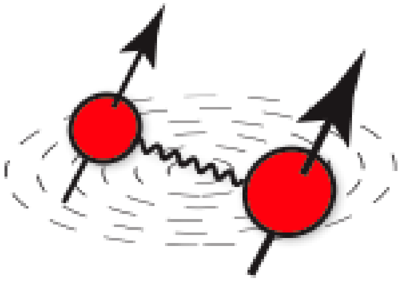
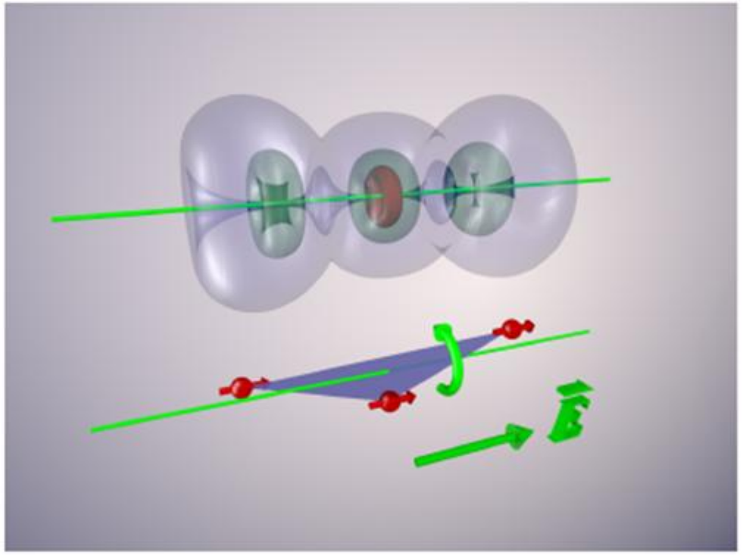
Few-body physics with 3D dipoles and with 3, 4, or 5 atoms

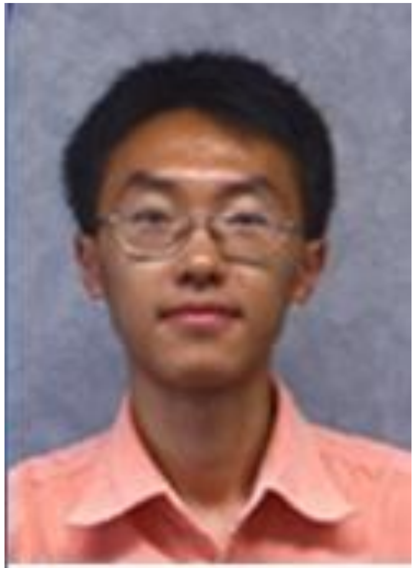
Chris H. Greene, JILA/CU → Purdue since August 2012



Yujun Wang, Jose D'Incao, Jia Wang, Javier von Stecher, and Brett Esry– all ex-*JILA and CU-Boulder Physics*

except for Jose@CU!





Yujun Wang,
UMd/JQI/NIST

Jose D'Incao, Senior Research Associate

Summer 2012 PhD

Topics for this talk:

1. **Overview of our philosophy *(Accurate versus Mulliken-style potential curves)***

2. **Universal findings for the problem of 3 identical bosonic or fermionic dipoles**

3. **Recent experiment and theory: universality for 3 identical bosonic atoms (with van der Waals interactions) plus - a 3-body D-wave resonance**

4. **Recent prediction: universality for 2 identical bosonic atoms + 1 distinguishable atom (with van der Waals interactions)**

5. **Recombination of 4 bosonic atoms and the connection to Efimov physics**

6. **Recombination resonances of 5 or more bosonic atoms**

3, 4, 5, …. To Avogadro's number
(but not beyond)

Our strategy

1. Single out one collision/fragmentation coordinate of the system – usually the hyperradius, R, to treat adiabatically

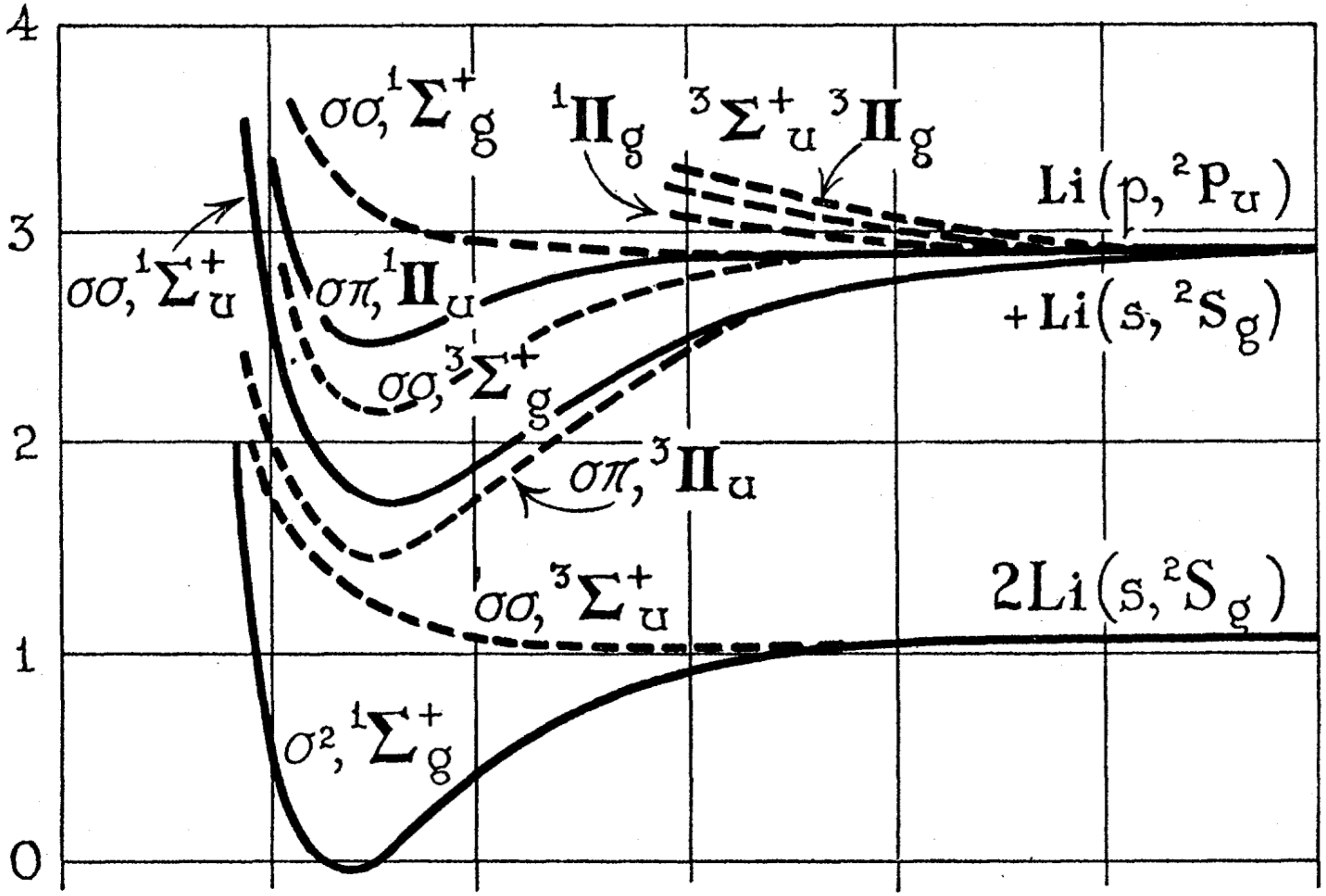
2. Find the fixed-R eigenenergies, plot the resulting potential energy curves $U_n(R)$. Then study their barriers and avoided crossings where inelasticity occurs

3. Solve for scattering observables such as N-body recombination, e.g., $A+A+A+A+A \rightarrow A_3+A_2$

4. Along the way, build intuition to the point where we can *intuit* the structure of the potential curves

Rev. Mod. Phys. 4, p.1 (1932)

ROBERT S. MULLIKEN



Lithium dimer potential curves observed (solid) and predicted (dashed)

1927 № 20

ANNALEN DER PHYSIK

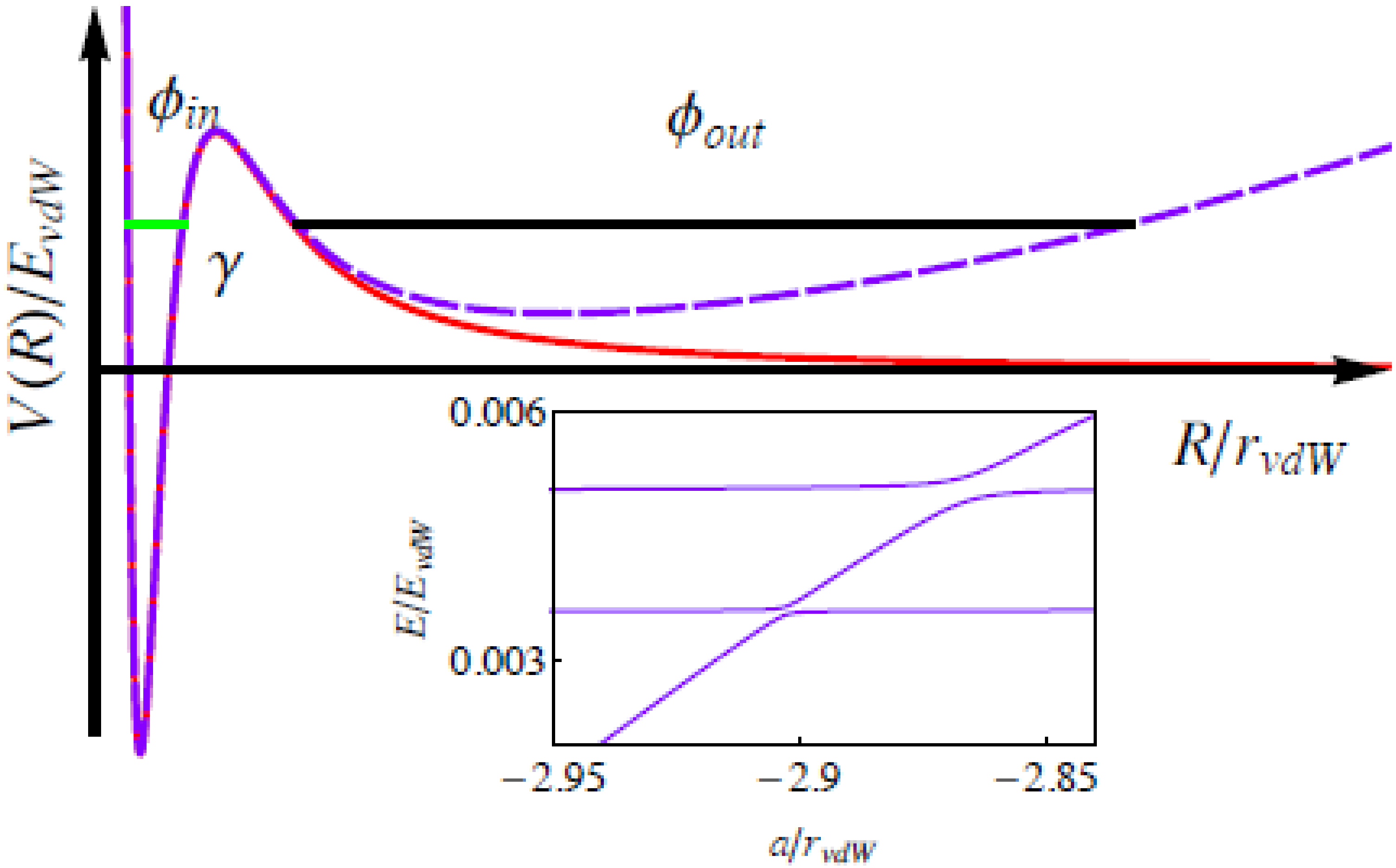
VIERTE FOLGE. BAND 84

1. *Zur Quantentheorie der Molekeln;*

von M. Born und R. Oppenheimer

Resonant five-body recombination in an ultracold gas of bosonic atoms

Mulliken-style potential energy versus hyperradius R for 5 free Cs atoms (solid red) or harmonically trapped (dashed)



arXiv:1205.1921

Alessandro Zenesini[1]‡, Bo Huang[1], Martin Berninger[1], Stefan Besler[1], Hanns-Christoph Nägerl[1], Francesca Ferlaino[1], Rudolf Grimm[1,2], Chris H Greene[3]§, Javier von Stecher[3,4]

Dipolar gases

We characterize dipolar two-body interactions by the "dipole length", which can be viewed as the characteristic range of the polar-polar interaction potential, namely:

$$d = \frac{\mu D^2}{\hbar^2}$$

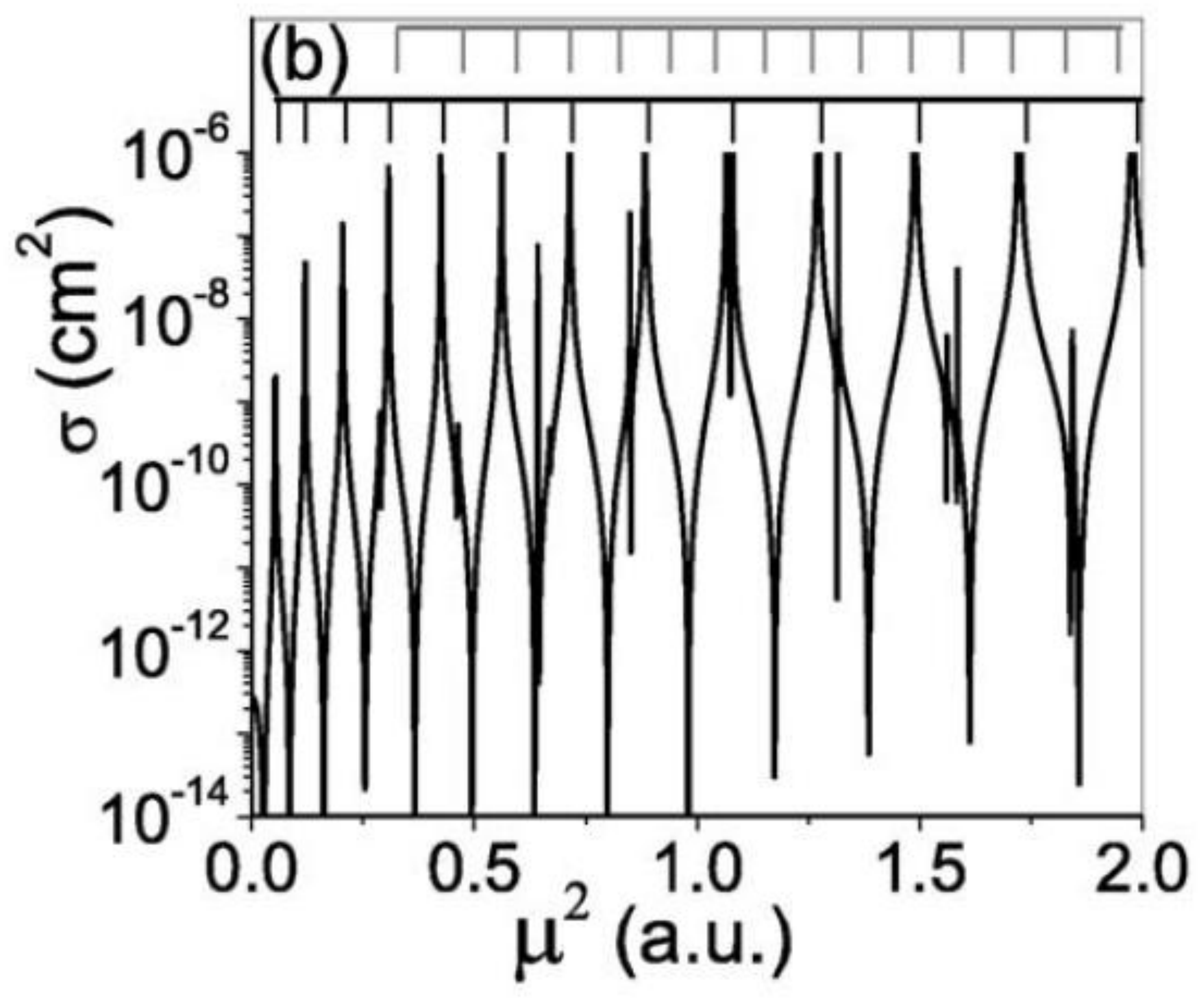
We are interested in the strongly dipolar limit: $kd_\ell \gg 1$

	$d_\ell^{\max}$	$kd_\ell^{\max}$ $(T = 100\mathrm{nK})$
RbK	$\approx 6 \times 10^3 a_0$	2.4
RbCs	$\approx 5 \times 10^4 a_0$	23
LiCs	$\approx 6 \times 10^5 a_0$	238
SrO	$\approx 1.1 \times 10^6 a_0$	467
IK	$\approx 2.8 \times 10^6 a_0$	1100

: Three-body dipolar physics becomes universal (Efimov states)

$$kd_\ell \gg 1$$

Dipole-dipole (2-body) resonances at low energy as the dipole strength is varied, e.g. by changing the aligning E-field strength



Ticknor and Bohn, PHYSICAL REVIEW A 72, 032717 2005

A case of two fermionic dipoles colliding in 3D
From Yujun Wang and CHG, Phys Rev A 85, 022704 (2012)

What is the low energy behavior?

We know that all odd partial waves should be present, and the leading term is known analytically, and because the dipole potential is anisotropic, the S-matrix is not diagonal in an L-representation, so define a diagonal phaseshift as:

$$\delta_l^{m_l} = \ln\left(S_{l,l}^{m_l}\right)/2i$$

Then the low energy expansion of this phaseshift for fermionic dipoles looks like:

$$\mathrm{Re}\left[\delta_l^{m_l}(k)\right] = -a_l^{m_l} k - b_l^{m_l} k^2 - V_l^{m_l} k^3 + O(k^4)$$

$$a_l^{m_l} = d_\ell \frac{D_3(m_l; l, l)}{2l(l+1)}.$$

=Leading term, universal, worked out by Bohn, Cavagnero, Ticknor, NJP 2009

$$\delta_l^{m_l} = \ln\left(S_{l,l}^{m_l}\right)/2i \qquad a_l^{m_l} = d_\ell \frac{D_3(m_l;l,l)}{2l(l+1)}.$$

$$\mathrm{Re}\left[\delta_l^{m_l}(k)\right] = -a_l^{m_l} k - b_l^{m_l} k^2 - V_l^{m_l} k^3 + O(k^4)$$

And also the term in k^2 turns out to be universal, but the term in k^3 depends on the short-range potential, and in particular $V_{L=1}$ is the first term that diverges when a bound state goes through E=0.

Jia Wang has since demonstrated this qualitative effect for other long range potentials, such as the van der Waals long range tail, where for L>1 the leading term is not the Wigner Law k^{2L+1}, but rather k^4, and the coefficient of k^4 is UNIVERSAL, but when a bound state goes through zero energy, it is the coefficient of the Wigner Law term that diverges.

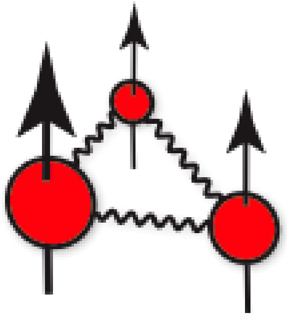
The three-dipole problem in 3D

A major extension of 3-body interactions to account for polar molecule resonances and recombination

Three dipoles in 3D

PRL 106, 233201 (2011)

Yujun Wang, D'Incao, & CHG



Strategy: **Hyperspherical Treatment**

$$\Psi_M(R,\Omega) = \sum_\nu F_\nu^M(R)\Phi_\nu^M(R;\Omega)$$

$$\Phi_\nu^M(R;\Omega) = \sum_J \sum_K \phi_{\nu K}^J(R;\theta,\varphi) D_{KM}^J(\alpha,\beta,\gamma)$$

Adiabatic eigenfunction, obeys

$$\sum_{K'} \frac{\hat{\Lambda}^2_{JK'}(\theta,\varphi)}{2\mu R}\phi_{\nu K'}^J(R;\theta,\varphi) + \sum_{i<j} v_{sr}(r_{ij})\phi_{\nu K}^J(R;\theta,\varphi)$$

$$+ \sum_{J'K'} \langle J'K'M'|v_{dd}(\vec{r}_{ij})|JKM\rangle \phi_{\nu K'}^{J'}(R;\theta,\varphi) = U_\nu(R)\phi_{\nu K}^J(R;\theta,\varphi)$$

Angular momentum is not conserved! Ouch!

$U_\nu(R)$: Adiabatic Potentials

$$\langle J'K'M'|v_{dd}(\vec{r}_{ij})|JKM\rangle = \frac{d_\ell}{\mu_{2b} r_{ij}^3}(-1)^{K+M}\delta_{MM'}\sqrt{(2J+1)(2J'+1)} \times$$

$$\left[\delta_{KK'}\begin{pmatrix} J & 2 & J' \\ K & 0 & -K' \end{pmatrix}\begin{pmatrix} J & 2 & J' \\ M & 2 & -M' \end{pmatrix}\right.$$

$$-\delta_{K-2,K'}\frac{3}{\sqrt{6}}\begin{pmatrix} J & 2 & J' \\ -K & 2 & K' \end{pmatrix}\begin{pmatrix} J & 2 & J' \\ -M & 0 & M' \end{pmatrix}\left(\frac{r_{ij}^x - i\, r_{ij}^y}{2}\right)^2$$

$$\left. -\delta_{K+2,K'}\frac{3}{\sqrt{6}}\begin{pmatrix} J & 2 & J' \\ K & 2 & -K' \end{pmatrix}\begin{pmatrix} J & 2 & J' \\ M & 0 & -M' \end{pmatrix}\left(\frac{r_{ij}^x + i\, r_{ij}^y}{2}\right)^2\right]$$

New elements needed for oriented dipoles: off-diagonal matrix elements in J

Some physics with 3 dipoles

References to our 2011 progress:

Yujun Wang, D'Incao, CHG, Efimov effect for three interacting bosonic dipoles

PRL 106, 233201 (2011)

Yujun Wang, D'Incao, CHG, Universal three-body physics for fermionic dip

PRL 107, 233201 (2011)

	$d_\ell^{\max}$	$kd_\ell^{\max}$ $(T = 100\text{nK})$
RbK	$\approx 6 \times 10^3 a_0$	2.4
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Summary of what we've found so far about the 3-dipole problem:

1. **For three polar bosons, there is an Efimov effect**, the first time this has been demonstrated for a system in 3D that has anisotropic interactions with no conservation of angular momentum. Also, the scattering length at which the Efimov state reaches zero energy is UNIVERSAL, and it has a barrier that makes it long-lived, and we…

$$a_{3b}^{*-}/d_\ell \approx -8.1$$

predict some important three-body scattering observables

$$K_3^{(a_s>0)} \approx \frac{67.1}{e^{2\eta}}\left\{\sin^2\left[s_0 \ln\left(\frac{a_s}{d_\ell}\right)+2.5\right]+\sinh^2\eta\right\}\frac{a_s^4}{m}, \quad (6)$$

$$K_3^{(a_s<0)} \approx \frac{4590\sinh(2\eta)}{\sin^2[s_0\ln(\frac{|a_s|}{d_\ell})+0.92]+\sinh^2\eta}\frac{a_s^4}{m}, \quad (7)$$

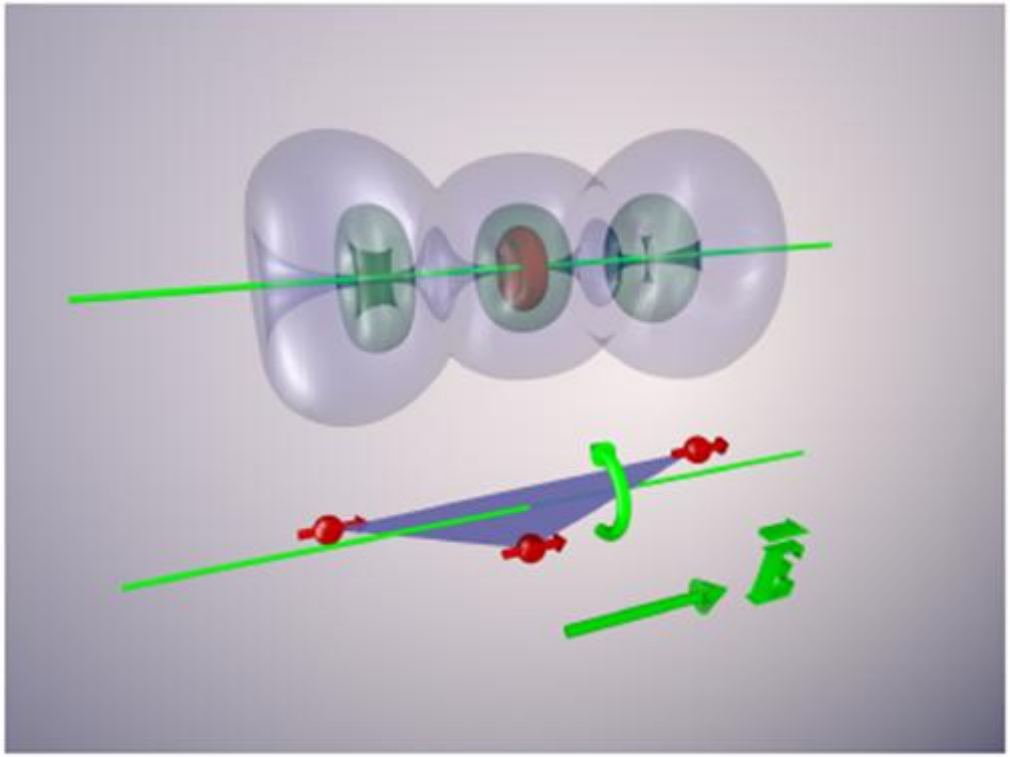
$$a_{Dd}^{(a_s>0)} \approx \left(1.46+2.15\cot\left[s_0\ln\left(\frac{a_s}{d_\ell}\right)+0.86+i\eta\right]\right)a_s, \quad (8)$$

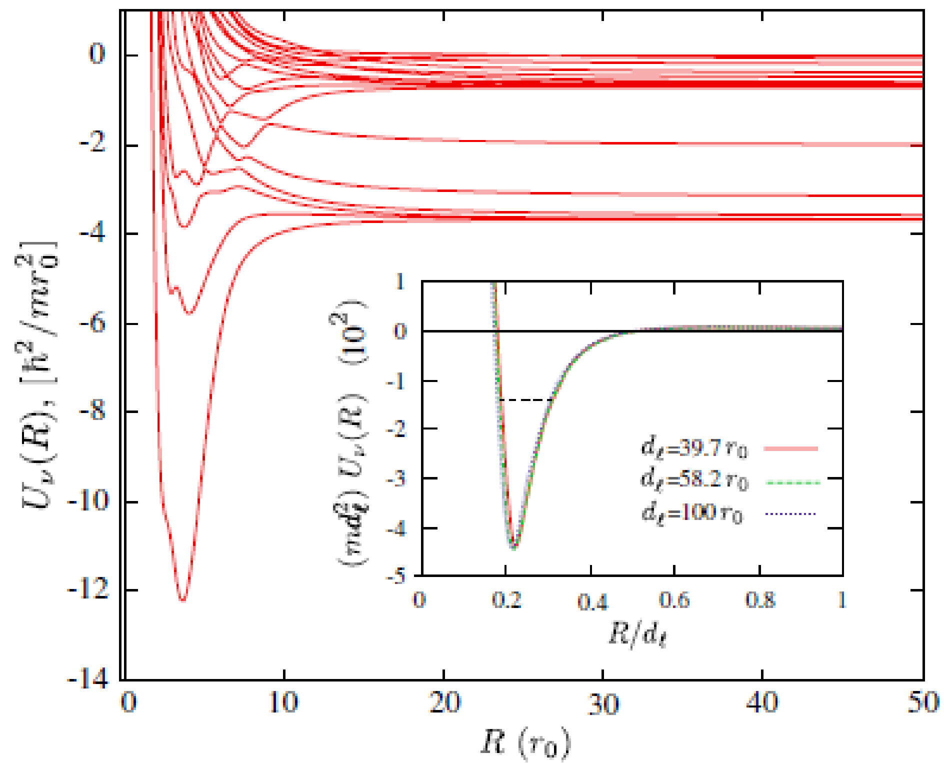
$$V_{\rm rel}^{(a_s>0)} \approx \frac{20.3\sinh(2\eta)}{\sin^2[s_0\ln(\frac{a_s}{d_\ell})+0.86]+\sinh^2\eta}\frac{a_s}{m}. \quad (9)$$

Note that this K_3 describes processes like AB+AB+AB →A_2B_2+AB

Summary of what we've found so far about the 3-dipole problem:

2. **<u>For three polar fermions, there is NO Efimov effect, but there is precisely ONE universal state,</u>** and again it has a barrier that makes it comparatively long-lived and independent of the short-range interactions





$$\mathrm{Re}[\delta(k_2)] \approx -ak_2 - bk_2^2 - Vk_2^3$$

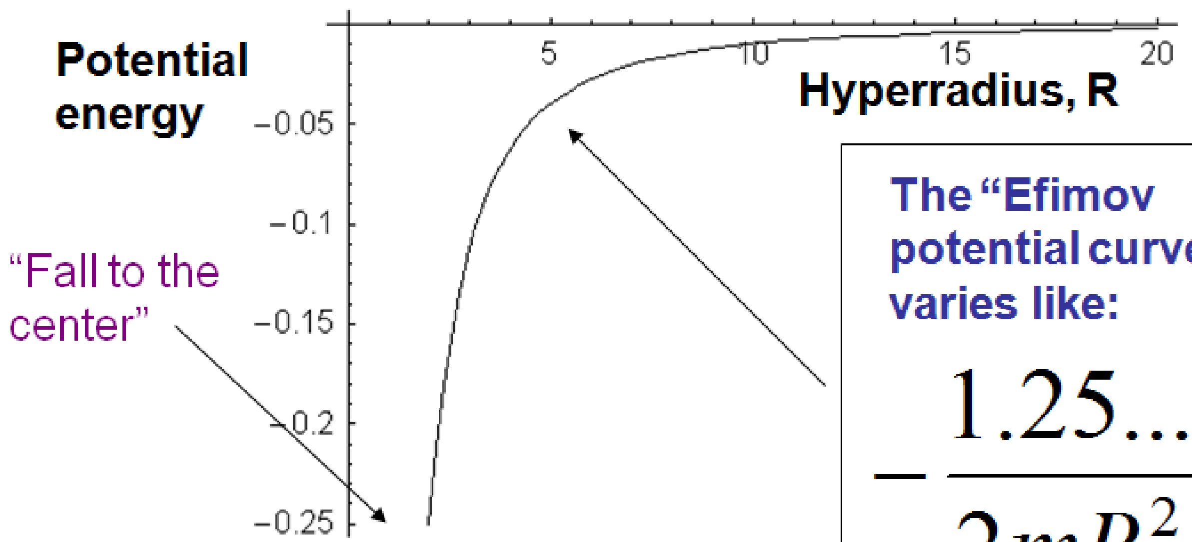
$$K_3 = \frac{C_3}{\mu} k^4$$

$$C_3 = \lambda V^{17/2} d_\ell^{-35/2}$$

FIG. 1 (color online). A typical set of adiabatic hyperspherical potentials $U_\nu(R)$ for three fermionic dipoles with $d_\ell/r0 = 58.2$ and $E_{2d} \to 0$. The inset shows rescaled, diabatized potentials exhibiting universal behavior for a few values of d_ℓ at a dipole-dipole resonance. The horizontal dashed line in the diabatic potential wells indicates the position of the universal three-dipole states.

Now, what about the **universality of the 3-body parameter** for homonuclear systems of 3 bosonic atoms? The 3-body parameter enters Efimov theory because the $-1/R^2$ potential must be terminated at small R somehow. Before the summer of 2011, it was thought to be more or less “random” for different systems.

To understand the Efimov effect, look at the effective potential energy curve at unitarity, as a function of the hyperradius:



The “Efimov potential curve” varies like:

$$-\frac{1.25\ldots}{2mR^2}$$

Mathematical Detail. Once you have this “effective dipole-type attractive potential curve”, the rest is ‘TRIVIAL’!

Here, ‘trivial’ means that the solutions are simply Bessel functions (of imaginary *order*, and imaginary *argument*).

PRL 108, 263001 (2012)

Origin of the Three-Body Parameter Universality in Efimov Physics

Jia Wang,[1] J. P. D'Incao,[1] B. D. Esry,[2] and Chris H. Greene[1]

=An interpretation of the unexpected BOMBSHELL paper of 2011, by:

M. Berninger, A. Zenesini, B. Huang, W. Harm, H.C. Nägerl, F. Ferlaino, R. Grimm, P.S. Julienne, and J.M. Hutson, Phys. Rev. Lett. **107**, 120401 (2011).

Other relevant theoretical work to interpret this result:

Cheng Chin's toy model (arXiv 2011)

And detailed hyperspherical calculations by Naidon, Endo, & Ueda:

``Physical Origin of the Universal Three-body Parameter in Atomic Efimov Physics" Pascal *Naidon*, Shimpei Endo, and Masahito *Ueda* *arXiv*:1208.3912 (largely confirms our interpretation)

The “three-body parameter” controlling the first Efimov resonance location had been thought to be more or less “random”, but there experimental evidence strongly suggests that it must be approximately universal:

1) ^{133}Cs (Berninger et al.) PRL 107, 120401 (2011) :
   $|a_-|/L_{vdW}$= 9.4, 11.1, 10.4, and 10.3

2) ^{7}Li (Hulet) Science 326, 1683 (2009) : $|a_-|/L_{vdW}$= 10.0

3) ^{7}Li (Khaykovich) PRL 103, 163202 (2009) : $|a_-|/L_{vdW}$= 8.9

4) ^{7}Li (Khaykovich) PRL 105, 103203 (2010) : $|a_-|/L_{vdW}$= 9.0

5) ^{39}K (Modungno) Nat. Phys. 5, 586 (2009): $|a_-|/L_{vdW}$= 25.4 or (revised interpretation, still speculative): $|a_-|/L_{vdW}$= 11.0

6) ^{85}Rb(Cornell-Jin group at JILA) 2012 PRL: $|a_-|/L_{vdW}$= 9.7(1)

3-body hyperspherical potential curves based on 2-body Lennard-Jones interaction potential with 10 s-wave bound states, around 100 total

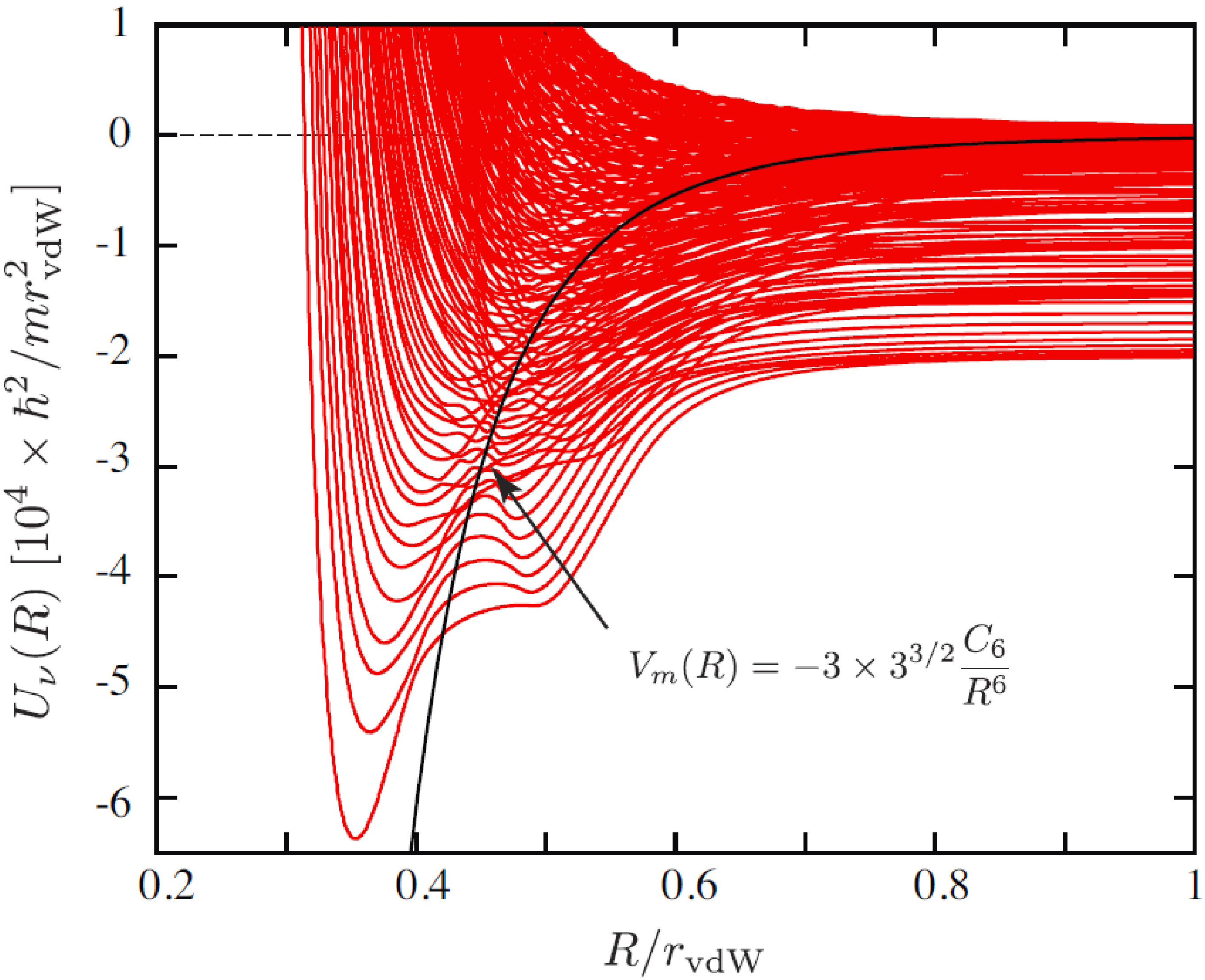
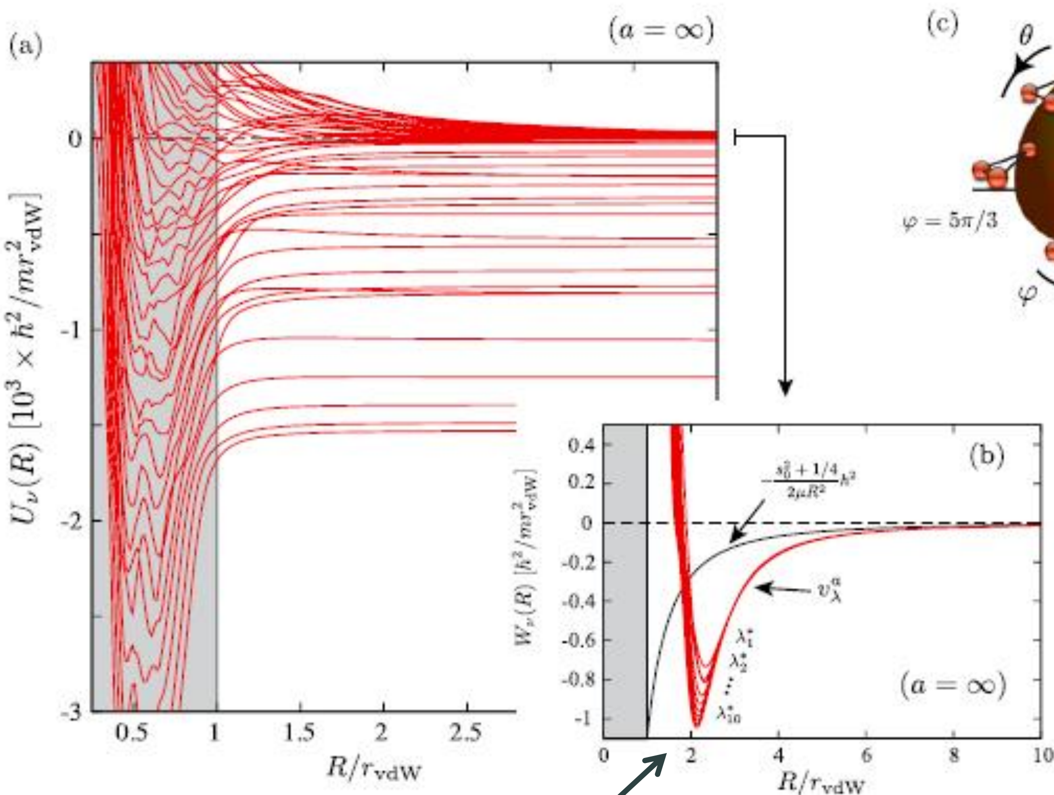
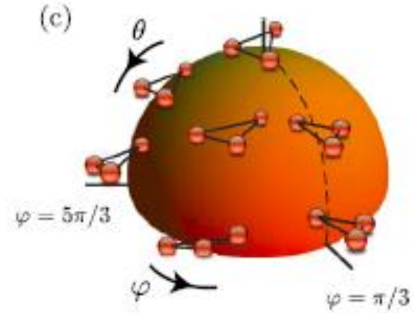
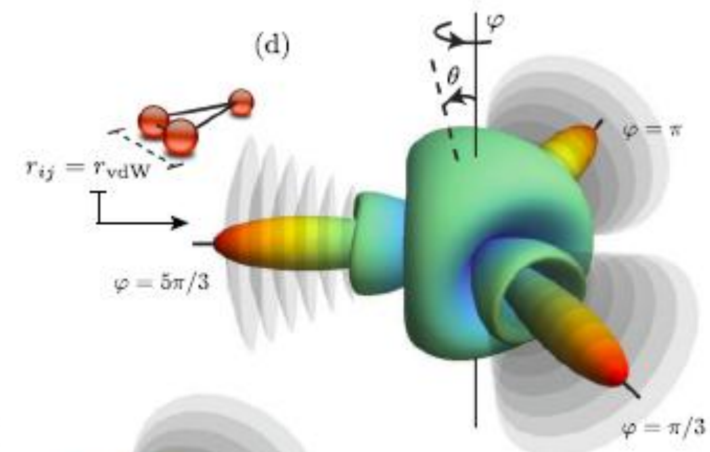


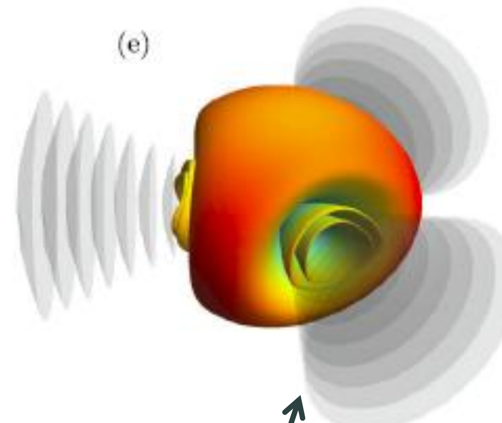
Figure 5.12: This figure shows the three-body potentials obtained using the $v_\lambda^a(\lambda = \lambda_{10}^*)$ model supporting a total of 100 bound states. Roughly speaking, the potential of Eq. (5.18) [16] (black solid line) can be seen as a diabatic potential since it passes near one of the series of avoided crossings.

Our study of hyperspherical potentials in the bosonic A+A+A system, showing that any two atoms "go over the van der Waals cliff" when they approach within their vdW radius, and this rise in kinetic energy produces a repulsive hyperspherical potential barrier









NO vdW force field, and NO suppression

vdW force field, note wavefunction suppression in 2-body valleys

Numerical evidence for the existence of a universal barrier when the two-body potential has a van der Waals tail

Summary of our extensive numerical tests and analysis. There is a universal Efimov potential curve that includes a universal short range barrier that fixes the 3-body parameter, shown here:

PRL 108, 263001 (2012) PHYSICAL REV

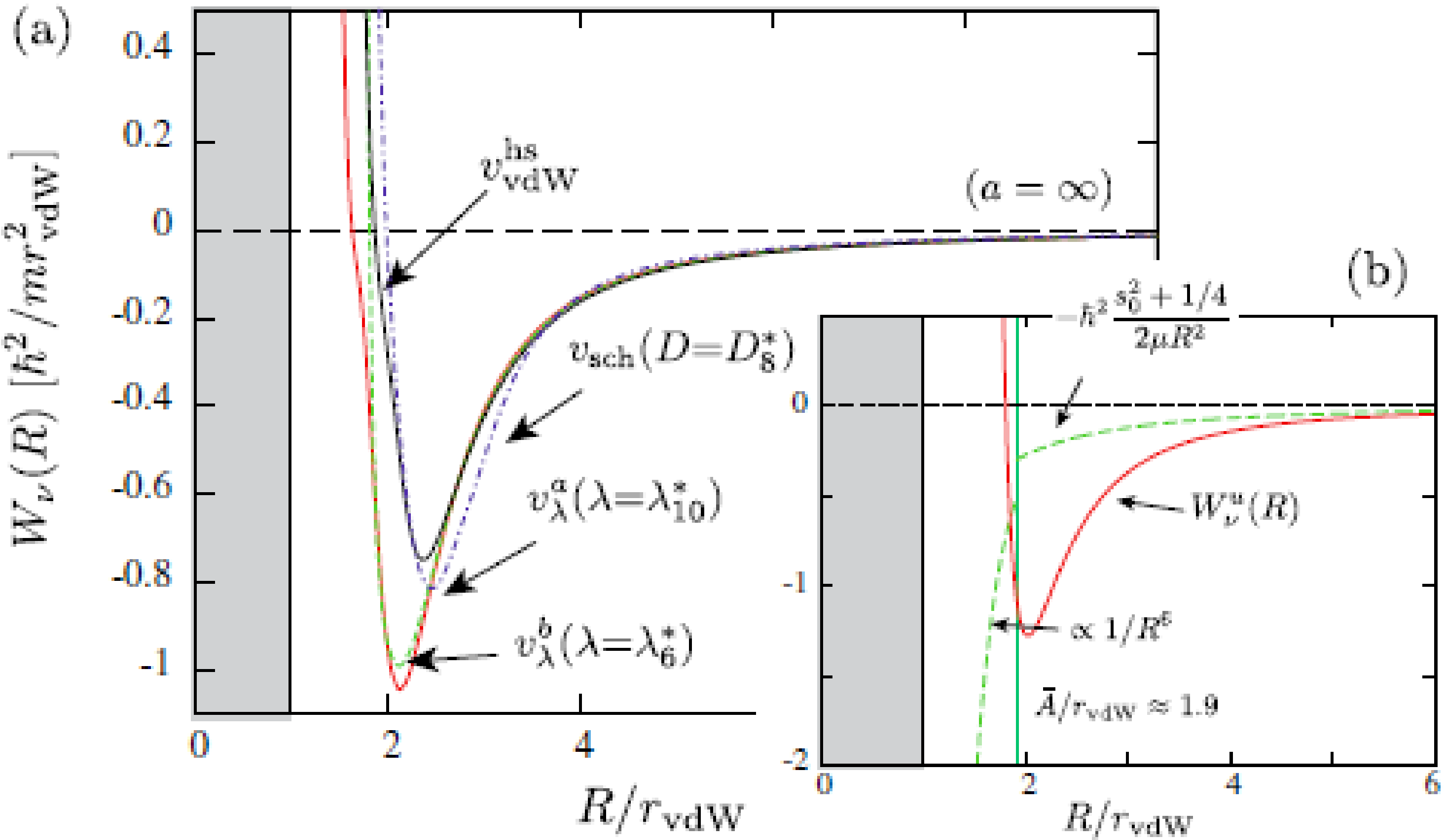


FIG. 3 (color online). (a) Efimov potential obtained from the different two-body potential models used here. The reasonably

Note that this barrier arises from a <u>classical</u> suppression of the wavefunction

PRL 2012 Jia Wang, D'Incao, Esry, CHG

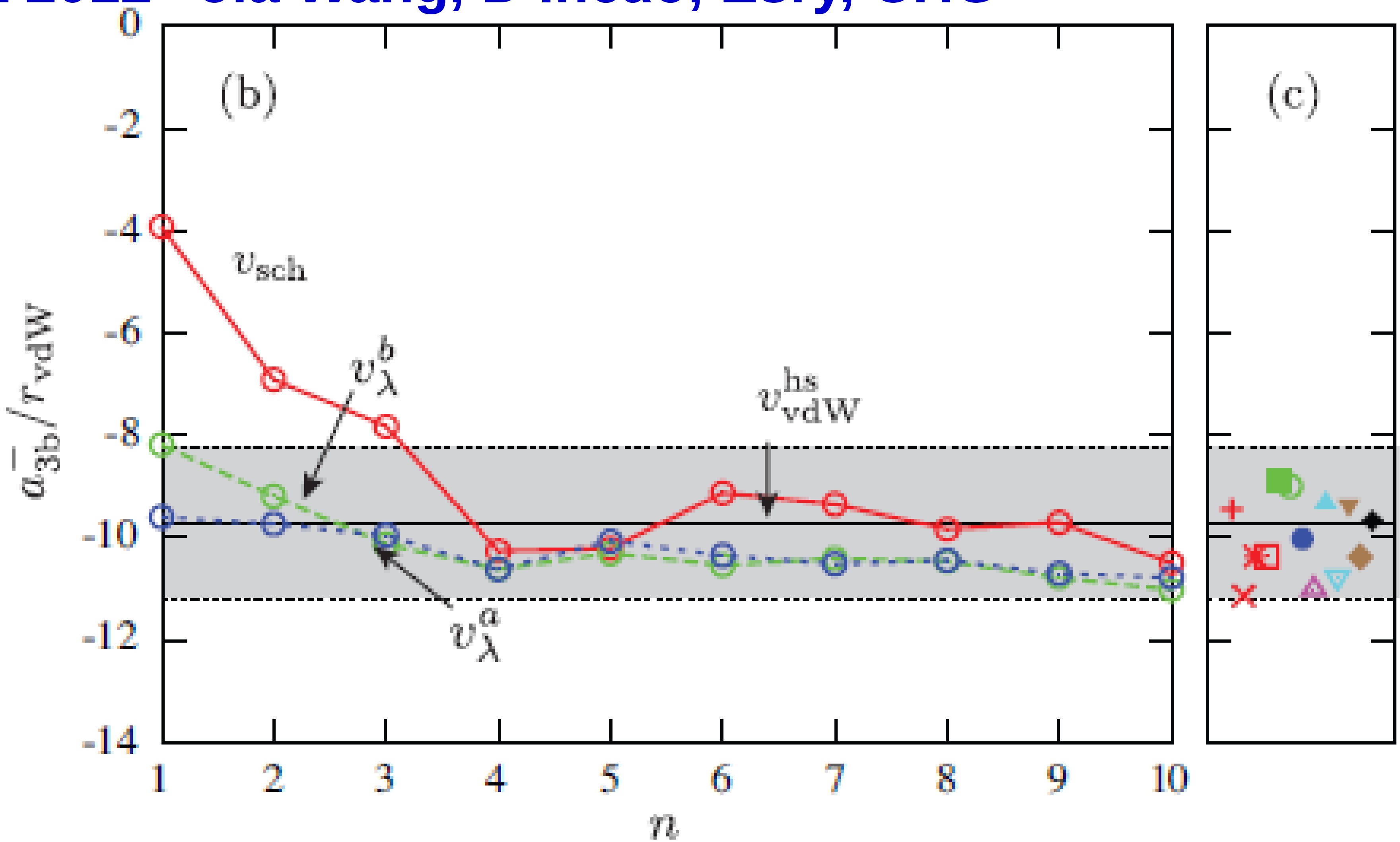


FIG. 4: Values for the three-body parameter (a) κ_* and (b) a_{3b}^- as functions of the number n of two-body s-wave bound states for each of the potential model studied here. (c) Experimental values for a_{3b}^- for ^{133}Cs [3] (red: ×, +, □, and ∗), ^{39}K [4] (magenta: △), ^{7}Li [5] (blue: •) and [6, 7] (green: ■ and ○), ^{6}Li [8, 9] (cyan: ▲ and ▽) and [10, 11] (brown: ▼ and ◊), and ^{85}Rb [12] (black: ⧫). The gray region specifies a band where there is a ±15% deviation from the $v_{\rm vdW}^{\rm hs}$ results. The inset of

Another finding: This property of 3-atom states is not expected to hold for nuclear systems, which have no van der Waals tail and few bound states.

Note that our detailed hyperspherical calculations have all assumed a single-channel interaction between each pair of atoms, which means that our conclusions are presumably valid for BROAD Fano-Feshbach resonances, but most likely inapplicable to NARROW resonances

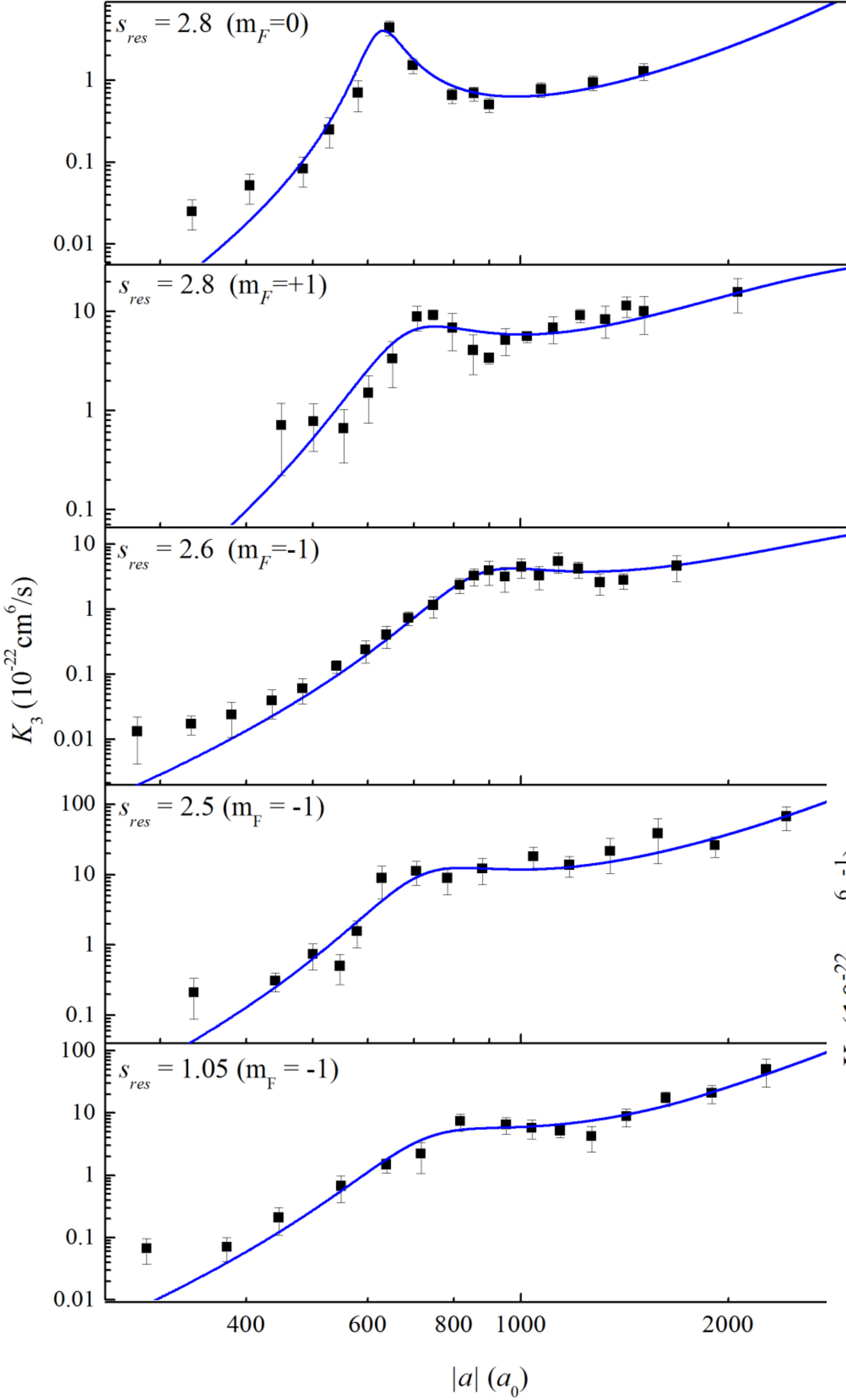
But very recently, this point has been tested experimentally

Universality of the three-body Efimov parameter for closed-channel dominated Feshbach resonances

Sanjukta Roy, Manuele Landini, Andreas Trenkwalder, Andrea Simoni*, Massimo Inguscio, Marco Fattori and Giovanni Modugno

This experiment by Sanjukta Roy, Giovanni Modugno, et al. has seen 7 Efimov resonances in ^{39}K, ranging from resonance width parameters: s_{res}=0.1 (narrow) up to s_{res}=2.8 (broad)

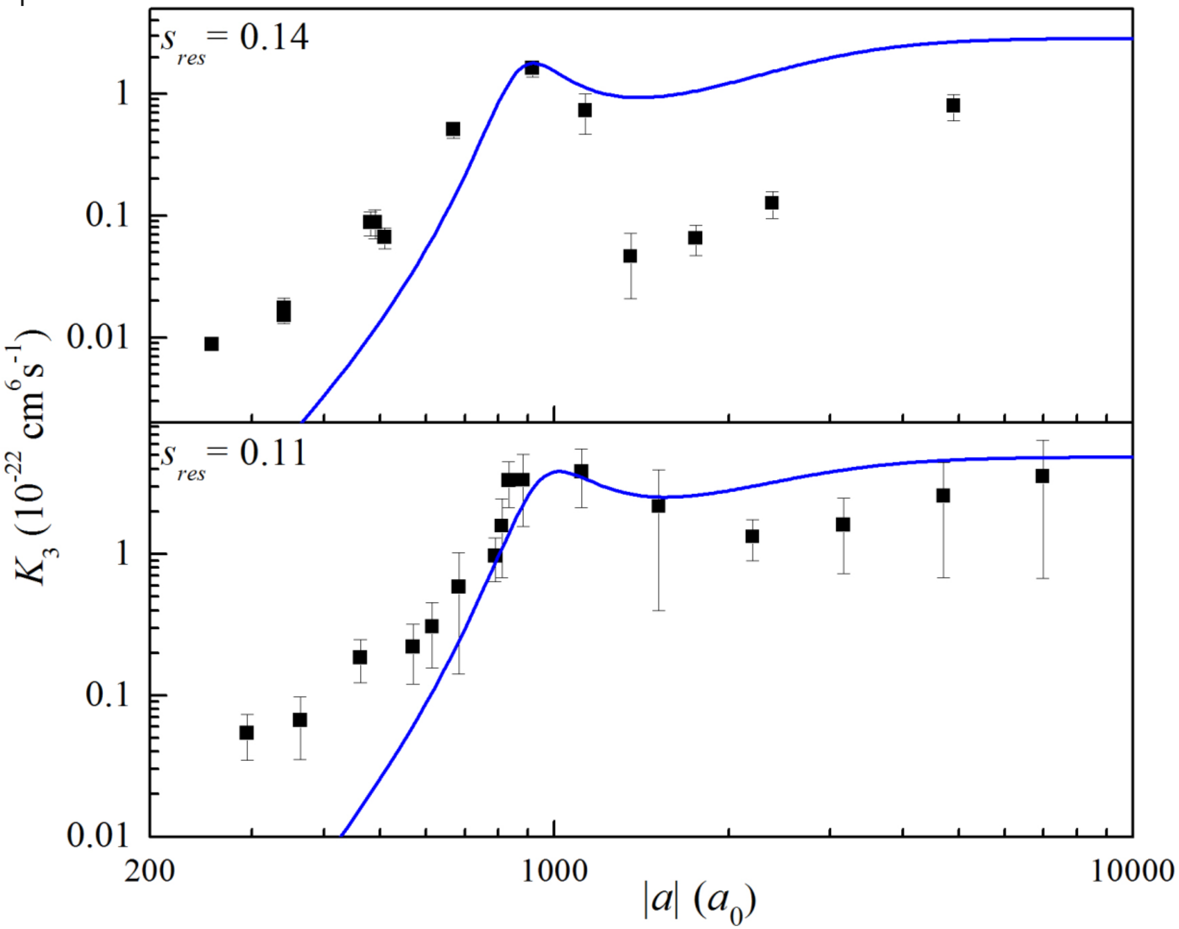
Preliminary results of Roy, Modugno, et al, not yet submitted for publication

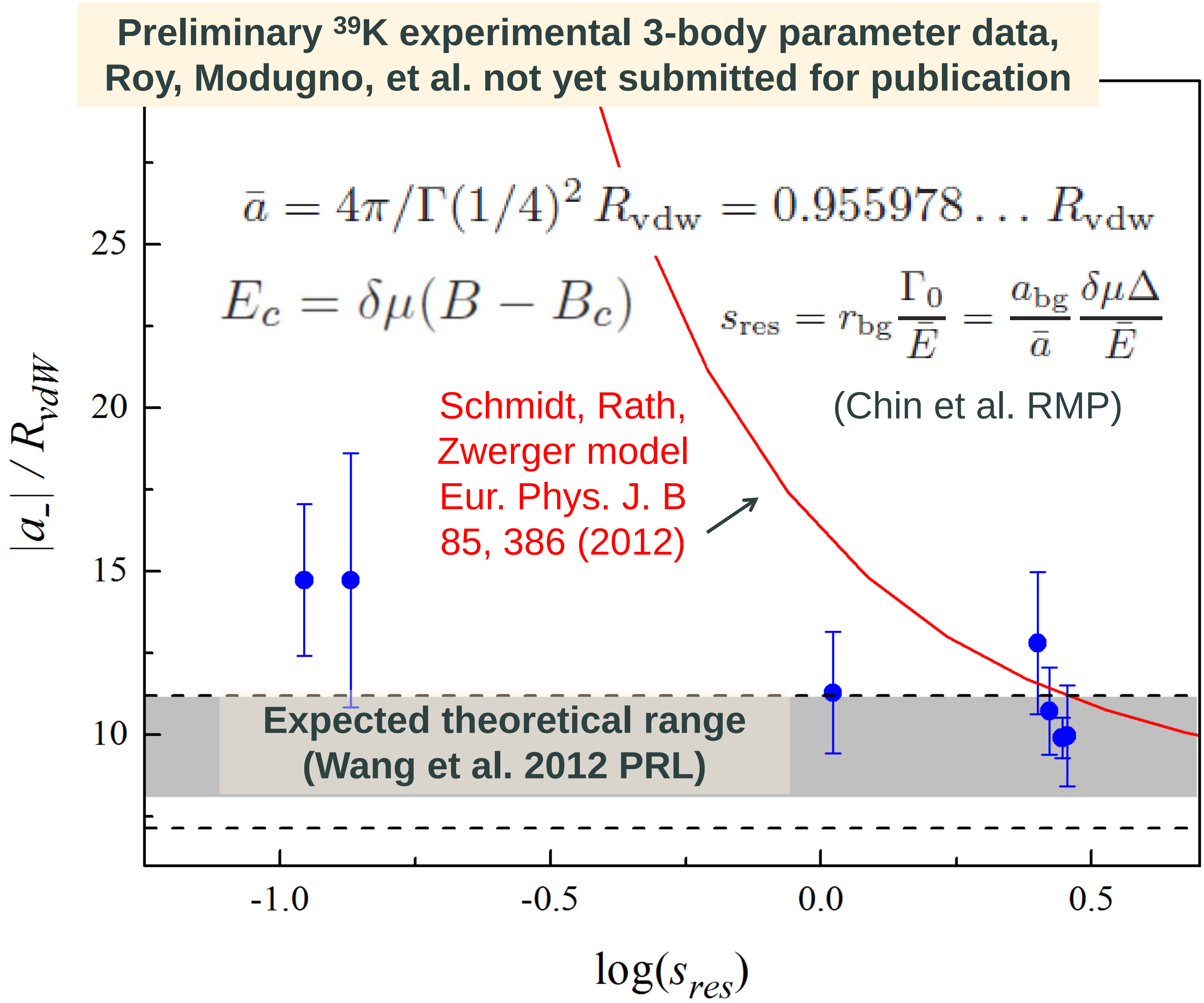


← **broad cases, s_{res}>1**

7 Efimov resonances in ^{39}K

Narrow cases s_{res}<0.15




Preliminary ^{39}K experimental 3-body parameter data,
Roy, Modugno, et al. not yet submitted for publication
$\bar{a} = 4\pi/\Gamma(1/4)^2 R_{vdw} = 0.955978\ldots R_{vdw}$
$E_c = \delta\mu(B - B_c)$
$s_{res} = r_{bg}\frac{\Gamma_0}{\bar{E}} = \frac{a_{bg}}{\bar{a}}\frac{\delta\mu\Delta}{\bar{E}}$
(Chin et al. RMP)
Schmidt, Rath, Zwerger model Eur. Phys. J. B 85, 386 (2012)
Expected theoretical range (Wang et al. 2012 PRL)
$|a_-| / R_{vdW}$
25
20
15
10
-1.0
-0.5
0.0
0.5
$\log(s_{res})$

Next, what can theory PREDICT for the heteronuclear Efimov effect?

Universal three-body parameter in heteronuclear atomic systems

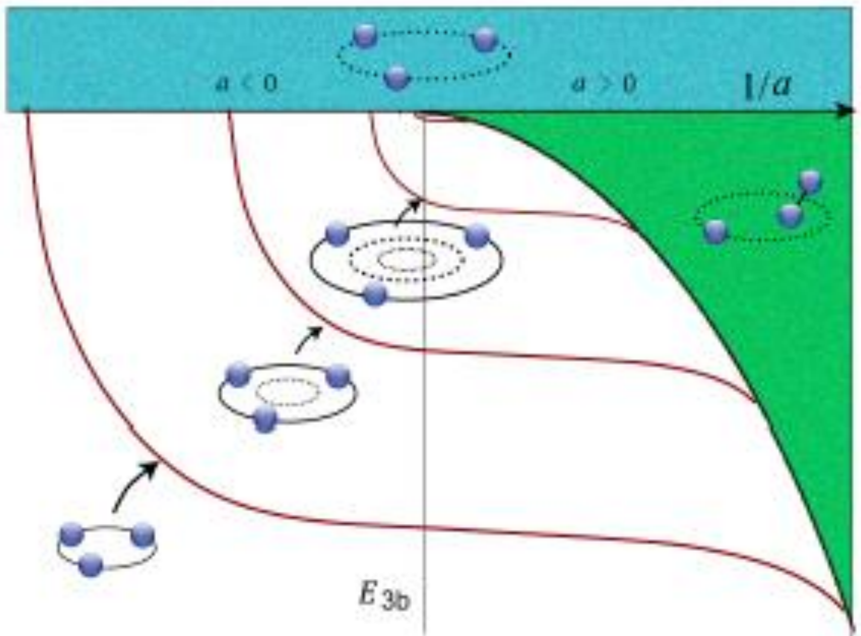
Yujun Wang, Jia Wang, J. P. D'Incao, & CHG

PRL 109, 243201 (2012)

Main result: we see that the Efimov physics is also universal for the case of 2 identical bosonic atoms (AA) and 1 distinguishable atom (X), but the parameter space is larger and more complicated. This is because the universality values predicted depend on the mass ratio, M_A/M_X, and on the background A-A scattering length, and on TWO different vdW radii (A-X and A-A).

arXiv:1207.6439

The Efimov effect: universality



Efimov spectrum

For three particles with two or three resonant interactions (scattering length $a \to \infty$), an infinite series of three-body bound states emerge with $E_n = E_0 e^{-2n\pi/s_0}$ [1].

Heteronuclear system AAX:
Efimov-favored when $m_A/m_X \gg 1$ such that $s_0 > 1$;
Efimov-unfavored when $m_A/m_X \lesssim 1$ such that $s_0 < 1$.

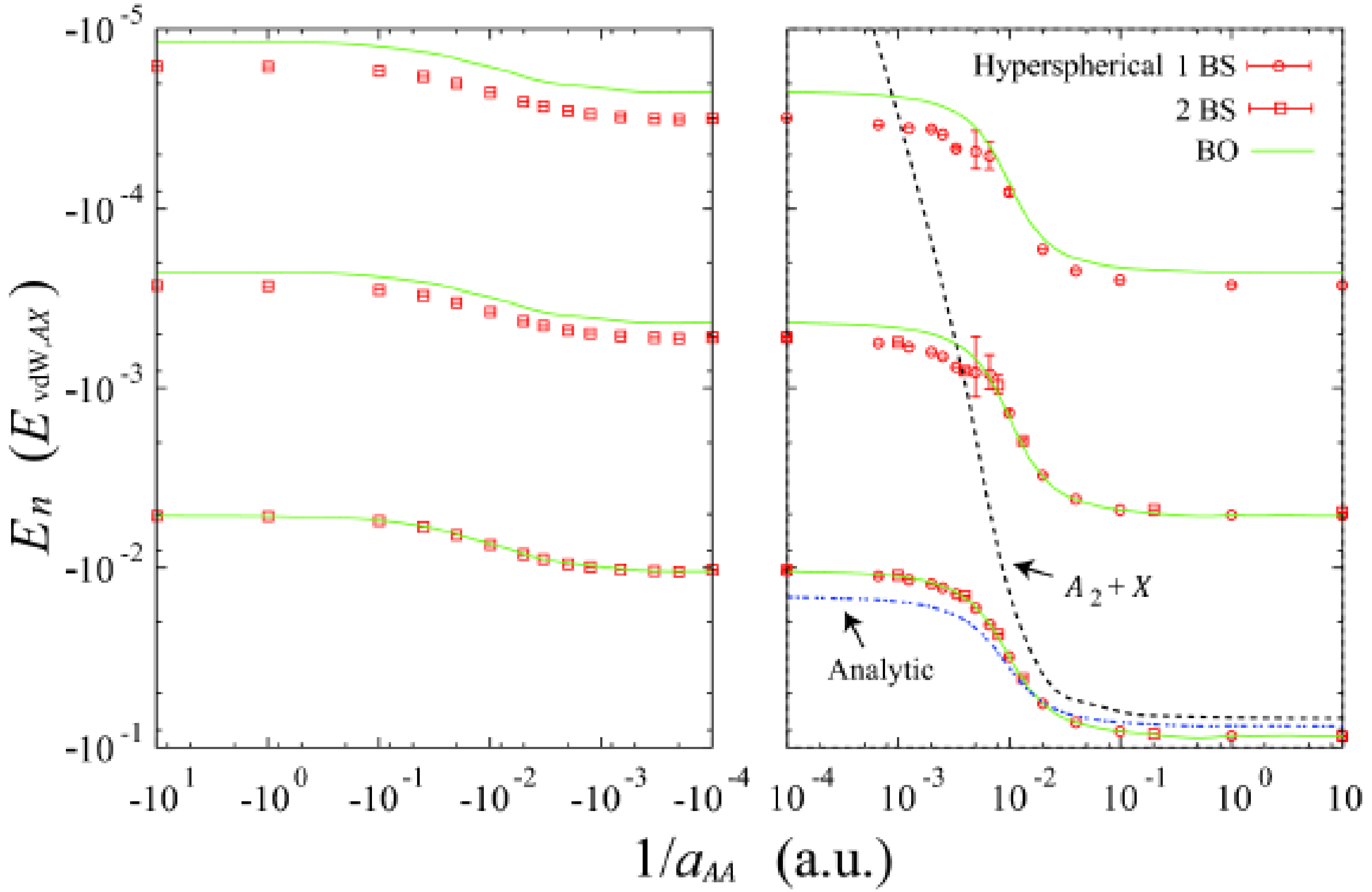
Three-body parameter can be expressed in three-body recombination observables a_-^* (first Efimov resonance) or a_0^* (first interference minimum).

For identical bosonic atoms, $a_-^* \approx -9.1 r_{\text{vdW}}$ [$r_{\text{vdW}} = (2\mu_2 C_6)^{1/4}/2$].

Universal three-body parameter for AAX?

Key finding: Our numerical evidence suggests that the 3-body parameter is UNIVERSAL for heteronuclear AAX systems also, but this universality depends on the AA scattering length, the mass ratio, the two van der Waals lengths, etc, and must be mapped out

Efimov-favored *AAX* systems — universal three-body parameter



Universal Efimov spectrum for YbYbLi [1]

Predictions of first Efimov resonance (negative a) and destructive interference Stueckelberg minimum (positive a)

	s_0	s_0^*	$a_{AA,bg}$ (a.u.)	a_0^* (a.u.)	a_-^* (a.u.)
$^{174}Yb_2{}^6Li$	2.246	2.382	104 [32, 33]	1.3×10^3	-8.4×10^2
$^{133}Cs_2{}^6Li$	1.983	2.155	2000 [34]	9.6×10^2	-1.4×10^3
$^{87}Rb_2{}^6Li$	1.633	1.860	100 [35]	3.8×10^2	-1.6×10^3
$^{41}K_2{}^6Li$	1.154	1.477	62 [36]	3.7×10^2	-2.4×10^3
$^{23}Na_2{}^6Li$	0.875	1.269	100 [37]	1.5×10^3	-1.3×10^4
$^{87}Rb_2{}^{40}K$	0.653	1.125	100	2.8×10^3	$< -3 \times 10^4$
$^{133}Cs_2{}^{87}Rb$	0.535	1.060	2000	2.3×10^3	$< -4 \times 10^4$
$^{41}K_2{}^{87}Rb$	0.246	0.961	62	$> 7 \times 10^3$	$< -1 \times 10^6$

TABLE I: The universal Efimov scaling constants s_0, s_0^* and the 3BPs $a_{AX} = a_0^*$ and $a_{AX} = a_-^*$ obtained by keeping a_{AA} fixed at its background value ($a_{AA,bg}$).

Predictions for Rb_2K and K_2Rb appear not to agree with the Barontini et al. experiment

G. Barontini, C. Weber, F. Rabatti, J. Catani, G. Thalhammer, M. Inguscio, and F. Minardi, Phys. Rev. Lett. **103**, 043201 (2009).

Something came out of Jia Wang's PhD thesis work – d-wave resonance features in 3-body recombination

Typically near threshold, we have come to expect that s-wave physics is always quite dominant, or more generally, the lowest partial wave, because centrifugal barriers increasingly suppress higher-L physics

But a resonance in a higher partial wave can overturn this expectation.

So consider one of the insights we've learned from Bo Gao's deep insights into scattering in a potential with a van der Waals tail. He pointed out that if you have a zero energy resonance state in one partial wave, $\mathbf{L_0}$, there will also be another one close to zero energy in the partial wave $\mathbf{L_0+4}$.

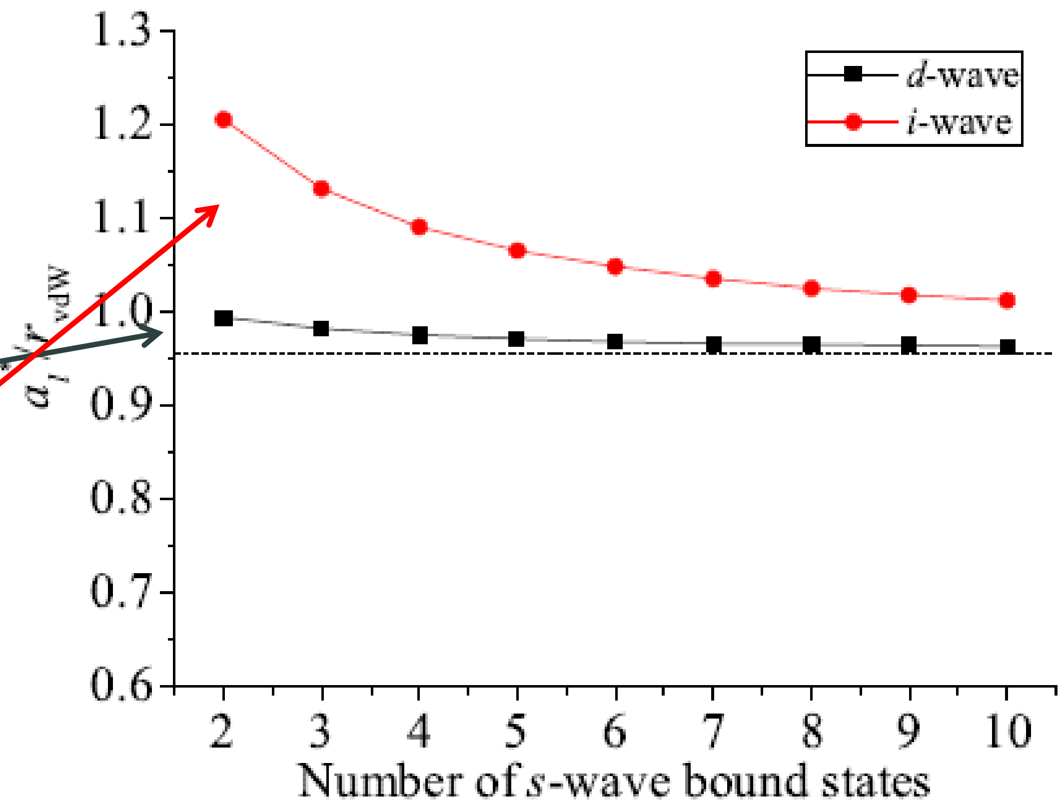
In other words, if you have a pole in the S-wave scattering length, there will usually be a near-threshold bound state or resonance in the G-wave. But midway between S-wave and G-wave is the D-wave, and so you might expect to be able to turn this around, and say that when the S-wave scattering length is SMALL, you could be close to a D-wave 2-body resonance near zero energy.

This observation led us to the following insights, in the paper
PHYSICAL REVIEW A **86**, 062511 (2012) **(arXiv:1209.4553)**

Universal three-body recombination via resonant d-wave interactions

Jia Wang,[1,2] J. P. D'Incao,[1] Yujun Wang,[1,*] and Chris H. Greene[1,3]

Value of the S-wave 2-body scattering length at which there is a zero energy **D-wave dimer (L=2) just bound** or **an I-wave (L=6) just bound**



Note: these values of a where one expects a D-wave or I-wave dimer to hit zero energy are for a single-channel broad resonance model only

FIG. 1: The value of the two-body s-wave scattering length a_l^* at the point where a d-wave ($l = 2$) dimer (black curve with square symbols) or an i-wave ($l = 6$) dimer just becomes bound (red curve with circular symbols), shown as functions of the number of two-body s-wave bound states.

Predicted 3-body recombination rate versus S-wave scattering length near a D-wave resonance for 2 different potentials
PRA 86, 062511 (2012)

2-body D-wave energy level

3-body state

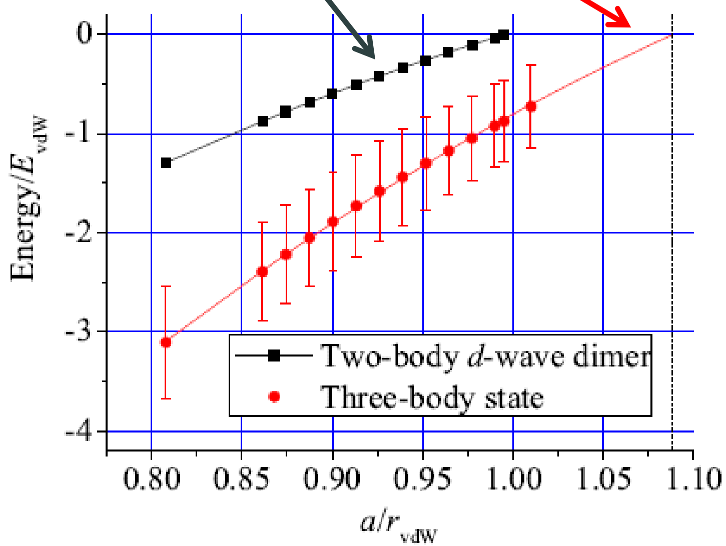


FIG. 5: Energy of the three-body bound state associated with a d-wave dimer as a function of scattering length a, both in van der Waals units.

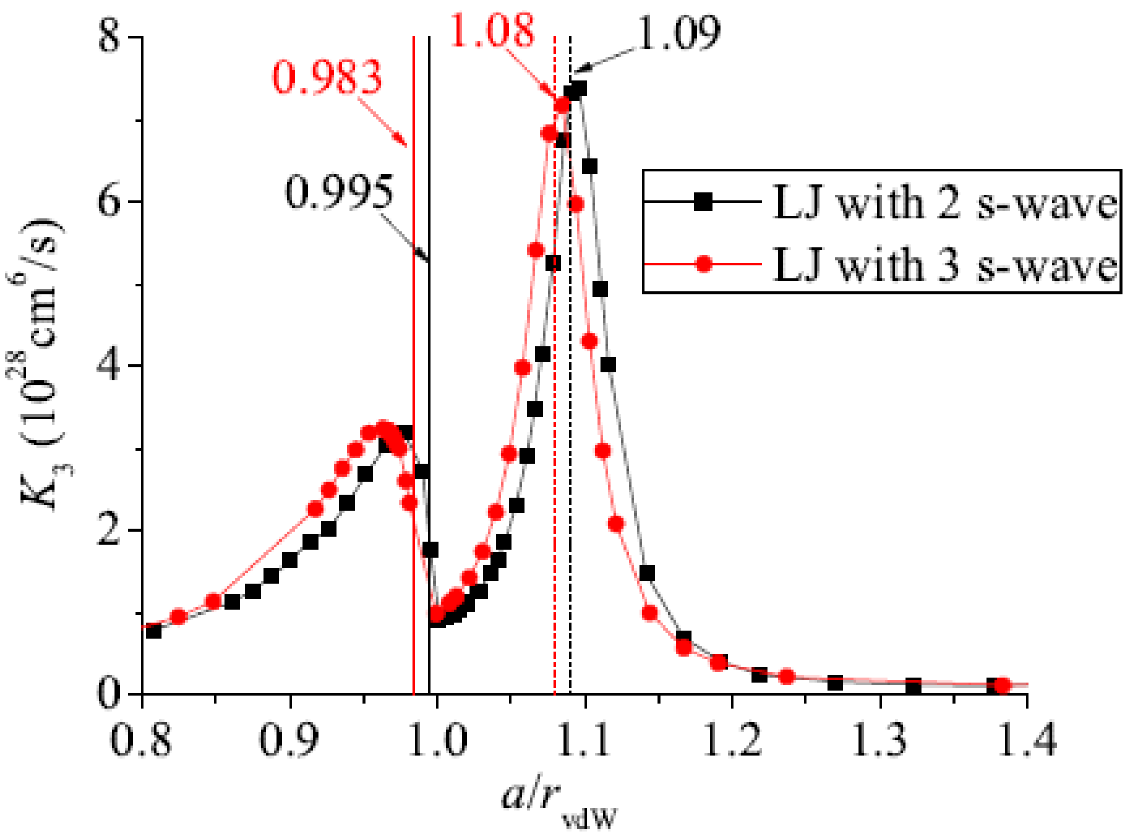


FIG. 6: The enhancements for the total three-body recombination rates at about $a = 0.995 r_{vdW}$ for Lennard-Jones potential with 2 and 3 s-wave bound states. K_3 is convert to cm^6/s by using van der Waals length $r_{vdW} = 101.0$ bohr and mass 132.905429 amu of ^{133}Cs

Note: 10% shift between the 2-body D-wave resonance and the 3-body loss peak!

How Efimov physics extends to more than 3 particles. This figure shows the schematic entrance channel **potential curve expected for N particles at negative 2-body scattering length**, From Mehta et al., 2009 PRL

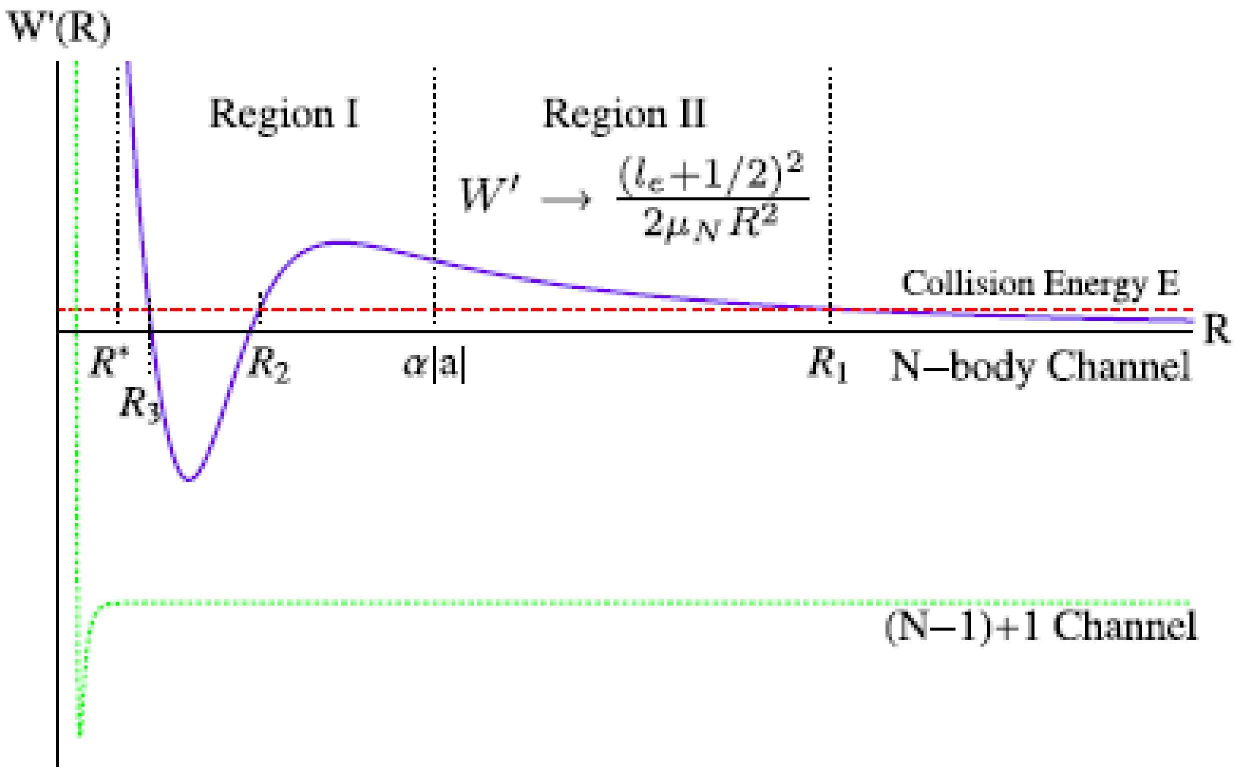


FIG. 1 (color online). A schematic representation of the N-boson hyperradial potential curves is shown. When a metastable N-boson state crosses the collision energy threshold at $E = 0$, N-body recombination into a lower channel with $N - 1$ atoms bound plus one free atom is resonantly enhanced.

But before we could actually calculate the rate of 4-body recombination in an ultracold gas, we had to develop some scattering theory:

PRL 103, 153201 (2009)

A general theoretical description of N-body recombination

N. P. Mehta,[1,2] Seth T. Rittenhouse,[1] J. P. D'Incao,[1] J. von Stecher,[1] and Chris H. Greene[1]

[1] *Department of Physics and JILA, University of Colorado, Boulder, CO 80309*

[2] *Grinnell College, Department of Physics, Grinnell, IA 50112**

(Dated: March 24, 2009)

We present a formula for the cross section and event rate constant describing recombination of N particles in terms of general S-matrix elements. Our result immediately yields the generalized Wigner threshold scaling for the recombination of N bosons. We find that four-boson recombination is resonantly enhanced by the presence of metastable states in the entrance channel. Hence, recombination into a trimer-atom channel could be an effective mechanism for the formation of Efimov trimers.

And here it is, THE FORMULA for N-body recombination, i.e. for the process:

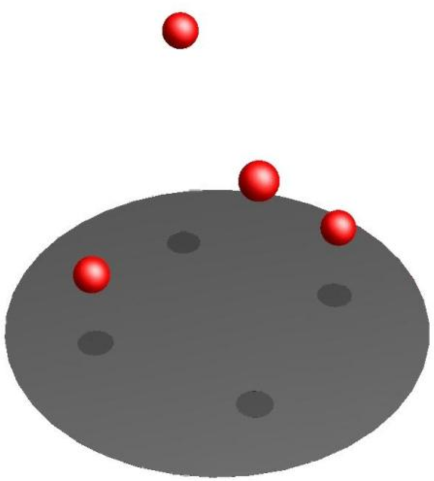
A+A+A+….+..A → A_{N-1}+A or A_{N-2}+A+A +…etc.

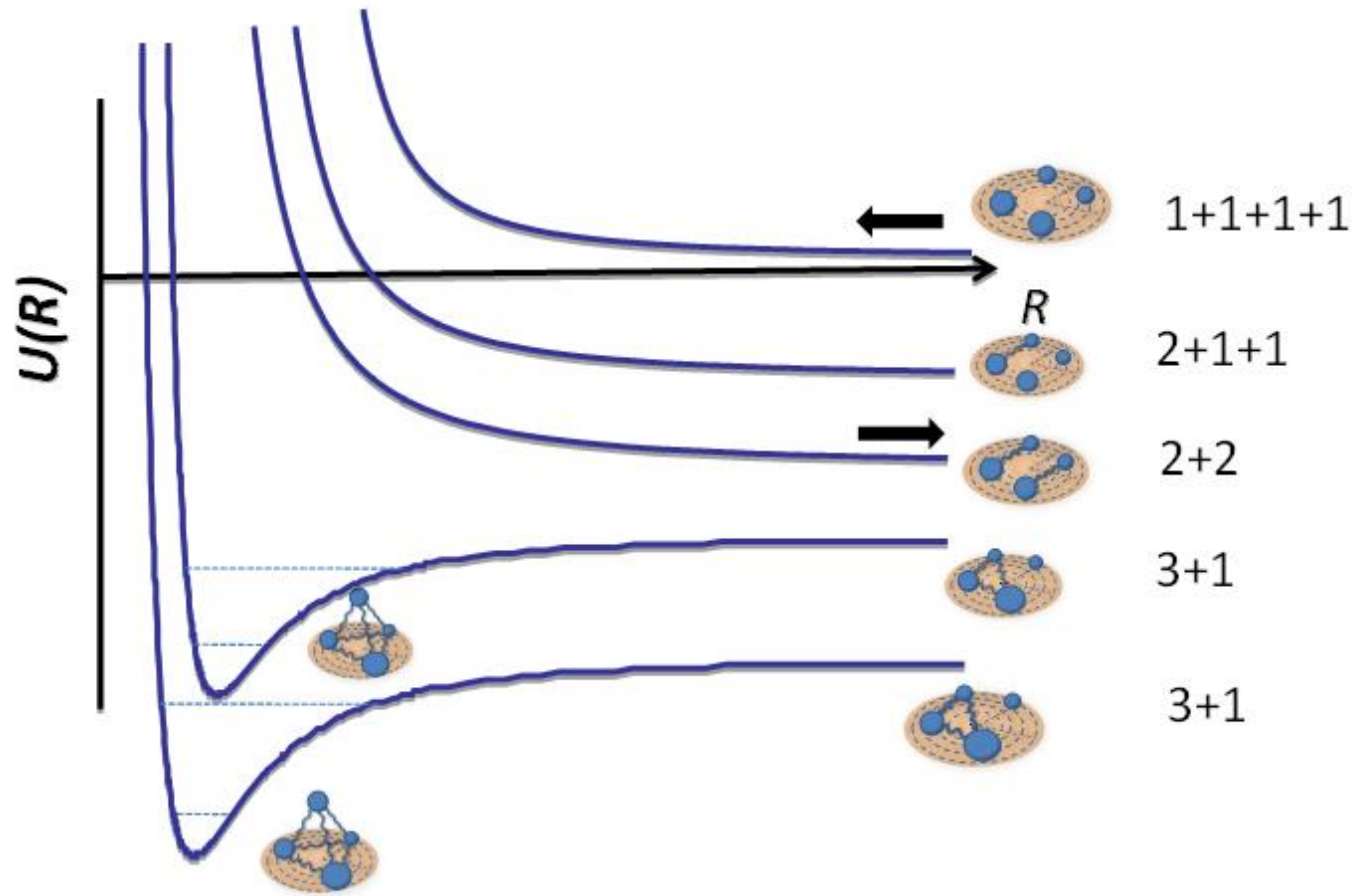
$$K_N^{0^+} = \frac{2\pi\hbar}{\mu_N} N! \left(\frac{2\pi}{k}\right)^{(3N-5)} \frac{\Gamma\left((3N-3)/2\right)}{2\pi^{(3N-3)/2}} \left|S_{f0}^{0^+}\right|^2$$

Hyperspherical Picture of 4-body recombination

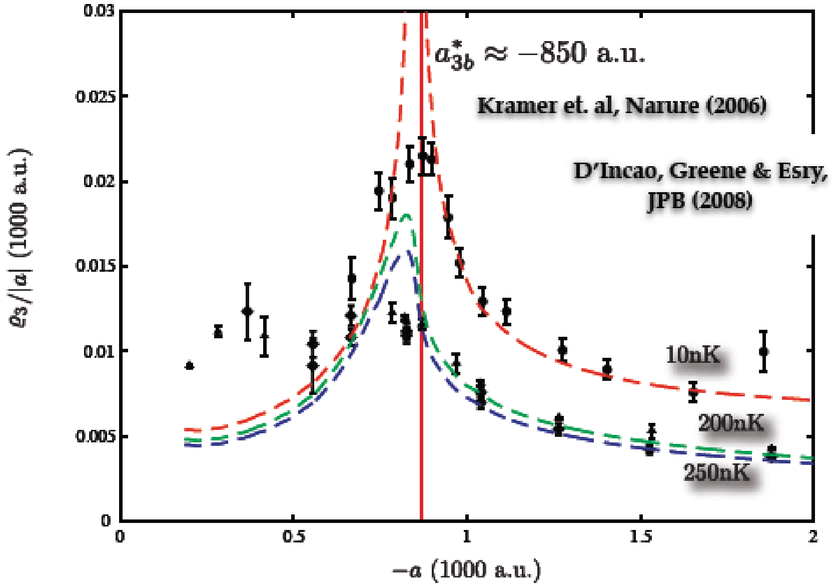
… think Born Oppenheimer

Fragmentation thresholds



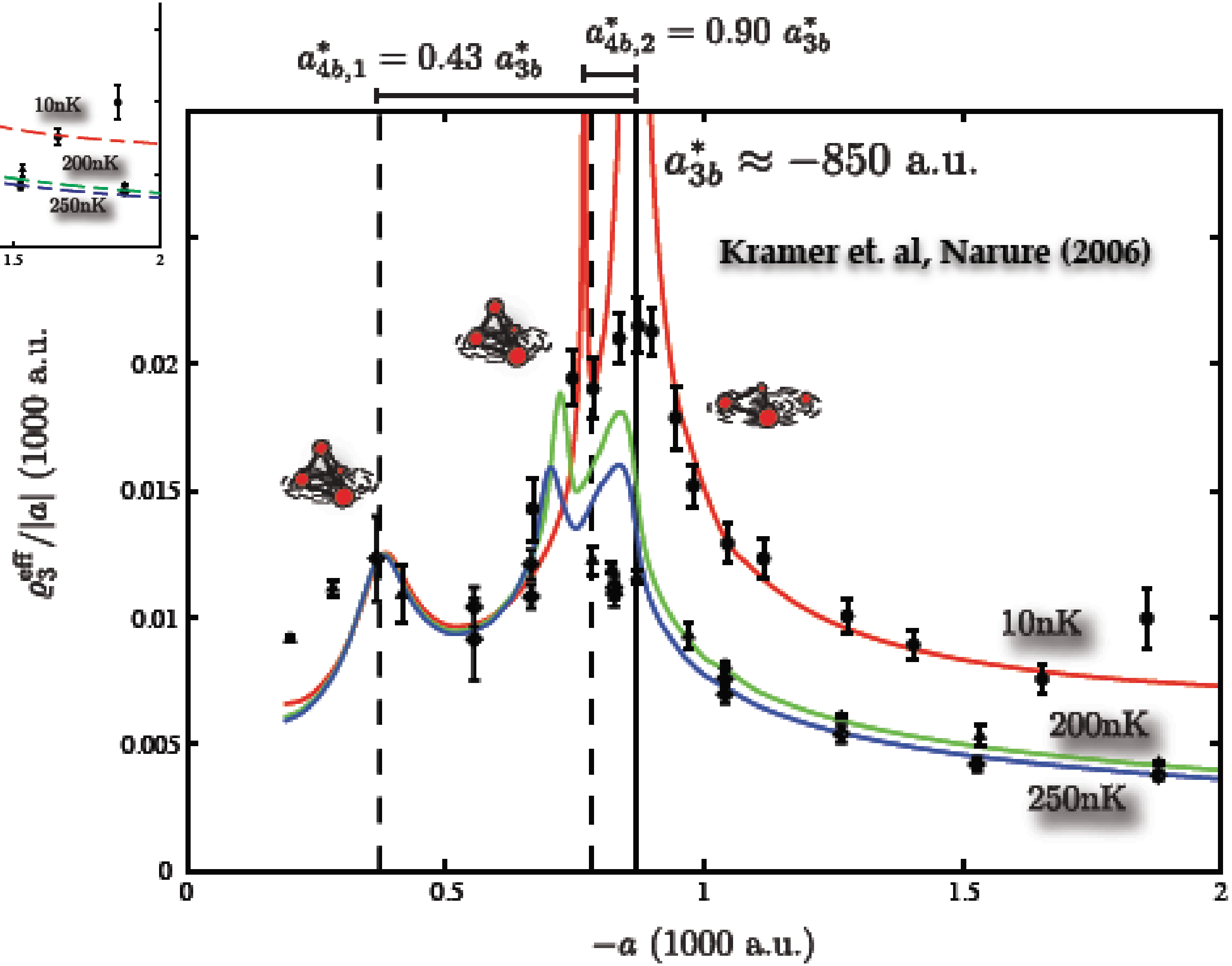


Considering only three-body recombination ...



Considering three- and four-body recombination ...

$$K_3^{\text{eff}}(a,t) = K_3(a) + n(t)K_4(a)$$



How to tackle 5-body recombination for 5 free bosonic atoms with pairwise additive forces?

i.e. the reaction A+A+A+A+A→ A_3+A_2 or A_4+A or…

Start with the time-independent Schroedinger equation:

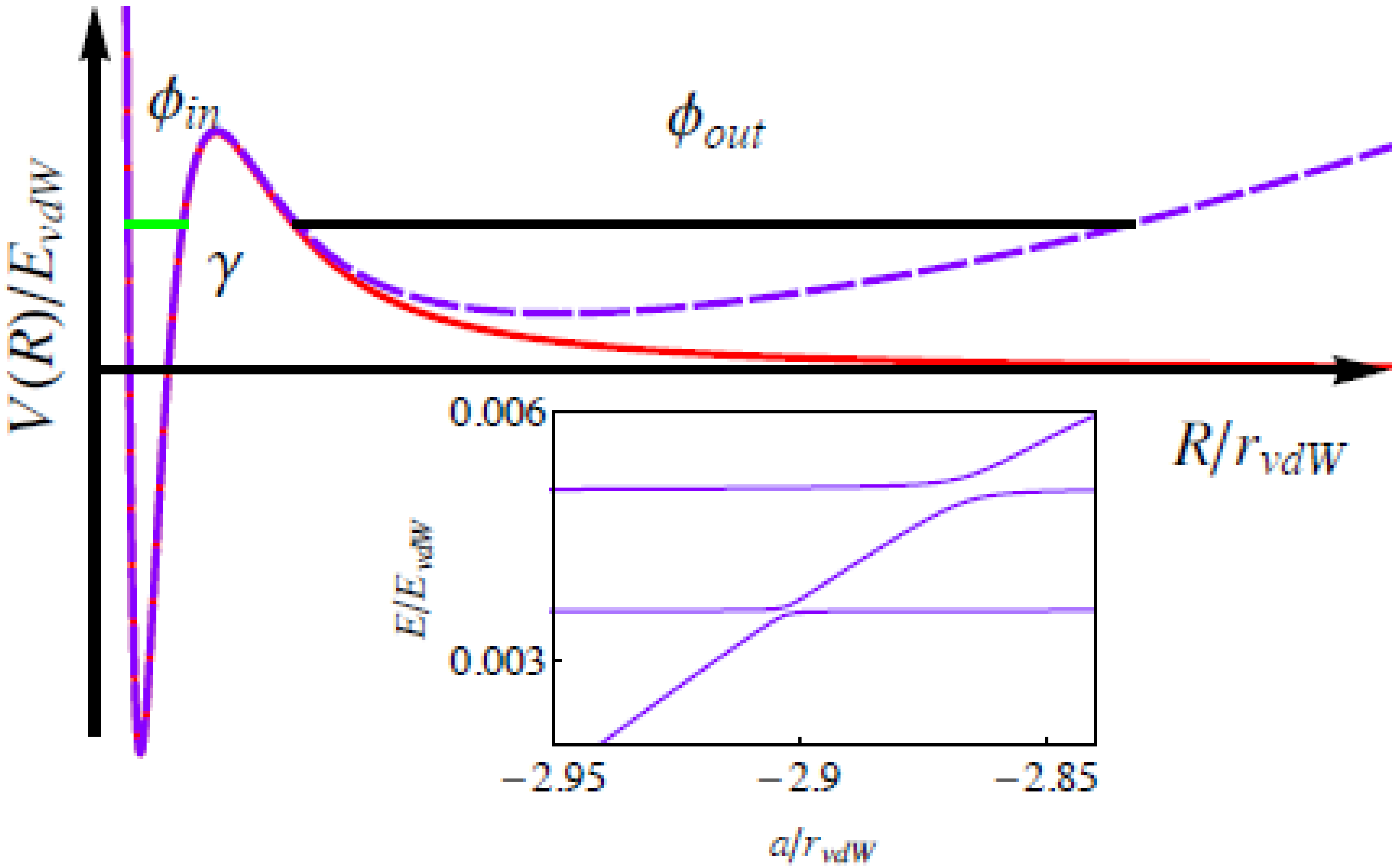
$$H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{p_3^2}{2m_3} + \frac{p_4^2}{2m_4} + \frac{p_5^2}{2m_5}$$

$$+ V(r_{12}) + V(r_{13}) + V(r_{14}) + V(r_{15}) + V(r_{23}) + V(r_{24}) + V(r_{25}) + V(r_{34}) + V(r_{35}) + V(r_{45})$$

After eliminating the center-of-mass degree of freedom, we're left with a 12-dimensional PDE to solve, which can be reduced to **a mere 9 dimensions** for J=0 states after going to the body frame.

Resonant five-body recombination in an ultracold gas of bosonic atoms

Mulliken-style potential energy versus hyperradius R for 5 free Cs atoms (solid red) or harmonically trapped (dashed)



arXiv:1205.1921

Alessandro Zenesini[1]‡, Bo Huang[1], Martin Berninger[1], Stefan Besler[1], Hanns-Christoph Nägerl[1], Francesca Ferlaino[1], Rudolf Grimm[1,2], Chris H Greene[3]§, Javier von Stecher[3,4]

Our “recent” preprint with the Innsbruck group: arXiv:1201.4310, defeated “in combat” with the editors and referees of PRL

Resonant Five-Body Recombination in an Ultracold Gas

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Institut für Experimentalphysik and Zentrum für Quantenphysik, Universität Innsbruck, and
Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, 6020 Innsbruck, Austria

Chris H. Greene and J. von Stecher*
Boulder, CO 80309, USA

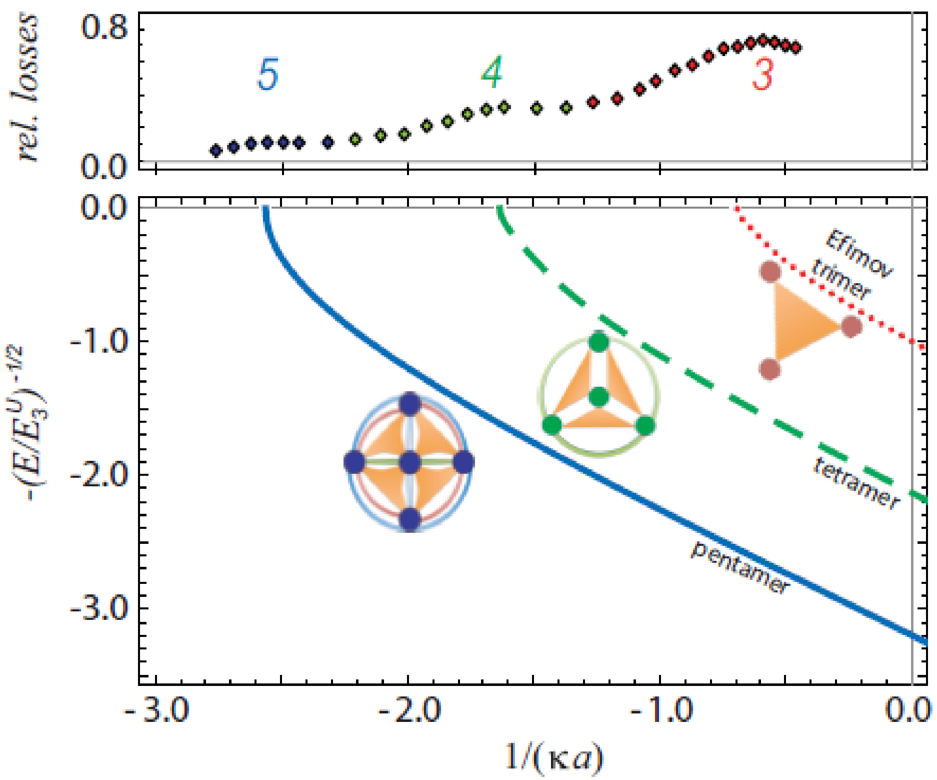


FIG. 1. (color online) N-body scenario in the region of negative two-body scattering length a. The lower panel shows the N-body binding energies as a function of the inverse scattering length. $E_3^U = (\hbar\kappa)^2/m$ is the trimer binding energy for resonant interaction. The dotted,

J. Phys. B: At. Mol. Opt. Phys. 43 (2010) 101002 (5pp) doi:10.1088/0953-4075/43/10/101002

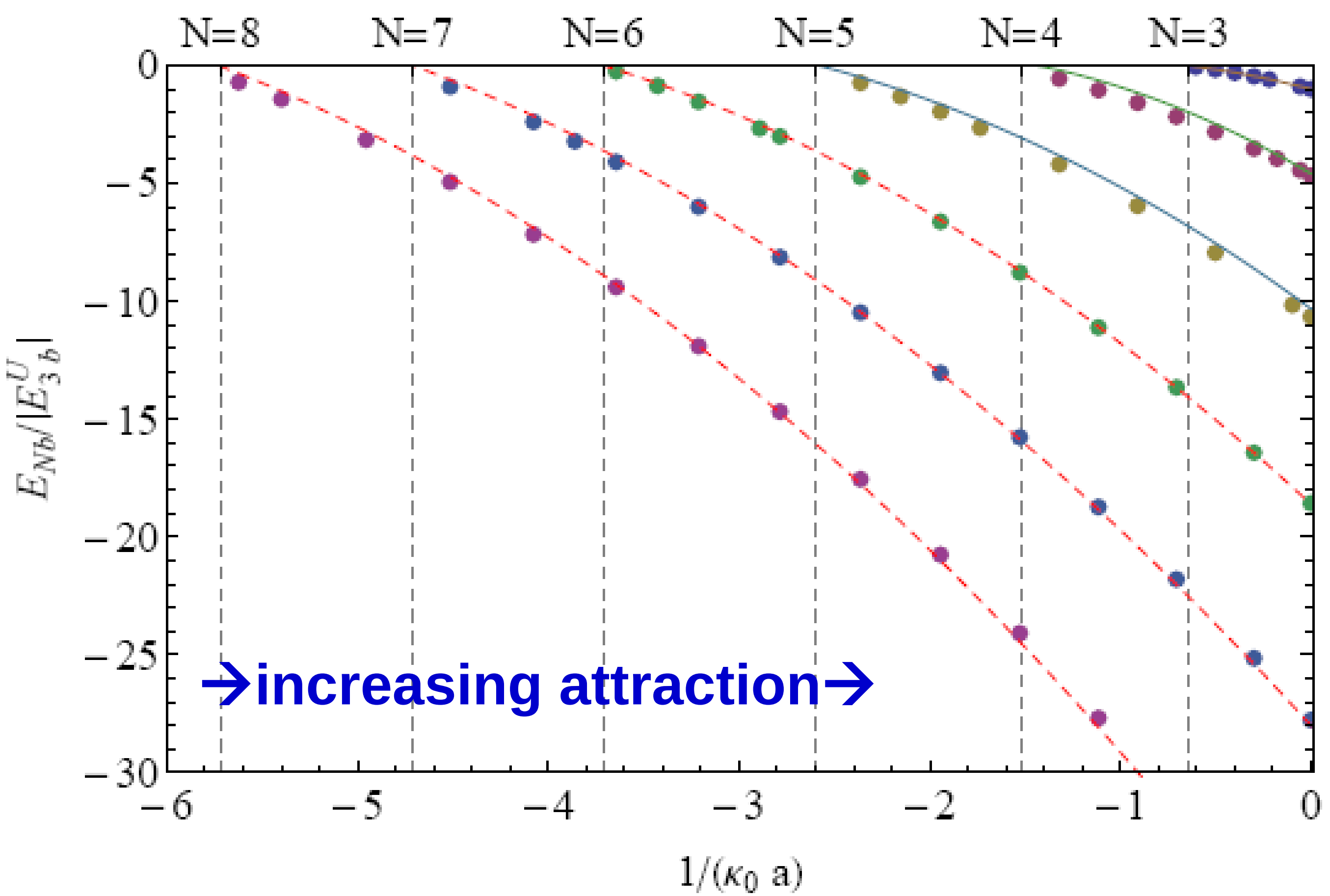
FAST TRACK COMMUNICATION

Weakly bound cluster states of Efimov character

MORE THAN 4 BOSONS: von Stecher's J. Phys. B article in 2010: combined study using correlated Gaussians, and diffusion Monte Carlo

Javier von Stecher

Clusters predicted up to N=13.



von Stecher - arXiv:0909.4056
and 2010 JPB

TABLE I: Energies at unitarity and scattering-length ratios that characterize weakly bound cluster states. The scattering length ratios can be transformed to an absolute scale using $1/(\kappa_0 a_{3b}) \approx 0.64$.

N	E_N^U/E_3^U	$a^*_{Nb}/a^*_{(N-1)b}$	N	E_N^U/E_3^U
4	4.66(4)	0.42(1)	9	49.9(6)
5	10.64(4)	0.60(1)	10	60.2(6)
6	18.59(5)	0.71(1)	11	70.1(7)
7	27.9(2)	0.78(1)	12	79.9(3)
8	38.9(3)	0.82(1)	13	88.0(7)

0.46(1) 0.65(2) 0.73(1) are latest revised/improved values from von Stecher,

PRL **107**, 200402 (2011)

Five- and Six-Body Resonances Tied to an Efimov Trimer

Javier von Stecher

Remarkable prediction, that **all** larger cluster resonances are determined once the 3-body parameter is known!

count for the experimental observations. Remarkably, the resonance position $a_{5,-} = 0.64(2)\, a_{4,-}$ is in agreement with the theoretical predictions $0.65(1)\, a_{4,-}$ [37, 38]. However, quantitatively, the experimental values for L_5 are about 15 times larger than the calculated ones. To account for this, we introduce a corresponding scaling factor. We find that this deviation may be explained by a small error in the WKB phase γ of about 10%, which remains in a realistic uncertainty range of our theory.

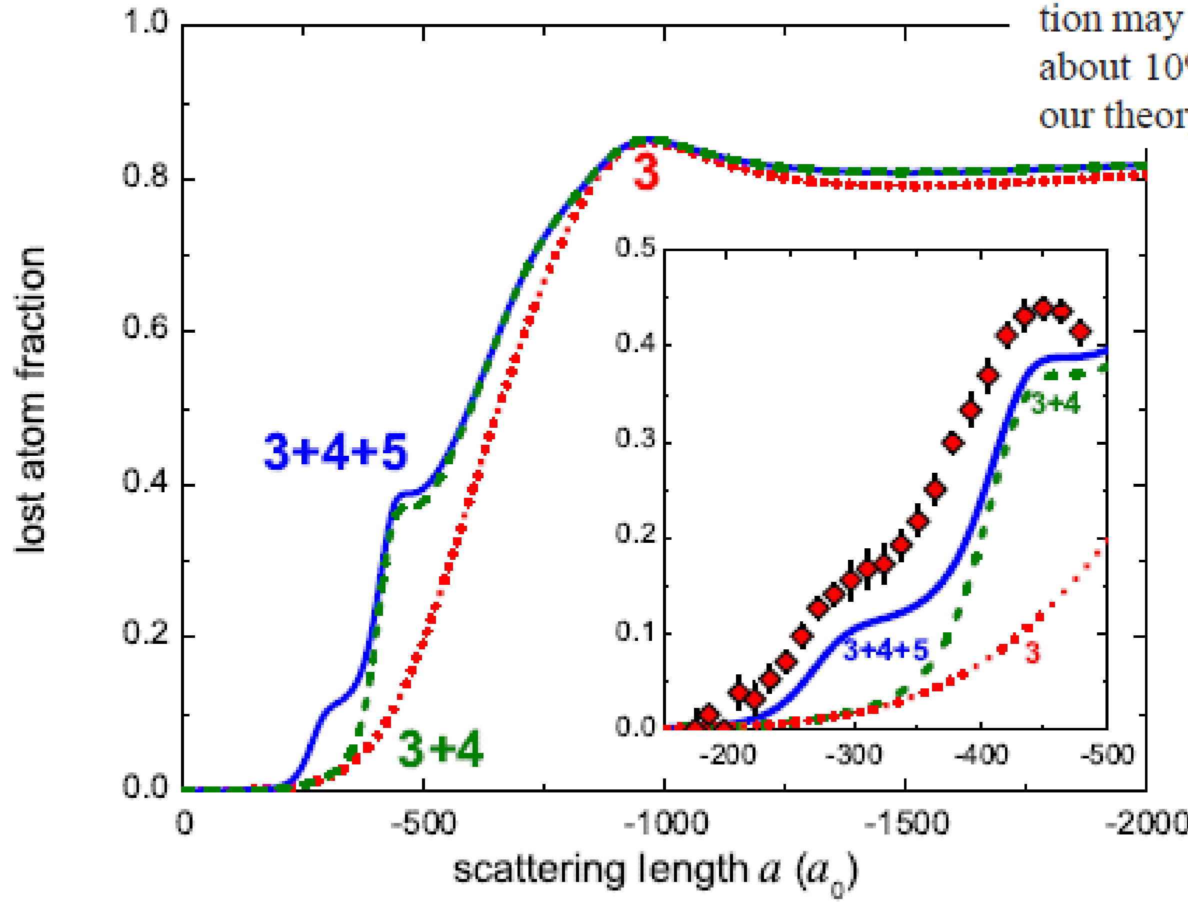
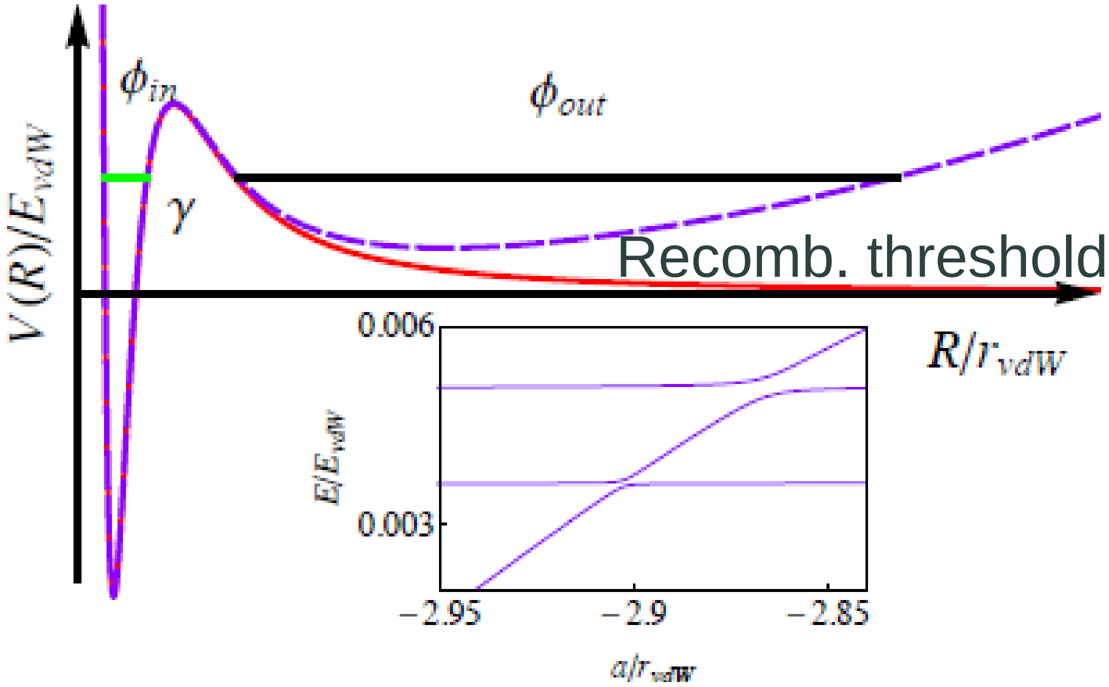


FIG. 4. (color online) Calculated and measured fraction of loss atoms from an atomic sample of initially 5×10^4 atoms at a temperature of 80 nK after a hold time of 100 ms. The red dotted line corresponds to the losses predicted for three-body recombination only, while the dashed green line and the blue solid line include also contributions from four- and five body recombination, as quantified in this work. A

Schematic qualitative hyperspherical potential curve for 5 bosons at negative scattering length, "Mulliken style"



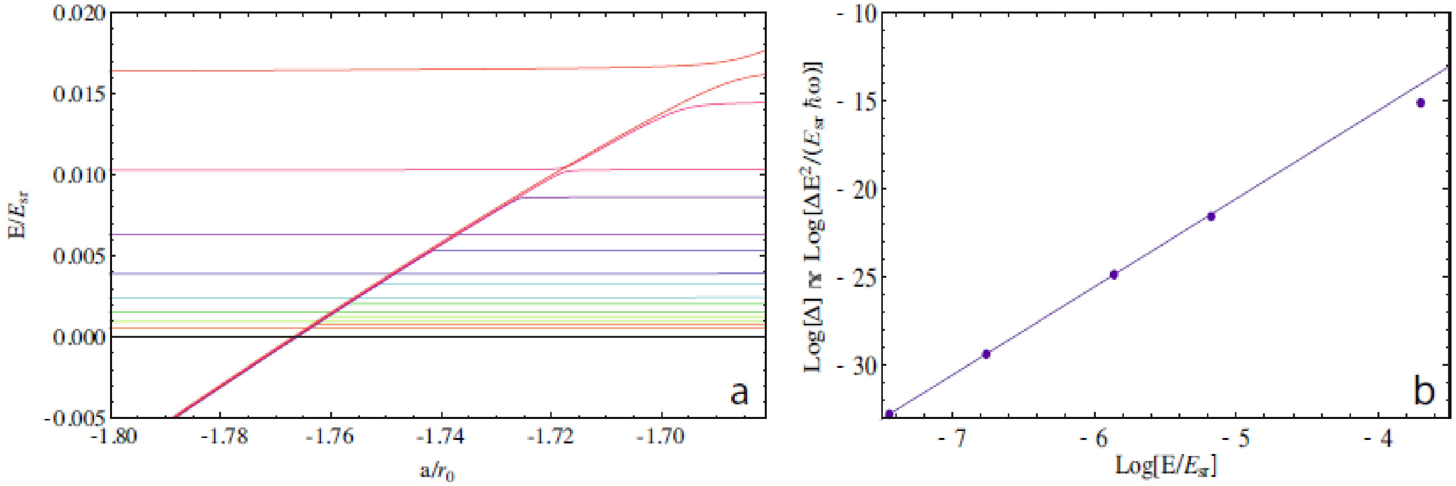
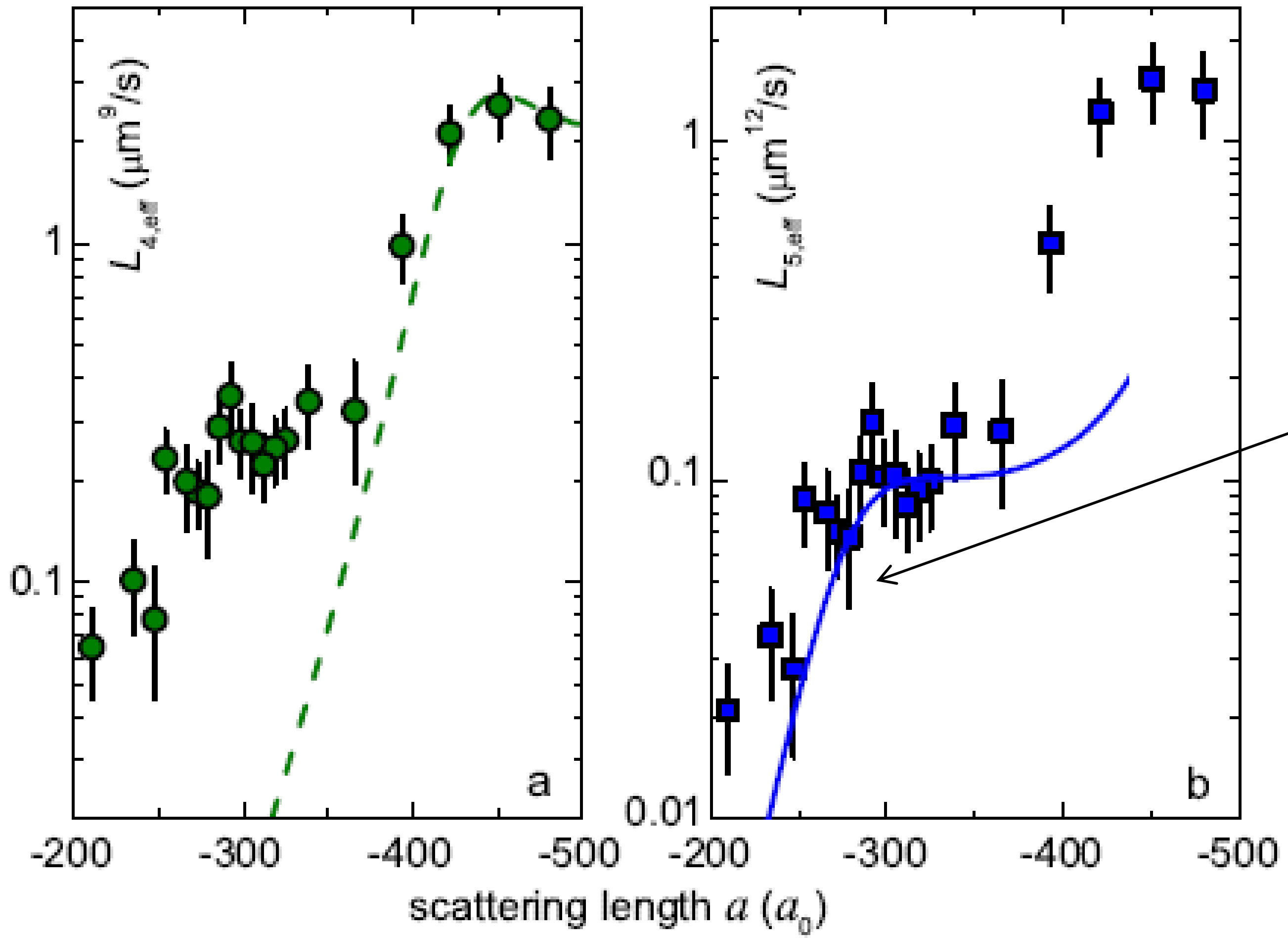


FIG. S1: (color online) (a) The lowest two eigenenergies of a trapped five body system are shown as functions of the scattering length for different trapping frequencies. Different colors represent different trapping frequencies. The combination of these states essentially describes the energy of the five-body state in the inner region of the potential $E_{mol}(a)$ (the diagonal curve). Here $E_{sr} = \hbar^2/(mr_0^2)$ and r_0 is the characteristic range of the two-body model potential that can be tuned to obtain the five-body resonance (i.e. $r_0 \sim 1.7 r_{vdw}$ where r_{vdw} is the van der Waals length). (b) The near-threshold behavior of Δ. The fitting of the lowest energy points leads implies that $\Delta \propto AE^b$. The lowest three points lead to $b \approx 5.004$ as expected from the known threshold behavior [4].

4-body recomb
only

5-body recomb
only



Position of the predicted 4-body resonance and now the 5-body resonance is in agreement with experiment! Kewl!

arXiv:1205.1921, Zenesini et al.

FIG. 3. (color online) Effective four- (a) and five-body recombination rates (b). The green dashed curve and the blue solid line follow the theoretical model for L_4 and L_5, respectively, with additional scaling factor for L_5; see text. The error bars include the statistical uncertainties from the fitting routine, the temperature and the trap frequencies.

Summary

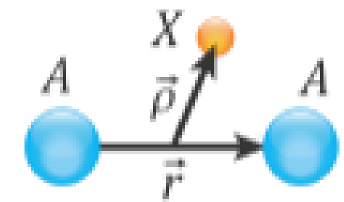
Universal states of 3 polar molecules predicted, including an Efimov effect for 3 identical bosonic dipoles

Three-body parameter universality is now understood, though not valid for all 3-body systems, e.g. probably not universal for 3 nucleons

Cluster resonances with N>3 atoms have been predicted and seen experimentally now up to N=5 atoms, i.e. a regime has been identified where the dominant loss process in the gas is from 5-body recombination

Efimov-favored AAX systems ($m_A \gg m_X$)

Hypersperical vs Born-Oppenheimer (BO)



$$\left[-\frac{1}{m_A}\nabla_r^2 - \frac{2m_A + m_X}{2m_A m_X}\nabla_\rho^2 + V_{AA}(r) + V_{AX}\left(\left|\boldsymbol{\rho} + \frac{\boldsymbol{r}}{2}\right|\right) + V_{AX}\left(\left|\boldsymbol{\rho} - \frac{\boldsymbol{r}}{2}\right|\right)\right]\Psi = E\Psi$$

In adiabatic hyperspherical representation $\Psi = \sum_\nu F_{\nu,E}(R)\Phi_\nu(R;\Omega)$:

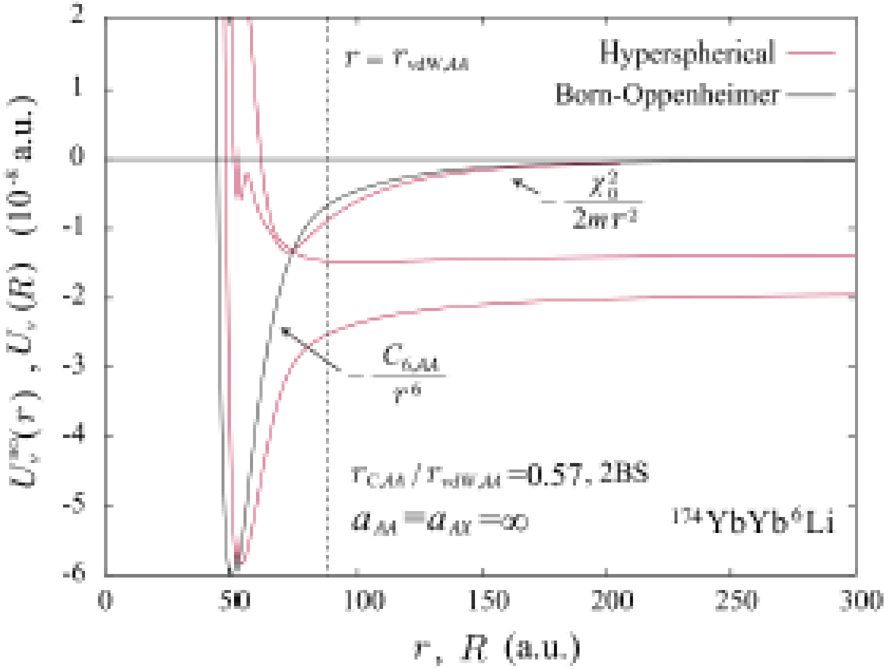
- Coupled hyperradial equations for solving $F_{\nu,E}$.
- Adiabatic hyperspherical potentials $U_\nu(R)$ characterize the energy landscape for fixed hyperradius R ($\mu R^2 = \frac{1}{2}m_A r^2 + \frac{2m_A m_x}{2m_A + m_X}\rho^2$).

In BO approximation $\Psi = F^{\mathrm{BO}}_{\nu,E}(\boldsymbol{r})\Phi^{\mathrm{BO}}_\nu(\boldsymbol{r};\boldsymbol{\rho})$:

- Single-channel potential $U^{\mathrm{BO}}_\nu(r)$ characterizes effective interaction between A atoms.

Efimov-favored AAX systems — Efimov potentials and solutions

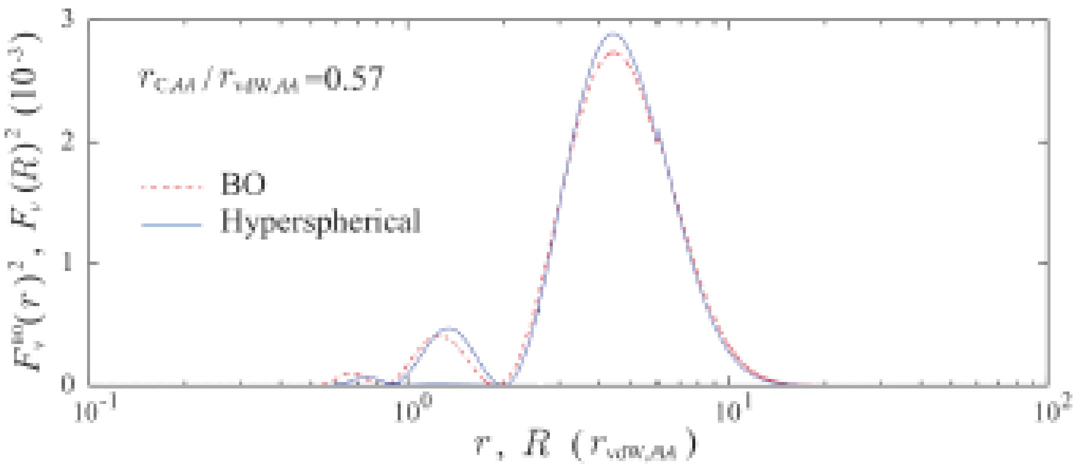
Hyperspherical potential reduces to BO potential when $m_A \gg m_X$:

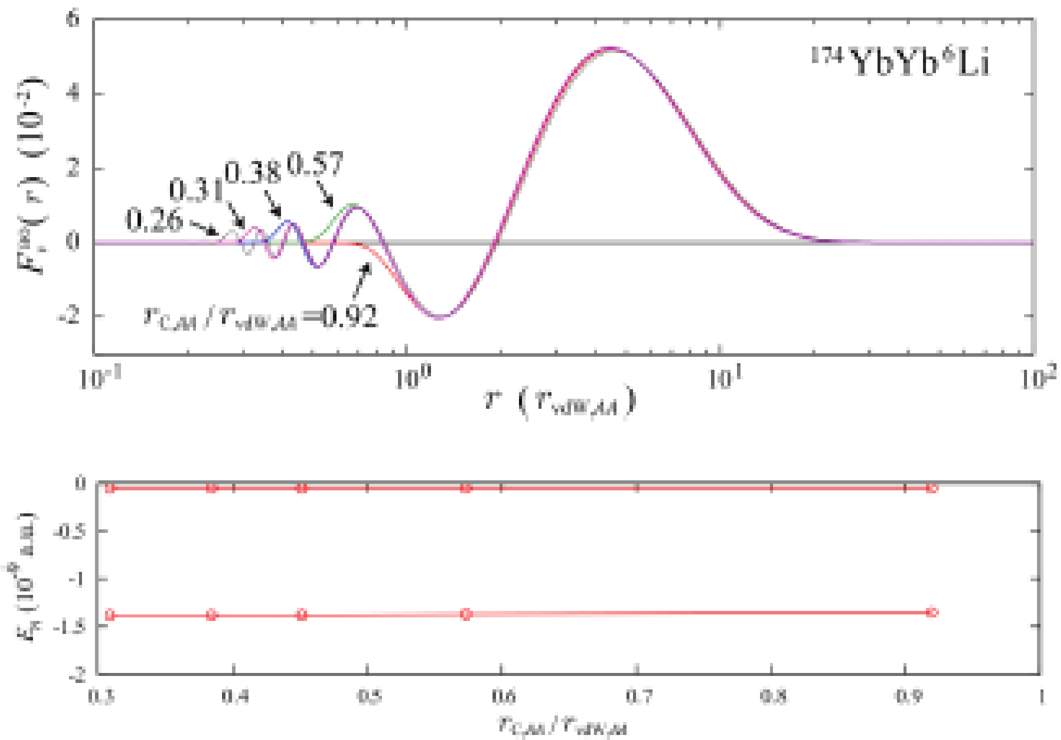


- BO potential ⇔ diabatic hyperspherical potential.
- Long-range Efimov behavior $U_\nu^{BO}(r) \simeq -\chi_0^2/2m_X r^2$ ($\chi_0 \approx 0.57$).
- Short-range van der Waals behavior $U_\nu^{BO}(r) \simeq V_{AA}(r) = -C_{6,AA}/r^6$.

Efimov state in hyperspherical and BO representations:

- Good agreement between hyperspherical and BO solutions.
- Efimov states can be studied in the BO approximation.





- Nodal positions in the van der Waals region are determined by a_{AA} and $r_{\text{vdW},AA}$.
- Universal Efimov state energies independent of r_c

r_+ determines the ground Efimov state energy ($s_0^2 \approx \chi_0^2 m_A/2m_X - 1/4$):
$E_{0,\text{analytic}} = -\frac{4}{Mr_+^2}\exp\left(-\frac{2}{s_0}\{\text{Arg}[\Gamma(1-is_0)]-\pi\}\right)$, r_+ can be found by

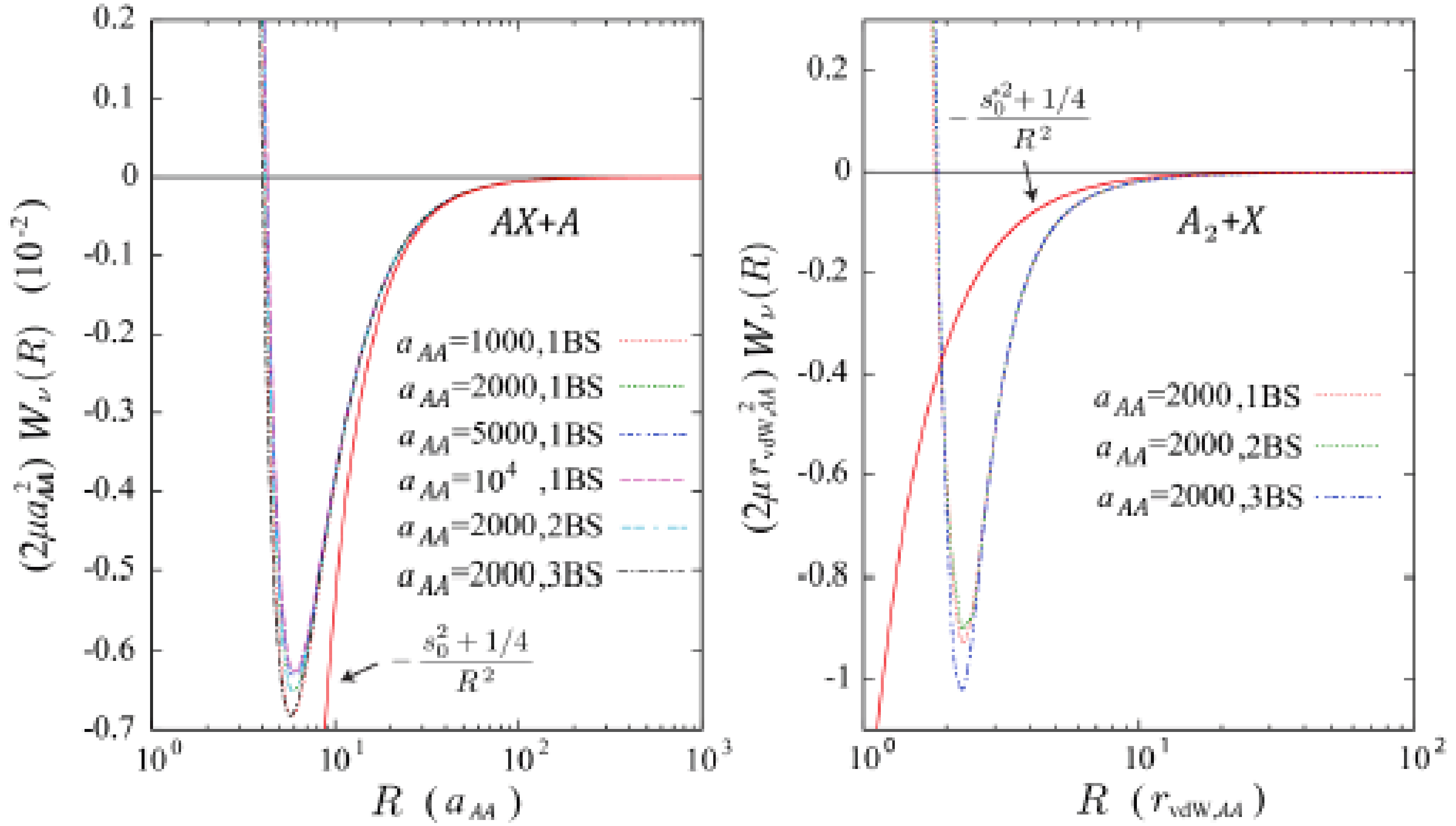
$$J_{-\frac{i\alpha s_0}{2}}\left(2\frac{r_{\text{vdW},AA}^2}{r_+^2}\right)N_{-\frac{i\alpha s_0}{2}}\left(2\frac{r_{\text{vdW},AA}^2}{r_-^2}\right)=N_{-\frac{i\alpha s_0}{2}}\left(2\frac{r_{\text{vdW},AA}^2}{r_+^2}\right)J_{-\frac{i\alpha s_0}{2}}\left(2\frac{r_{\text{vdW},AA}^2}{r_-^2}\right),$$

$$N_{\frac{1}{4}}\left(2\frac{r_{\text{vdW},AA}^2}{r_-^2}\right)=\left[1-\sqrt{2}\frac{a_{AA}}{r_{\text{vdW},AA}}\frac{\Gamma(5/4)}{\Gamma(3/4)}\right]J_{\frac{1}{4}}\left(2\frac{r_{\text{vdW},AA}^2}{r_-^2}\right).\ (\alpha\approx 2)$$

An approximate analytical model for Efimov ground state.

Efimov-unfavored AAX systems — universal potentials

When $|a_{AX}| \gg a_{AA} \gg r_0$, the effective adiabatic hyperspherical potentials show different universal Efimov scaling for two-resonant-interaction channel $(AX + A)$ and three-resonant-interaction channel $(A_2 + X)$:



Effective adiabatic hyperspherical potentials for CsCsRb ($a_{CsRb} = \infty$) [1]