

Polar molecules in one-dimensional optical lattices

Luis Santos Institute of Theoretical Physics and Center of Excellence QUEST Leibniz Universität Hannover



Santa Barbara, March 14, 2013

Ultra-cold gases in optical lattices: lattice models

Bose-Hubbard Hamiltonian

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[Fisher *et al.*, PRB **40**, 546 (1989) ; Jaksch *et al.*, PRL **81**, 3108 (1998)]

$$H = -t \mathop{a}_{i} \stackrel{e}{\otimes} b_{i+1}^{\dagger} + H.c. \stackrel{i}{\otimes} + \frac{U_0}{2} \mathop{a}_{i} n_i (n_i - 1)$$

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Mott insulator (gapped incompressible insulator)

Superfluid

[M. Greiner et al., Nature 415, 39 (2002)]



LASER

Ultra-cold gases in optical lattices: lattice models

Bose-Hubbard Hamiltonian

[Fisher *et al.*, PRB **40**, 546 (1989) ; Jaksch *et al.*, PRL **81**, 3108 (1998)]

$$H = -t \mathop{a}_{i} \mathop{\oplus}_{i} b_{i+1}^{\dagger} + H.c. \underbrace{+}_{2} - \underbrace{+}_{i} n_{i} (n_{i} - 1)$$

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Resemble spin models, e.g. hard-core bosons (n=0,1)

$$\left(H_{XX} \gg -2t \mathop{\text{a}}_{\langle ij \rangle} \left(S_i^x S_j^x + S_i^y S_j^y\right)\right)$$

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Dipolar lattice gases: intersite effects

For a dipolar gas inter-site interactions become important

Extended Bose-Hubbard model $H = -t \sum_{i} \left[b_i^+ b_{i+1} + H.c. \right] + \frac{U_0}{2} \sum_{i} n_i \left(n_i - 1 \right) + \frac{U_1 \sum_{i} n_i n_{i+1}}{2}$

Dipolar interactions have been shown already to play an important role in dipolar Chromium BECs in optical lattices (Stuttgart) Inter-site destabilization [Müller et al., PRA 84, 053601 (2011)] Time-of-flight-induced collapse of in-lattice stable BECs [Billy et al., PRA 86, 051603(R) (2012)]

Also in recent works on spin dynamics of Chromium in lattices (Talk of B. Laburthe)



Supersolid

Interlayer SF

Dipolar lattice gases: intersite effects

For a dipolar gas inter-site interactions become important

Density wave

Extended Bose-Hubbard model $H = -t\sum_{i} \left[b_i^+ b_{i+1} + H.c.\right] + \frac{U_0}{2} \sum_{i} n_i (n_i - 1) + U_1 \sum_{i} n_i n_{i+1}$

Supersolid, density-waves, <u>s</u>elf-assembled crystals, metastable states, interlayer superfluids and more...

[Baranov et al, Chemical Reviews 112, 5012 (2012)]



Dipolar lattice gases: intersite effects

For a dipolar gas inter-site interactions become important

1D polar gases

The intersite interactions lead to a very rich physics for 1D chains and ladders

- Devil's staircase [Burnell et al., PRB 80, 174519 (2009)]
 - Haldane insulator [Dalla Torre et al., PRL 97, 260401 (2006)]
- Disorder [Deng et al., arXiv (2012)]
 - Simulation of spin-orbital models [Sun et al., PRB 86, 155159 (2012)]
 - More... See. [Chemical Reviews 112, 5012 (2012]



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Disorder in optical lattices

Speckle

[Billy et al., Nature **453**, 891 (2008)]





Bichromatic lattices

[Roati et al., Nature **453**, 895 (2008)]





Quasi-periodic



This talk

1D polar gases at unit filling

1D polar gases with uniform bound disorder

1D polar gases in a quasi-periodic lattice



1D polar bosons in optical lattices

Extended Bose-Hubbard model $H = -t\sum_{i} \left[b_i^+ b_{i+1} + H.c.\right] + \frac{U_0}{2} \sum_{i} n_i (n_i - 1) + U_1 \sum_{i} n_i n_{i+1}$

U CAN

Average filling $\langle n \rangle = 1$

/ m=1

Holstein-Primakoff transformation maps occupation into spin-1 $S_i^z = 1 - n_i \longrightarrow \text{m} = 0$ $S_i^+ = \sqrt{2 - n_i} b_i \qquad \text{m} = -1$ $S_i^- = b_i^+ \sqrt{2 - n_i}$



1D polar bosons in optical lattices

Extended Bose-Hubbard model $H = -t\sum_{i} \left[b_i^+ b_{i+1} + H.c.\right] + \frac{U_0}{2} \sum_{i} n_i (n_i - 1) + U_1 \sum_{i} n_i n_{i+1}$

The system resembles to a $H = J \sum_{i} \left[S_{i}^{x} \cdot S_{i+1}^{x} + S_{i}^{y} \cdot S_{i+1}^{y} + \Delta S_{i}^{z} \cdot S_{i+1}^{z} + D(S_{i}^{z})^{2} \right]$ large extent an AF spin-1 chain with uniaxial singleion anisotropy J = 2t $D = U_{0}/4t$ $\Delta = U_{1}/2t$

(imperfect mapping: due to n>2 and extra terms ~ $S_i^- \cdot F(S_i^z + S_{i+1}^z) \cdot S_{i+1}^+$)

AF spin-1 chains

$$H = J \sum_{i} \left[S_{i}^{x} \cdot S_{i+1}^{x} + S_{i}^{y} \cdot S_{i+1}^{y} + \Delta S_{i}^{z} \cdot S_{i+1}^{z} + D(S_{i}^{z})^{2} \right]$$

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1D polar gases in optical lattices: Haldane-insulator phase

 $H = -t \sum_{i} \left[b_i^{+} b_{i+1}^{+} + H.c. \right] + \frac{U_0}{2} \sum_{i} n_i (n_i - 1) + U_1 \sum_{i} n_i n_{i+1}$

Polar bosons in one-dimensional disordered optical lattices

o,2 Edmond Orignac,3 Anna Minguzzi,4 and Luis Santos viz Universitüt Hannover, Appelstr. 2, D-30167 Hannover, Gern Ilo" and Spin-CNR, Universitä deeli Studi di Salerno, Salerno, de Lyon, CNRS-UMR5672, 69364 Lyon Cedex 7, Fra boratoire de Physique et Modélisation, d CNRS. Lab Maison des Magistères, B.P. 166, 38042 Grenoble, France (Dated: March 5, 2012)

i-disorder on the ground-sta to between disorder and int i Haldane-insulator phase with finite parity order, w phase becomes a Bose-glass at very weak disorder. For quasi-disorder, the Haldane

 U_1/t

Introduction. The interplay between disorder and inte Introduction. The interprise of strongly-correlations plays a crucial role in the physics of strongly-correlation systems [1]. Disorder in non-interacting systems [2], and the physical strong systems are strongly and strongly nons pays a crucial role in the physics of strongly-conten-systems [1]. Disorder in non-interacting systems of An-derson localization [2], which in one dimes on occurs for vanishingly small disorder [3]. For the particular case of ¹-soons in a lattice potential, interactions have been shown, shi ni 1D [4–7] and higher dimensions [8, 9], to induce in

an in (2 (e+7) and inglet dimensions (e, 5), to induce in e presence of disorder a phase diagram characterized by ree phases: a superfluid (SF) phase, a gapless localized in-impressible phase known as Bose-Class (BG), and a Mott-sulator (MI) occuring at commesurate lattice fillings. Ultra-cold atoms in optical lattices offer an extraordinarily

trollable scenario for the detailed analysis of the compe introluzite scenario for the detailed analysis of the compe-ion between disorder and interactions. Disorder in the on-e energies may be implemented in various ways in these stems, including the use of speckle [10–13], binary mix-res [14–17], and bichromatic combinations of two mutuincommensurate lattices [18] Recently localization has en experimentally observed in non-interacting cold gases in 1D and 3D speckle [19, 20], and bichromatic potentials [21]. Bichromatic lattices constitute a peculiar type of disorder, rather a quasi-disorder, realizing the so-called Aubry-André model [22]. The effects of interactions in 1D lattices with model [22]. The effects of interactions in 1D fainces with quasi-disorder have been recently studied [23–25]. Particu-larly interesting is the existence of a gapped localized phase, the so-called incommensurate density wave (ICDW), which results from the quasi-periodicity of the potential.

Polar gases are attracting a growing attention mostly moti-rated by experiments on atoms with large magnetic moments as Chromium [26] and Dysprosium [27], and especially by event groundbreaking experiments on polar molecules [28]. Due to the dipole-dipole interaction, these gases present an exedingly rich physics [29, 30]. Polar lattice gases are partic ceedingly net physics [29, 30]. Polar fattice gases are partic-ularly interesting, mostly due to the qualitatively new features introduced by dipole-induced inter-site interactions [31]. In particular, intersite interactions may allow for the realization of the so-called Haldane-insulator (HI) phase [32], a gapped

of the so-called matadane-insulator (fil) phase [32], a gapped phase characterized by a nonlocal string order parameter. In this Letter we show that the interplay between on-site and inter-site interactions and disorder leads to a rich physics for lattice bosons with nearest neighbor interactions. In par-

ticular, in the presence of uniform disorder the HI phase is preserved, although with finite parity, up to a finite disorder, where a phase transition into a BG is produced. On the con-trary, in the presence of quasi-disorder the HI is connected, without any intermediate critical region, to a gapped general ise occurring for non-polar g

Model. As mentioned above, the main qua feature of polar lattice gases concerns the signific interactions. Polar interactions between sites play sites apart decay as 1/i3. Although interactic play a role in the physics of polar gases, especially in wi concerns the existence of crystalline phases at any fraction filling (Devil's staircase [33, 34]) for very weak on-site into actions and sufficiently large dipoles, the most relevant pror erties of polar lattices gases may be understood from a m

 $H = -t \sum_{i} (b_{i}^{\dagger}b_{i+1} + H.c.) + \frac{U}{2} \sum_{i=1}^{N} n_{i}(n_{i} - 1)$ $+V\sum_{i=1}^{i}n_{i}n_{i+1} + \sum_{i=1}^{i}\epsilon_{i}n_{i}$ (II)

Mott-insulator

...1021...1201...1200...

Haldaneinsulator

Universität

Hannover

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...101...121...101...121...

[Dalla-Torre, Berg and Altman, PRL 97, 260401 (2006)]

Density wave0202020202020202...

Mar 2012

arXiv:1203.0505v1

String and parity order

"fluid of AF ordered defects"

 $\mathcal{O}n_{i} = 1 - n_{i}$ String order $\mathcal{O}_{S}^{2} \equiv \lim_{|i-j| \to \infty} \left\langle -\mathcal{O}n_{i} \exp\left[i\rho \sum_{l=i+1}^{j-1} \mathcal{O}n_{l}\right] \mathcal{O}n_{j} \right\rangle \neq 0$

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Parity order $O_P^2 \equiv \lim_{|i-j| \to \infty} \left\langle \exp\left[i\rho \sum_{l=i+1}^{j-1} dn_l\right] \right\rangle = 0$...1101...121...101...121... ...1011...211...011...211... ...0111...112...110...112...

For the Mott insulator $O_S^2 = 0$ $O_P^2 \stackrel{1}{} 0$

[Endres et al., Science **334**, 200 (2011)]



This talk

1D polar gases at unit filling

1D polar gases with uniform bound disorder

1D polar gases in a quasi-periodic lattice



$$M = -t \sum_{i} [b_{i}^{+}b_{i+1} + H.c.] + \frac{U_0}{2} \sum_{i} n_i (n_i - 1) + \sum_{i} \varepsilon_i n_i$$

Bounded box disorder: $-D < e_i < D$
(similar to speckle)

For non-polar bosons: Bose-Glass (gapless, compressible insulator)

[Giamarchi and Schulz, PRB **37**, 325 (1988); Fisher et al., PRB **40**, 546 (1989); Rapsch, Schollwöck and Zwerger, EPL **46**, 559 (1999)]



[From PRB 40, 546 (1989)]



For non-polar bosons: Bose-Glass (gapless, compressible insulator)

[Giamarchi and Schulz, PRB **37**, 325 (1988); Fisher et al., PRB **40**, 546 (1989); Rapsch, Schollwöck and Zwerger, EPL **46**, 559 (1999)]



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Polar bosons in one-dimensional disordered optical lattices

5.² Edmond Orignac.³ Anna Minguzzi,⁴ and Luis Santos¹ ni: Universität Hannover, Appelatr. 2, D.30167 Hannover, Germany and Superior and Construction and Construction and Construction and Superior de Lyon, CNRS-UMR527, 03964 Lyon Codes, 7, Fra and CNRS, Laboratoire de Physique et Modélisation, 93, Maison des Magistères, B.P. 166, 38042 Grenoble, France

quasi-disorder on the ground-state properties of ultra-cold polar box epiloy between disorder and inter-stein interactions leads to rich pl i Haldane-insulator phase with finite parity order, whereas the dens need sometilzed incomments and incident in the Haldane insulator comment and sometilzed incomments.

 U_1/t

Introduction: The interplay between disorder and intertions plays a reutial role in the physics of strongly-correl derson localization [2], which in one dimension occurs vanishingly small disorder [3]. For the particular case "soons in a lattice potential, interactions have been sho or presence of disorder a plase diagram characterized response of disorder a plase diagram characterized responses: a superfluid (SF) plase, a gaptess local, and a superschilder plase known as howe (Tasa, RG), and a M

2 Mar 2012

arXiv:1203.0505v1

ntrollable scenario for the detailed analysis of the competition of between disorder and interactions. Disorder in the on several gas may be implemented in whom we say in these several states of the several states of th

1D and 3D speckle [19, 20], and behromatic potentials [21]. Bichromatic lattices constitute a peculiar type of disorde rather a quasi-disorder, realizing the so-called Aubry-Andu Bichromatic lattices and the second second second second larly interesting is the existence of a gapped localized phase the so-called incommensurate density wave (ICDW), which results from the quasi-periodicity of the potential. Wated by experiments on atoms with large magnetic moment wheels by experiments on atoms with large magnetic moment

vated by experiments on atoms with large magnetic moments as Chronium [26] and Dysproximm [27], and especially by secont groundbreaking experiments on polar molecules [28]. The provide second second second second second second large interesting, mostly due to the qualitatively new features introduced by dipole-induced miner-site interestions [31]. In subject is the second second second second second of the so-called Haldane-insulator (HD) phase [32], a gapped base characterized by a nonlocal string order parameter. In this Letter we show that the interplay between onyoids and the second second second second second second of lattice boots with nearest neighbor interestions. In par-

 $\frac{SP}{NG}$ $\frac{SP$

To: 1: Phase diagrams of bosons with nearest-neighbor interaction a) unperturbed case; (b) staggered on-site energy; (c) uniform c ruler; (d) quasi-disorder. Figures (b)–(d) are obtained for U/t =see text for details.

ticular, in the presence of uniform disorder the HI phase in preserved, although with finite parity, up to a finite disorder where a phase transition into a BG is produced. On the conwithout any intermediate critical region, to a gapped generalized ICDW phase occurring for non-polar gases. Other phases are discussed in densition types of disorder, feature of polar lattice gases concerns the significant inter-site interactions. Polar interactions between sites placed j > 0.

feature of polar lattice gases concerns the significant inter-site interactions. Polar interactions between sites placed j > 0sites apart decay as $1/j^2$. Although interactions for j > 1 do play a role in the physics of polar gases, especially in what concerns the existence of crystalline phases at any fractional filling (Devi') subscription of the phase site of the phase action and sufficiently large dipoles, the most relevant propwith only nearest-neighbor interactions.

 $H = -t \sum_{i} (b_{i}^{\dagger}b_{i+1} + H.c.) + \frac{U}{2} \sum_{i=1}^{N} n_{i}(n_{i} - 1)$ $+V\sum_{i=1}^{i}n_{i}n_{i+1} + \sum_{i=1}^{i}\epsilon_{i}n_{i},$ (1)

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Bounded box disorder: $-D < e_i < D$

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 $H = -t\sum_{i} \left[b_{i}^{+}b_{i+1} + H.c. \right] + \frac{U_{0}}{2}\sum_{i} n_{i} (n_{i} - 1) + U_{1}\sum_{i} n_{i} n_{i+1} + \sum_{i} \varepsilon_{i} n_{i}$

Hannover

[From EPL 46, 559 (1989)]



$$H = -t\sum_{i} \left[b_{i}^{+}b_{i+1} + H.c. \right] + \frac{U_{0}}{2}\sum_{i} n_{i} (n_{i} - 1) + U_{1}\sum_{i} n_{i} n_{i+1} + \sum_{i} \varepsilon_{i} n_{i}$$

Bounded box disorder: $-D < e_i < D$



SF vanishes for growing NN interactions (SF-BG at K=3/2) [Giamarchi and Schulz, EPL 3, 1287 (1987)]

BG goes all the way to zero at the MI-HI for K<3/2 [Deng et al., arXiv:1302.0528]



$$H = -t\sum_{i} \left[b_i^+ b_{i+1}^- + H.c. \right] + \frac{U_0}{2} \sum_{i} n_i (n_i - 1) + U_1 \sum_{i} n_i n_{i+1}^- + \sum_{i} \varepsilon_i n_i^-$$

Bounded box disorder: $-D < \theta < D$



DW disappears even for very small disorder. (Imry-Ma argument) [Imry and Ma, PRL **35**, 1399 (1975)]

Imry-Ma argument

...0202020202020202...

[Imry and Ma, PRL **35**, 1399 (1975)] $\mathcal{O}_{DW} = \lim_{|i-j| \to \infty} \langle (-1)^{i-j} \delta n_i \delta n_j \rangle$ Gap for flipping a spin



Energy of the domain coming from the noise: $<\Delta\epsilon > \sim \Delta L^{1/2}$

Energy of the domain walls ~ V

Formation of domains of size $L \sim (V/\Delta)^{1/2}$

Domain formation destroys the order

The gap is destroyed because now the excitations are simply moving walls, not creating them



$$H = -t\sum_{i} \left[b_i^+ b_{i+1} + H.c. \right] + \frac{U_0}{2} \sum_{i} n_i (n_i - 1) + U_1 \sum_{i} n_i n_{i+1} + \sum_{i} \varepsilon_i n_i$$

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Bounded box disorder: $D < 0 < D$

Bounded box disorder: $-D < e_i < D$





"Glassy" Haldane insulator at finite disorder

0

Parity order was zero in the HI due to
the ,,fluid" character of the defects
$$O_P^2 \equiv \lim_{|i-j| \to \infty} \left\langle \exp\left[i\rho \sum_{l=i+1}^{j-1} cln_l\right] \right\rangle = 0$$

$$dn_i = 1 - n_i$$

$$\dots 1101\dots 121\dots 01\dots 121\dots$$

$$\dots 1011\dots 211\dots 01\dots 211\dots$$

$$\dots 0111\dots 112\dots 11\dots$$

0

Disorder leads to the localization of defects reducing their mobility (while still keeping their AF ordering)

Parity becomes non-zero due to the glassy defects

2



$$H = -t\sum_{i} \left[b_i^+ b_{i+1} + H.c. \right] + \frac{U_0}{2} \sum_{i} n_i \left(n_i - 1 \right) + U_1 \sum_{i} n_i n_{i+1} + \sum_{i} \varepsilon_i n_i$$

Bounded box disorder: $D < 0 < D$

Bounded box disorder: $-D < e_i < D$





This talk

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1D polar gases in a quasi-periodic lattice



$$H = -t \mathop{\stackrel{\circ}{\underset{i}{\otimes}}}_{i} \mathop{\stackrel{\circ}{\otimes}}_{i+1} + H.c. \underbrace{H} + \frac{U_0}{2} \mathop{\stackrel{\circ}{\underset{i}{\otimes}}}_{i} n_i (n_i - 1) + \mathop{\stackrel{\circ}{\underset{i}{\otimes}}}_{i} e_i n_i$$

Bichromatic lattices $\stackrel{\circ}{\longrightarrow}$ $\stackrel{\circ}{\longleftarrow}$ $\stackrel{\circ}{\longleftarrow}$ $e_j = D\cos(2\rho r j + j)$
 $r = q_2 / q_1$

Extra phase: incommesurate density wave (ICDW) [Roscilde PRA 77, 063605 (2008); Roux et al, PRA 78, 023628 (2008)]



Super-wells develop over a characteristic length scale 1/(1-r) (,,super site") due to the beating between the two periods of the two lattices.



$$H = -t \bigotimes_{i} \stackrel{\circ}{\oplus} b_{i+1}^{+} + H.c. \stackrel{\circ}{=} + \frac{U_0}{2} \bigotimes_{i} n_i (n_i - 1) + \bigotimes_{i} e_i n_i$$

Bichromatic lattices
$$\bigoplus_{\omega_2} \stackrel{\circ}{\longrightarrow} e_j = D\cos(2\rho r j + j) \stackrel{\circ}{=} \frac{1}{r} = q_2 / q_1$$

Extra phase: incommesurate density wave (ICDW) [Roscilde PRA 77, 063605 (2008); Roux et al, PRA 78, 023628 (2008)]



[From PRA 78, 023628 (2008)]

A ICDW phase for <n>=1-r may be interpreted as filling each ,,super-site" with one particle.

For n=r it is like having one hole siting at each ,,super-site"



$$H = -t \mathop{\overset{\circ}{\underset{i}{\otimes}}}_{i} \stackrel{\circ}{b}_{i+1} + H.c. \stackrel{\circ}{\underline{b}} + \frac{U_0}{2} \mathop{\overset{\circ}{\underset{i}{\otimes}}}_{i} n_i (n_i - 1) + \mathop{\overset{\circ}{\underset{i}{\otimes}}}_{i} e_i n_i$$

comatic lattices
$$\stackrel{\circ}{\longrightarrow} \stackrel{\circ}{\longleftarrow} \stackrel{\circ}{\underbrace{\longleftrightarrow}}_{0_2} e_j = D\cos(2\rho r j + j) \frac{U_0}{r} e_j n_i$$

Extra phase: incommesurate density wave (ICDW) [Roscilde PRA 77, 063605 (2008); Roux et al, PRA 78, 023628 (2008)]



Bichro

[From PRA 78, 023628 (2008)]

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$$H = -t \bigotimes_{i} \stackrel{o}{\otimes} b_{i}^{+} b_{i+1} + H.c. = \frac{U_0}{2} \bigotimes_{i} n_i (n_i - 1) + \bigotimes_{i} e_i n_i$$

Bichromatic lattices
$$\bigoplus_{\omega_2} \stackrel{o}{\otimes} e_j = D\cos(2\rho r j + j)$$
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Extra phase: incommesurate density wave (ICDW) [Roscilde PRA 77, 063605 (2008); Roux et al, PRA 78, 023628 (2008)]

As a result a gapped ICDW appears with a structure factor peaking at the beating periodicity



[From PRA 78, 023628 (2008)]



$$H = -t \mathop{\overset{\circ}{\underset{i}{\otimes}}}_{i} \stackrel{\circ}{\ominus} b_{i+1}^{+} + H.c. \stackrel{\circ}{\ominus} + \frac{U_0}{2} \mathop{\overset{\circ}{\underset{i}{\otimes}}}_{i} n_i (n_i - 1) + \mathop{\overset{\circ}{\underset{i}{\otimes}}}_{i} e_i n_i$$

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Extra phase: incommesurate density wave (ICDW) [Roscilde PRA 77, 063605 (2008); Roux et al, PRA 78, 023628 (2008)]

Away from these fillings the ICDW disappears

Bichron



[From PRA 78, 023628 (2008)]



$$H = -t \mathop{\stackrel{\circ}{\underset{i}{\otimes}}}_{i} \mathop{\stackrel{\circ}{\otimes}}_{i+1} + H.c. \underbrace{H} + \frac{U_0}{2} \mathop{\stackrel{\circ}{\underset{i}{\otimes}}}_{i} n_i (n_i - 1) + \mathop{\stackrel{\circ}{\underset{i}{\otimes}}}_{i} e_i n_i$$

Bichromatic lattices $\stackrel{\circ}{\longrightarrow}$ $\stackrel{\circ}{\longleftarrow}$ $e_j = D\cos(2\rho r j + j)$
 $r = q_2 / q_1$

Extra phase: incommesurate density wave (ICDW) [Roscilde PRA 77, 063605 (2008); Roux et al, PRA 78, 023628 (2008)]



For a filling <n>=1 a possible gapped phase may occur as well (generalized ICDW)

$$H = -t \sum_{i} \left[b_{i}^{+} b_{i+1} + H.c. \right] + \frac{U_{0}}{2} \sum_{i} n_{i} (n_{i} - 1) + U_{1} \sum_{i} n_{i} n_{i+1} + \sum_{i} \varepsilon_{i} n_{i}$$

Bichromatic lattices



A generalized ICDW phase appears for <n>=1 for a sufficiently large 2nd lattice



$$H = -t\sum_{i} \left[b_{i}^{+} b_{i+1} + H.c. \right] + \frac{U_{0}}{2} \sum_{i} n_{i} (n_{i} - 1) + U_{1} \sum_{i} n_{i} n_{i+1} + \sum_{i} \varepsilon_{i} n_{i}$$

Bichromatic lattices

$$e_{j} = D\cos(2\rho r j + j)$$

$$r = q_{2} / q_{1} = (\sqrt{5} - 1)/2$$



The DW phase survives for a small disorder (no Imry-Ma argument here)

$$H = -t\sum_{i} \left[b_{i}^{+} b_{i+1} + H.c. \right] + \frac{U_{0}}{2} \sum_{i} n_{i} \left(n_{i} - 1 \right) + U_{1} \sum_{i} n_{i} n_{i+1} + \sum_{i} \varepsilon_{i} n_{i}$$

Bichromatic lattices

$$e_{j} = D\cos(2\rho r j + j)$$

$$r = q_{2} / q_{1} = (\sqrt{5} - 1)/2$$



"Glassy" HI phase due to the pinining of defects



$$H = -t\sum_{i} \left[b_{i}^{+} b_{i+1} + H.c. \right] + \frac{U_{0}}{2} \sum_{i} n_{i} \left(n_{i} - 1 \right) + U_{1} \sum_{i} n_{i} n_{i+1} + \sum_{i} \varepsilon_{i} n_{i}$$

Bichromatic lattices

$$e_{j} = D\cos(2\rho r j + j)$$

$$e_{j} = D\cos(2\rho r j + j)$$

$$r = q_{2} / q_{1} = (\sqrt{5} - 1)/2$$



The ,,glassy" HI phase connects adiabatically with the ICDW (which has also O_S^2 , $O_P^2 > 0$)



$$H = -t\sum_{i} \left[b_{i}^{+}b_{i+1} + H.c. \right] + \frac{U_{0}}{2}\sum_{i} n_{i} (n_{i} - 1) + U_{1}\sum_{i} n_{i} n_{i+1} + \sum_{i} \varepsilon_{i} n_{i}$$

Bichromatic lattices



This may open an interesting route (protected by a gap) for the creation of the HI phase

Summary

Polar gases behave very differently in uniformly disordered lattices and quasi-periodic lattices Uniform disorder: glassy HI (dissapearing into a BG) and rapidly vanishing DW

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Quasi-disorder: finite DW, glassy-HI and ICDW adiabatically connected





People









X. Deng (Leibniz Uni.)

E. Orignac (ENS, Lyon)

R. Citro A. Minguzzi (Univ. Salerno) (Univ. Grenoble)

T. Vekua





G. Sun

+ G. Jackeli (MPI-FKP, Stuttgart)