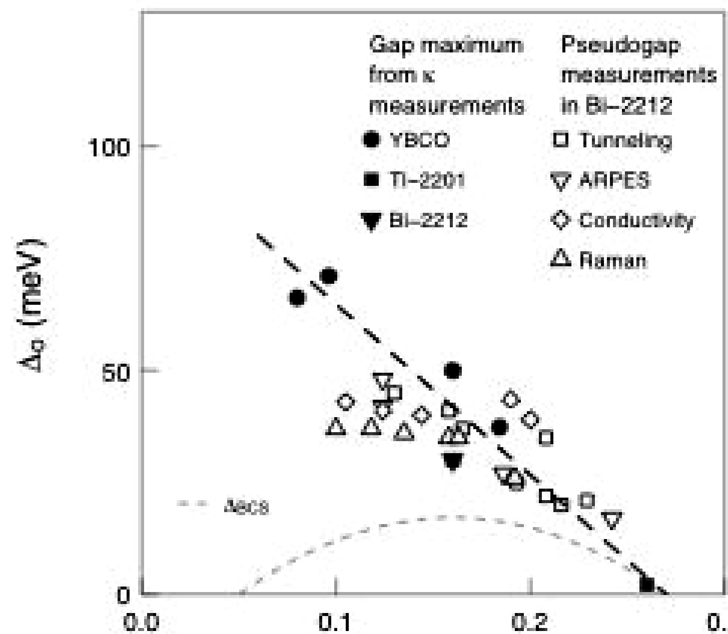


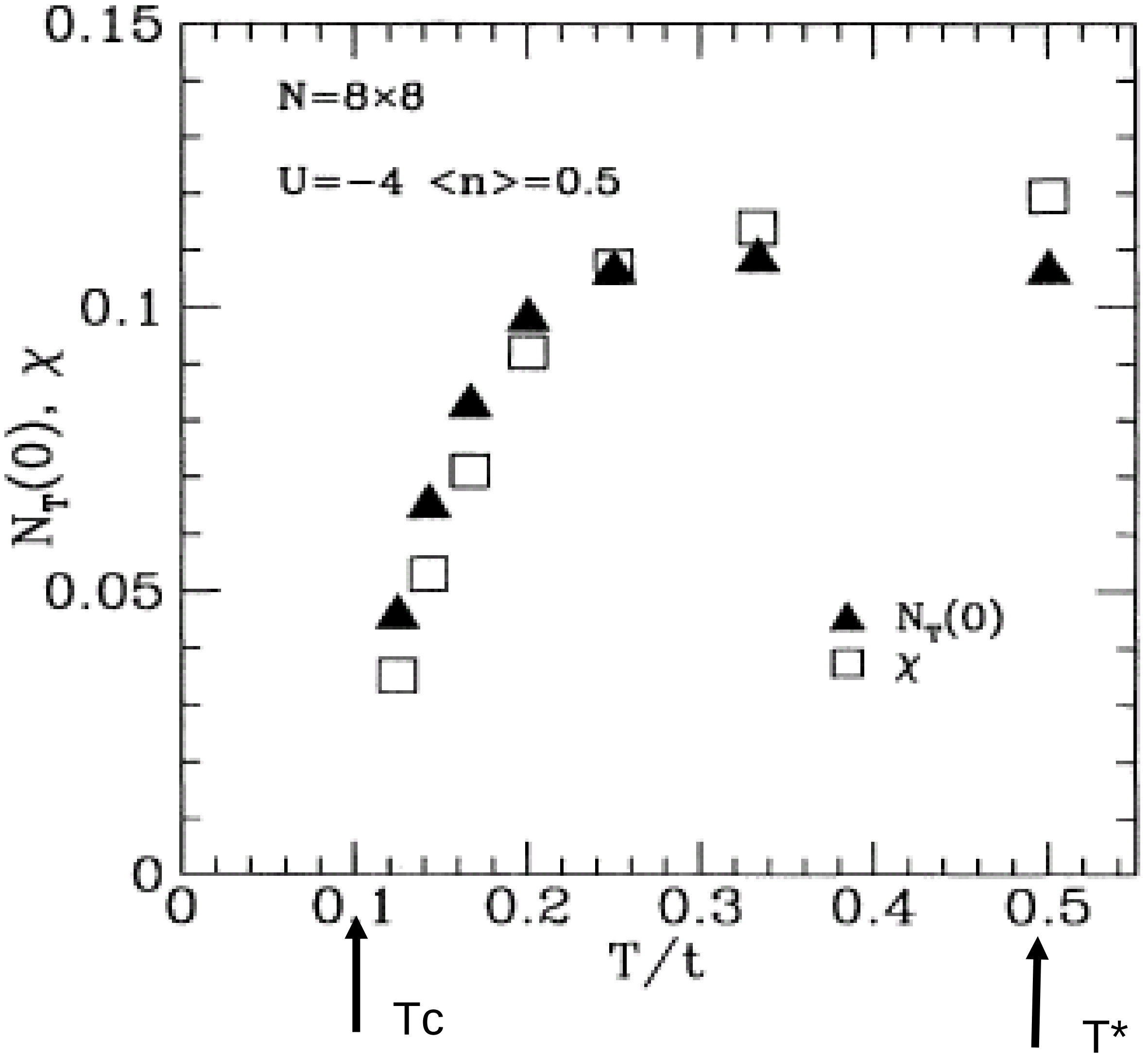
FIG. 2. Tunneling spectra measured as a function of temperature on underdoped Bi2212. The conductance scale corresponds to the 293 K spectrum, the other spectra are offset vertically for clarity.



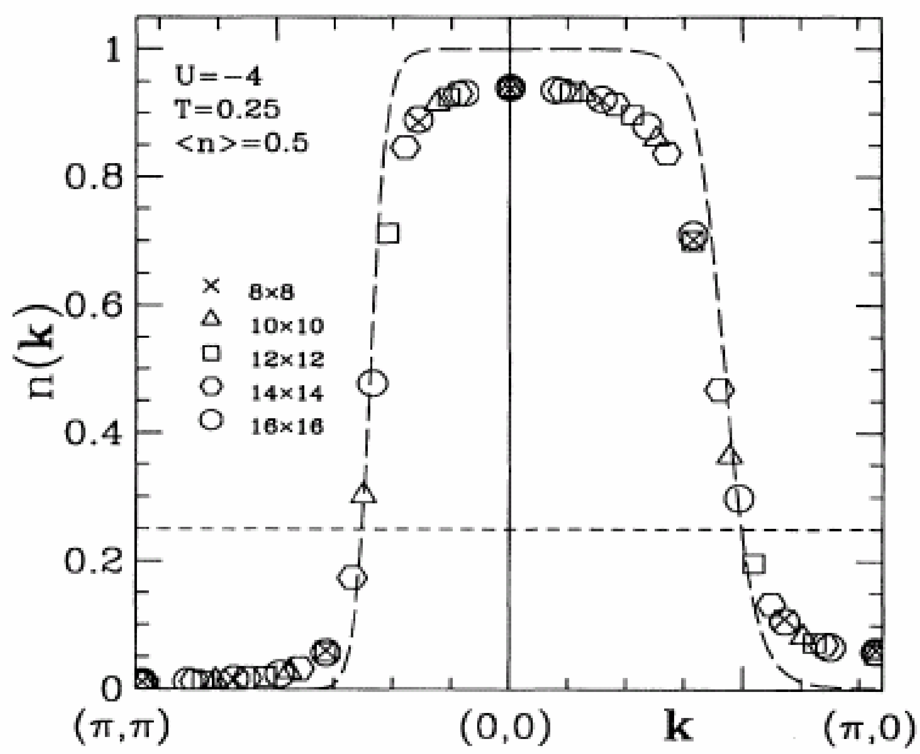
Sutherland et al, (2003)

STM: Renner et al, PRL (1998)

SUPPRESSION IN LOW ENERGY DENSITY OF STATES

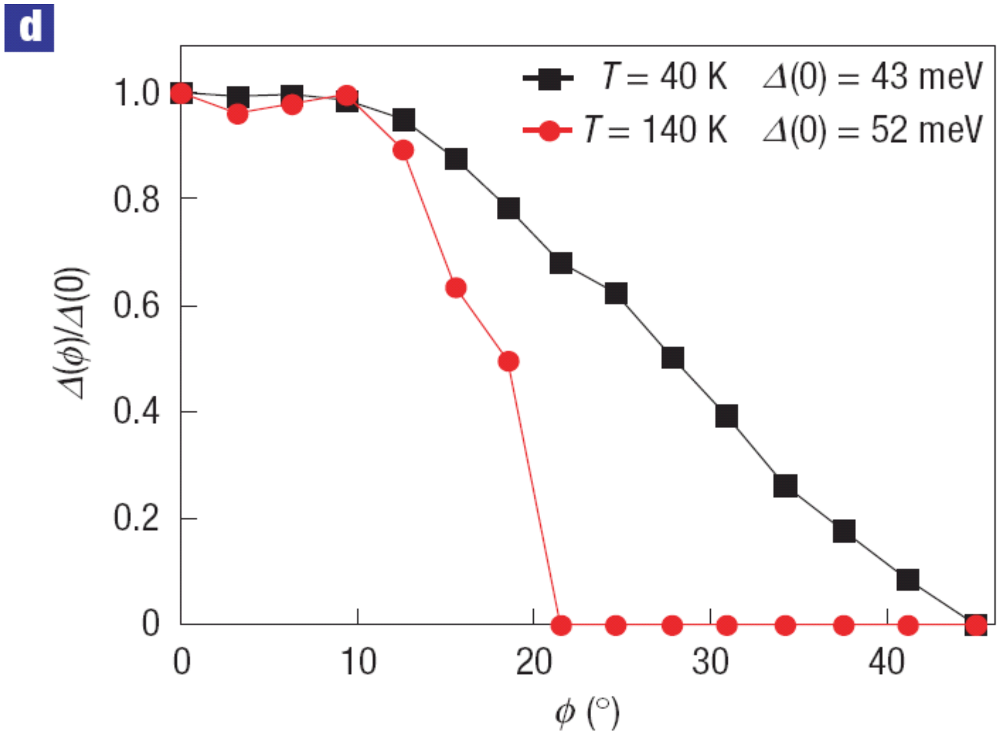


Trivedi and Randeria PRL 75, 312 (1995)



U=-4
T=0.25
<n>=0.5
8×8
10×10
12×12
14×14
16×16
n(k)
1
0.8
0.6
0.4
0.2
0
(π,π)
(0,0)
k
(π,0)

NEW PHASE AT T=0?
NODAL METAL?

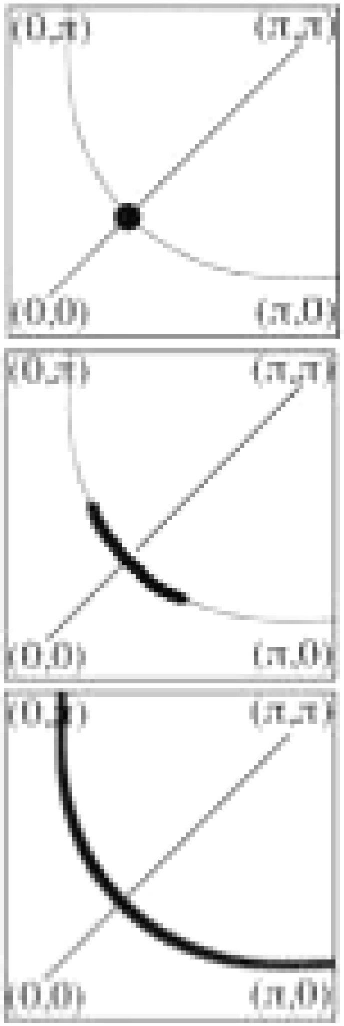


Kanigel et al, Nature Physics 2, 447 (2006)

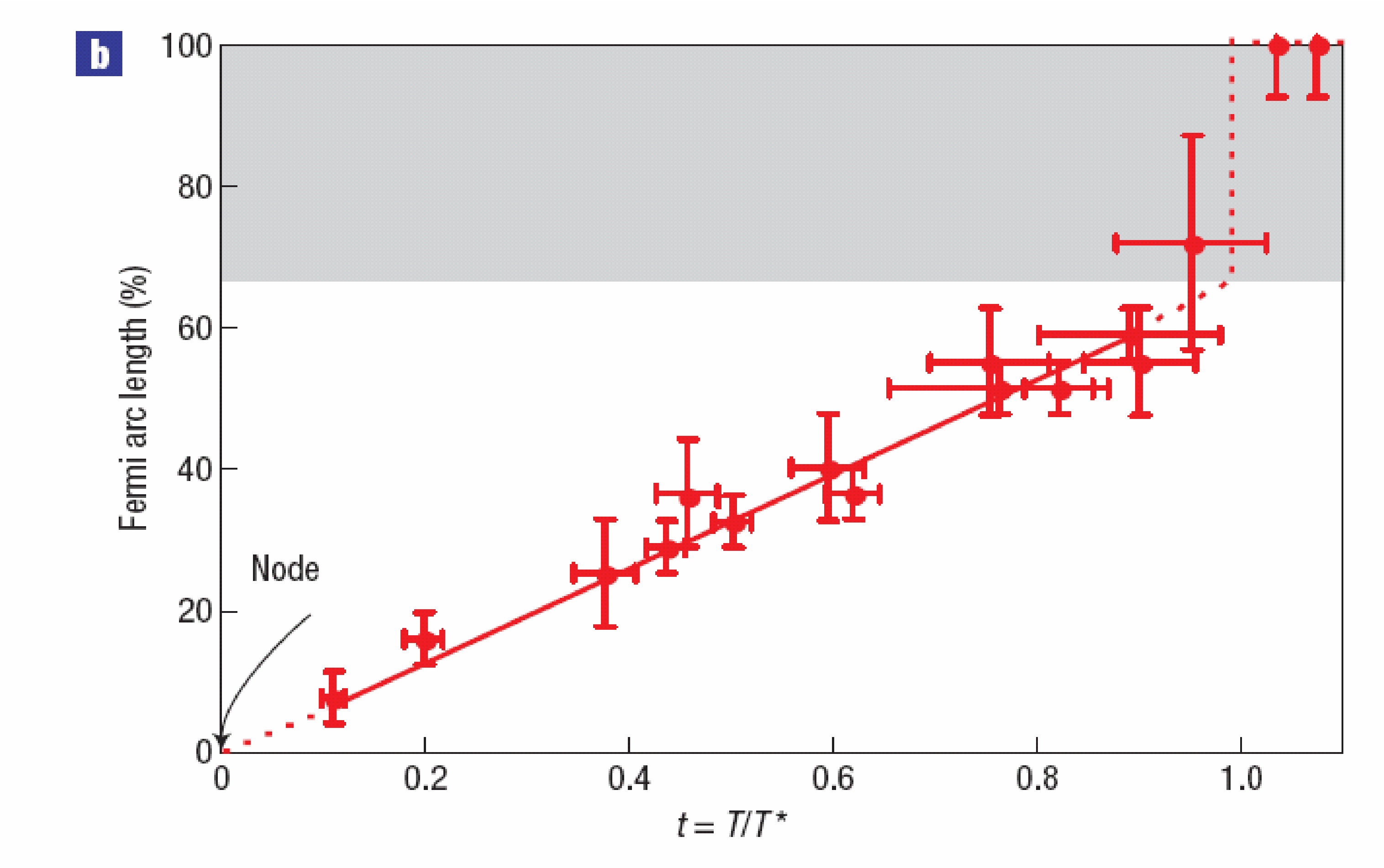
SC
T = 0

pseudogap
T* > T > Tc

T > T *



Norman et al, Nature (1998)



NODAL METAL?? ***NEW PHASE OF MATTER***

Kanigel et. al Nature Physics, 2, 447 (2006)