Realizing Topological Phases with Dipolar Spins

Alexey V. Gorshkov

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Joint Quantum Institute (JQI)
NIST and University of Maryland

KITP conference
New Science with Ultracold Molecules
March 14, 2013

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- symmetry protected topological phases [PRB 87, 081106(R) (2013)]:

  S. Manmana, K. Hazzard, A. M. Rey - JILA [related: arXiv:1301.5636]

  M. Stoudenmire - UCI

- fractional Chern insulators [PRL 109, 266804 (2012) & arXiv:1212.4839]:

  N. Yao, C. Laumann, S. Bennett, E. Demler, M. Lukin - Harvard

  A. Läuchli, P. Zoller - Innsbruck

  J. Ye - JILA

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Motivation

Interacting dipoles

Electric

Magnetic

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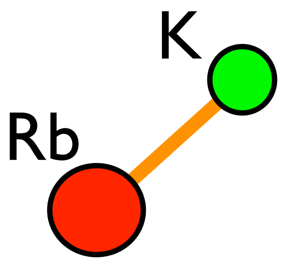

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[ultracold, rovibrational ground state: Ye, Jin, Weidemuller, Inouye, Nagerl, etc...]

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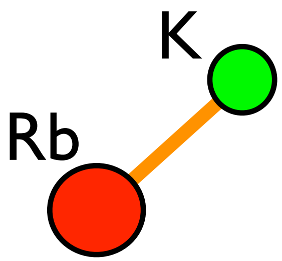

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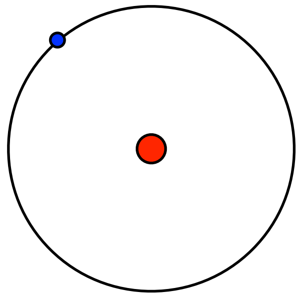

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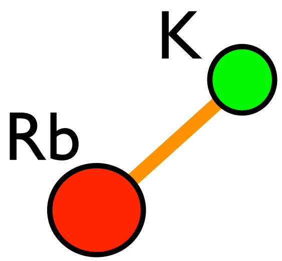

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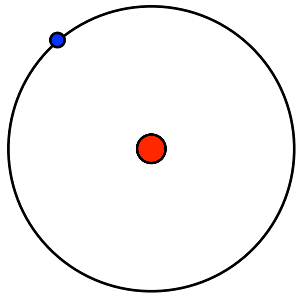

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large electronic angular momentum J

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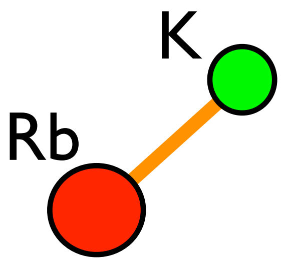

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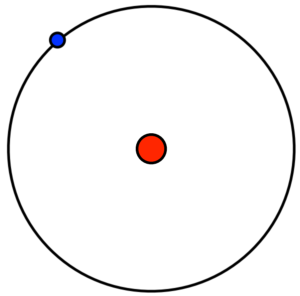

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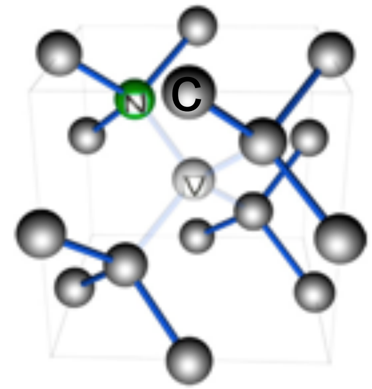

[drawn by Wrachtrup et al.]

NV centers

[Wrachtrup, Jelezko, Lukin, etc...]

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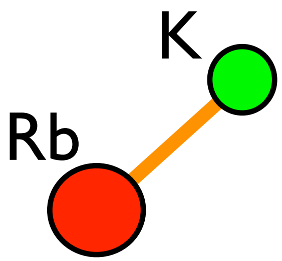

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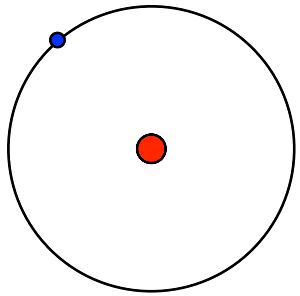

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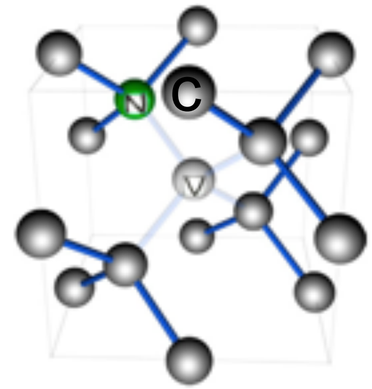

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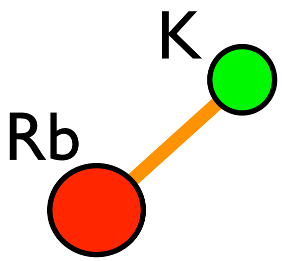

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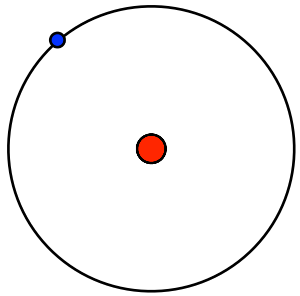

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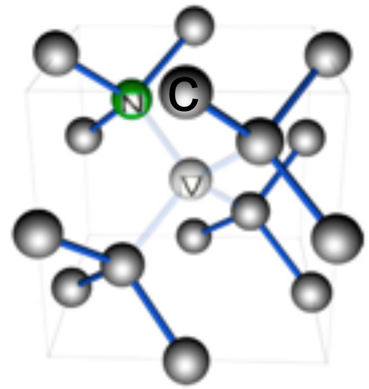

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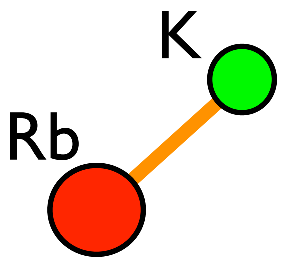

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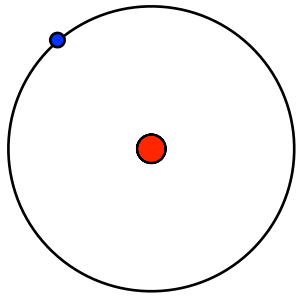

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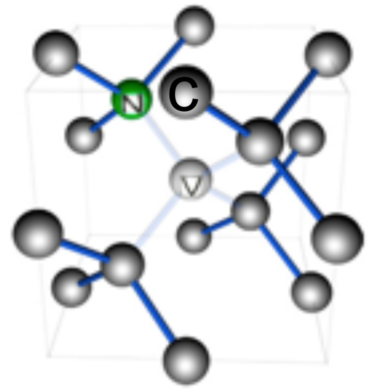

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- KRb already loaded into a 3D optical lattice at JILA!

  [Jun Ye's talk; Chotia et al, PRL (2012)]

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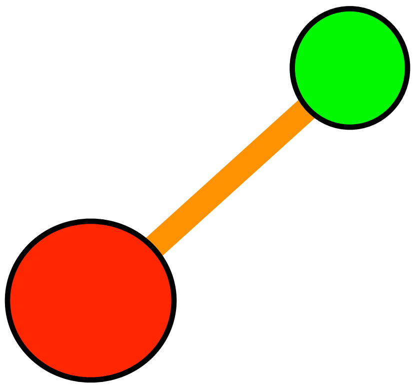

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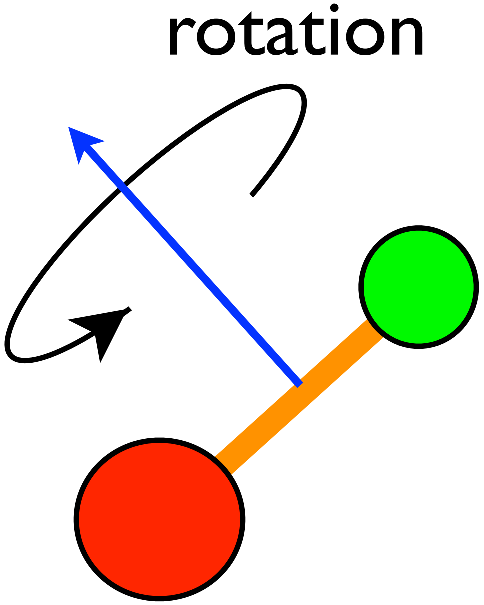

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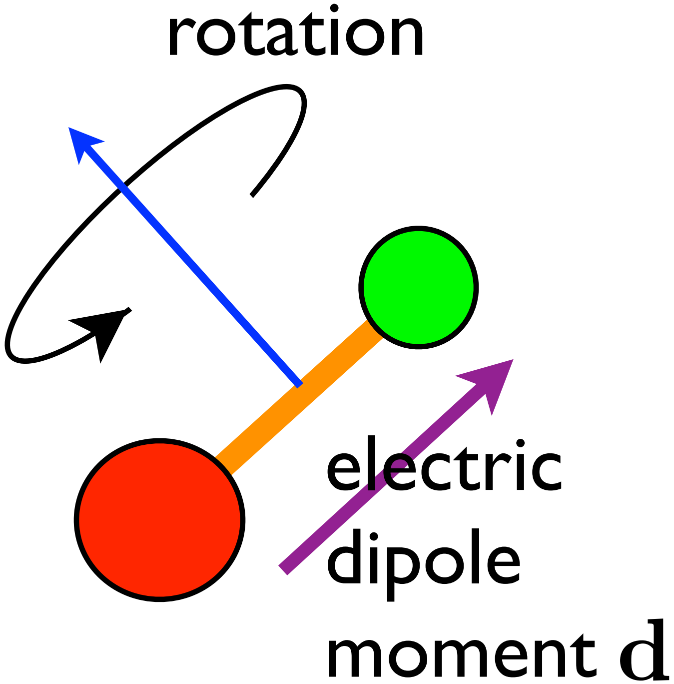

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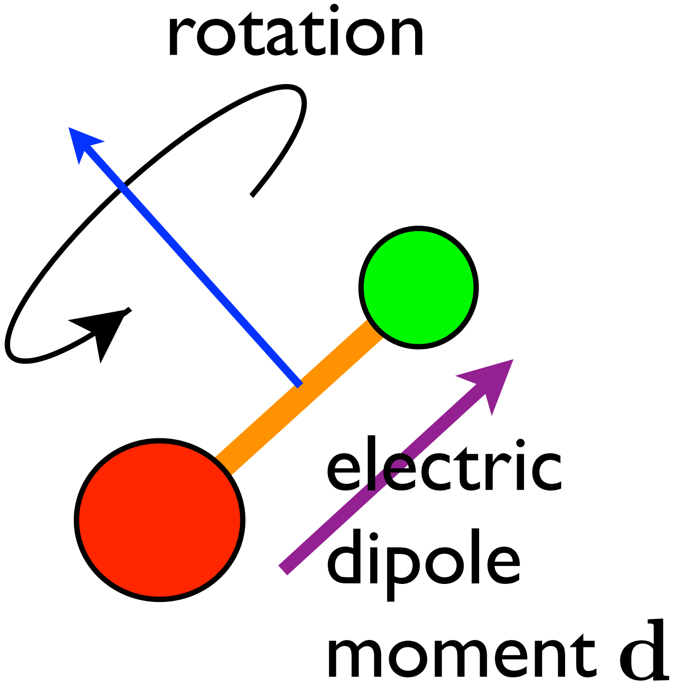

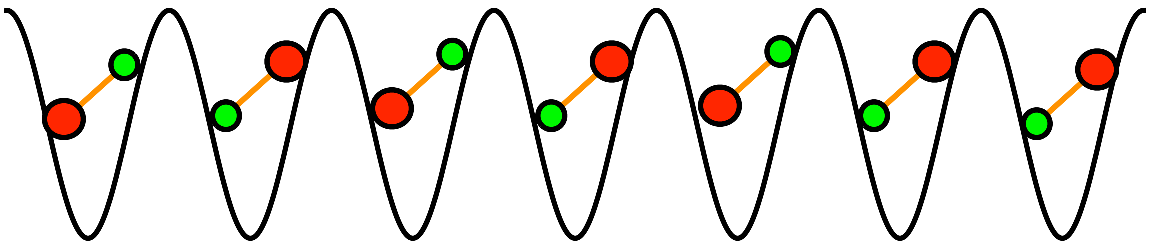

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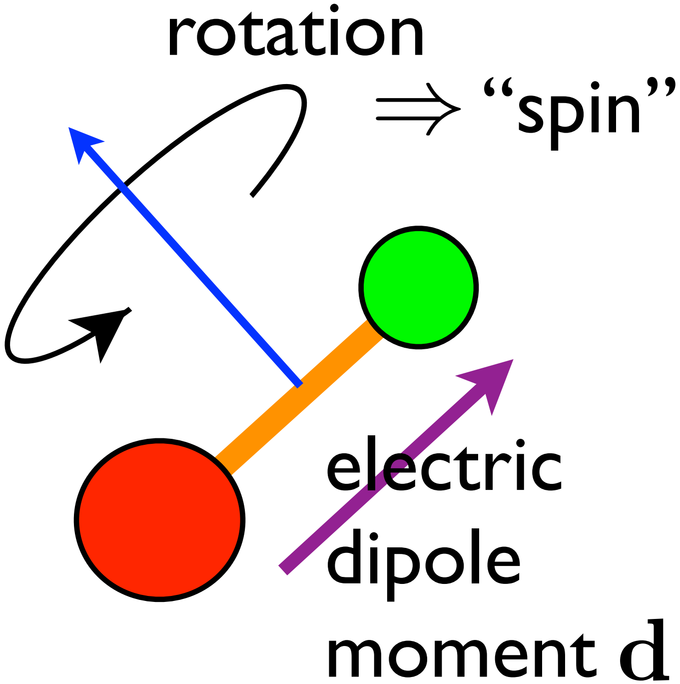

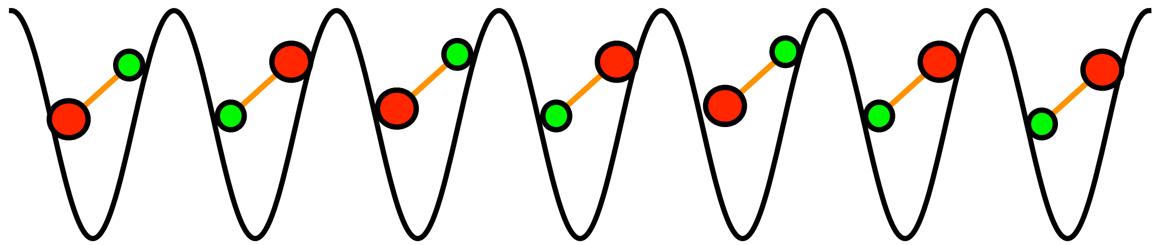

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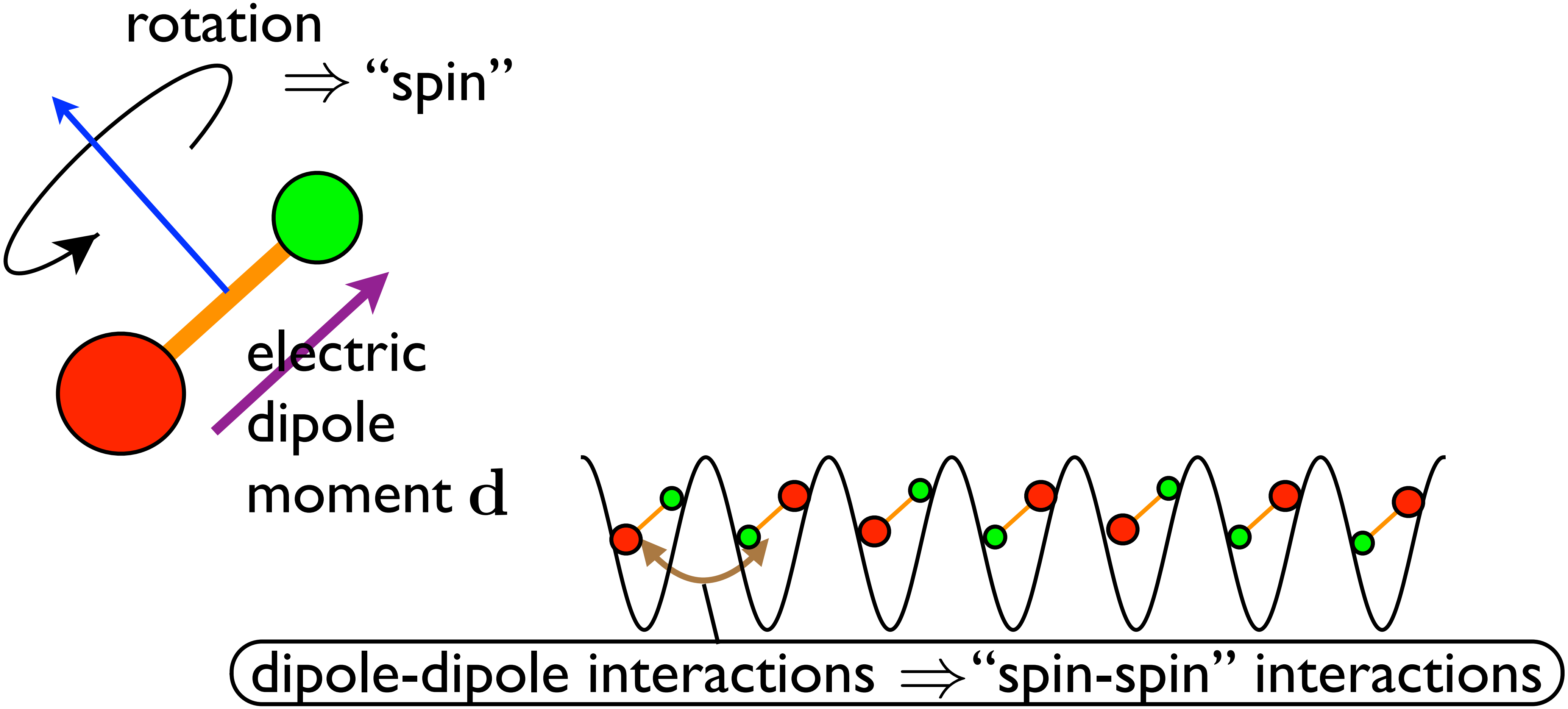
rotation
⇒ "spin"
electric
dipole
moment $\mathbf{d}$
dipole-dipole interactions ⇒ "spin-spin" interactions

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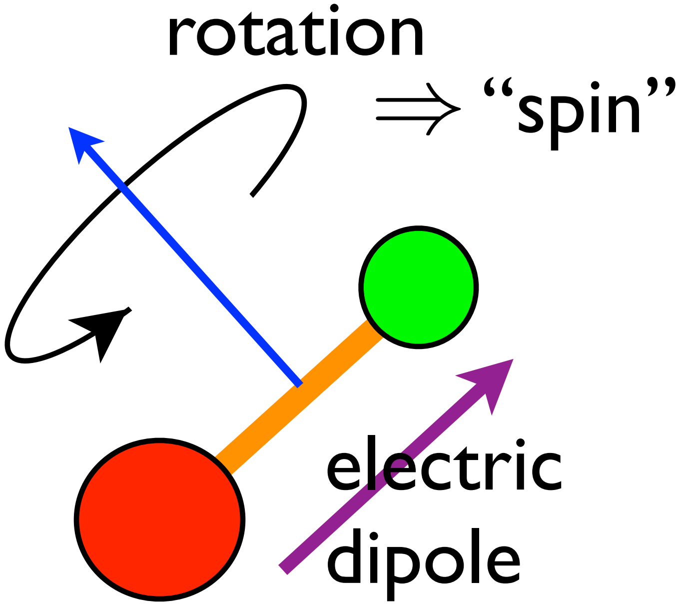
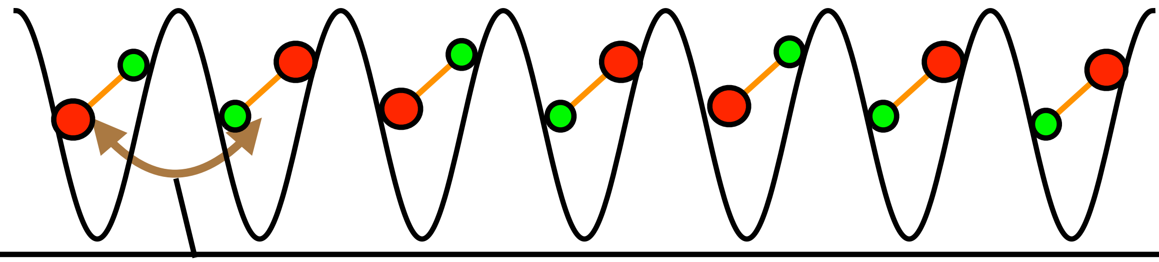


highly tunable exotic spin models



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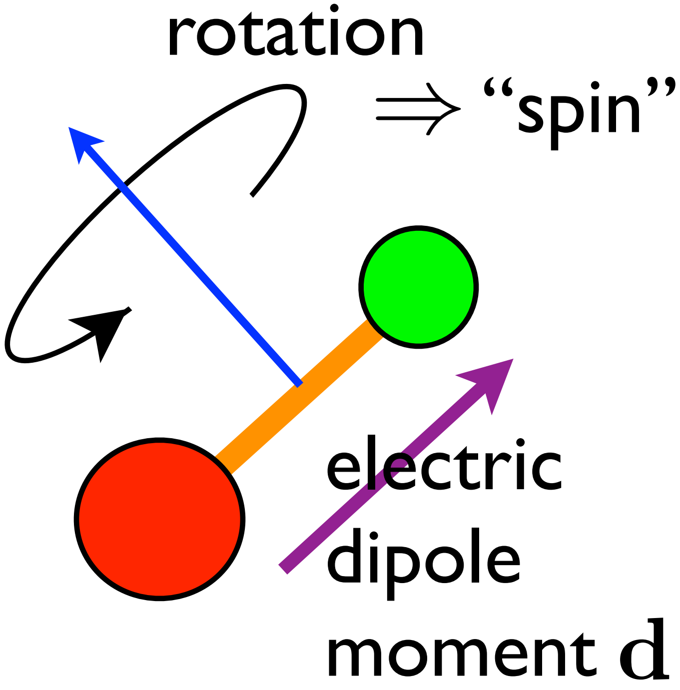

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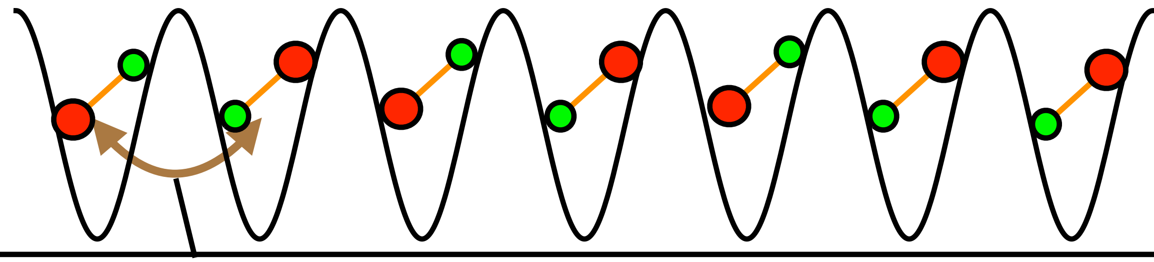

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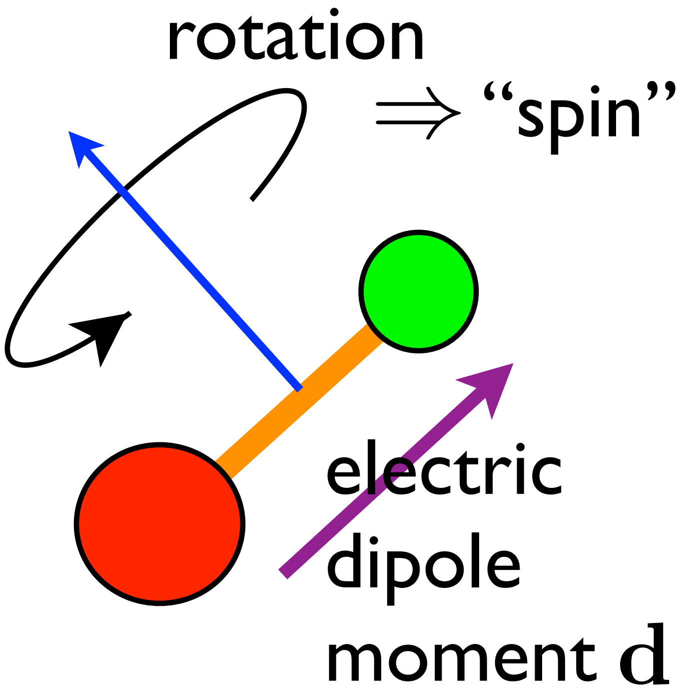
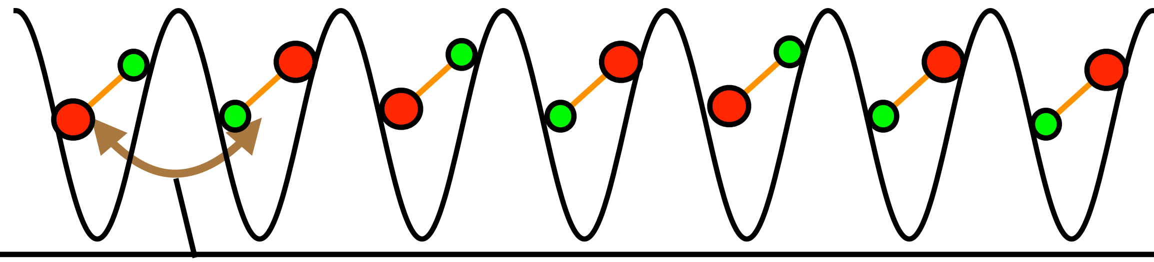


highly tunable exotic spin models

Our achievements:

- stronger interactions
- much higher tunability



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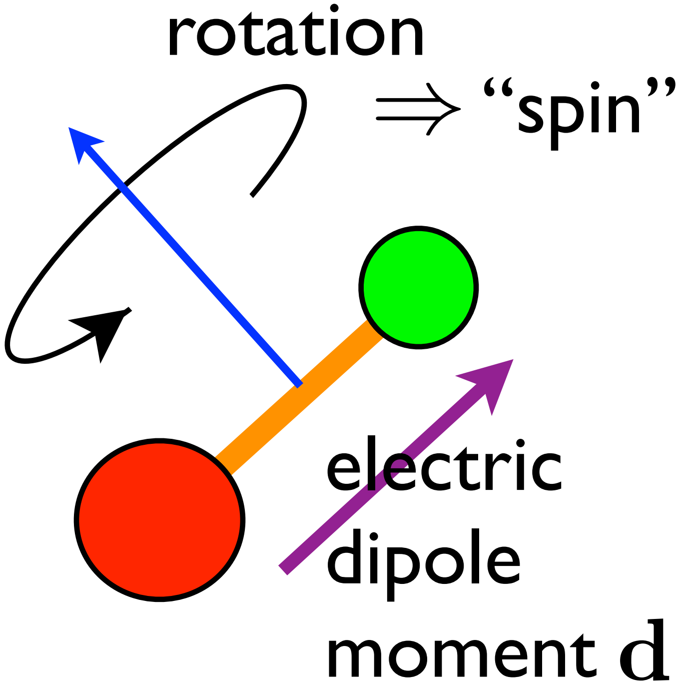

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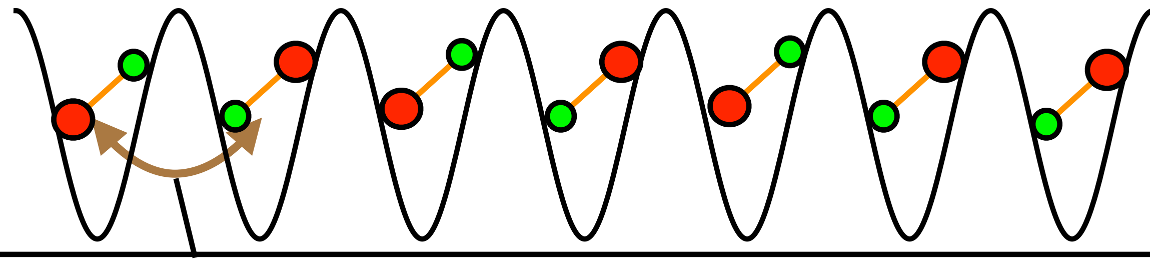

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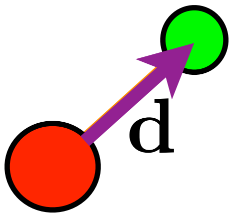
$$H_0 = B\mathbf{N}^2$$



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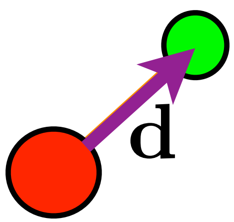
$$\mathbf{N}^2|N, N_z\rangle = N(N+1)|N, N_z\rangle$$



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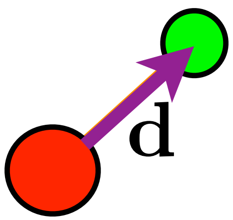


$$N_z = -1 \quad 0 \quad 1$$

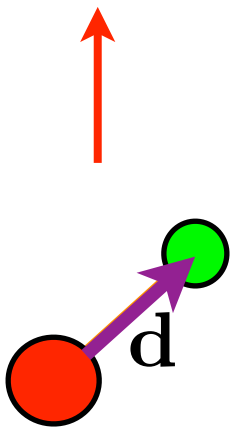
$$N = 1$$

$2B$

$$N = 0$$

Rigid rotor

$$\mathbf{E} = E\hat{\mathbf{z}}$$



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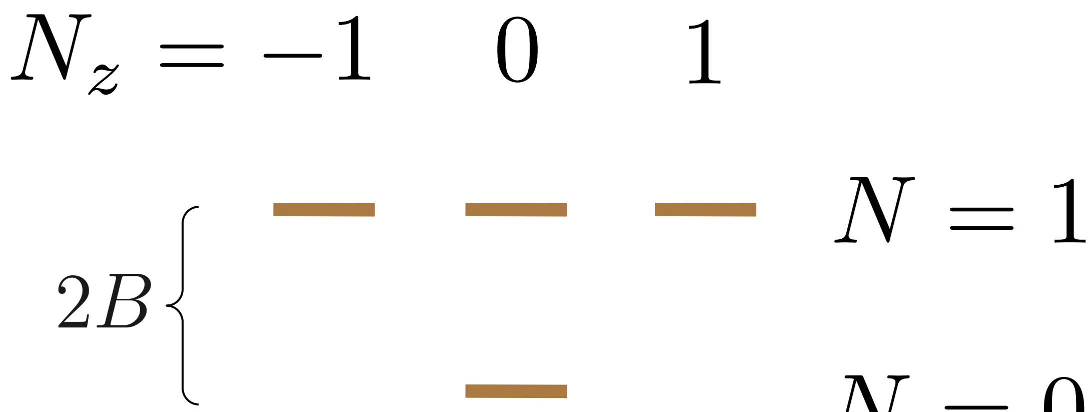

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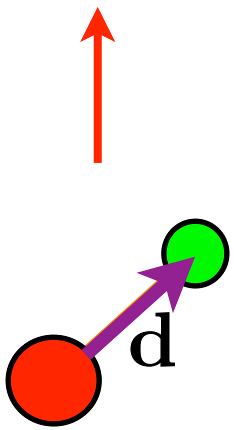

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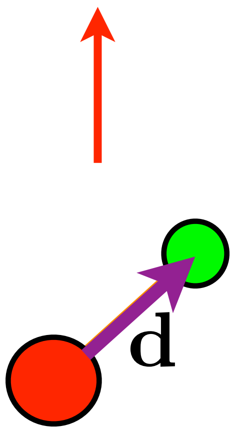

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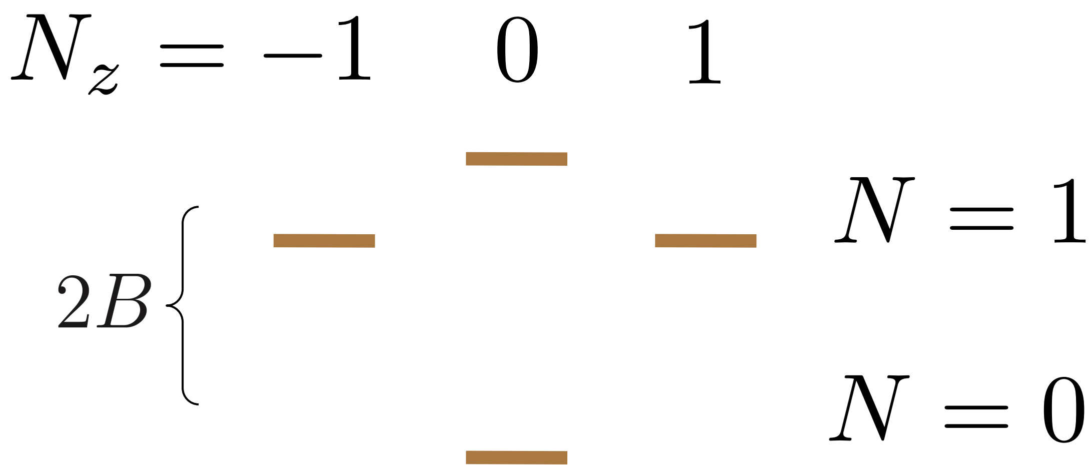

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Simplest spin Hamiltonian

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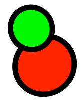
AVG et al, PRL, PRA 2011; Kuns et al, PRA 2011

Simplest spin Hamiltonian



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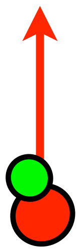

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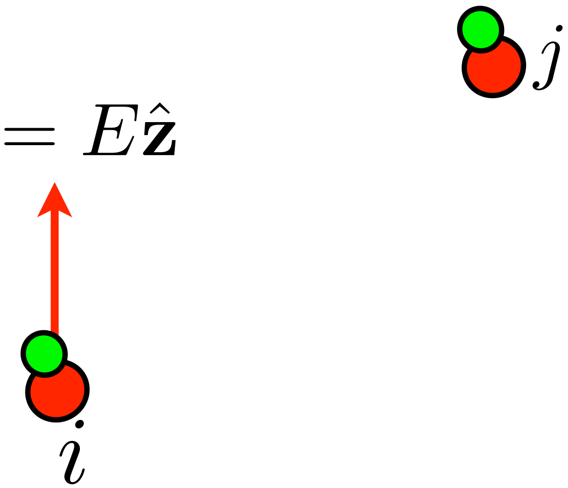

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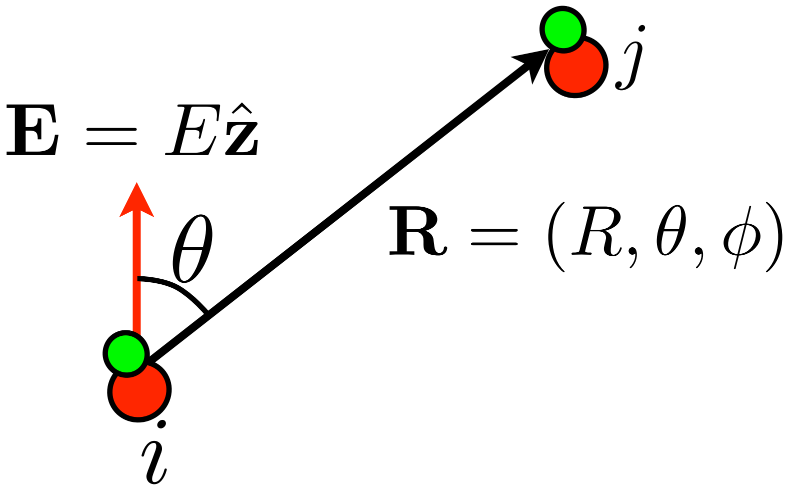

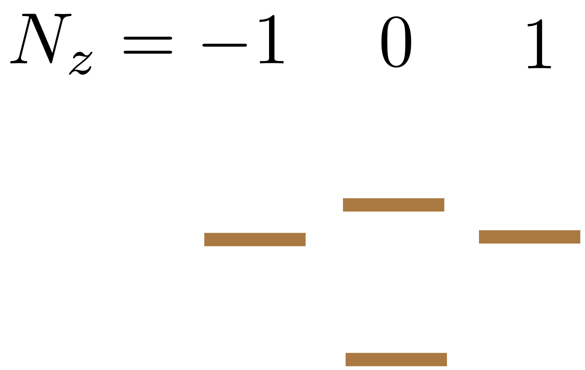

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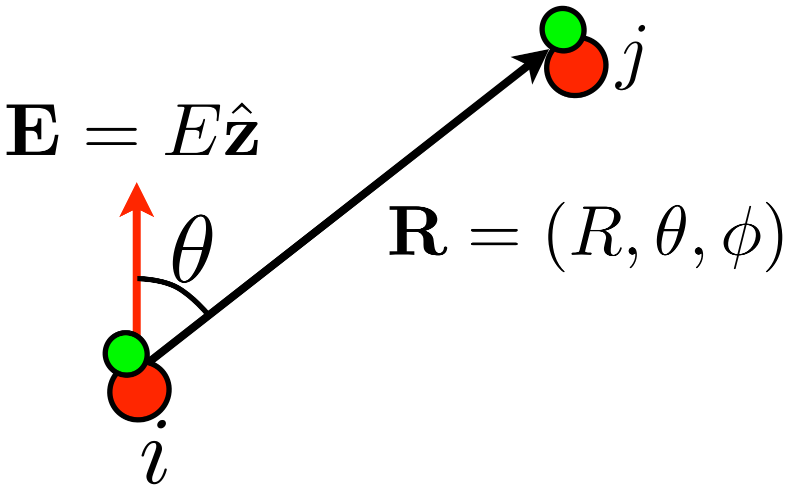

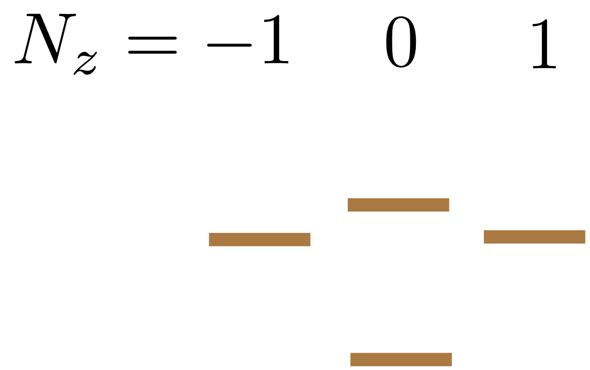

$$R^3 H_{dd} = \mathbf{d}_i \cdot \mathbf{d}_j - 3(\mathbf{d}_i \cdot \hat{\mathbf{R}})(\mathbf{d}_j \cdot \hat{\mathbf{R}})$$

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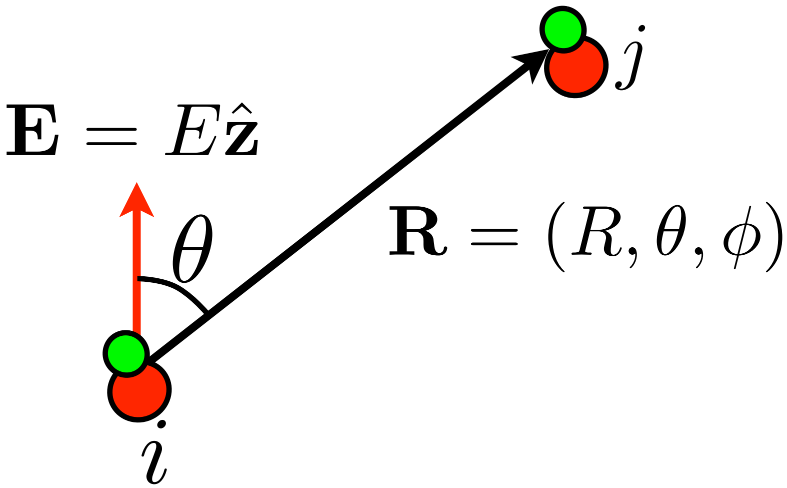
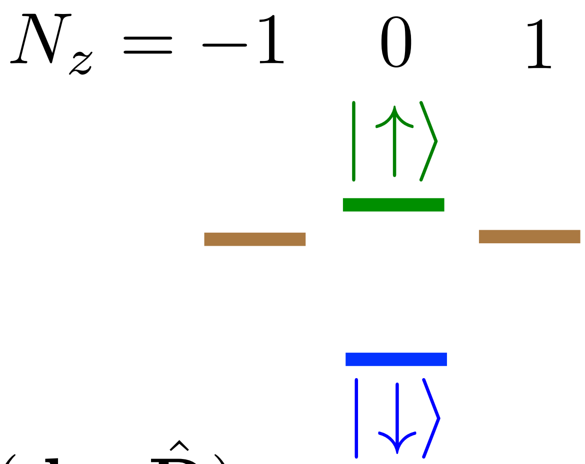
Simplest spin Hamiltonian





$$R^3 H_{dd} = \mathbf{d}_i \cdot \mathbf{d}_j - 3(\mathbf{d}_i \cdot \hat{\mathbf{R}})(\mathbf{d}_j \cdot \hat{\mathbf{R}})$$

AVG et al, PRL, PRA 2011; Kuns et al, PRA 2011

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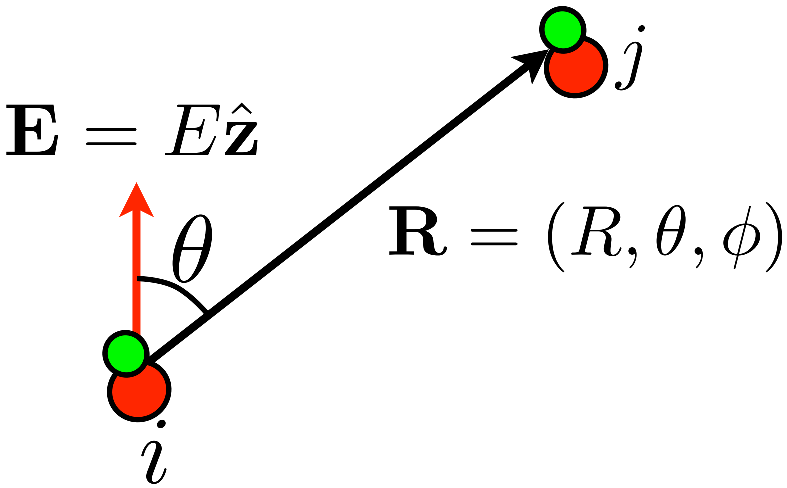

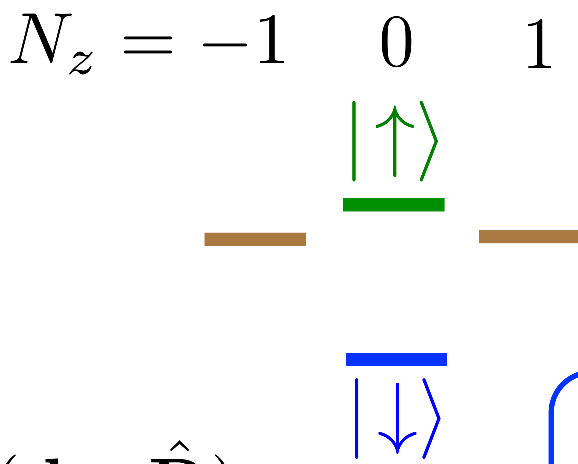

$$R^3 H_{dd} = \mathbf{d}_i \cdot \mathbf{d}_j - 3(\mathbf{d}_i \cdot \hat{\mathbf{R}})(\mathbf{d}_j \cdot \hat{\mathbf{R}})$$

$\boxed{H_{dd} \text{ weak}}$

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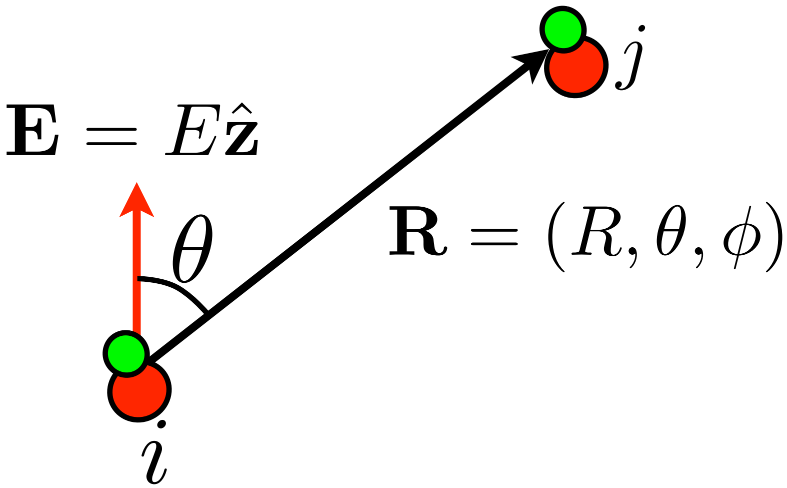

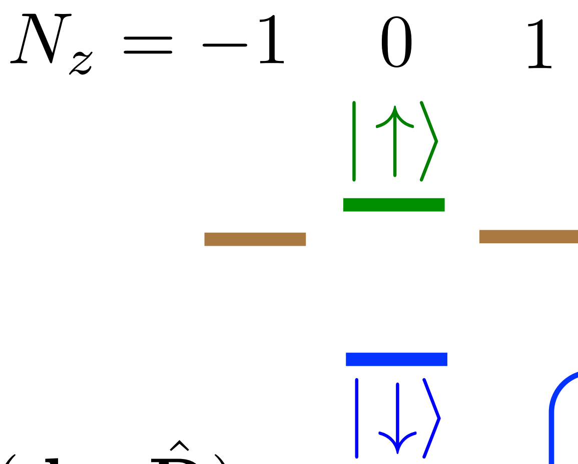

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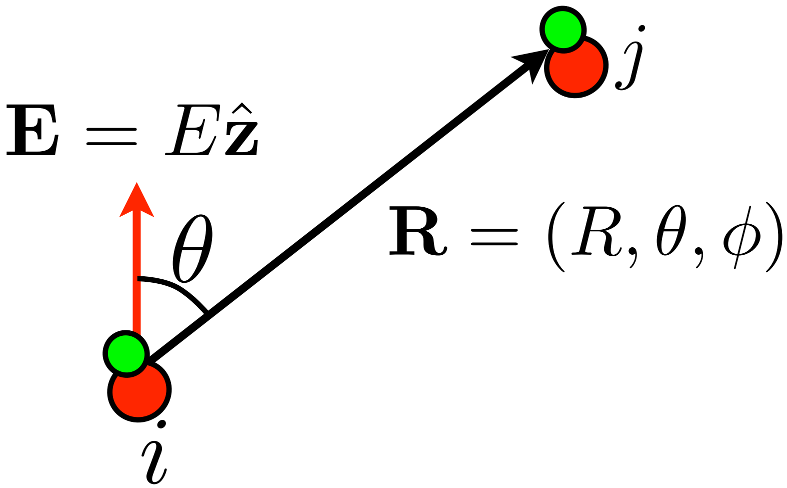
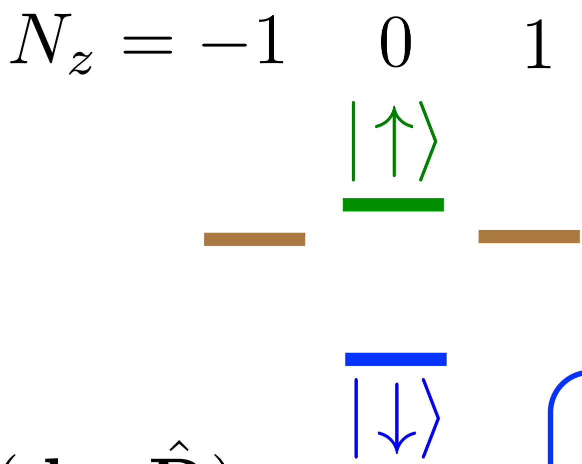
H_{dd}
weak

- project on $|\uparrow\rangle$, $|\downarrow\rangle$

AVG et al, PRL, PRA 2011; Kuns et al, PRA 2011

Simplest spin Hamiltonian





H_{dd}
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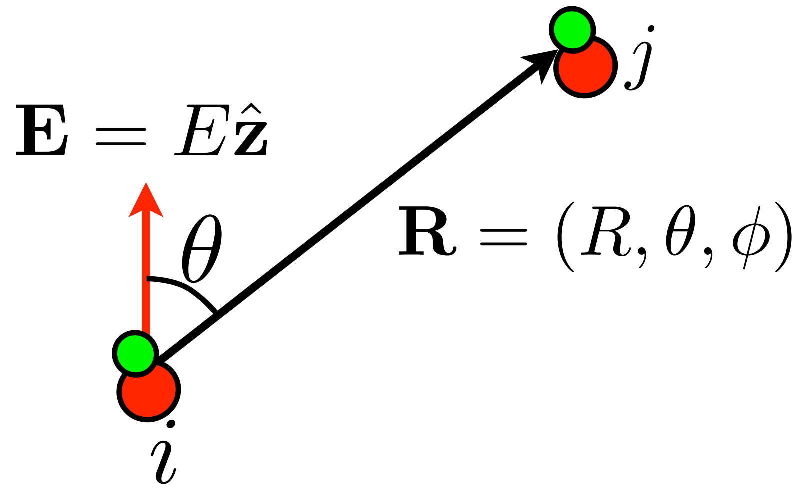
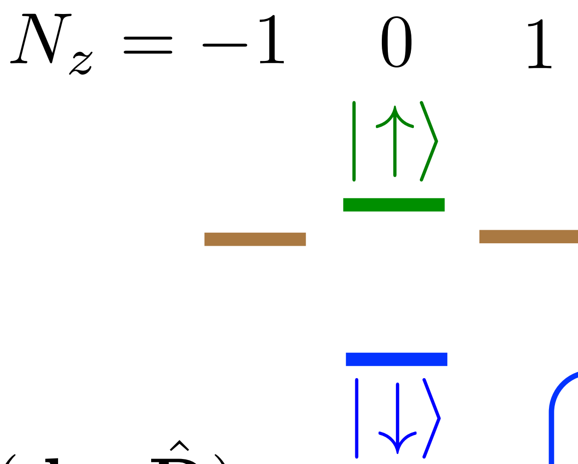
$$R^3 H_{dd} = Y_{2,0}(\theta)[J_z S_i^z S_j^z + J_{xy}(S_i^x S_j^x + S_i^y S_j^y)]$$

$$Y_{2,0} \propto 1 - 3\cos^2\theta$$

AVG et al, PRL, PRA 2011; Kuns et al, PRA 2011

Simplest spin Hamiltonian





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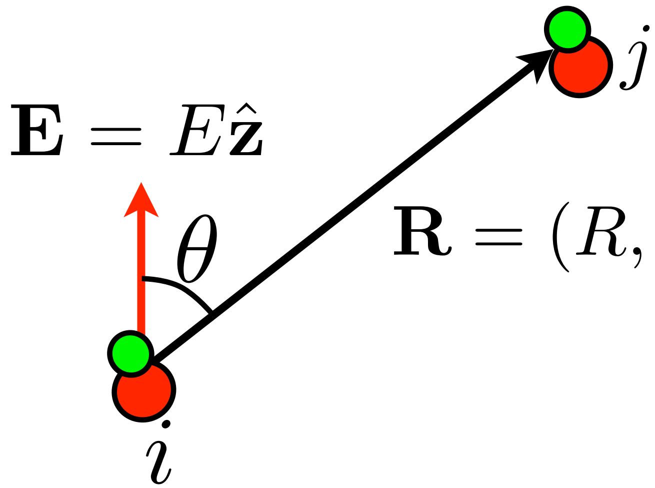
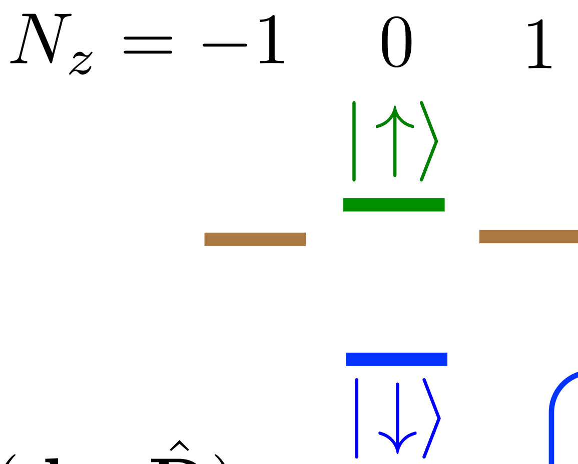


$\mathbf{R} = (R, \theta, \phi)$



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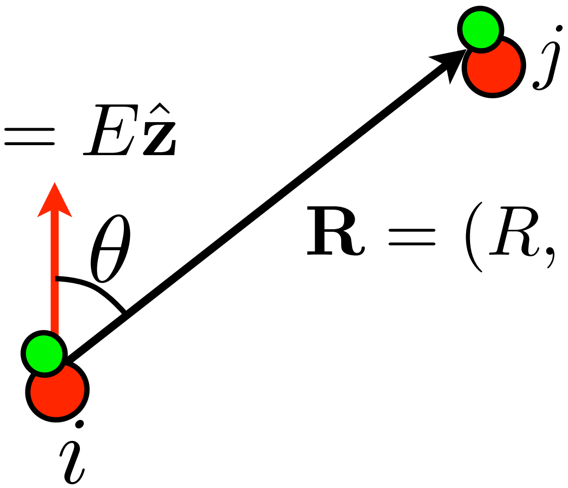
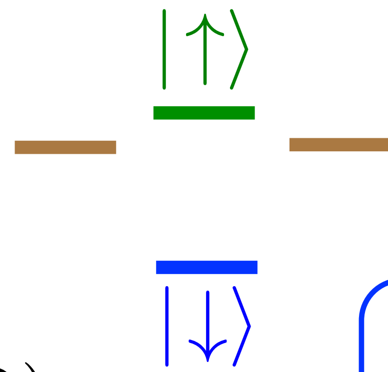
Simplest spin Hamiltonian

$$\mathbf{E} = E\hat{\mathbf{z}}$$



$$\mathbf{R} = (R, \theta, \phi)$$

$$N_z = -1 \quad 0 \quad 1$$



$\boxed{H_{dd} \text{ weak}}$

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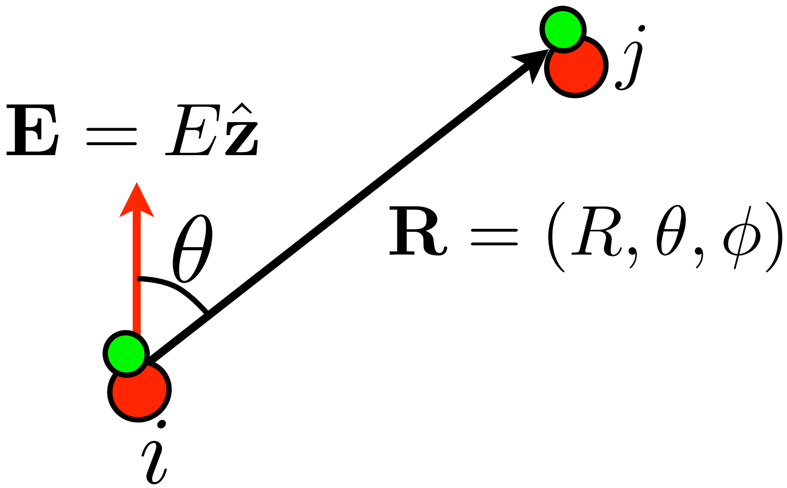

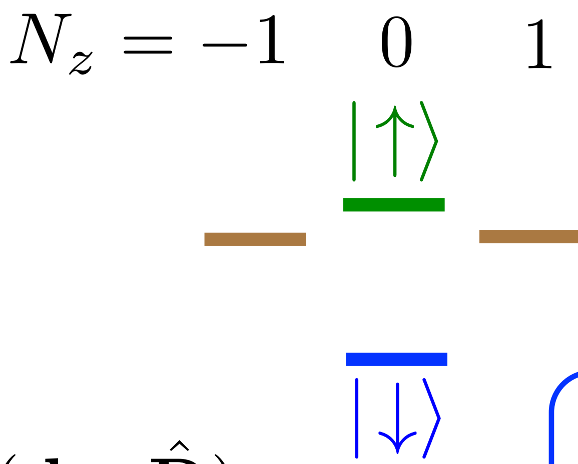

H_{dd} weak

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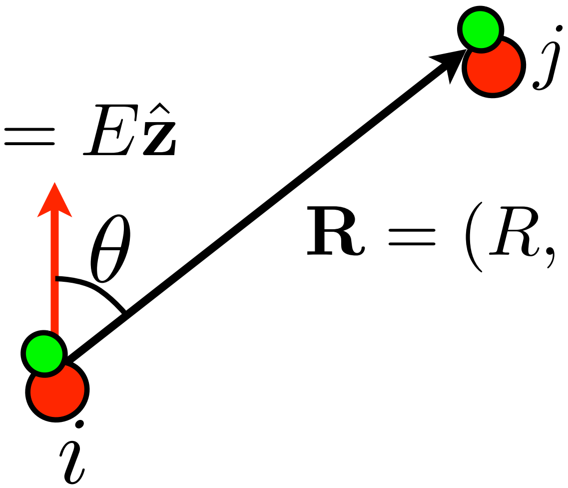

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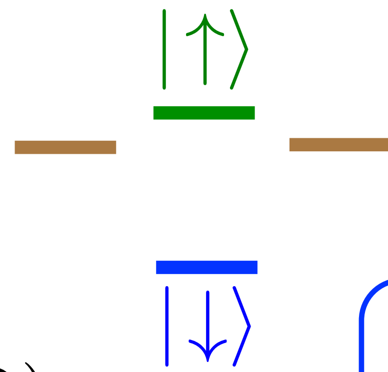

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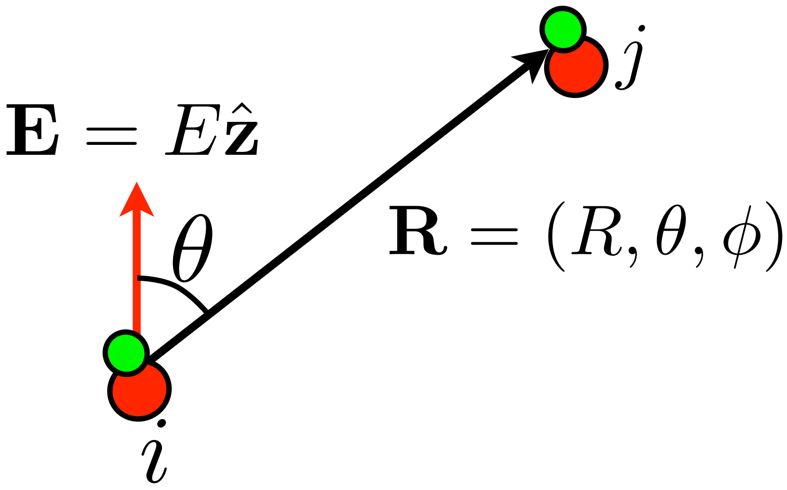

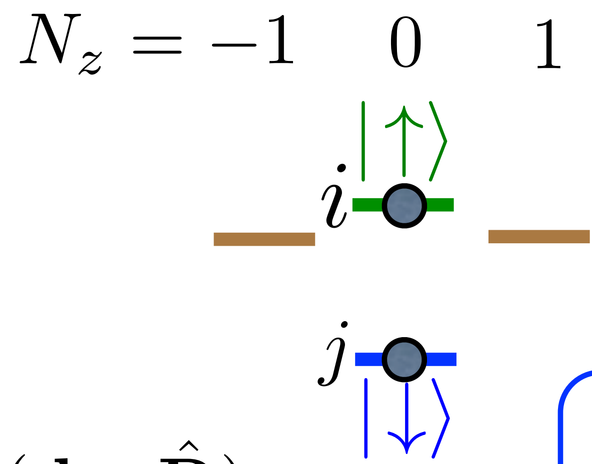

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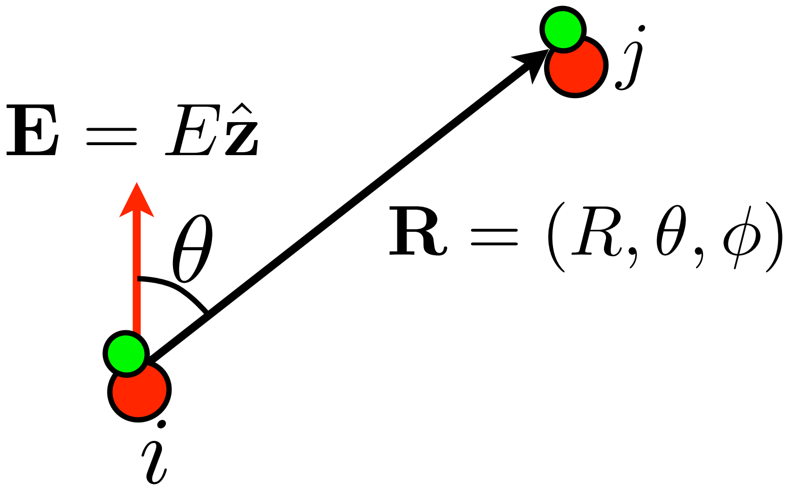

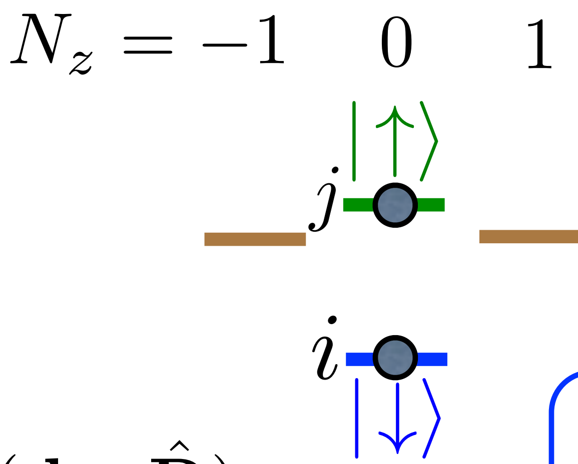

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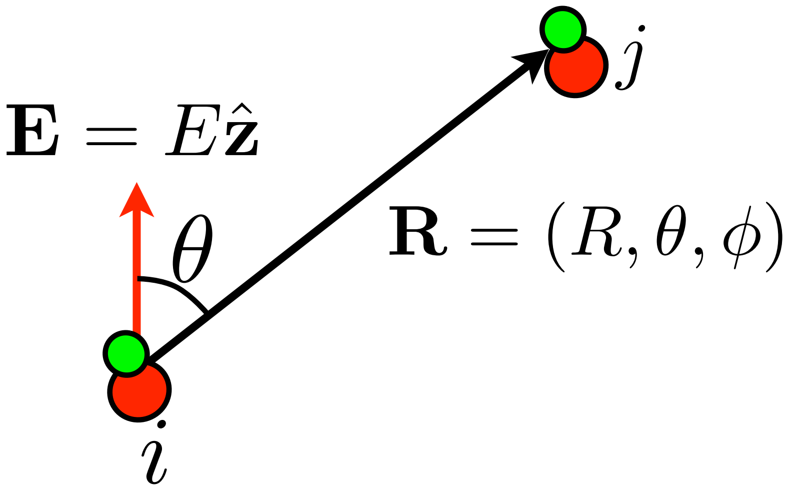

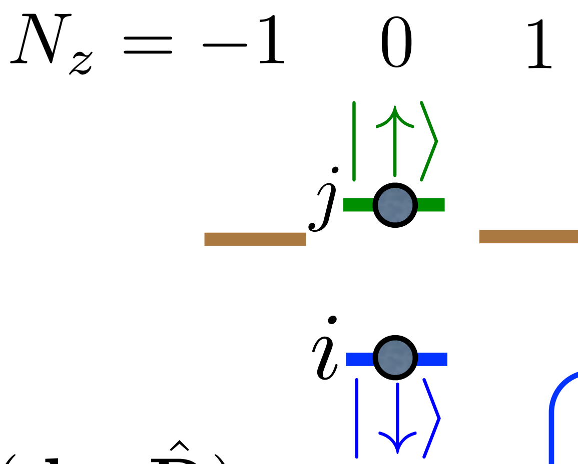

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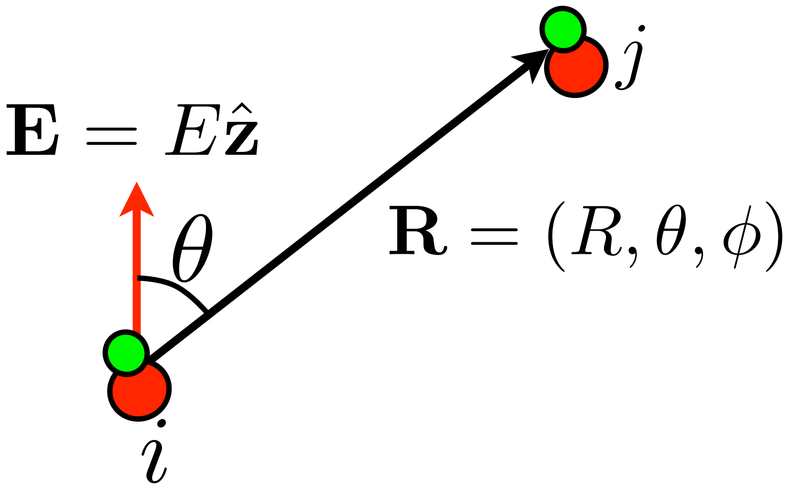
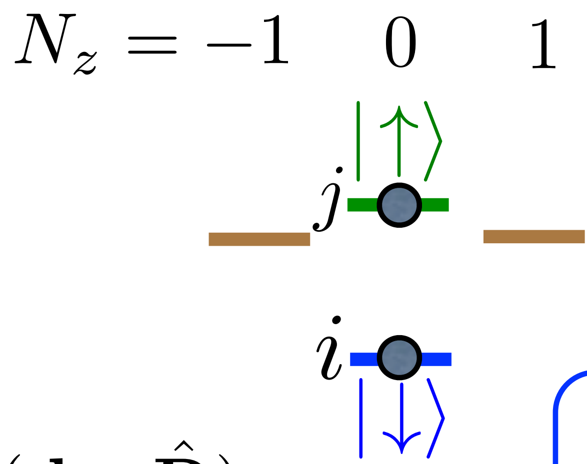
(• if add hopping ⇒ t-J model ⇒ d-wave superfluidity?)

AVG et al, PRL, PRA 2011; Kuns et al, PRA 2011

Simplest spin Hamiltonian





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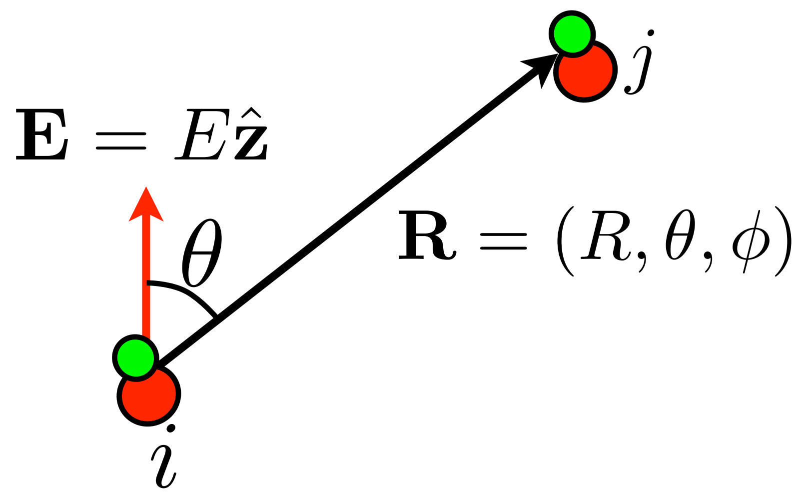

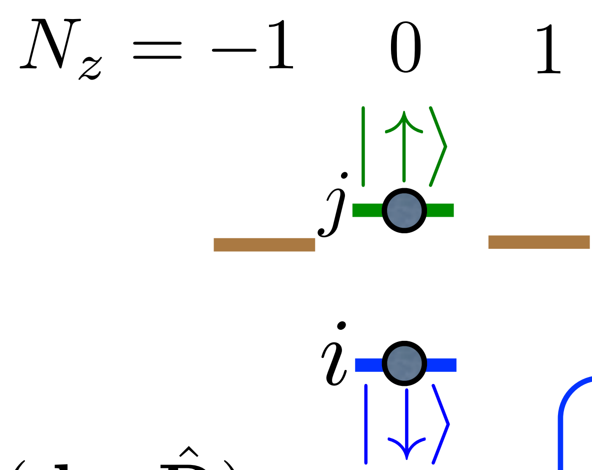

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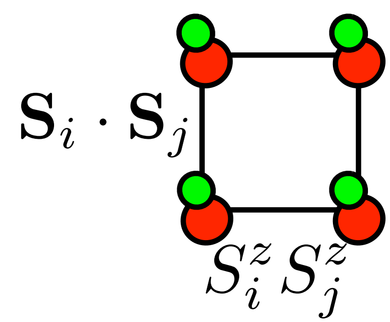

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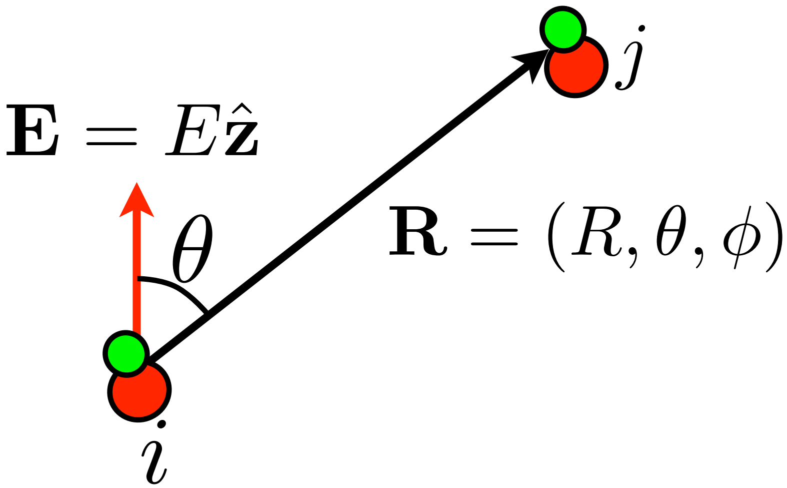

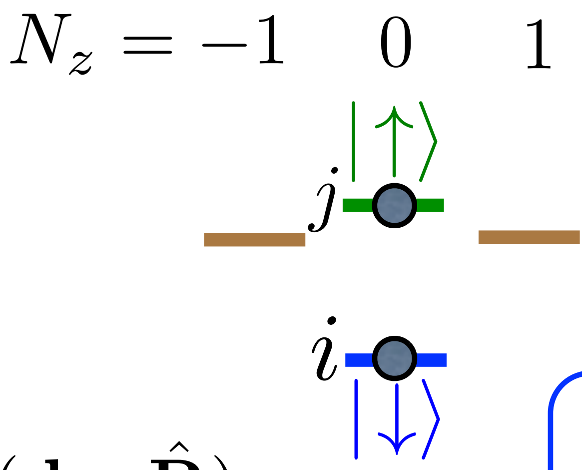

H_{dd} weak

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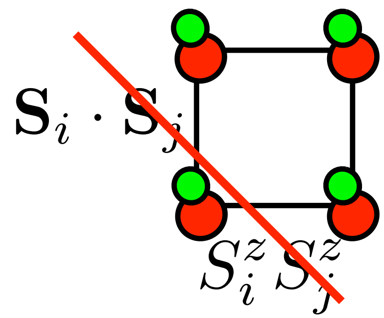

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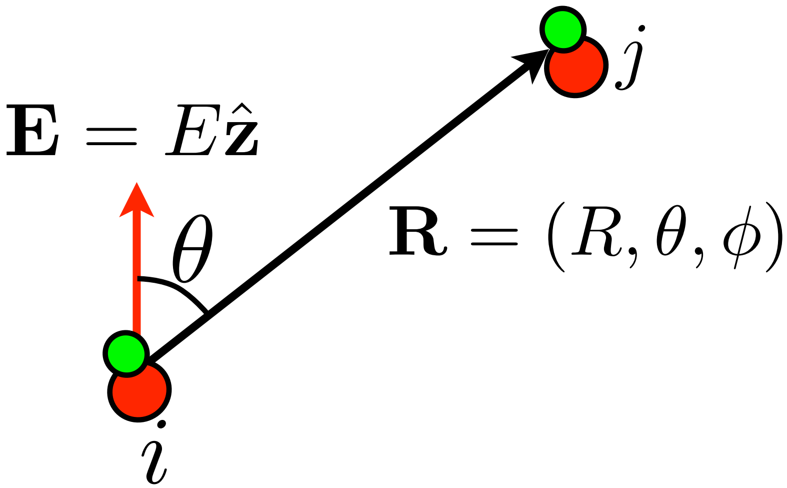

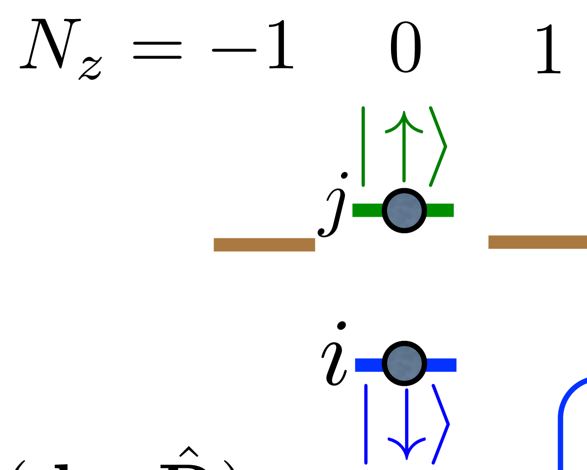

$$\boxed{\begin{array}{c} H_{dd} \\ \text{weak} \end{array}}$$

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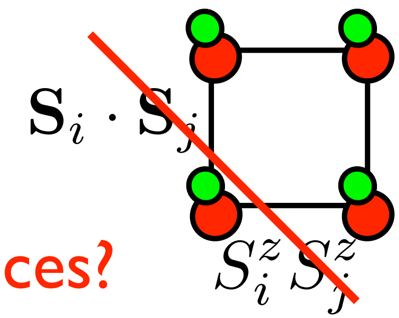
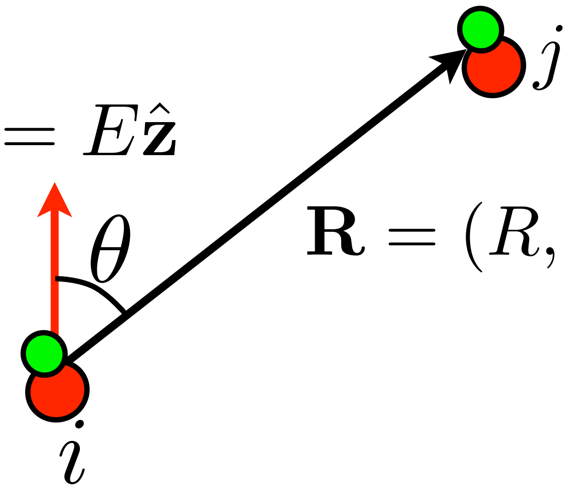


- can J_z and J_{xy} come with different angular dependences?

AVG et al, PRL, PRA 2011; Kuns et al, PRA 2011

Simplest spin Hamiltonian

$\mathbf{E} = E\hat{\mathbf{z}}$



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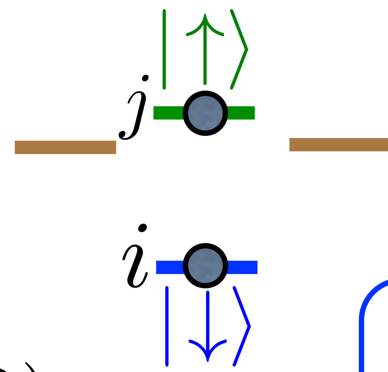

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H_{dd}
weak

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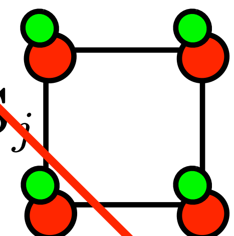
$\cancel{\mathbf{S}_i \cdot \mathbf{S}_j}$



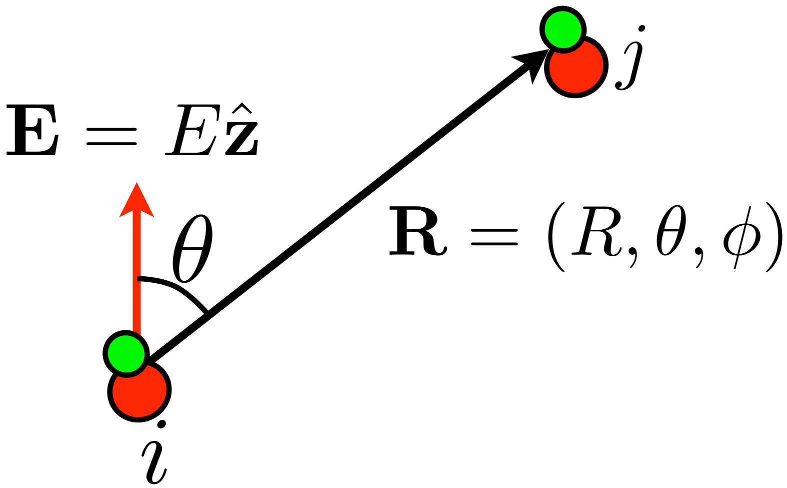
$\cancel{S_i^z S_j^z}$

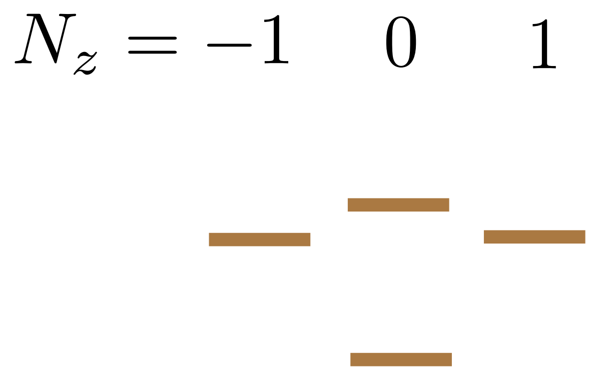
- can J_z and J_{xy} come with different angular dependences?

YES!

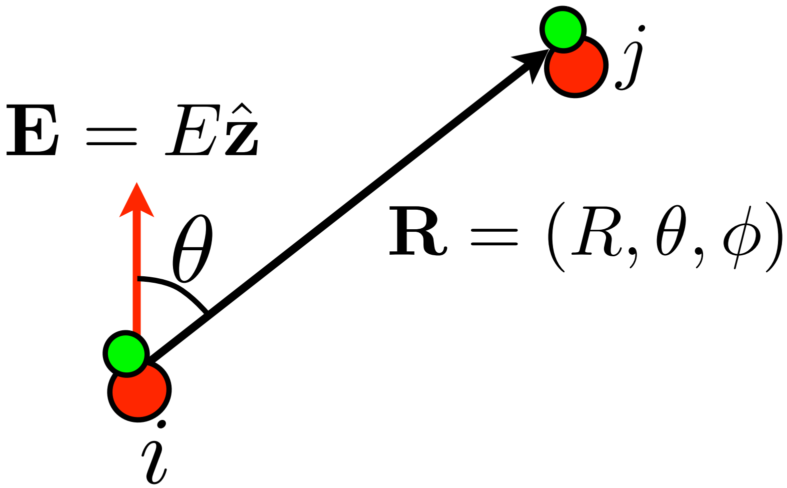
AVG et al, PRL, PRA 2011; Kuns et al, PRA 2011

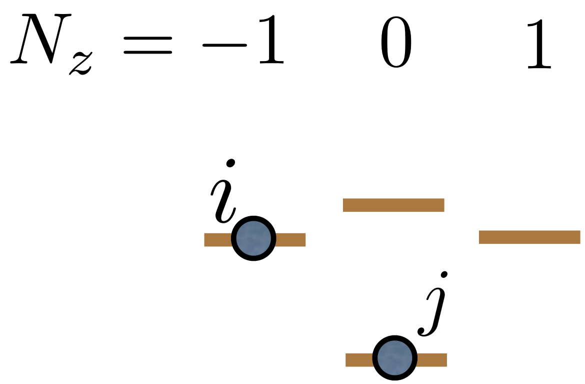
Spin Hamiltonian from dressed states



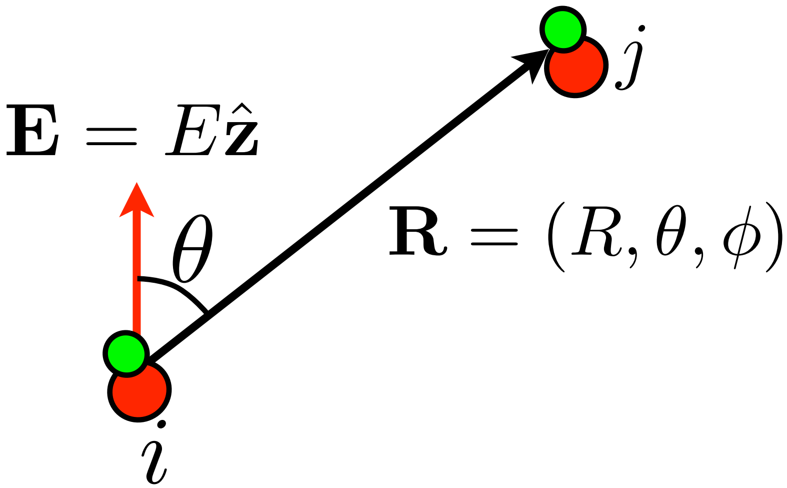


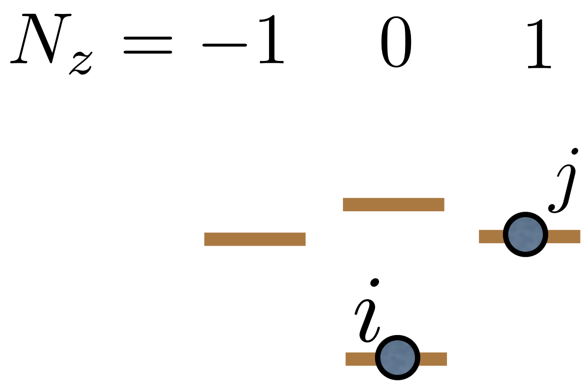
Spin Hamiltonian from dressed states



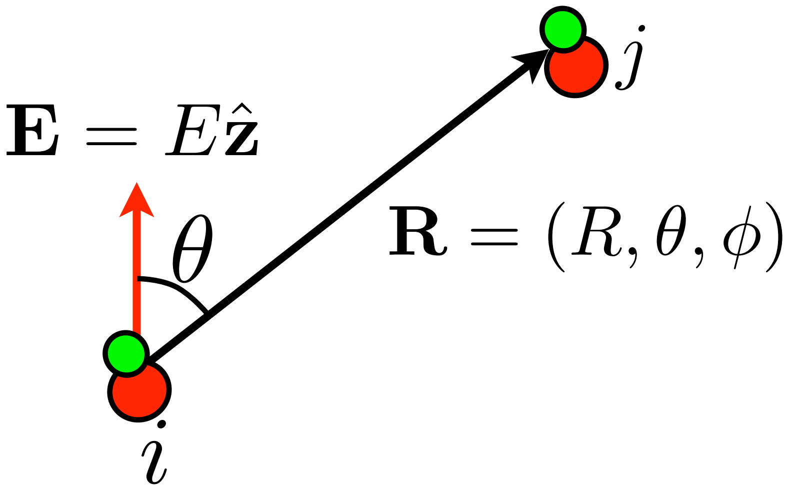


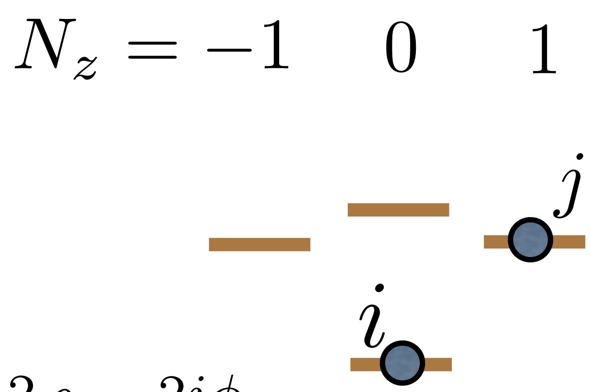
Spin Hamiltonian from dressed states





Spin Hamiltonian from dressed states





$$Y_{2,-2} \propto \sin^2\theta e^{-2i\phi}$$

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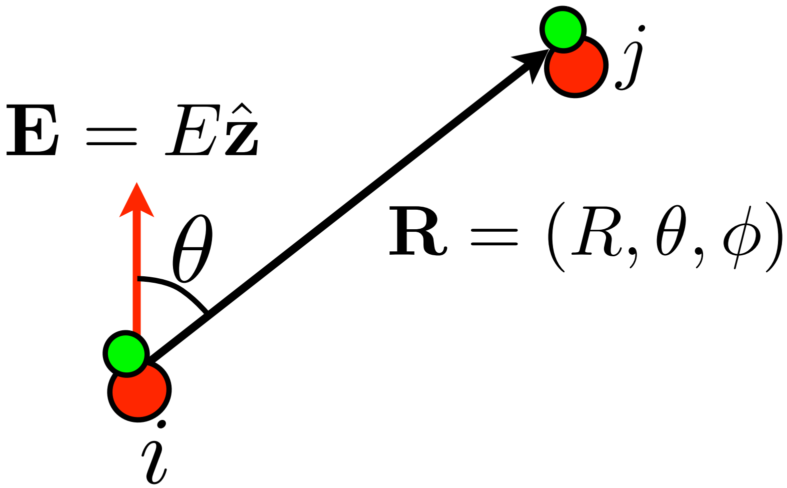

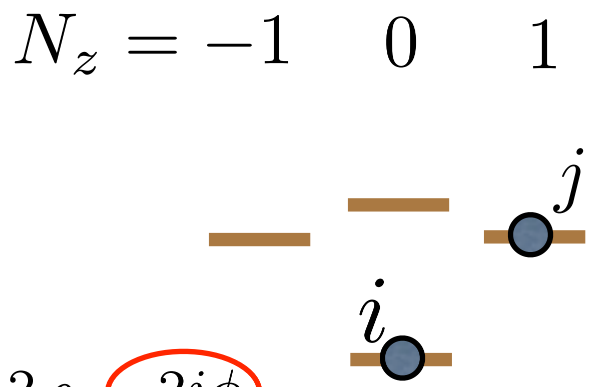

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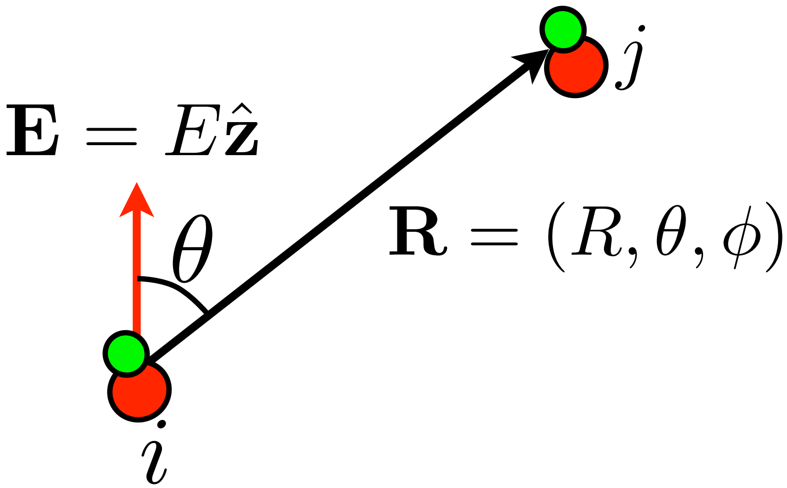

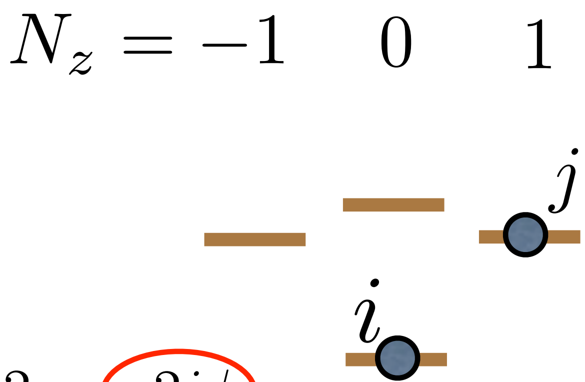

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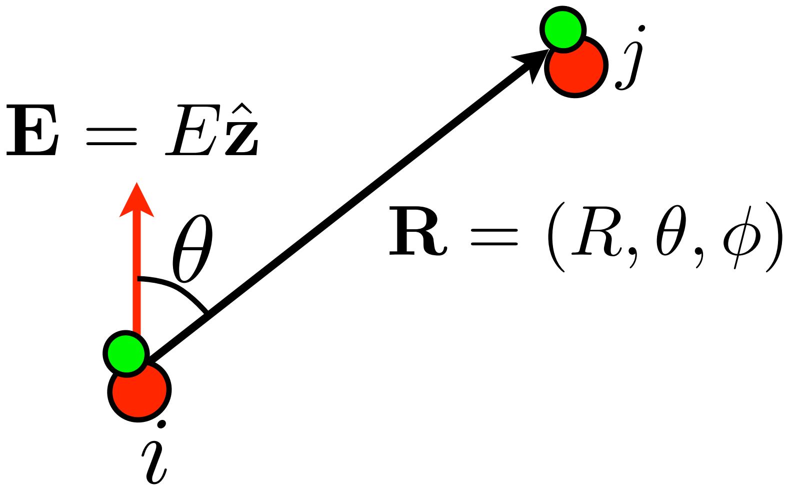

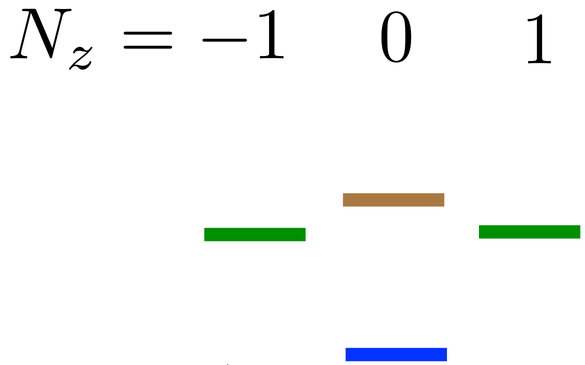

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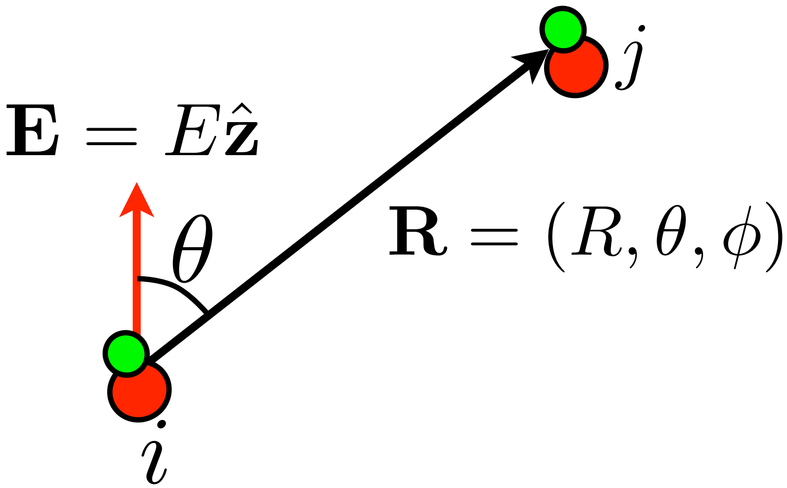

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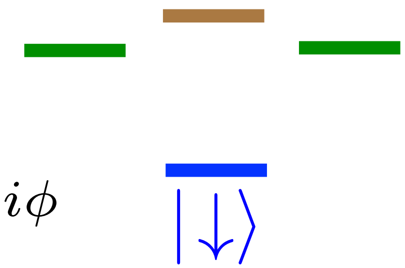

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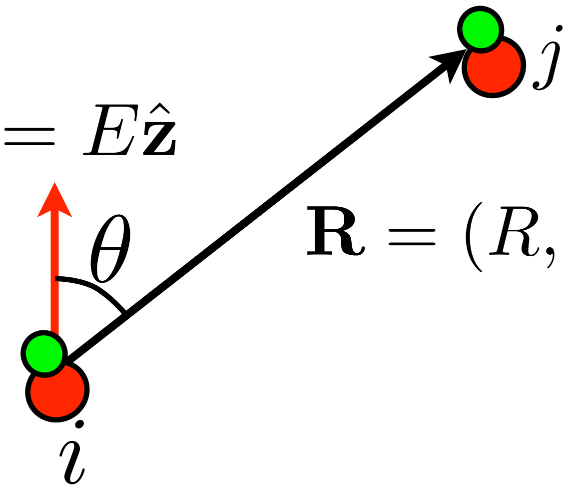
Spin Hamiltonian from dressed states

$\mathbf{E} = E\hat{\mathbf{z}}$

$\mathbf{R} = (R, \theta, \phi)$



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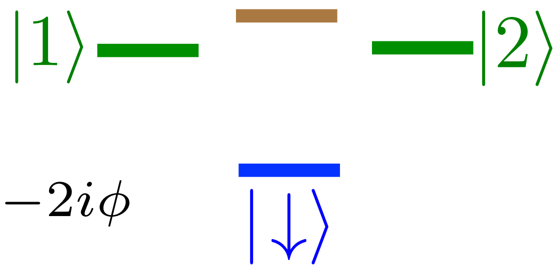

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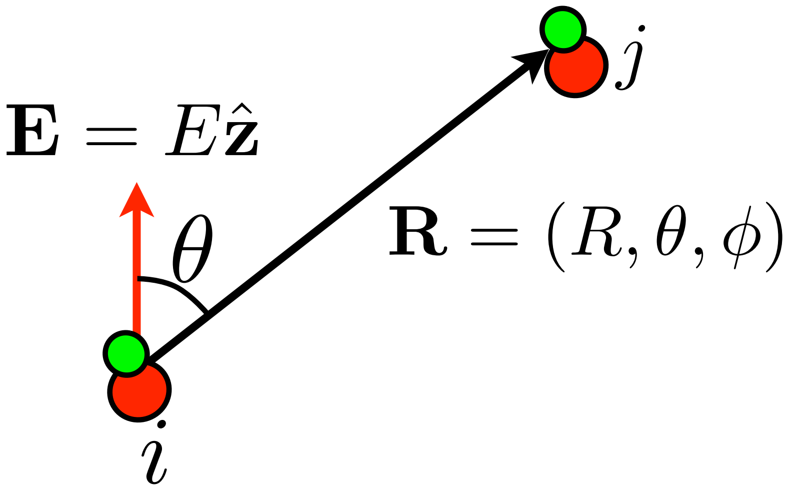

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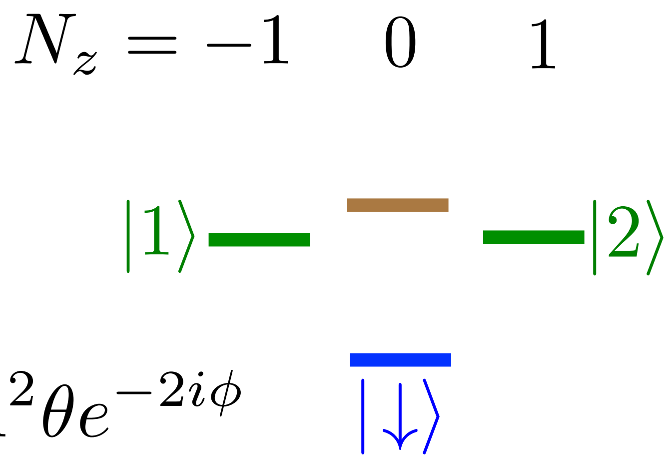

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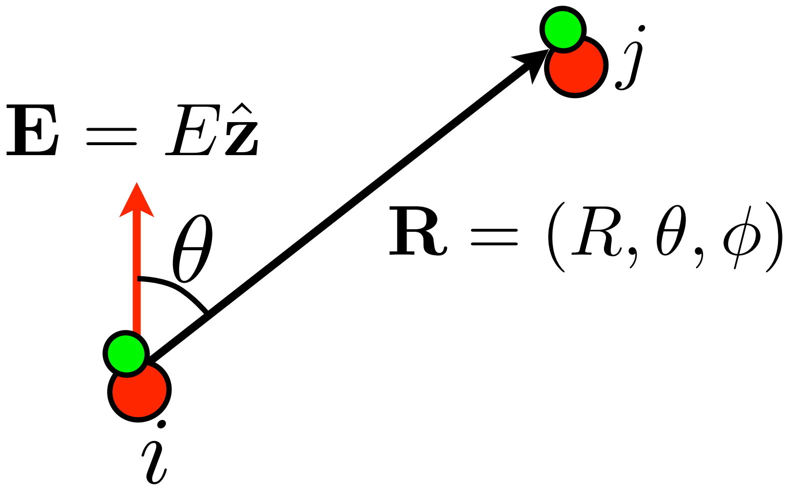

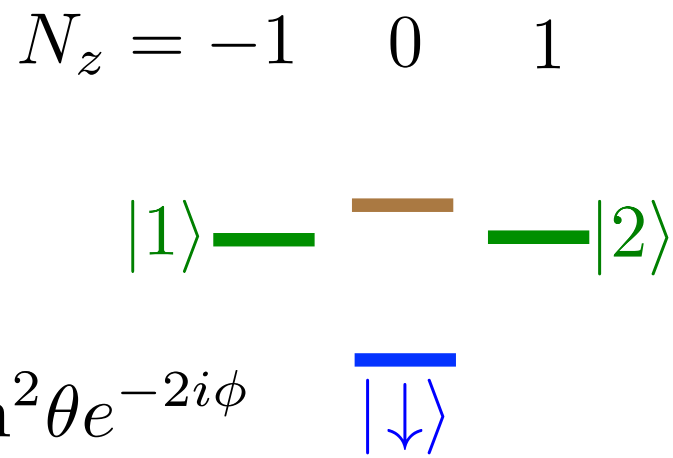

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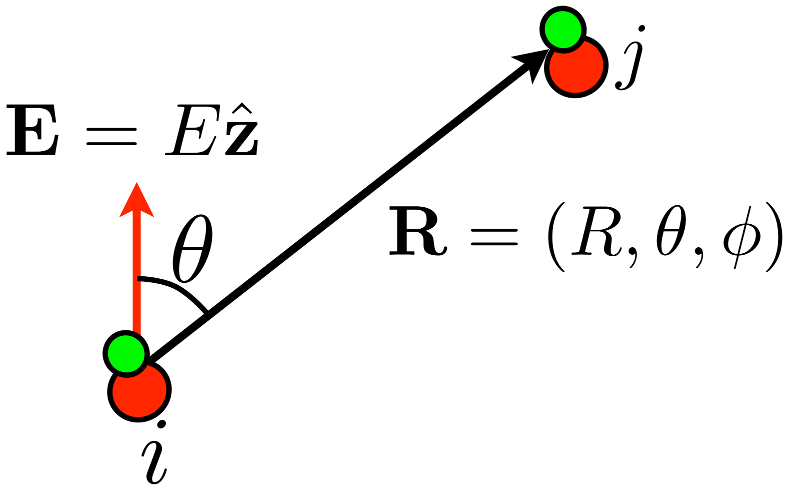

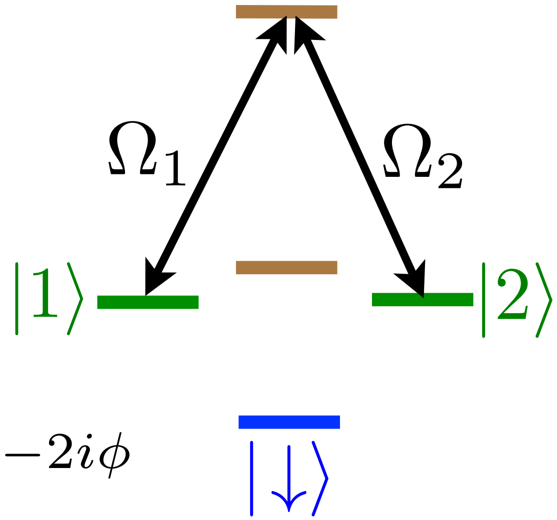

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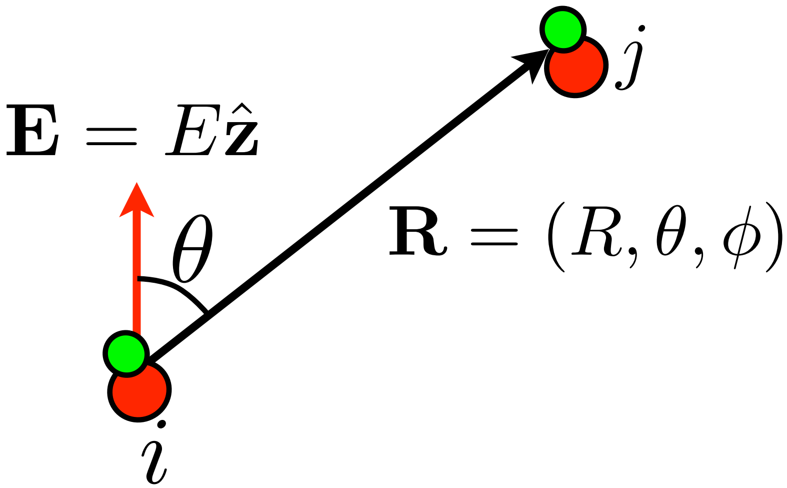

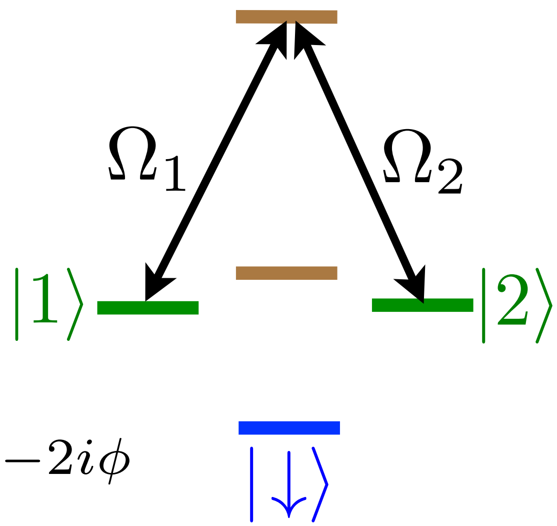

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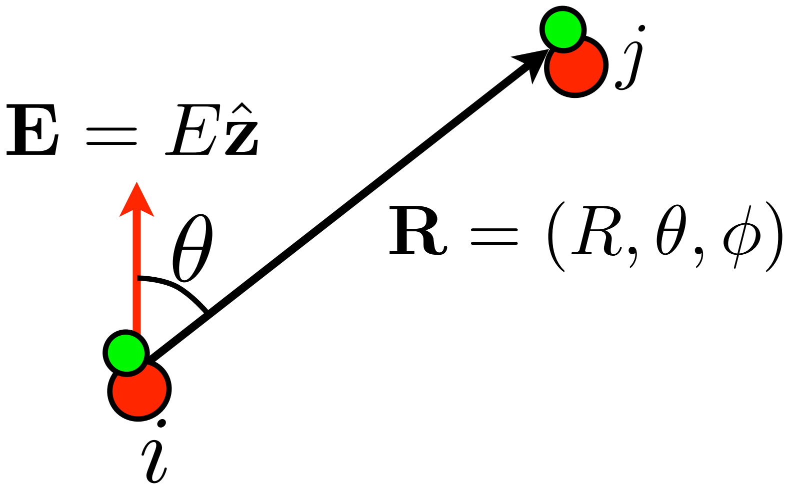

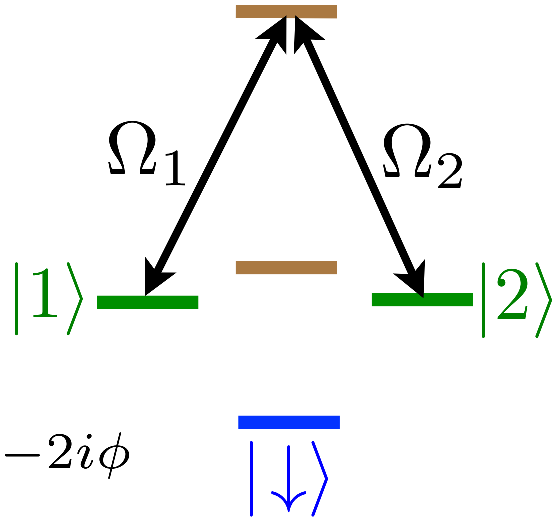

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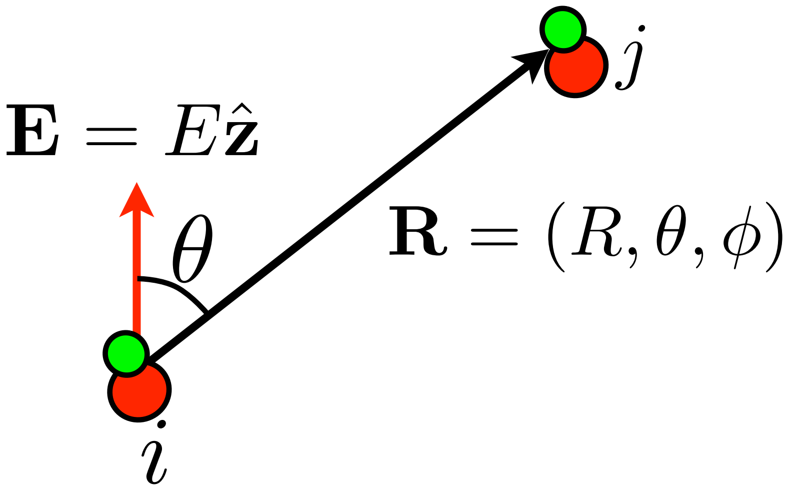

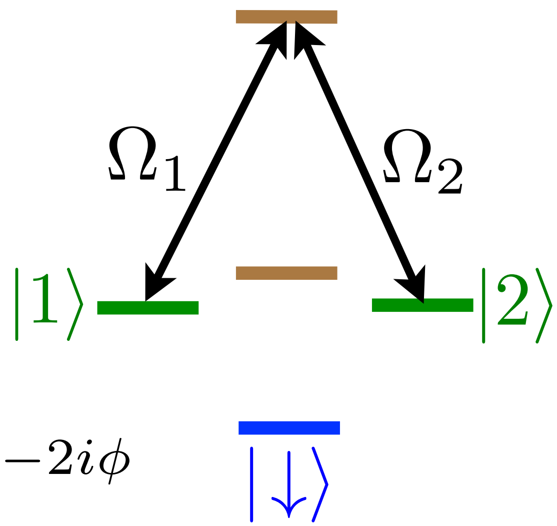

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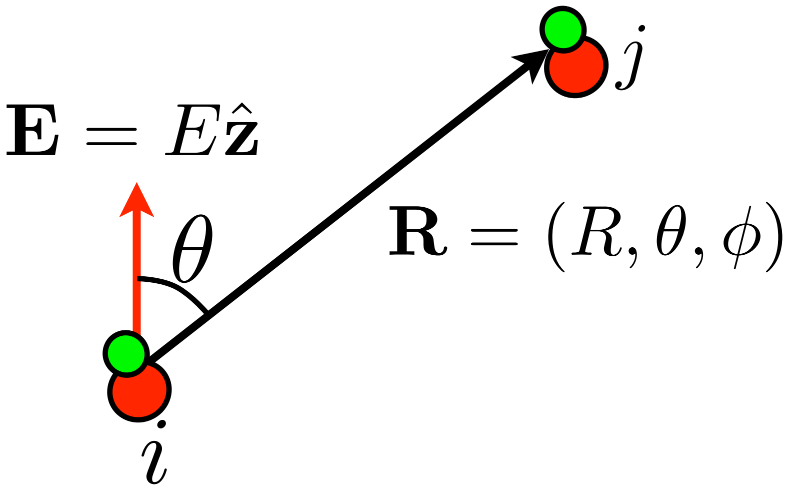

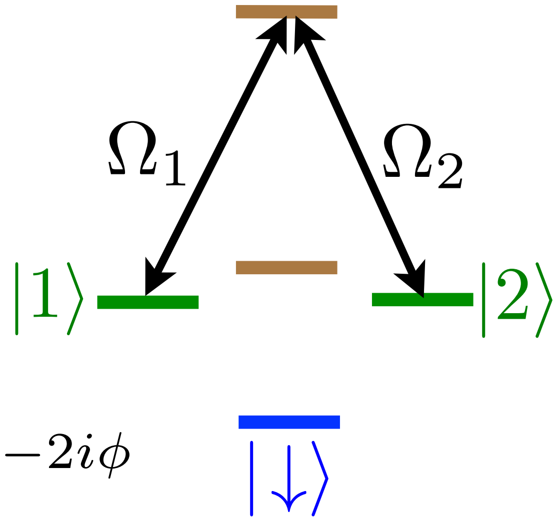

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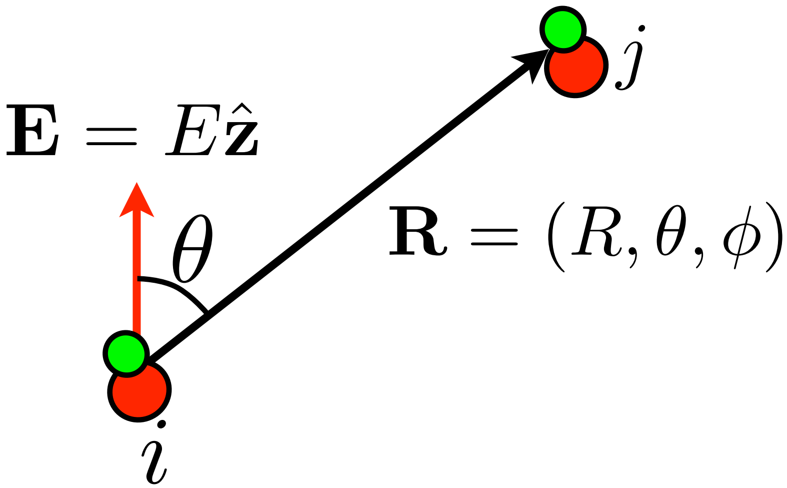

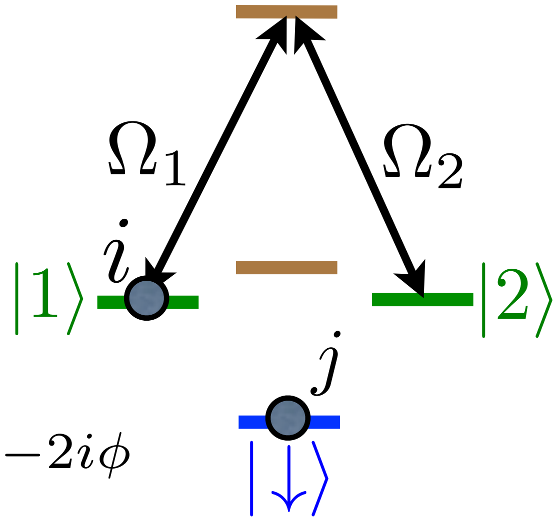

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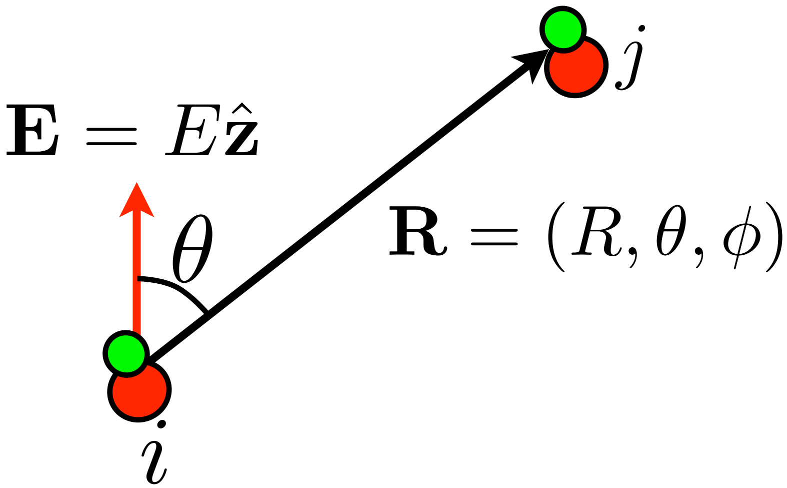

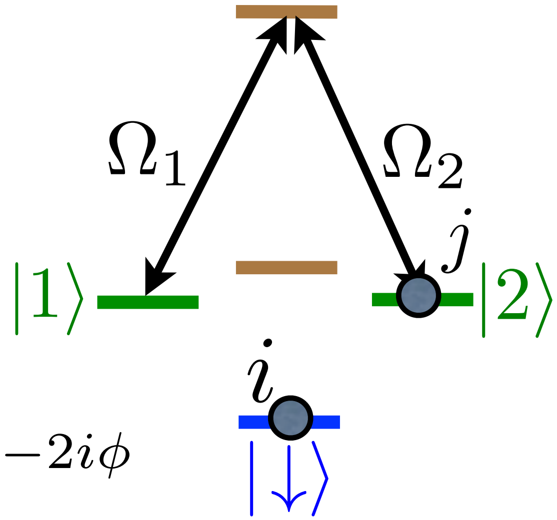

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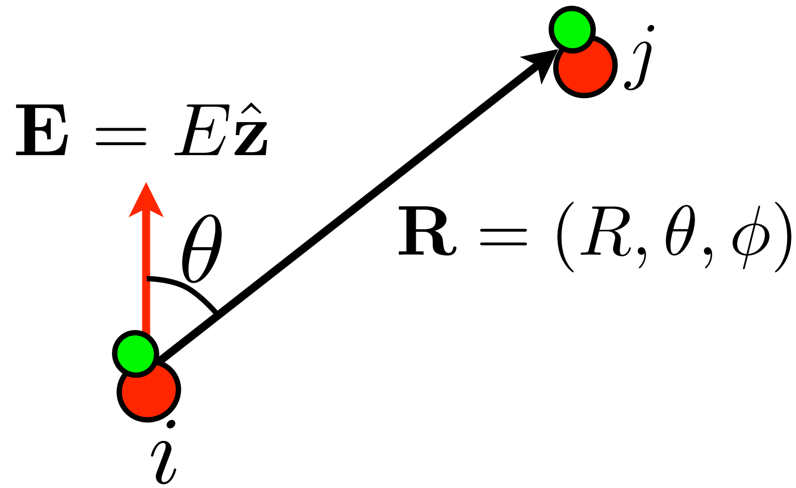

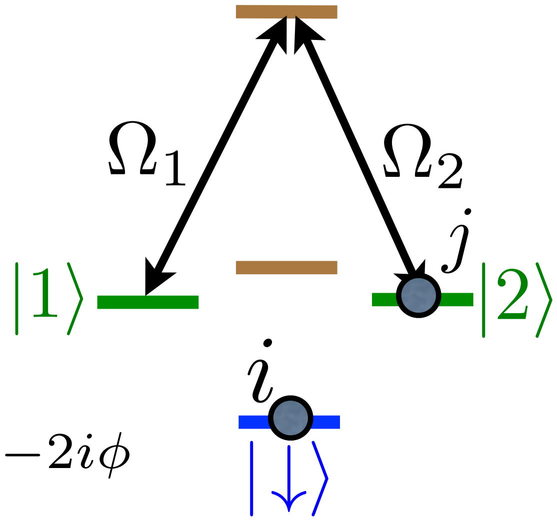

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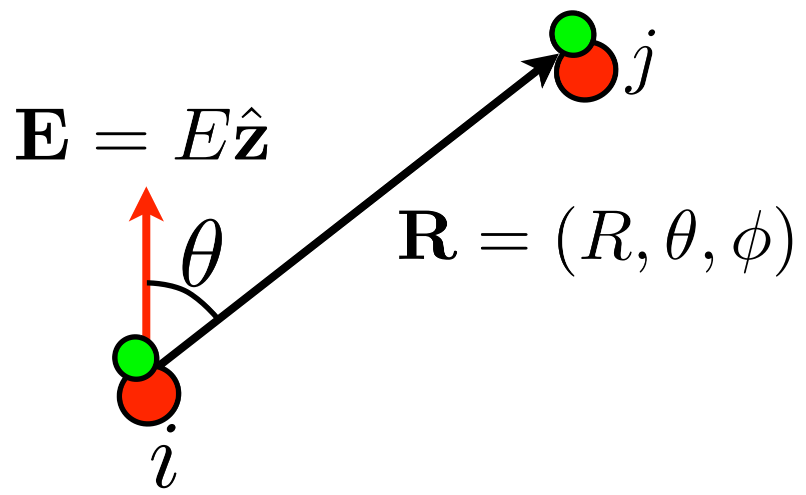

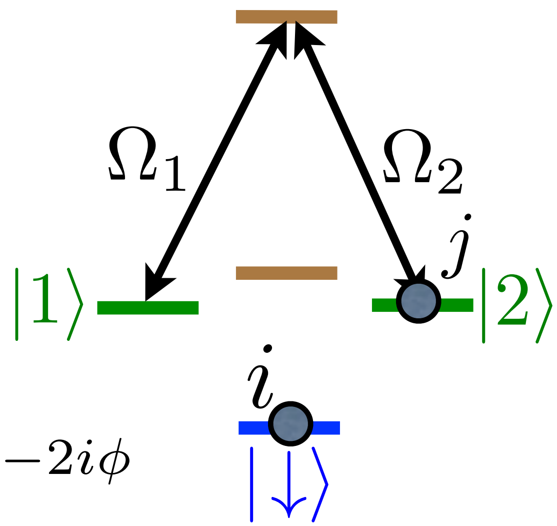

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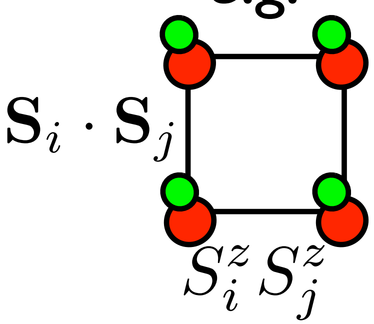

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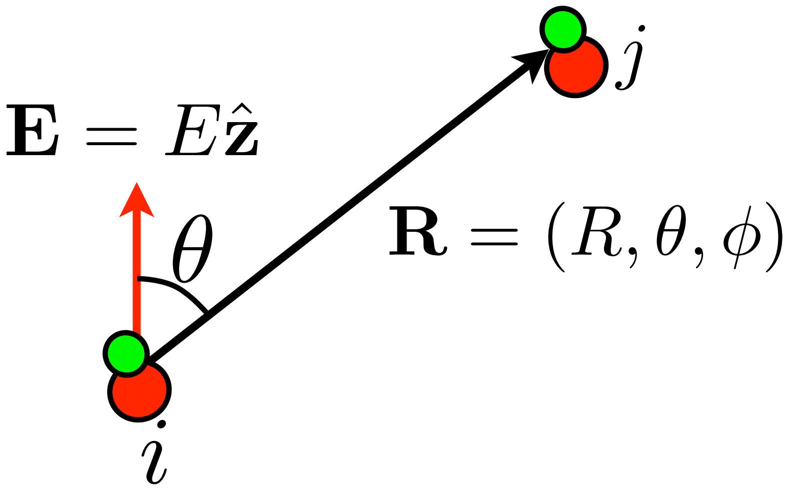

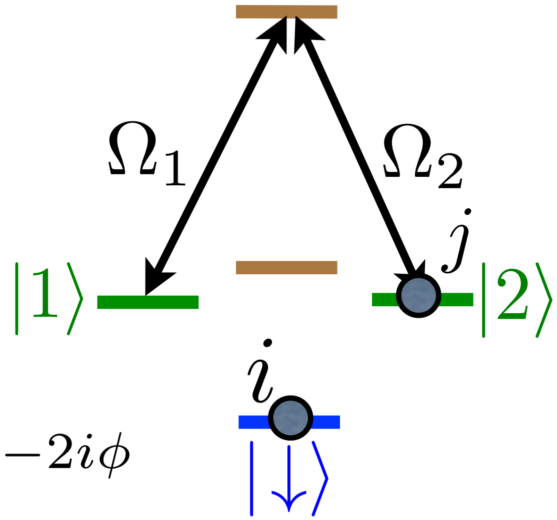

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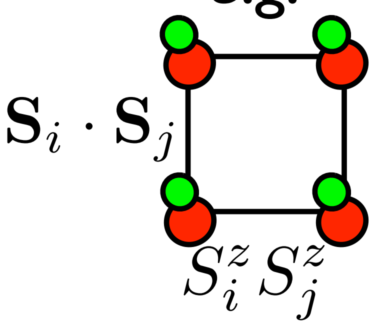

- what can we implement with this?

Symmetry protected topological (SPT) phases

Shnyder et al 2008, Kitaev 2009, Gu & Wen 2009

Symmetry protected topological (SPT) phases

- topological ≈ no local order parameter, exotic

local order parameter:
e.g. ↑↓↑↓↑↓

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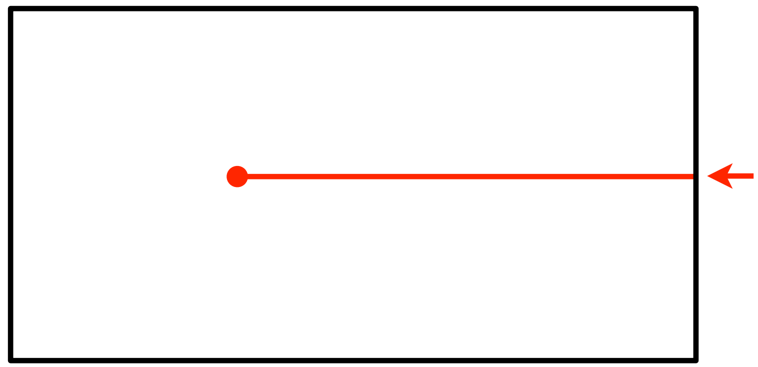

line of phase transitions

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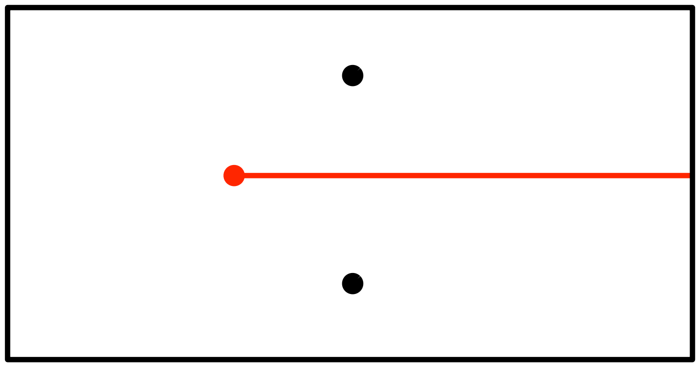

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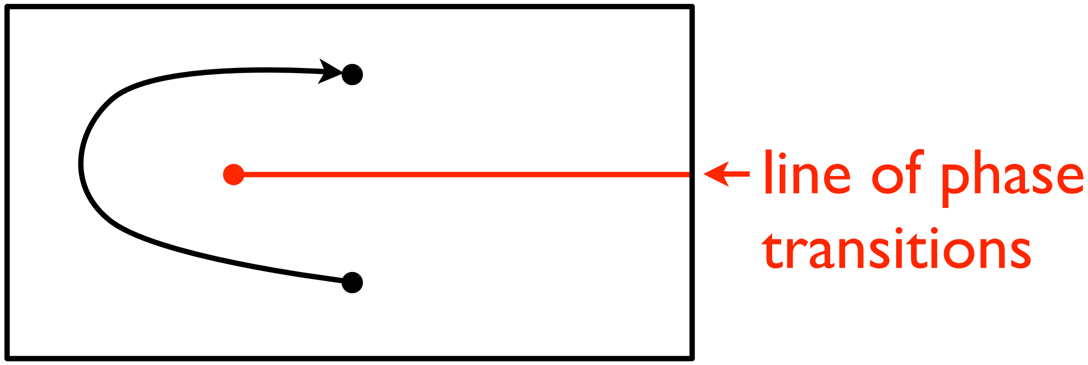

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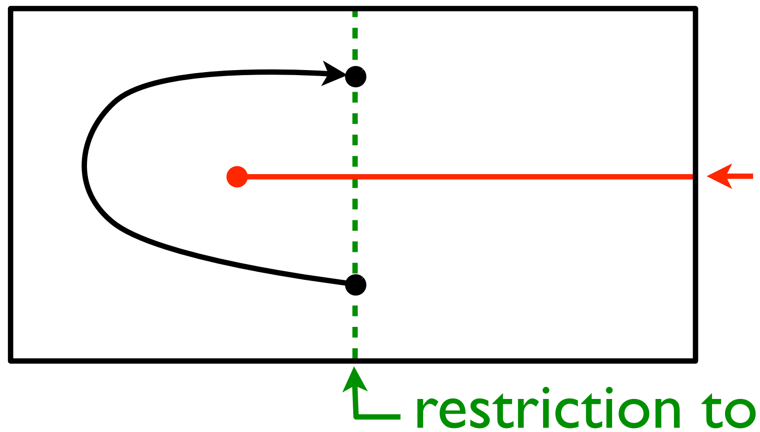

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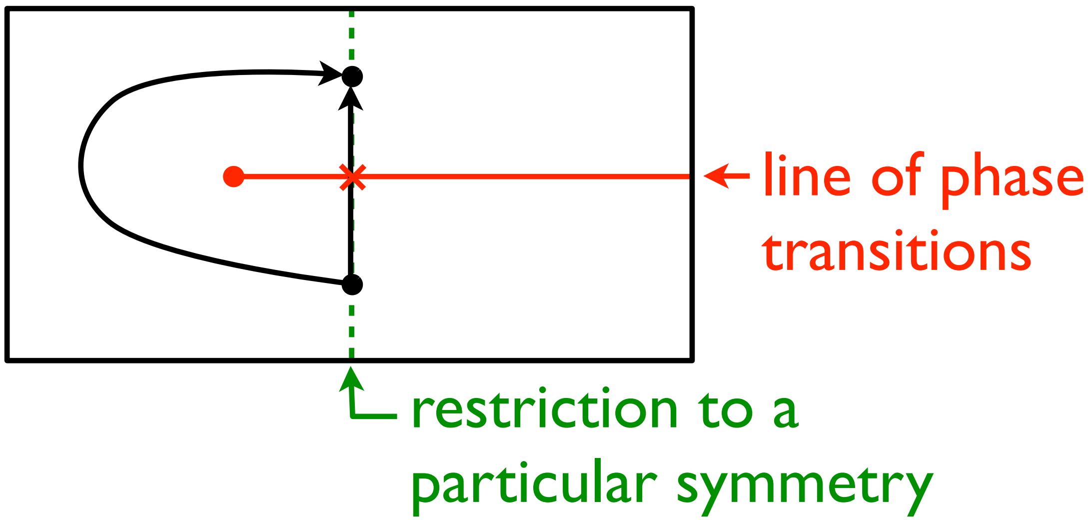

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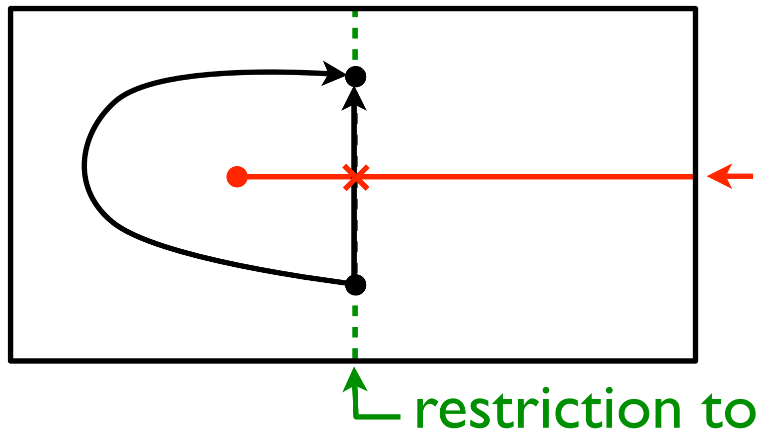


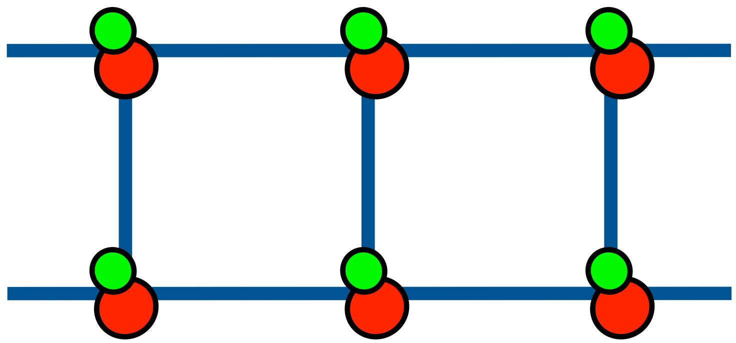
line of phase transitions

restriction to a particular symmetry

- motivation: quest to classify and understand all phases of matter

Shnyder et al 2008, Kitaev 2009, Gu & Wen 2009

SPT phases in spin-1/2 ladders



Liu, Wen, et al, PRB 2012; Chen, Gu, Wen, PRB 2011; Pollman et al, PRB 2010

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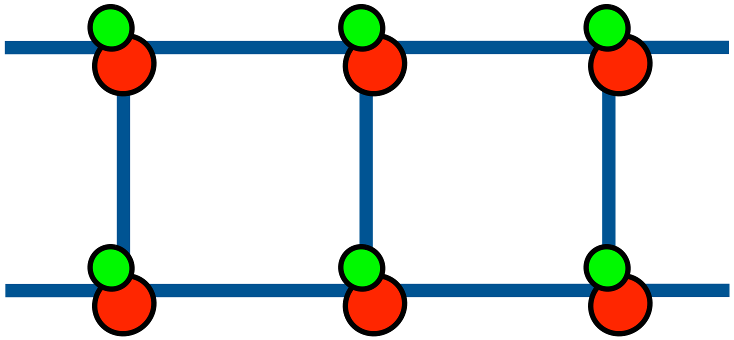

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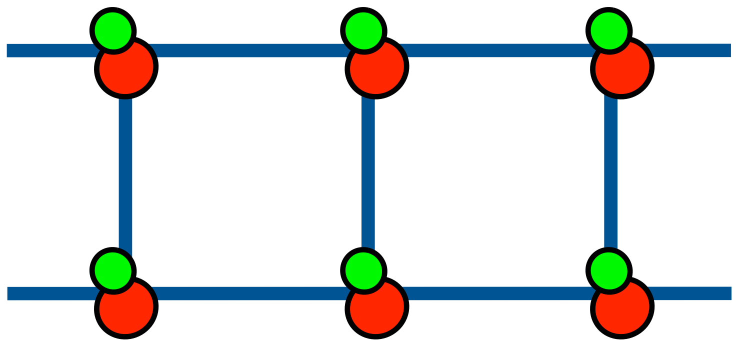

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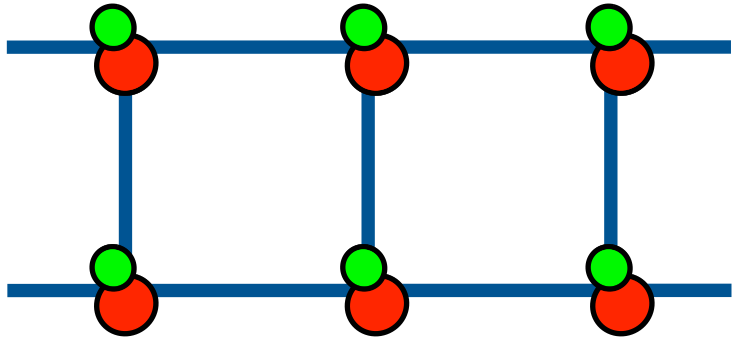

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$\{E, R_x, R_y, R_z\}$

identity

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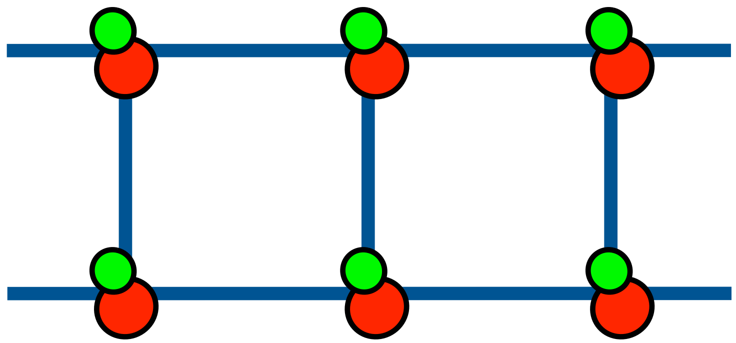
SPT phases in spin-1/2 ladders



- symmetry:

$$\underbrace{D_2}_{\{E,\, R_x,\, R_y,\, R_z\}} \times \underbrace{\sigma}_{\text{exchange of the two legs}}$$

exchange of the two legs

$\{E, R_x, R_y, R_z\}$

E: identity

R_x: π-pulse around x-axis on all spins

Liu, Wen, et al, PRB 2012; Chen, Gu, Wen, PRB 2011; Pollman et al, PRB 2010

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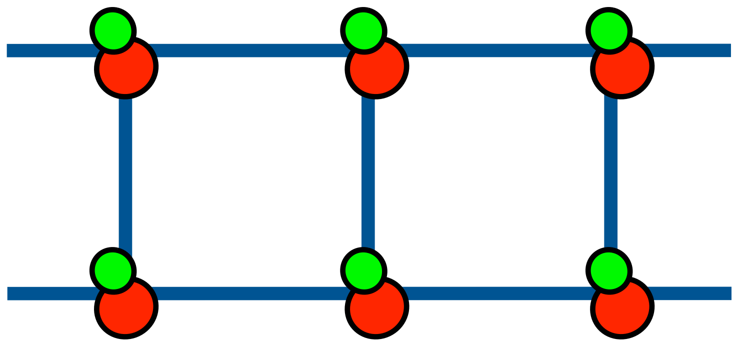

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7 non-trivial SPT phases

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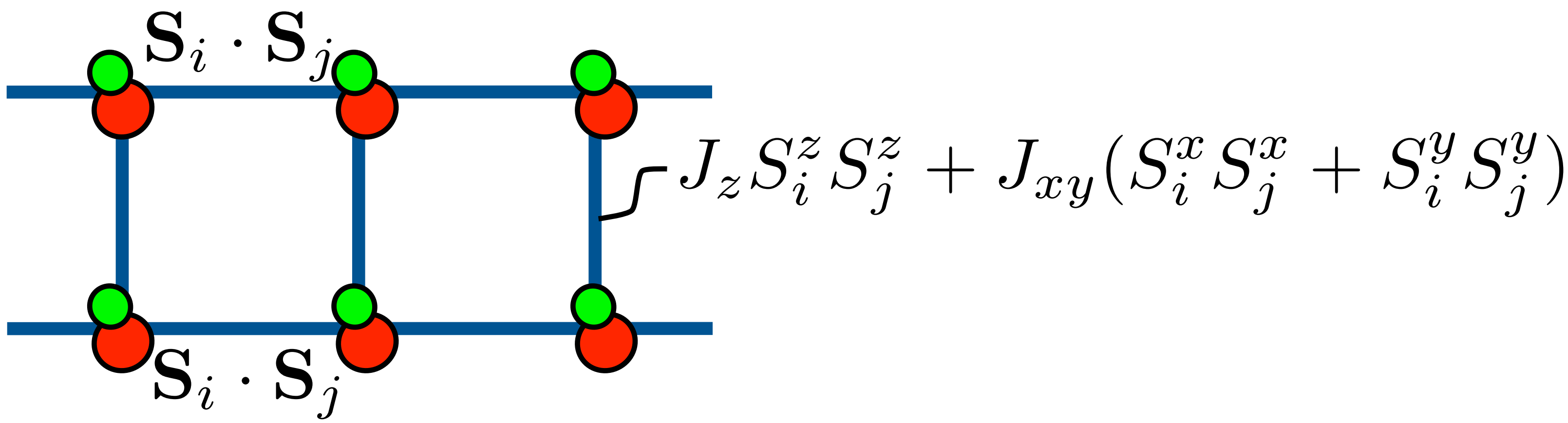

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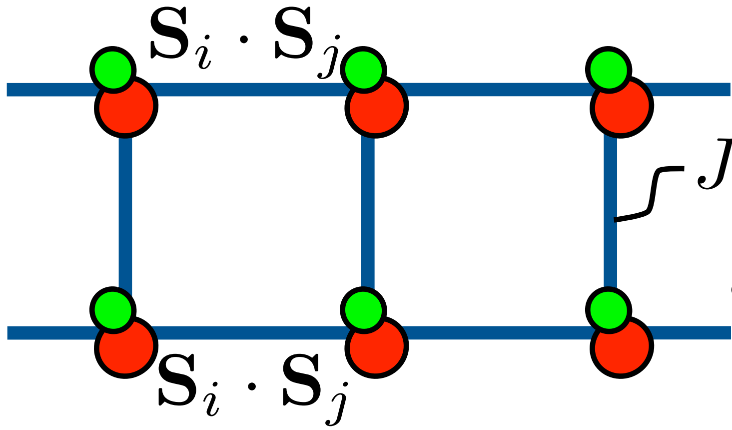

$J_z S_i^z S_j^z + J_{xy}(S_i^x S_j^x + S_i^y S_j^y)$

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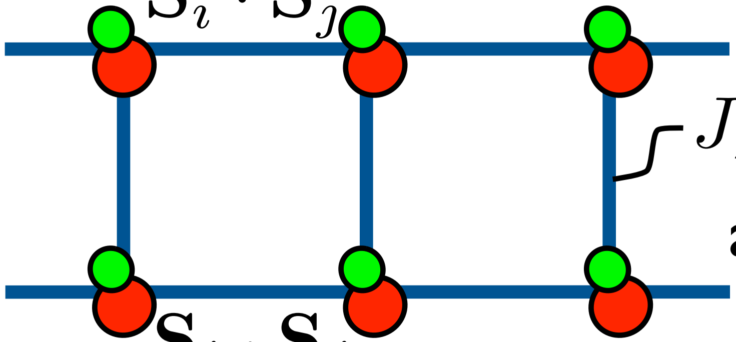

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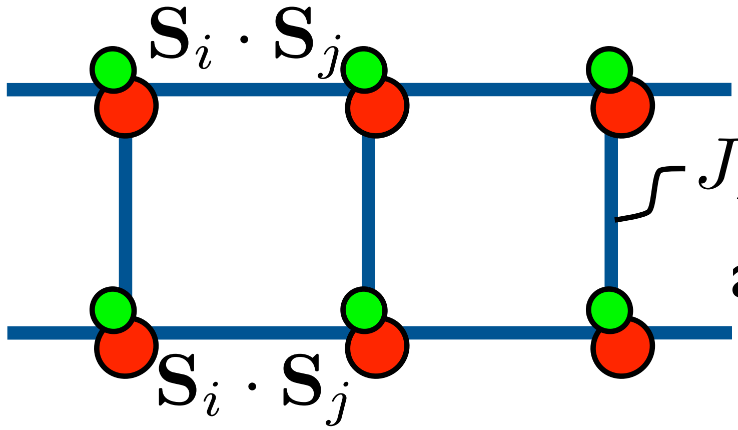

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One SPT phase:

The other SPT phase:

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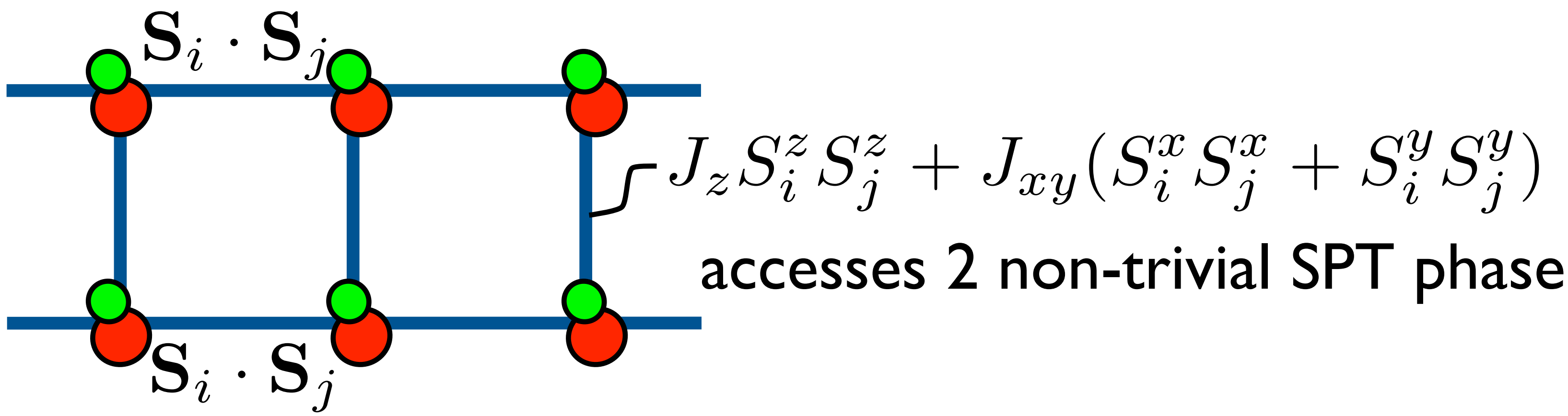

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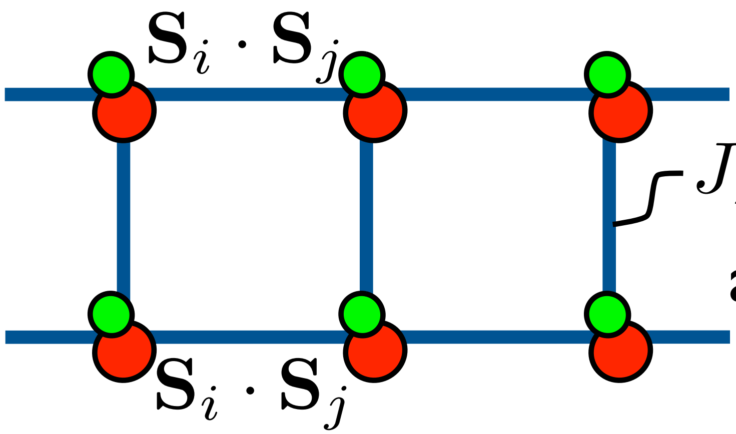

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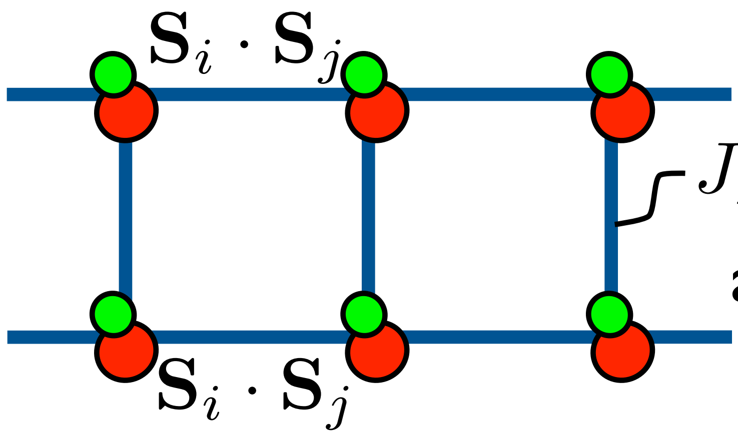

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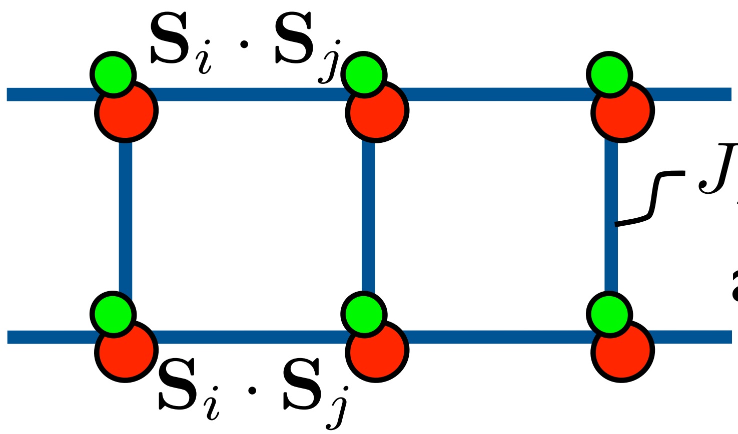

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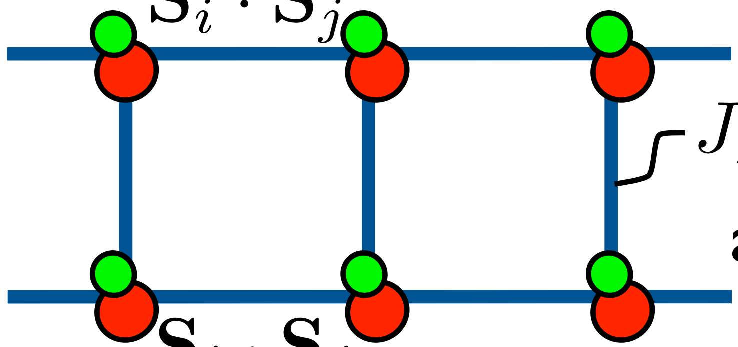

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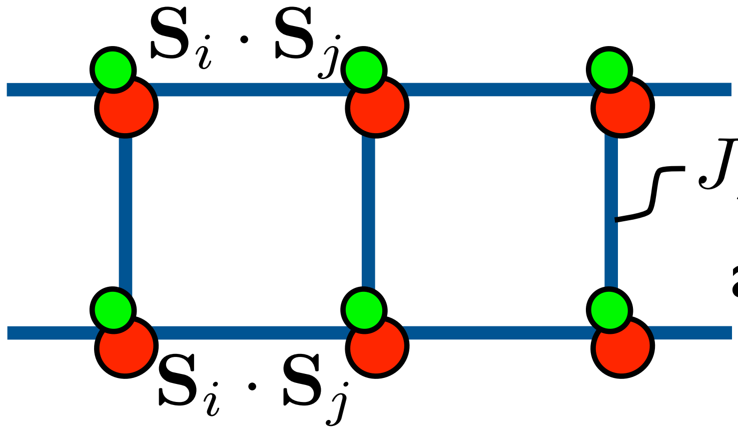

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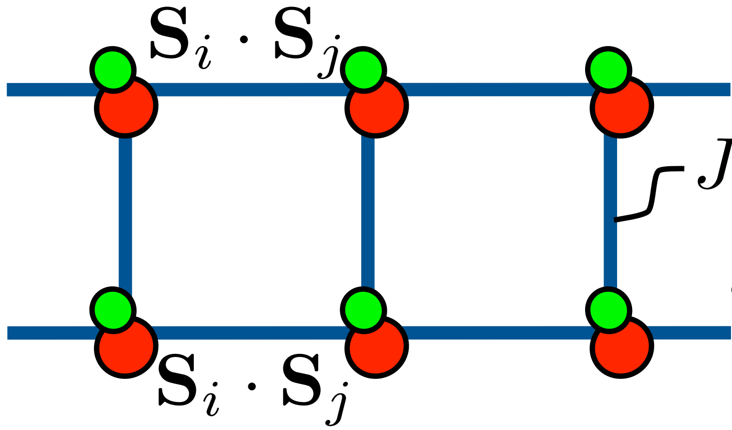

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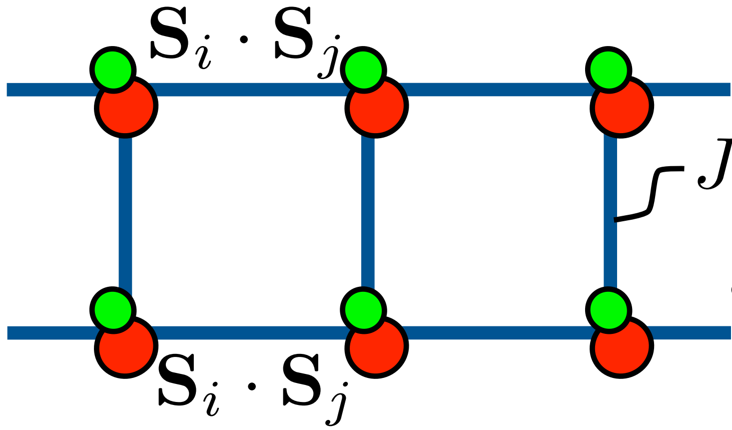

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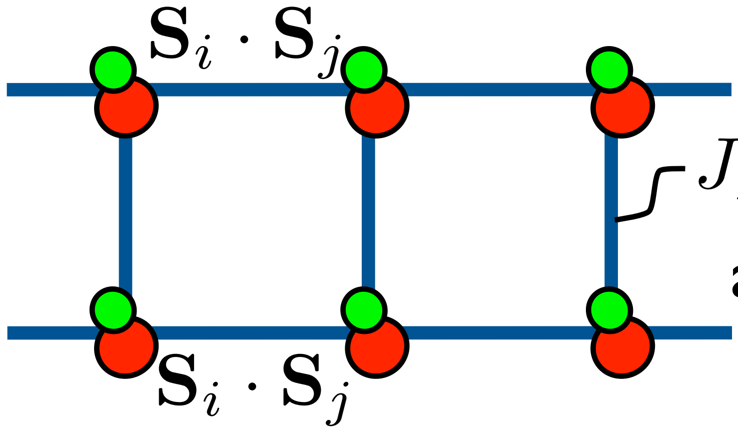

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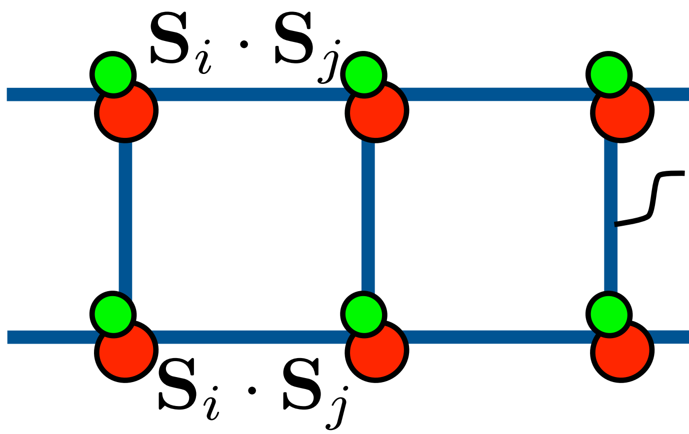

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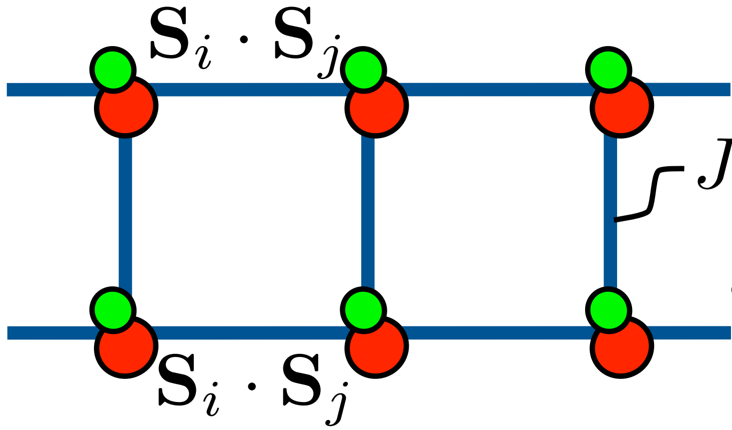

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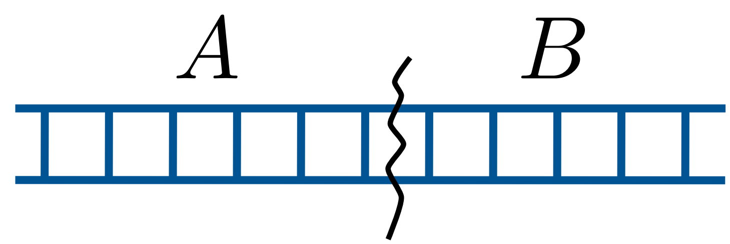

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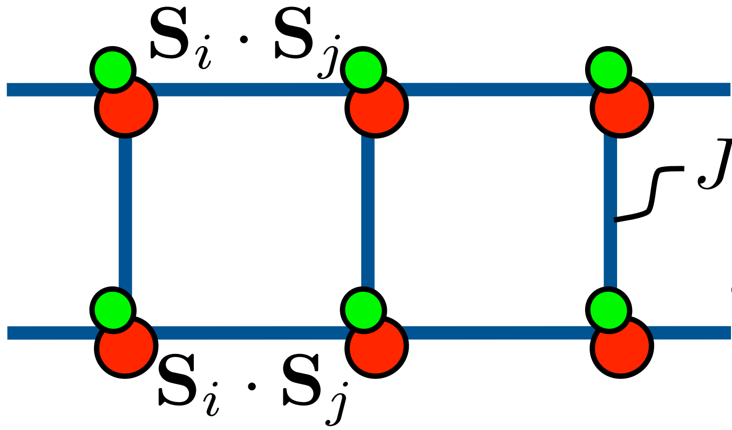

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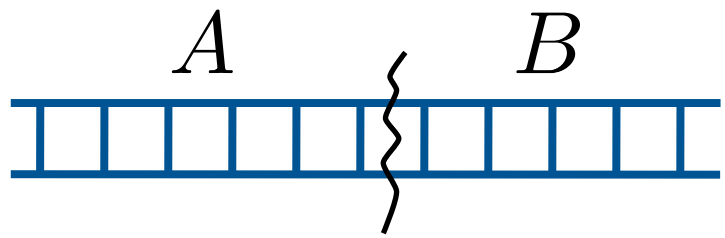

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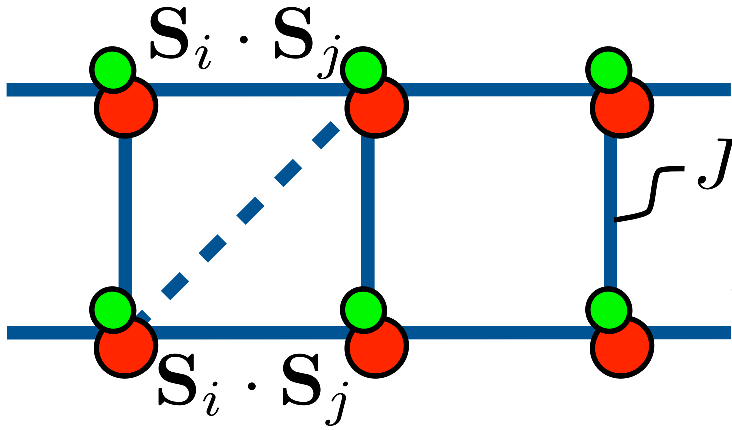

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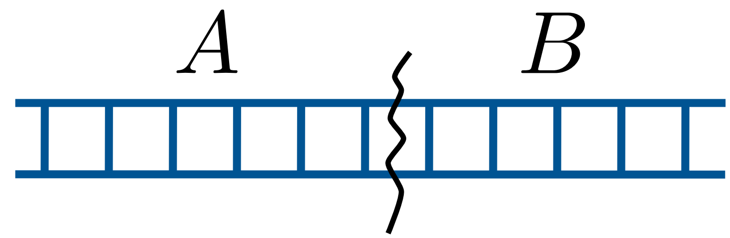

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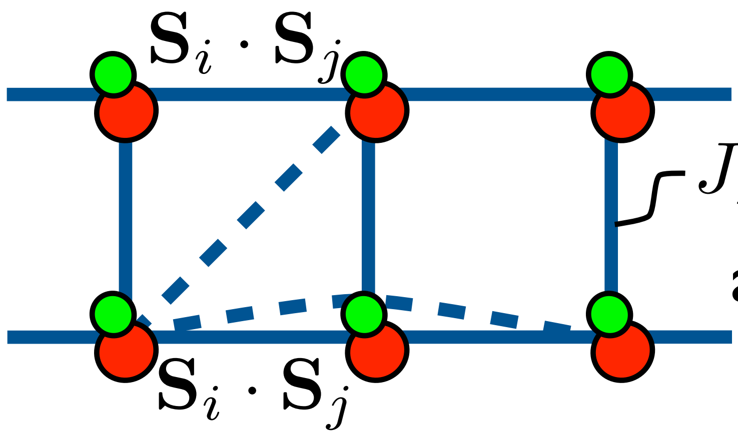

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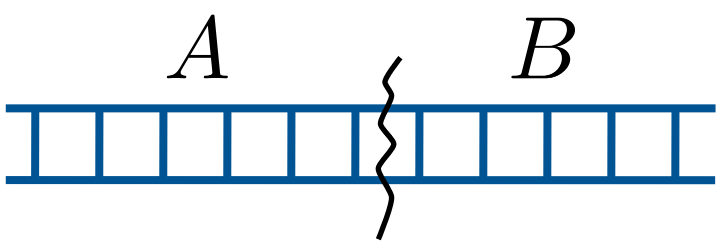

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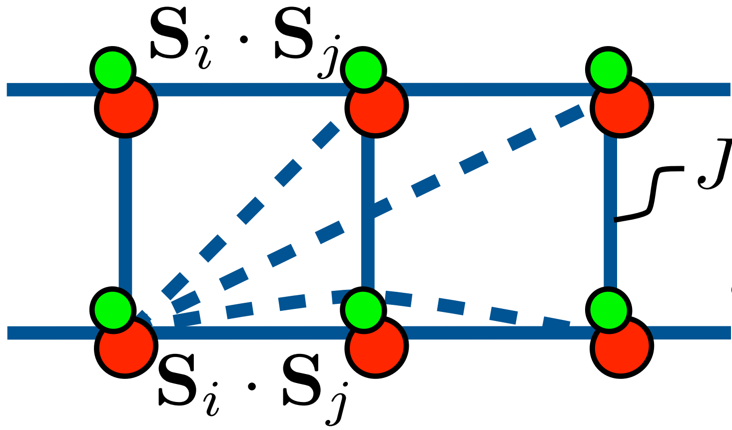

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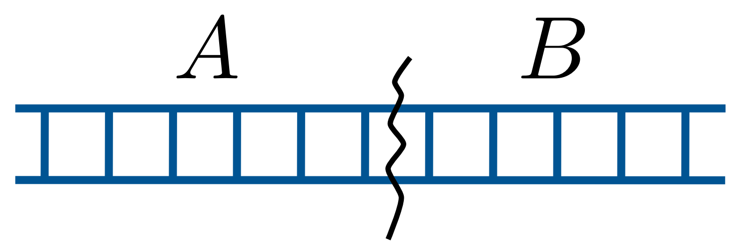

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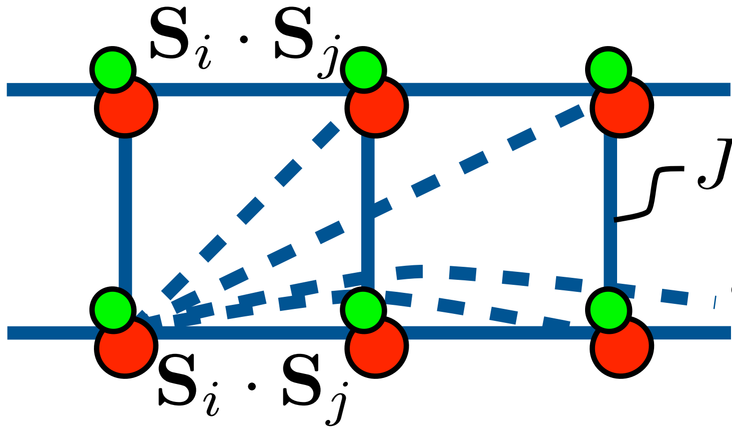

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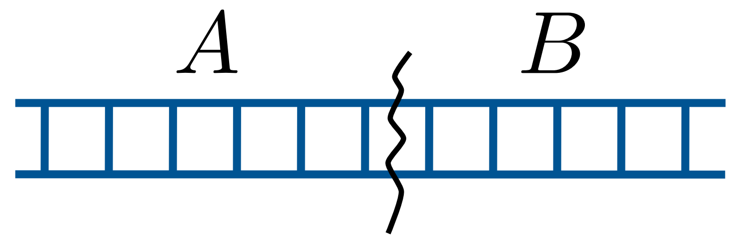

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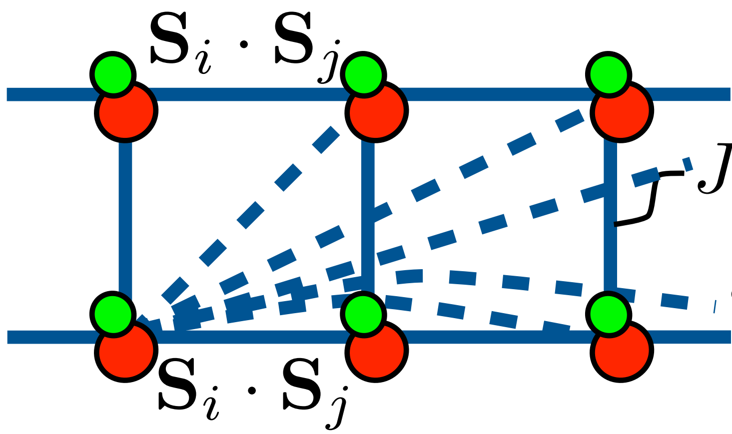

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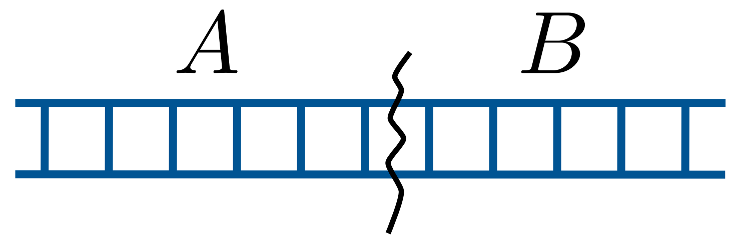

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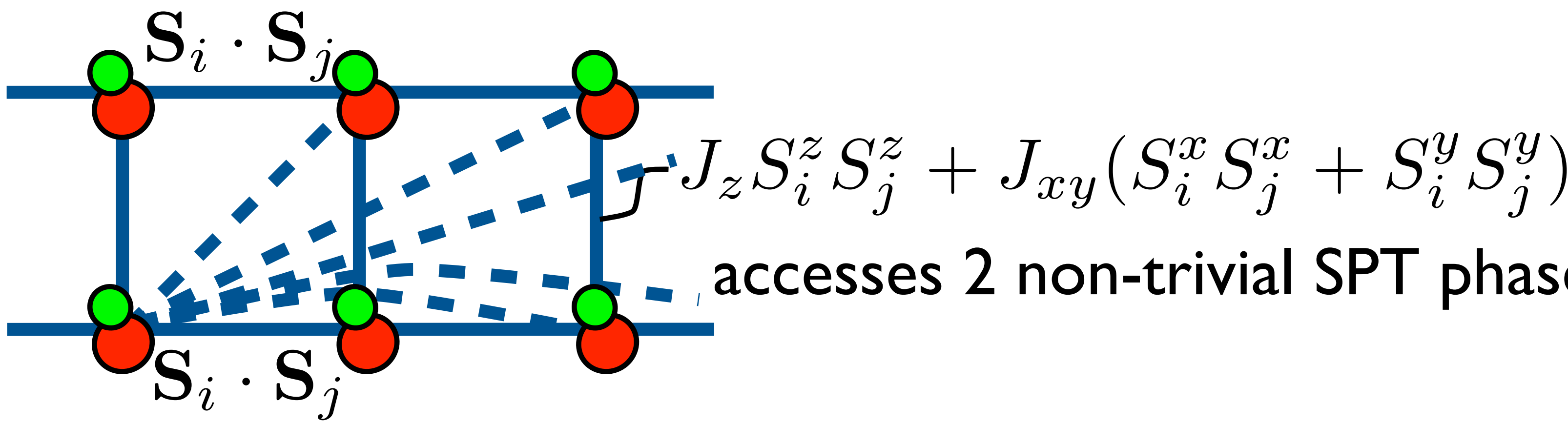

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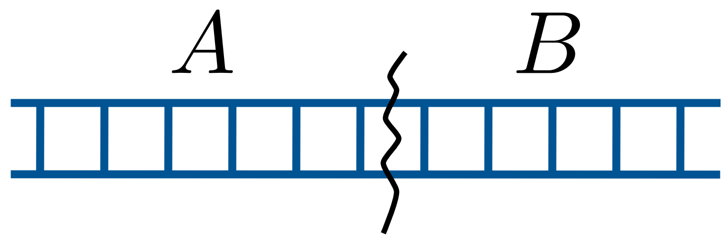

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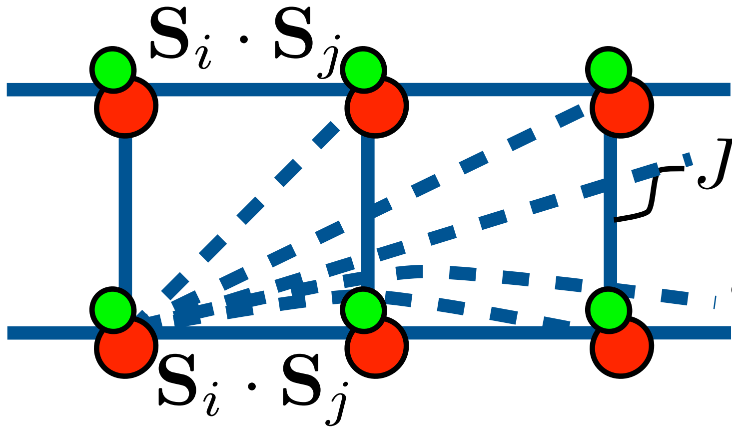

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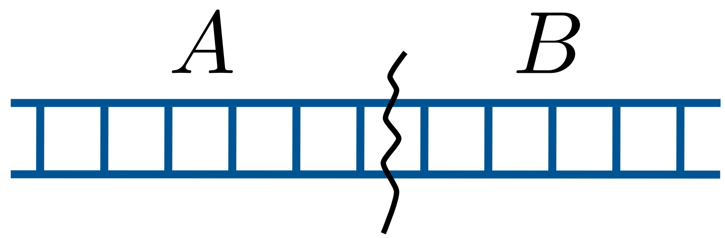

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[PRB 87, 081106(R) (2013)]

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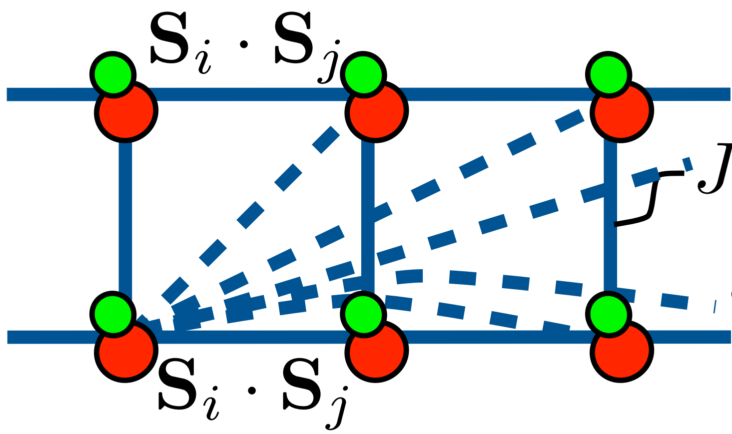

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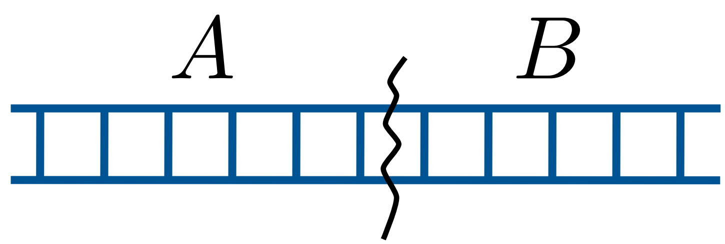

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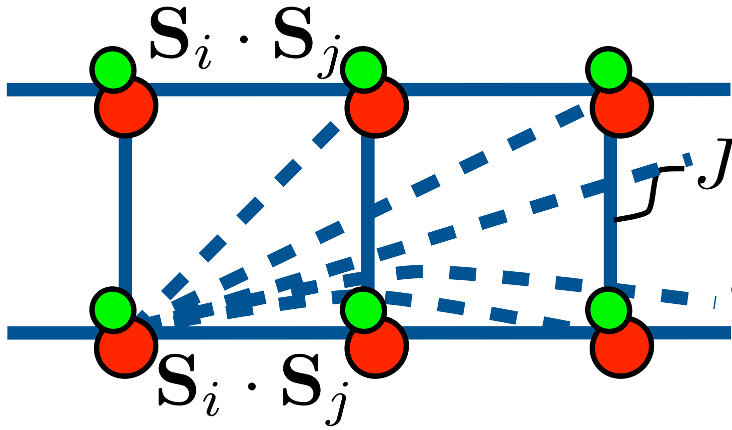

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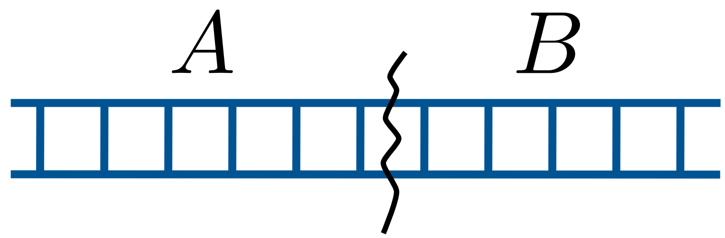

eigenvalues of $\mathrm{tr}_B\left[|\Psi\rangle\langle\Psi|\right]$ are doubly degenerate

- do SPT phases survive?

YES!
[PRB 87, 081106(R) (2013)]

Liu, Wen, et al, PRB 2012; Chen, Gu, Wen, PRB 2011; Pollman et al, PRB 2010

Conclusions and Outlook

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- in general, peculiar effects of long-range interactions
  [see e.g. poster by H.P. Buchler & other talks and posters]

Topological flat bands
and
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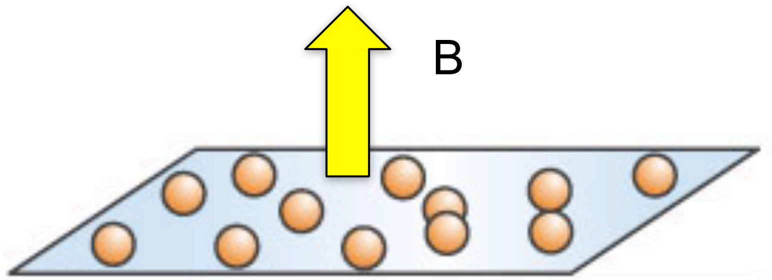

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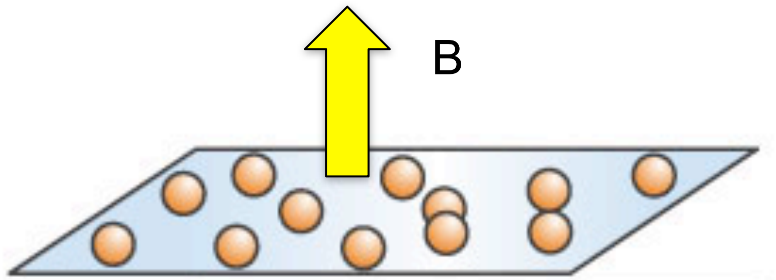

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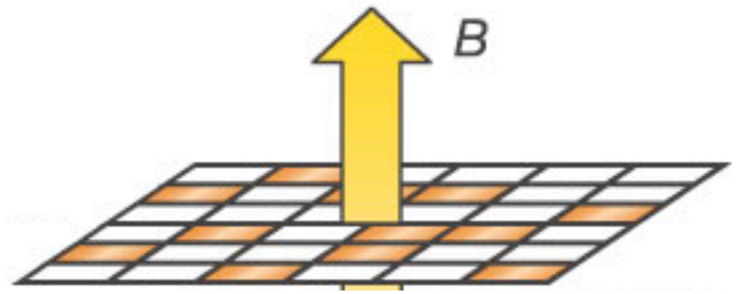

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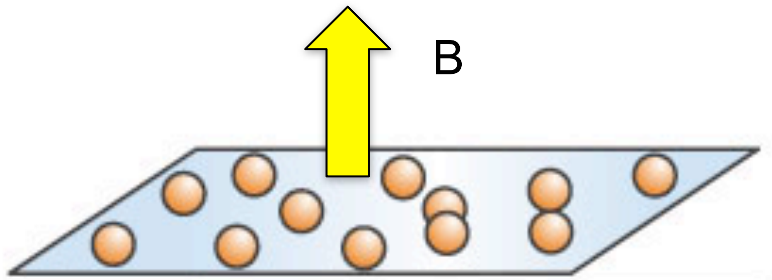

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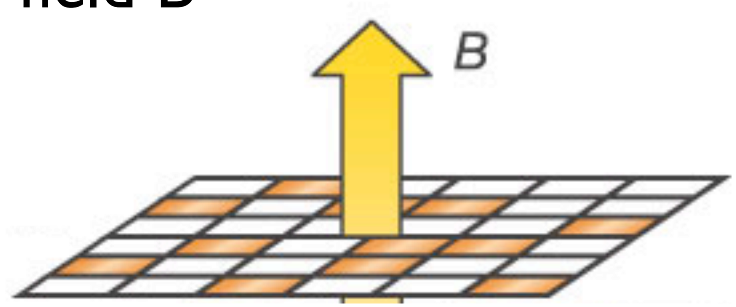

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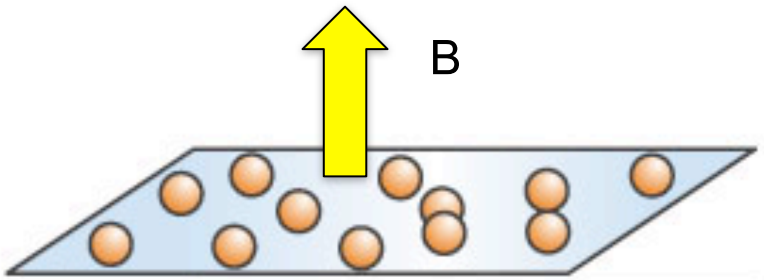

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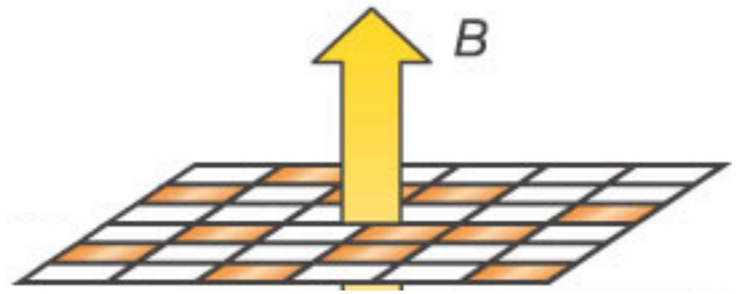

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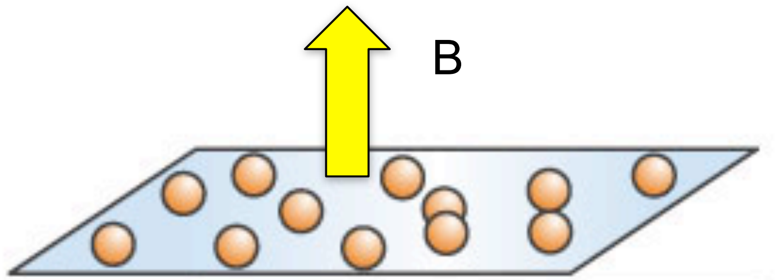

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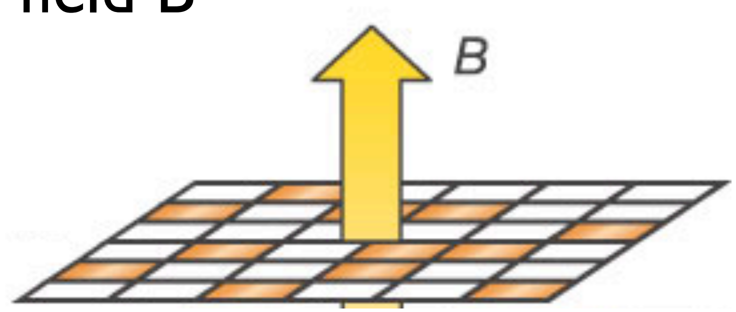

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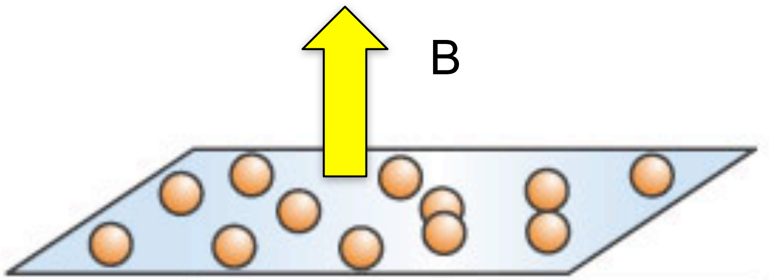

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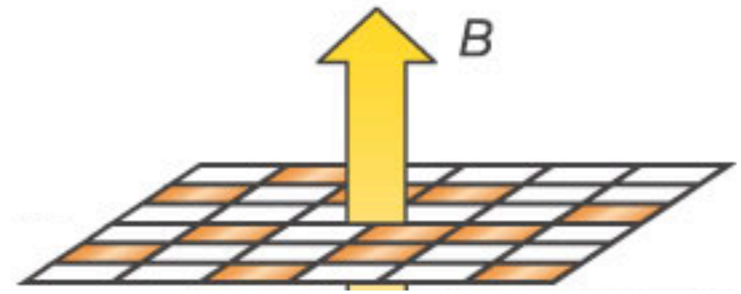

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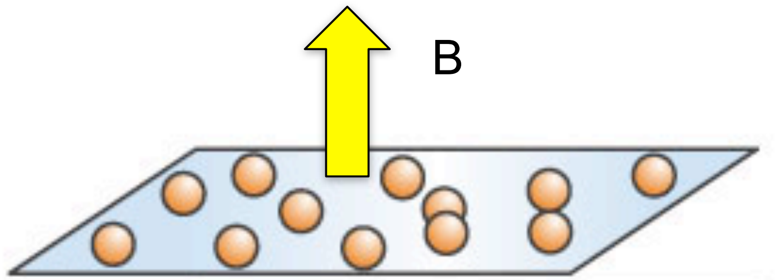

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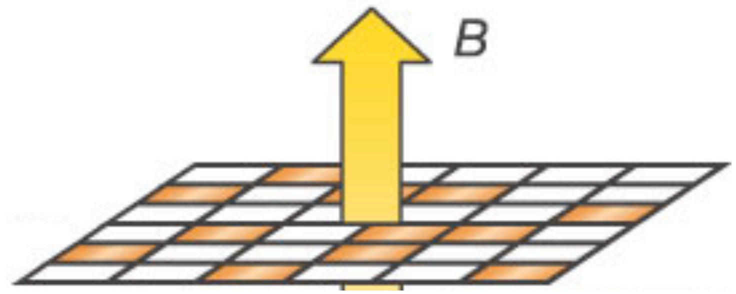

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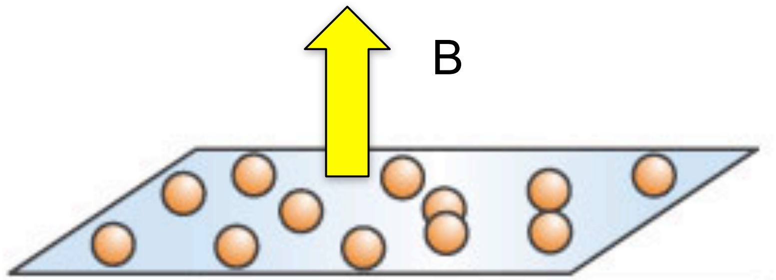

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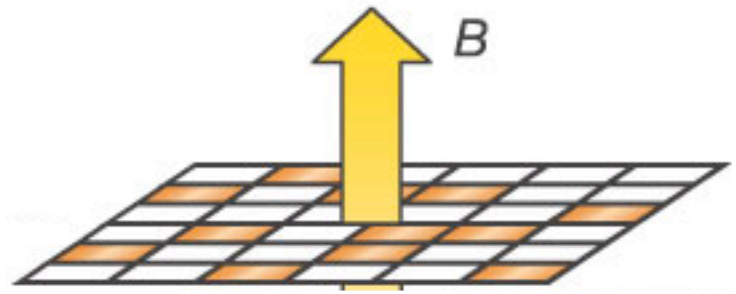

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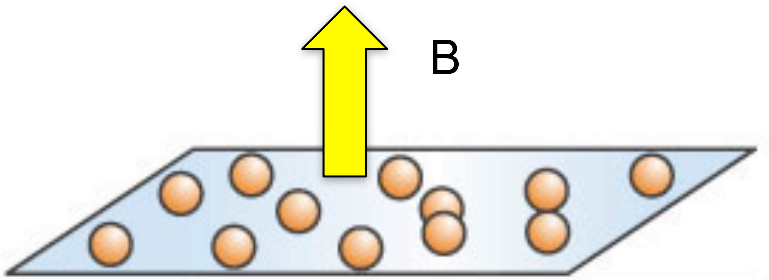

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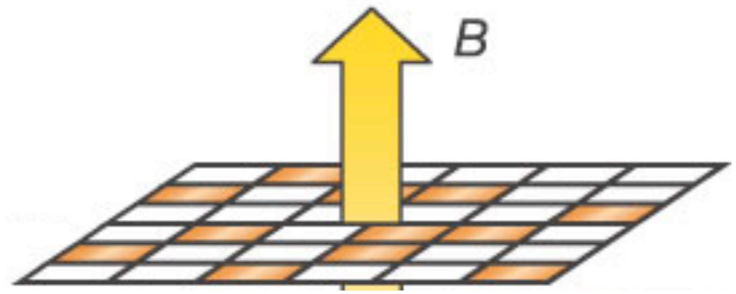

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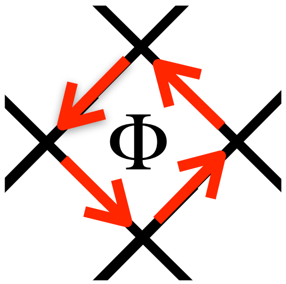

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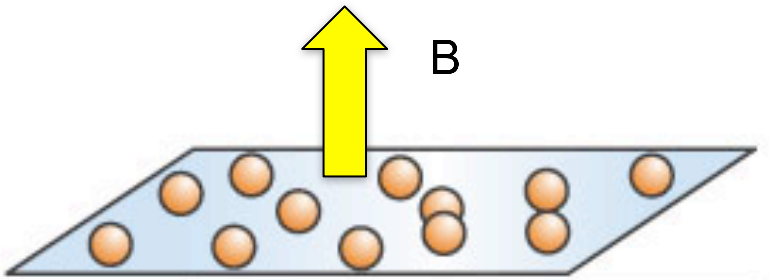

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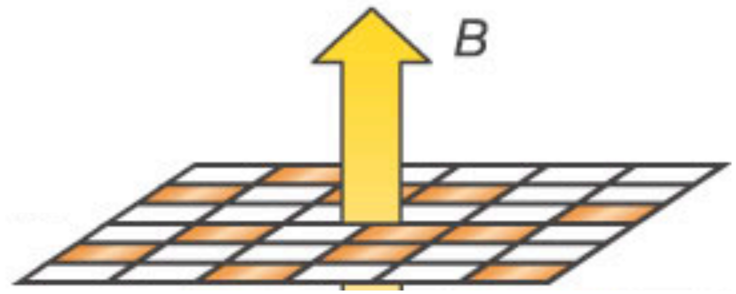

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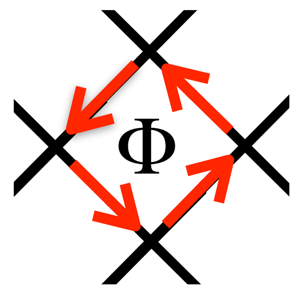

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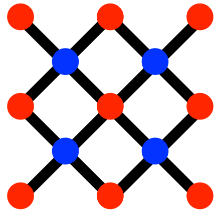

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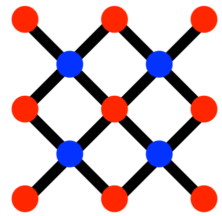

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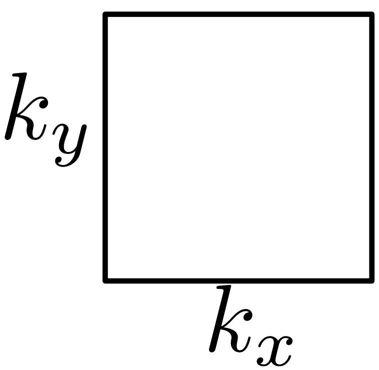

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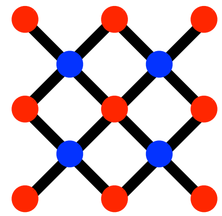

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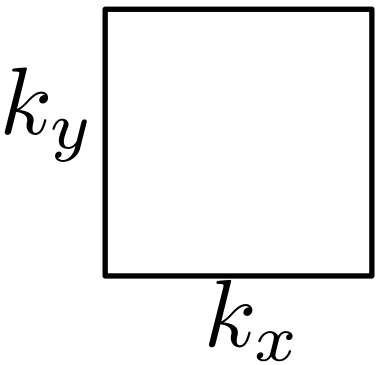

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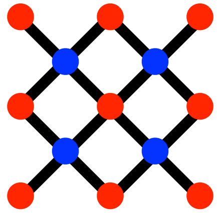

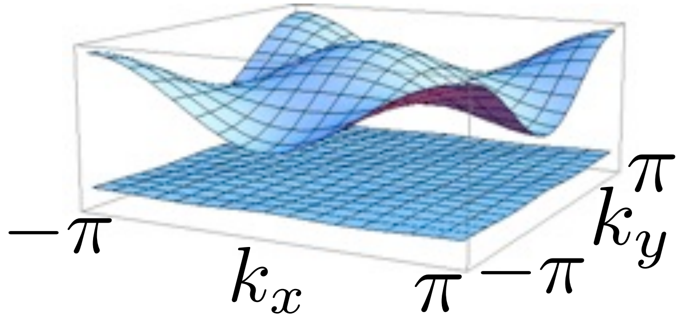

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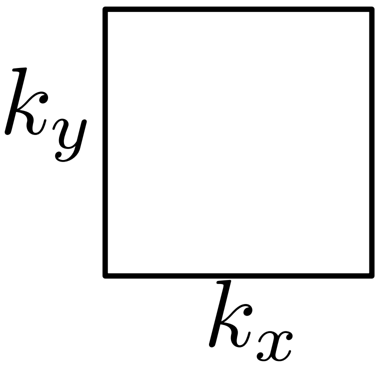

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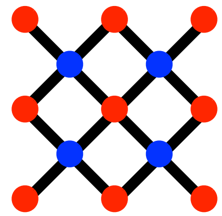

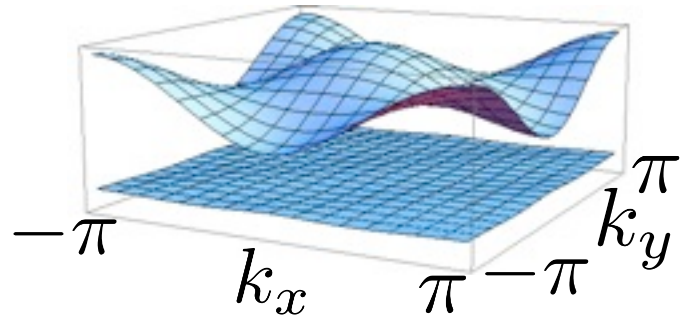

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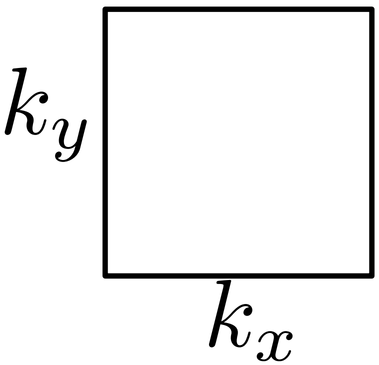

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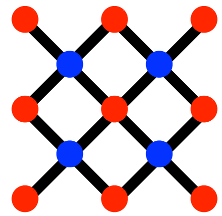

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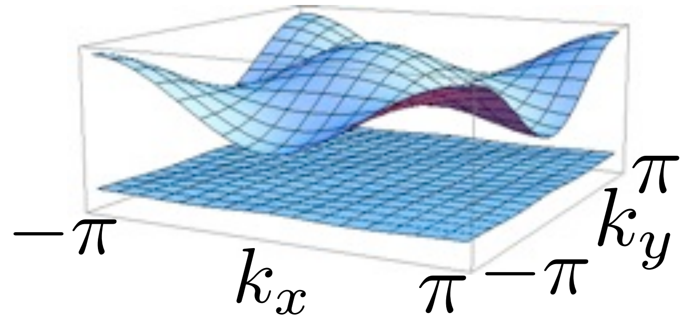

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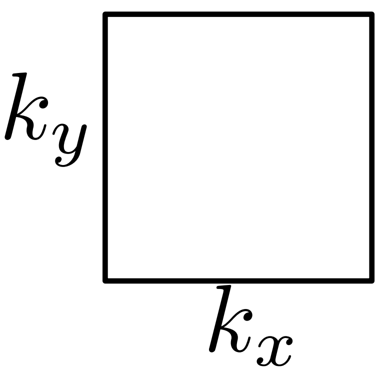

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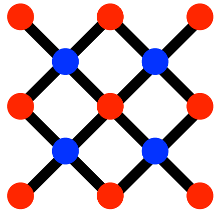

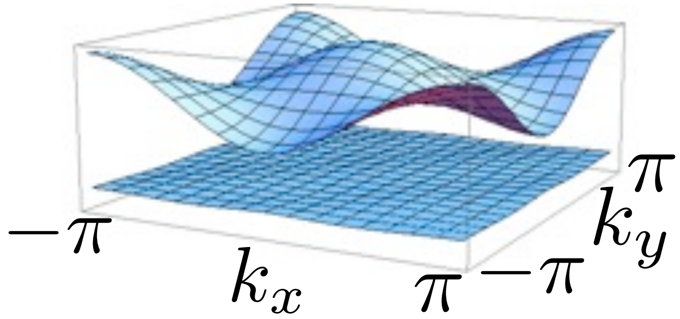

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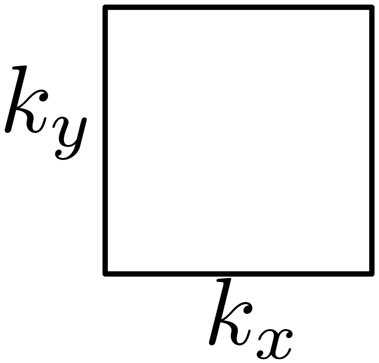

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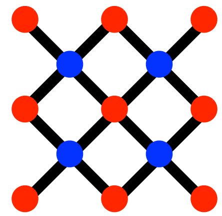

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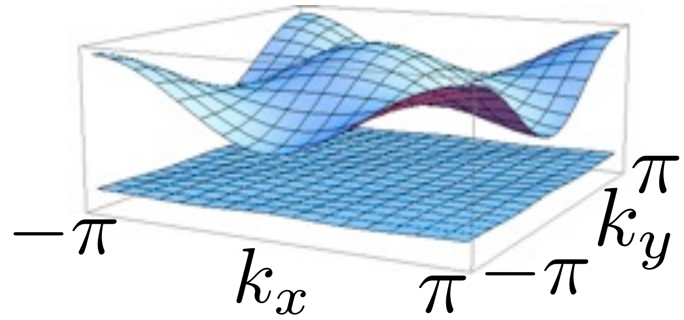

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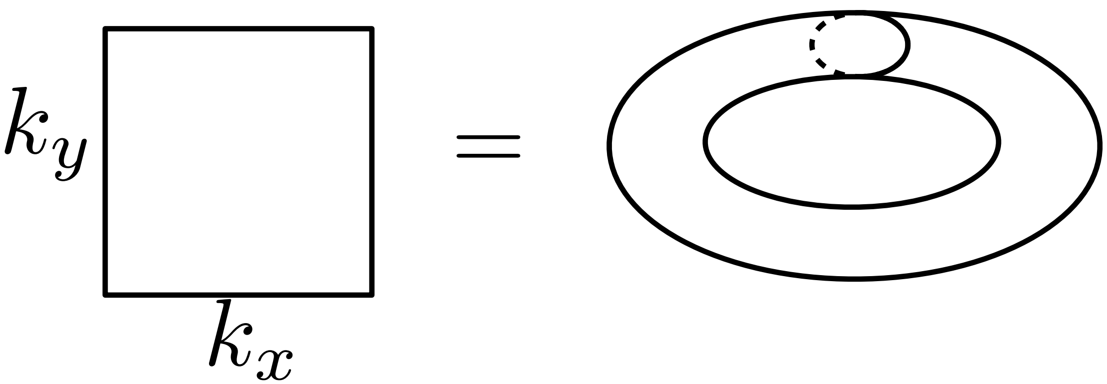

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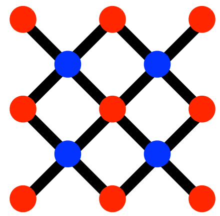

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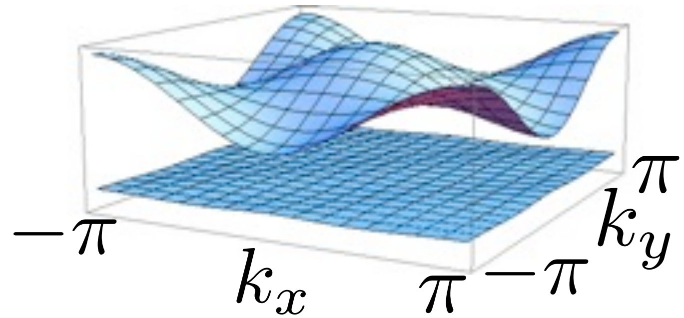

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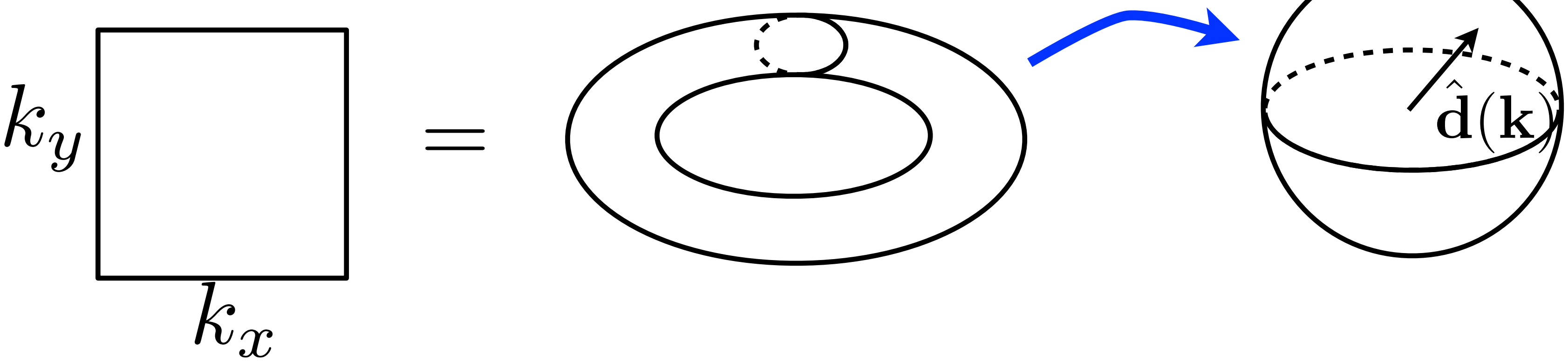

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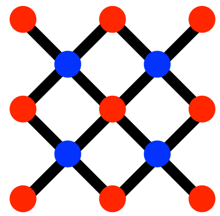

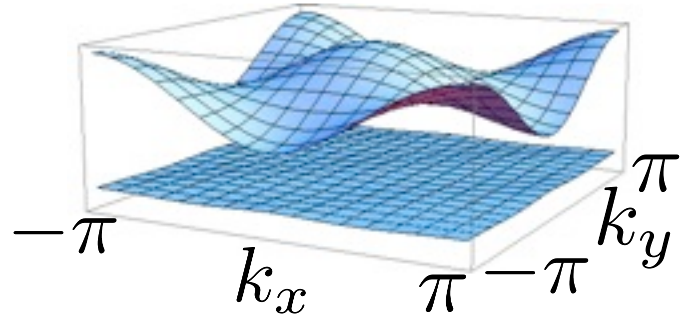

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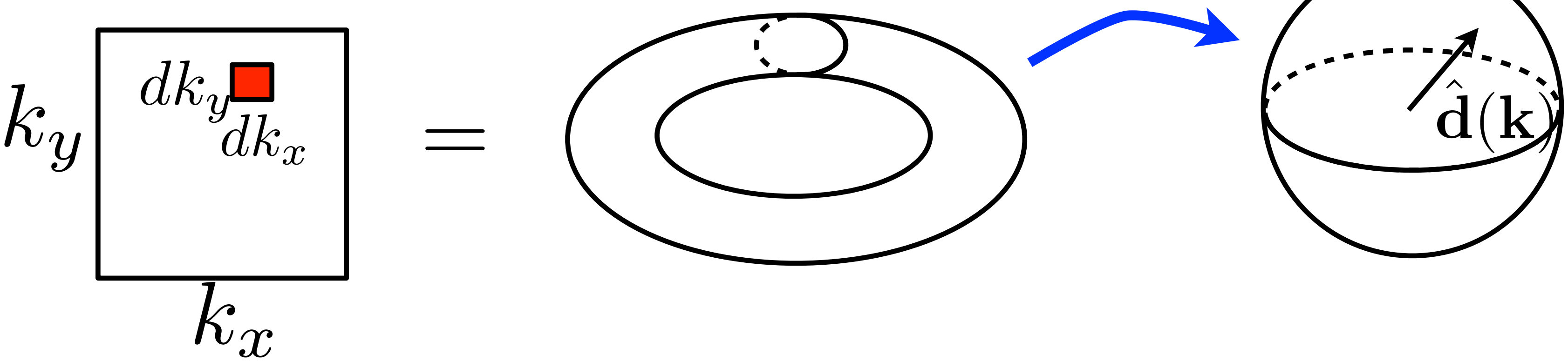

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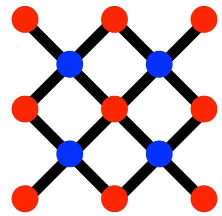

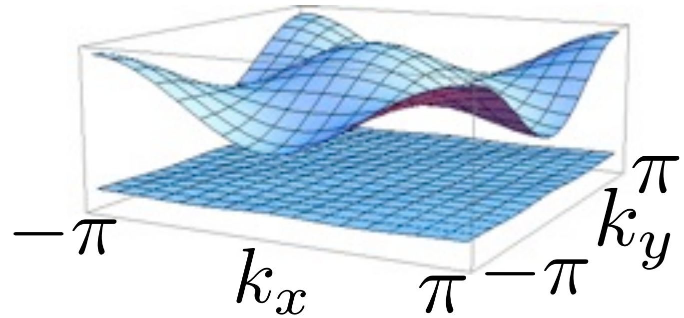

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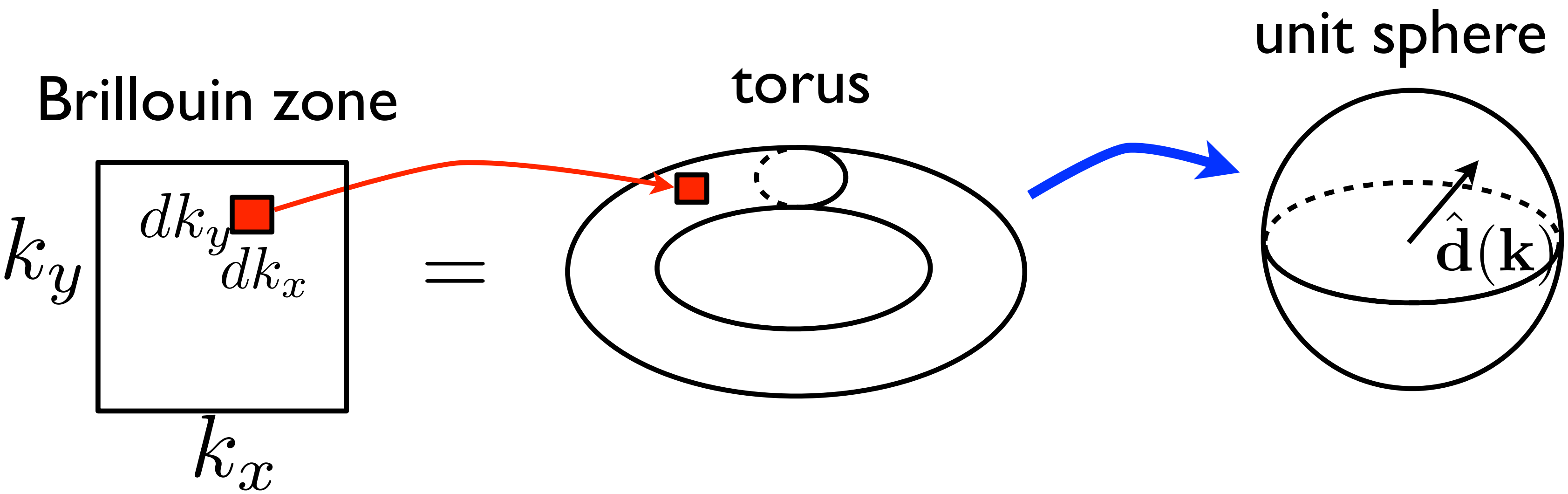

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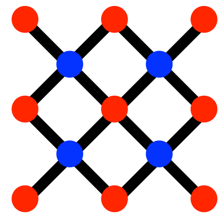

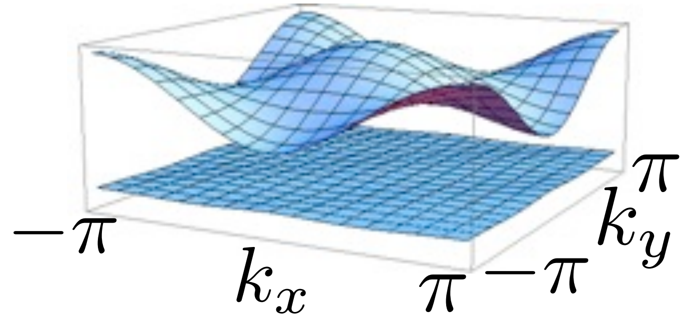

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- topological flat band

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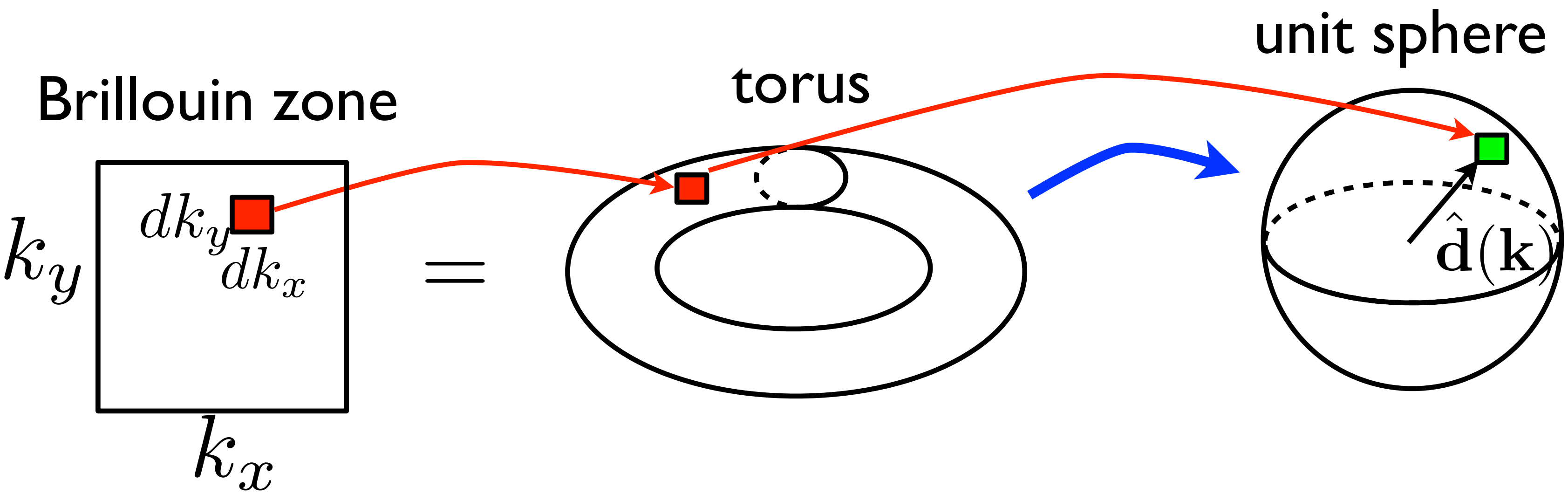

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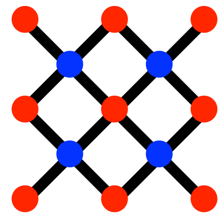

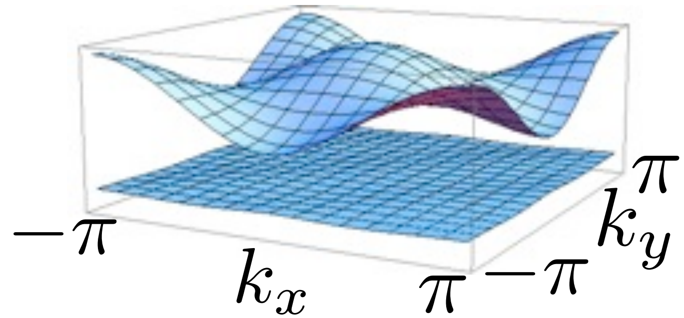

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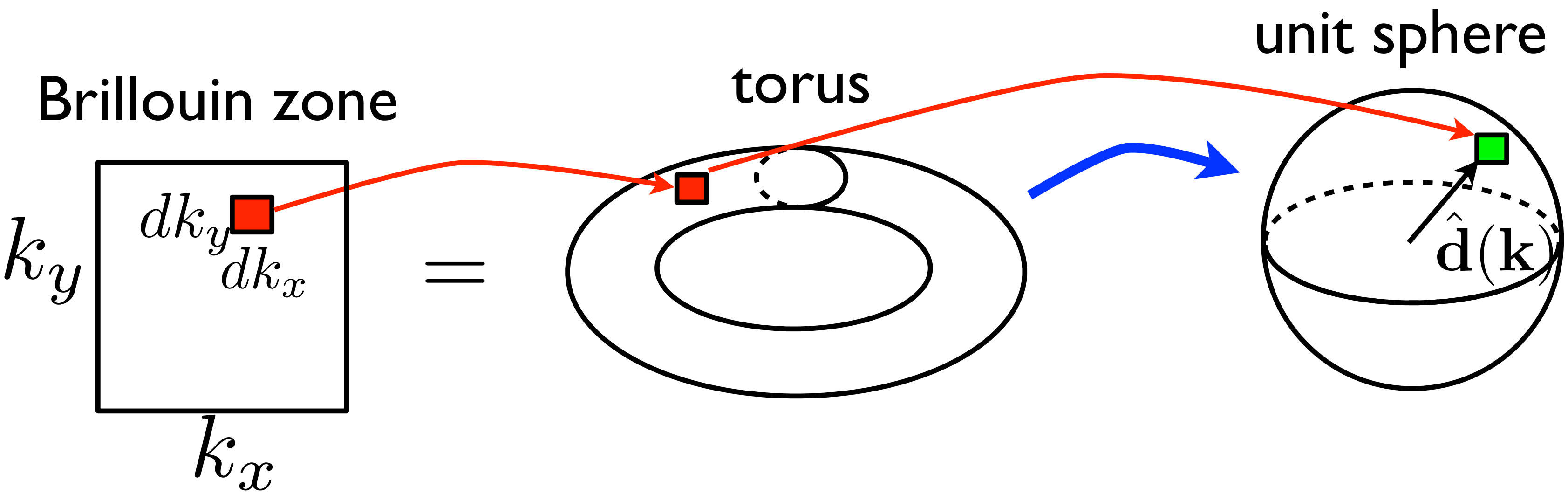

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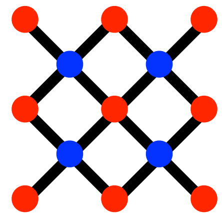

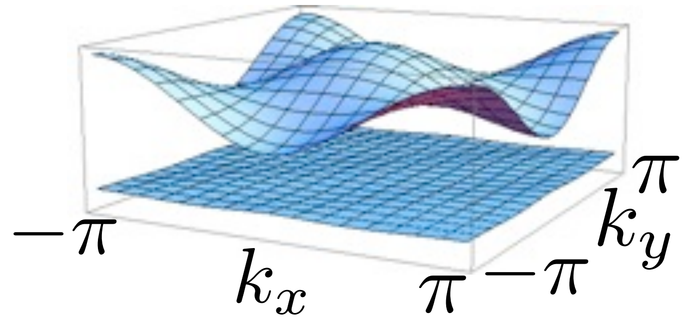

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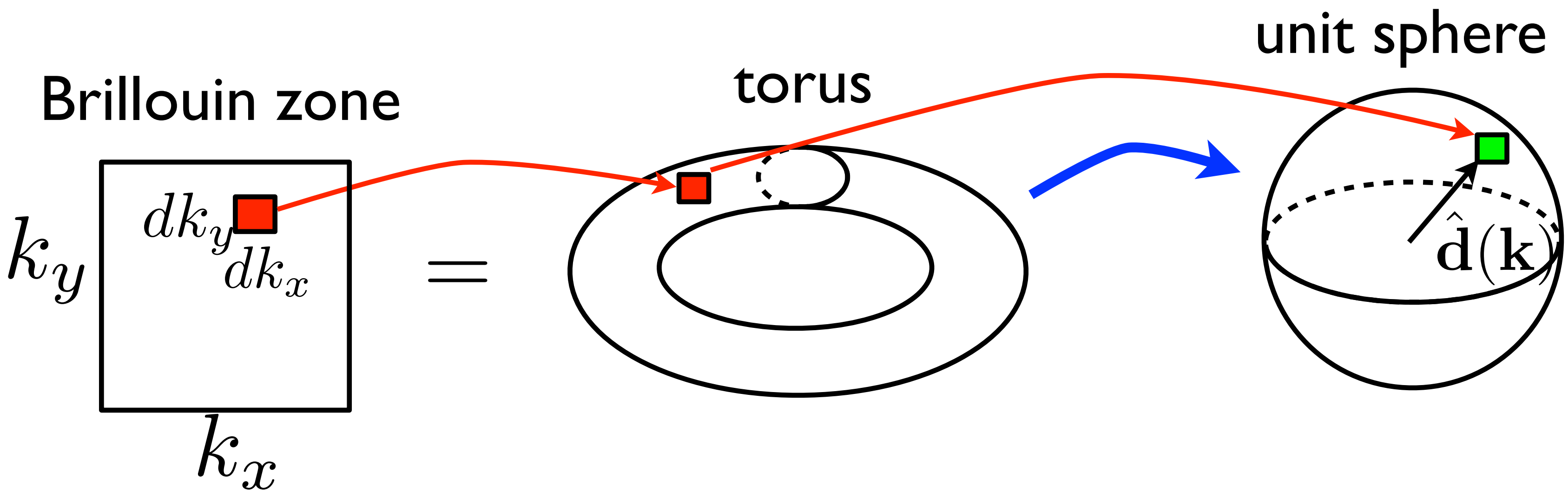

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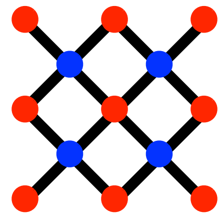

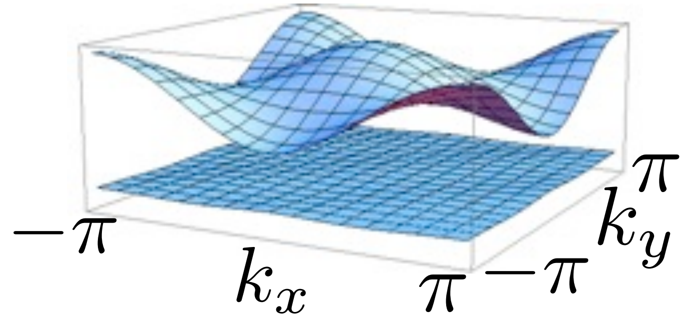

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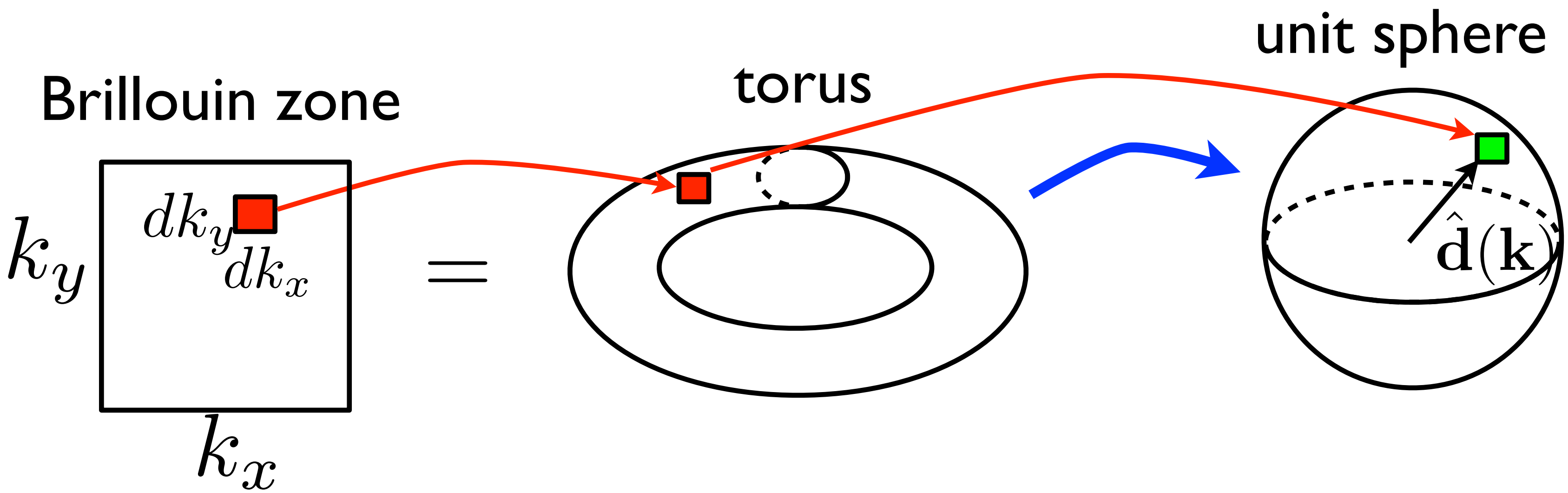

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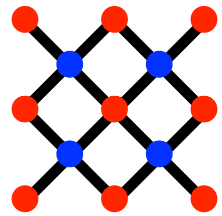

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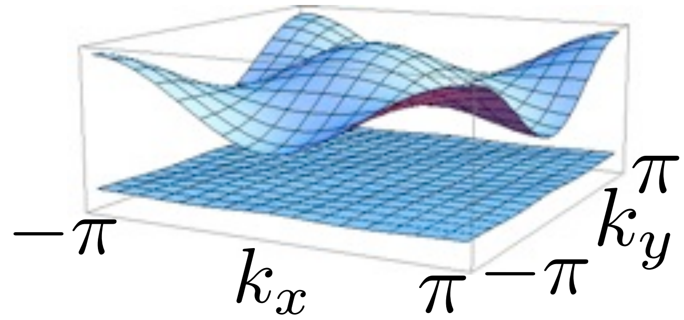

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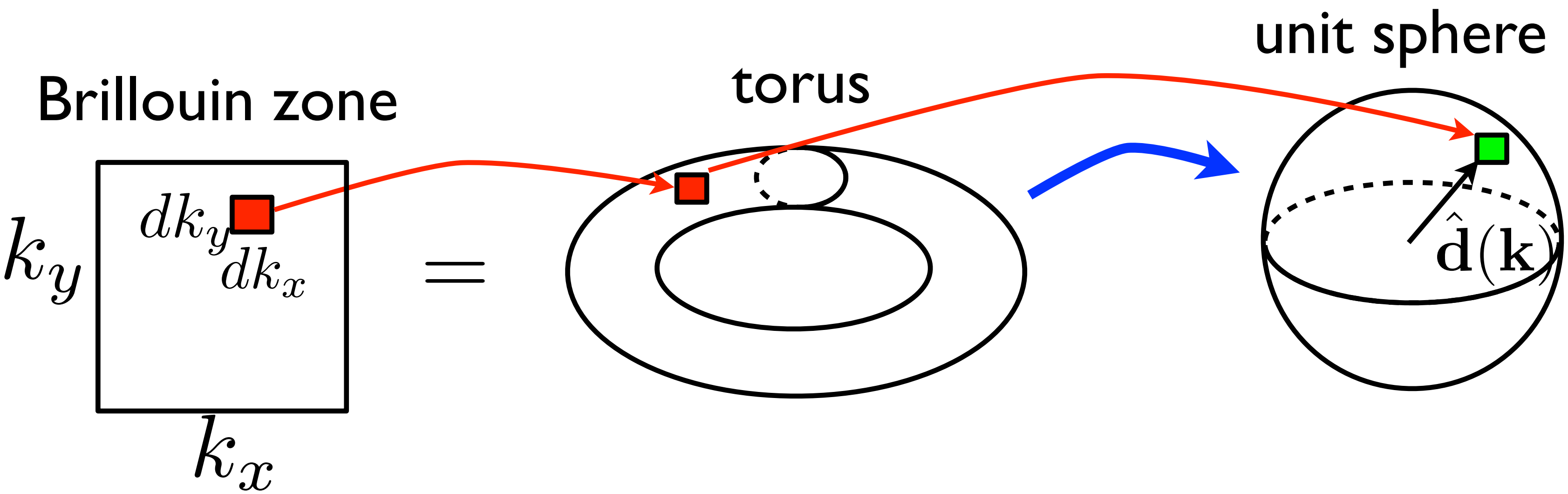

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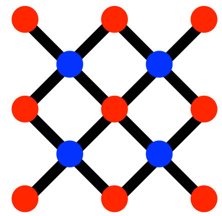

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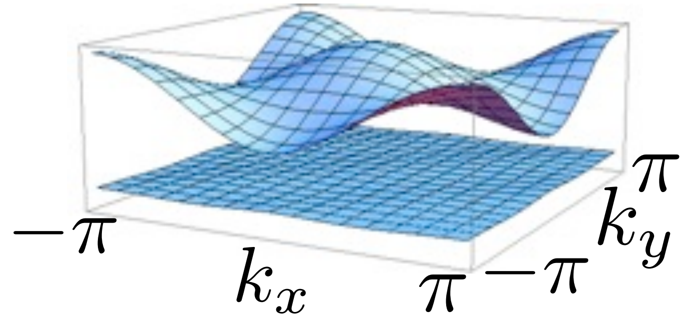

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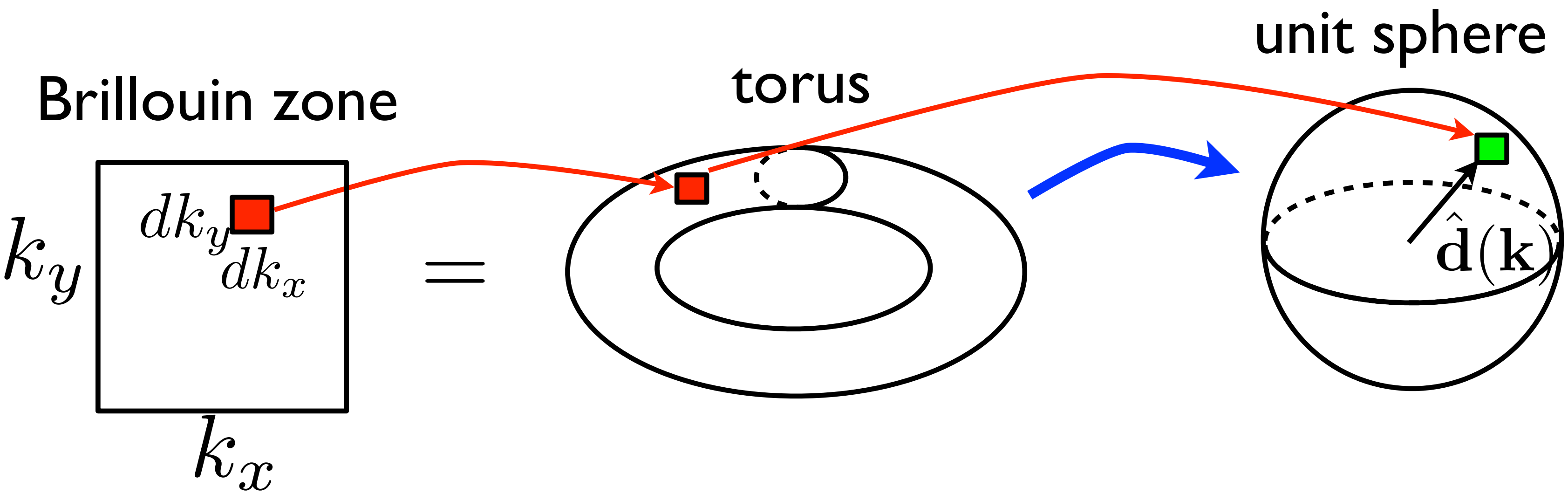

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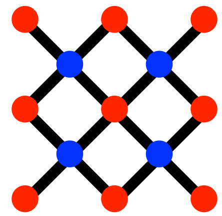

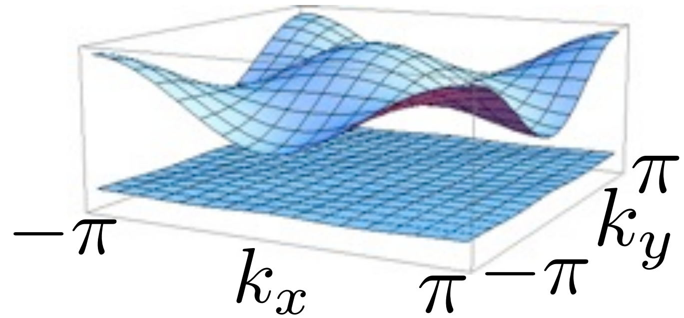

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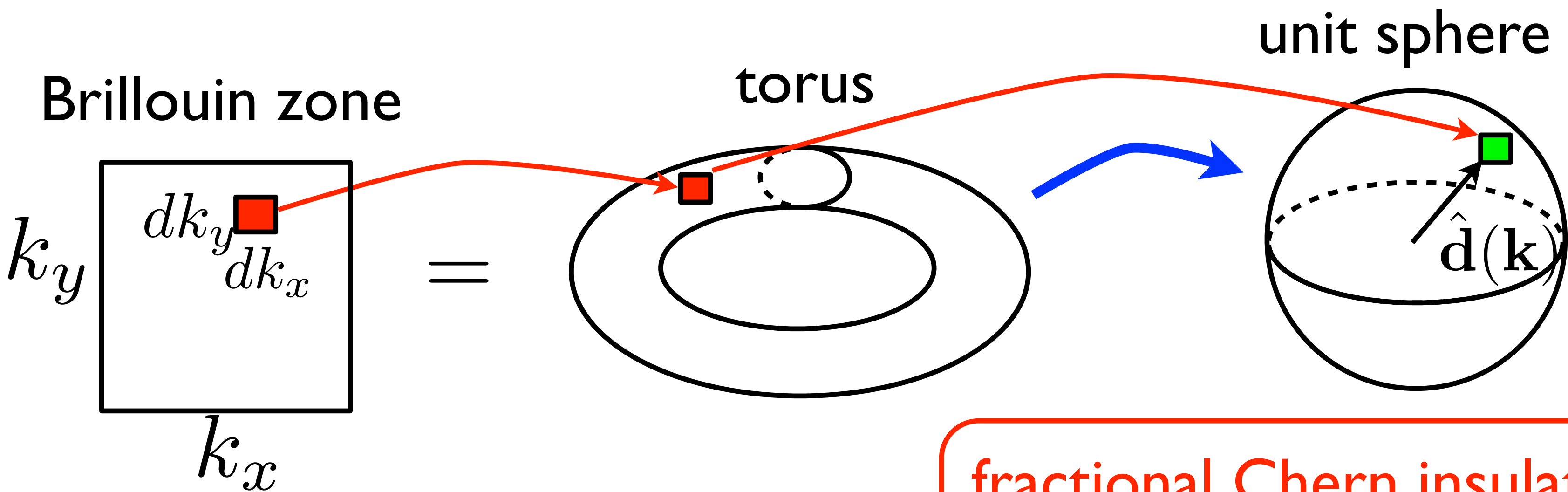

fractional Chern insulator (FCI)

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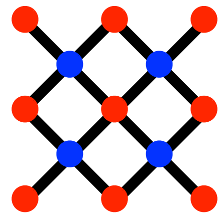

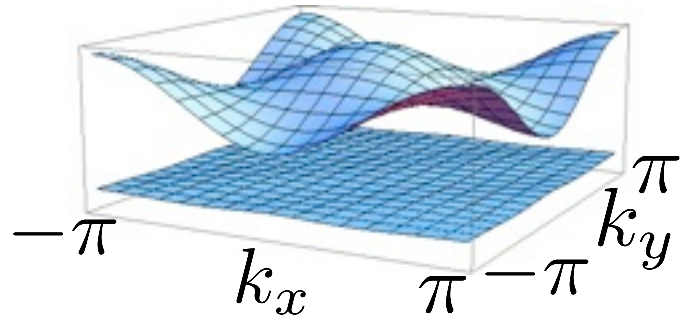

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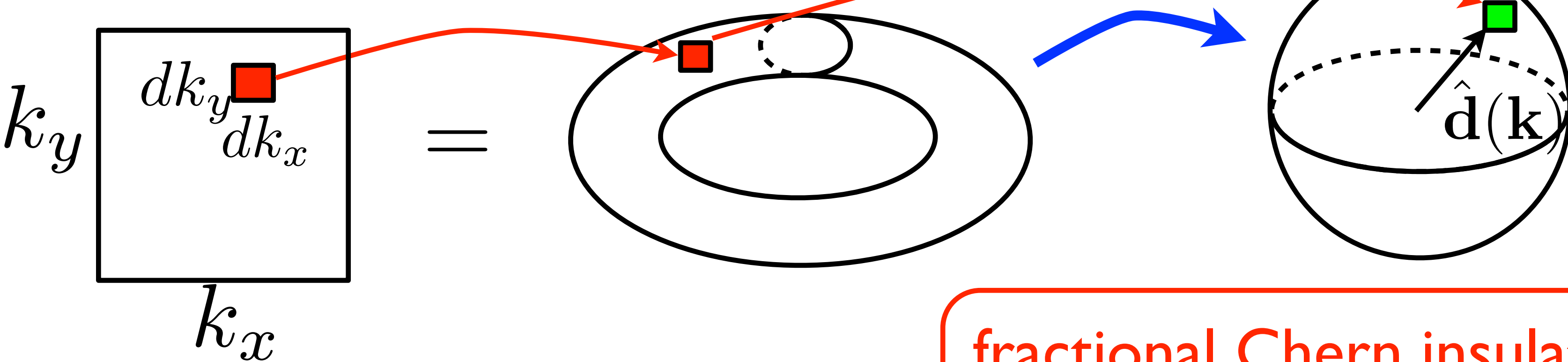
Brillouin zone

torus

unit sphere



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Topological flat bands and fractional Chern insulators

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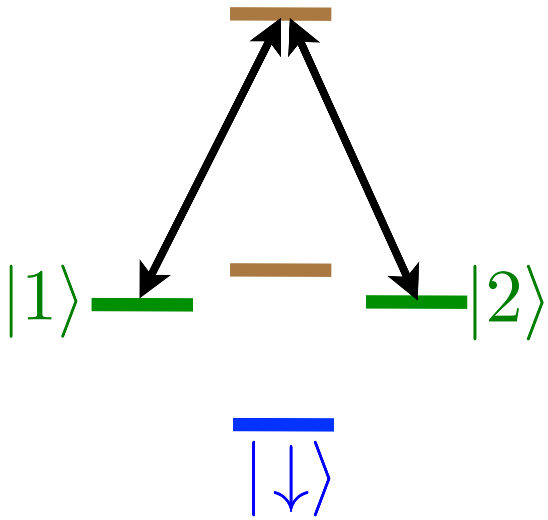
- dipolar spin systems (e.g. polar molecules) naturally admit topological flat bands and fractional Chern insulator (~FQHE) ground states!

Fractional Chern insulator with polar molecules

Fractional Chern insulator with polar molecules

Recall SPT discussion:



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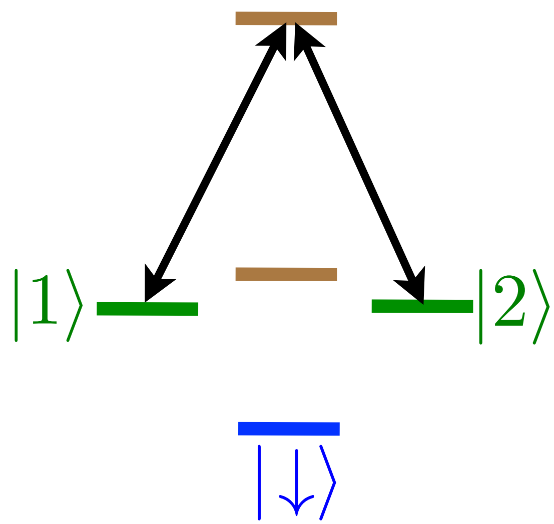

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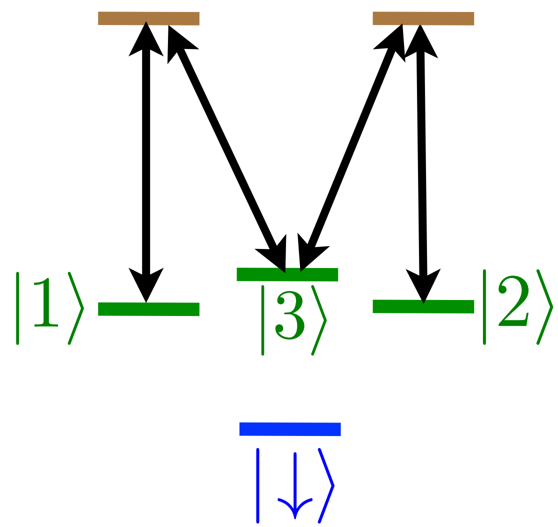

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[PRL 109, 266804 (2012)
& arXiv:1212.4839]

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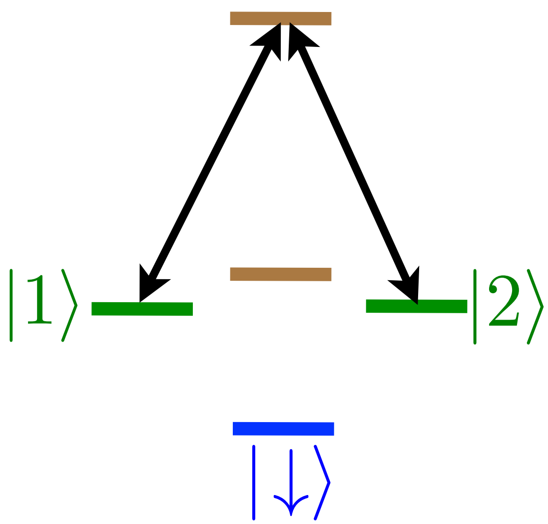

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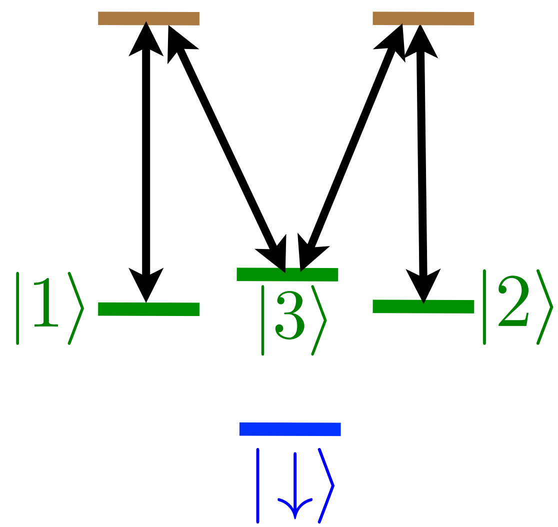

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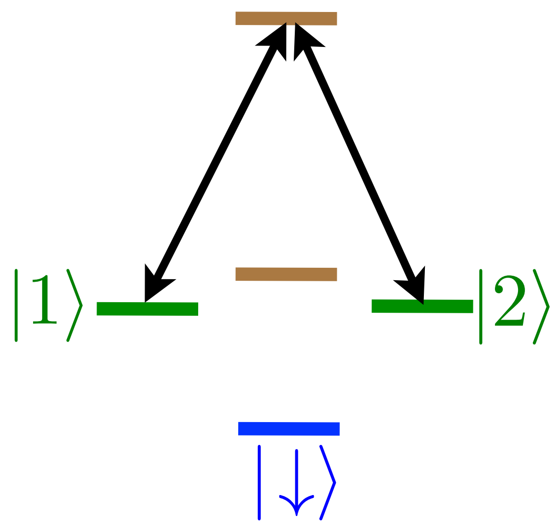

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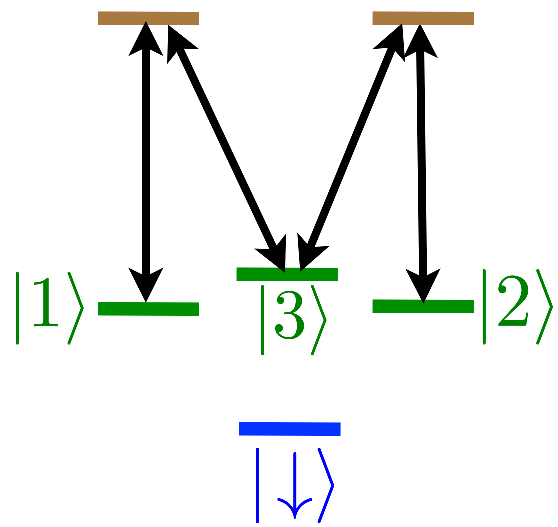

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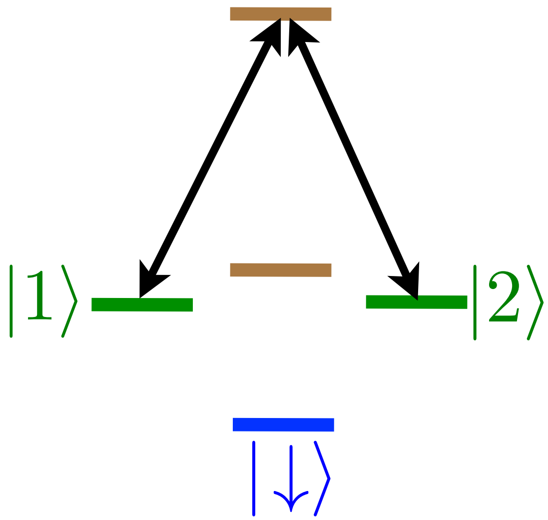

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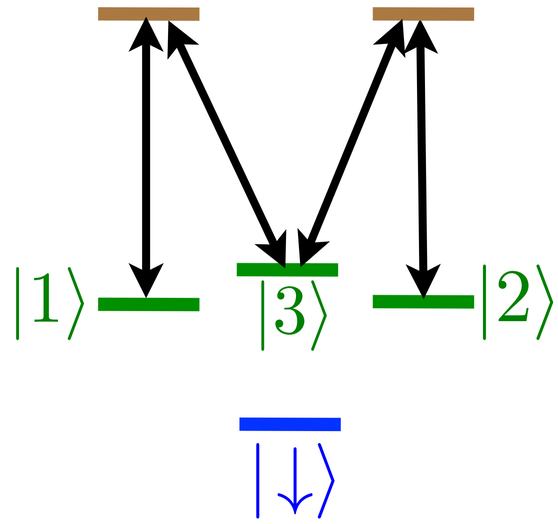

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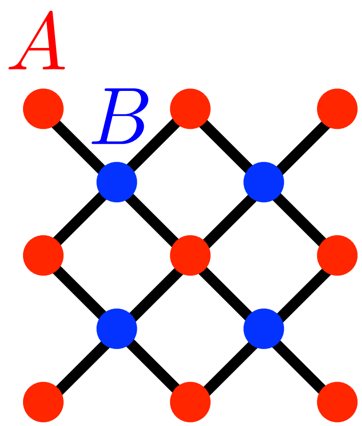

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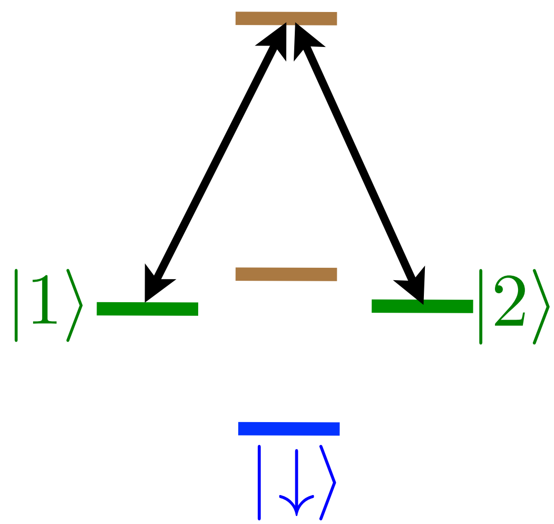

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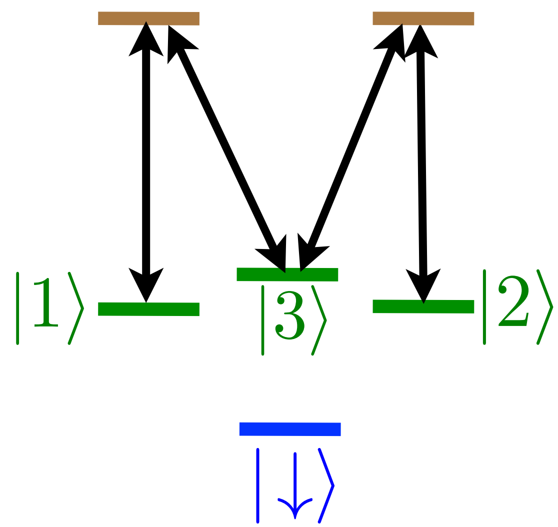

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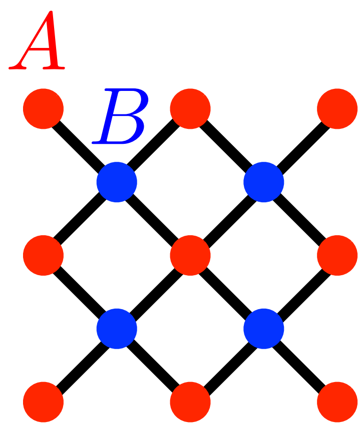

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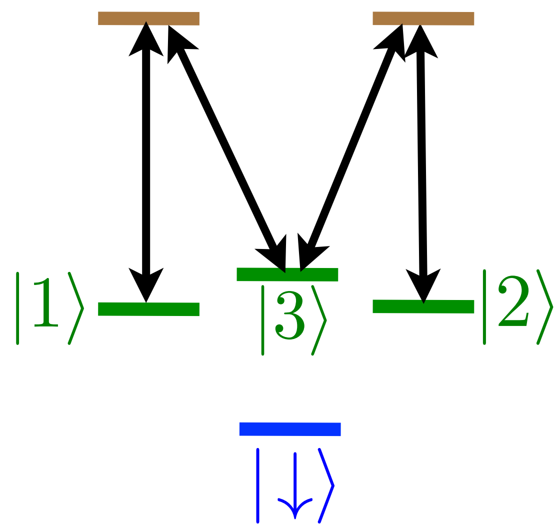

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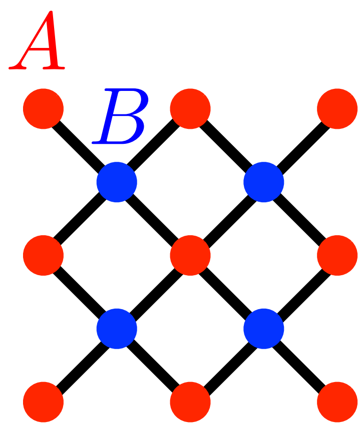

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Fractional Chern insulator with polar molecules

- weak DC electric field

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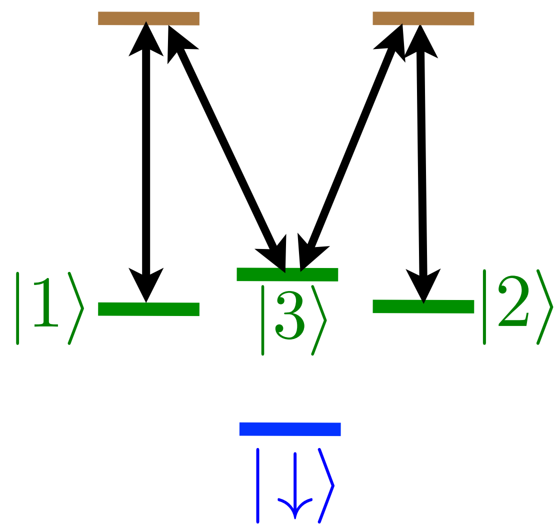

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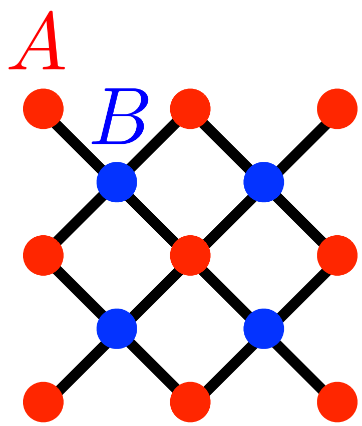

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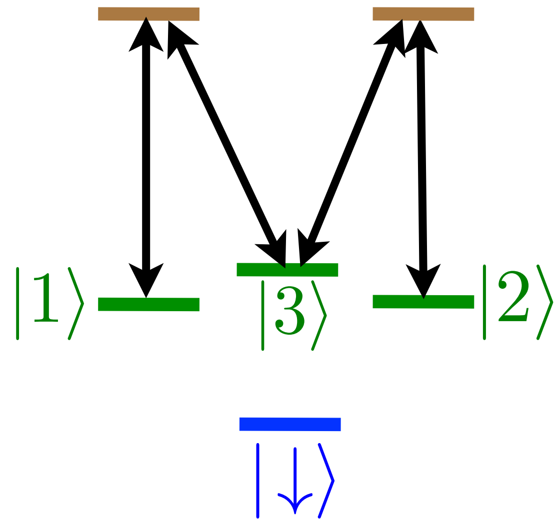

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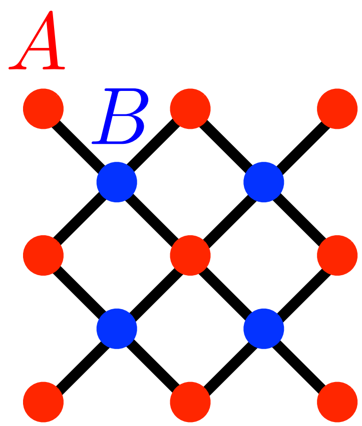

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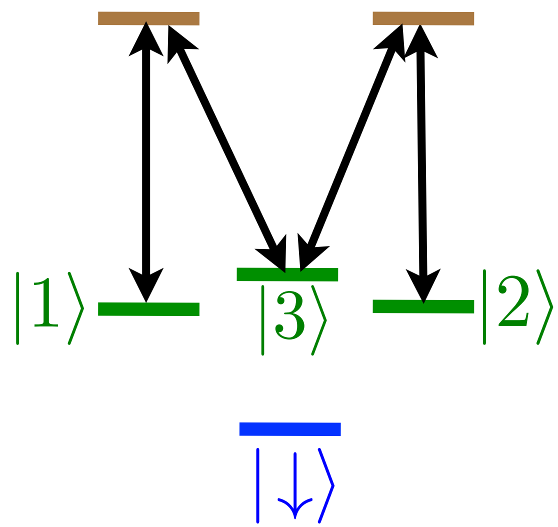

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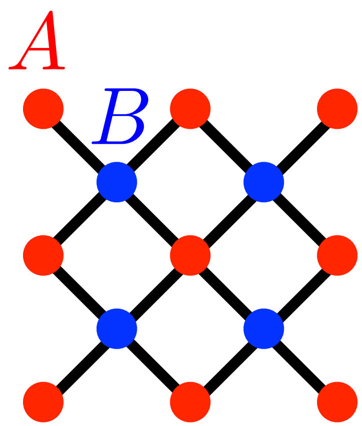

[PRL 109, 266804 (2012) & arXiv:1212.4839]

Fractional Chern insulator with polar molecules

- weak DC electric field

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spin-1/2 = hardcore boson

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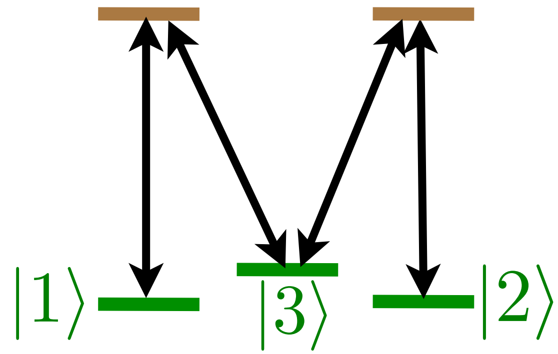

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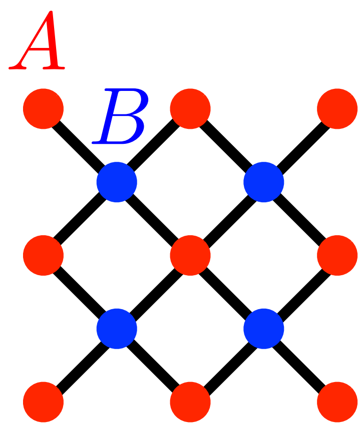

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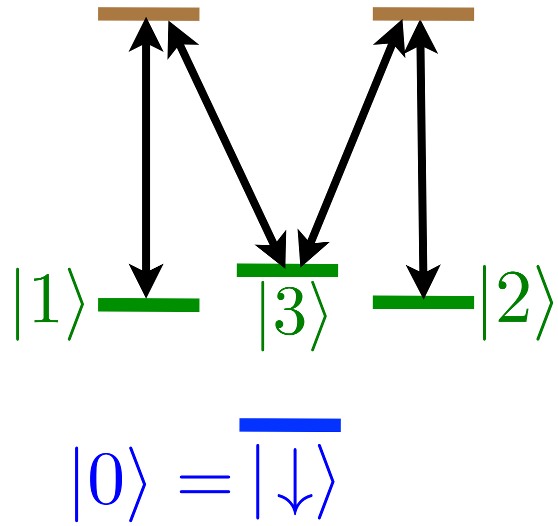

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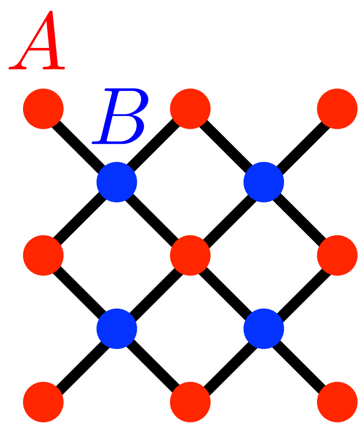

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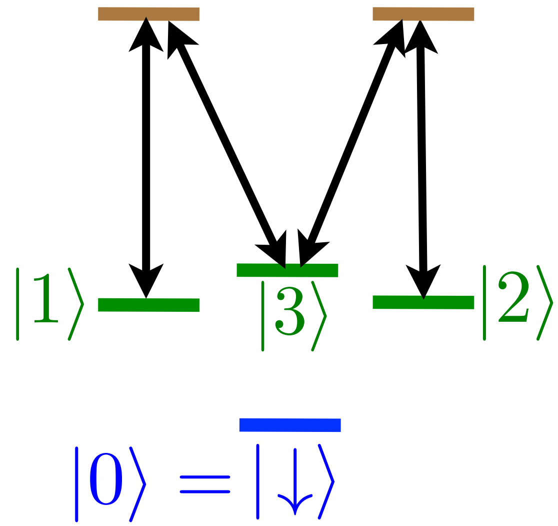

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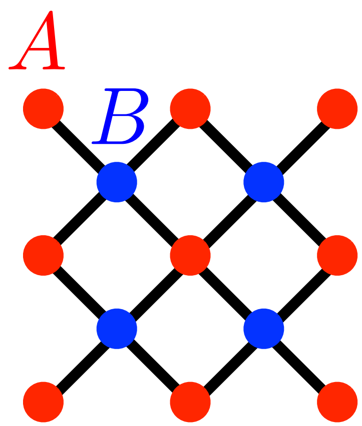

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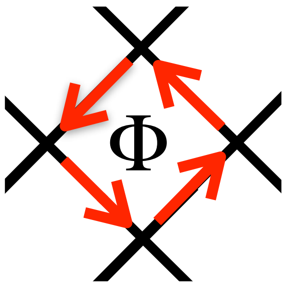

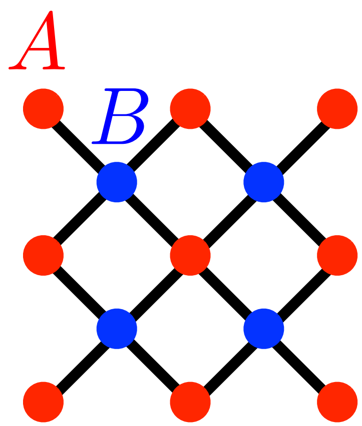

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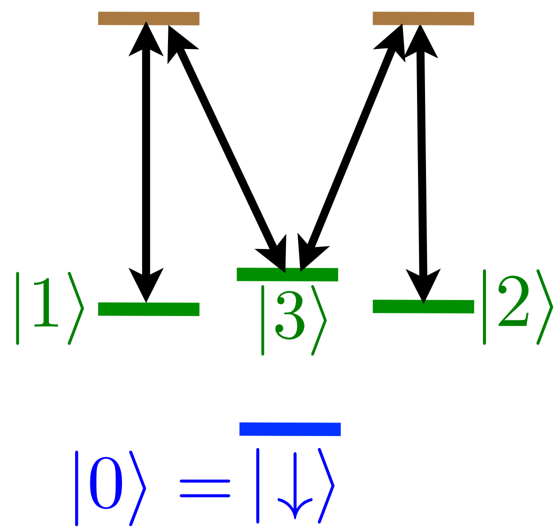

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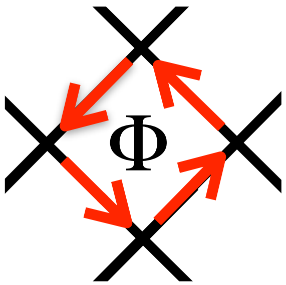

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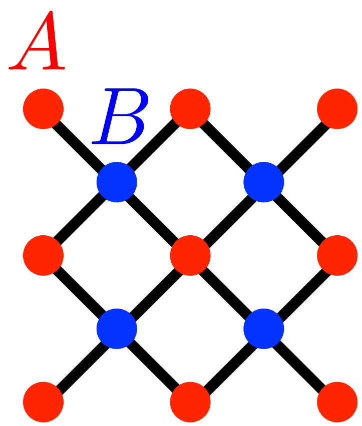

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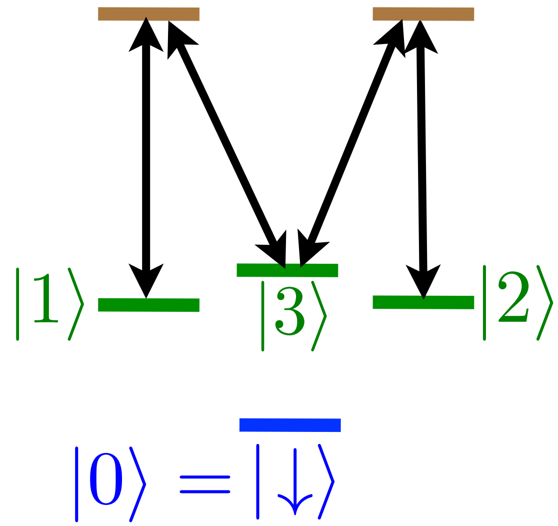

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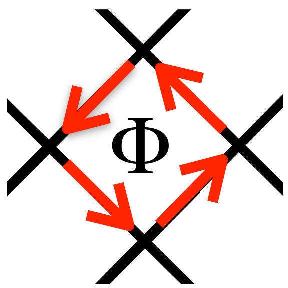
& $Y_{2,2} \propto e^{i2\phi} \Rightarrow$ 

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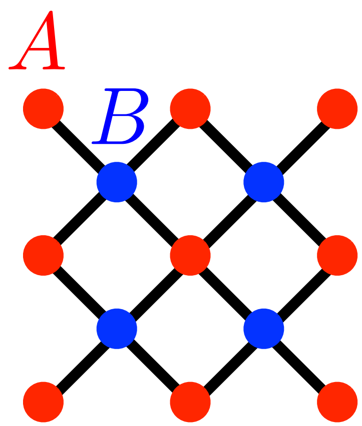

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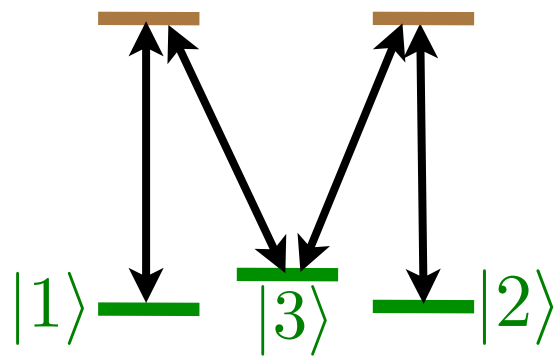

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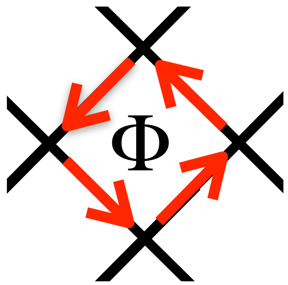
& $Y_{2,2} \propto e^{i2\phi} \Rightarrow$ 

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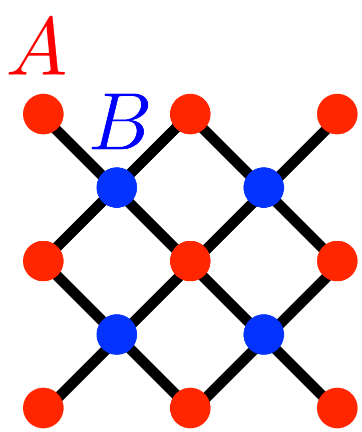

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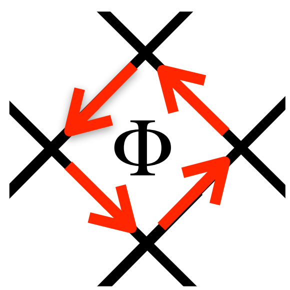

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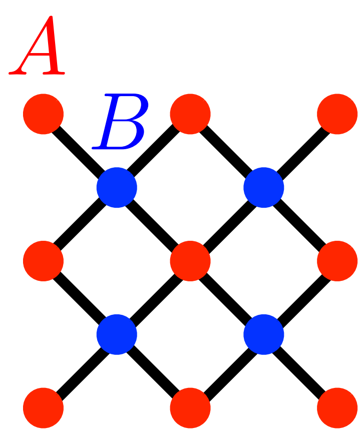

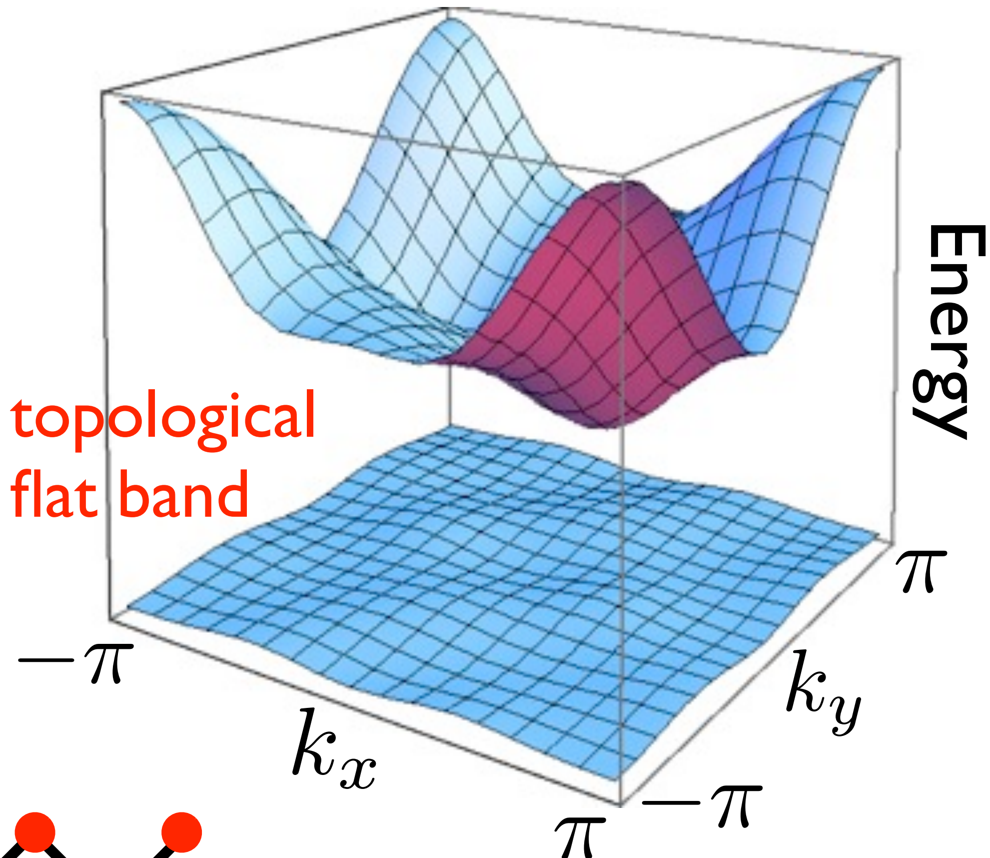

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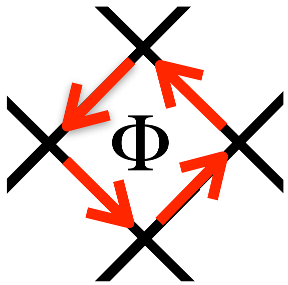
& $Y_{2,2} \propto e^{i2\phi} \Rightarrow$ 

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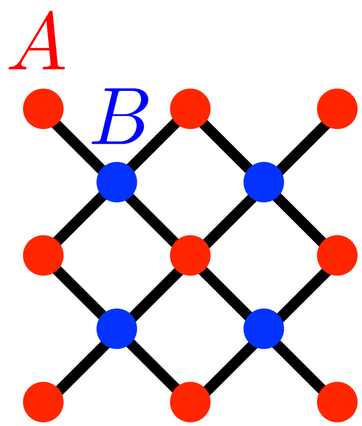

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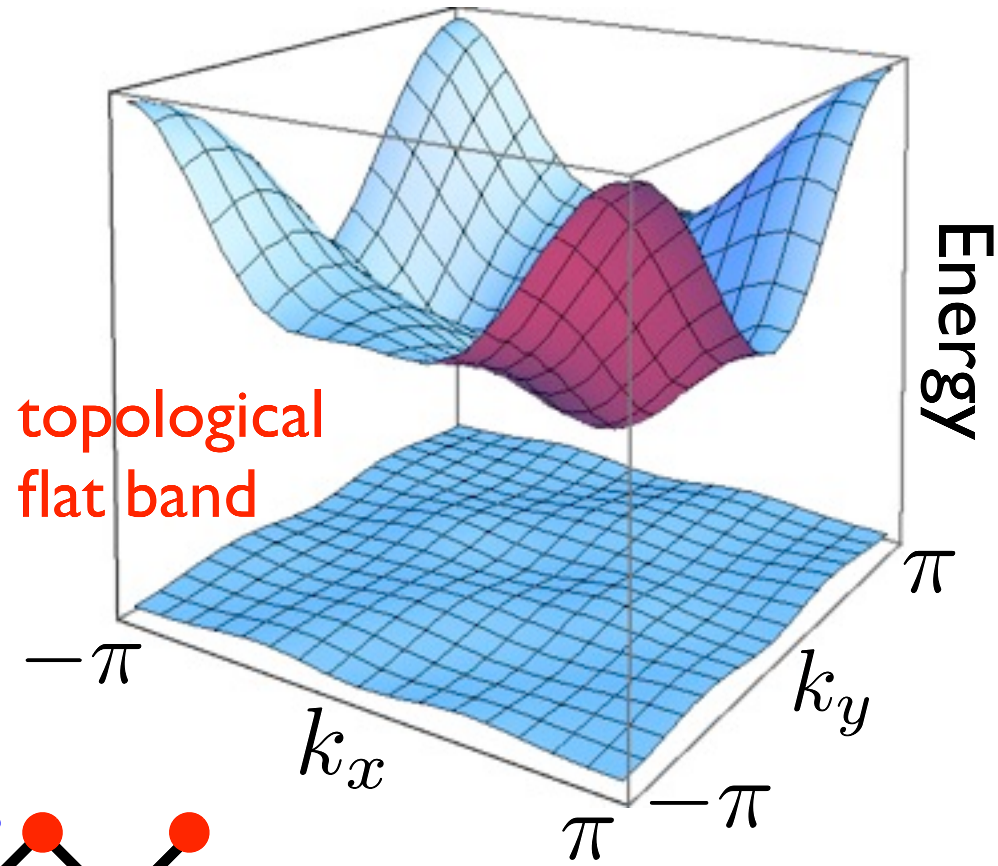

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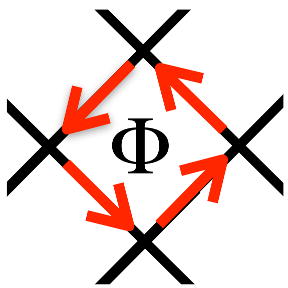
& $Y_{2,2} \propto e^{i2\phi} \Longrightarrow$ 

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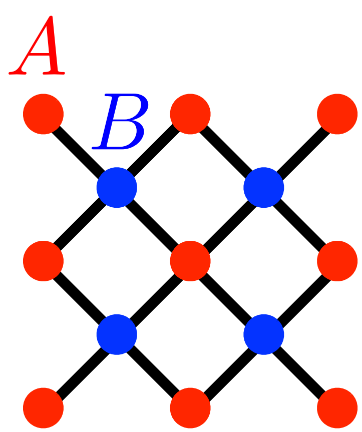

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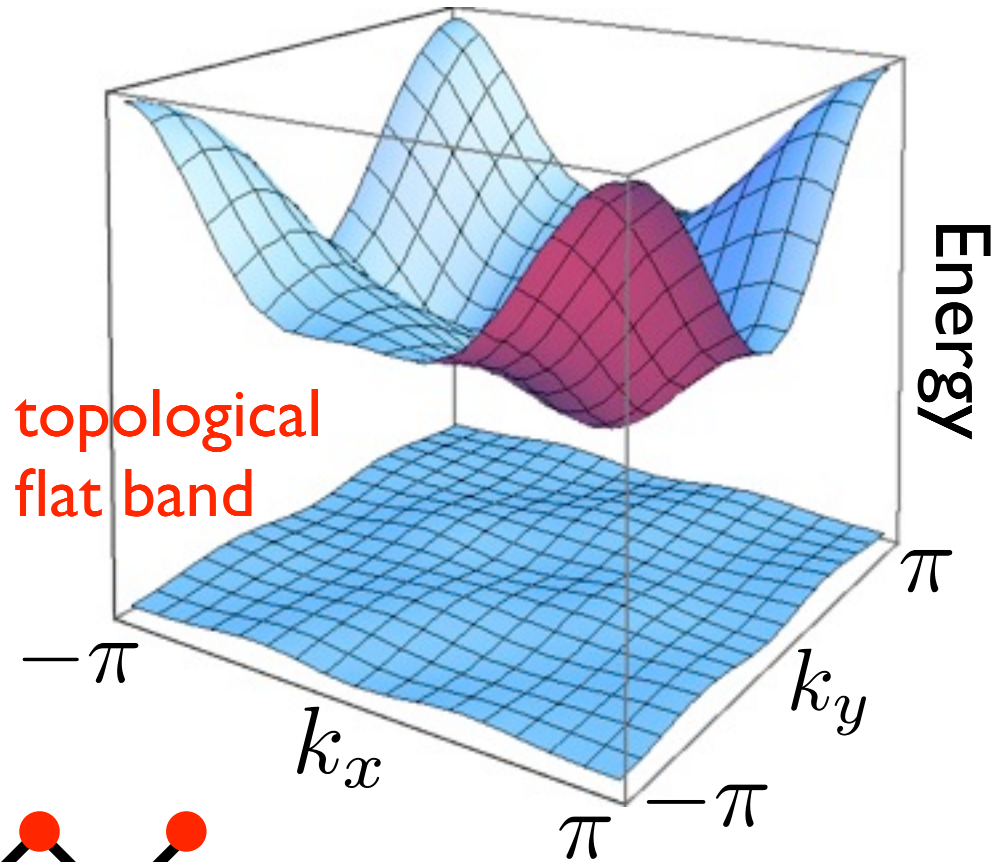

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Thank You