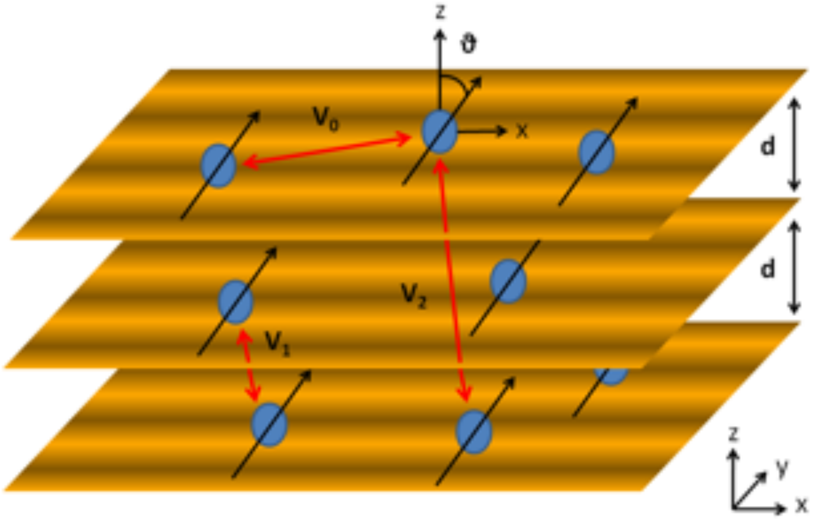
Dipolar fermions in 1D and 2D geometries

Georg M. Bruun
Aarhus University

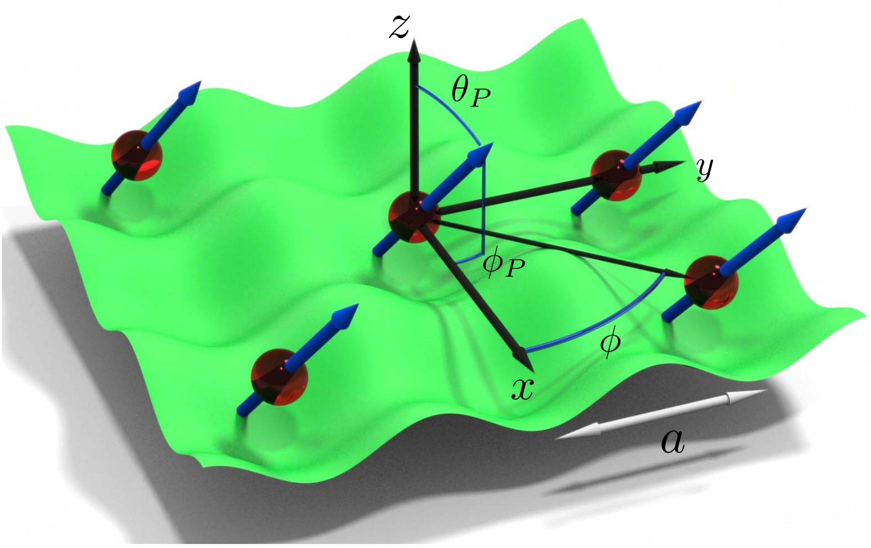
Outline

- Dipoles and stripe order in a multilayer setup



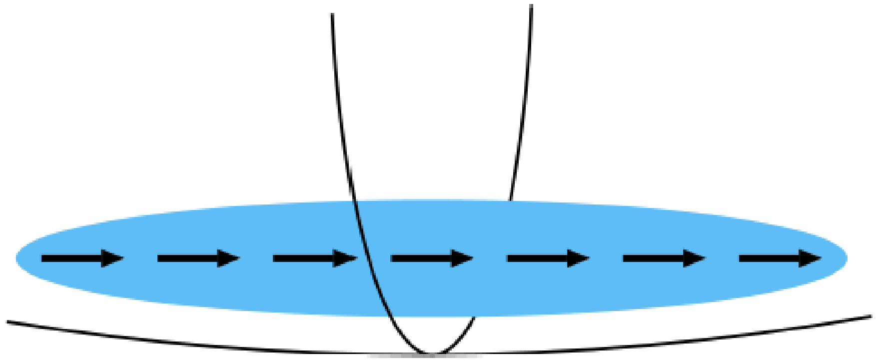
J. K. Block, N. Zinner, and GMB, NJP **14** 105006 (2012)

- Dipoles in a 2D square lattice: density order and superfluidity



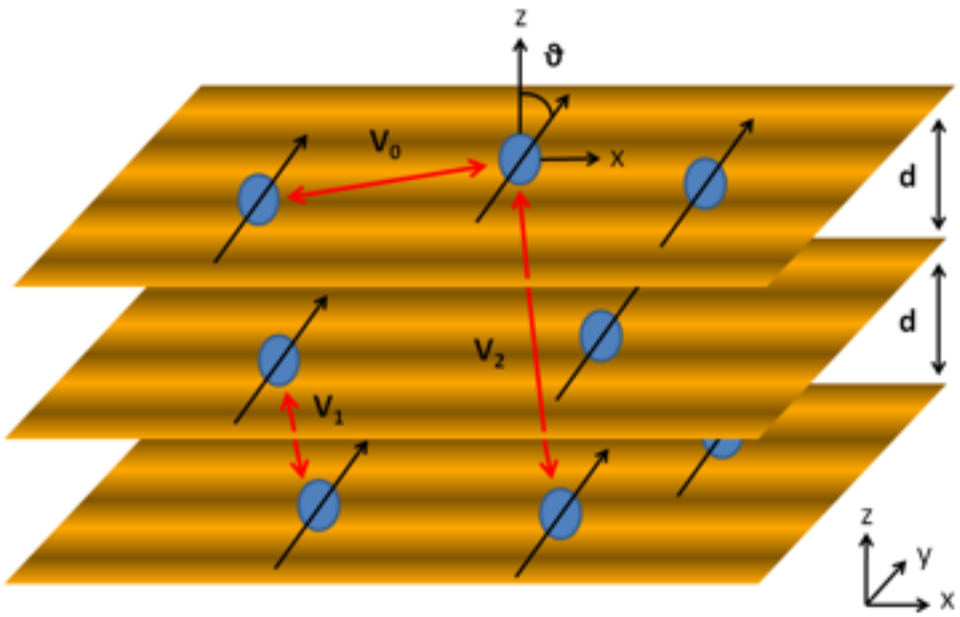
A.-L. Gadsbølle and GMB, PRA **86**, 033623 (2012)
A.-L. Gadsbølle and GMB, PRA **85**, 021604(R) (2012)

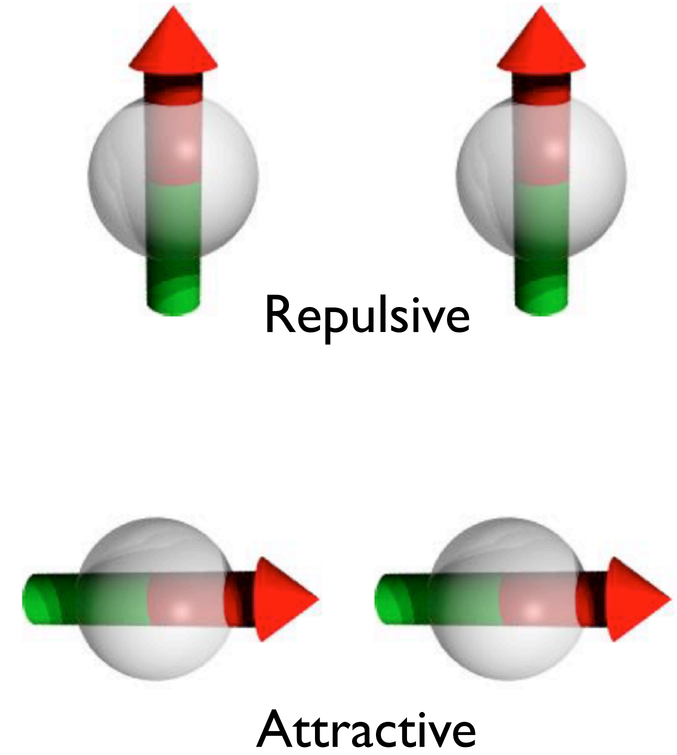
- Self-bound states of 1D dipolar Fermi gas



F. Deuretzbacher, GMB, C. Pethick, L. Santos ...

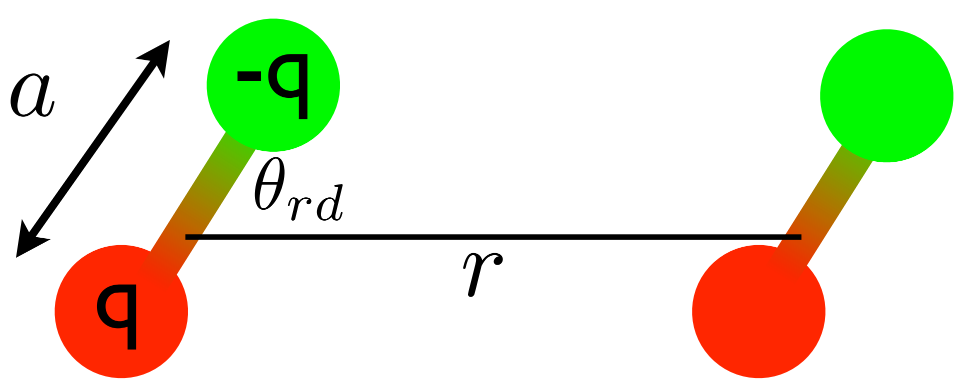
Dipoles and stripe order in a multilayer setup





Dipole-dipole interaction

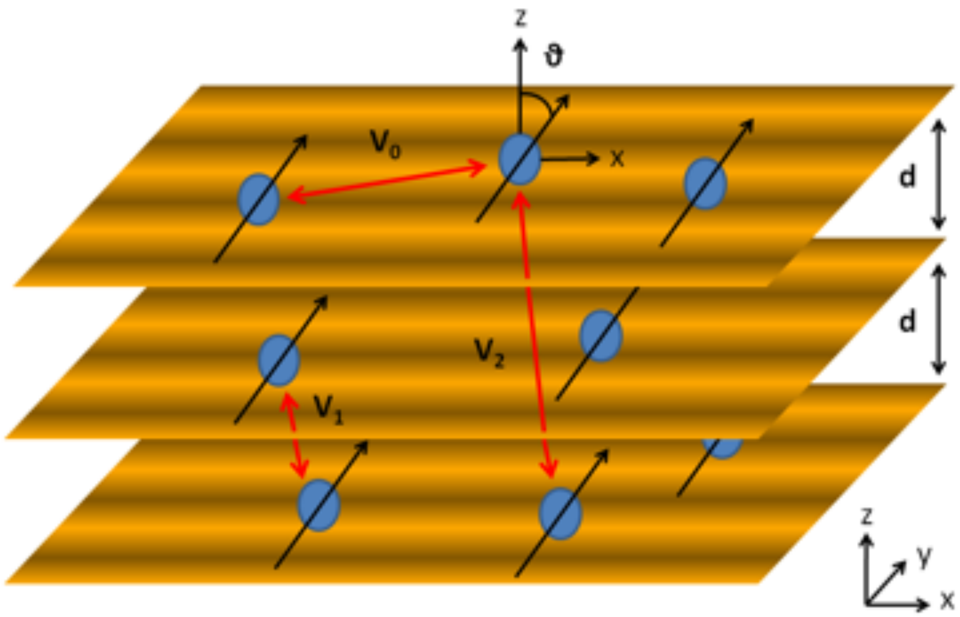
$$V(\mathbf{r}) = D^2 \frac{1 - 3\cos^2\theta_{rd}}{r^3}$$

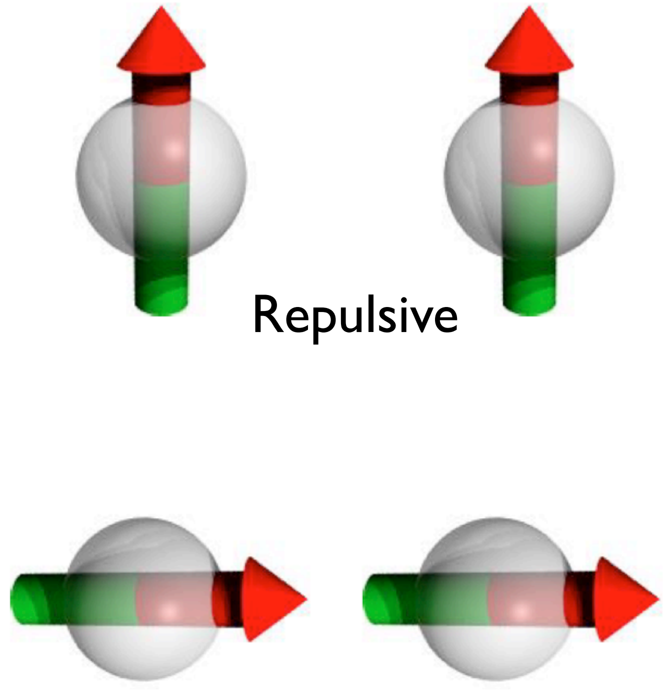


$$d = q \cdot a$$

$$D^2 = \frac{d^2}{4\pi\epsilon_0}$$

Dipoles and stripe order in a multilayer setup





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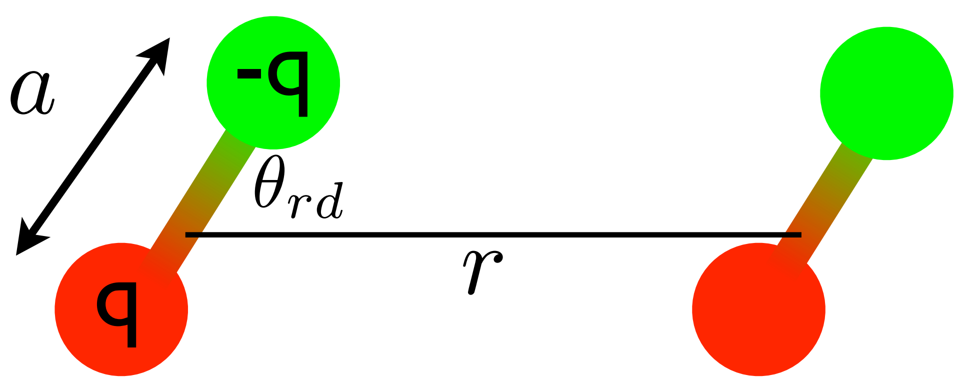

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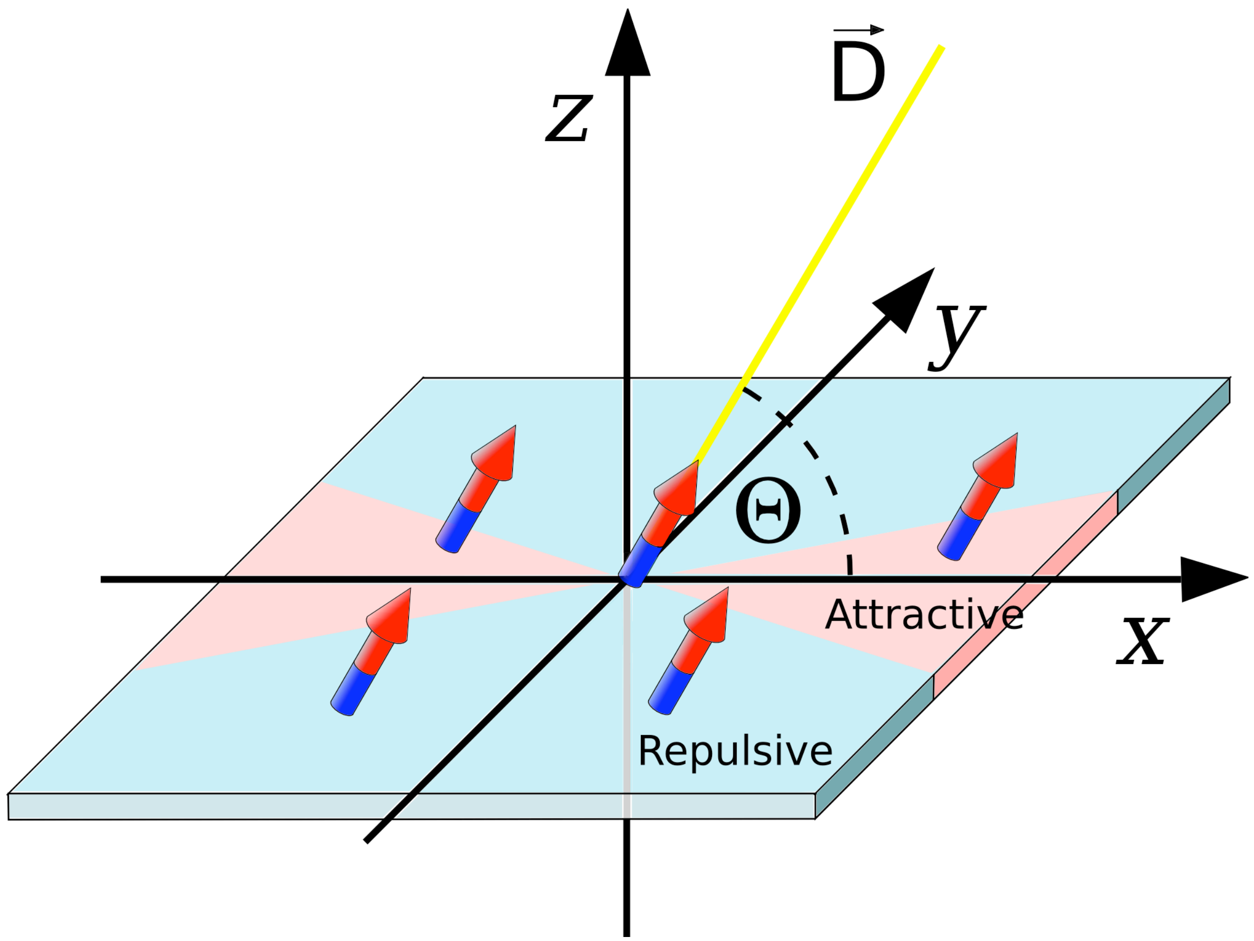
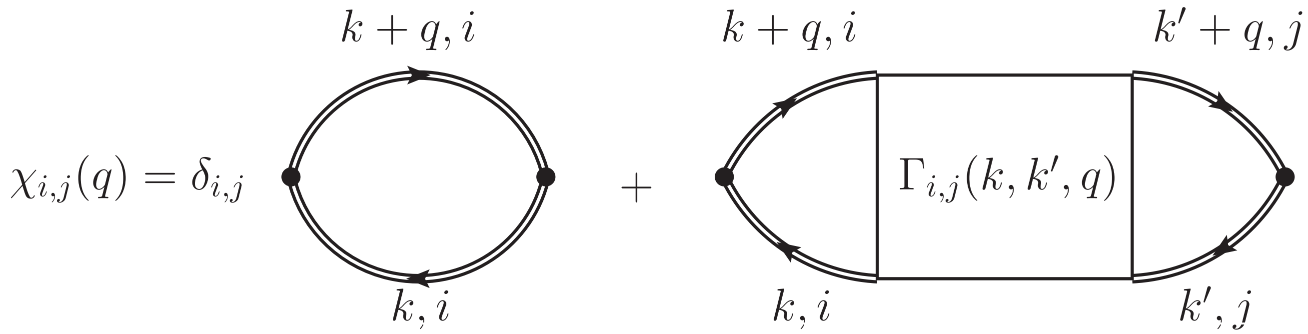


Density waves (stripes)=pole in density-density response function

$$\chi^R_{ij}(\mathbf{r}-\mathbf{r}',t-t') = -i\theta(t-t')\langle\big[\hat{\rho}_i(\mathbf{r},t),\hat{\rho}_j(\mathbf{r}',t')\big]\rangle$$

Layer index



RPA:

K. Sun, C. Wu, and S. Das Sarma, PRB **82**, 075105 (2010)
Y. Yamaguchi, T. Sogo, and T. Ito PRA **82**, 013643 (2010)

Local field factor:

M. M. Parish and F. M. Marchetti, PRL **108**, 145304 (2012)
F. M. Marchetti and M. M. Parish, arXiv:1207.4068

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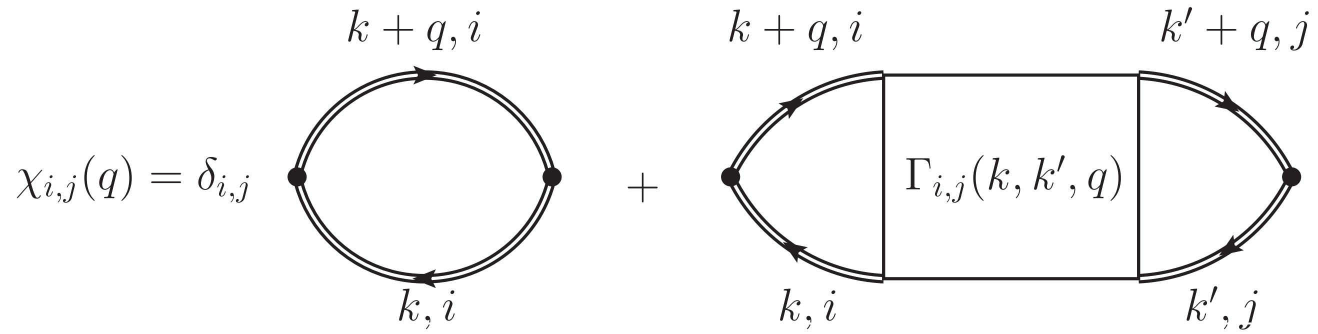

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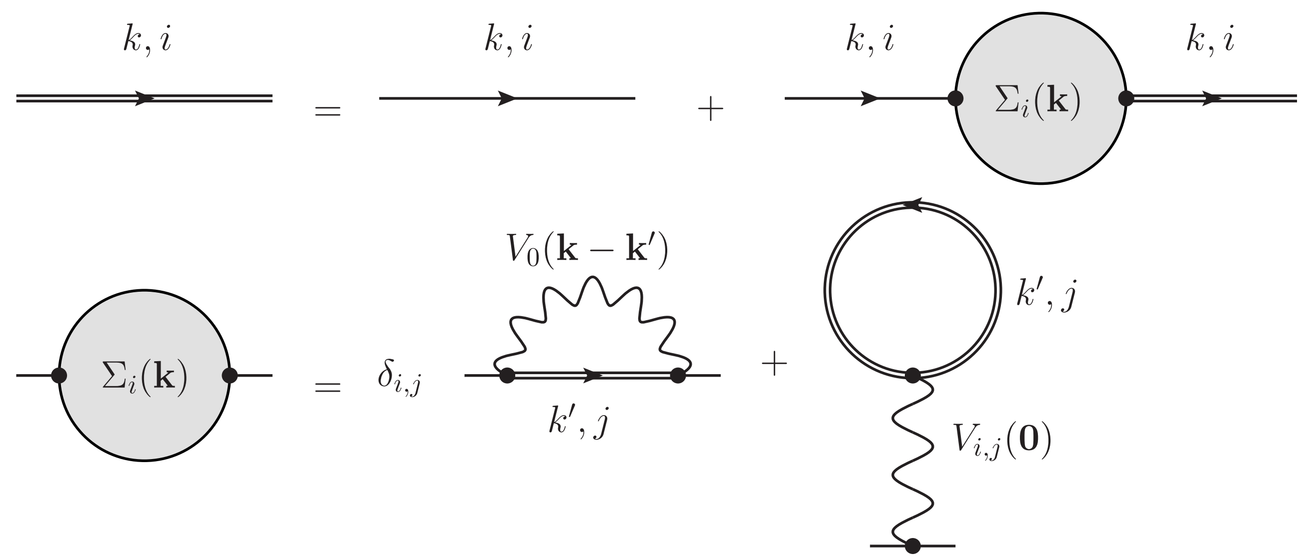
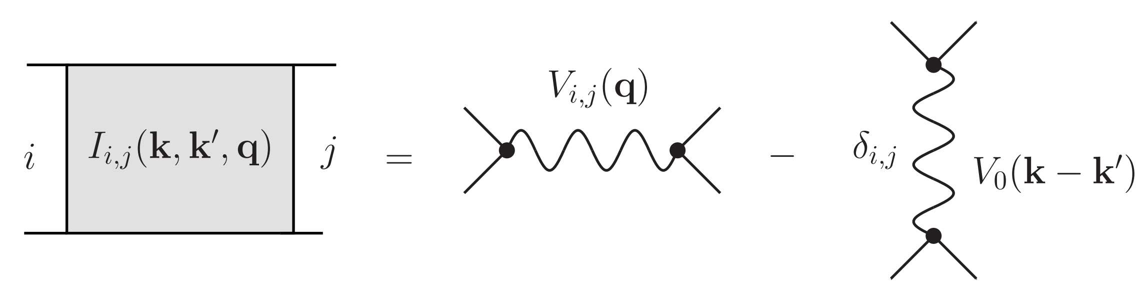
Self-consistent Hartree-Fock (conserving)

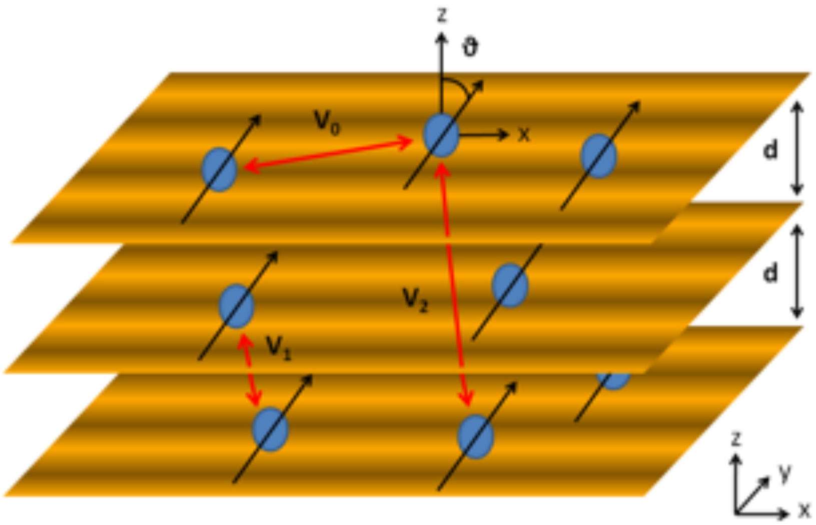


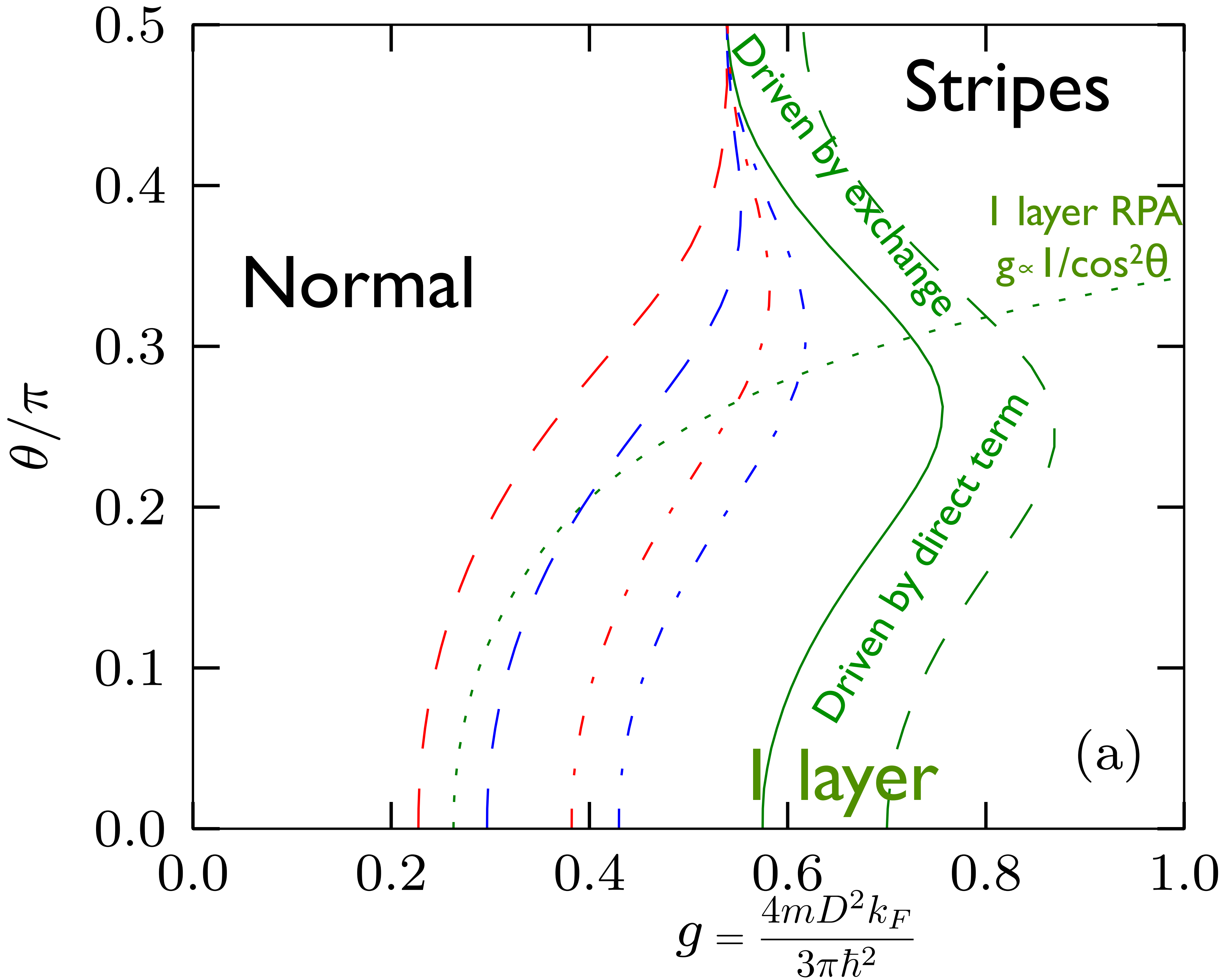


M. Bababi and E. Demler, PRB **84**, 235124 (2011)
L. M. Sieberer and M. A. Baranov, PRA **8**, 063633 (2011)

Results:

$$g = \frac{4mD^2k_F^0}{3\pi\hbar^2}$$





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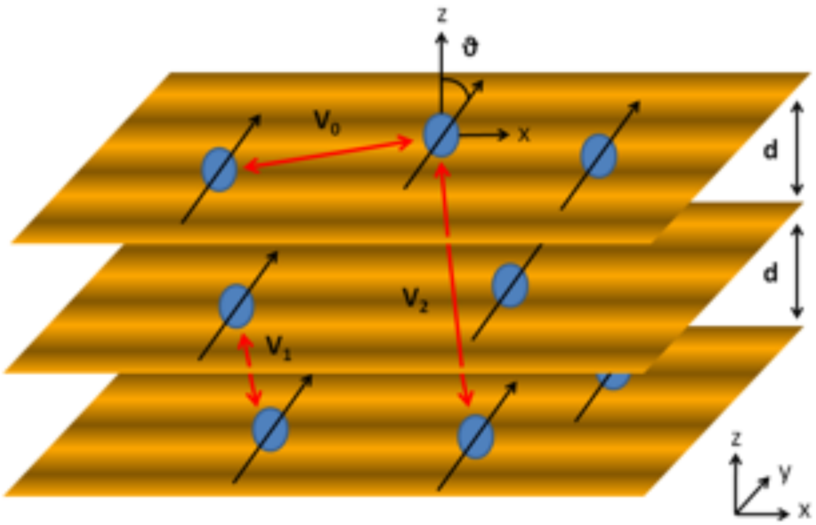

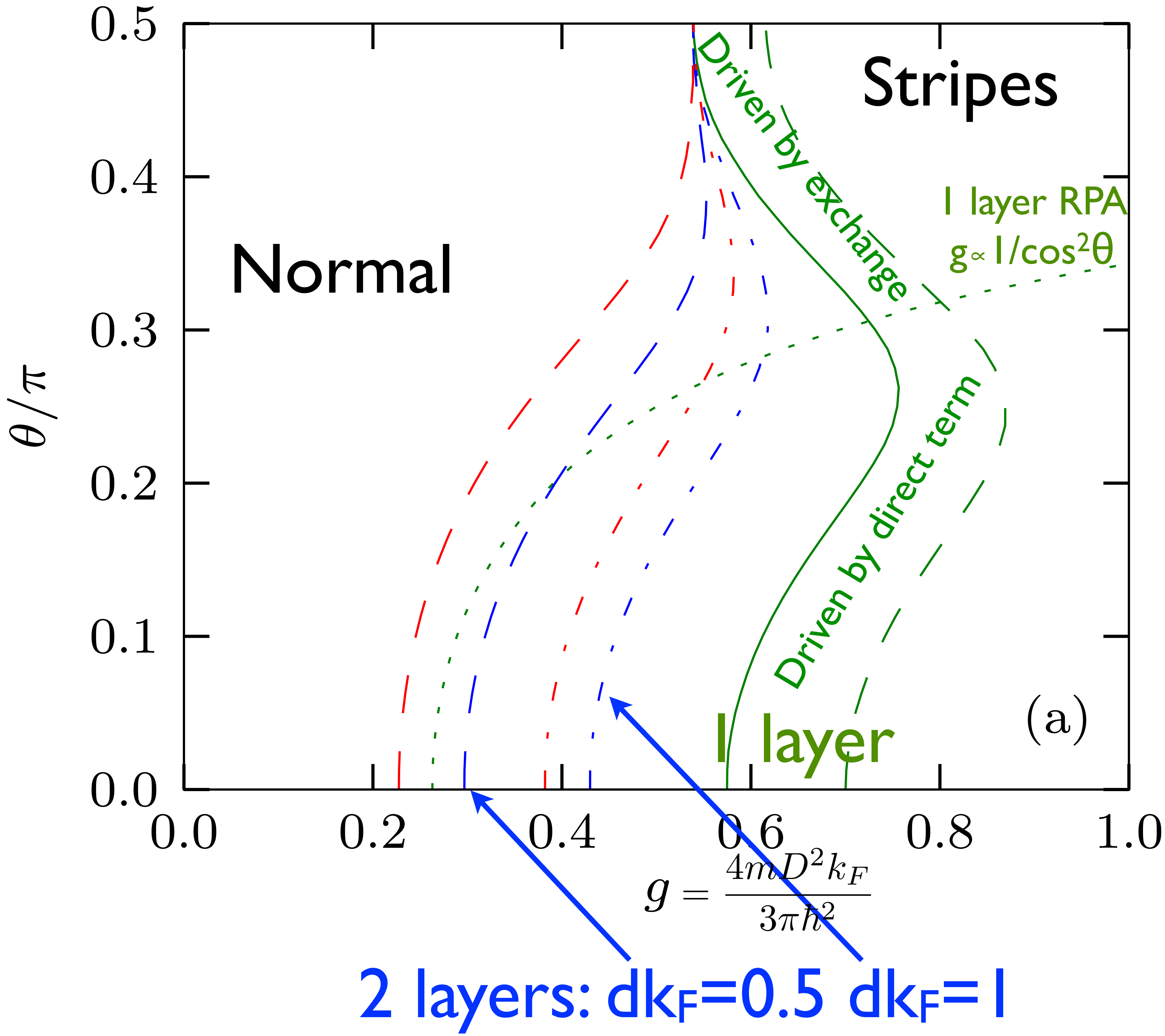

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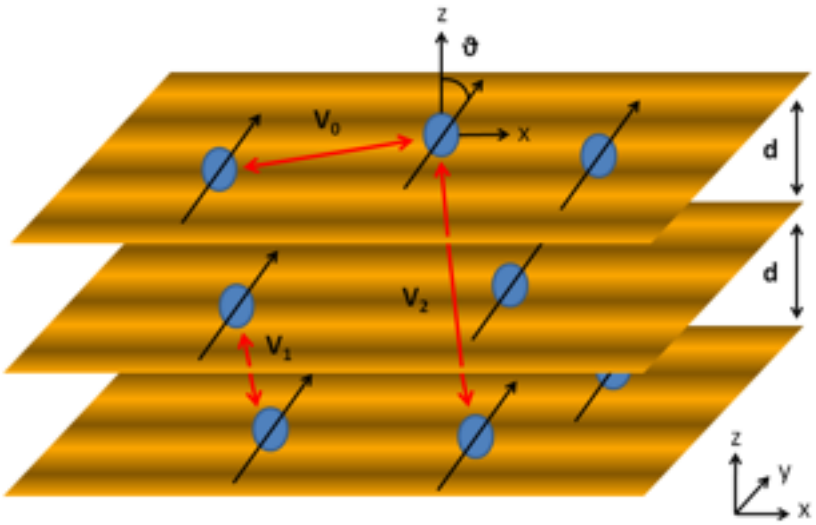

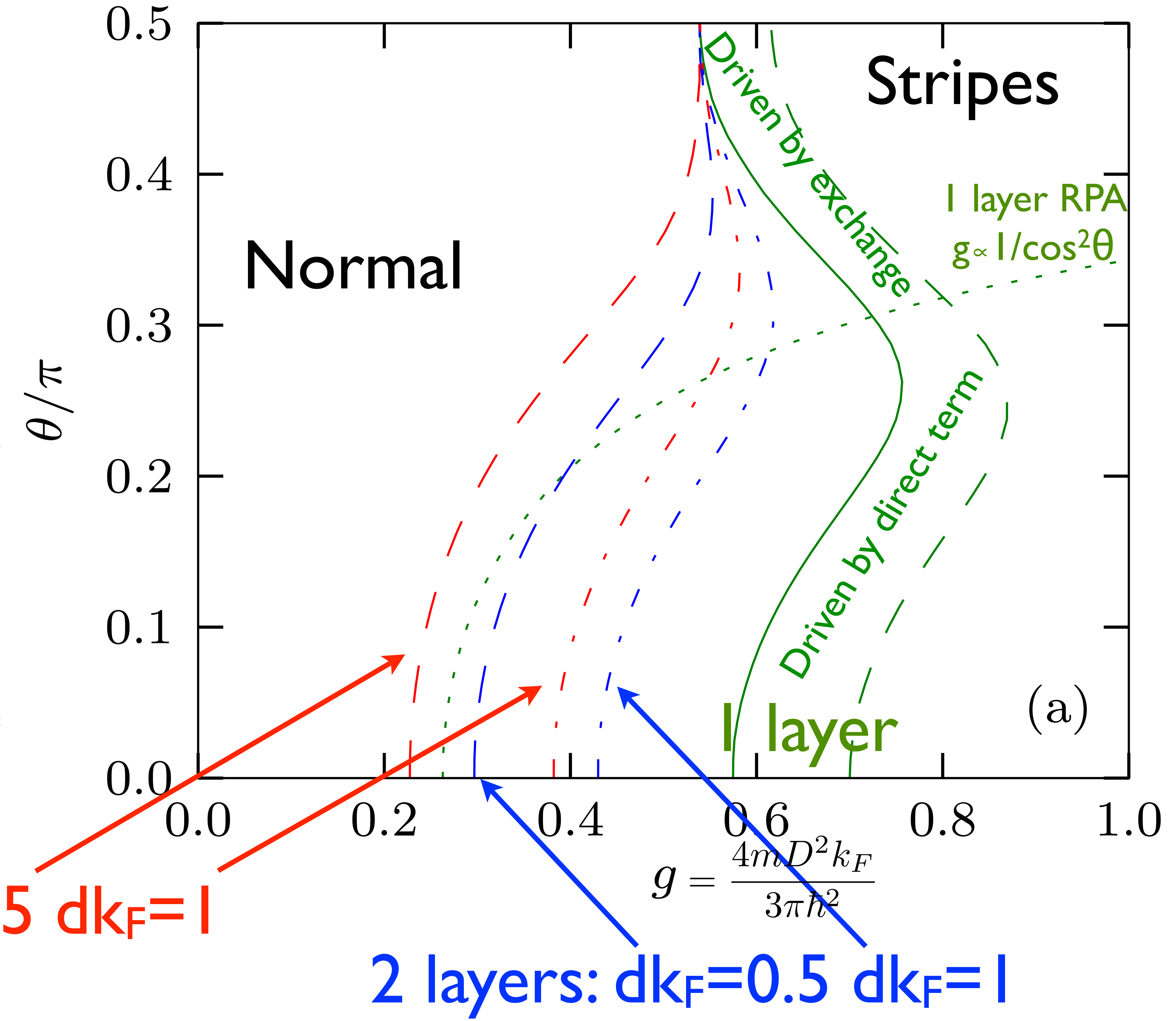

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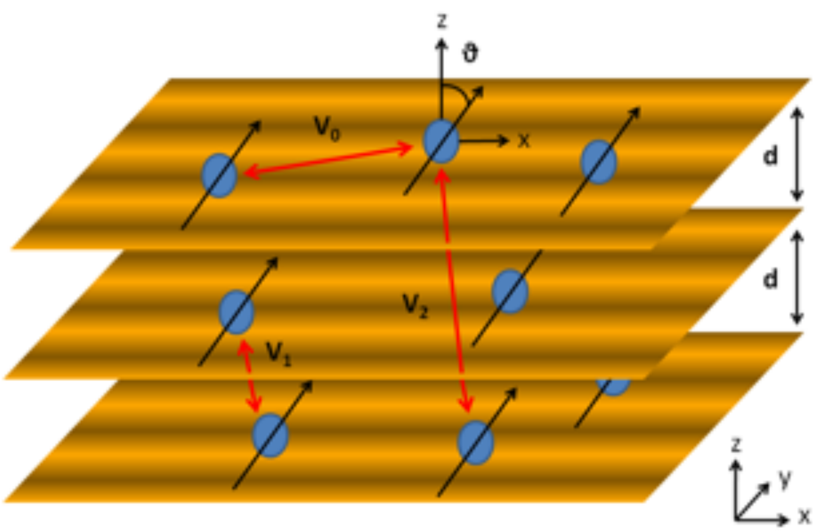

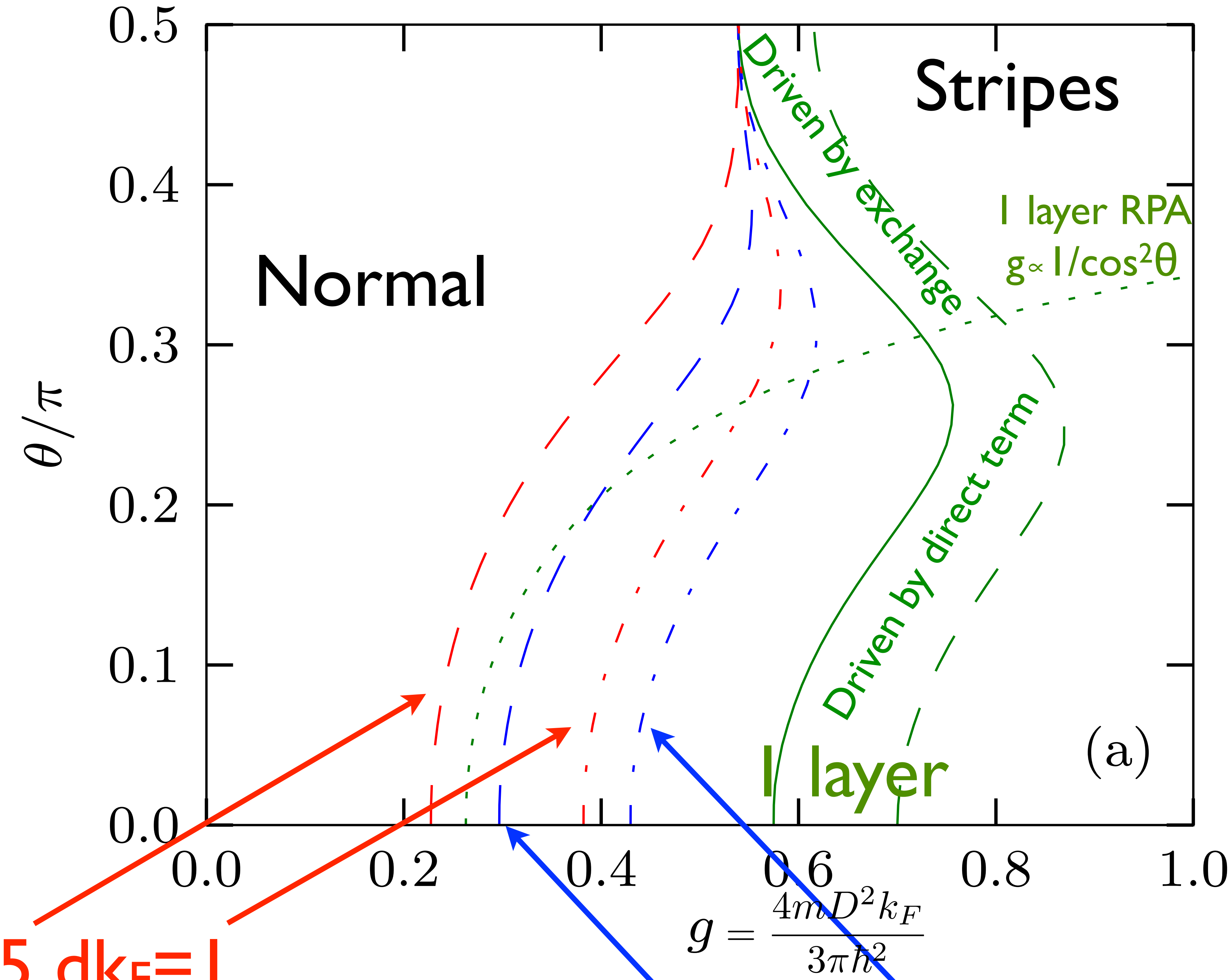

3 layers: dk_F=0.5 dk_F=1

2 layers: dk_F=0.5 dk_F=1

Interlayer correlations favor stripes
Effect largest for θ=0; it vanishes for θ=π/2

RPA gives for small layer separation: $g_c \propto 1/N$ $N \gg 1$

N. Zinner and GMB,
Eur. J. Phys. D **65** 133 (2011)

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Stripes in different layers always in-phase

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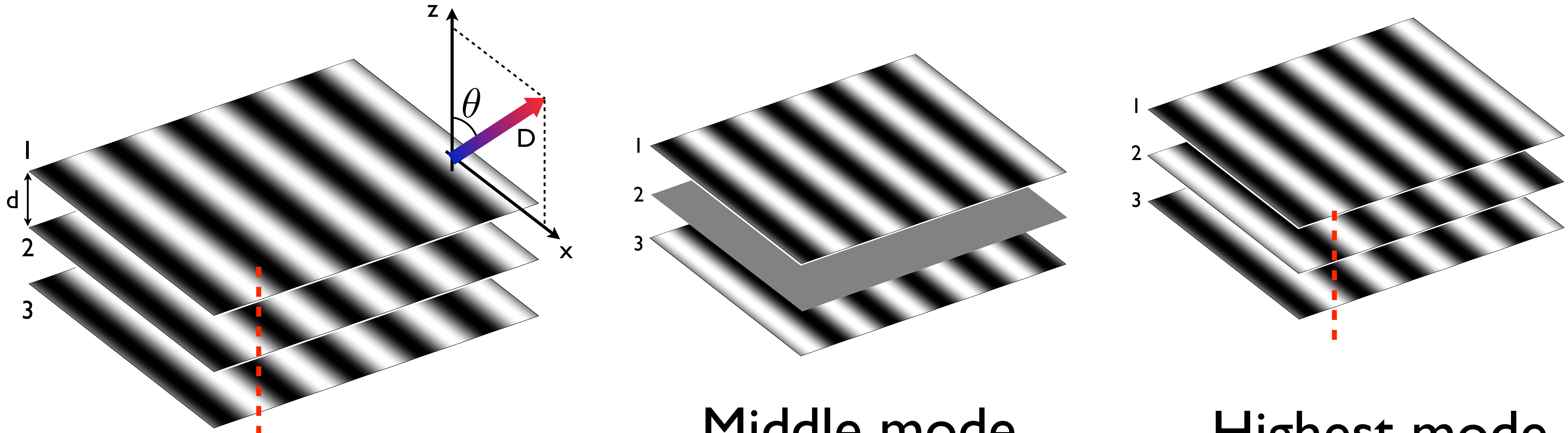
Eur. J. Phys. D **65** 133 (2011)



Lowest mode

Middle mode

Highest mode

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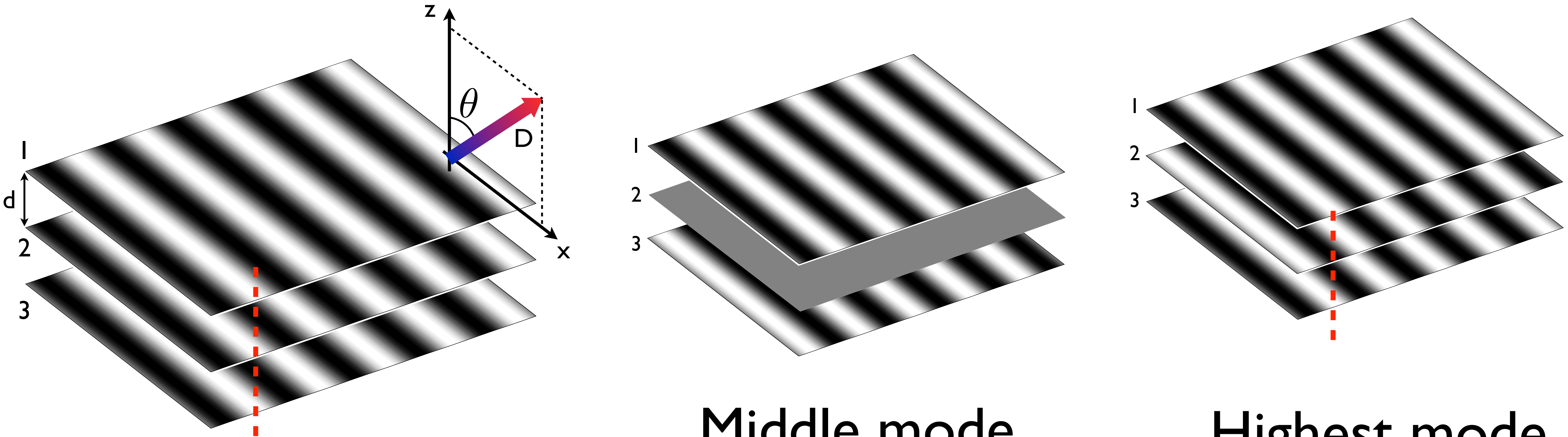

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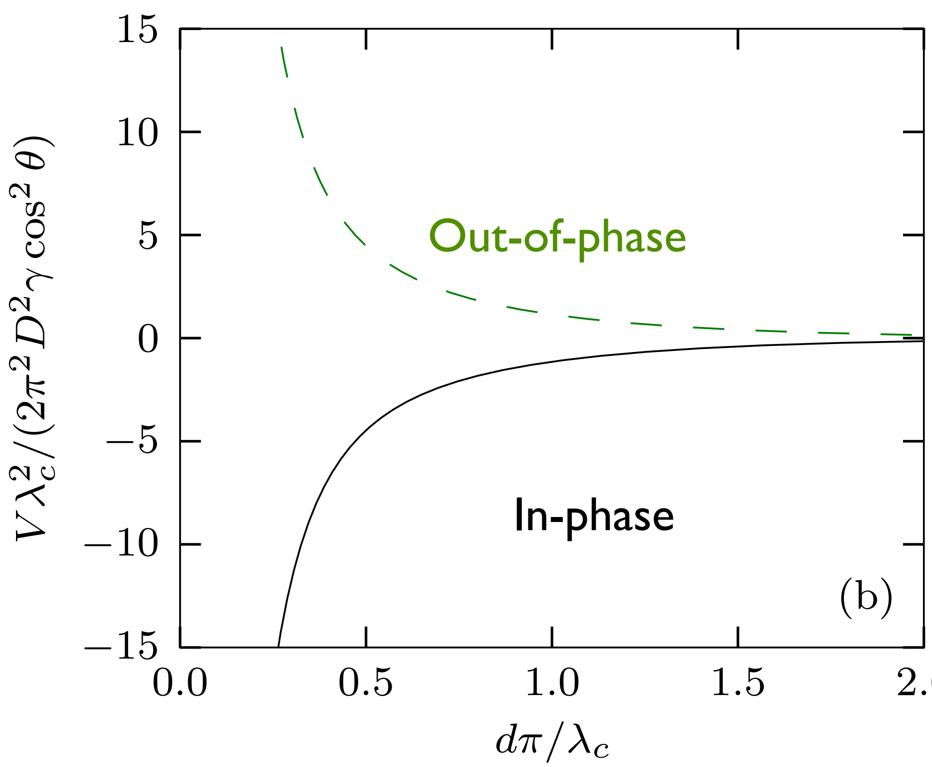
Classical interaction energy between layers

$$V_{\text{classical}} = \mp \frac{2\pi^2 D^2 \gamma \cos^2\theta}{\lambda_c^2}\left[\text{csch}^2(\pi d/\lambda_c) + \text{sech}^2(\pi d/\lambda_c)\right]$$



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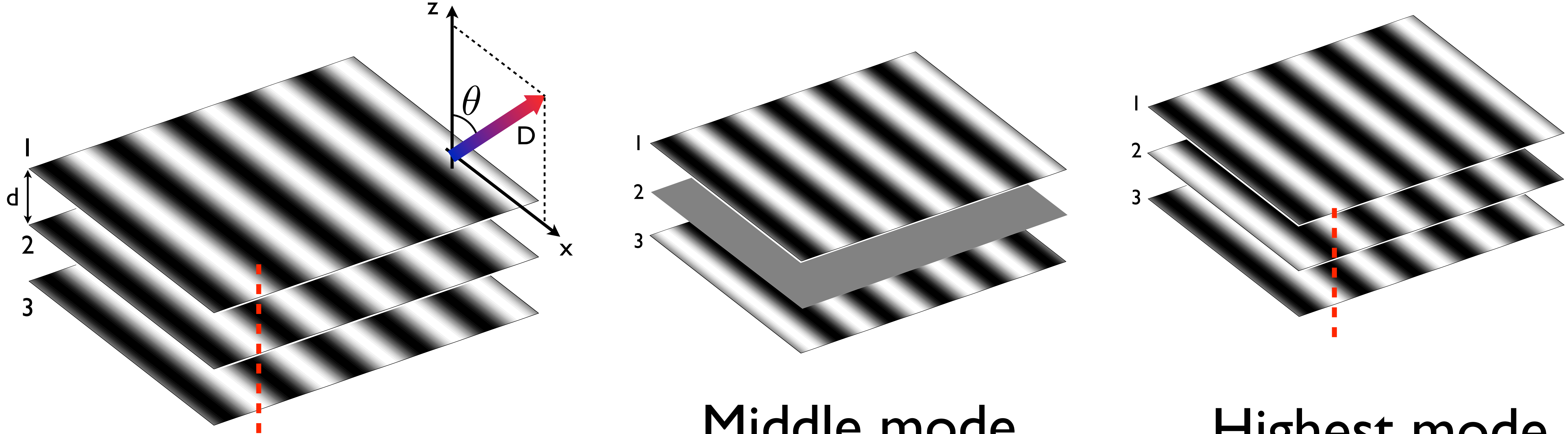

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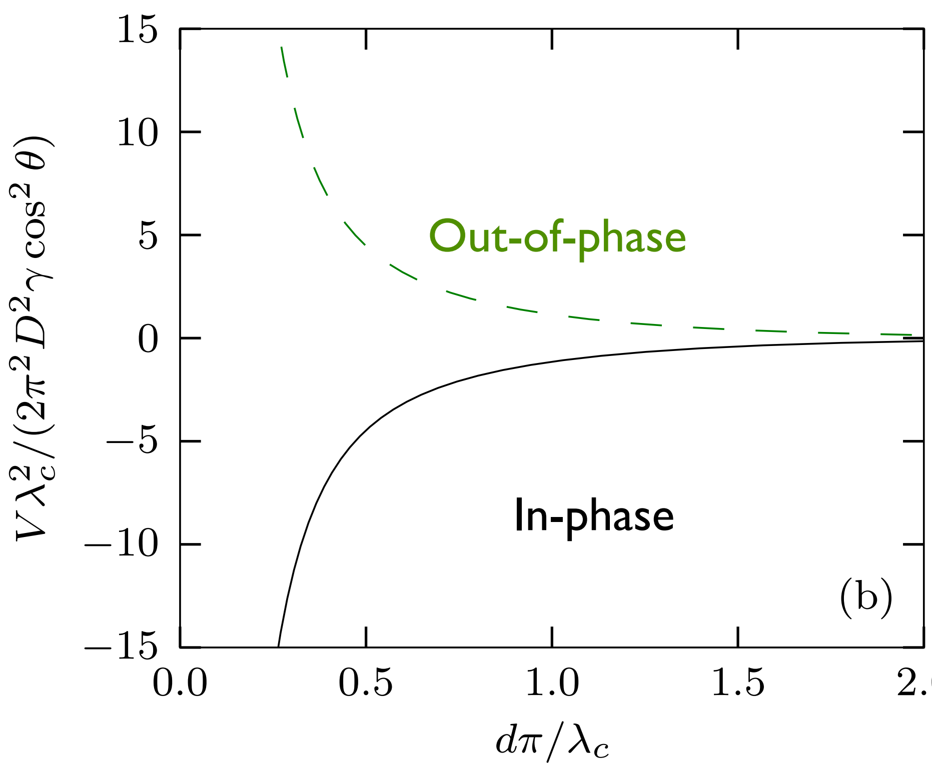
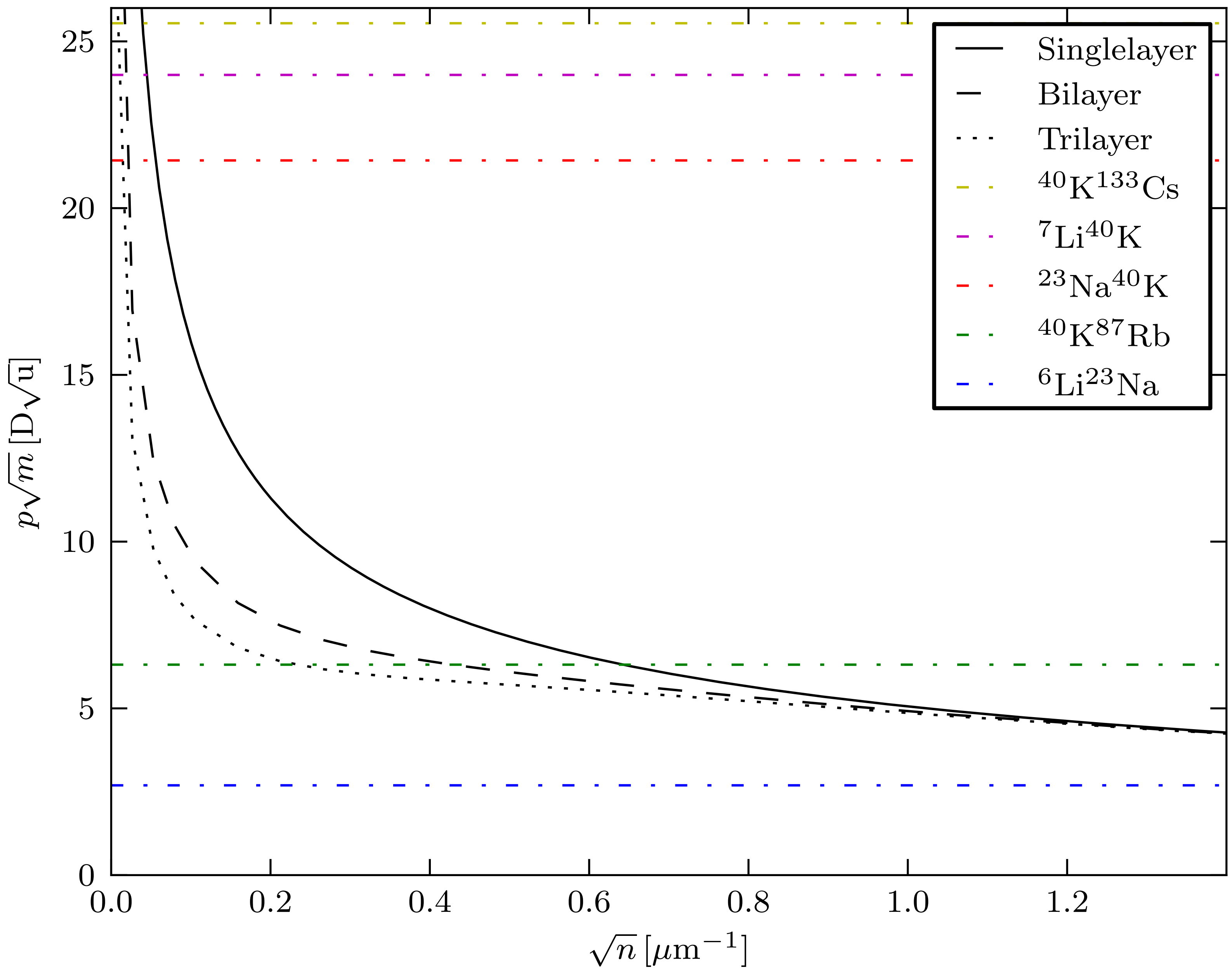


Experimental Realization



$$d = 1064nm/2$$

$$\theta = 0$$

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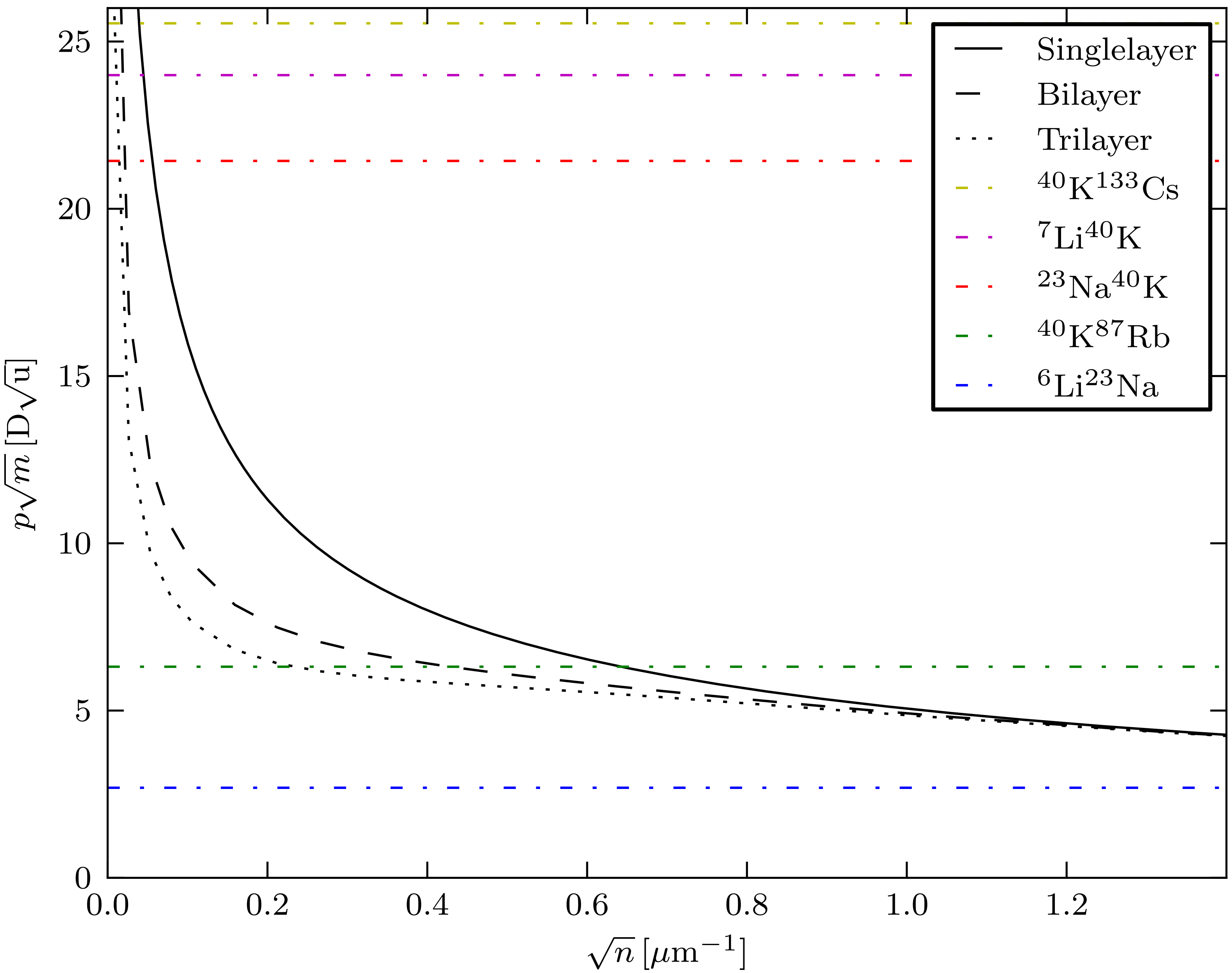

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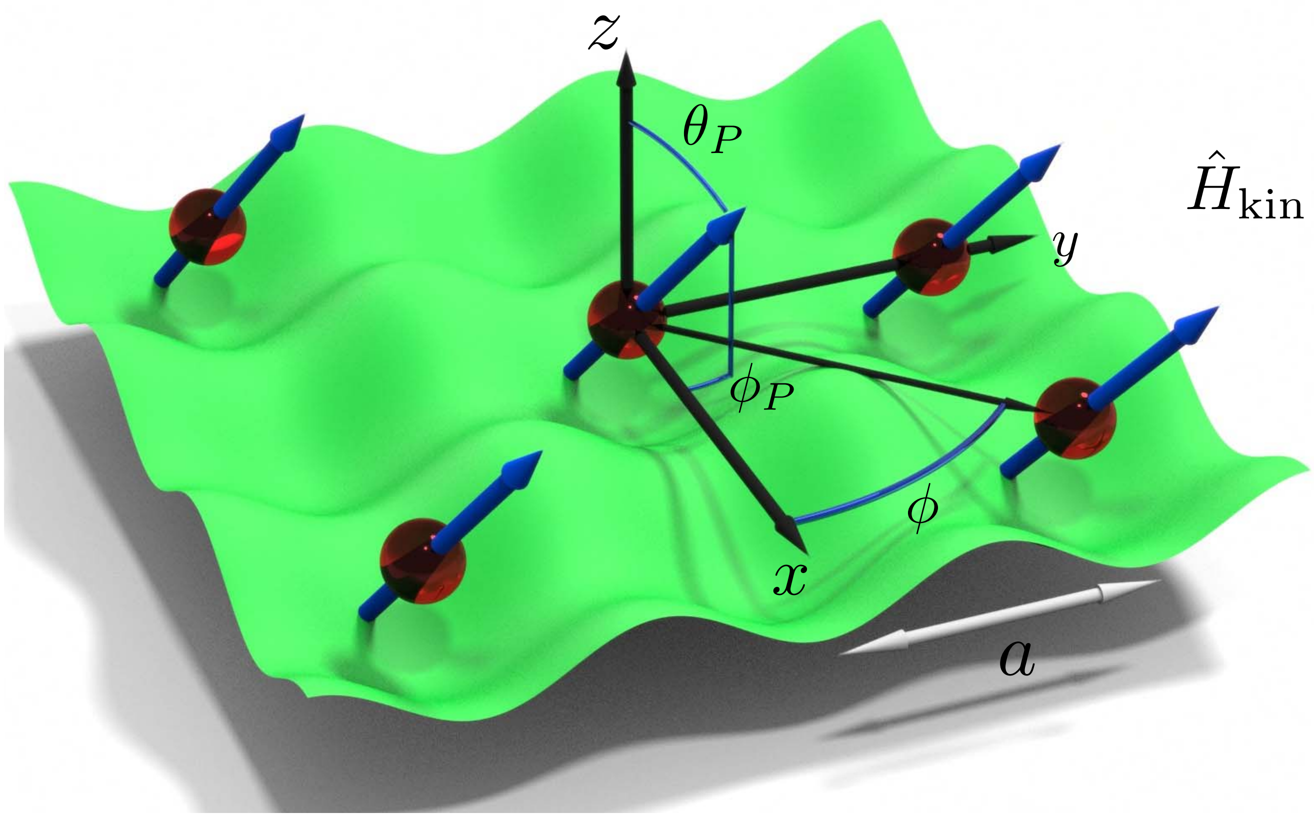
Multilayer effects relevant for:
$^{7}Li^{40}K$, $^{23}Na^{40}K$, $^{40}K^{133}Cs$, $^{6}Li^{87}Rb$, and $^{6}Li^{133}Cs$

Dipoles in a 2D square lattice



Trap included <u>exactly</u>

$$\hat{H}_{\text{kin}} = -t \sum_{\langle ij \rangle} \left(\hat{c}_i^\dagger \hat{c}_j + h.c. \right) + \sum_i \left(\frac{1}{2} m\omega^2 r_i^2 - \mu \right) \hat{n}_i$$

$$\hat{V} = \frac{1}{2} \sum_{i \neq j} V_D(\mathbf{r}_{ij}) \hat{n}_i \hat{n}_j$$

Very rich physics: density order, bond solid order, p-wave superfluidity, supersolidity, liquid crystals ...

K. Mikelsons and J. K. Freericks, PRA **83**, 043609 (2011); L. He and W. Hofstetter, PRA **83**, 053629 (2011); S. G. Bhongale *et al.*, PRL **108**, 145301 (2012); C. Lin, E. Zhao, and W.V. Liu, PRB **81**, 045115 (2010); ...

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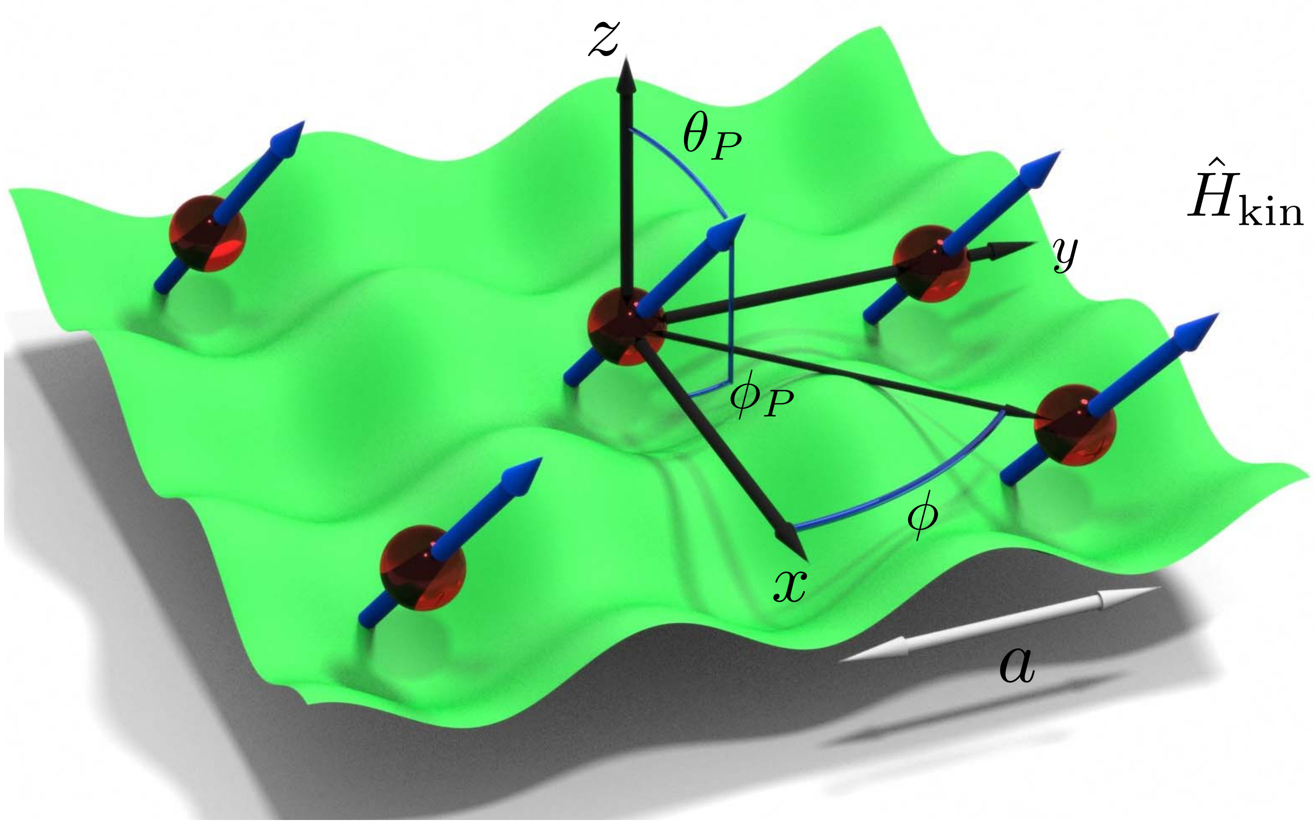

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Capture phases and their competition at T=0 with mean-field theory: Bogoliubov-de Gennes equations

$$\sum_j \begin{pmatrix} L_{ij} & \Delta_{ij} \\ \Delta^*_{ji} & -L_{ij} \end{pmatrix} \begin{pmatrix} u^j_\eta \\ v^j_\eta \end{pmatrix} = E_\eta \begin{pmatrix} u^i_\eta \\ v^i_\eta \end{pmatrix},$$

Include density- and superfluid order

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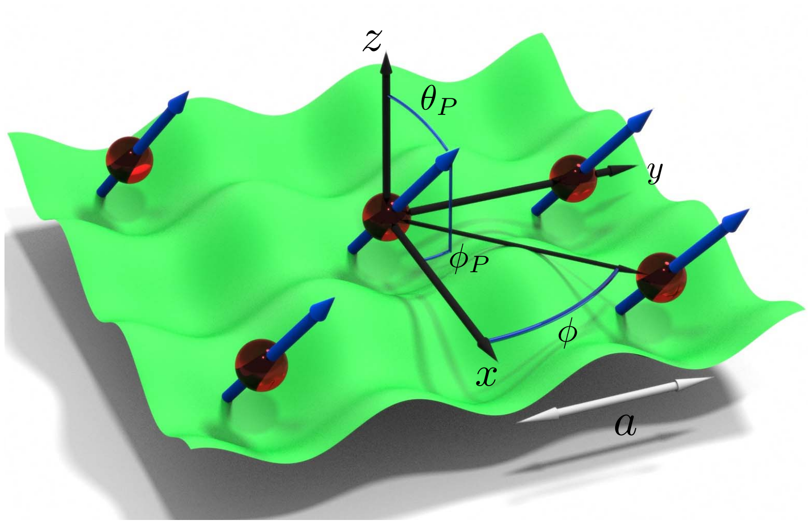
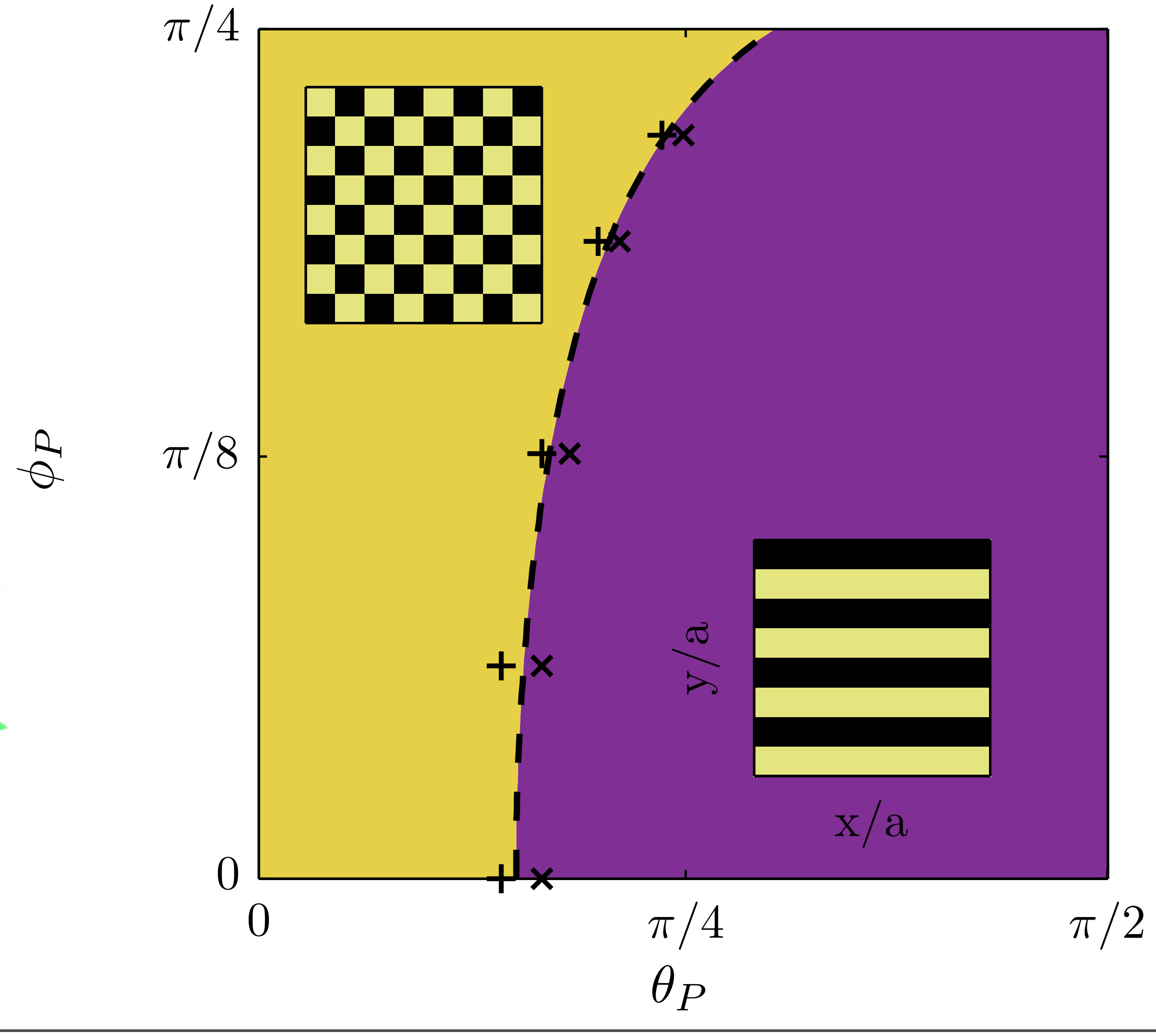
Strong coupling limit, half filling: $g \gg t$

$g = D^2/a^3$





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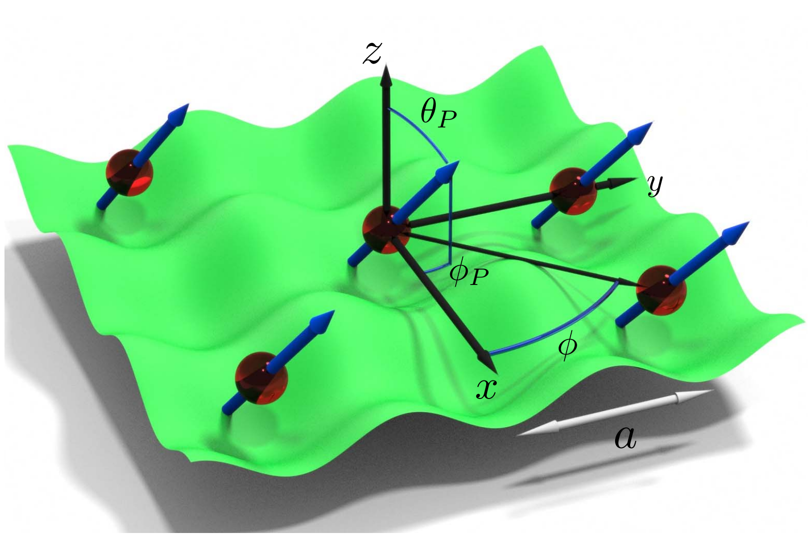

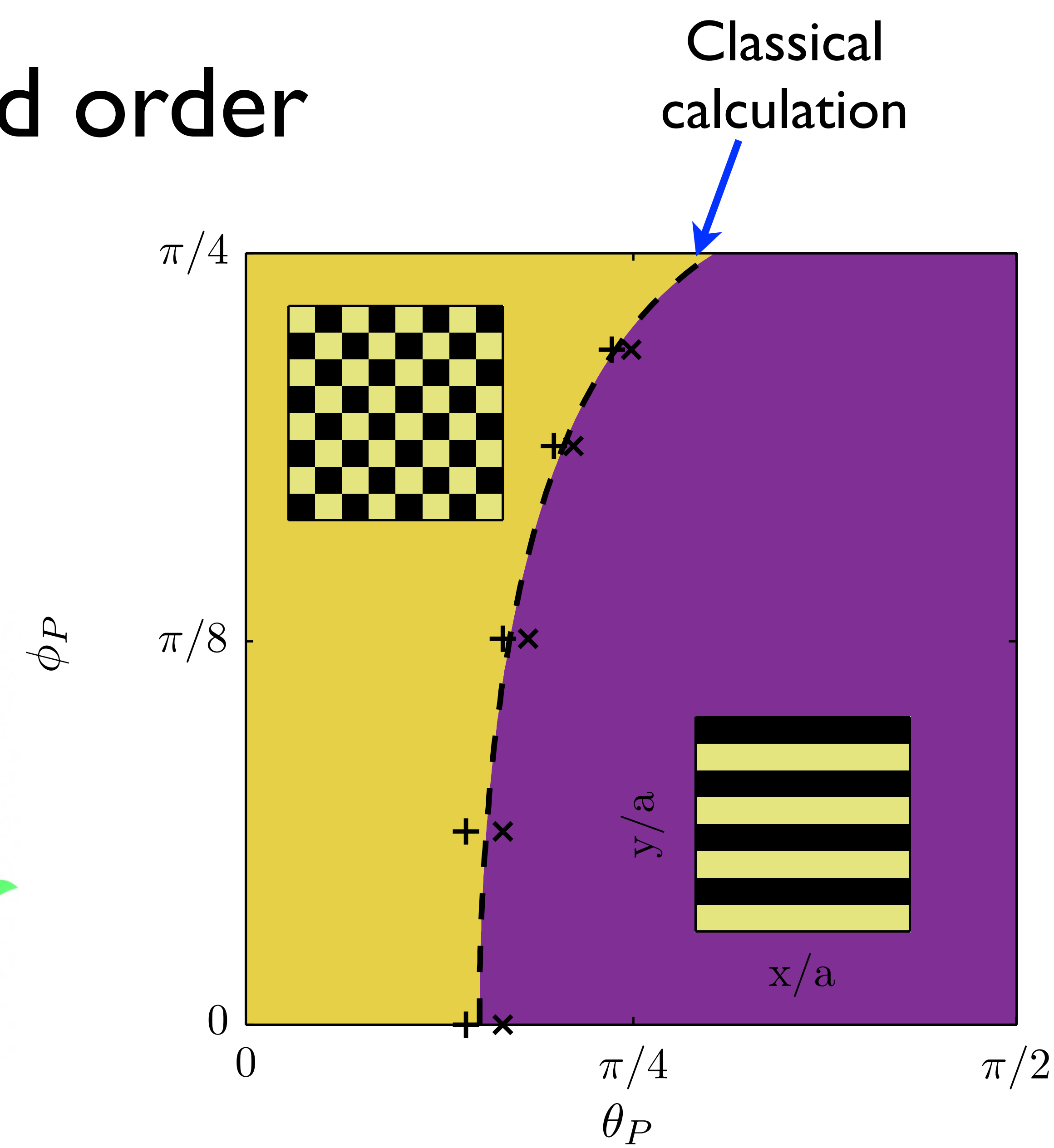

Melting of density ordered phases

$$1 = \frac{1}{N_L} \sum_{k_y > 0} \frac{\tilde{V}_D(0, \pi/a)(f_{1\mathbf{k}} - f_{2\mathbf{k}})}{\sqrt{(2t \cos k_y)^2 + [\tilde{V}_D(0, \pi/a) M/2]^2}}$$

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Strong coupling
Half filling

$$g \gg t$$

$$T_c = -\frac{1}{4} \times \begin{cases} \tilde{V}_D(0,\pi) & \text{Stripes} = 1.27g \\ \tilde{V}_D(\pi,\pi) & \text{CB} \quad = 0.66g \end{cases}$$

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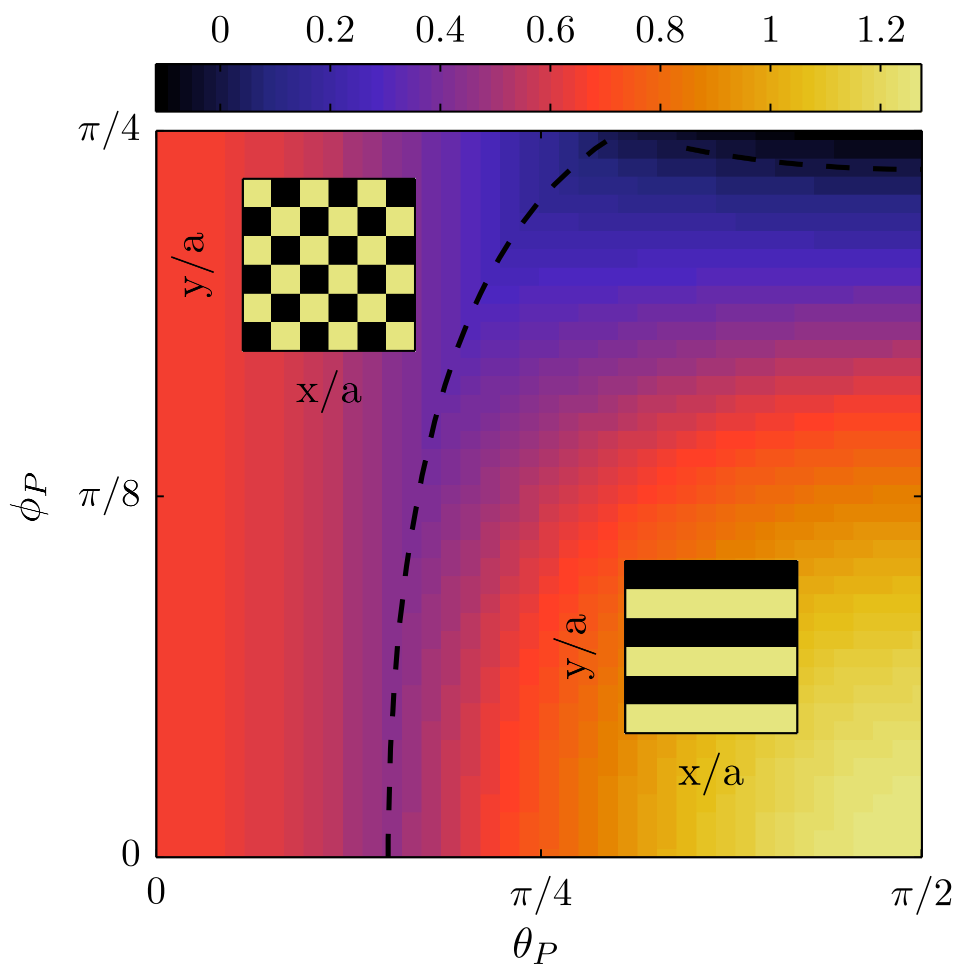

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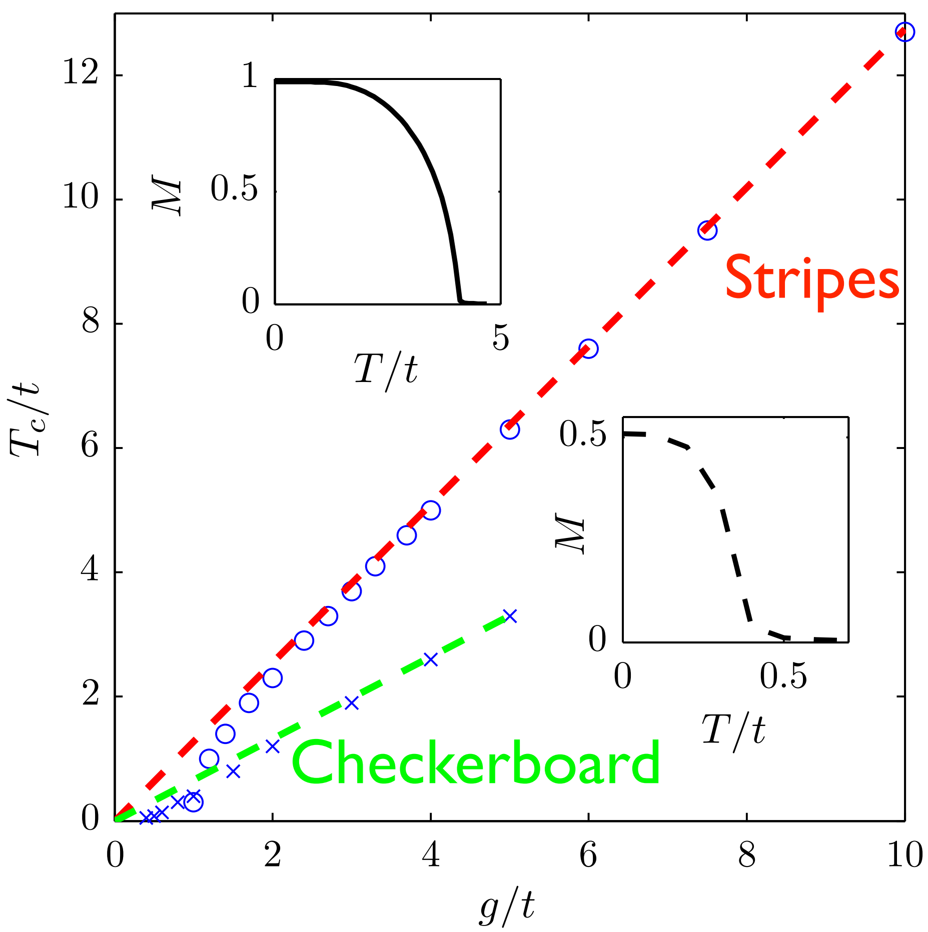
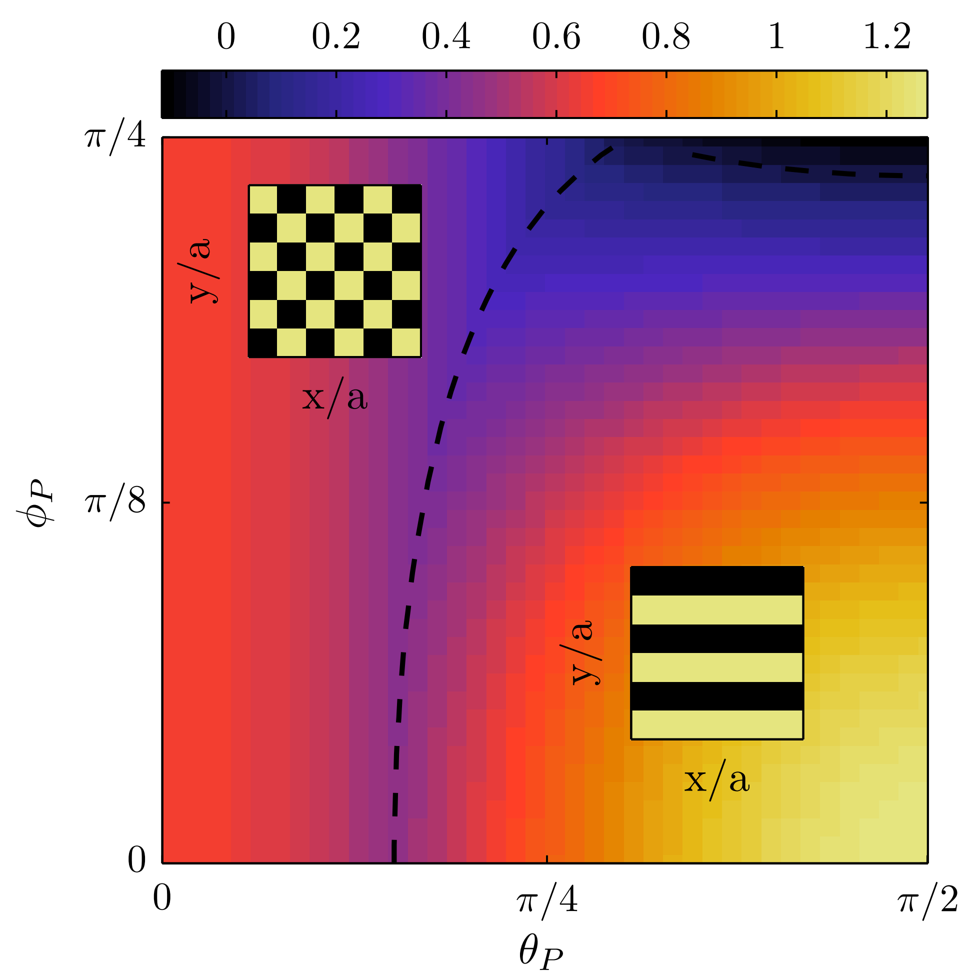
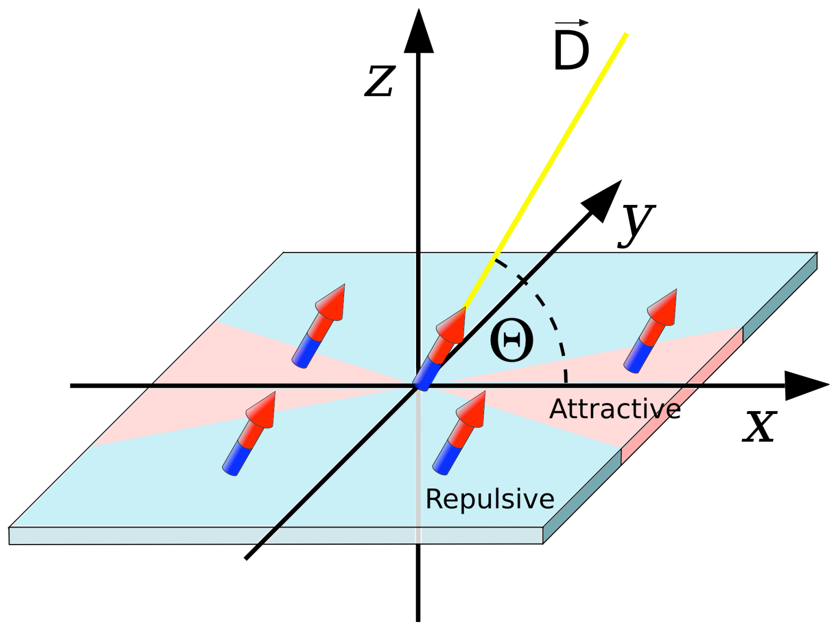




Away from half-filling and $\theta_P > \arcsin(1/\sqrt{3})$



p-wave pairing

GMB and E. Taylor, PRL **101**, 245301 (2008)

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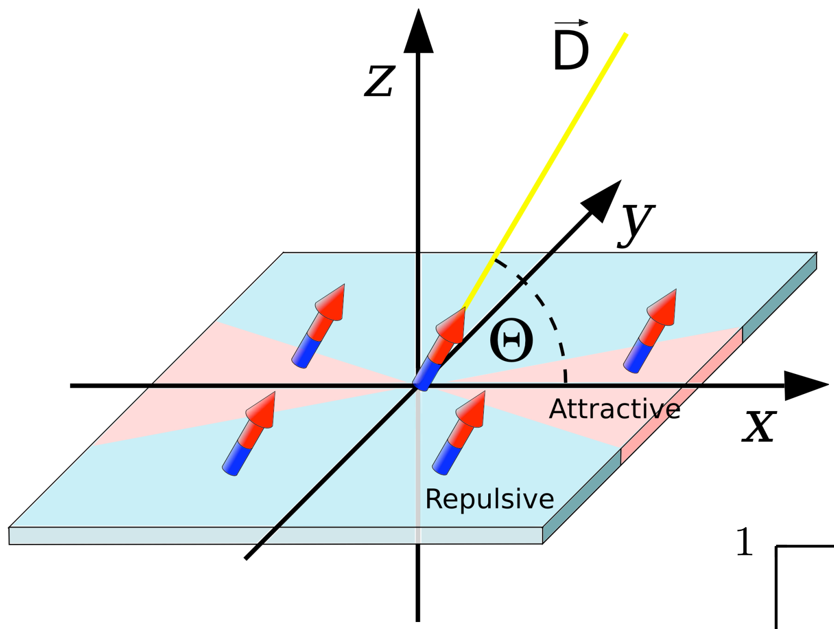

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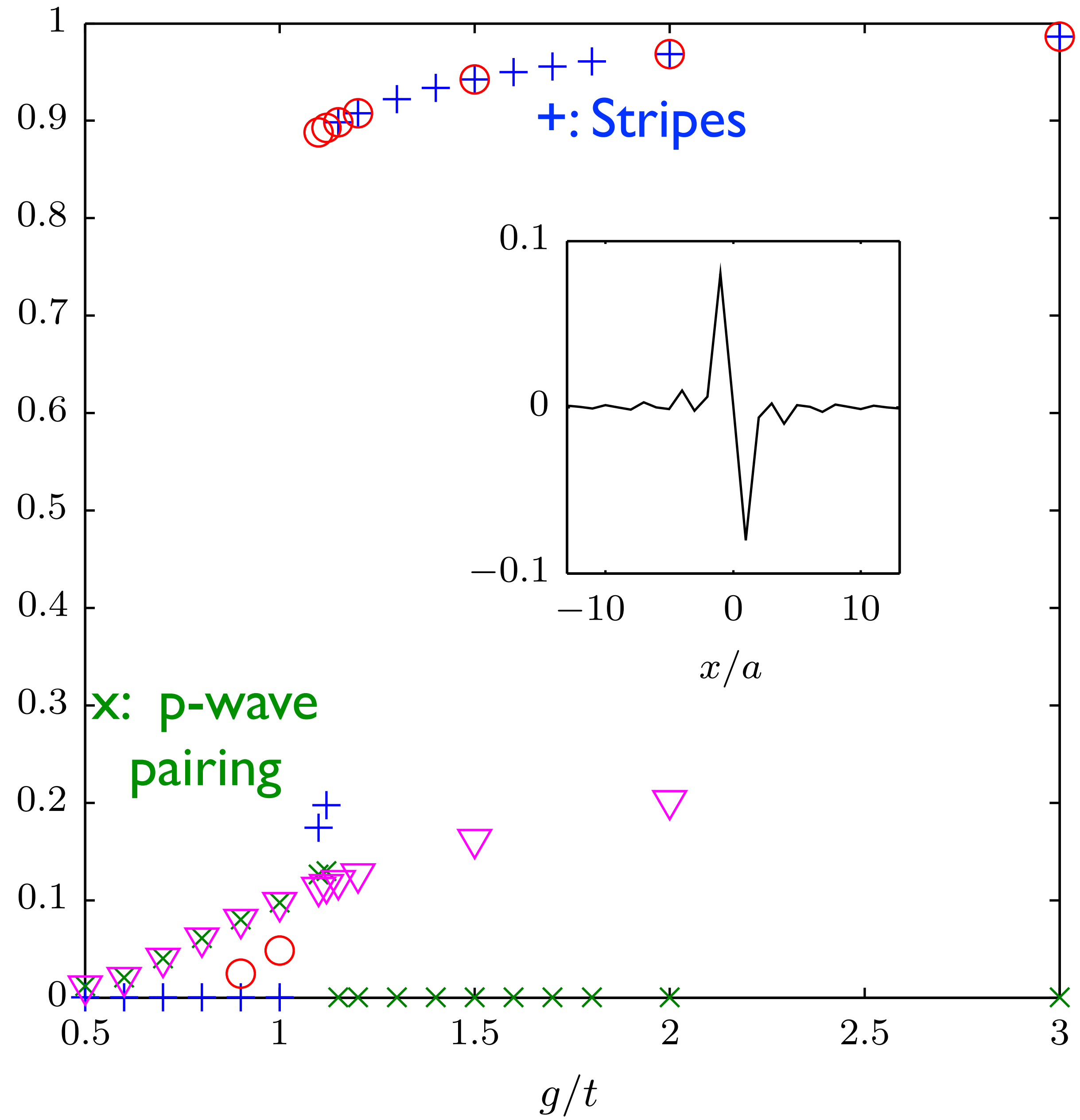

Berezinskii-Kosterlitz-Thouless melting of superfluid phase

Twist of order parameter: $\Delta_{ij} \rightarrow \Delta_{ij} e^{i(x_i + x_j)\delta\theta/a}$

Phase stiffness

Cost in free energy: $F_\Theta - F_0 \simeq \frac{J_x}{2} \sum_i \delta\Theta^2 = \frac{N}{2} \rho_{s,x} m^* {v_s}^2$

Superfluid fraction

Superfluid velocity

$$v_s = \frac{\delta\Theta}{2m^* a}$$

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Equivalent to gauge transform

$$\hat{H}_\Theta = e^{-i\delta\theta\sum_l \hat{x}_l/a}\hat{H}e^{i\delta\theta\sum_l \hat{x}_l/a}$$

E. H. Lieb and R. Seiringer, PRB **66**, 134529 (2009)

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$$v_s = \frac{\delta\Theta}{2m^* a}$$

Equivalent to gauge transform

$$\hat{H}_\Theta = e^{-i\delta\theta\sum_l \hat{x}_l/a}\hat{H}e^{i\delta\theta\sum_l \hat{x}_l/a}$$

E. H. Lieb and R. Seiringer, PRB **66**, 134529 (2009)

$$T_{\mathrm{BKT}} = \pi\frac{\bar{J}}{2} = \frac{\pi}{4}\frac{N}{N_L}\bar{\rho}_s t$$

Berezinskii-Kosterlitz-Thouless melting of superfluid phase

Twist of order parameter: $\Delta_{ij} \to \Delta_{ij} e^{i(x_i + x_j)\delta\theta/a}$

Phase stiffness

Cost in free energy: $F_\Theta - F_0 \simeq \frac{J_x}{2} \sum_i \delta\Theta^2 = \frac{N}{2} \rho_{s,x} m^* {v_s}^2$

Superfluid fraction

Superfluid velocity

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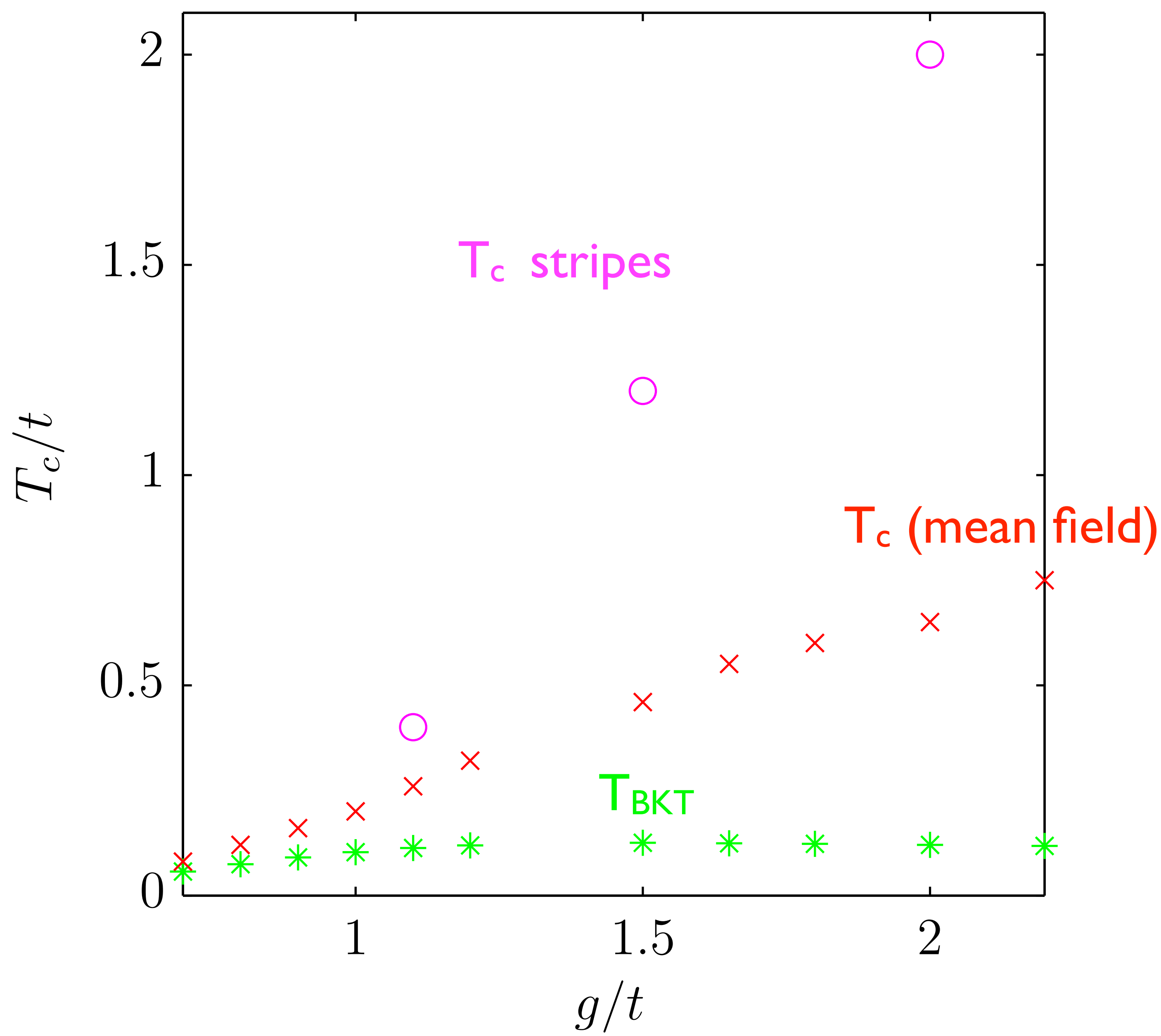
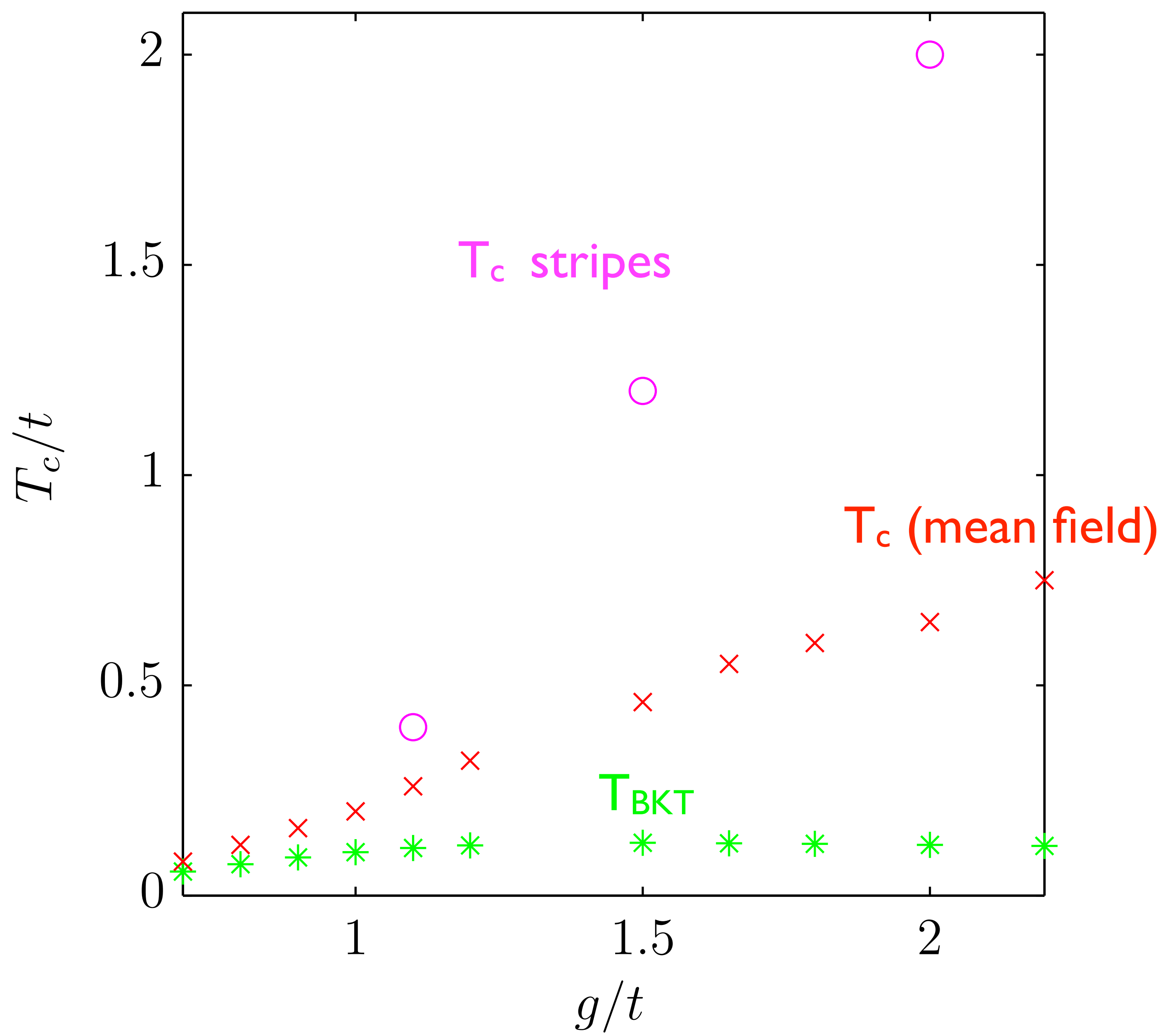
$$T_{\rm BKT} = \pi \frac{\bar{J}}{2} = \frac{\pi}{4} \frac{N}{N_L} \bar{\rho}_s t$$

No trap: $\rho_{s,x} = \frac{1}{N} \sum_{\mathbf{k}} \left[n_{\mathbf{k}} \cos k_x a - \frac{2t}{T} f_{\mathbf{k}} (1 - f_{\mathbf{k}}) \sin^2 k_x a \right]$



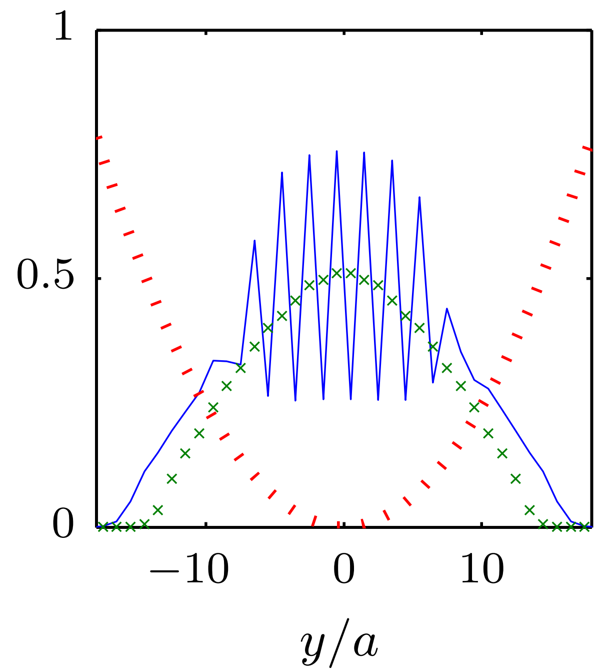
T_c stripes
T_c (mean field)
T_BKT
T_c/t
g/t

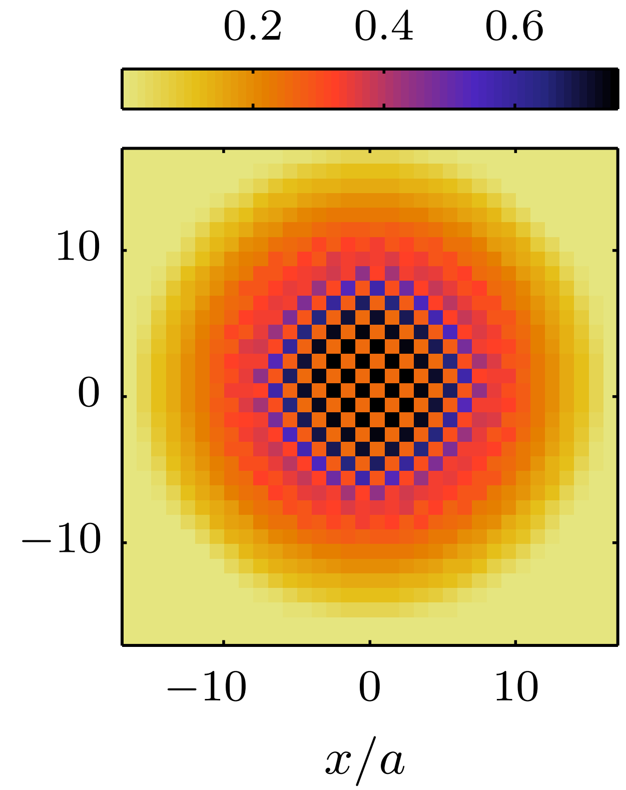


Strong coupling: $T_{\mathrm{BKT}} \sim t$

Trapping potential: ring and island structures

$$\theta_P = 0$$

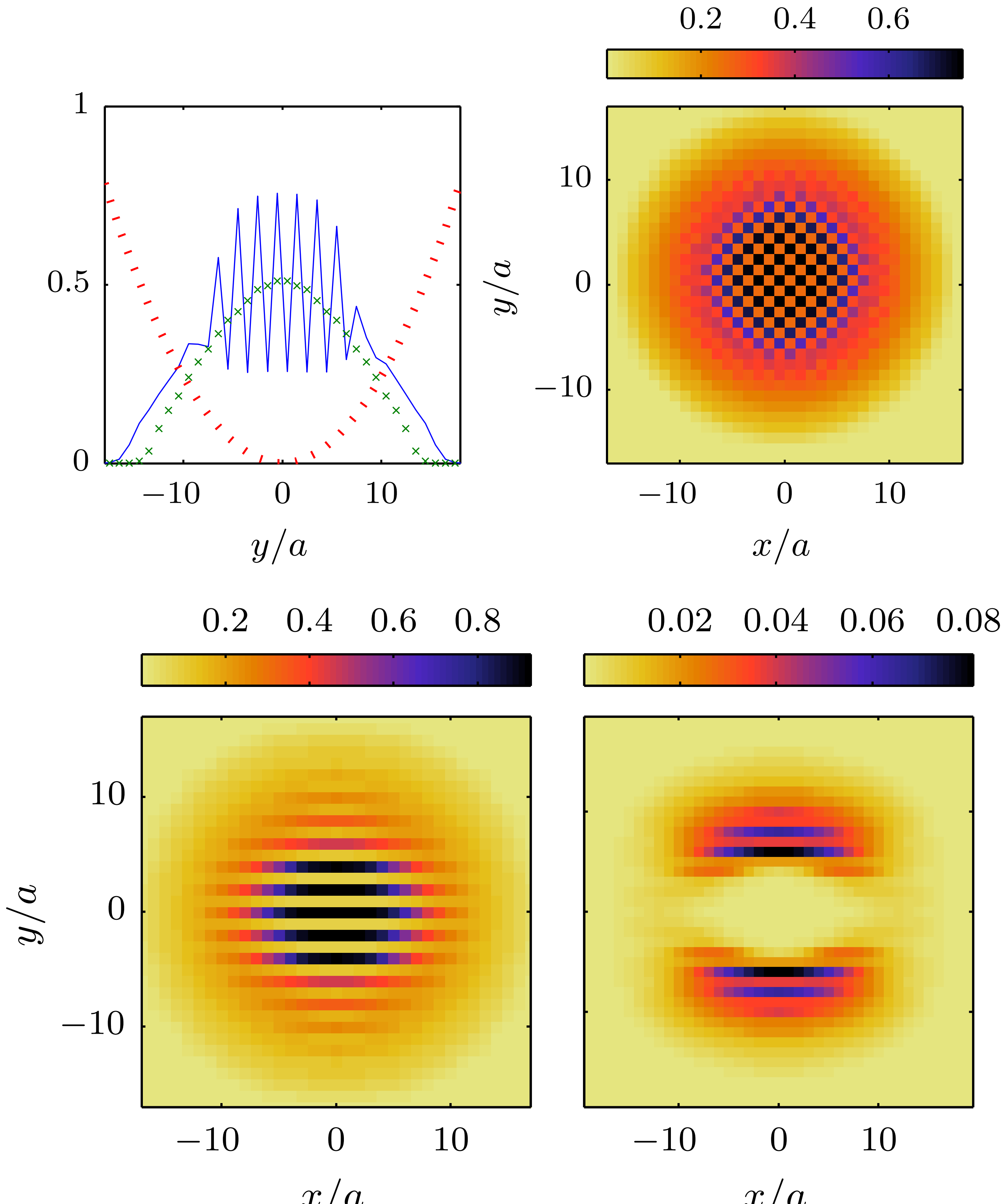




Trapping potential: ring and island structures

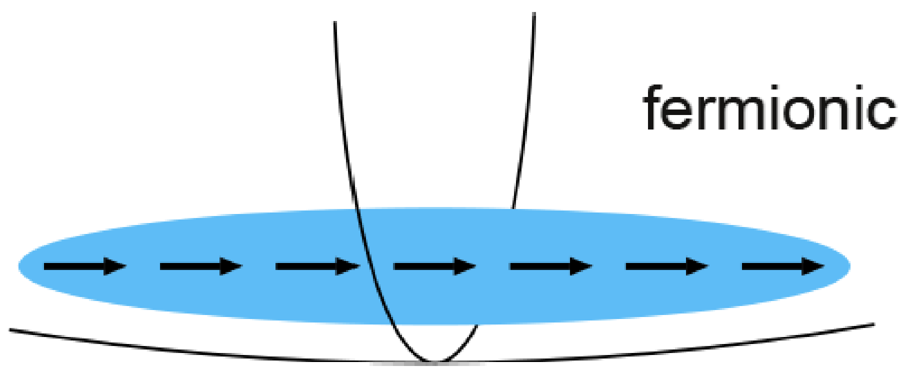
$\theta_P = 0$

$\theta_P = \pi/2$



Self-bound states of a 1D dipolar Fermi gas

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + \sum_{i<j} V_{\mathrm{dd}}(x_i - x_j)$$



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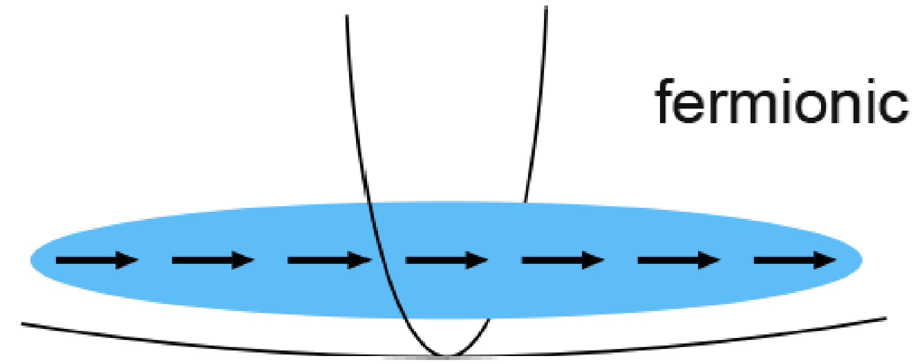

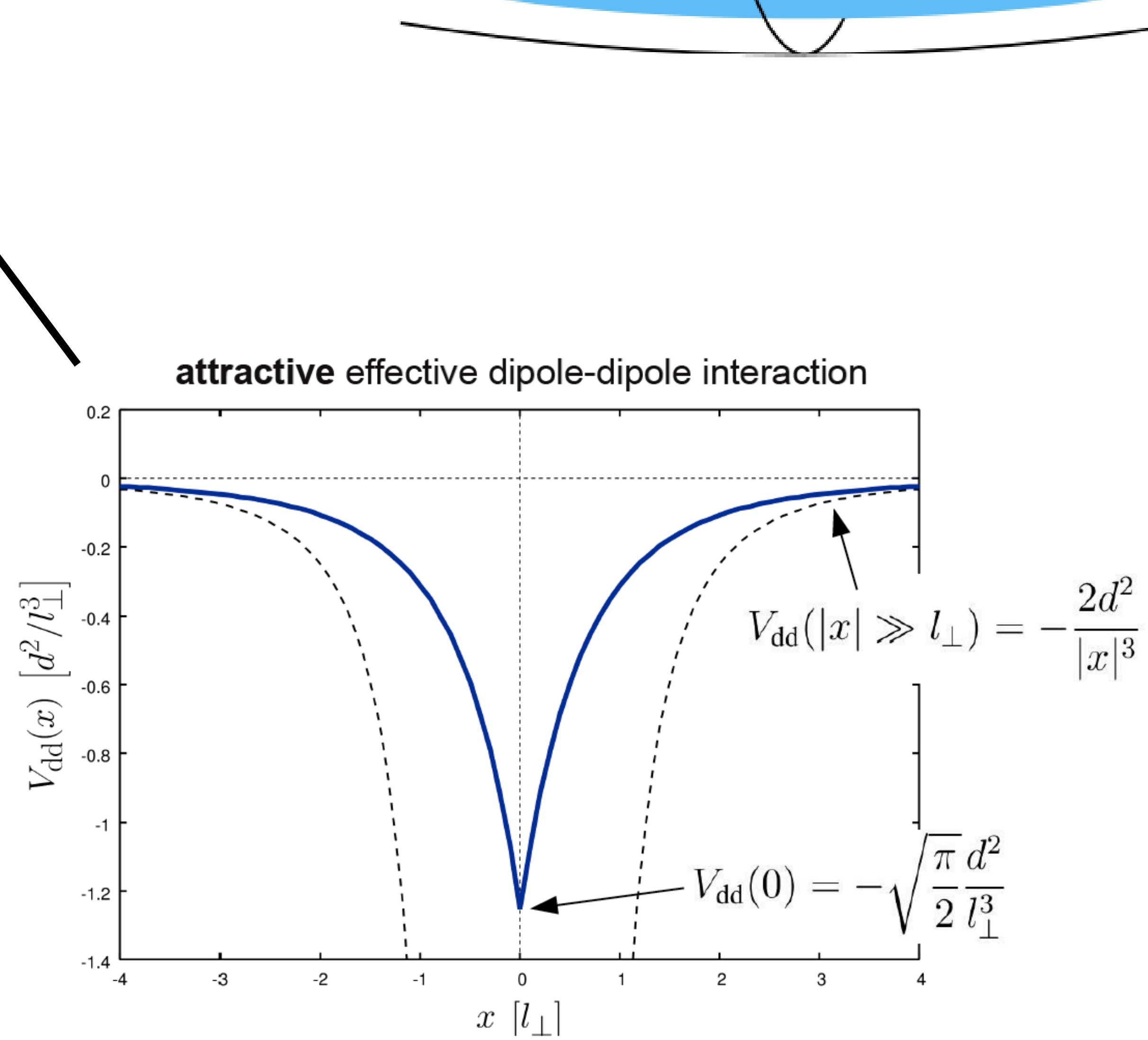

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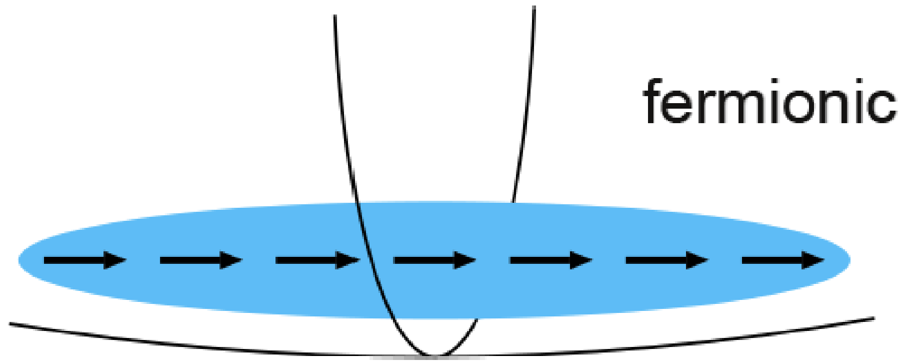
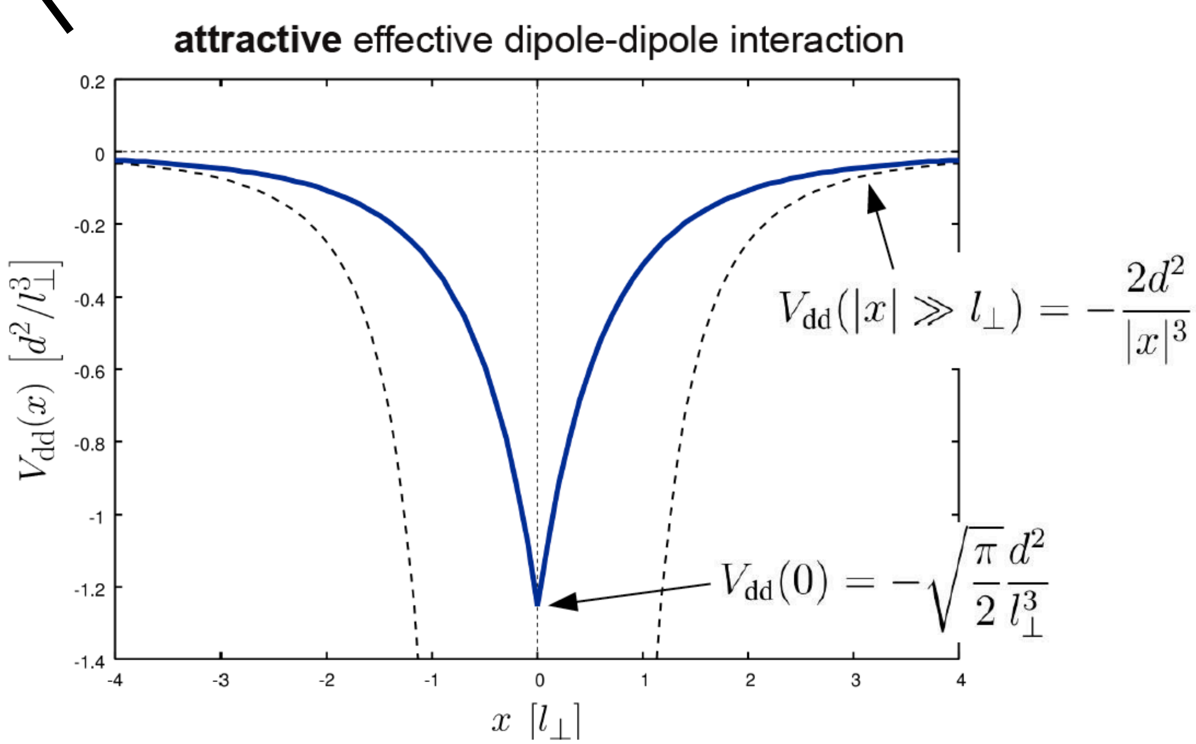
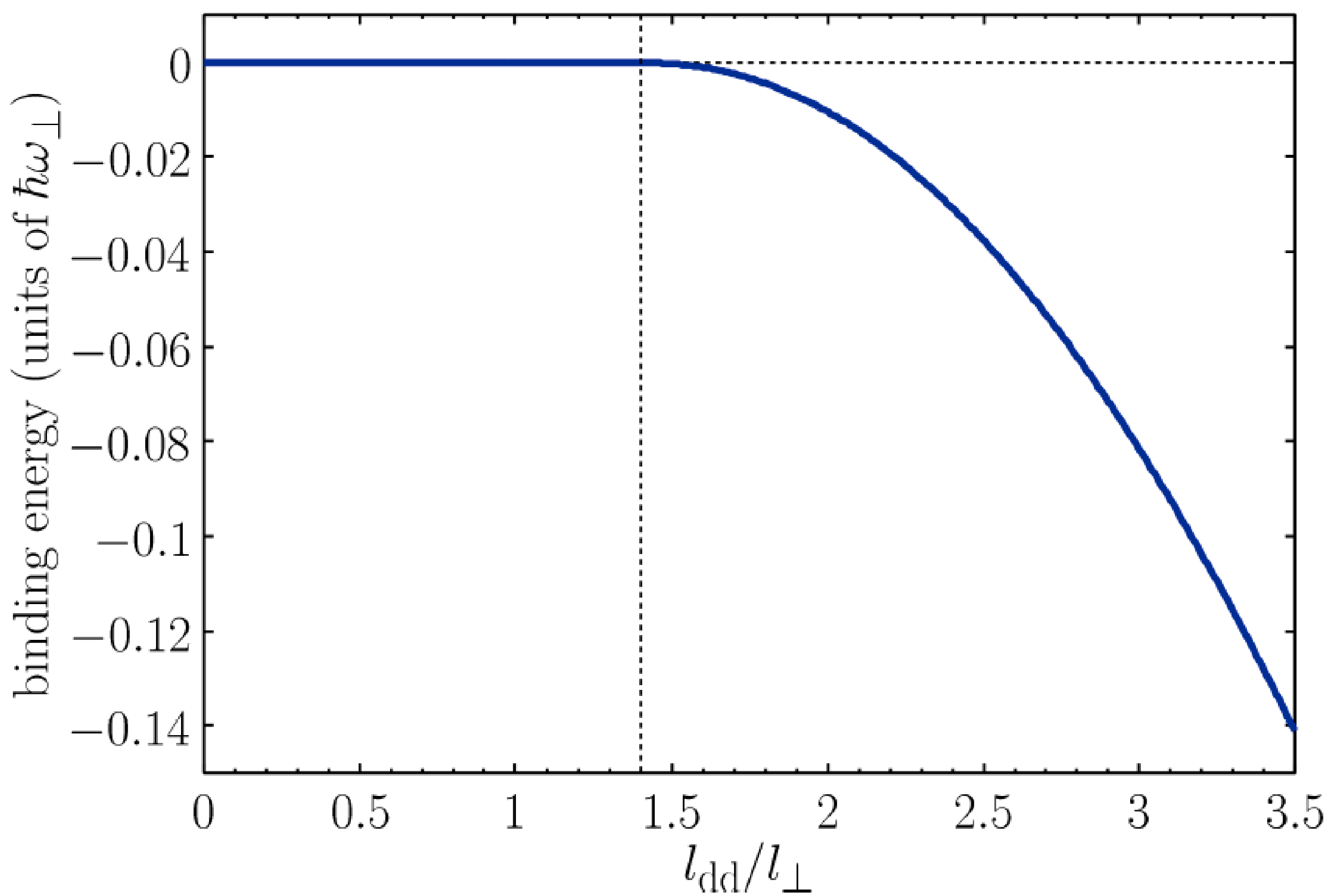


"Short range" in 1D with range $\sim l_\perp$

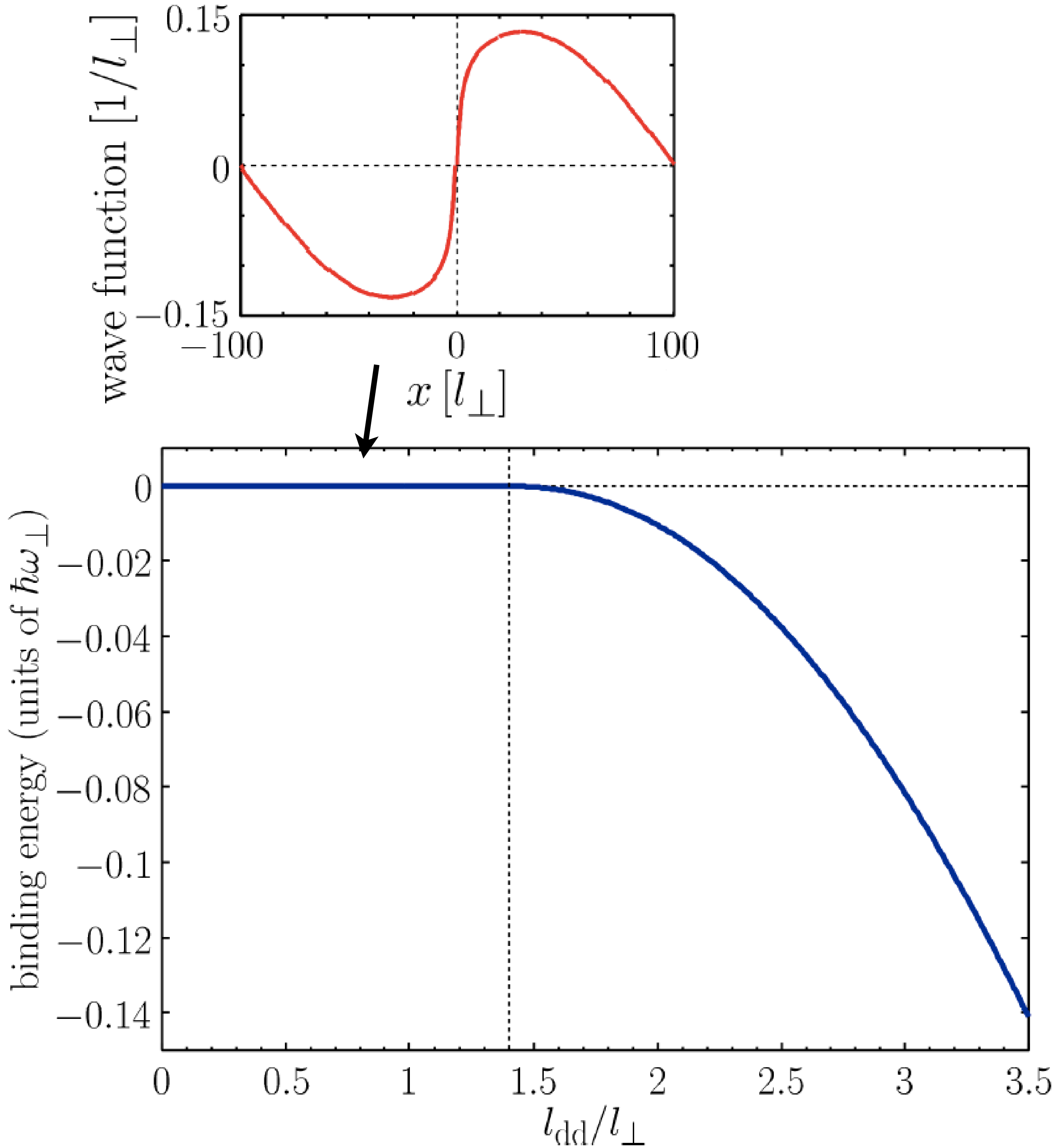


2-body problem:



$$l_{\mathrm{dd}} = d^2 m/\hbar^2$$

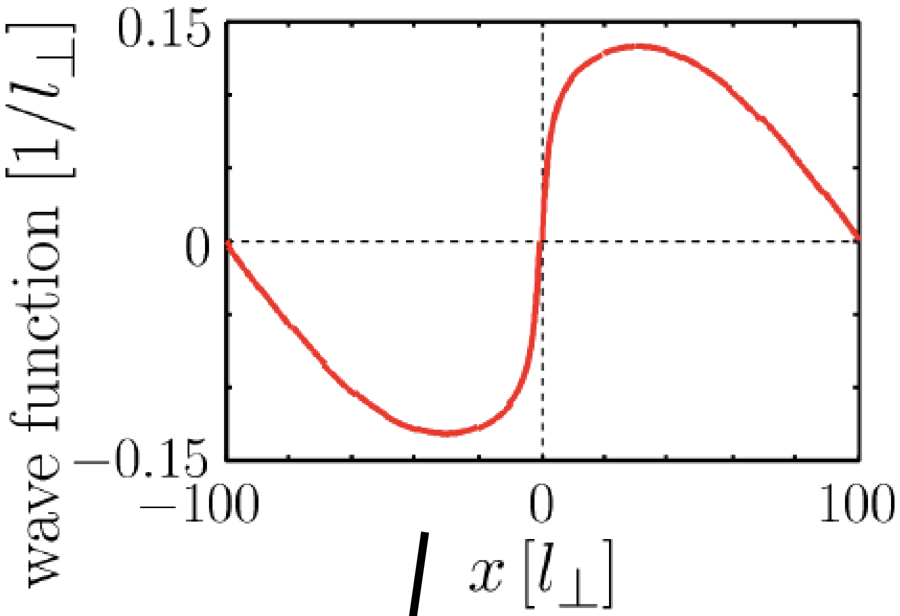
2-body problem:

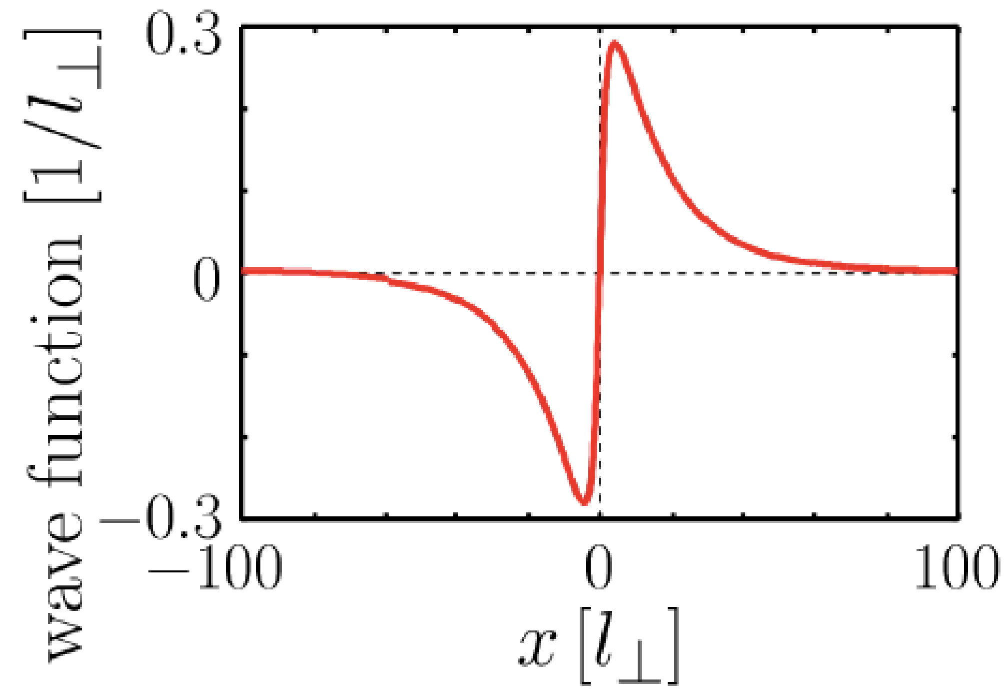


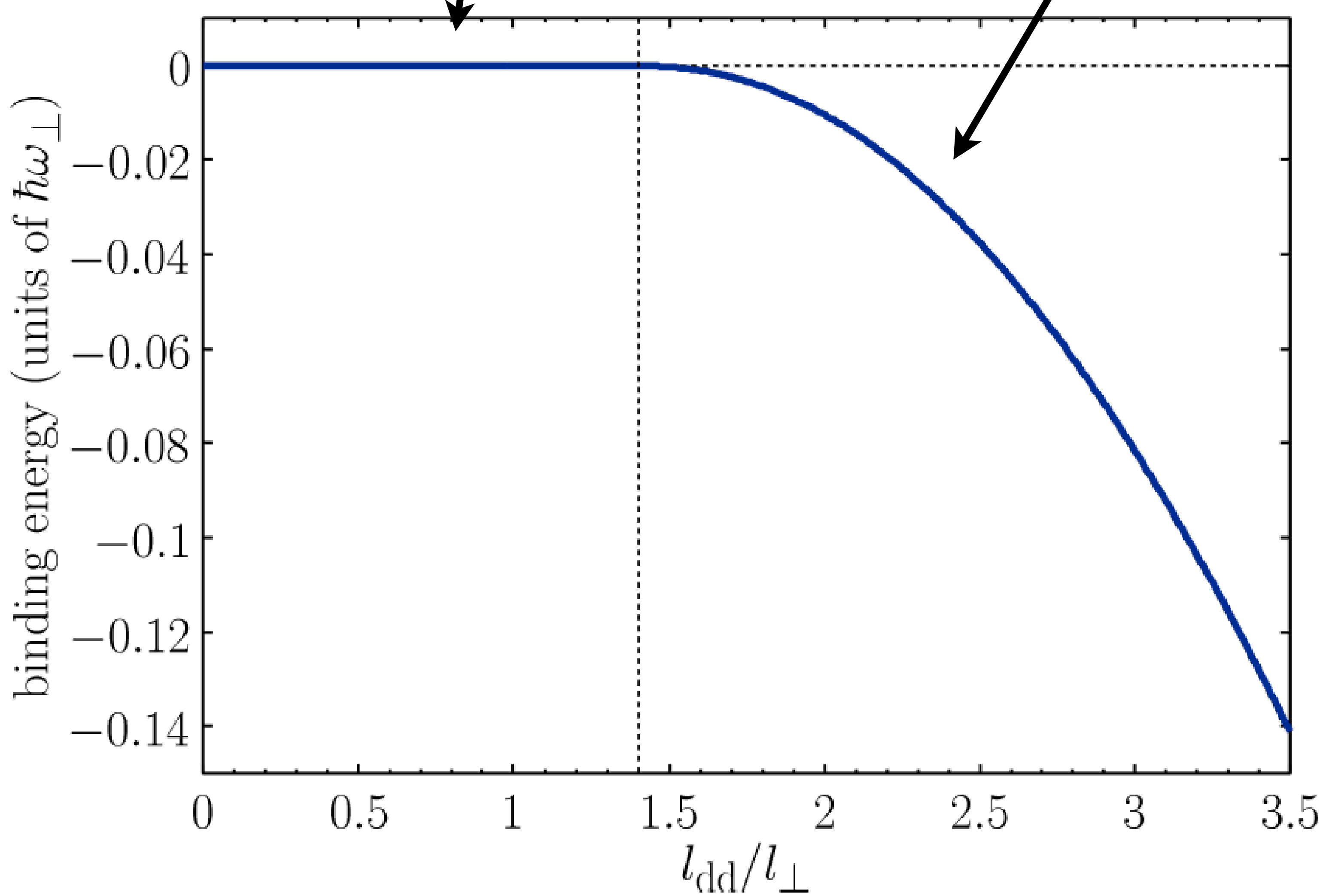
$$l_{\mathrm{dd}} = d^2 m/\hbar^2$$

2-body problem:

Bound state for $l_{\rm dd}/l_\perp \gtrsim 1.4$







Outside range of potential:

$$\psi(x) \propto \mathrm{sgn}(x) e^{-x/\rho}$$

$$\rho = 1/\sqrt{mE_B}$$

$$l_{\rm dd} = d^2 m/\hbar^2$$

N-body problem:

Interaction short range ⇒ use exact results for 1D

fermions interacting via a "p-wave" contact interaction

Mapping from a fermionic to a bosonic N-body state

F. Calogero and A. Degasperis, PRA **11** 265 (1975)
S. A. Bender, K. D. Erker and B. E. Granger, PRL **95**, 230404 (2010)
T. Cheon and T. Shigehara, PRL **82**, 2536 (1999)

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$$\psi \propto \prod_{i<j} \mathrm{sgn}(x_i - x_j) e^{-|x_i - x_j|/\rho} \qquad \rho = -2/m g_{1D}^B$$

$$n(0) \propto |g_{1D}^B| N^2$$

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Collapse for N→∞. Stopped by short range physics for x~$l_\perp$

F. Calogero and A. Degasperis, PRA **11** 265 (1975)
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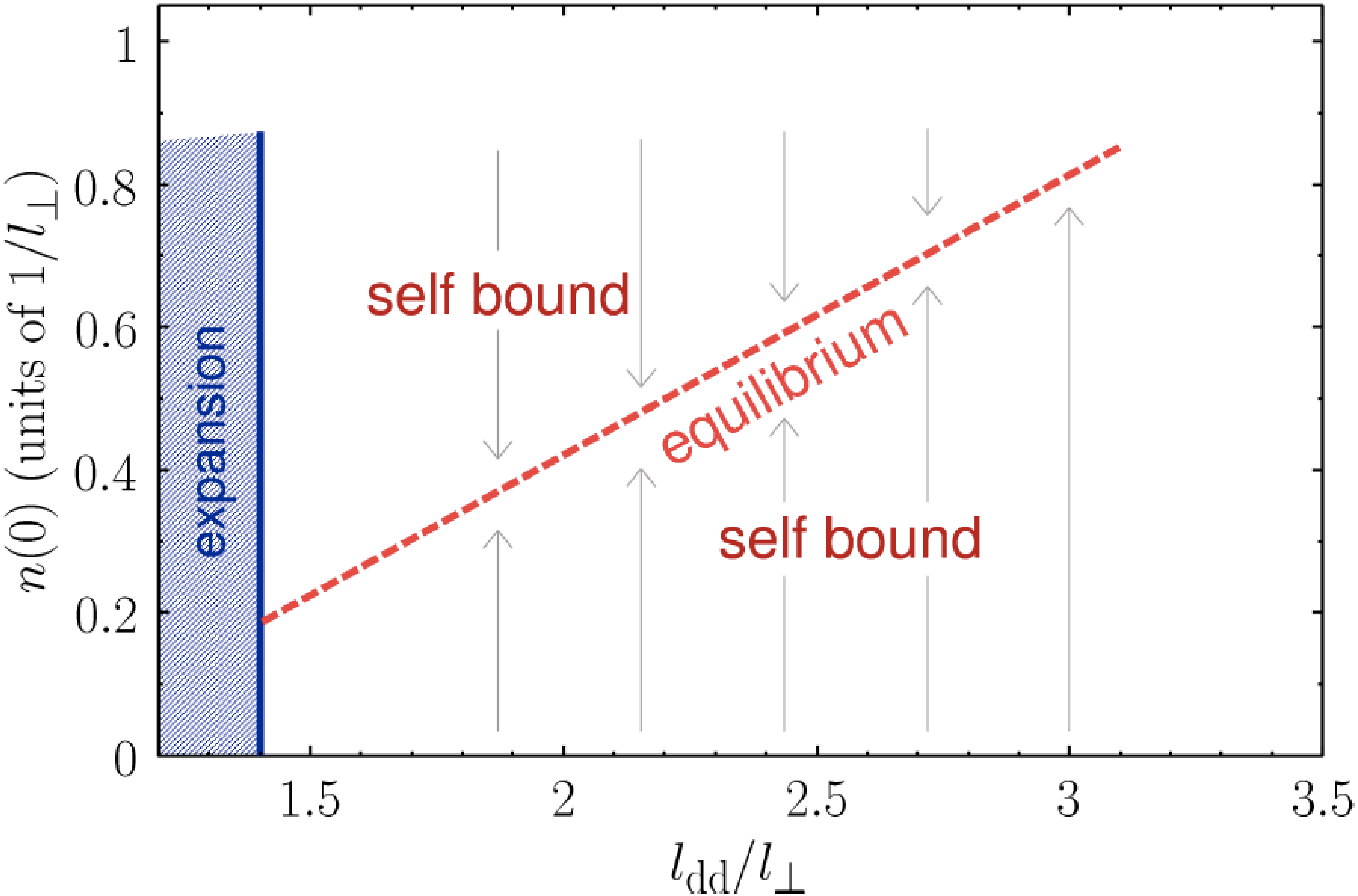
Tentative phase diagram



Hartree-Fock calculations gives the energy per particle:

$$\epsilon(\kappa_F) = \frac{\kappa_F^2}{8} - \frac{8 l_{\rm dd}}{3\pi^2 l_\perp}\left\{\kappa_F - \frac{3}{2}\left[I_a(\kappa_F) - \frac{I_l(\kappa_F)}{4\kappa_F}\right]\right\}$$

kinetic energy

direct energy

exchange energy

$$E_{\rm d} + E_{\rm ex} \to 0$$

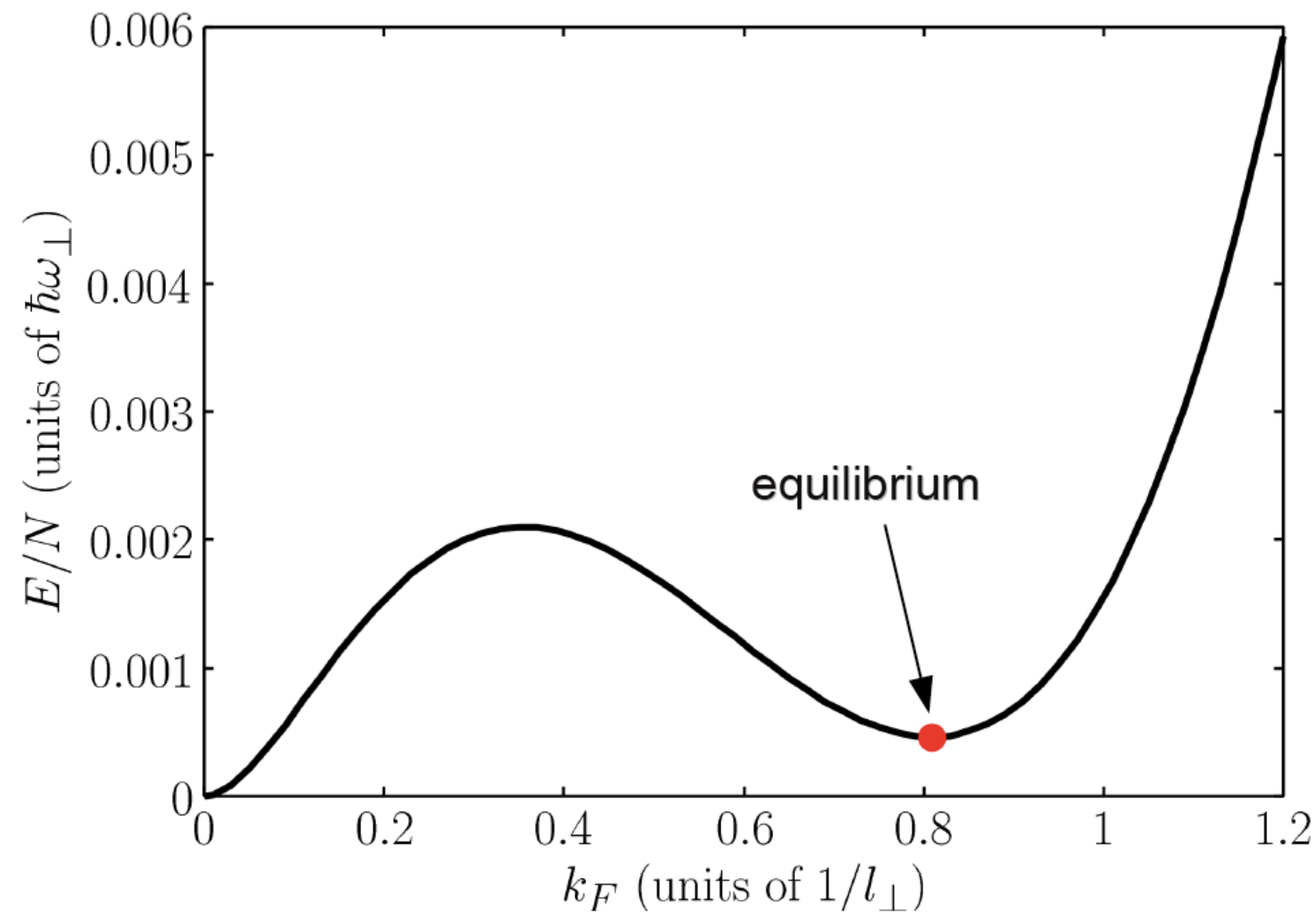
for $\kappa_F \to 0$

(short range)

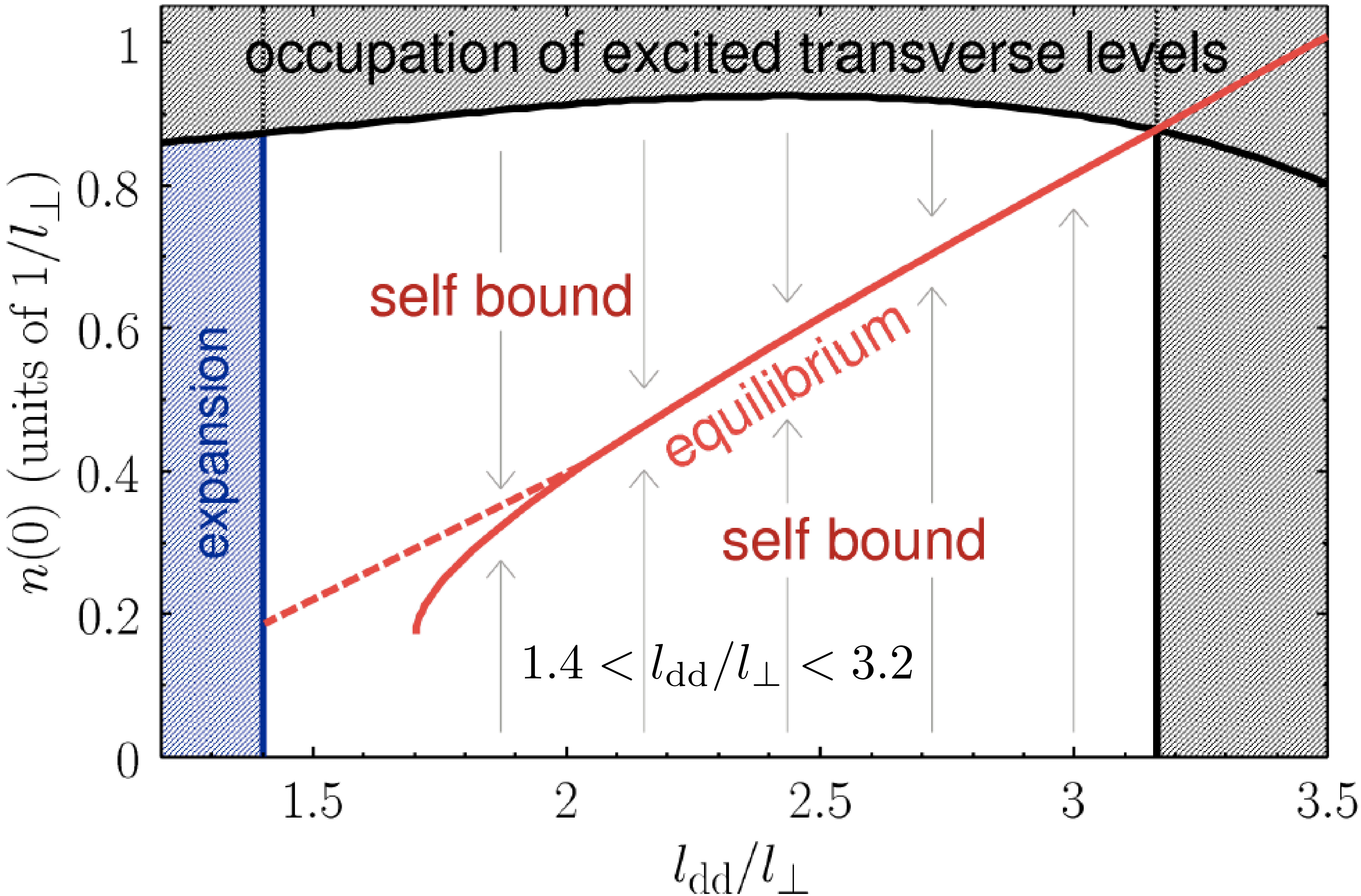
$$E_{\rm d} + E_{\rm ex} \propto -\kappa_F$$

for $\kappa_F \gg 1$

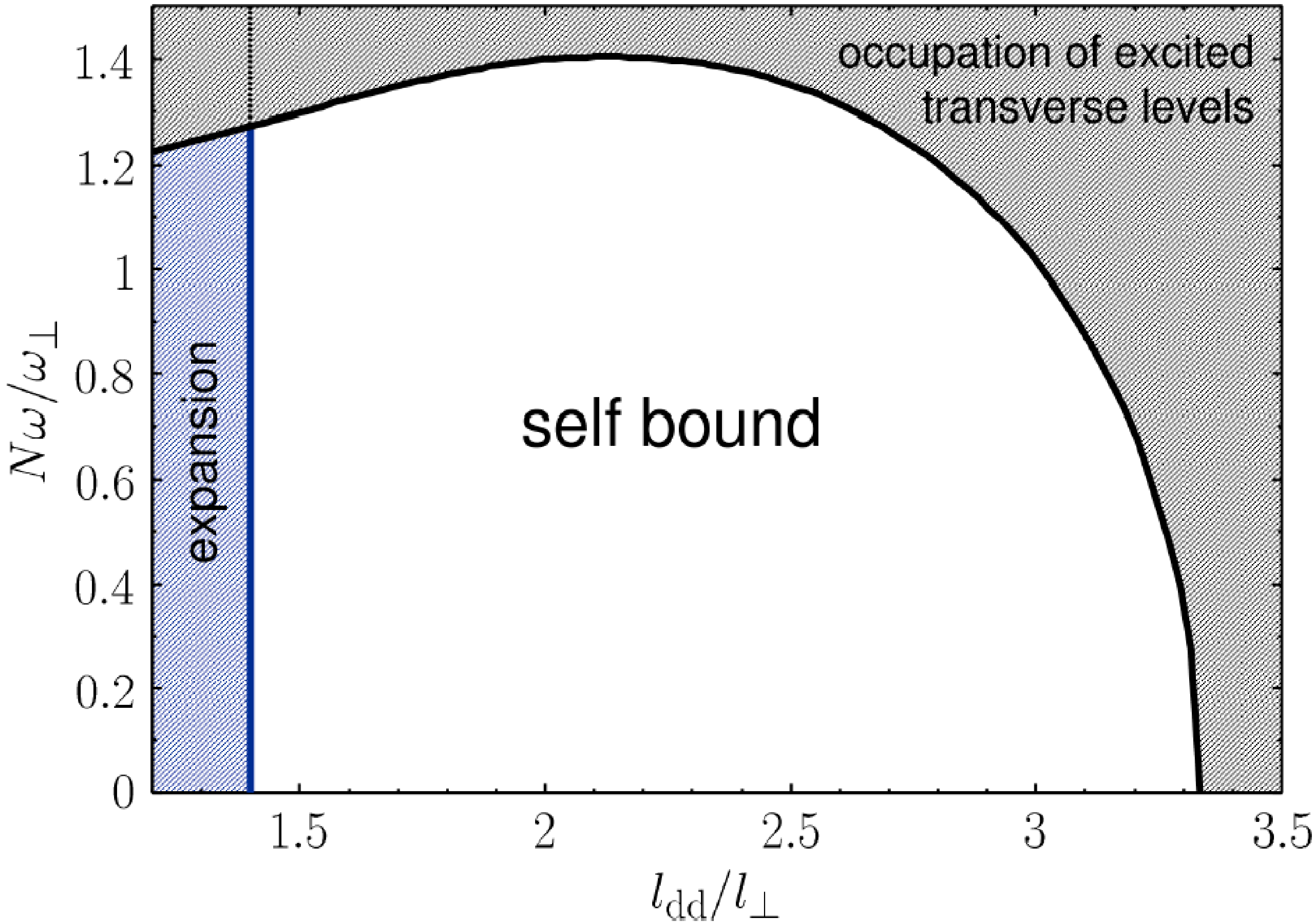
(Hartree)



Phase diagram



Release of the trap:



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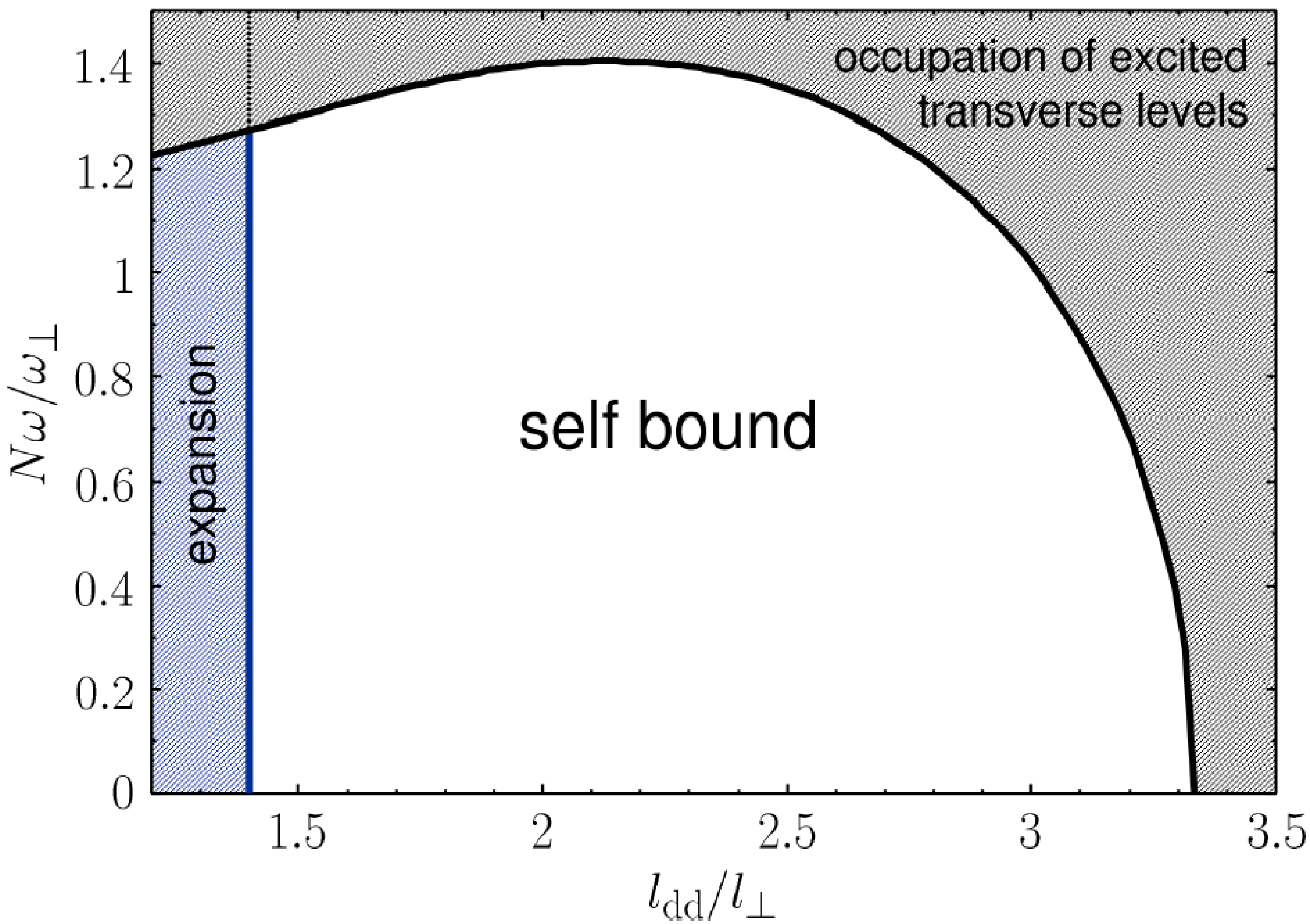

Experiment with $^{23}Na^{40}K$ (2.72Debye fully polarised):

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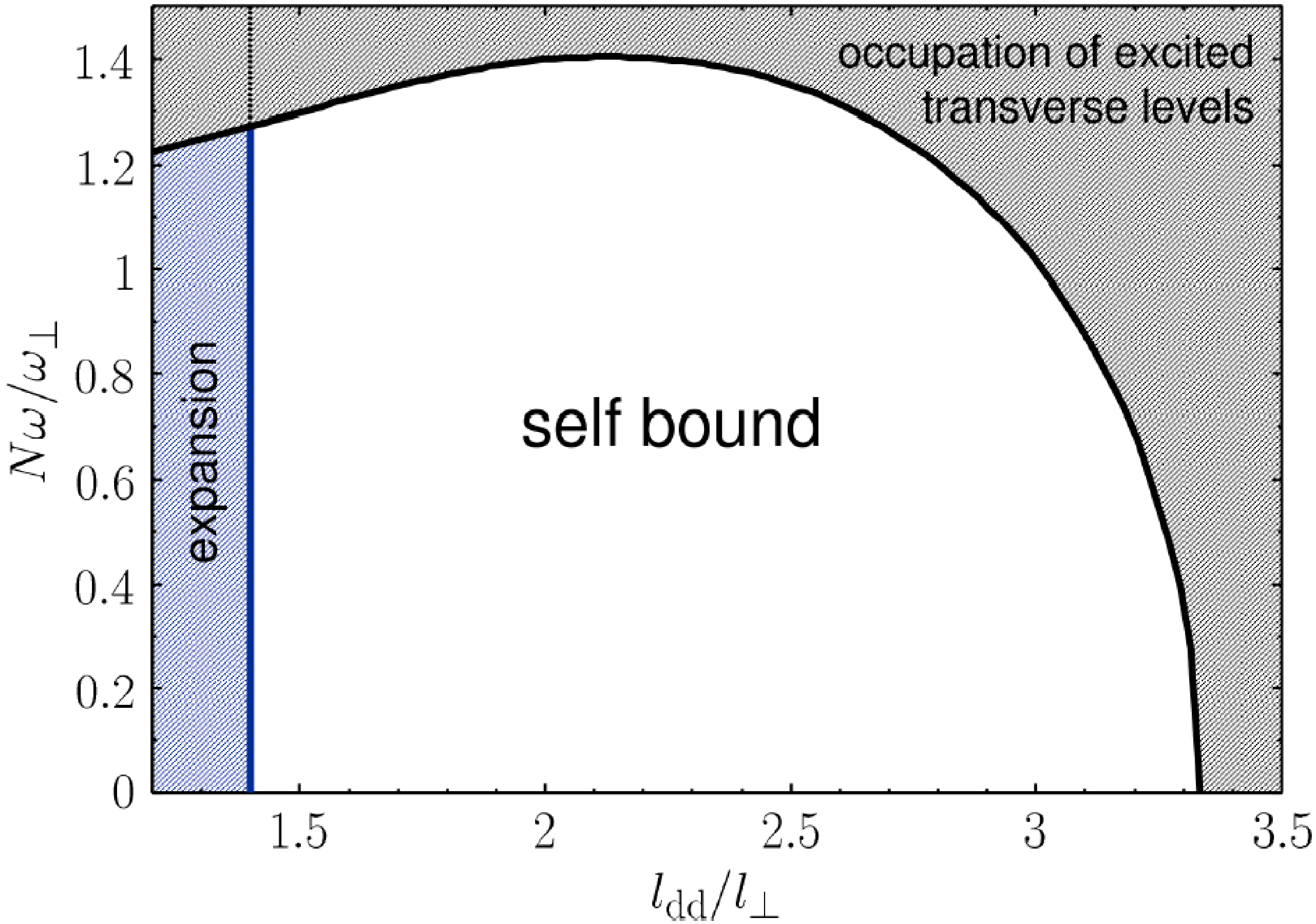

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1Debye partially polarised $l_{\rm dd} = 1\mu\mathrm{m}$

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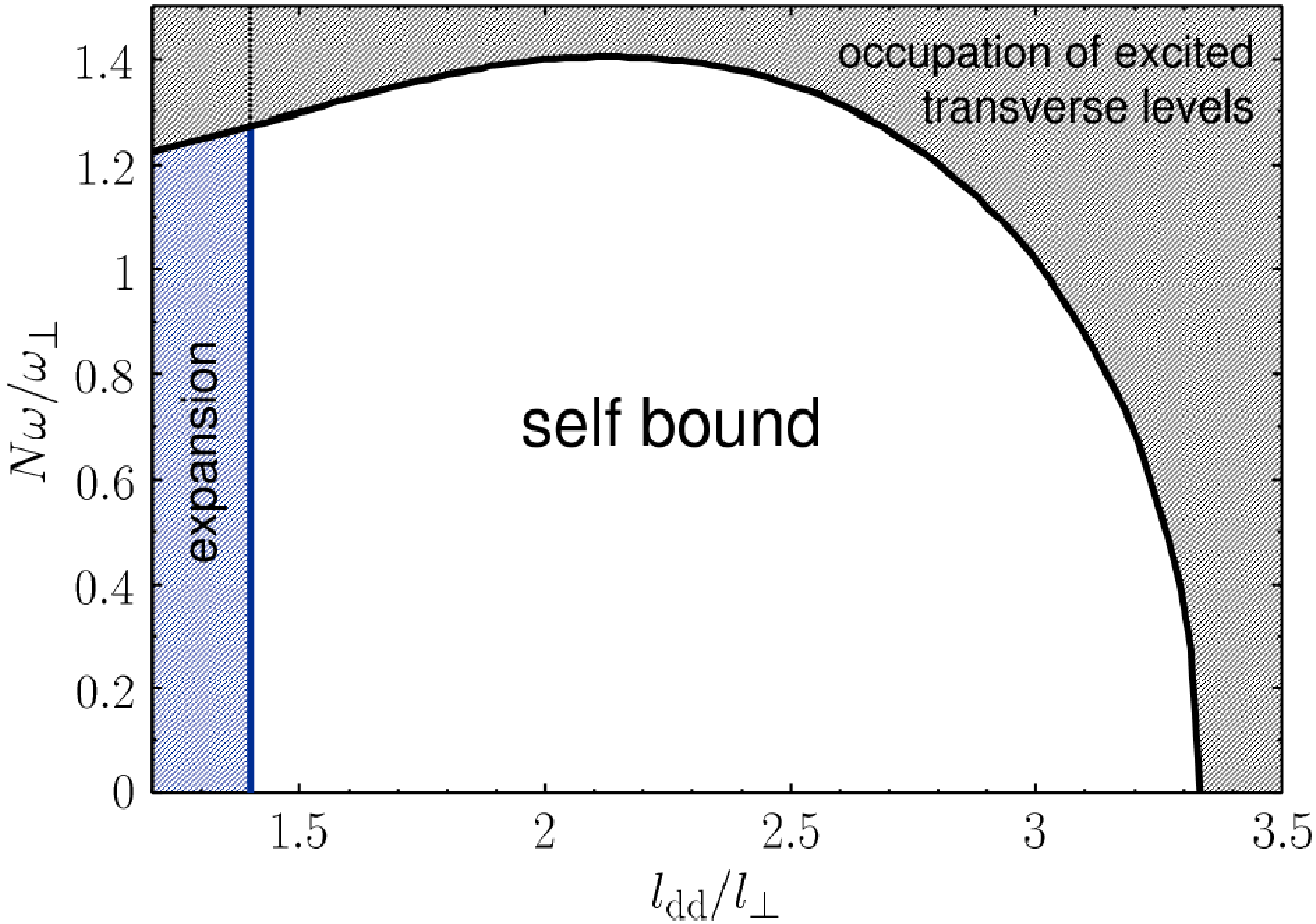

Experiment with $^{23}Na^{40}K$ (2.72Debye fully polarised):

1Debye partially polarised $l_{dd} = 1\mu m$

$$\omega = 2\pi \times 10 Hz$$

$$\omega_\perp = 2\pi \times 1 kHz$$

~100 molecules bound