Nematic order-by-disorder and new transitions in spinor condensates

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•Nematic Order by Disorder in Spin-2 BECs

Ari Turner, Ryan Barnett, Eugene Demler, and A.V.

Phys. Rev. Lett. **98**, 190404 (2007).

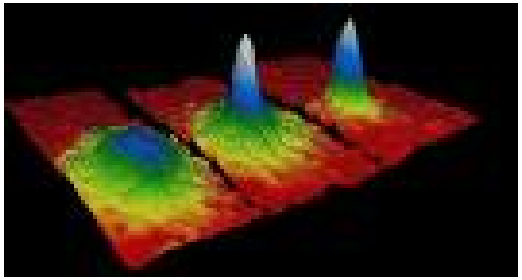
•New K-T like transitions in spinor condensates {Blackboard}

-With *Daniel Podolsky and Shailesh Chandrasekharan* (in preparation)

Spinor Condensates

- Bose-Einstein condensates of atoms with spin (spin 0, 1, 2, 3). Optical traps – *release* spin degree of freedom.

- Examples:
  - Spin 1 Rb_{87} , Na_{23}
  - Spin 2 Rb_{85};, (excited state) Rb_{87} and Na_{23}
  - Spin 3 Cr_{52} (Stuttgart), Cs_{133} (Innsbruck)

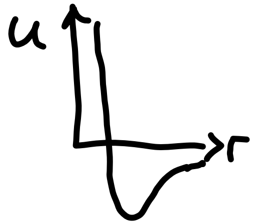


- Phenomena
  - Coexistence of superfluidity and magnetism
  - Novel orders and transitions
  - Dynamics

Spinor Bose-Einstein Condensates I

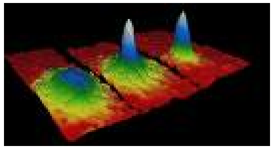
Scalar Condensate:
Boson creation operator: $\Psi^{\dagger}(r)$

Interaction: $U(r-r')$



Cold dilute gas: parameterize interactions by s-wave scattering length a_0:

$$H_{int} = \frac{4\pi\hbar^2}{m} a_0 \, \delta^3(r_1 - r_2)$$



Small parameter: **na^3. Mean field ground state:**

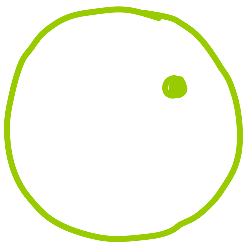
$$|\Psi_0\rangle \cong e^{\int \varphi_0 \hat{\Psi}^{\dagger}_r d^3r} |0\rangle \; ; \; \langle \Psi \rangle_0 = \varphi_0$$

- Atoms with spin S:
  - (S+1) fully symmetric scattering channels with different scattering lengths (a_0, a_2, a_4,…a_{2S})
  - Condensate structure can be complex.

Spin Structure of Condensates

- Single boson condensate implies spin structure: $\langle \hat{\varphi}_m \rangle = \varphi_m$

- Representing spin structure:
  - Spin ½: any state represented by a direction



(u, v)

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{bmatrix} \cos\frac{\theta}{2}\, e^{i\varphi/2} \\ \sin\frac{\theta}{2}\, e^{-i\varphi/2} \end{bmatrix}$$

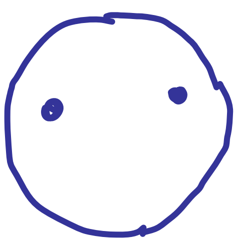
Schwinger bosons:

$$(u b^\dagger_\uparrow + v b^\dagger_\downarrow)|0\rangle$$

–Higher Spins, more complicated. Not necessarily along a direction. Convenient to use Schwinger bosons: (Barnett, Turner, Demler).

–Eg. S=1, represented by 2 points on the sphere. Integer spin S=> S+1 points.

(u_2, v_2)



(u_1, v_1)

$$\boxed{(u_1 b^\dagger_\uparrow + v_1 b^\dagger_\downarrow)(u_2 b^\dagger_\uparrow + v_2 b^\dagger_\downarrow)|0\rangle}$$

Spinor Bose Einstein Condensates II

- **Example: spin-1** $H_{int} = \frac{4\pi\hbar^2}{m}\left[a_0 P_{S=0} + a_2 P_{S=2}\right]\delta^3(r_1 - r_2)$

–Although (a_2-a_0 small compared to a_0, has a crucial effect in determining condensate structure).

Three Bose fields: $(\psi_{+1}, \psi_0, \psi_{-1})$

Instead use:
Transform as vectors.

$\vec{\psi} = (\psi_x, \psi_y, \psi_z)$

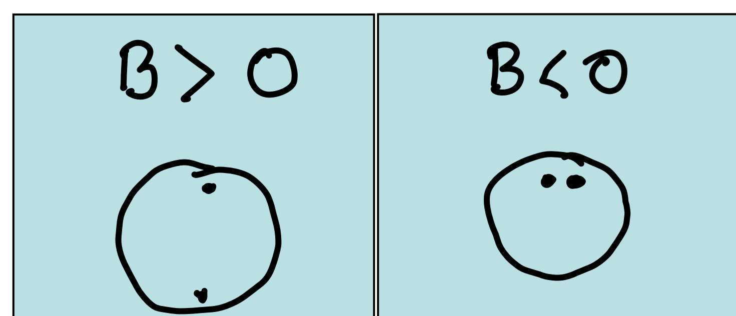
$$\begin{cases} \psi_{\pm 1} = \psi_x \pm i\psi_y \\ \psi_0 = \psi_z \end{cases}$$

Spin Operator: $\vec{S} = -i\,\vec{\psi}^{\dagger} \times \vec{\psi}$

Interaction Hamiltonian:

$$H_{int} = A\left[\vec{\psi}^{\dagger}\cdot\vec{\psi}\right]^2 + B\left[\vec{S}\cdot\vec{S}\right]$$

$$B \propto (a_2 - a_0)$$



Uniaxial nematic – Na23

Ferromagnet – Rb87

Mean Field Theory for S=2

- Mermin-d wave Sc. (74), Ciobanu, Yip, Ho (00), Barnett, Turner, Demler (06).

$$H_{int} = \frac{4\pi\hbar^2}{m}\left[a_0 P_0 + a_2 P_2 + a_4 P_4\right]\delta(r_1 - r_2)$$

Define d-wave fields:

$$\vec{\psi} = (\psi_{x^2-y^2}, \psi_{xy}, \psi_{yz}, \psi_{xz}, \psi_{3z^2-1})$$

$$\psi_{\pm 2} = \left[\psi_{x^2-y^2} \pm i\,\psi_{xy}\right]\frac{1}{\sqrt{2}}$$

$$\psi_{\pm 1} = \frac{i}{\sqrt{2}}\left[\psi_{xz} \pm i\,\psi_{yz}\right]$$

$$\psi_0 = \psi_{3z^2-1}$$

Five component d-wave vector does not transform simply under rotations.
Use a traceless, symmetric 3x3 matrix representation – tensor under SO(3)

$$\chi = \begin{bmatrix} \psi_{x^2-y^2} - \frac{1}{\sqrt{3}}\psi_{z^2} & -\psi_{xy} & \psi_{yz} \\ & -\psi_{x^2-y^2} - \frac{1}{\sqrt{3}}\psi_{z^2} & \psi_{xz} \\ & & +\frac{2}{\sqrt{3}}\psi_{z^2} \end{bmatrix}$$

$\langle \chi \rangle$ Complex

Mean Field Theory for S=2

Spin Operator: $\vec{S} = \mathrm{Tr}\left[\chi^{\dagger}\vec{s}\,\chi\right]$; $\left[s^{a}\right]_{bc} = \epsilon_{abc}$ {anti-symmetric}

$$H_{int} = \alpha\left[\mathrm{Tr}\,\chi^{\dagger}\chi\right]^{2} \underbrace{- g_{y}\,\vec{S}\cdot\vec{S}}_{\text{Favors spin.}} + \underbrace{2\,(g_{x} - g_{y})}_{>0,\ \text{favors T breaking}}\,\mathrm{Tr}\,\chi^{2}\,\mathrm{Tr}\,\chi^{\dagger 2}$$

$$g_{x} = \frac{4\pi\hbar^{2}}{10m}(a_{0} - a_{4}); \quad g_{y} = \frac{4\pi\hbar^{2}}{7m}(a_{2} - a_{4})$$

Nematic Phase of S=2

$$H_{int} = \alpha \left[\mathrm{Tr}\, \chi^{\dagger}\chi \right]^2 - g_y \vec{S}\cdot\vec{S} + 2(g_x - g_y)\, \mathrm{Tr}\,\chi^2\, \mathrm{Tr}\,\chi^{\dagger 2}$$

Enhanced SO(5) symmetry at g_y=0: $H_{int} = \alpha \left[\vec{\psi}^{\dagger}\cdot\vec{\psi} \right]^2 + 2(g_x - g_y) |\vec{\psi}\cdot\vec{\psi}|^2$

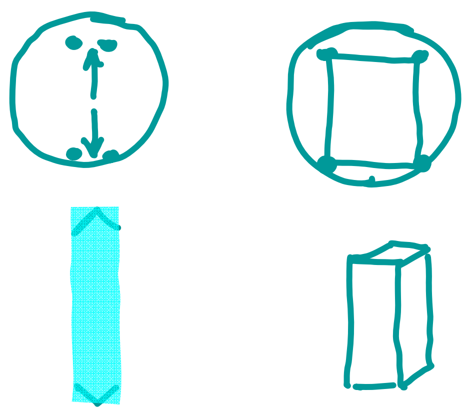
For $g_x < g_y$ want $\vec{\psi} = e^{i\varphi} \times$ {real vector}

$\chi = e^{i\varphi} \times$ {Real Traceless Symmetric matrix} $\langle \vec{S} \rangle = 0$

All nematic states **degenerate** at mean field level [Barnett, Turner, Demler] even away from g_y=0.

<ins>Accidental degeneracy *removed* by fluctuations.</ins>

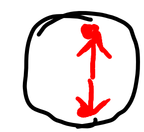
- Which nematic state realized?
- Energy scale of selection?
- Good setting for ‘order by disorder’ – known H, weak fluctuations.



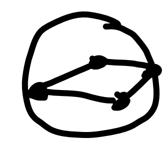
Nemate States

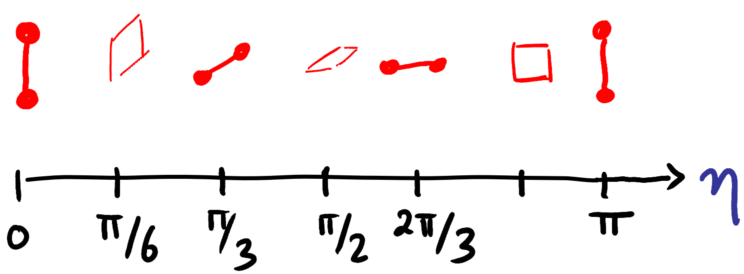
- General nematic: $\chi = \Phi \begin{bmatrix} \cos(\eta + 2\pi/3) & 0 & 0 \\ 0 & \cos(\eta + 4\pi/3) & 0 \\ 0 & 0 & \cos\eta \end{bmatrix}$

$\eta = 0 \Rightarrow \begin{bmatrix} -1/2 & & \\ & -1/2 & \\ & & 1 \end{bmatrix}$ uniaxial nematic along $\hat{z}$



$\eta = \pi/2 \Rightarrow \begin{bmatrix} \sqrt{3}/2 & & \\ & -\sqrt{3}/2 & \\ & & 0 \end{bmatrix}$ "square" biaxial nematic $\hat{x}\hat{y}$





General η, biaxial nematic.

Hard to realize in liquid crystals.

Reason – if order is weak:

$\Delta F_3 = g \operatorname{Tr} \chi^3 = g \cos 3\eta$

$\Rightarrow$ uniaxial for any g.

NOT permitted here $\{U(1)_{\text{charge}}\}$

Fluctuations I

$\chi = \bar{\chi}_\eta + \delta\chi$; expand H to $[\delta\chi]^2 \longrightarrow$ normal modes $\omega_\alpha(k)$

Quantum fluctuations: $\Delta E[\eta] = \sum_{k,\alpha} \hbar\omega_\alpha(k)$

Also, Song, Semenoff, Zhou.

thermal fluctuations: $\Delta F[\eta]$

Define modes:

Rotations: $a_1^\dagger = \chi_{23}^\dagger \qquad a_2^\dagger = \chi_{13}^\dagger \qquad a_3^\dagger = \chi_{12}^\dagger$

Phase rotations: $p_\eta^\dagger = \sqrt{\frac{2}{3}}\left[\cos\left(\eta + \frac{2\pi}{3}\right)\chi_{11}^\dagger + \cos\left(\eta + \frac{4\pi}{3}\right)\chi_{22}^\dagger + \cos\eta\, \chi_{33}^\dagger\right]$

Accidental degen: $q_\eta^\dagger = \sqrt{\frac{2}{3}}\left[\sin\left(\eta + \frac{2\pi}{3}\right)\chi_{11}^\dagger + \sin\left(\eta + \frac{4\pi}{3}\right)\chi_{22}^\dagger + \sin\eta\, \chi_{33}^\dagger\right]$

- $H_{Bogoliubov} = \left[\frac{k^2}{2m} + A_p\right] p_k^\dagger p_k - A_p\{p_k^\dagger p_{-k}^\dagger + h.c.\} + (p \to q, a_1, a_2, a_3)$

- **Independent modes**

Fluctuations II

- Bogoliubov spectrum

$$\hbar\omega_K = \sqrt{\epsilon_K^2 + 2\epsilon_K A_\alpha} \simeq \hbar K \underbrace{\sqrt{\frac{A_\alpha}{m}}}_{\text{sound velocity } c_\alpha}$$

1. $c_p = \sqrt{\frac{\alpha + 2(g_x - g_y)}{} \frac{n_0}{m}}$. related to compressibility; sound

2. $c_q = \sqrt{2(g_y - g_x)\frac{n_0}{m}}$ independent of η

BUT $c_{1,2,3}$ depend on η

3. $c_j = \sqrt{\frac{2n_0}{m}\left[-g_x + g_y \cos\left(2\eta + \frac{2\pi}{3} j\right)\right]}$

Note

$$\left.\begin{array}{l} g_y \to -g_y \\ \eta \to \eta + \pi/2 \end{array}\right\} \text{symm.}$$

Fluctuations III

T=0:

$$\begin{cases} E(\eta) = \frac{1}{2} \sum_{K} \sum_{j=1}^{3} \hbar \omega_j(K) - \left[\frac{\hbar^2 K^2}{2m} + m c_j^2 \right] \\ \frac{\Delta E(\eta)}{V} = \frac{8 m^4}{15 \pi^2 \hbar^3} \sum_{j=1}^{3} c_j^5 \end{cases}$$

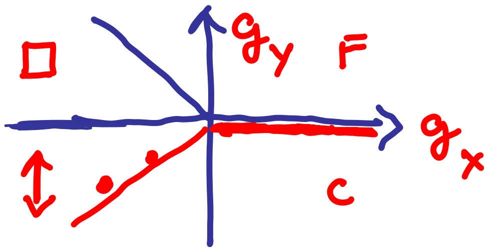
Finite T:

$$\begin{cases} F(\eta) = k_B T \sum_{K} \sum_{j=1}^{3} \log \{ 2 \sinh (\hbar \omega_j(k) / 2 k_B T) \} + \text{const.} \\ \frac{\Delta F(\eta)}{V} \cong - k_B T \left(\frac{m}{\hbar} \right)^3 \sum_{j=1}^{3} c_j^3 \quad \text{if } m u^2 \ll k_B T \; (< k_B T_c) \end{cases}$$

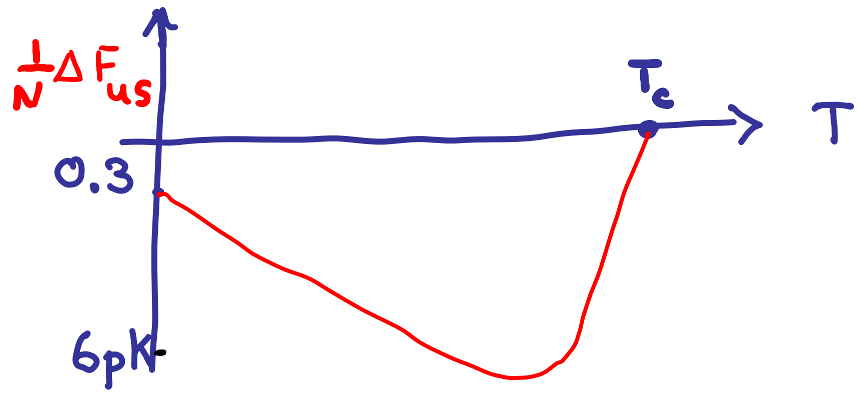
Fluctuations IV

Both contributions same effect.

Favour uniaxial state for $g_y<0$
Favour sq. biaxial state for $g_y>0$



Free energy difference Rb_{87} (Sengstock 04):



For Na_{23}, quantum splitting larger ~3pK

Magnetic energy scale~1nK, T~10nK.

Landau Theory:

Fluctuations generate sixth order term that breaks degeneracy:

$$F_6 = g_6 \, \mathrm{tr}\, \chi^3 \chi^{\dagger 3}$$

$$\cong g_6 \cos 6\eta$$

$g_6 > 0: \ \eta = \pi/6, \pi/2, \frac{5\pi}{6}$ $\square$

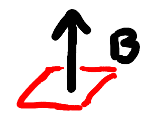
$g_6 < 0; \ \eta = 0, \pi/3, 2\pi/3$

Experimental Prospects

Competition- external magnetic field:

Spin conservation – no linear effect.
Quadratic Zeeman coupling – favours square state ⊥ to **B.**

$$E = -c_Q B^2 \hat{S}_z^2 \qquad \{Rb_{87};\ c_Q > 0\}$$

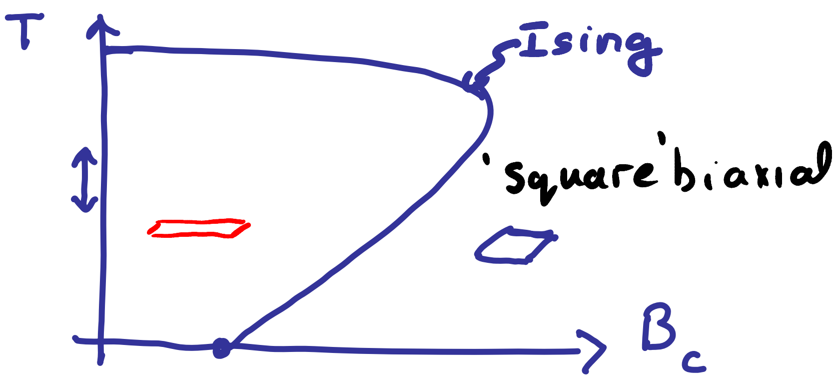


Induces a transition at B_c(T).

•Uniaxial order *stronger* at higher temperature.

•Transition in Ising universality class.

•B_c~25 mGauss (expts. 350mGauss, square biaxial observed)



- Typical timescale ~ 0.5sec.
- Energy scales bigger in optical lattices, for other atoms…