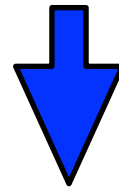


Topological Phases in Polar-Molecule Quantum Magnets (and then overview & discussion)

Alexey V. Gorshkov

Institute for Quantum Information and Matter (IQIM),
Caltech

IQIM



(summer 2013)

jqi

NIST

Joint Quantum Institute (JQI)
NIST and University of Maryland



Postdoc and graduate student positions available!

(theoretical many-body physics,
quantum information, AMO physics)

KITP workshop on
Fundamental Science and Applications of Ultra-cold Polar Molecules
January 24, 2013

Topological Phases in Polar-Molecule Quantum Magnets

- **symmetry protected topological phases** [[arXiv:1210.5518](#)]:
S. Manmana, K. Hazzard, A. M. Rey - JILA
M. Stoudenmire - UCI
- **fractional Chern insulators** [[PRL 109, 266804 \(2012\)](#) & [arXiv:1212.4839](#)]:
N. Yao, C. Laumann, S. Bennett, E. Demler, M. Lukin - Harvard
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Motivation

Interacting dipoles

Electric

Magnetic

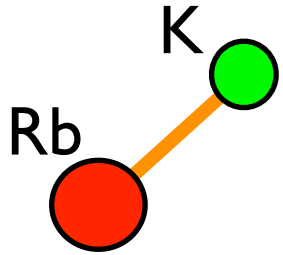


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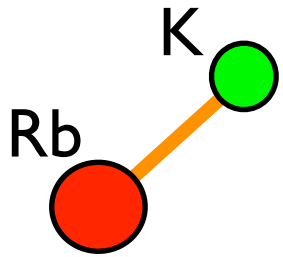
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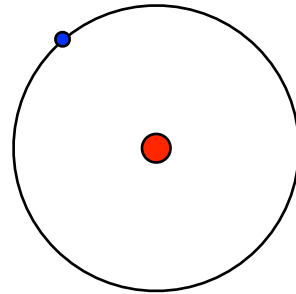
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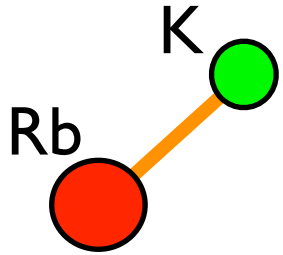
Rydberg atoms

[Grangier, Saffman,
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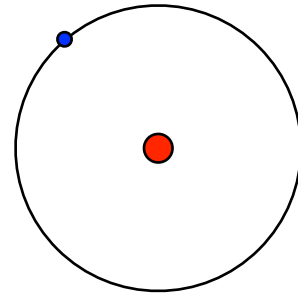
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Magnetic

large
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angular
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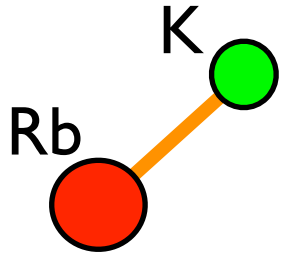
magnetic atoms
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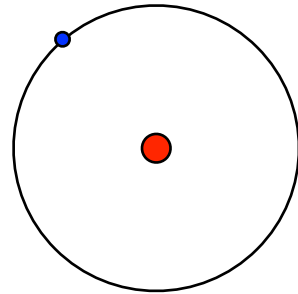
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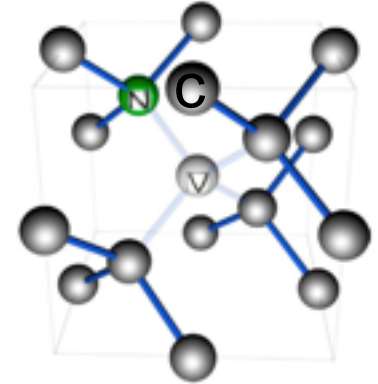
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NV centers

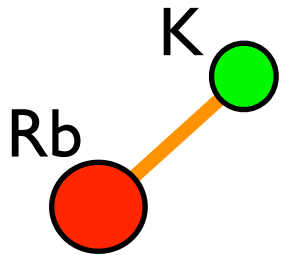
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[drawn by Wrachtrup et al.]

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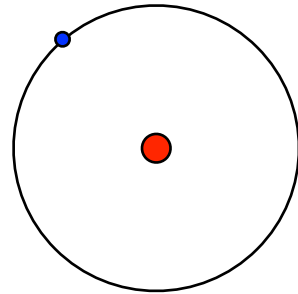
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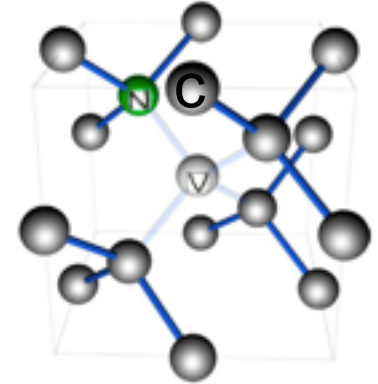
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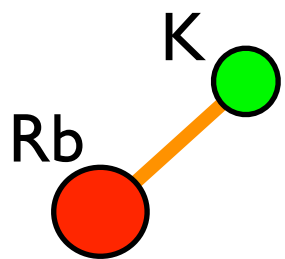
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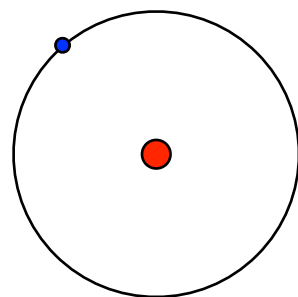
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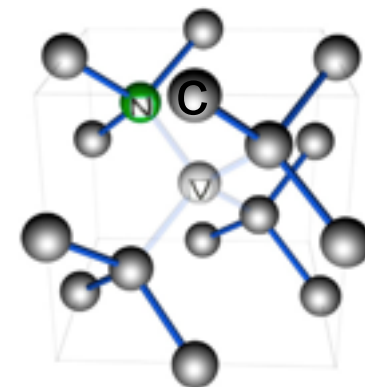
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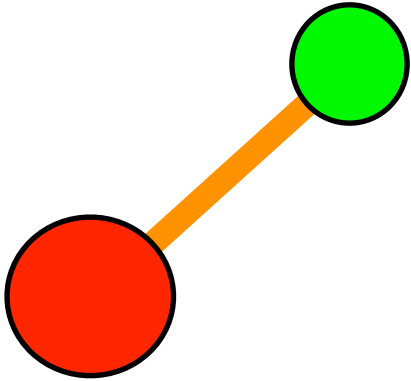
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- KRb already loaded into a 3D optical lattice at JILA!

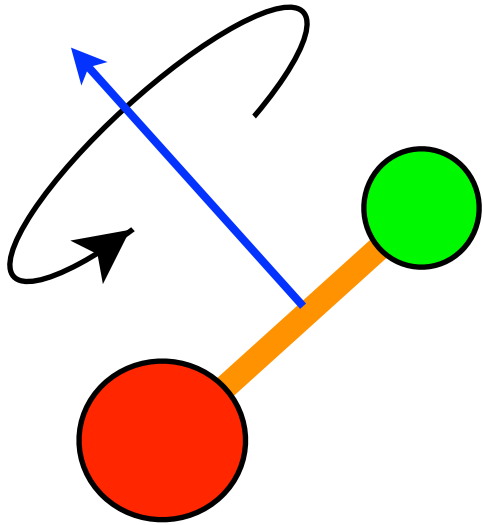
[Chotia et al, PRL (2012)]

Motivation

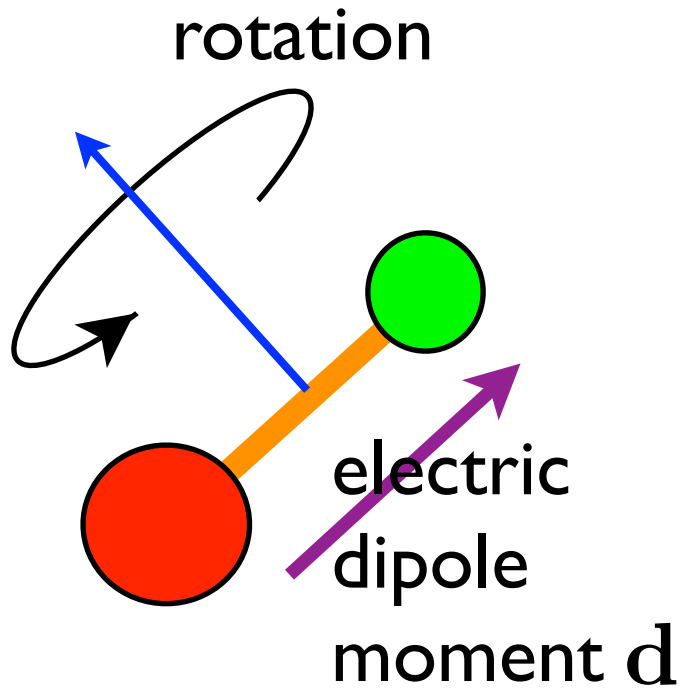


Motivation

rotation

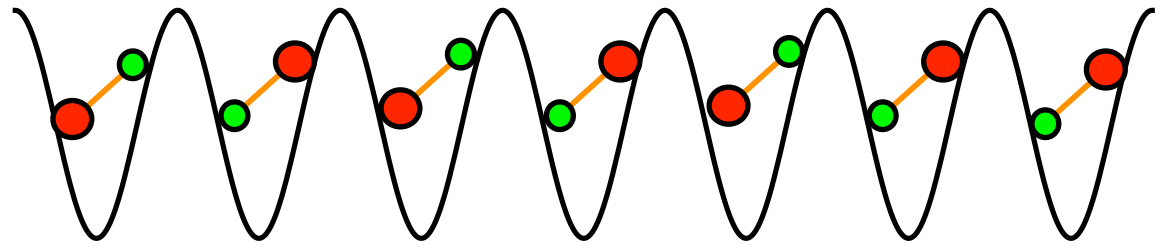
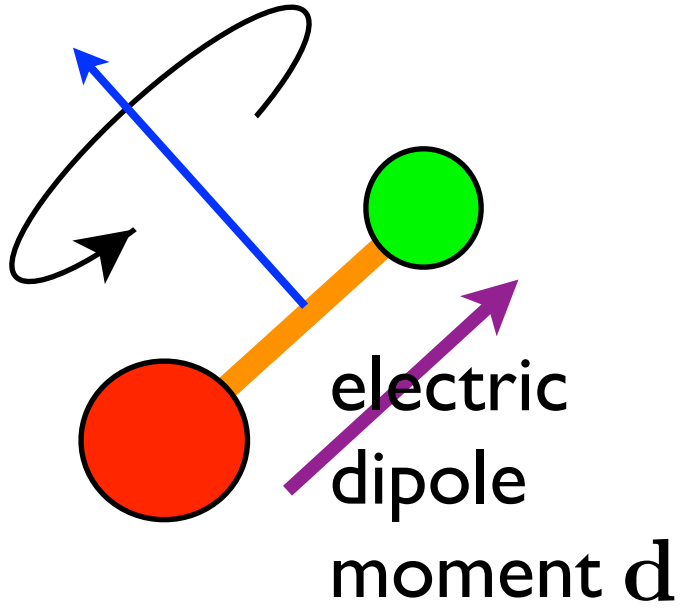


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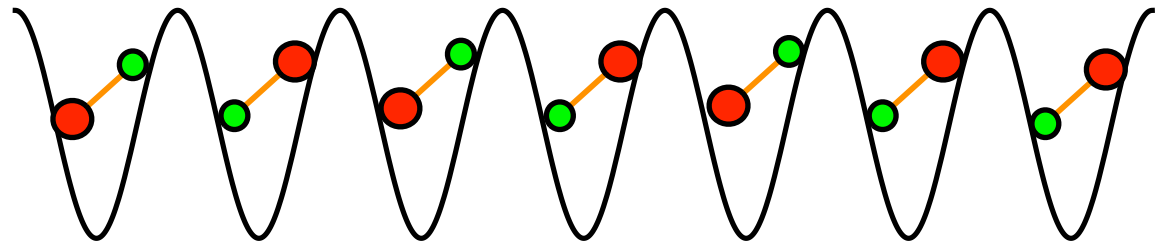
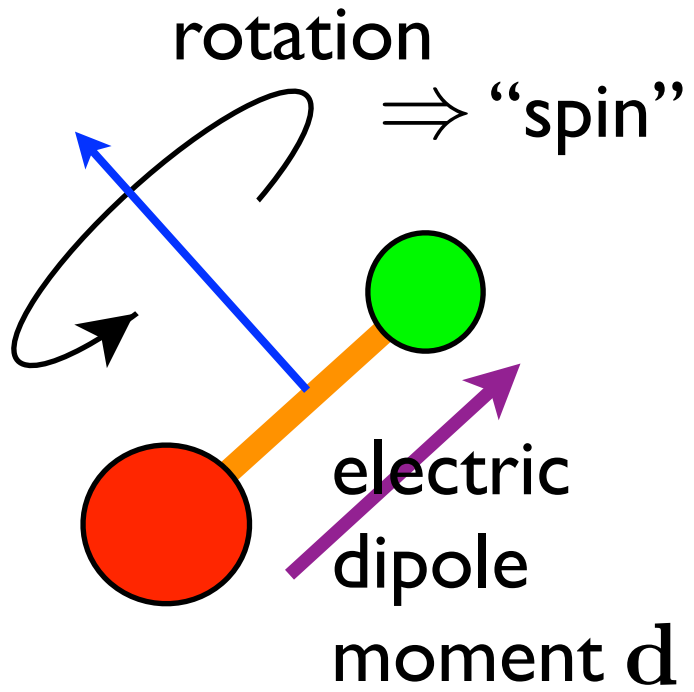


Motivation

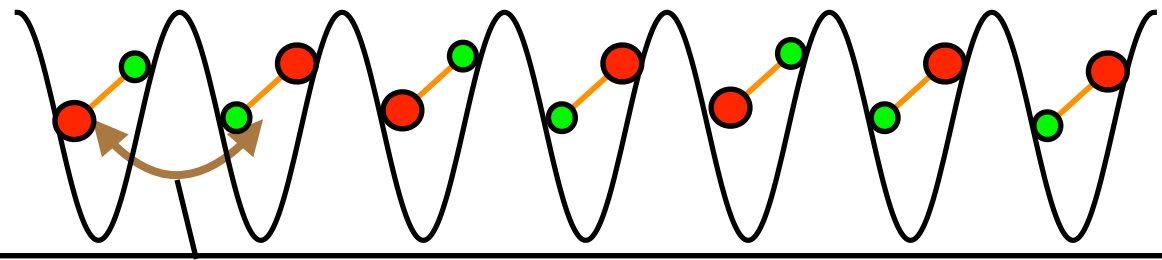
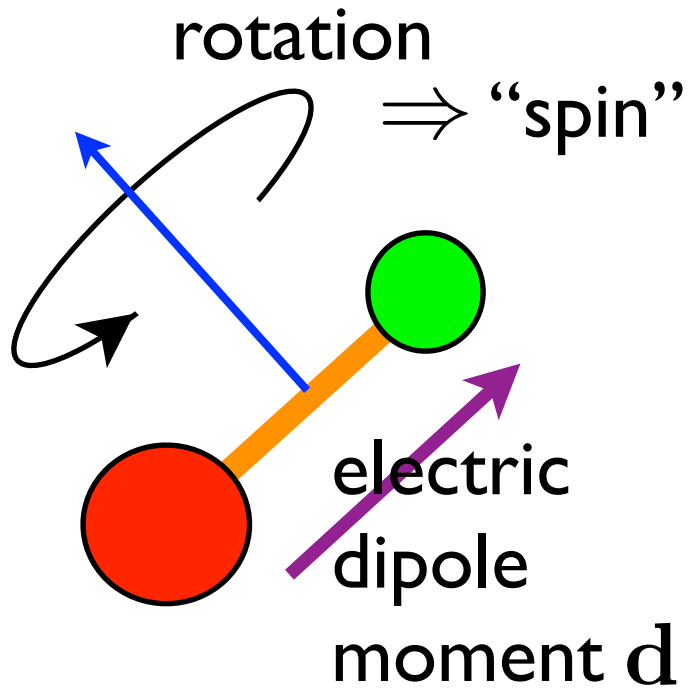
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Motivation



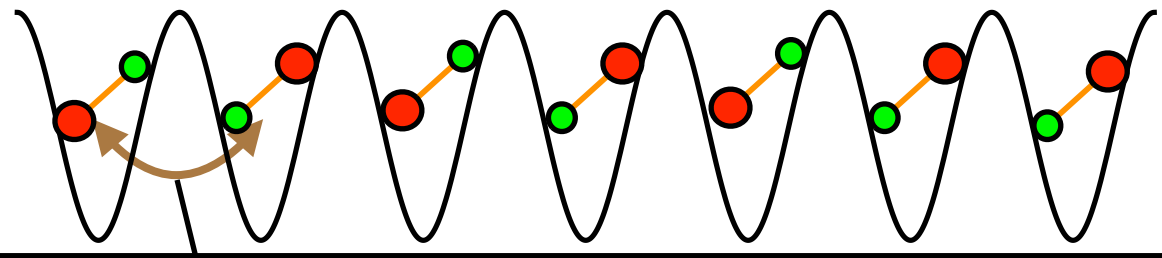
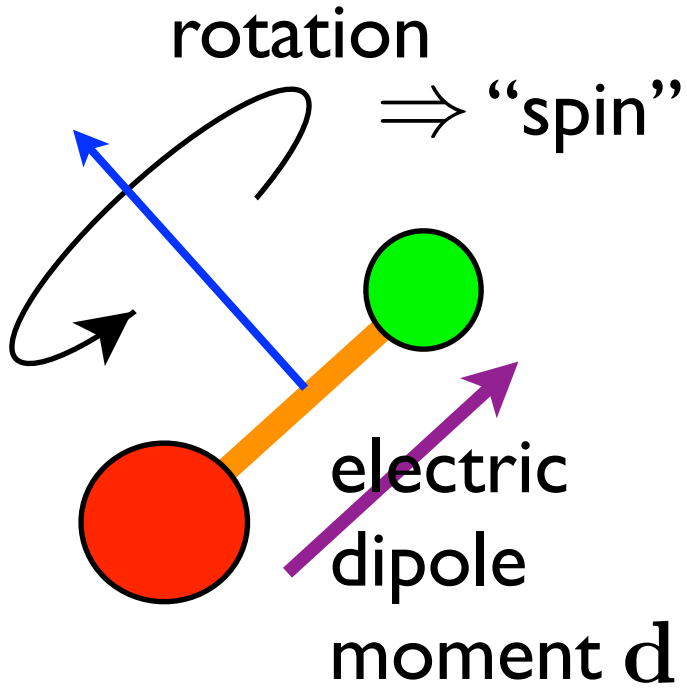
Motivation



dipole-dipole interactions \Rightarrow "spin-spin" interactions

Motivation

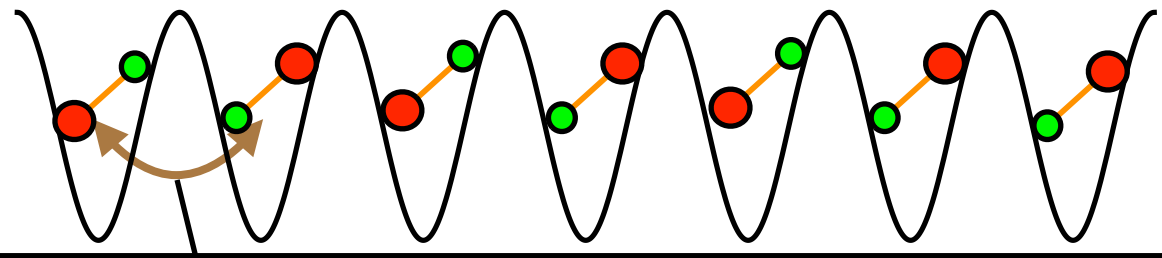
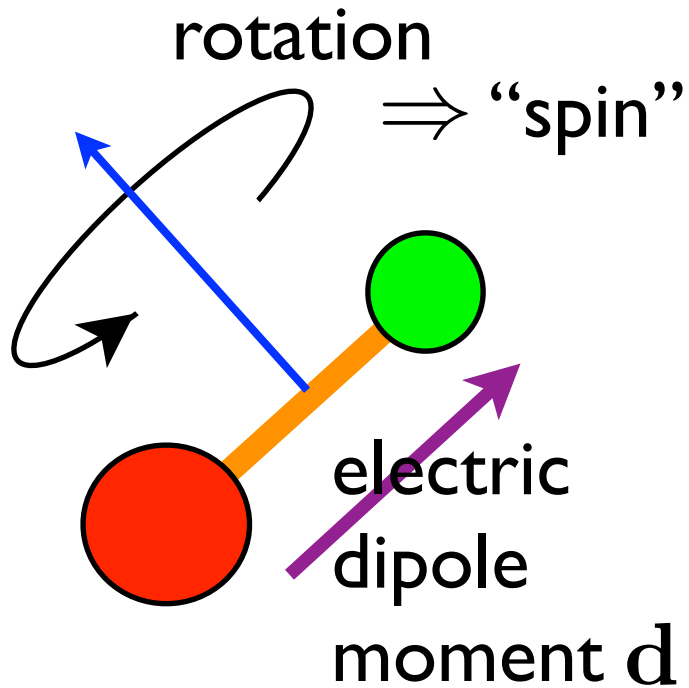
highly tunable exotic spin models



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Motivation

highly tunable exotic spin models



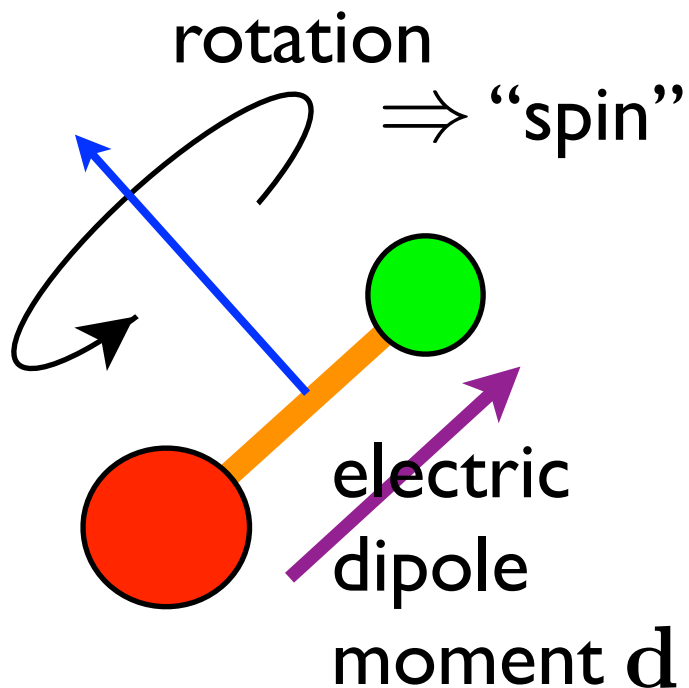
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Barnett et al., PRL (2006)
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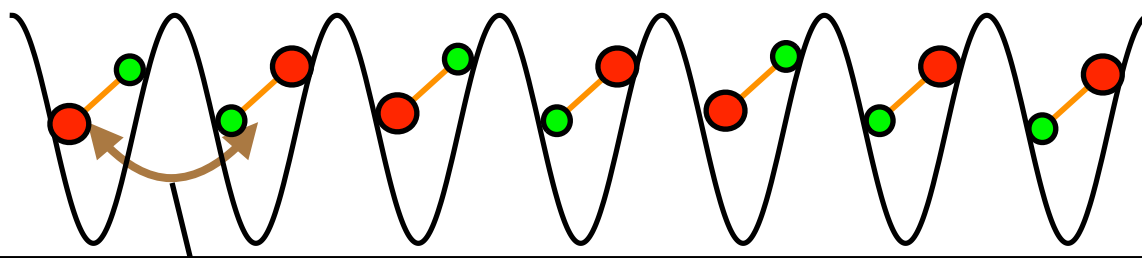
Motivation

highly tunable exotic spin models



Our achievements:

- stronger interactions
- much higher tunability



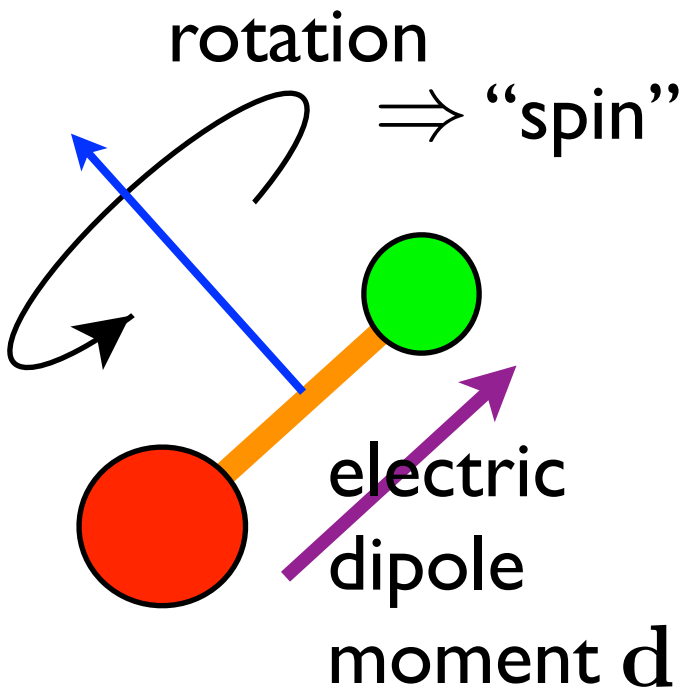
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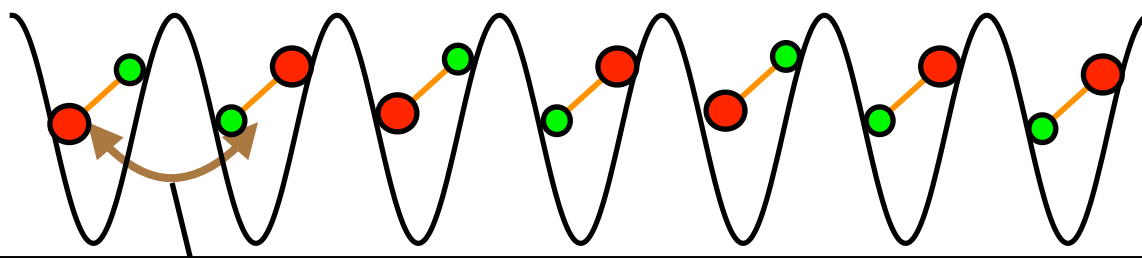
Motivation

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Our achievements:

- stronger interactions
- much higher tunability \Rightarrow exotic physics



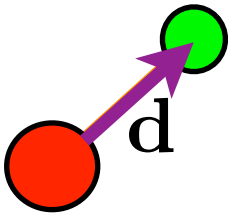
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Rigid rotor

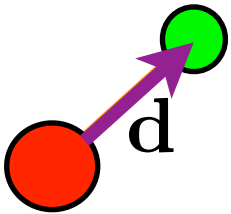
$$H_0 = B\mathbf{N}^2$$



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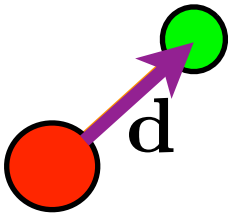
$$\mathbf{N}^2|N, N_z\rangle = N(N + 1)|N, N_z\rangle$$



Rigid rotor

$$H_0 = BN^2$$

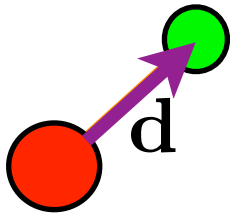
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$$\begin{array}{ccc} N_z = & -1 & 0 & 1 \\ 2B \left\{ \begin{array}{l} \text{---} & \text{---} & \text{---} & N = 1 \\ & \text{---} & & N = 0 \end{array} \right. \end{array}$$

Rigid rotor

$$\mathbf{E} = E\hat{z}$$



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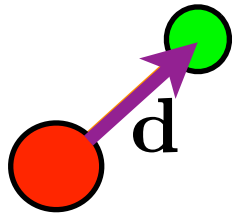
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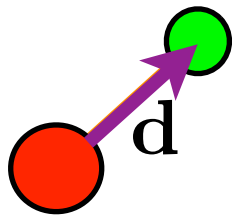
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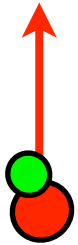
Simplest spin Hamiltonian

$$\mathbf{E} = E\hat{\mathbf{z}}$$

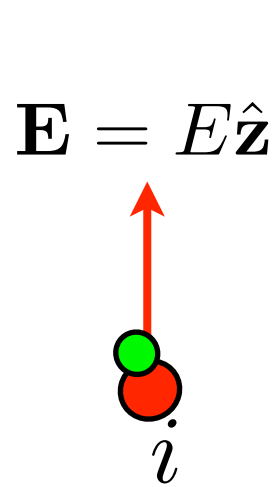


Simplest spin Hamiltonian

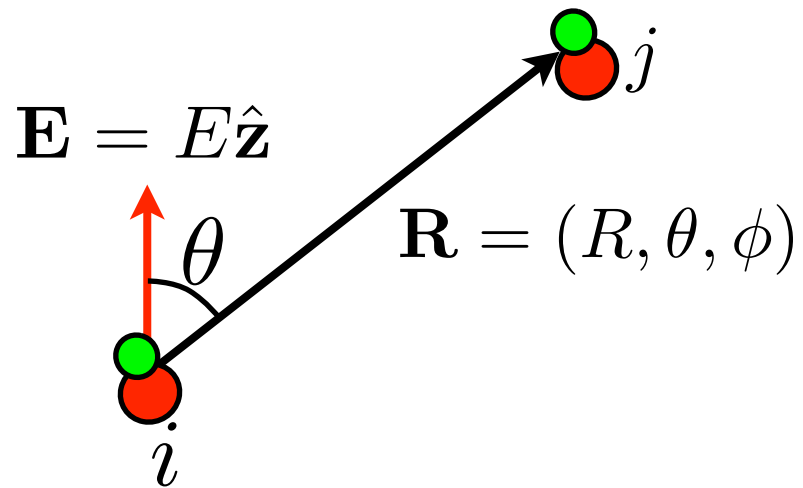
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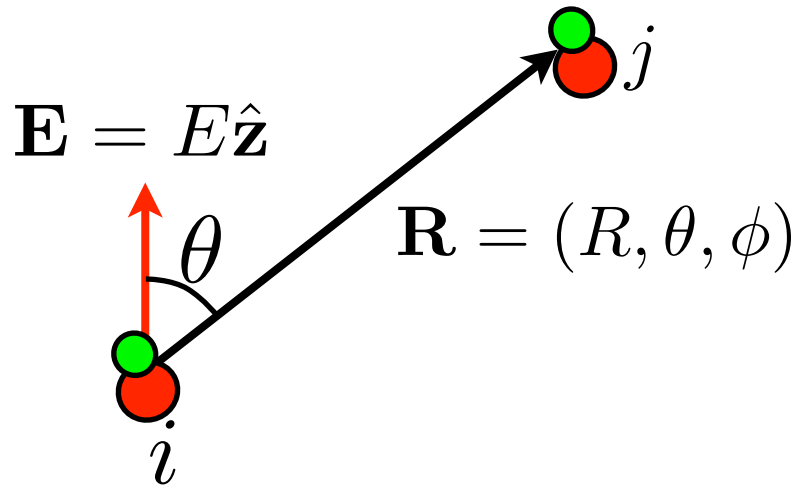
Simplest spin Hamiltonian



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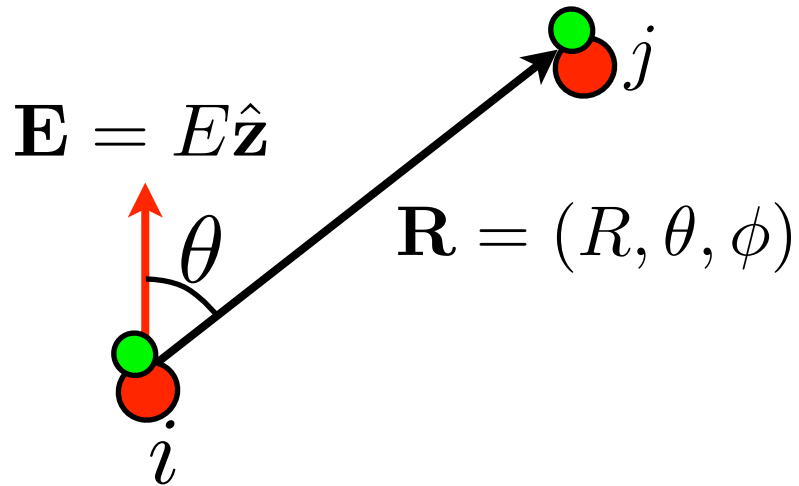


Simplest spin Hamiltonian

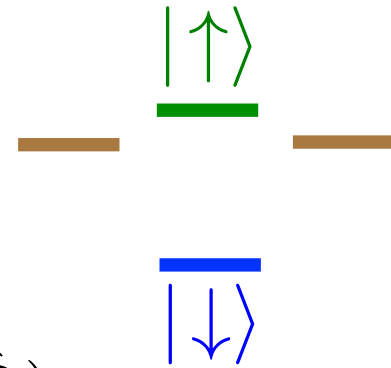


$$R^3 H_{dd} = \mathbf{d}_i \cdot \mathbf{d}_j - 3(\mathbf{d}_i \cdot \hat{\mathbf{R}})(\mathbf{d}_j \cdot \hat{\mathbf{R}})$$

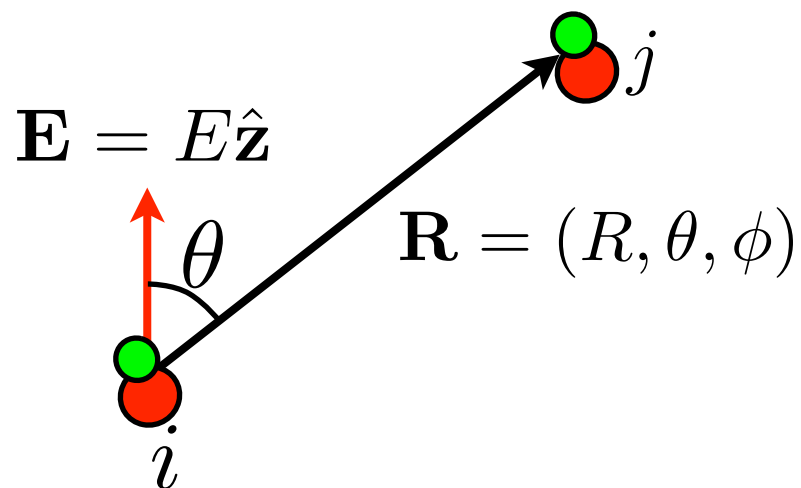
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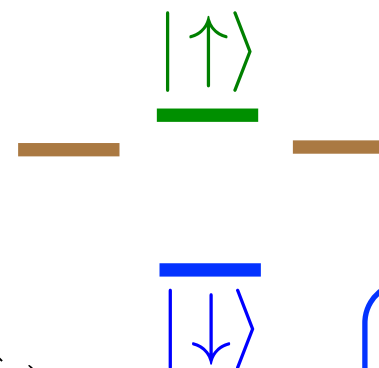
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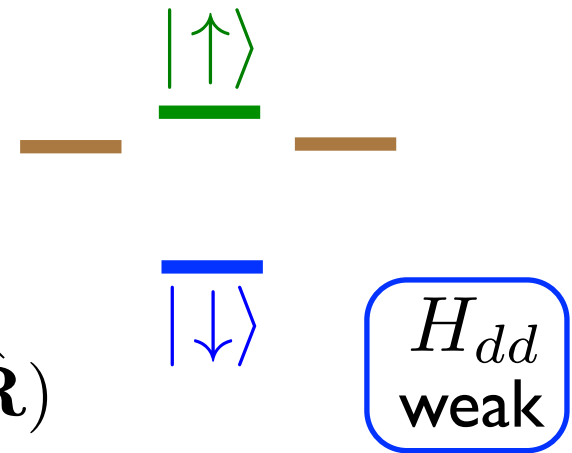
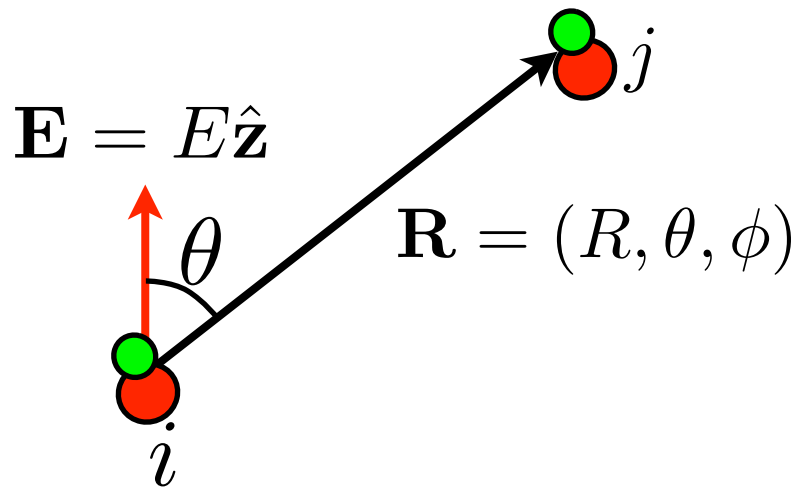


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H_{dd}
weak

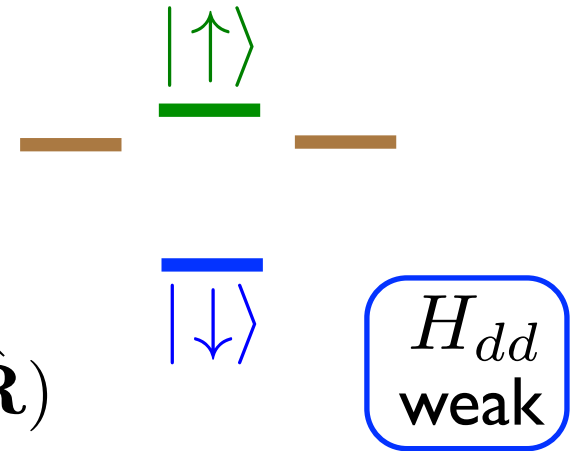
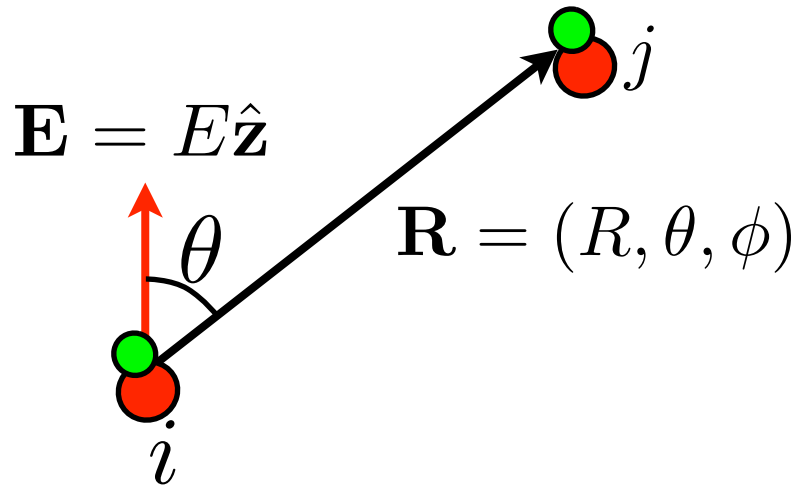
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- project on $|\uparrow\rangle, |\downarrow\rangle$

Simplest spin Hamiltonian



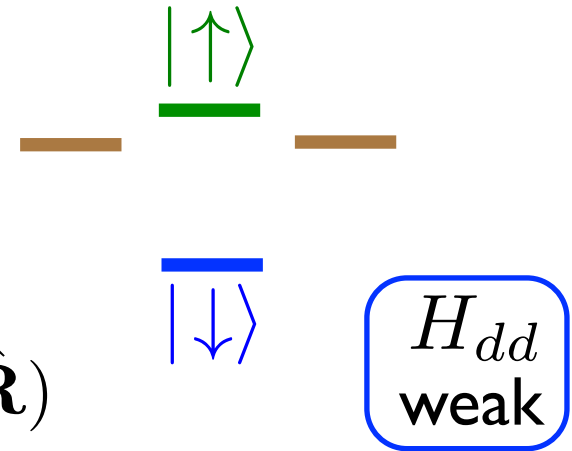
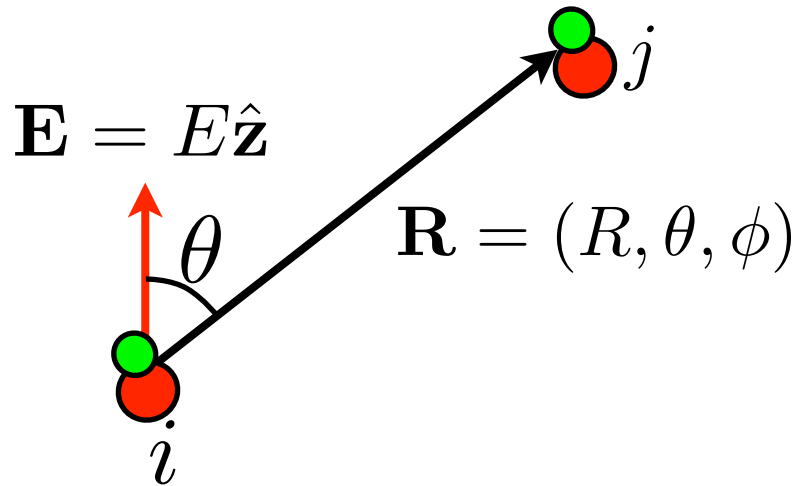
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$$Y_{2,0} \propto 1 - 3 \cos^2 \theta$$

Simplest spin Hamiltonian



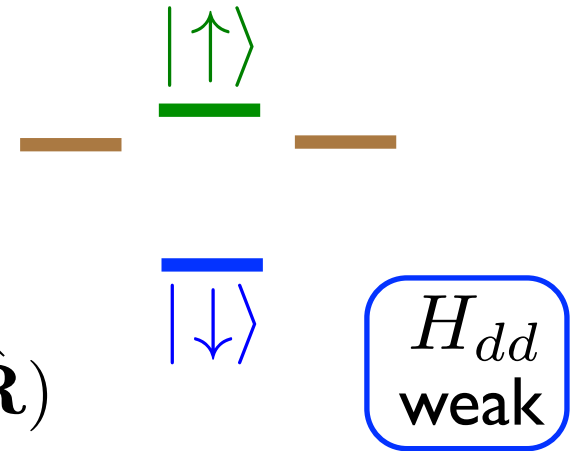
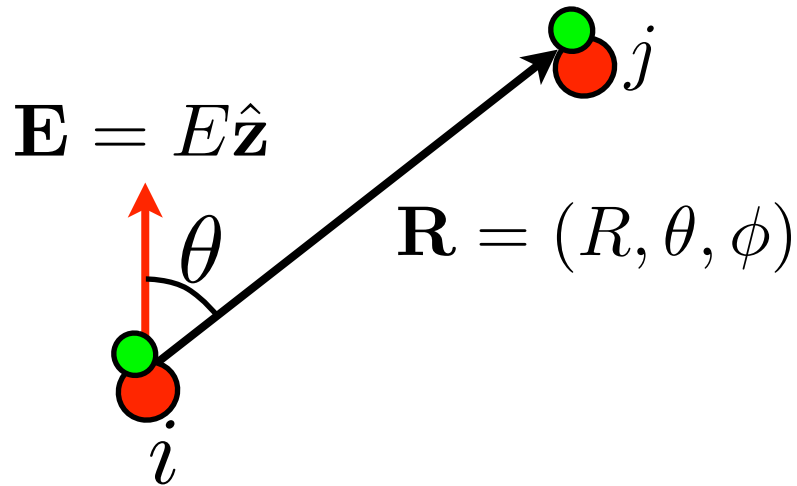
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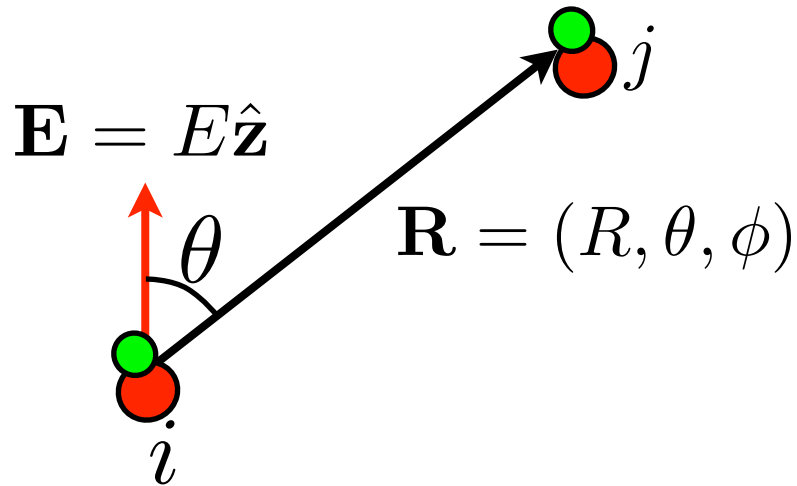
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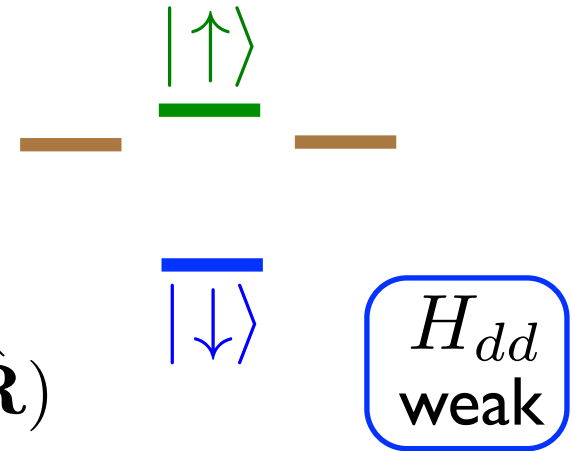
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Simplest spin Hamiltonian



$$R^3 H_{dd} = \mathbf{d}_i \cdot \mathbf{d}_j - 3(\mathbf{d}_i \cdot \hat{\mathbf{R}})(\mathbf{d}_j \cdot \hat{\mathbf{R}})$$

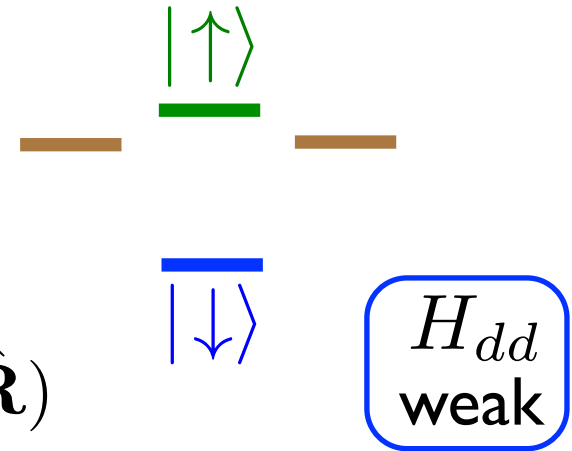
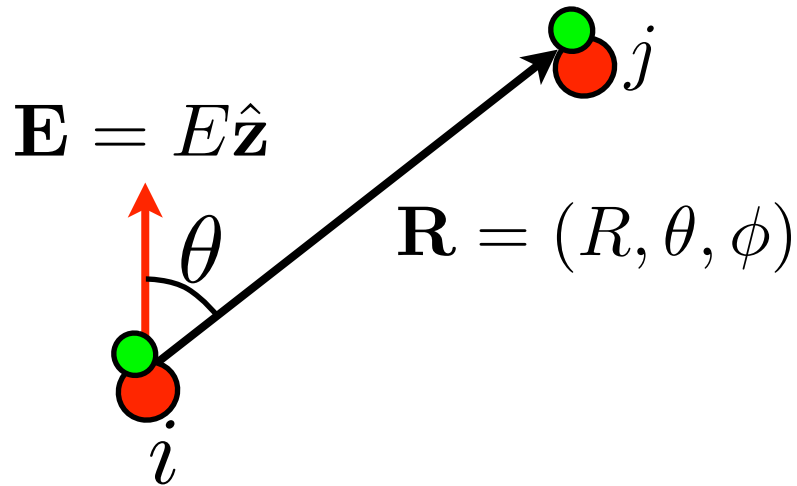


- project on $|\uparrow\rangle, |\downarrow\rangle$

$$R^3 H_{dd} = Y_{2,0}(\theta) [J_z S_i^z S_j^z + J_{xy} (S_i^x S_j^x + S_i^y S_j^y)]$$

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Simplest spin Hamiltonian



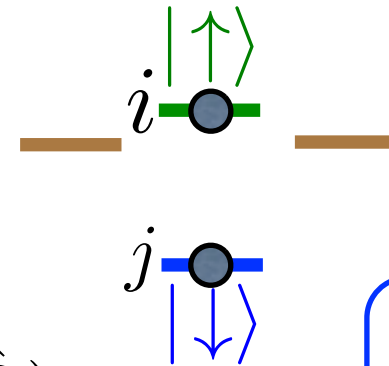
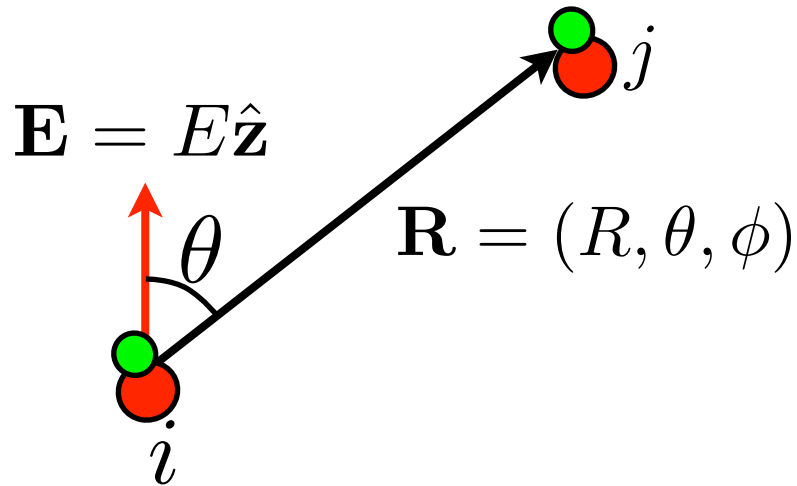
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Simplest spin Hamiltonian



H_{dd}
weak

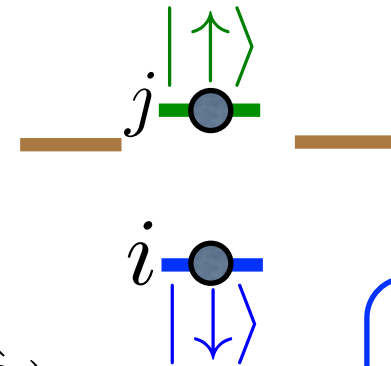
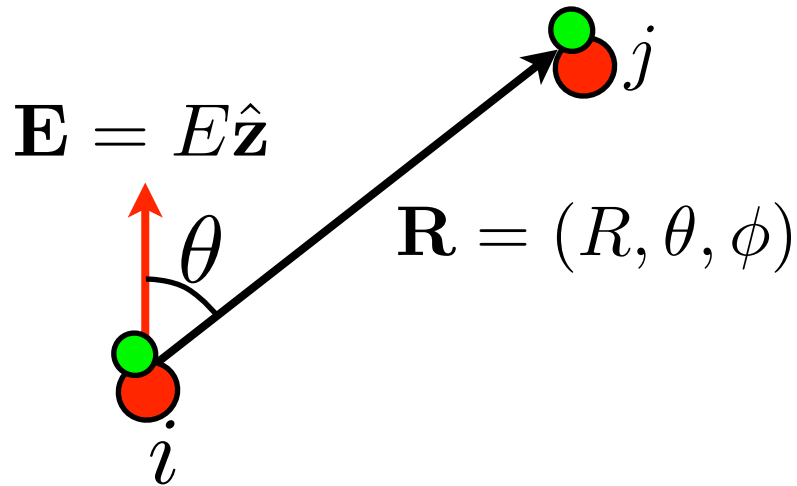
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Simplest spin Hamiltonian



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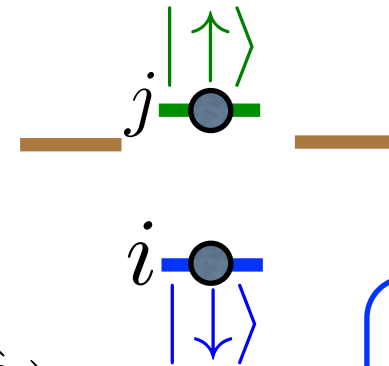
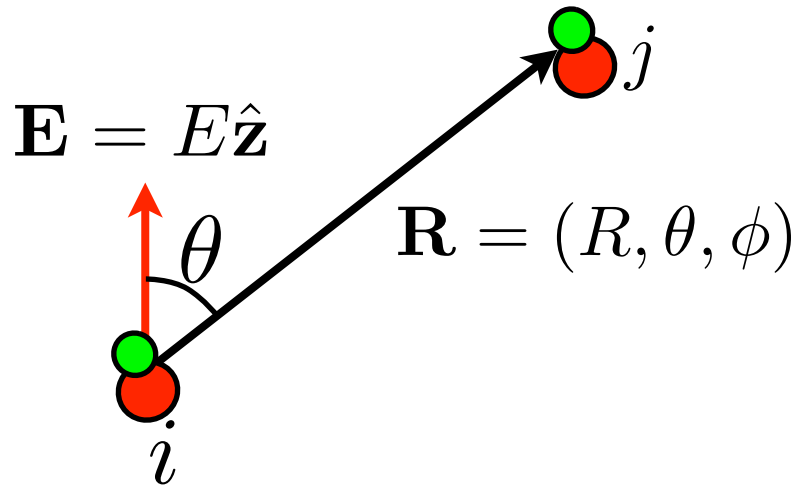
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Simplest spin Hamiltonian



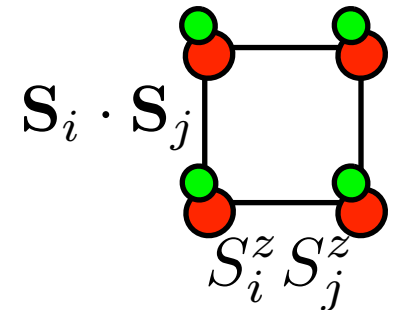
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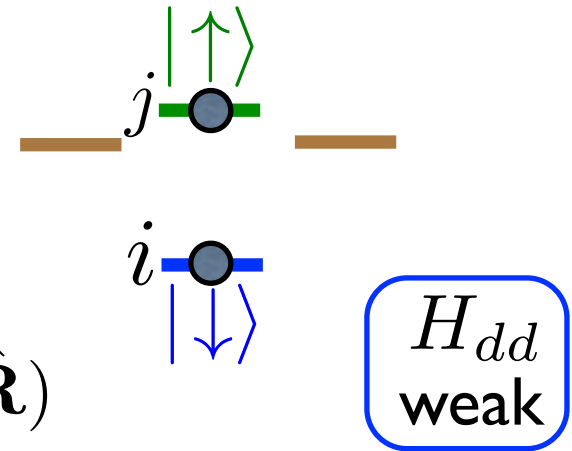
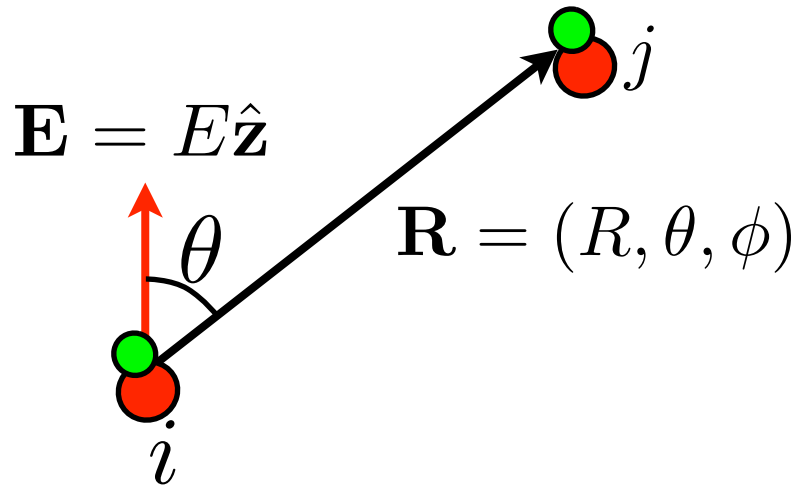
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Simplest spin Hamiltonian

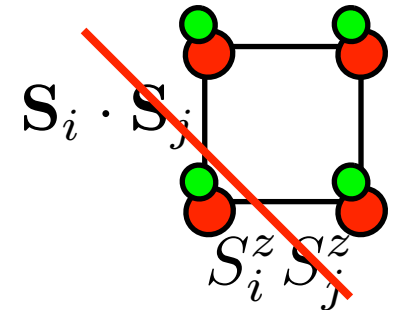


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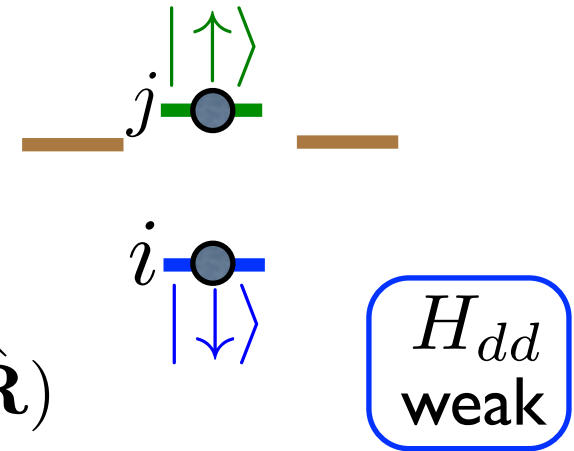
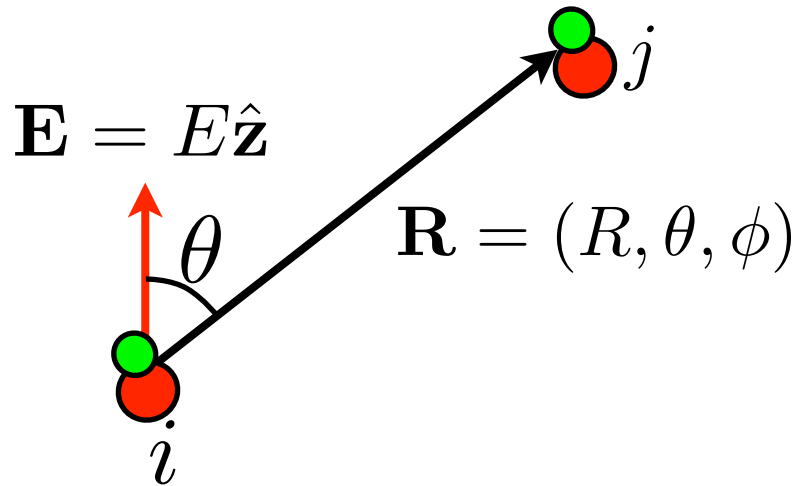
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Simplest spin Hamiltonian



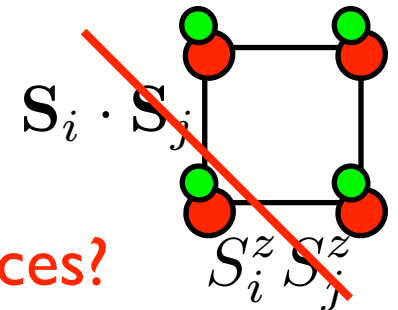
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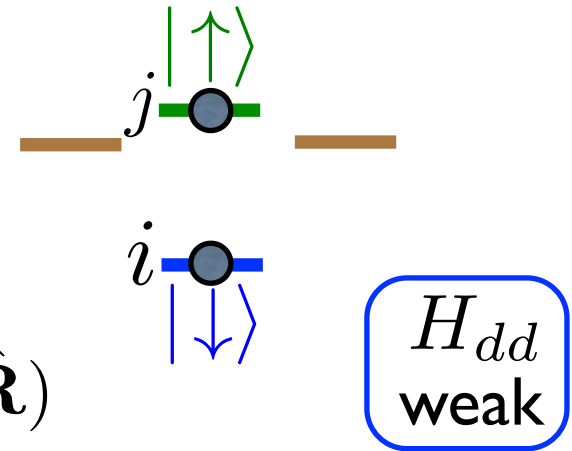
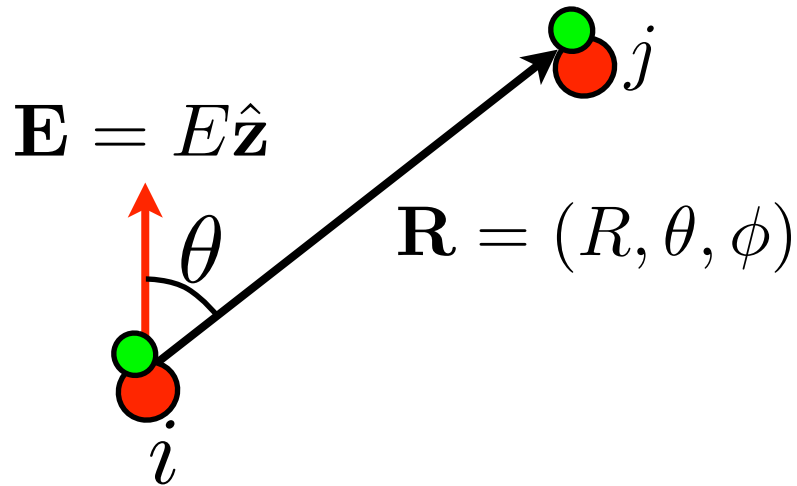
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- can J_z and J_{xy} come with different angular dependences?



Simplest spin Hamiltonian



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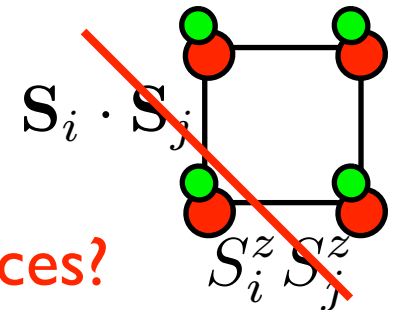
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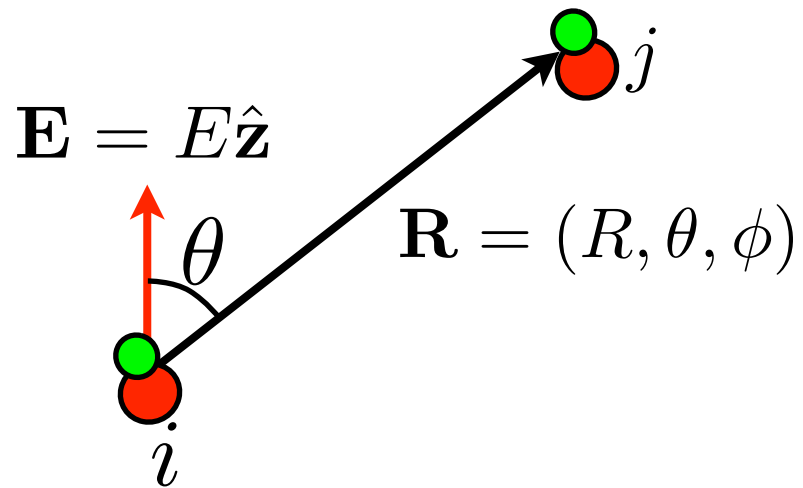
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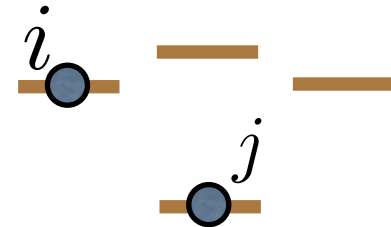
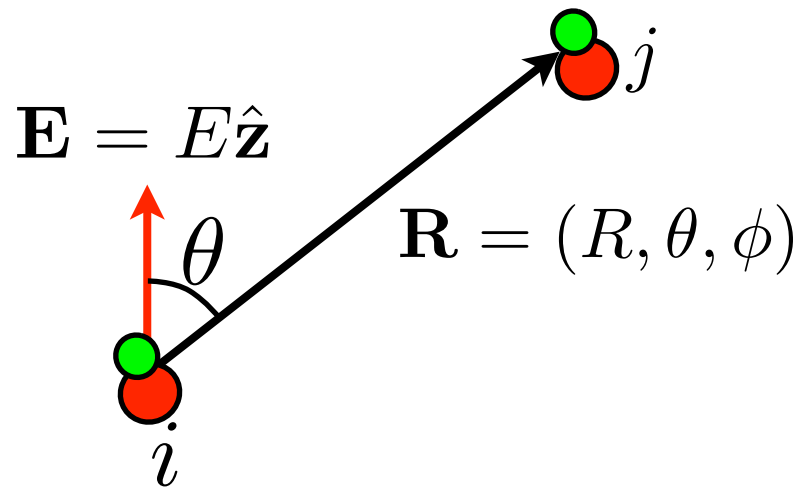
YES!



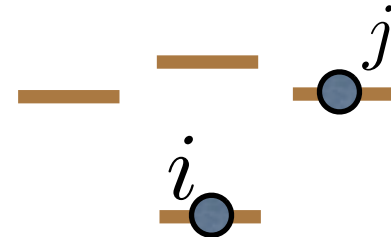
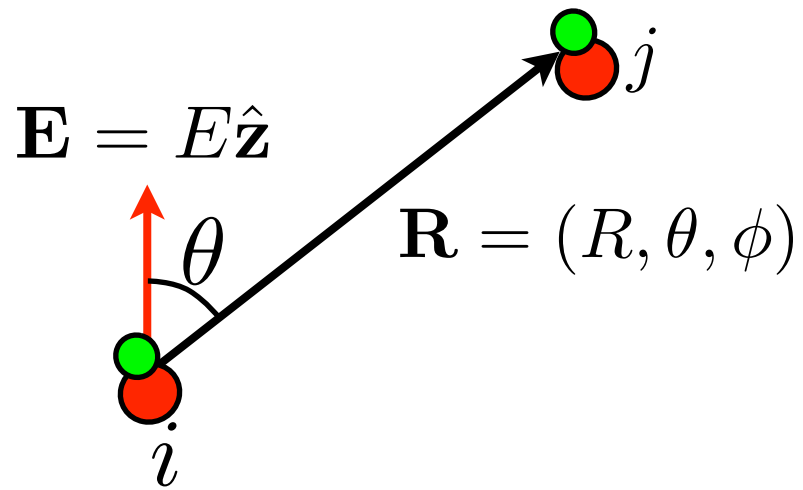
Spin Hamiltonian from dressed states



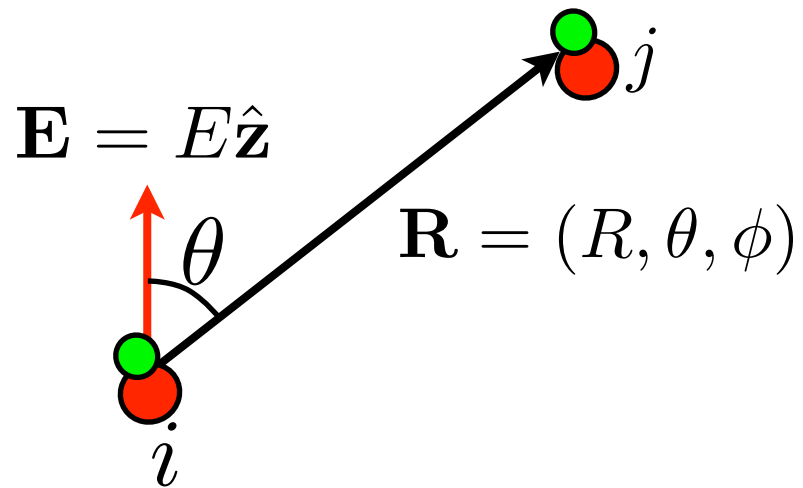
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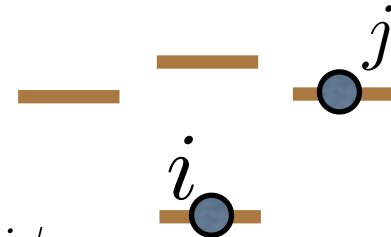
Spin Hamiltonian from dressed states



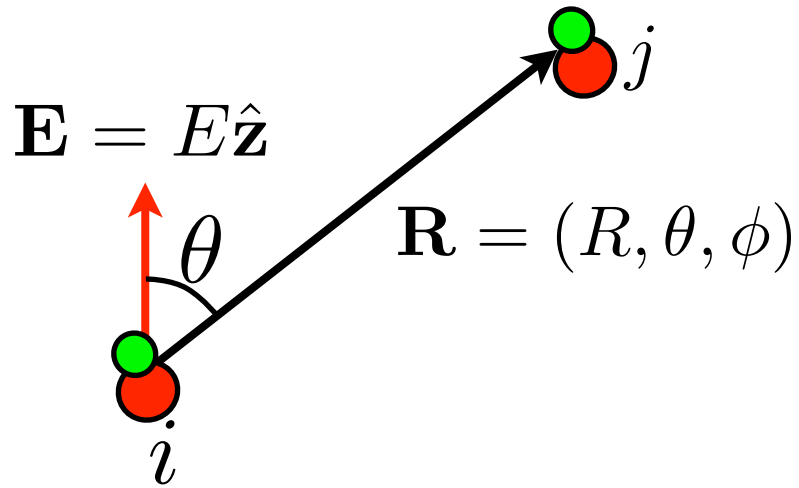
Spin Hamiltonian from dressed states



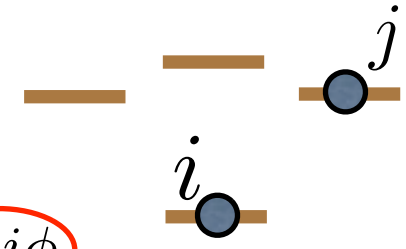
$$Y_{2,-2} \propto \sin^2 \theta e^{-2i\phi}$$



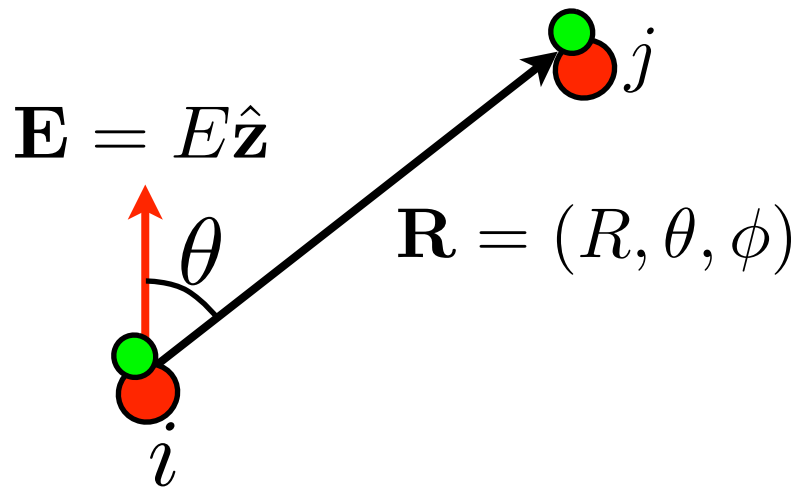
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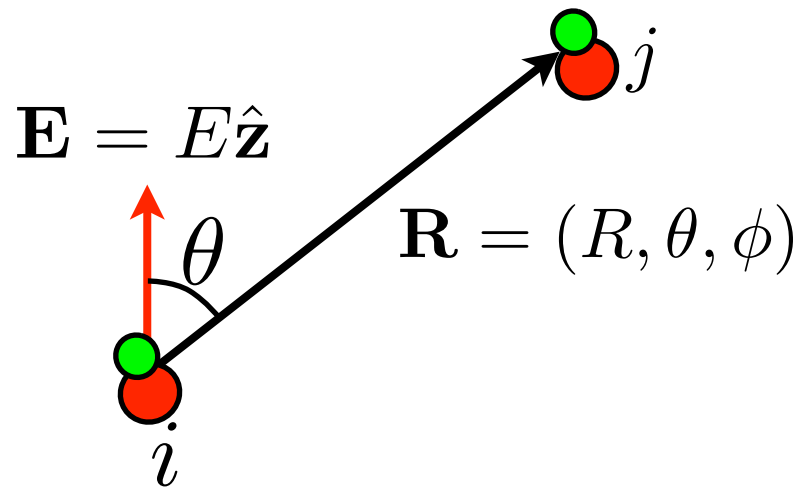
Spin Hamiltonian from dressed states



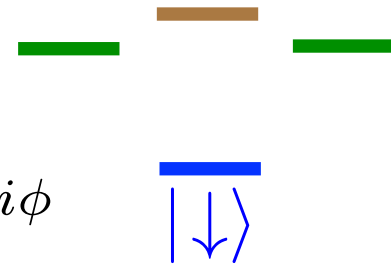
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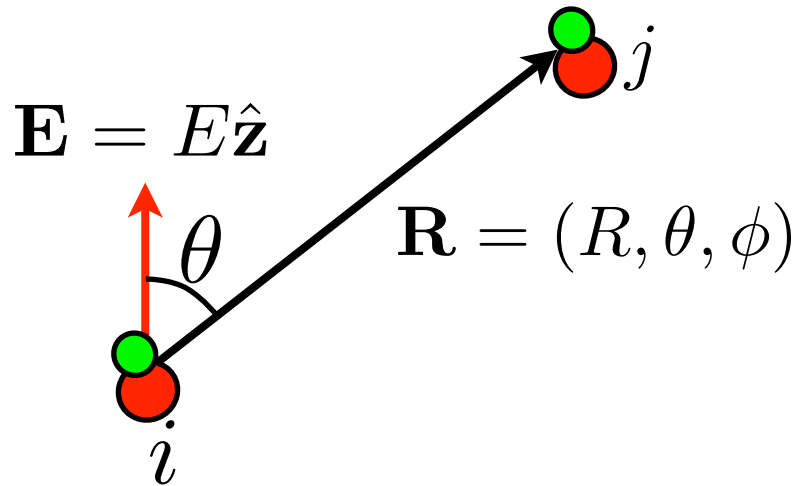
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Spin Hamiltonian from dressed states

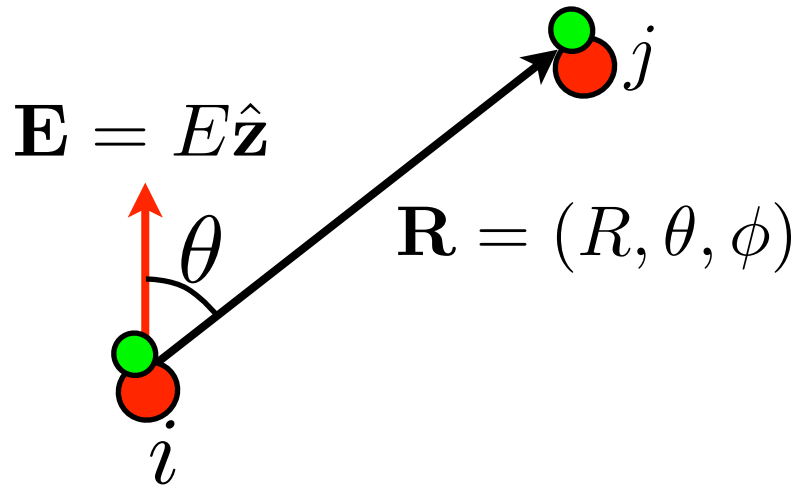


$$|1\rangle \text{---} \text{---} \text{---} |2\rangle$$

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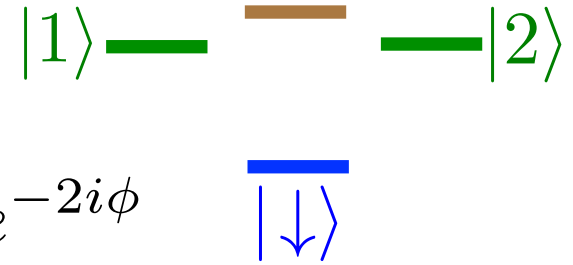
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Spin Hamiltonian from dressed states

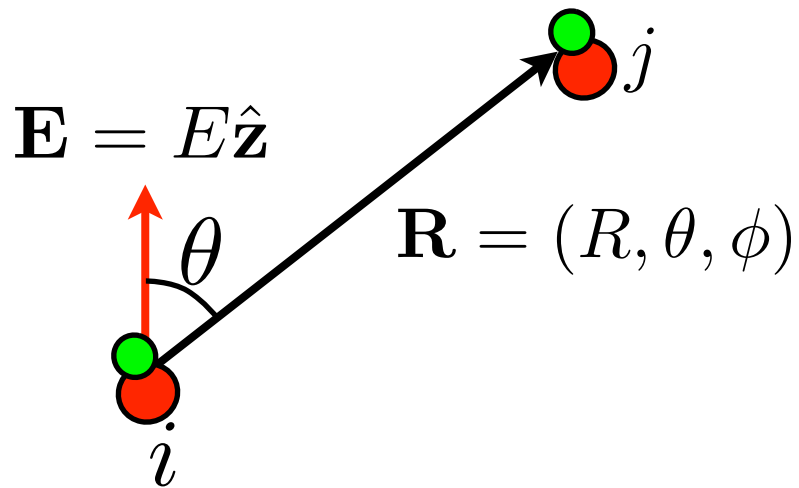


$$|\uparrow\rangle = \Omega_2|1\rangle - \Omega_1|2\rangle$$

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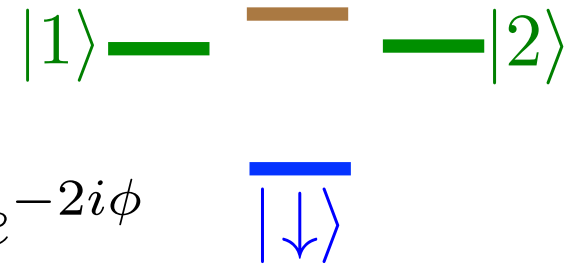


Spin Hamiltonian from dressed states

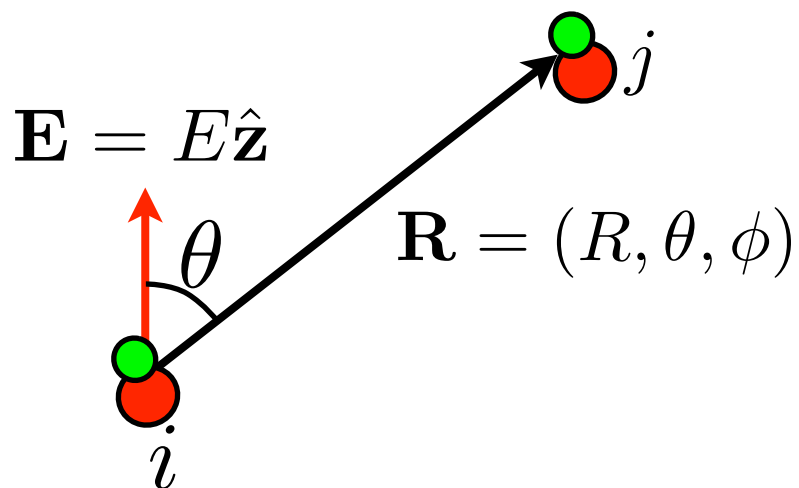


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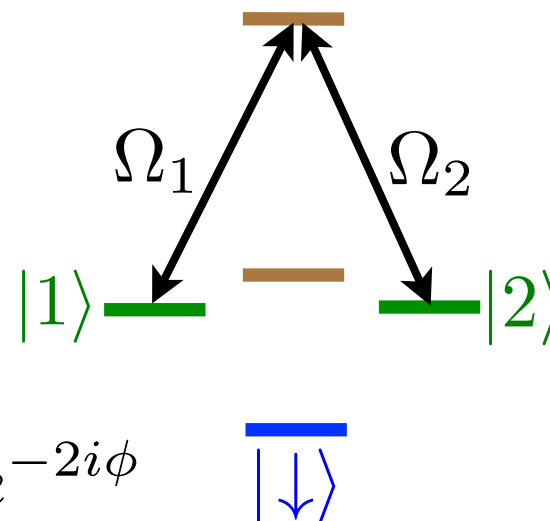


Spin Hamiltonian from dressed states

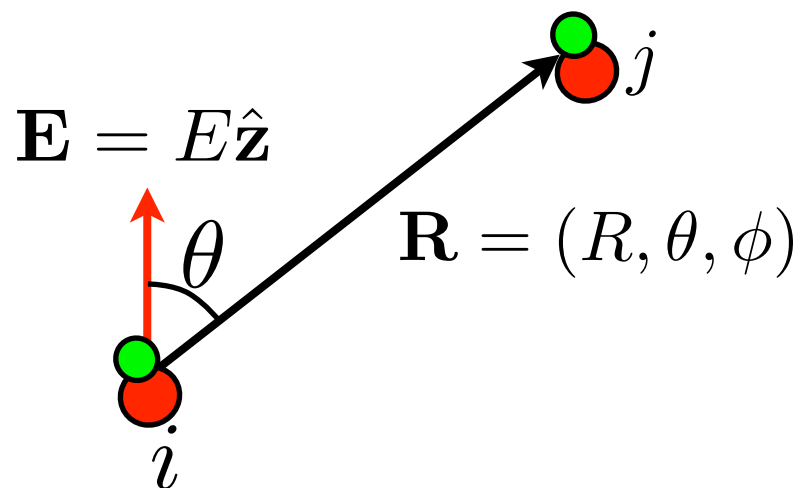


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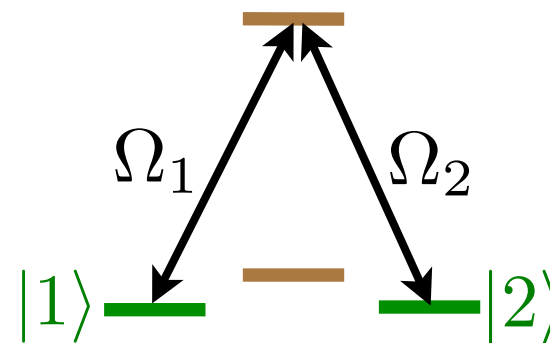


Spin Hamiltonian from dressed states



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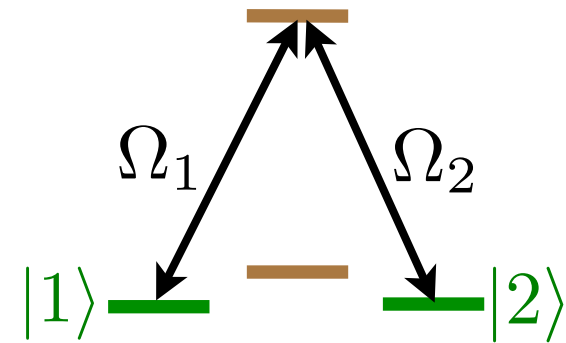
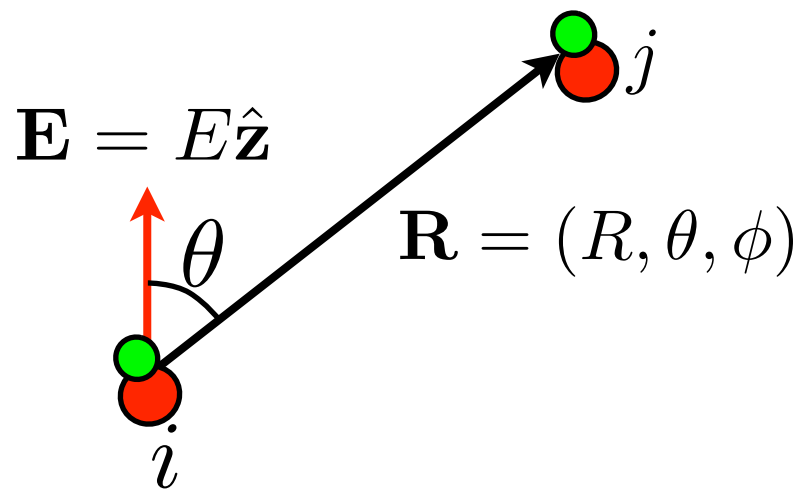
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Spin Hamiltonian from dressed states



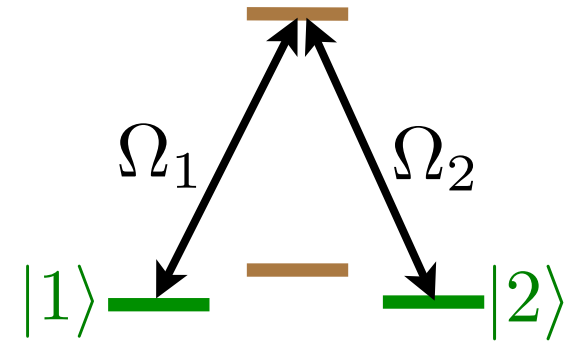
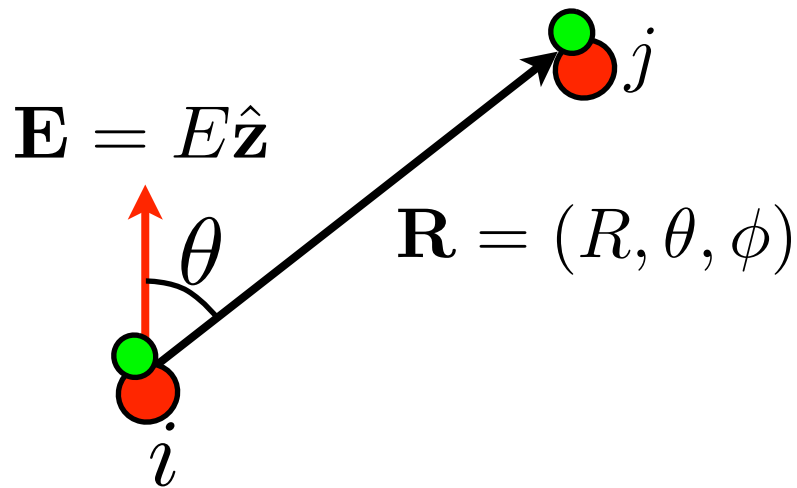
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Spin Hamiltonian from dressed states



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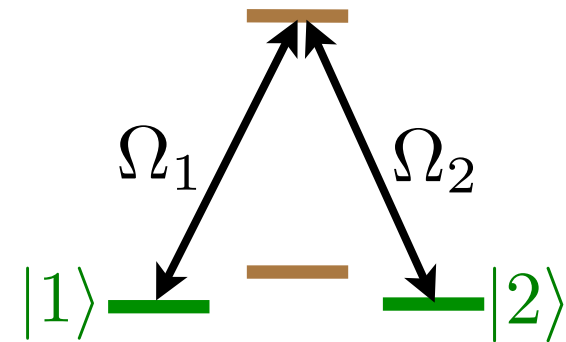
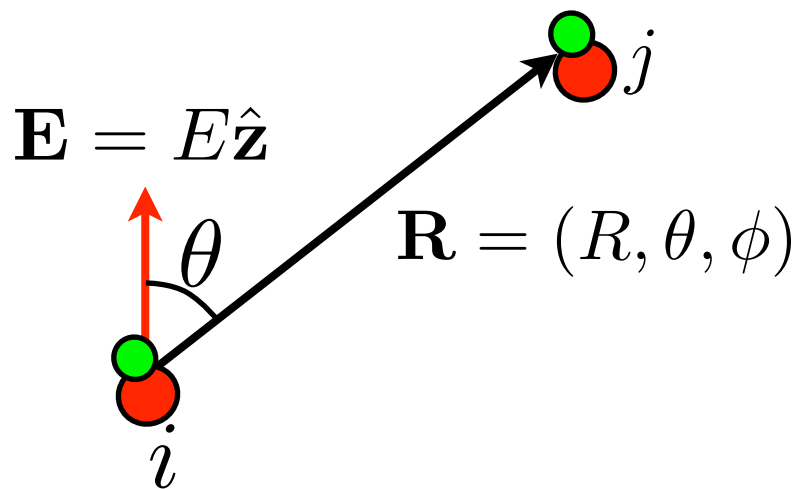


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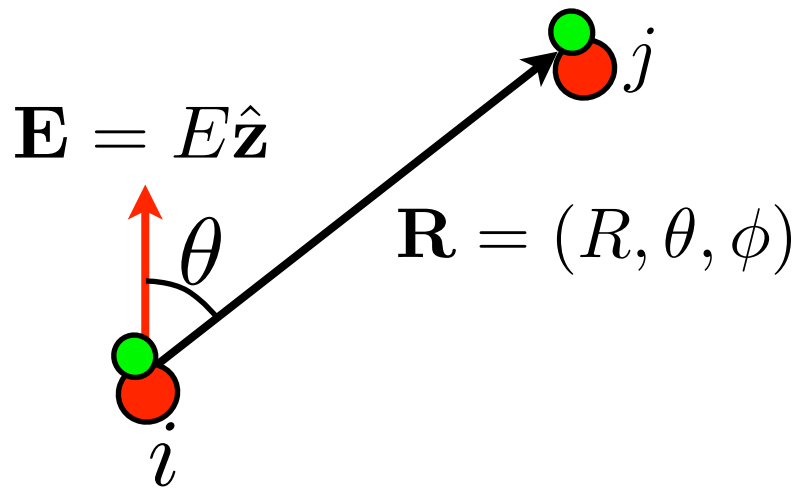


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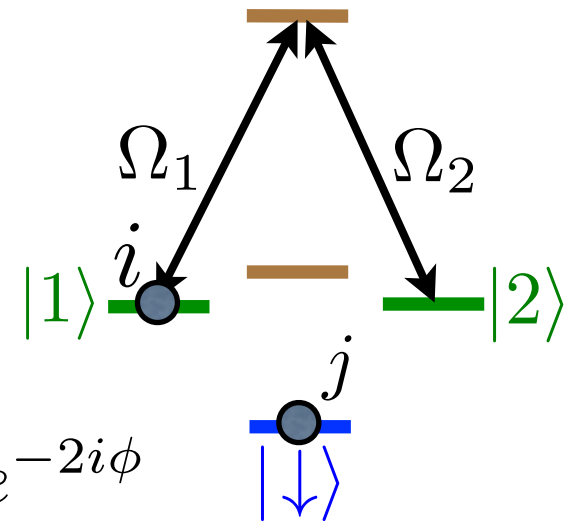
Spin Hamiltonian from dressed states



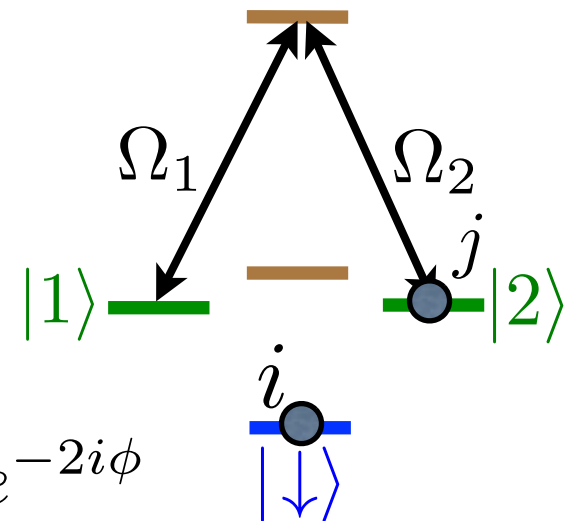
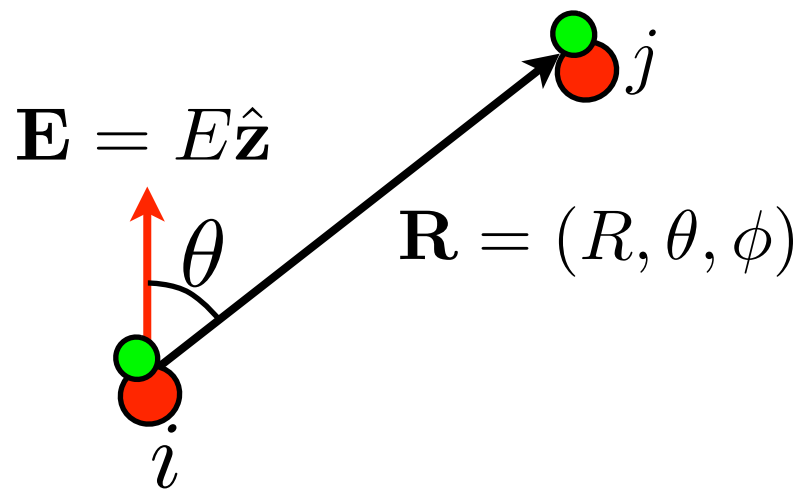
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Spin Hamiltonian from dressed states



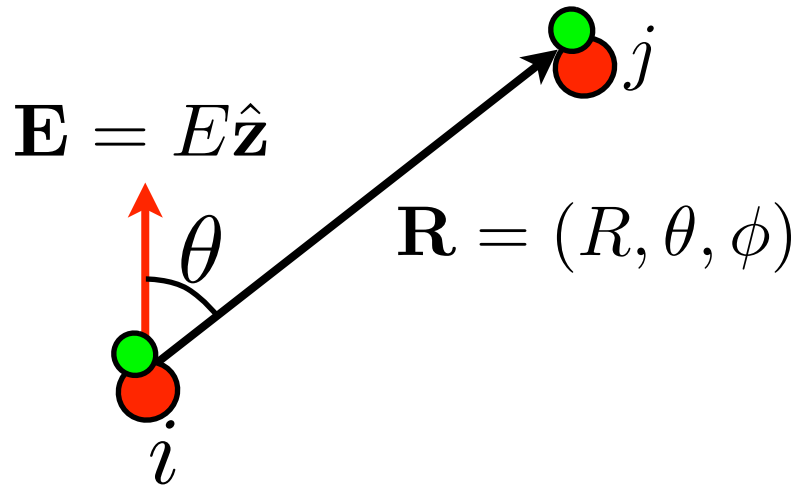
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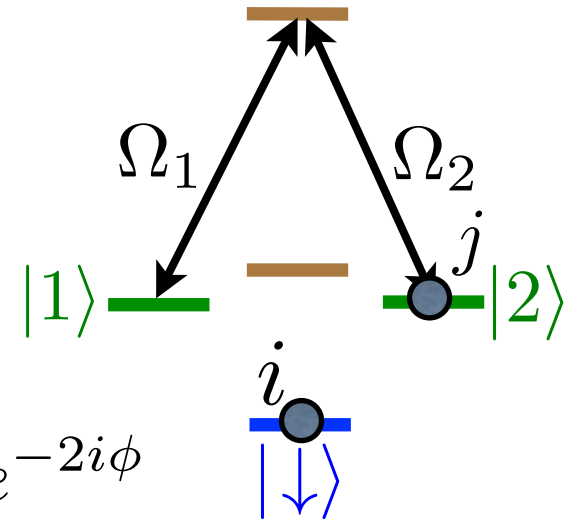
Spin Hamiltonian from dressed states



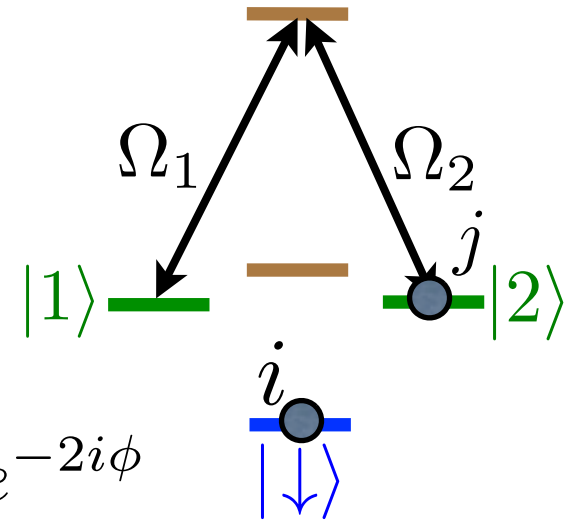
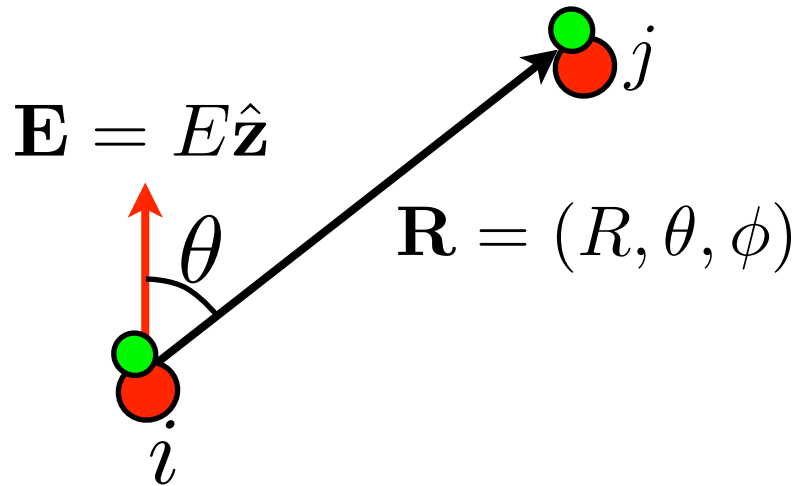
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Spin Hamiltonian from dressed states

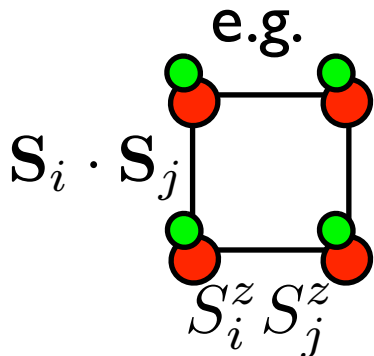


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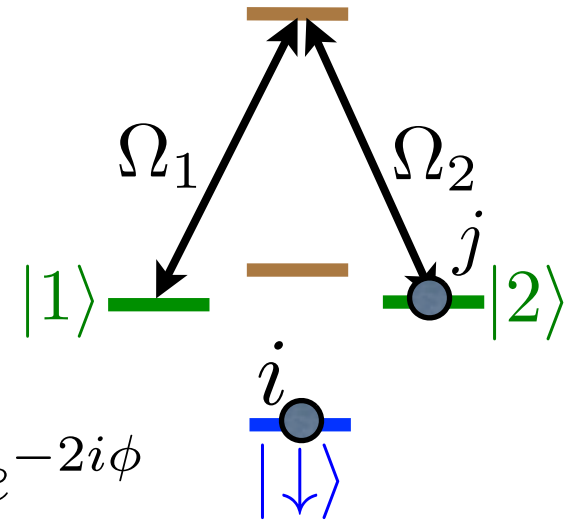
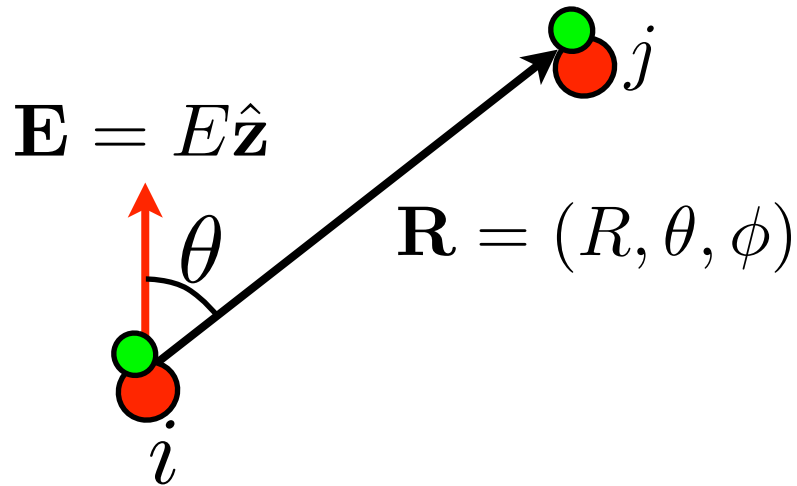
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Spin Hamiltonian from dressed states

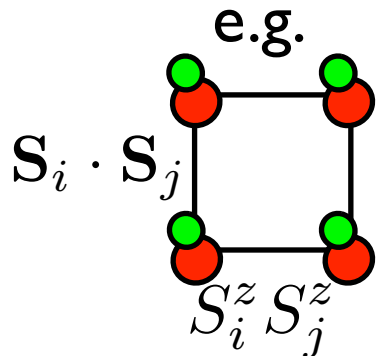


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- what can we implement with this?

Symmetry protected topological (SPT) phases

Shnyder et al 2008, Kitaev 2009, Gu & Wen 2009

Symmetry protected topological (SPT) phases

- **topological** \approx no local order parameter, exotic

local order
parameter:
e.g. $\uparrow \downarrow \uparrow \downarrow \uparrow \downarrow$

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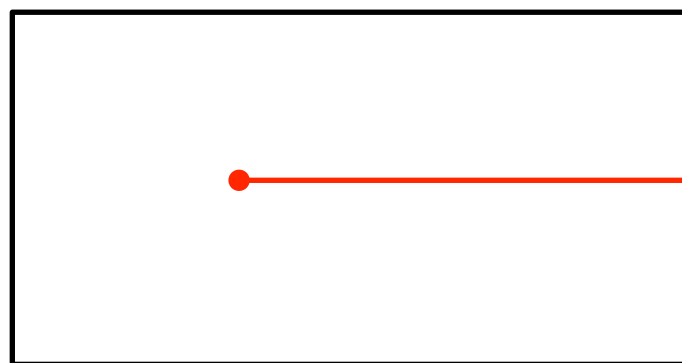
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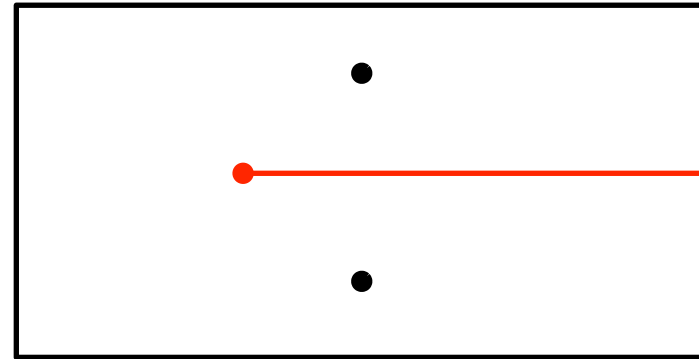
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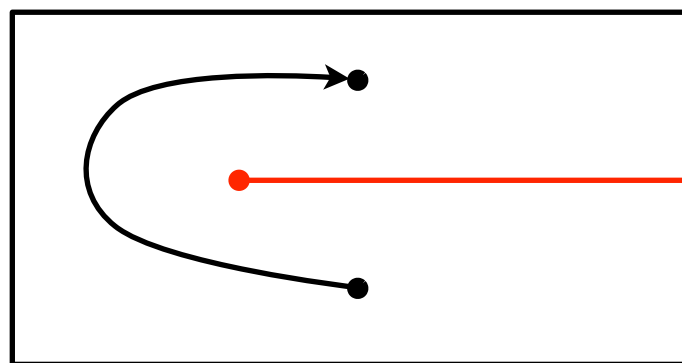
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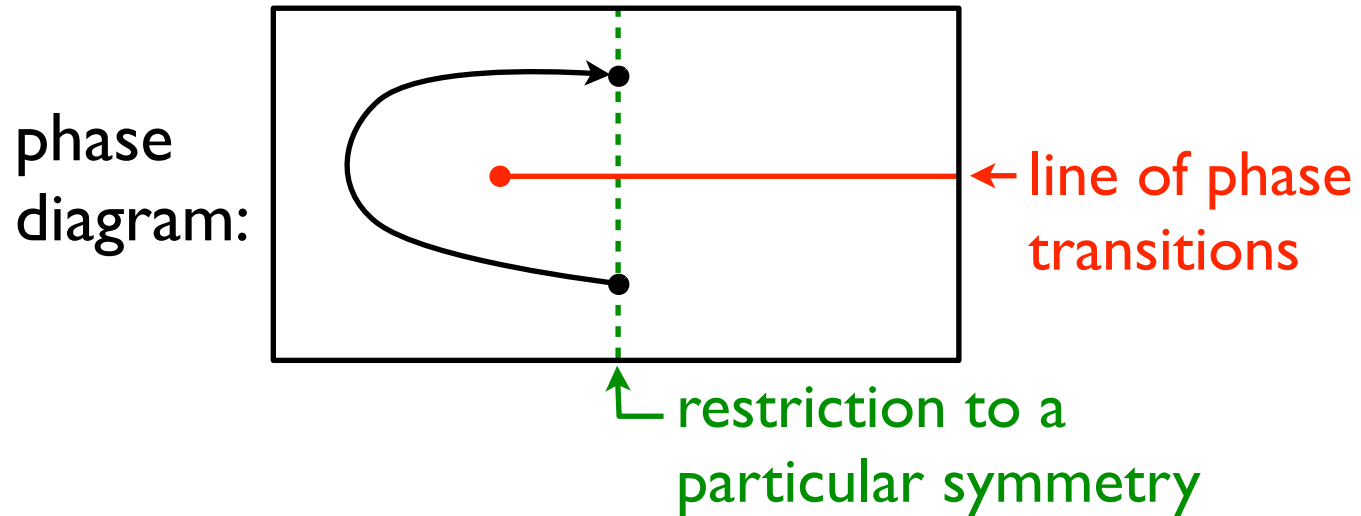
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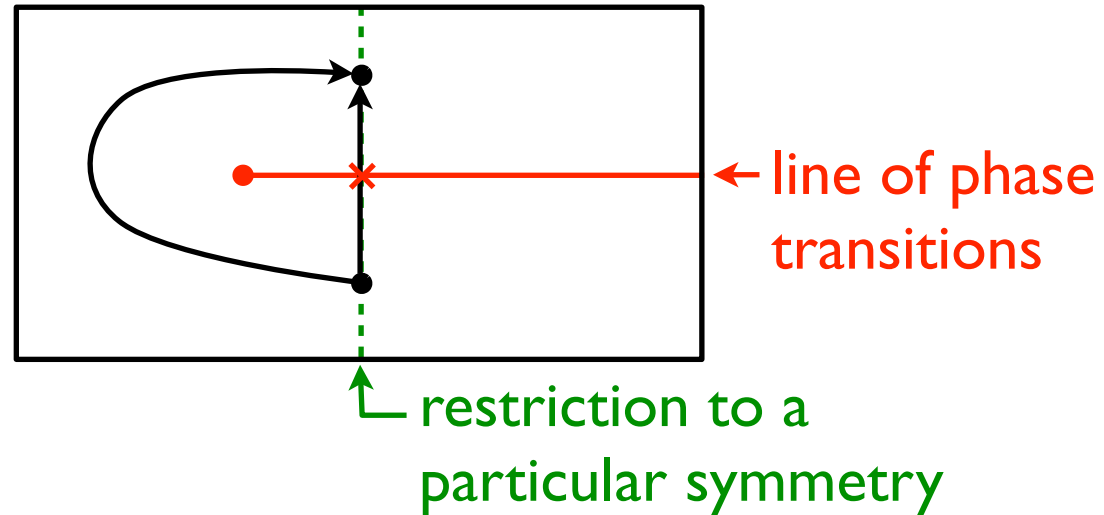
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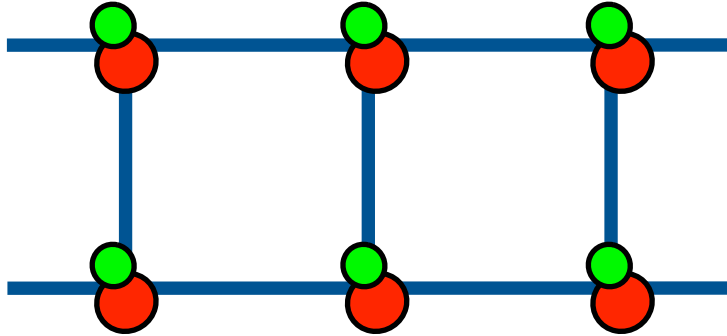
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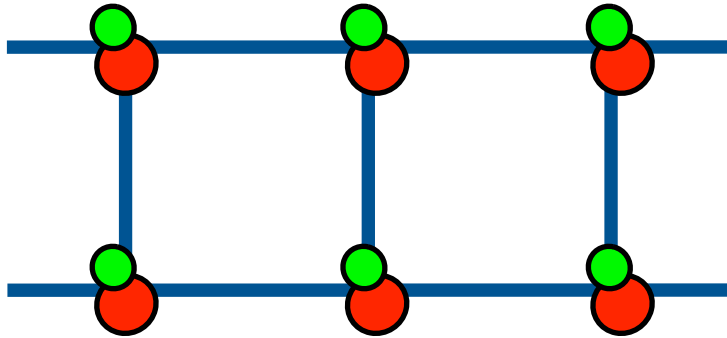
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SPT phases in spin-1/2 ladders



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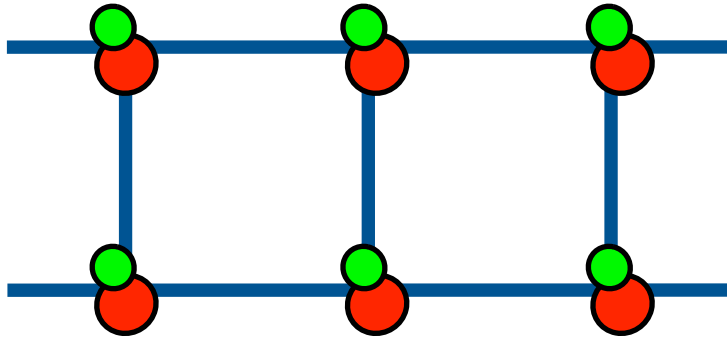
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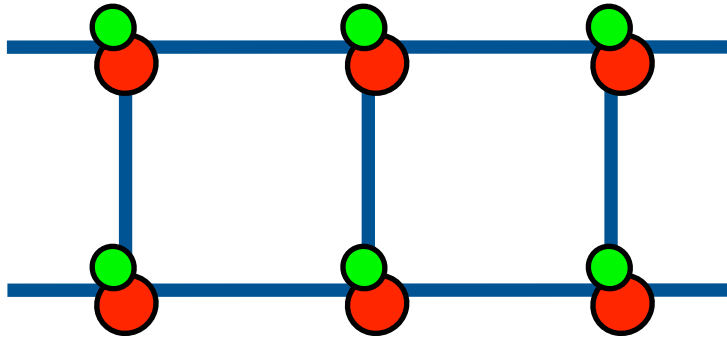


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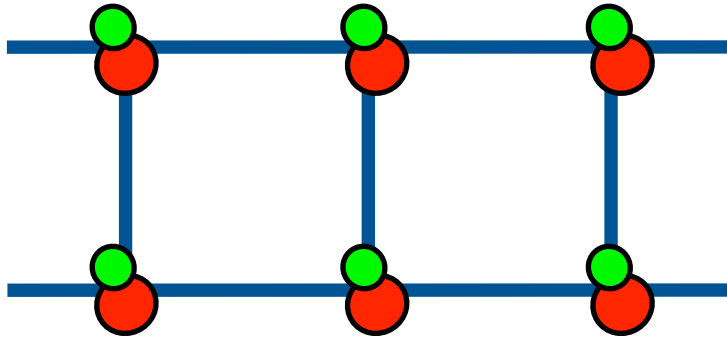


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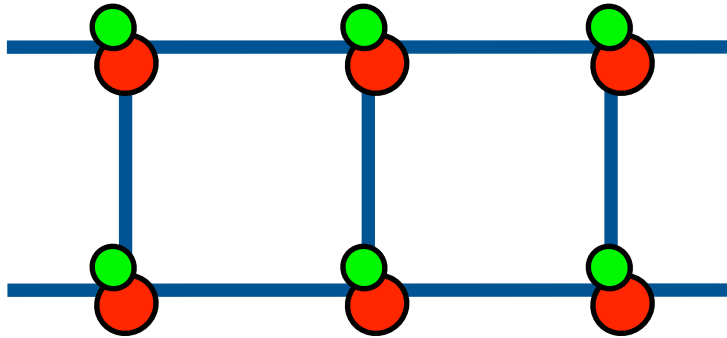
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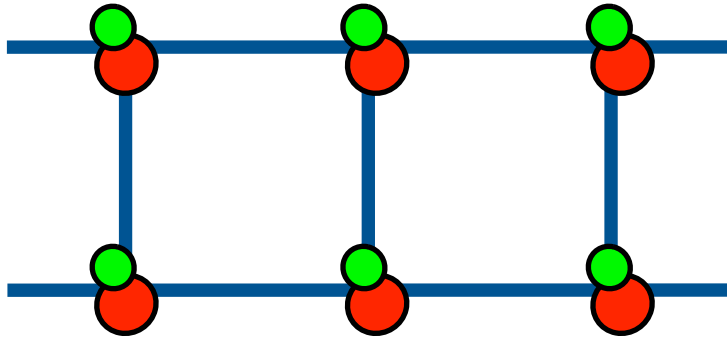
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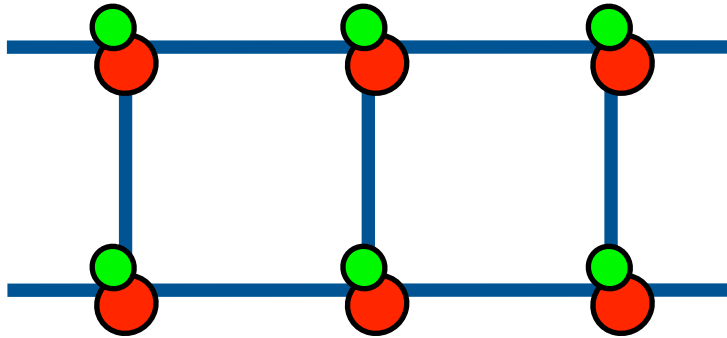
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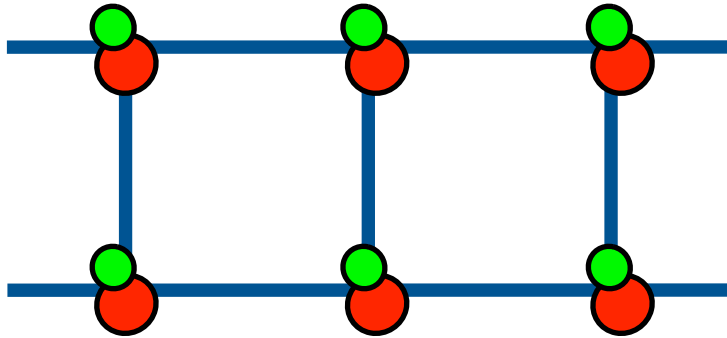
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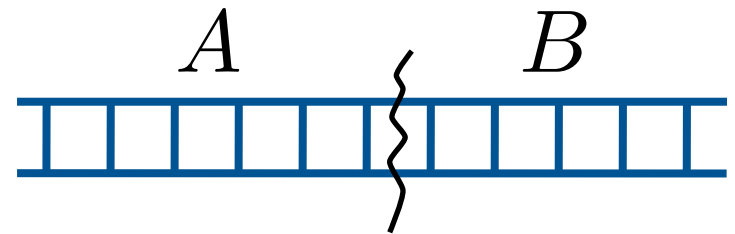
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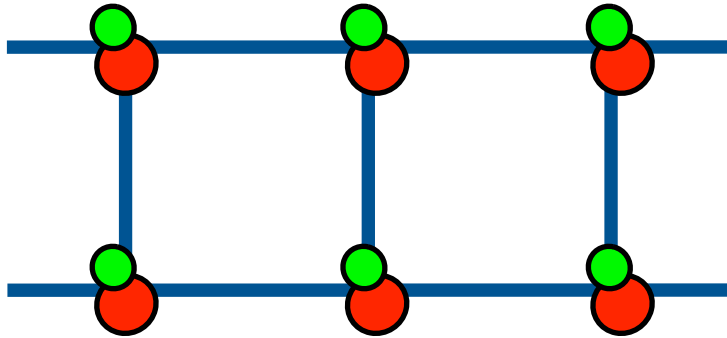
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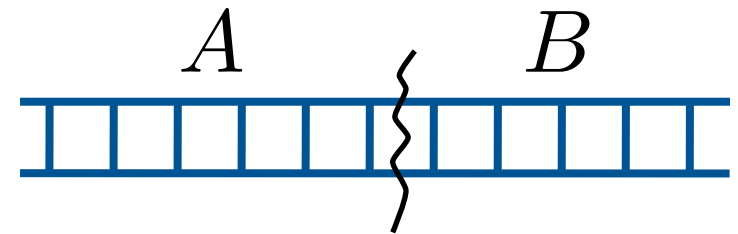
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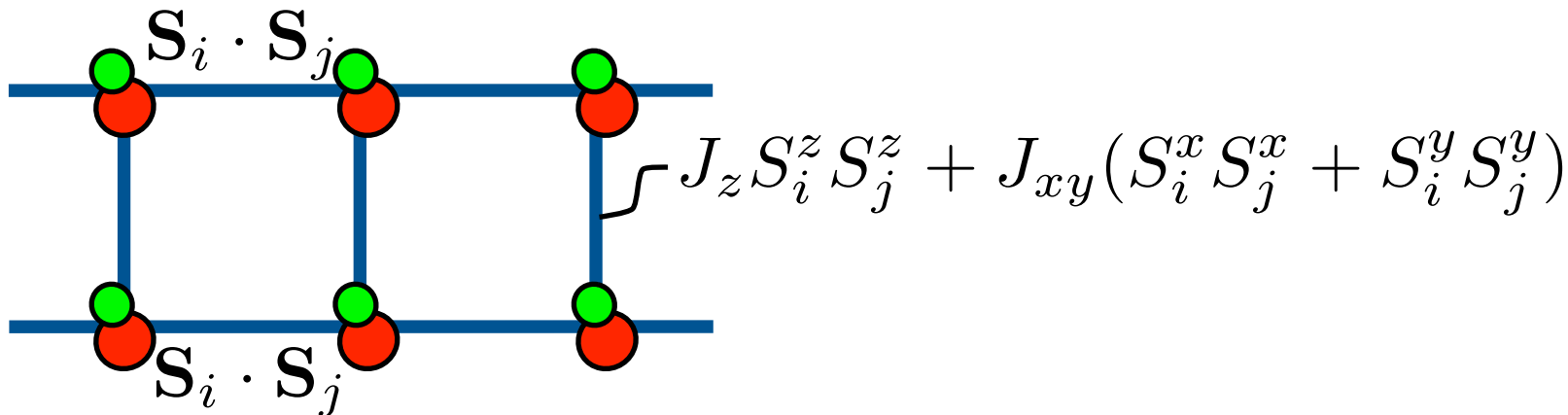
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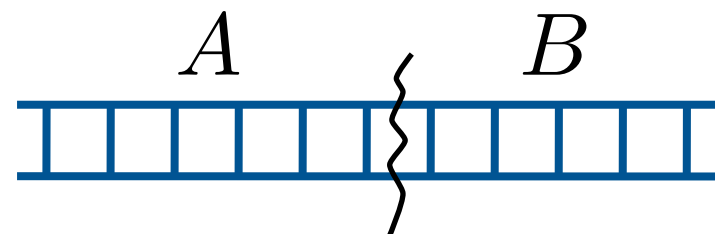
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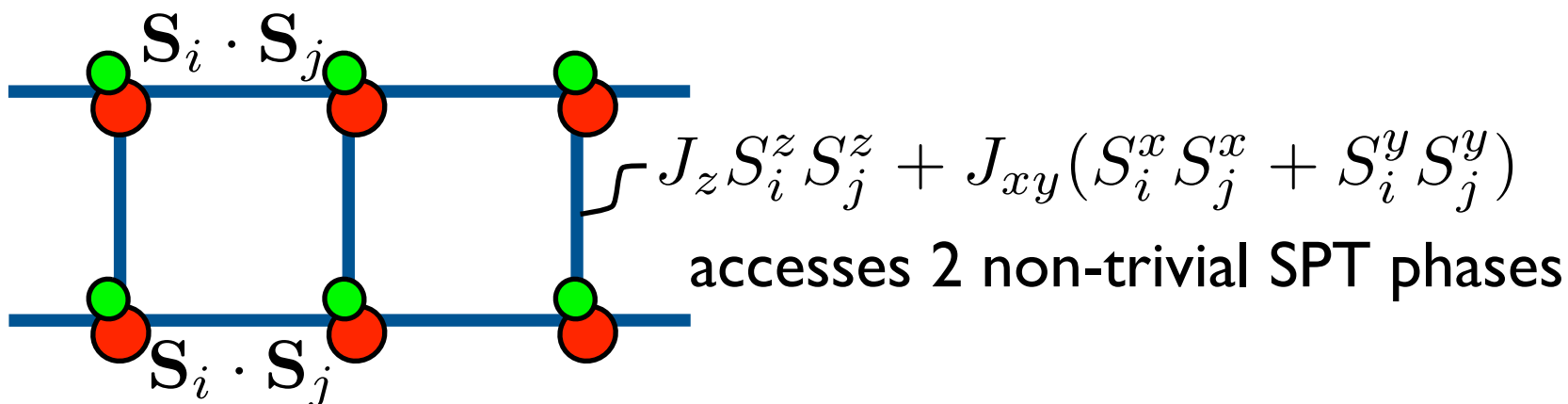
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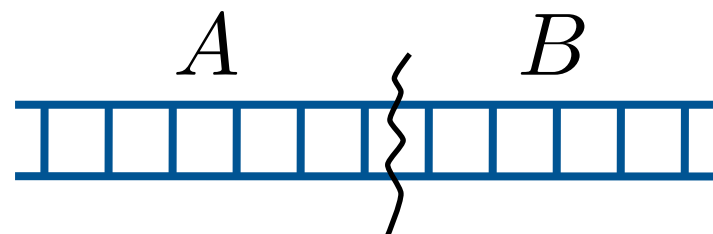
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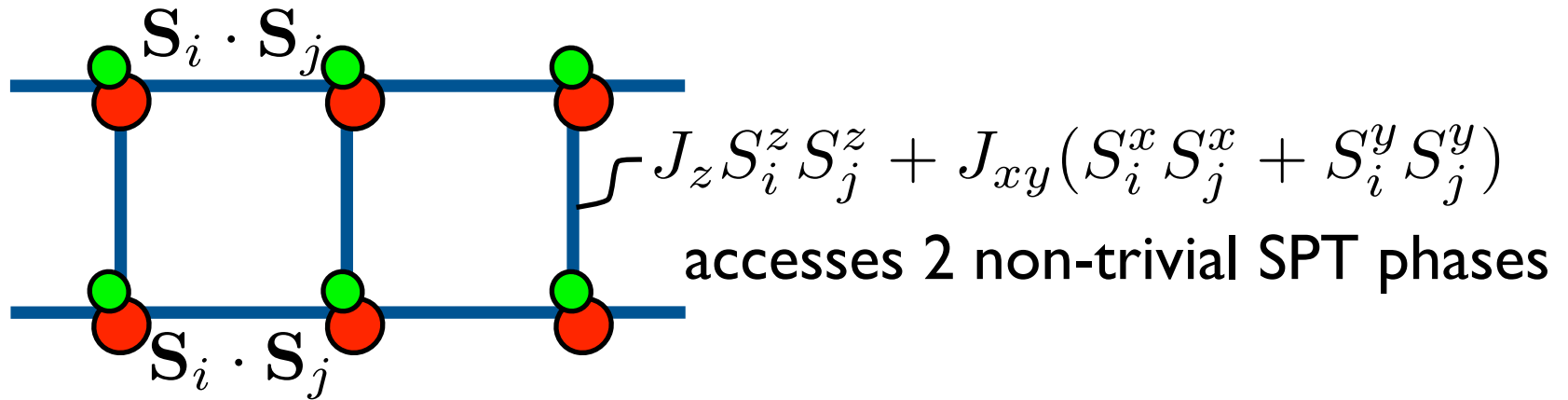
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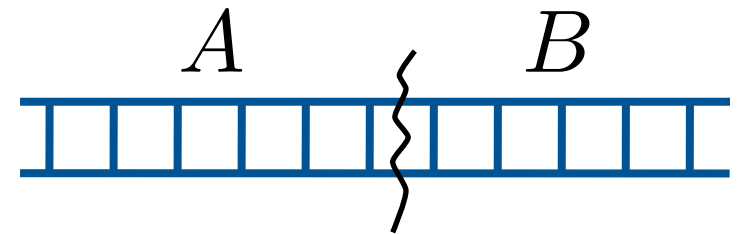


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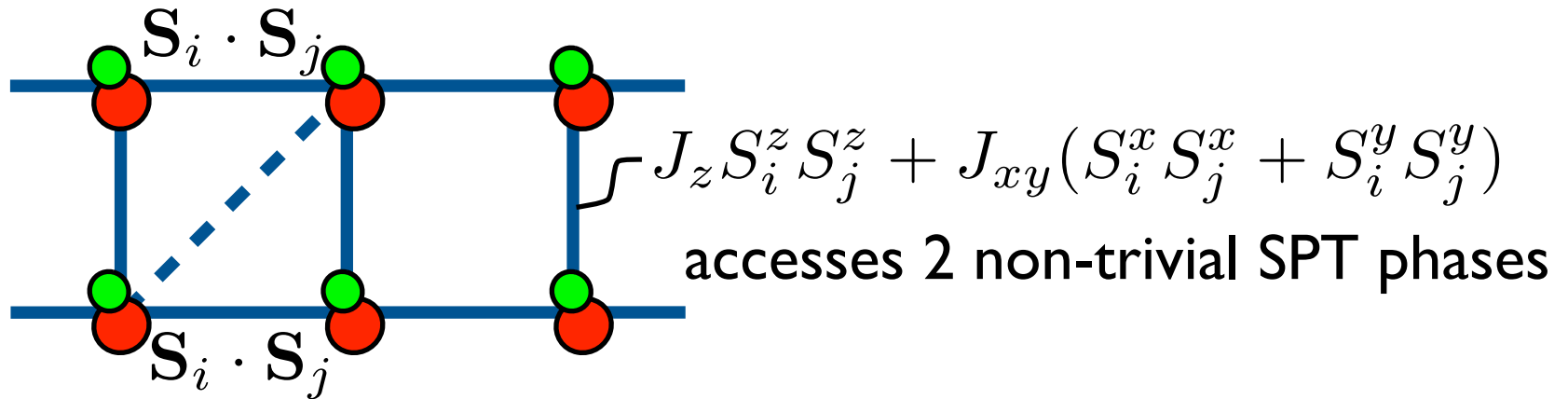


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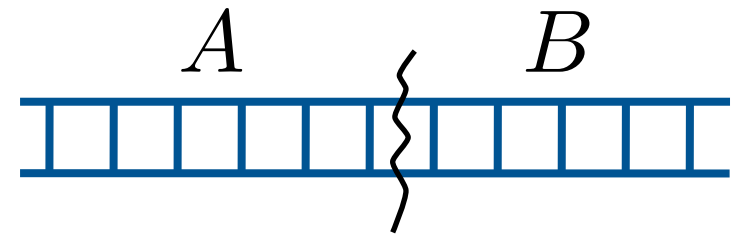


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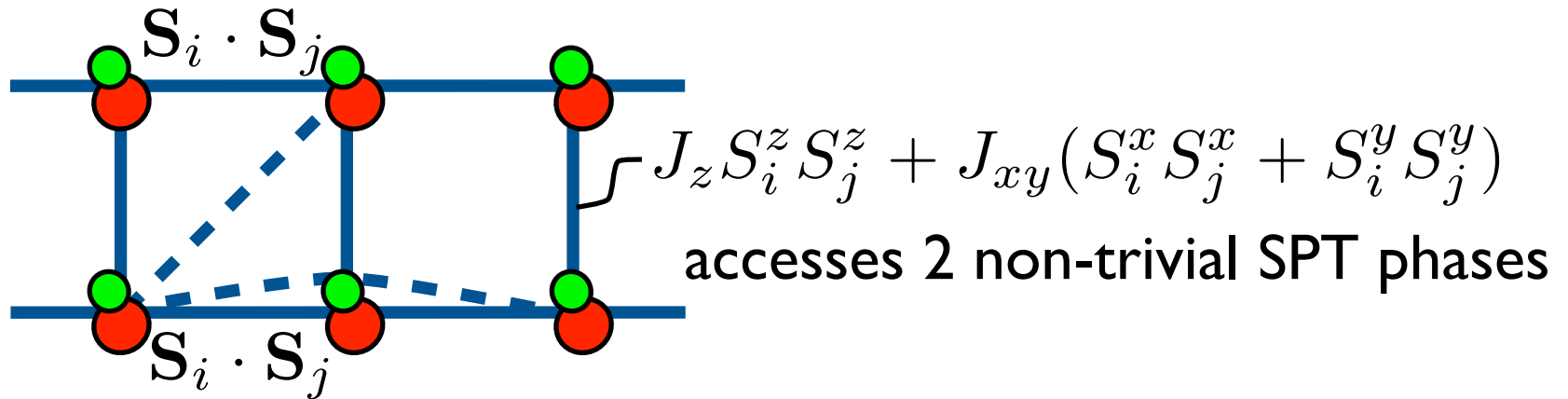


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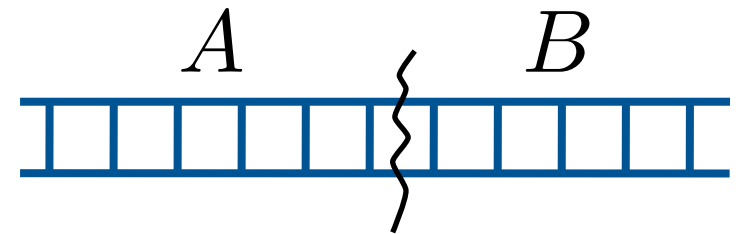


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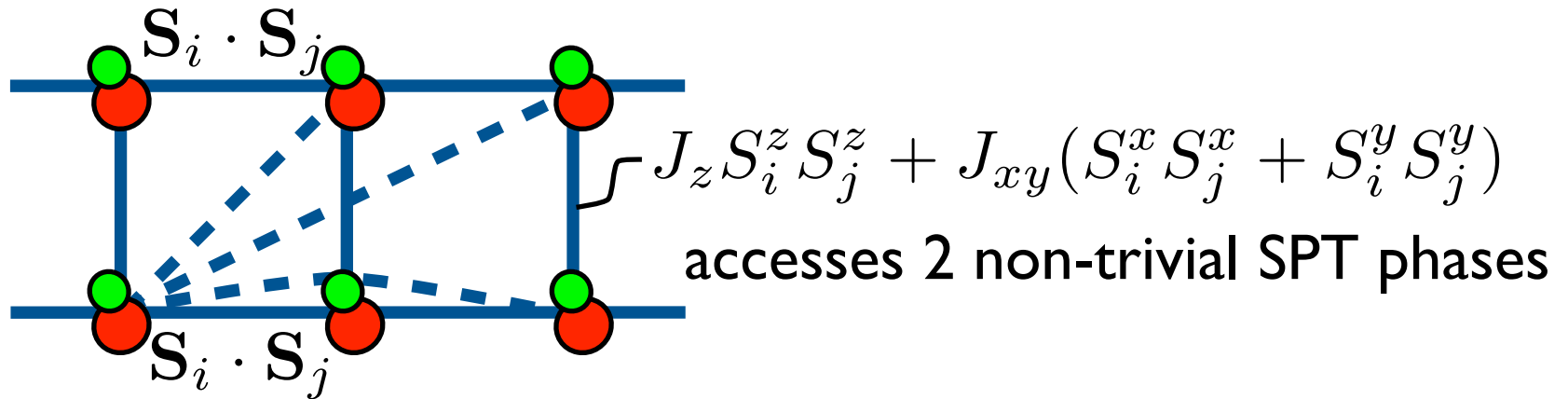


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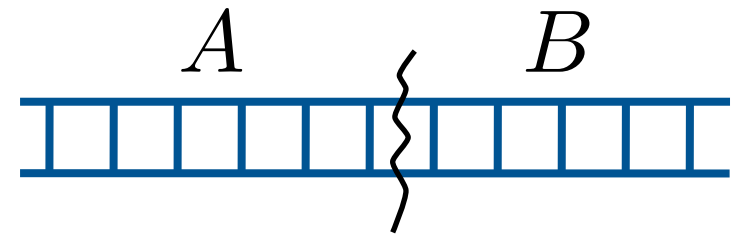


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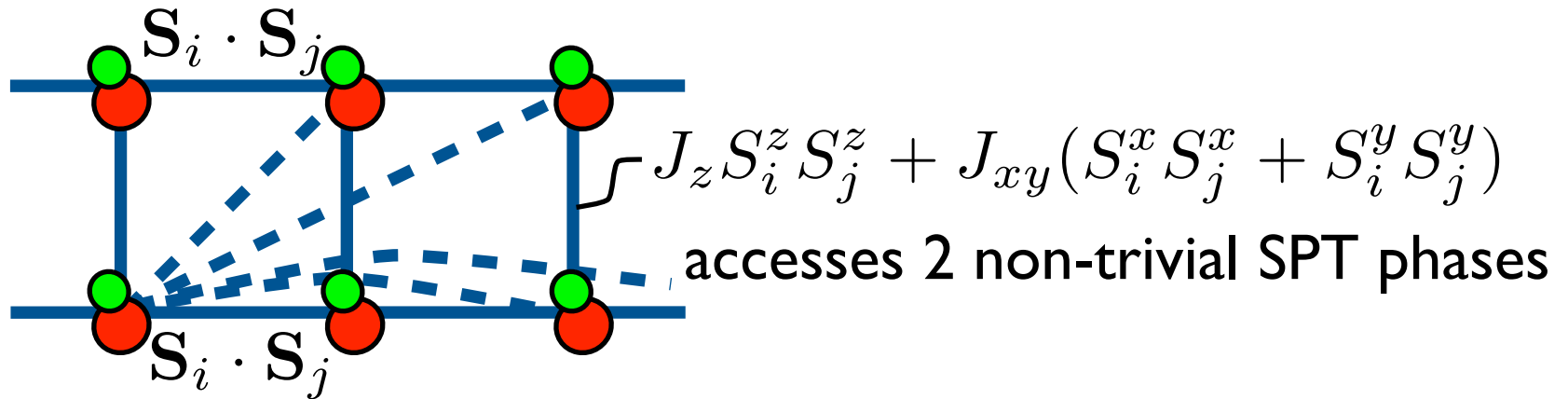


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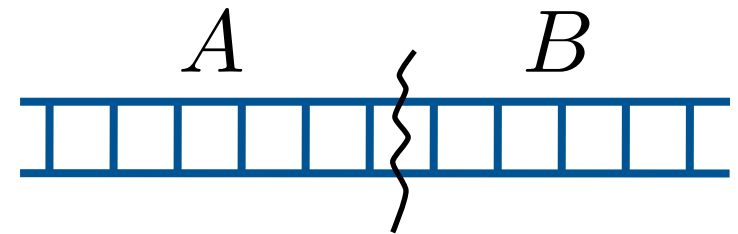


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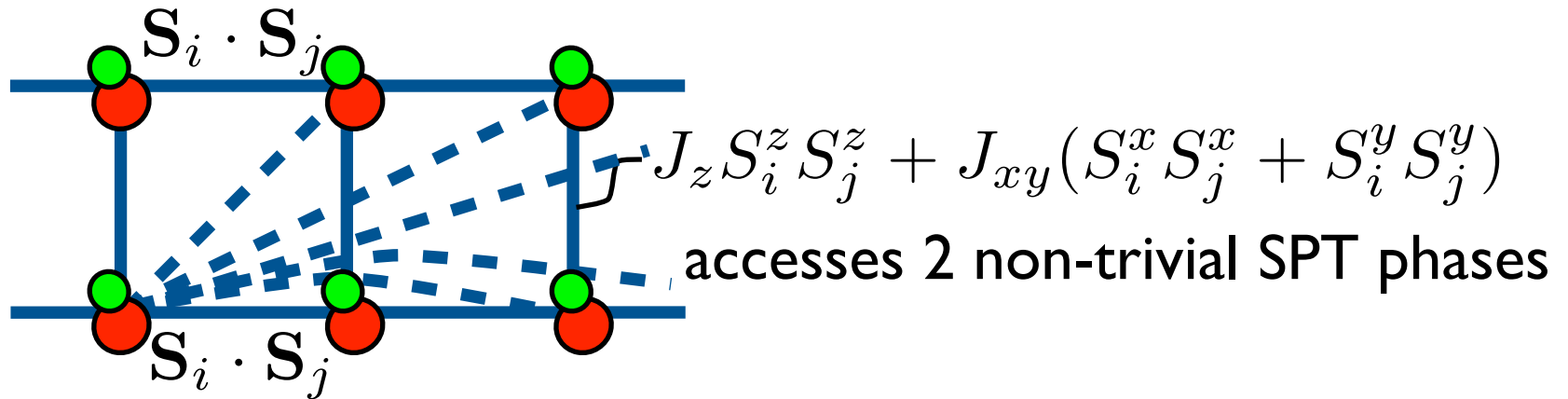


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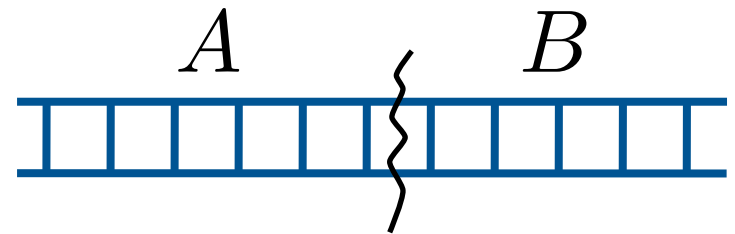


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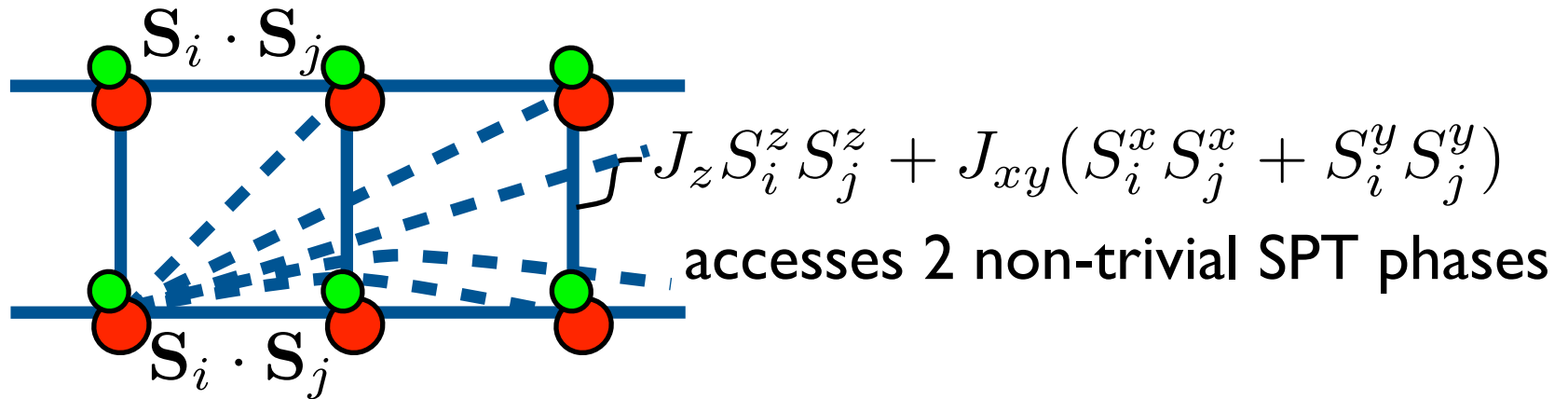


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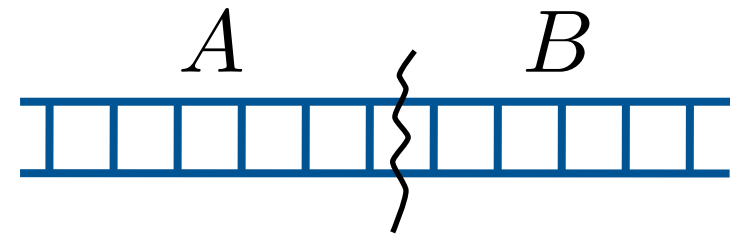
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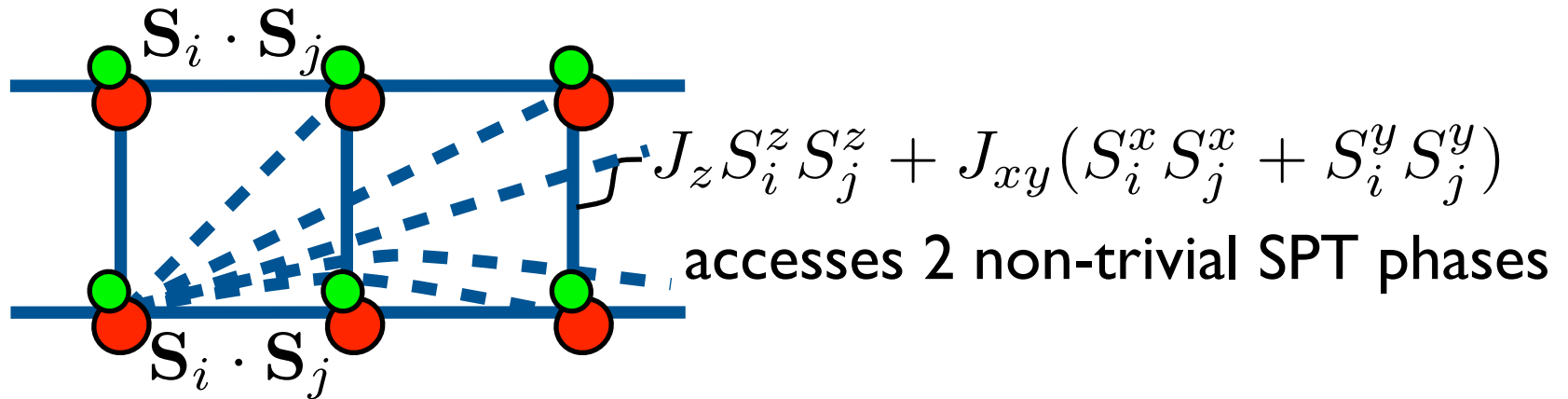
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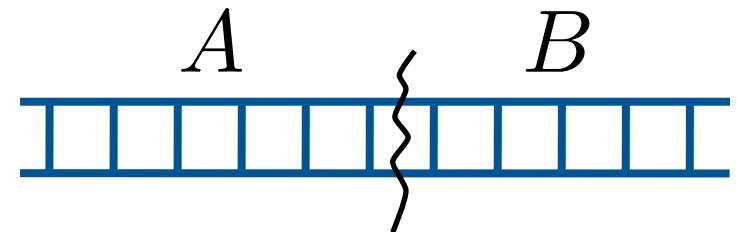


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[arXiv:1210.5518]

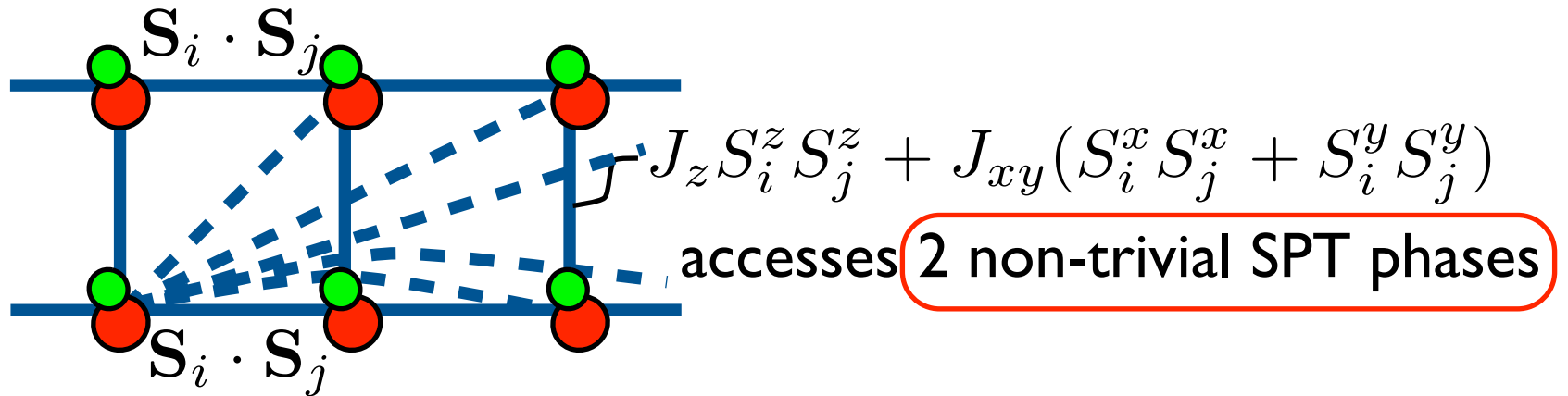
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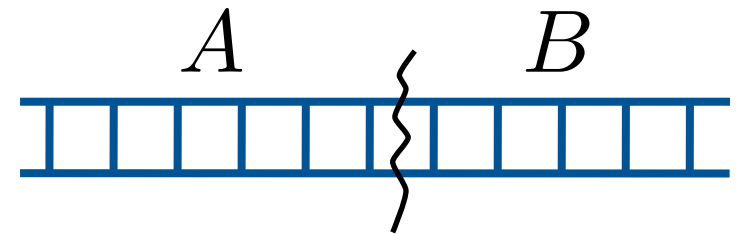


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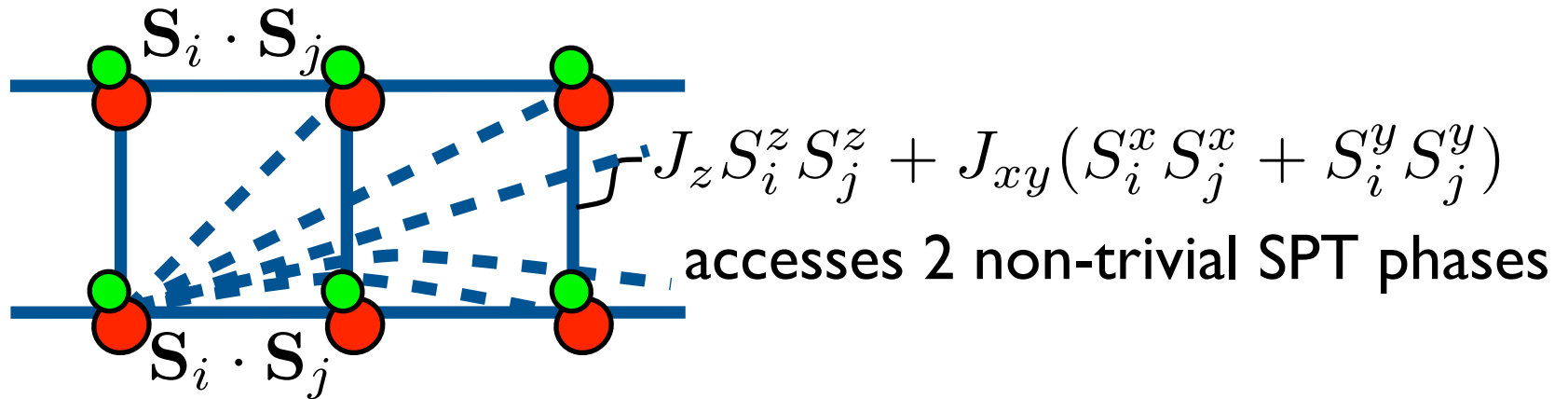
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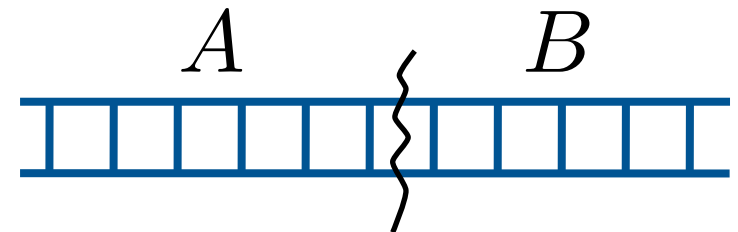
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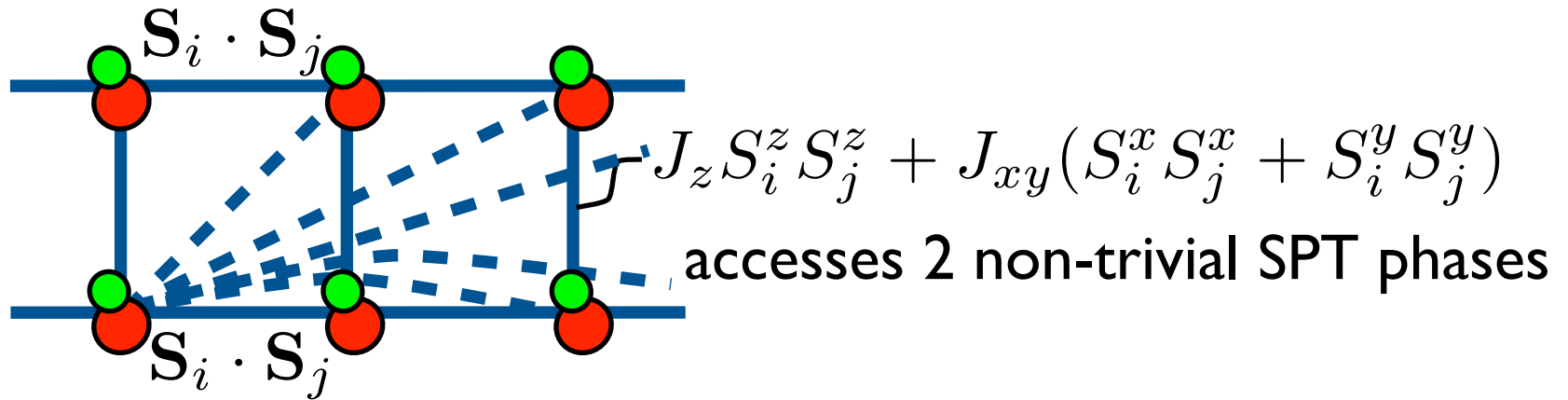
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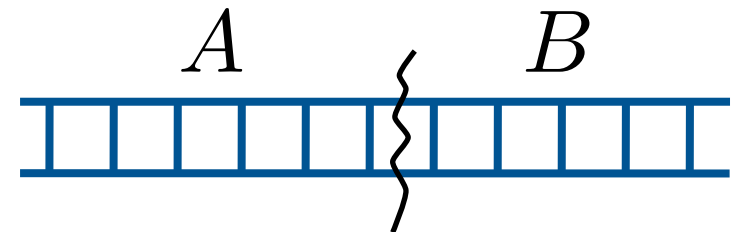


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- in general, peculiar effects of long-range interactions

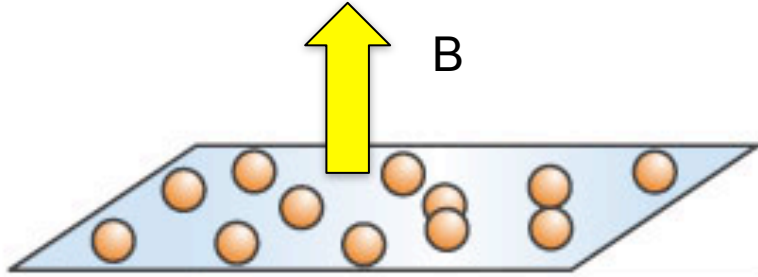
Topological flat bands
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Motivation

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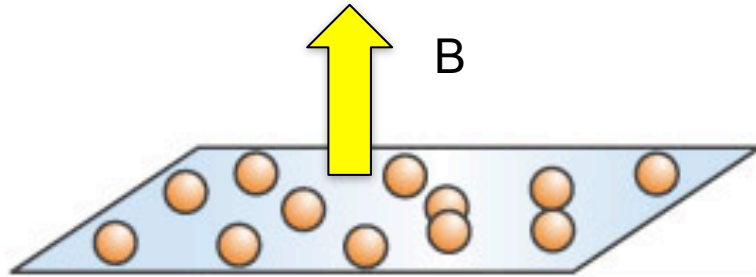
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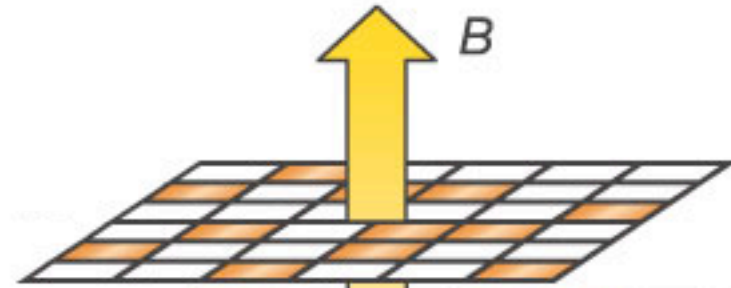
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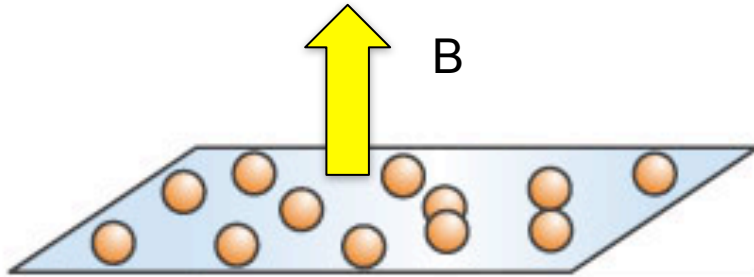
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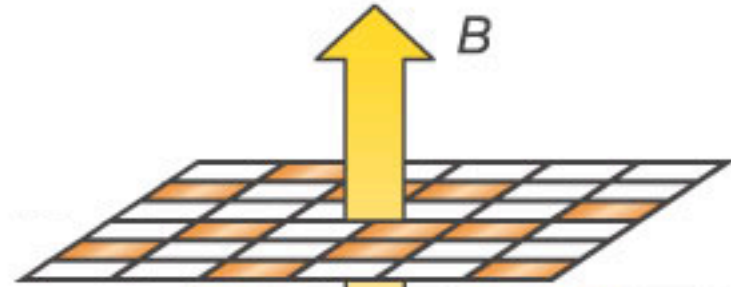
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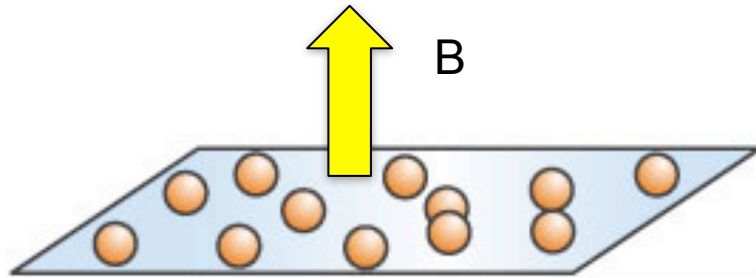
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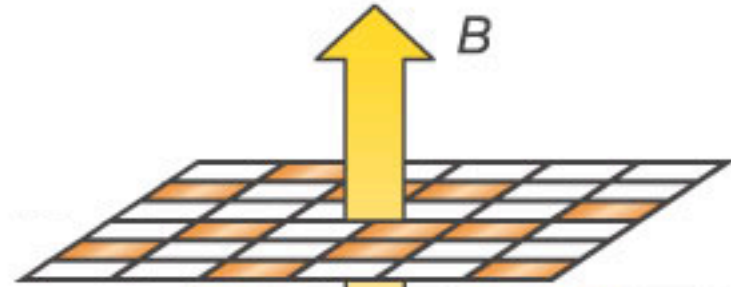
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integer quantum
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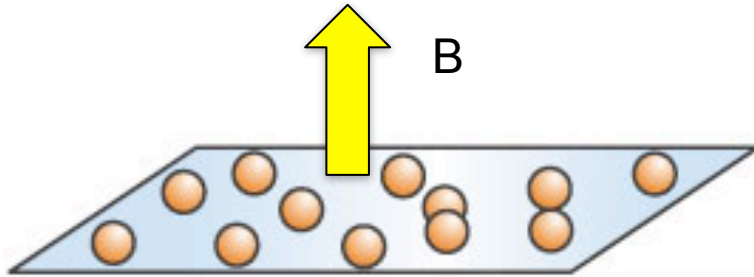
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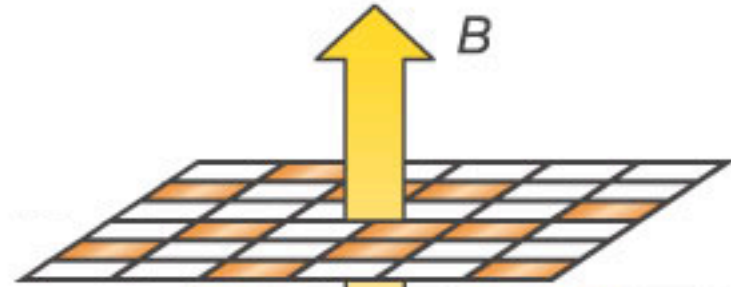
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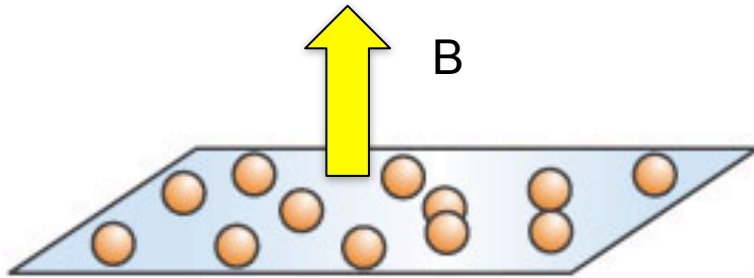


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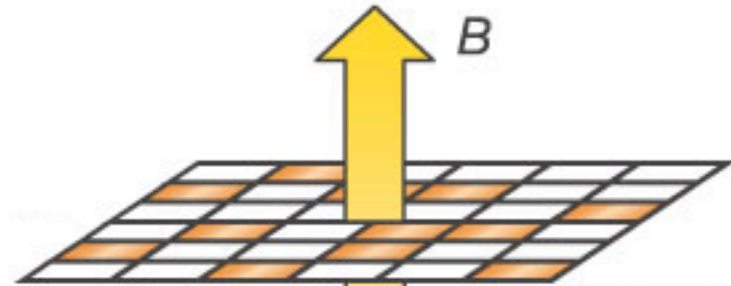
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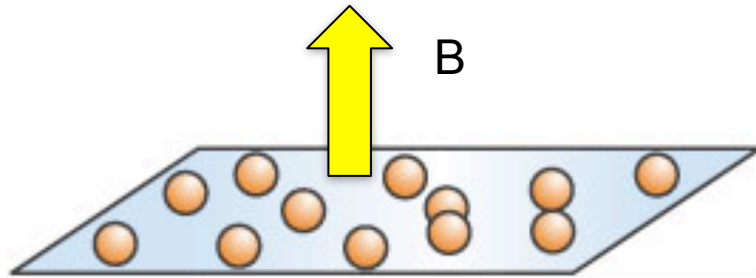
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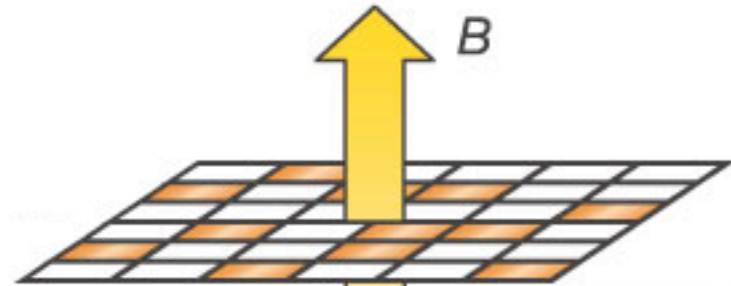
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- fractional quantum Hall effect (FQHE):
 - 2D electron gas in a strong magnetic field B



[Eisenstein & MacDonald, Nature (2004)]



lowest Landau level

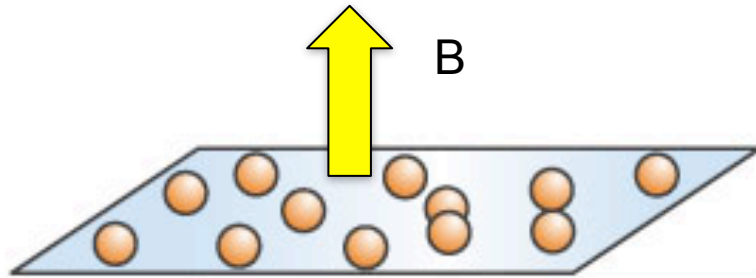
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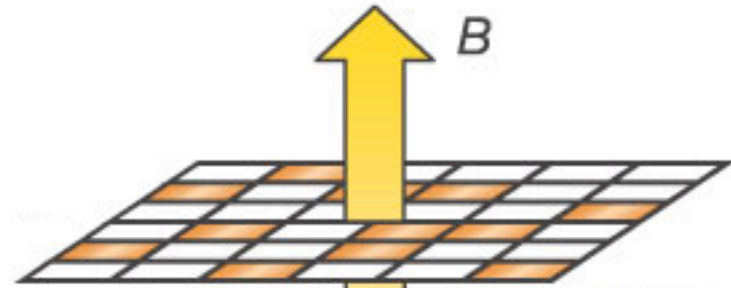
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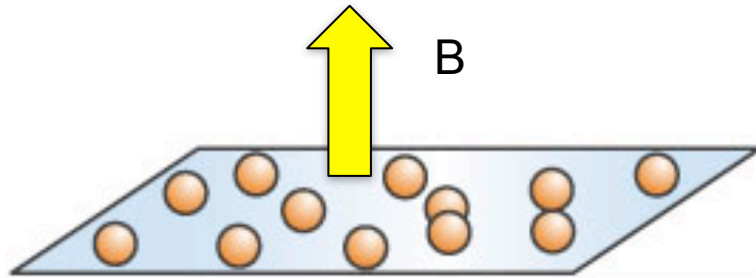
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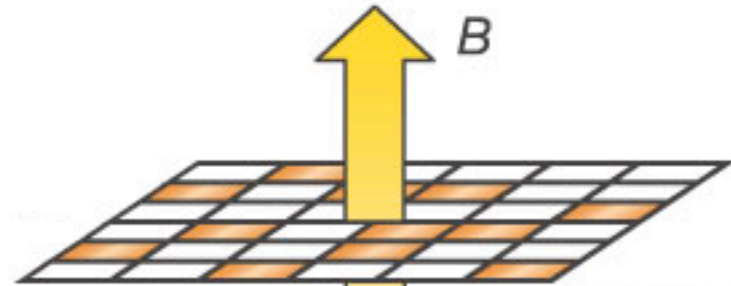
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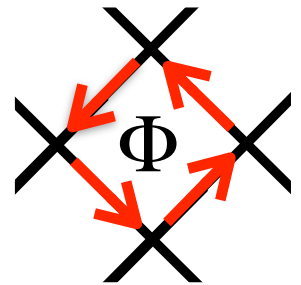
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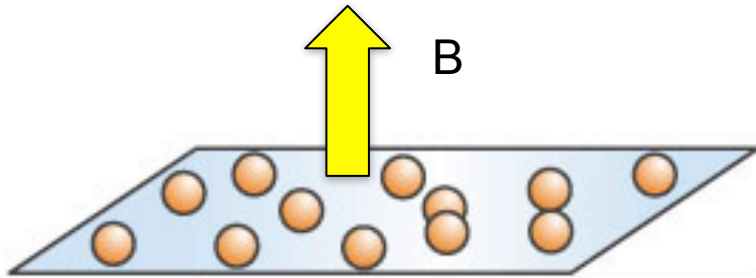
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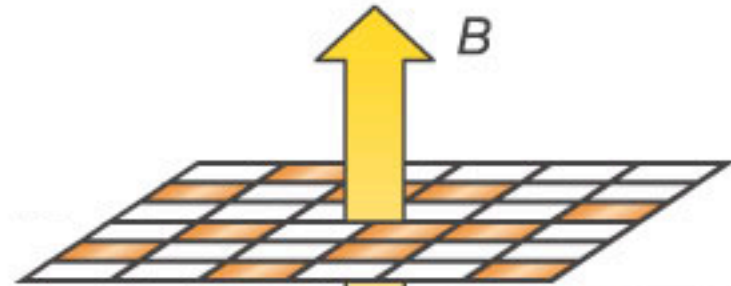
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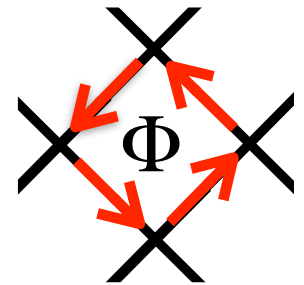
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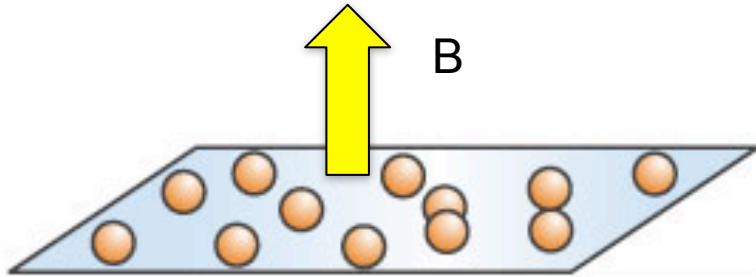
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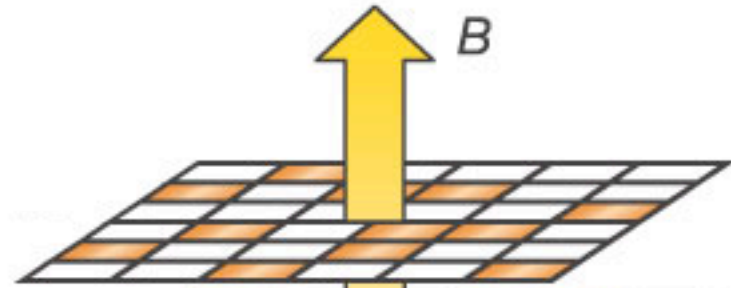
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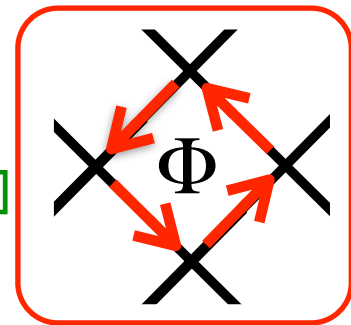
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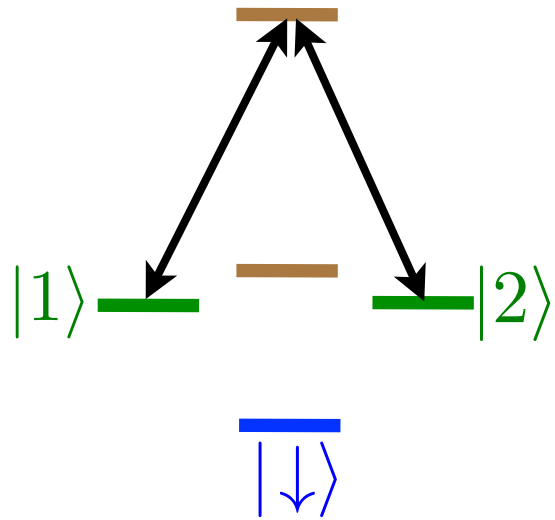
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- realistic physical system missing
- dipolar spin systems (e.g. polar molecules) naturally admit topological flat bands and fractional Chern insulator (\sim FQHE) ground states!

Fractional Chern insulator with polar molecules

Fractional Chern insulator with polar molecules

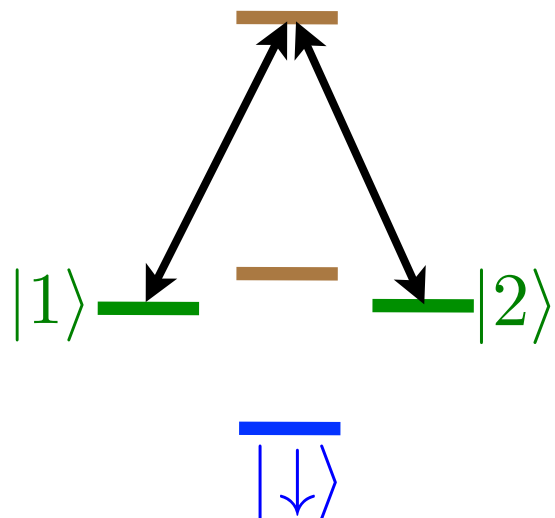
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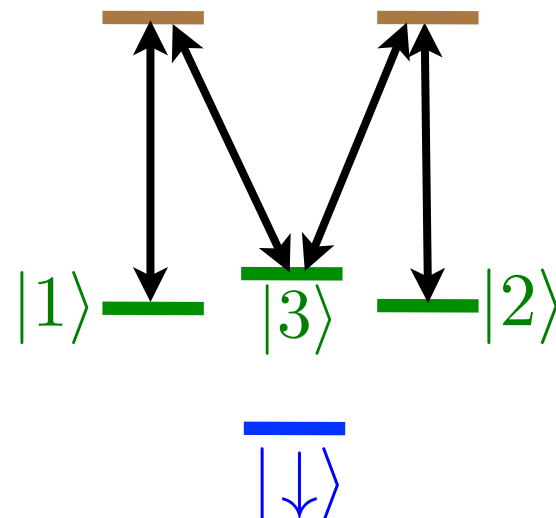
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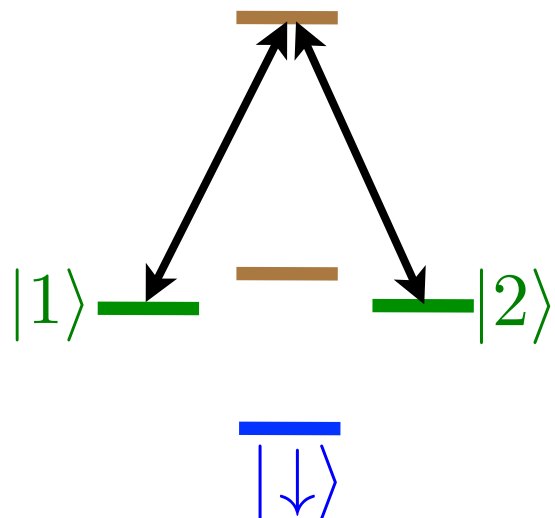


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[PRL 109, 266804 (2012)
& arXiv:1212.4839]

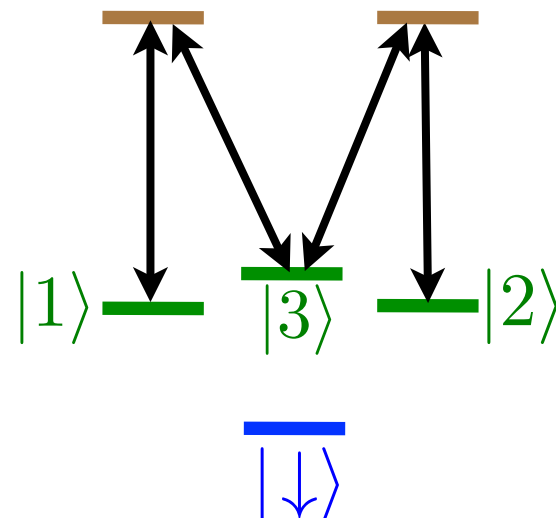
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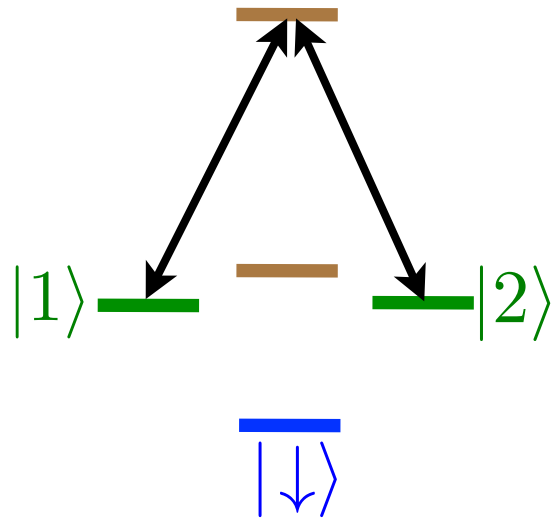
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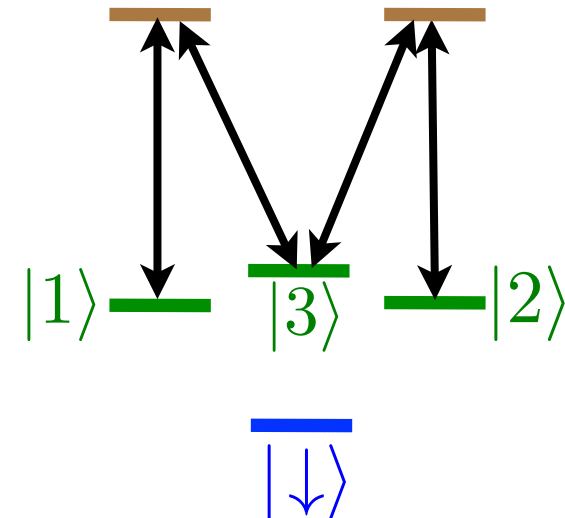
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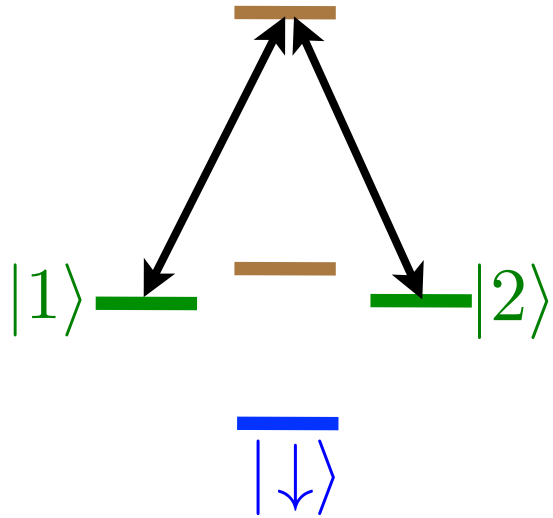
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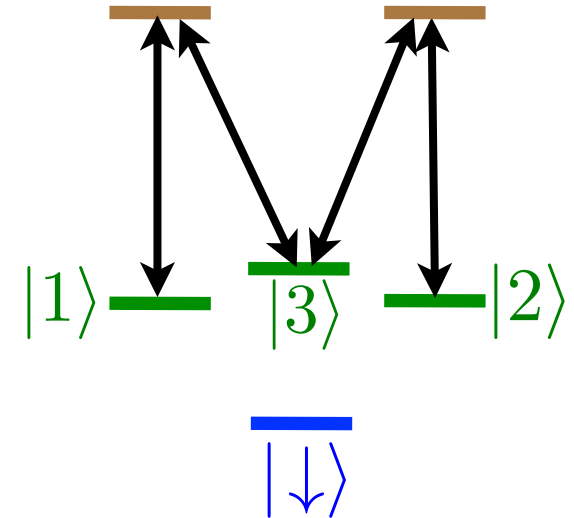
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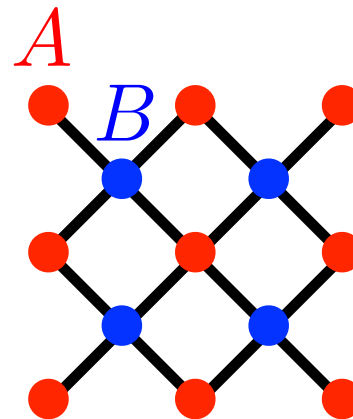
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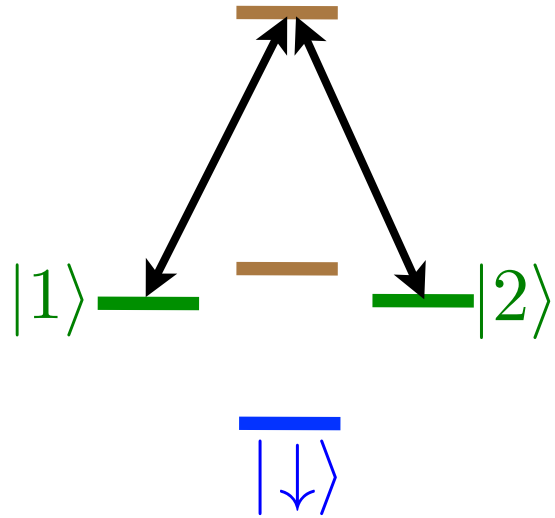
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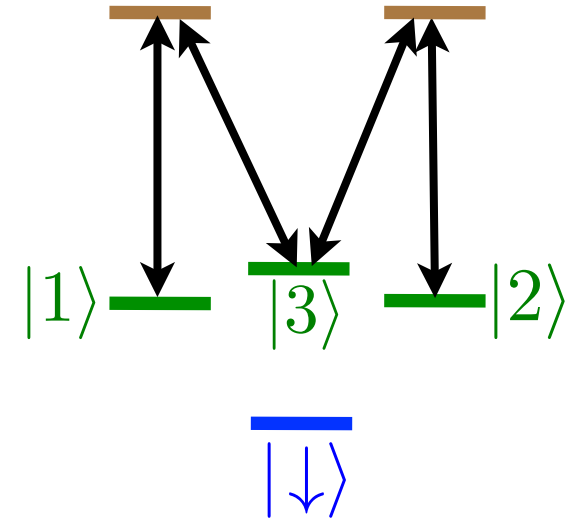
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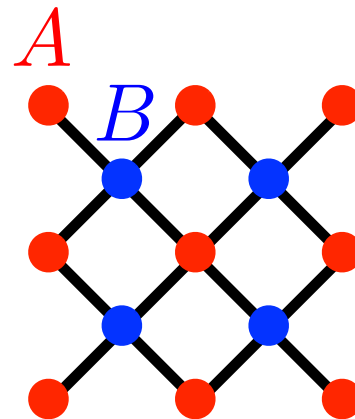
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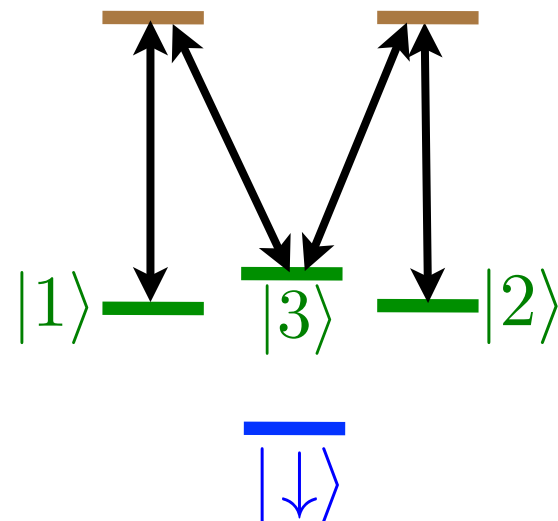
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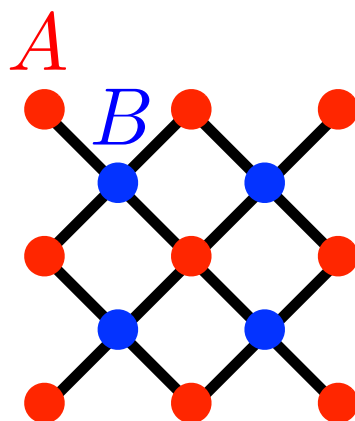
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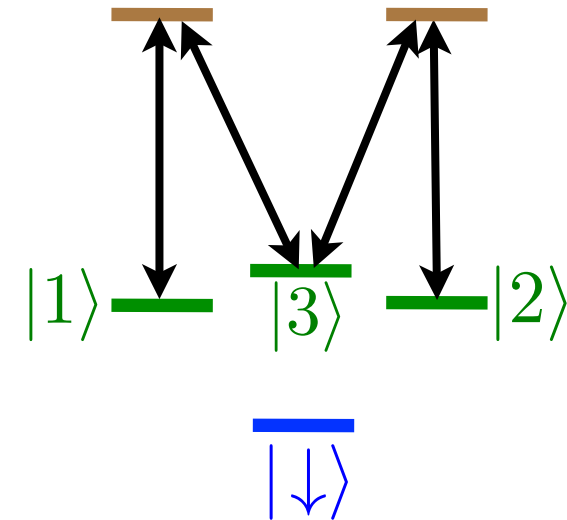


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Fractional Chern insulator with polar molecules

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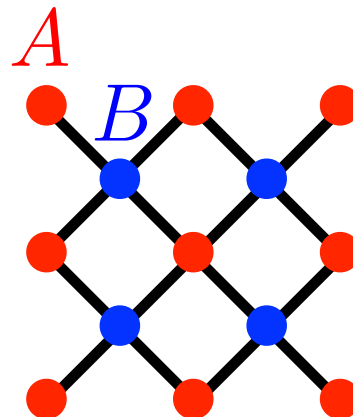
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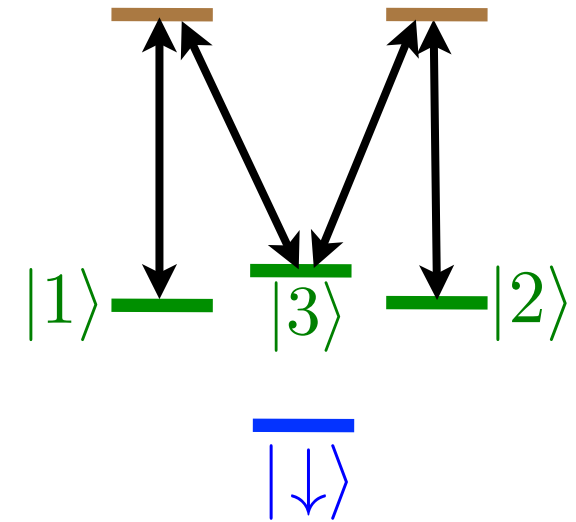
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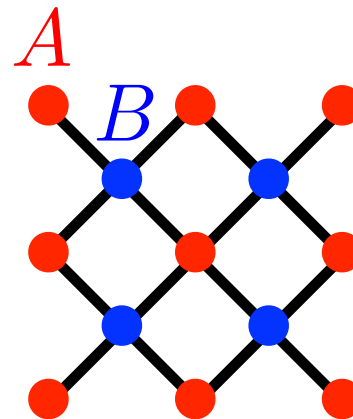
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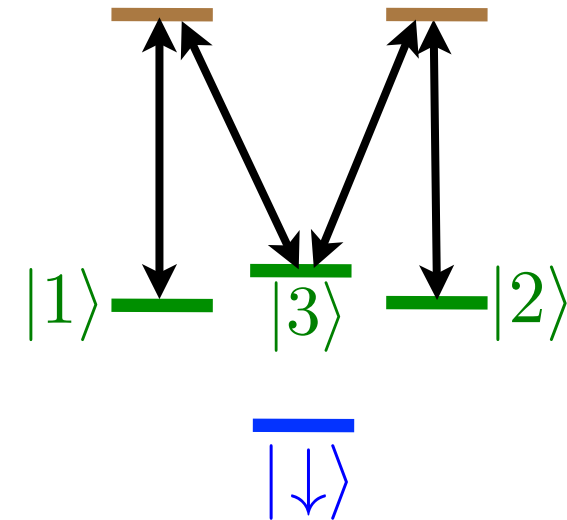
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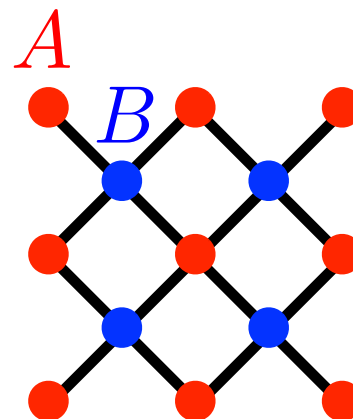
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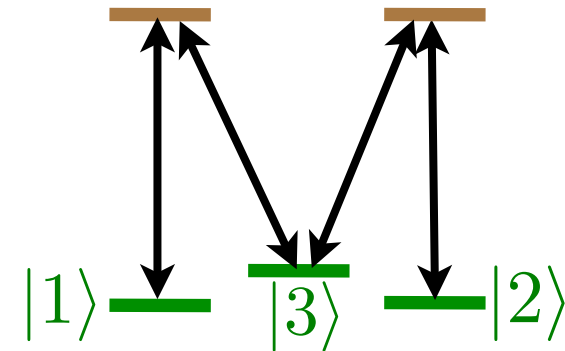
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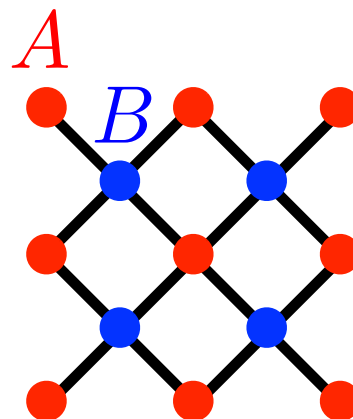


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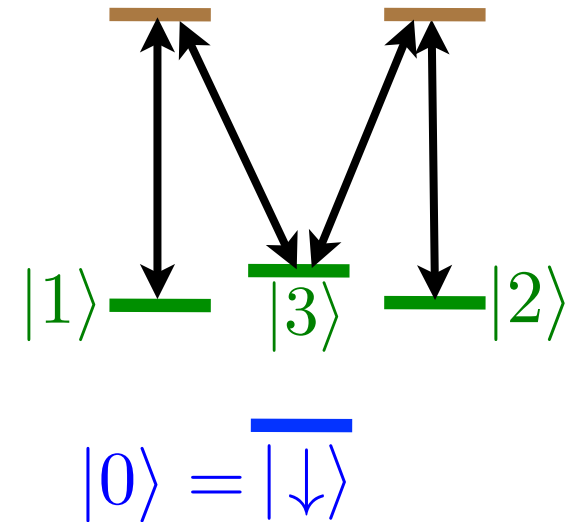
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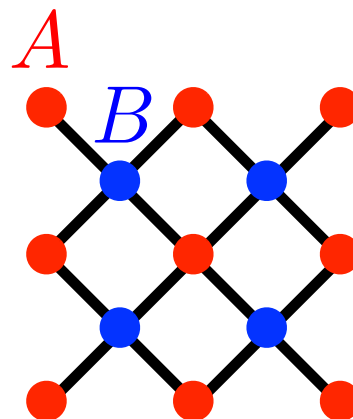
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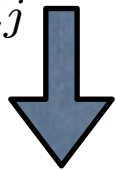
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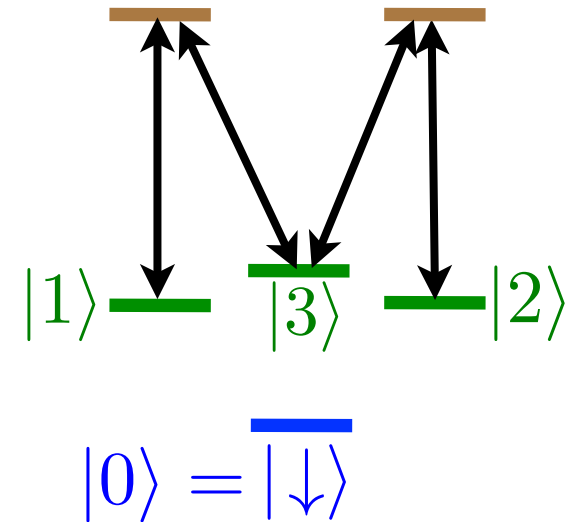
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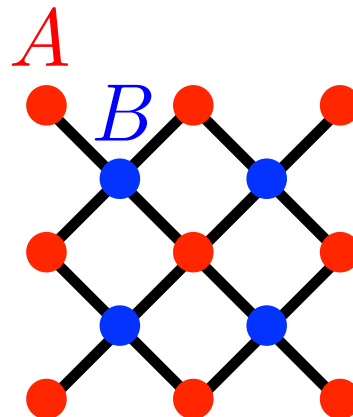
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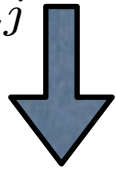
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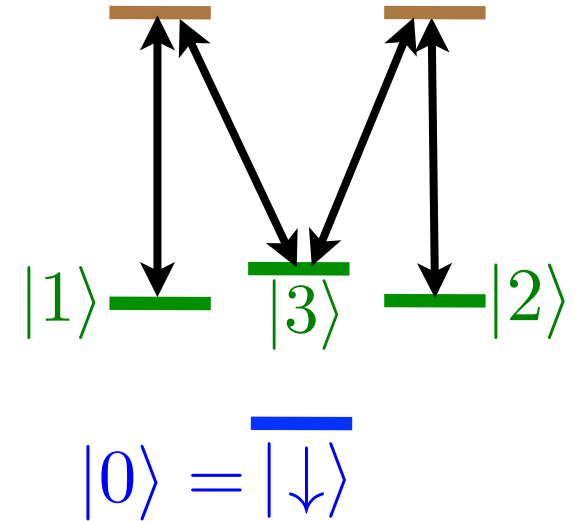
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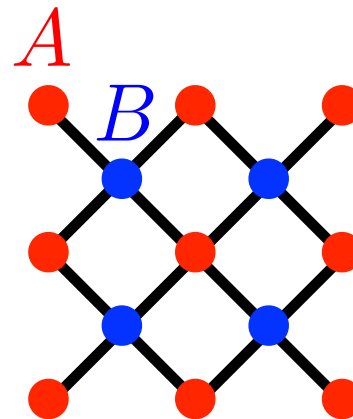
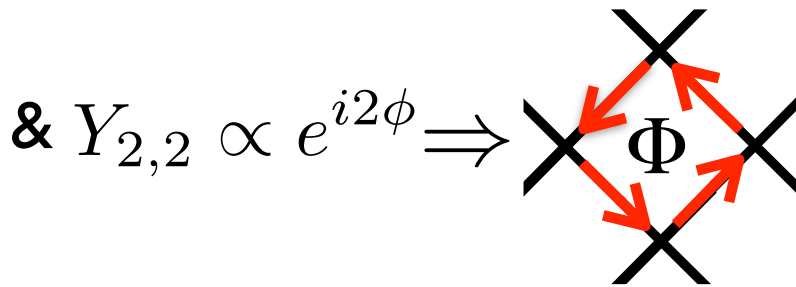


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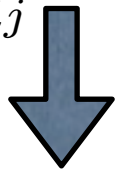
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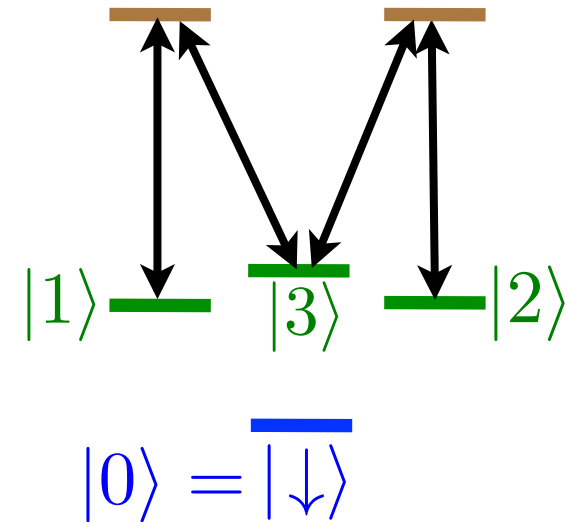
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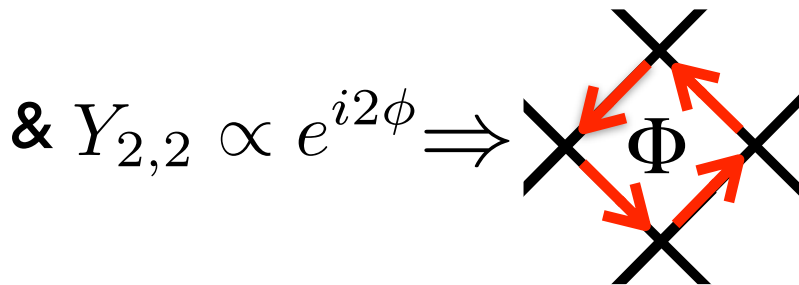


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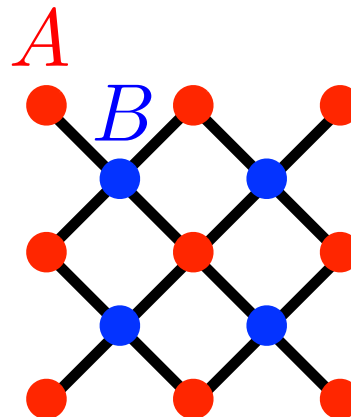


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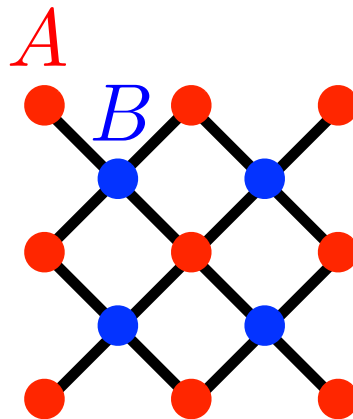
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$$\& Y_{2,2} \propto e^{i2\phi} \Rightarrow \begin{array}{c} \times \\ \swarrow \quad \searrow \\ \times \quad \Phi \quad \times \\ \nwarrow \quad \nearrow \\ \times \end{array}$$

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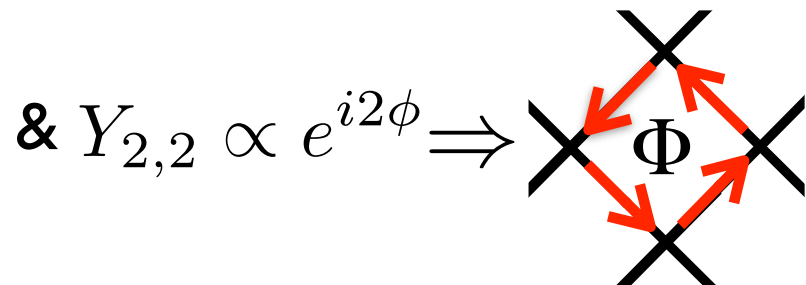


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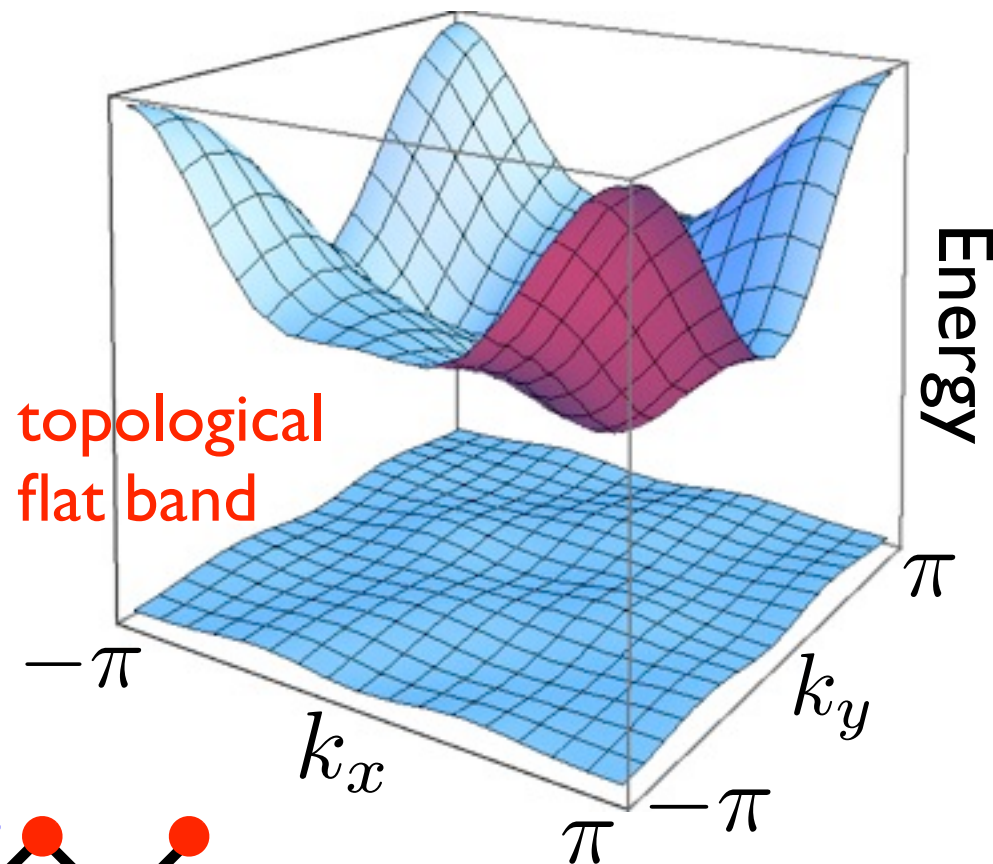
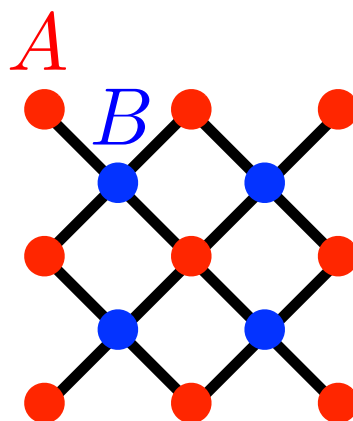
Fractional Chern insulator with polar molecules

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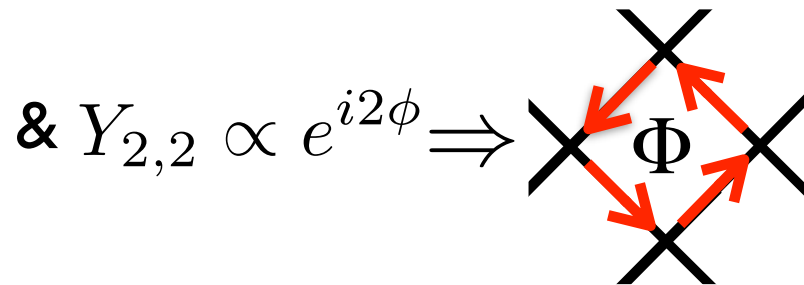


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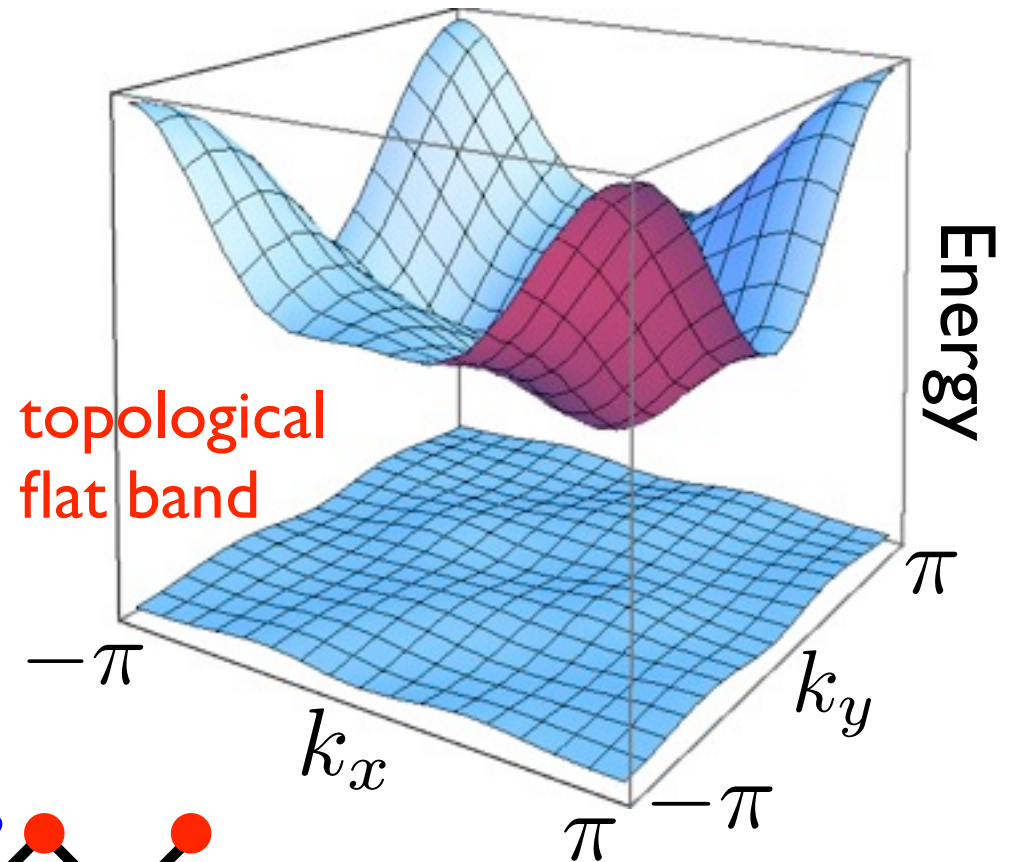
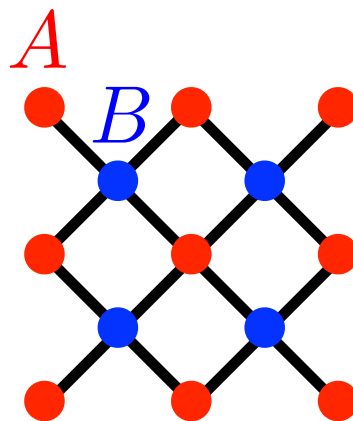
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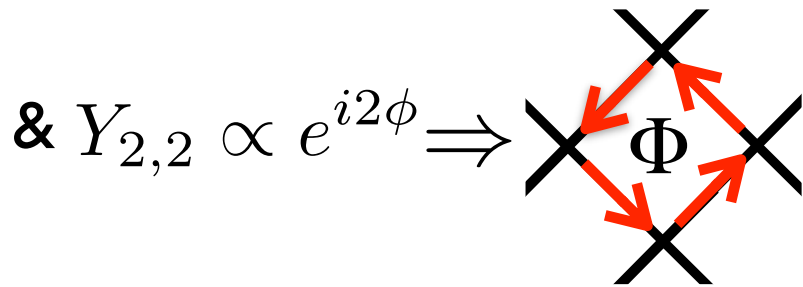


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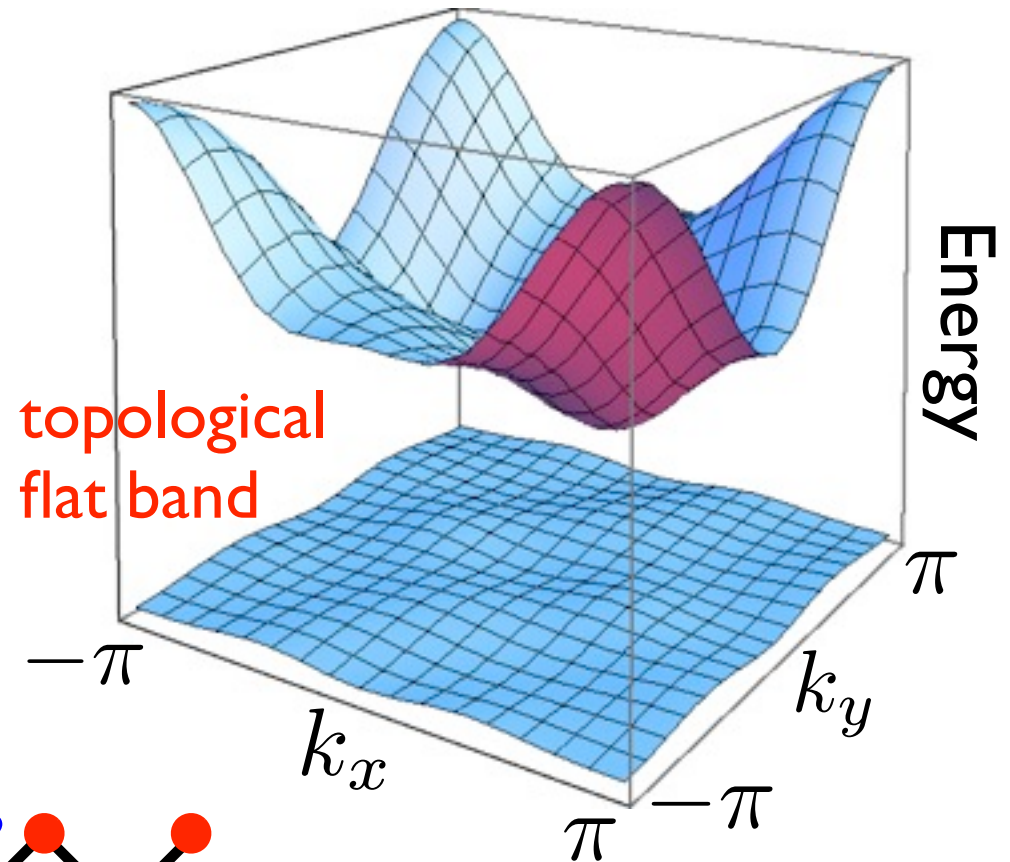
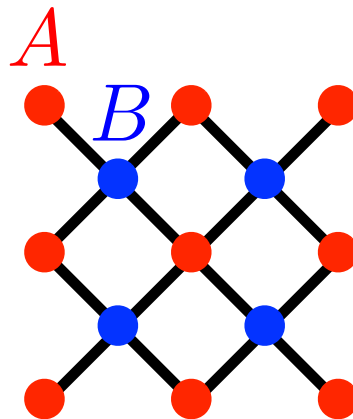
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(informal)
Overview and discussion
of
many-body physics with polar molecules

M.A. Baranov, M. Dalmonte, G. Pupillo, and P. Zoller,
Condensed Matter Theory of Dipolar Quantum Gases,
Chem. Rev. 112, 5012 (2012); arXiv:1207.1914 [cond-mat.quant-gas].

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Field of many-body physics with polar molecules owes its promise to:

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- strong, long-range, anisotropic interactions
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But:

- very challenging experiments (e.g. compared to trapped ions or neutral atoms)

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- study all of these separately and in combinations, with DC magnetic and electric field control, microwave and RF control, optical control

Geometry

- 3D
- 2D (pancakes)
- 1D (tubes)
- Lattices:
 - 1D (lattice in a tube),
 - 2D (lattice in a pancake),
 - 3D

3D

Baranov, Dalmonte, Pupillo, Zoller, Chem. Rev. 112, 5012 (2012)

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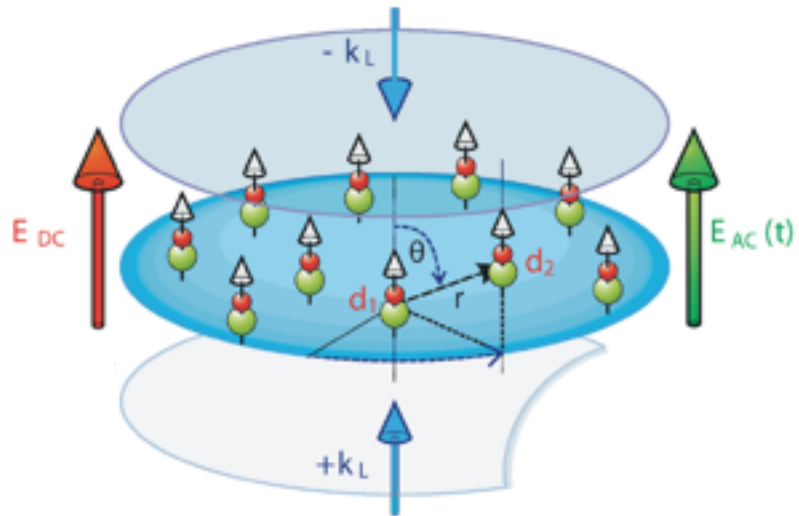
To get **strong interactions:** confine to lower-D and/or use lattice

Baranov, Dalmonde, Pupillo, Zoller, Chem. Rev. 112, 5012 (2012)

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Baranov, Dalmonte, Pupillo, Zoller, Chem. Rev. 112, 5012 (2012)

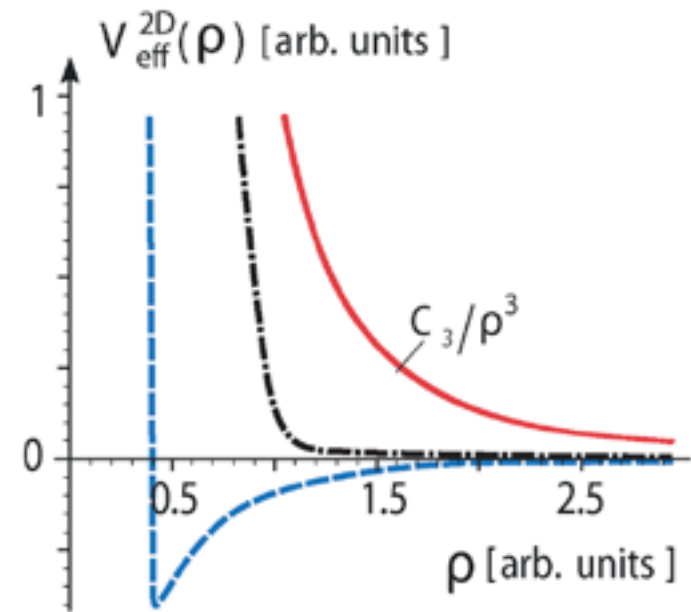
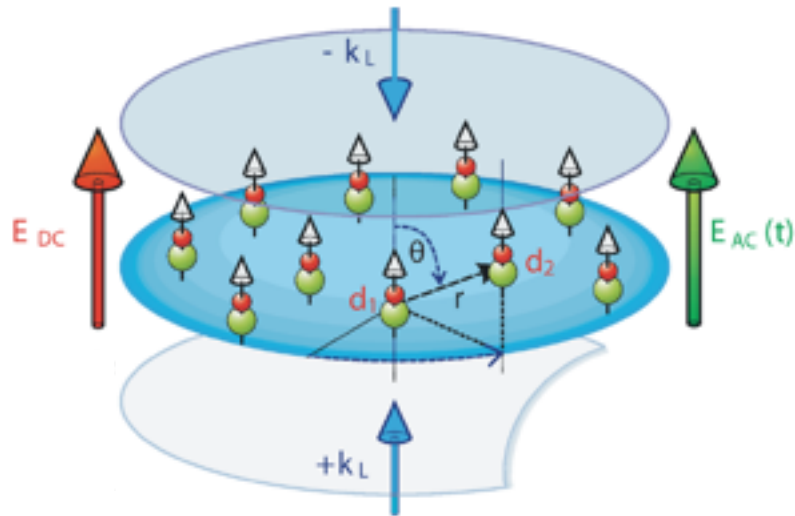
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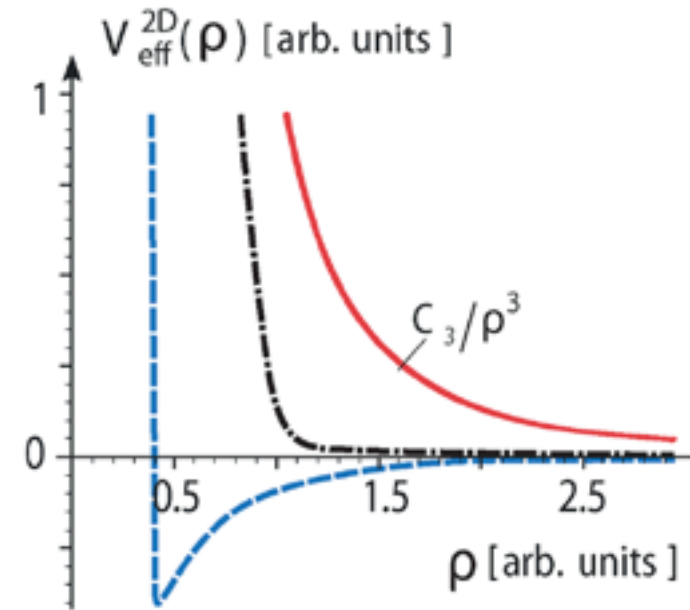
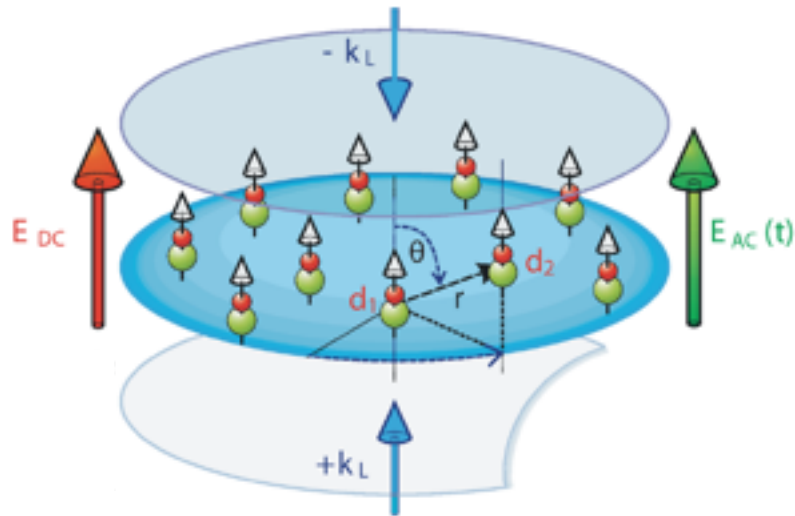
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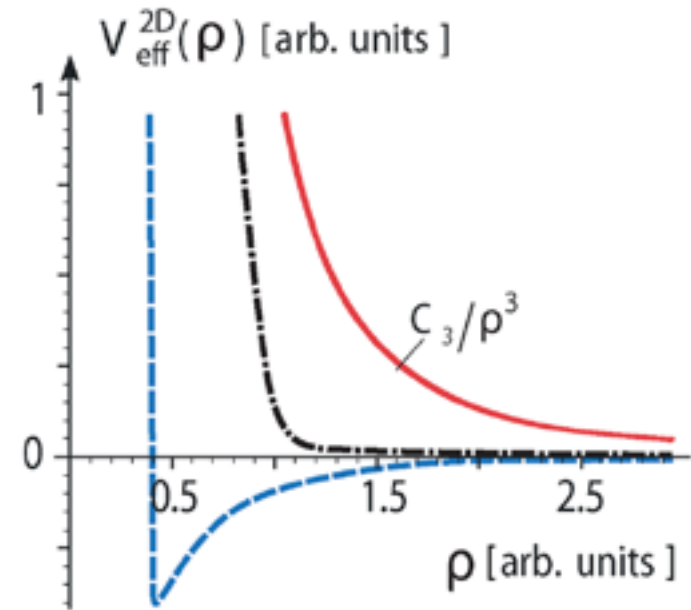
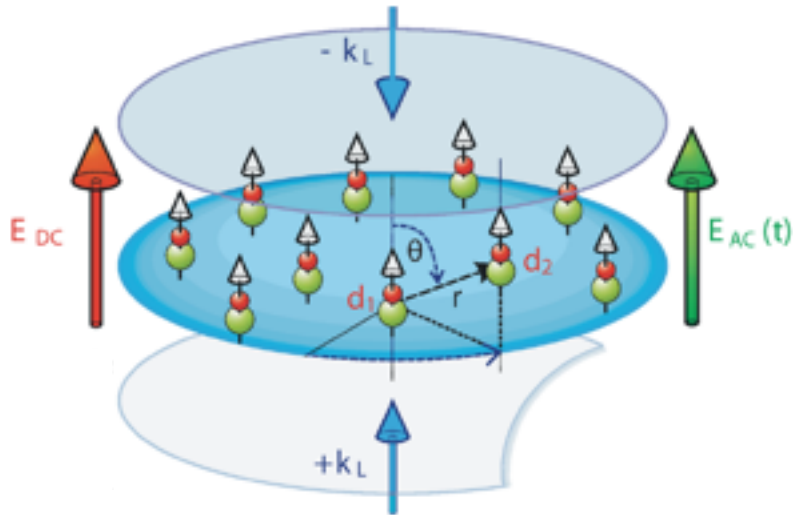


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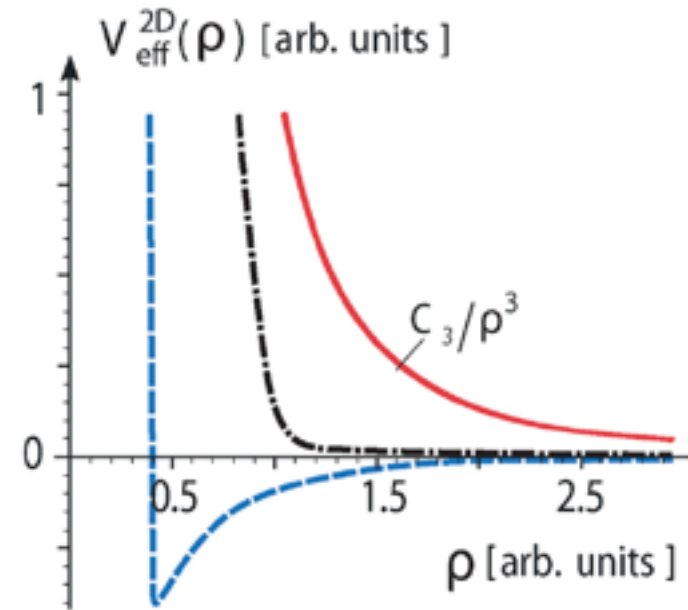
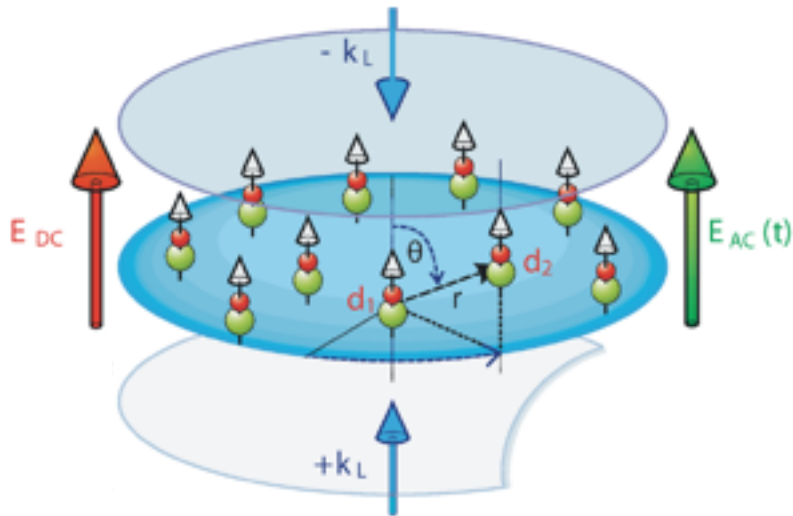


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Baranov, Dalmonte, Pupillo, Zoller, Chem. Rev. 112, 5012 (2012)

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with hopping



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Baranov, Dalmonte, Pupillo, Zoller, Chem. Rev. 112, 5012 (2012)

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Baranov, Dalmonte, Pupillo, Zoller, Chem. Rev. 112, 5012 (2012)

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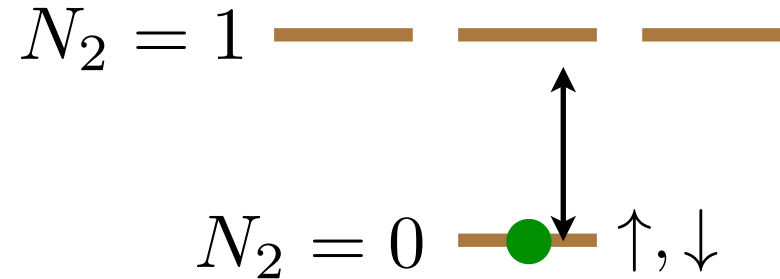
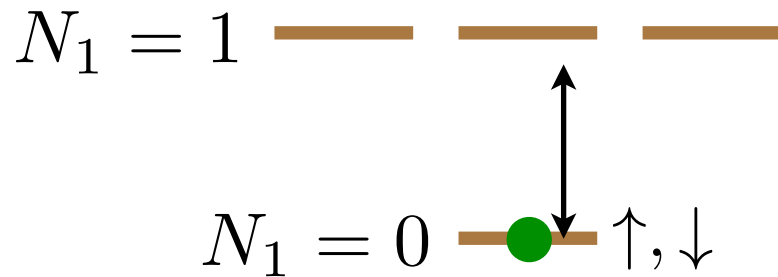
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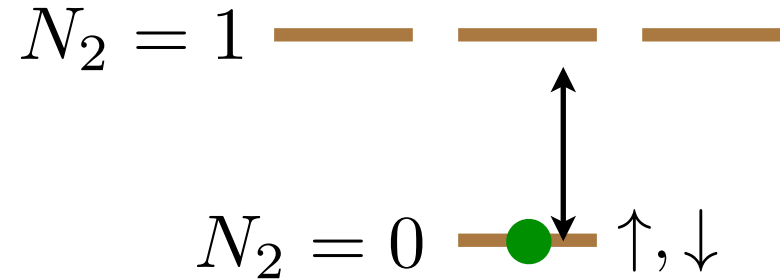
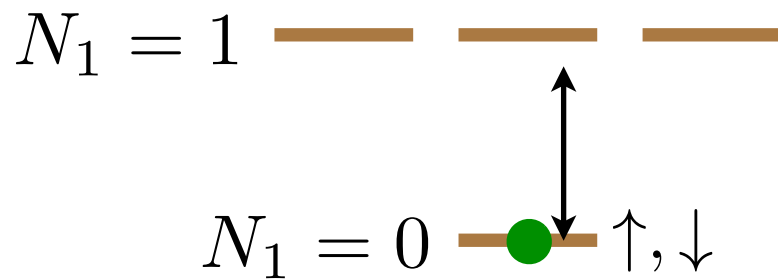


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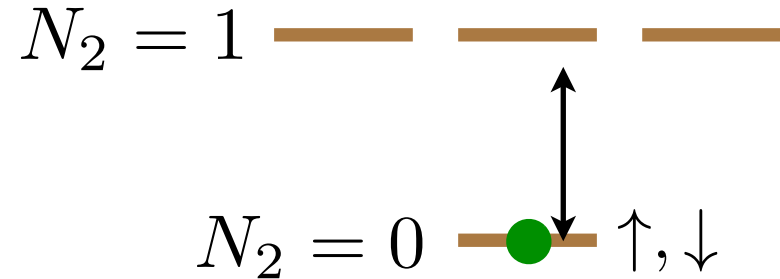
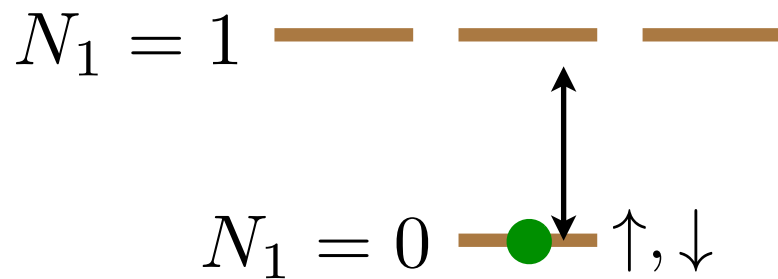
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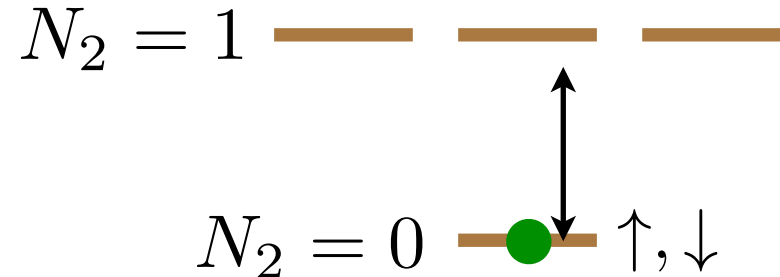
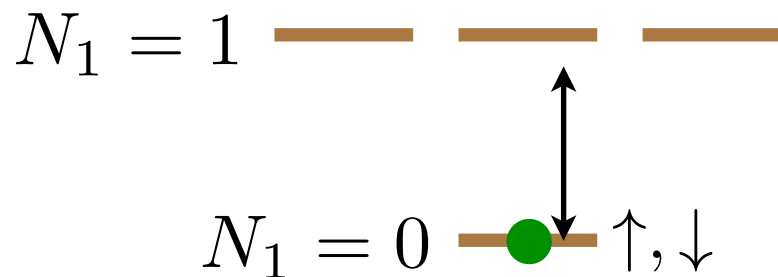
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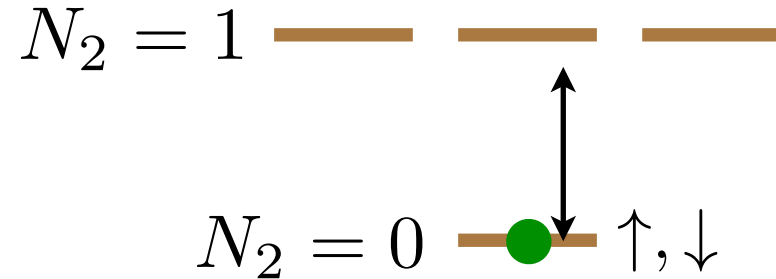
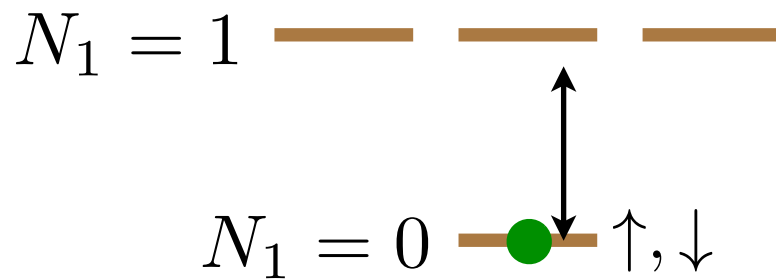
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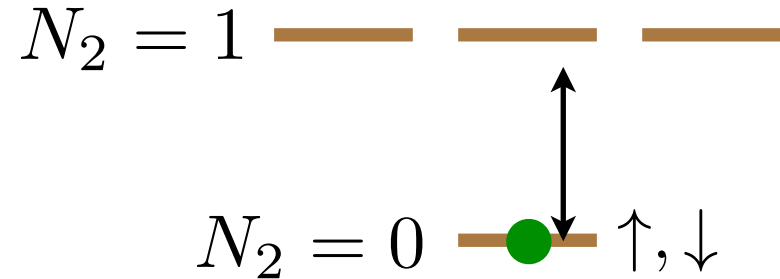
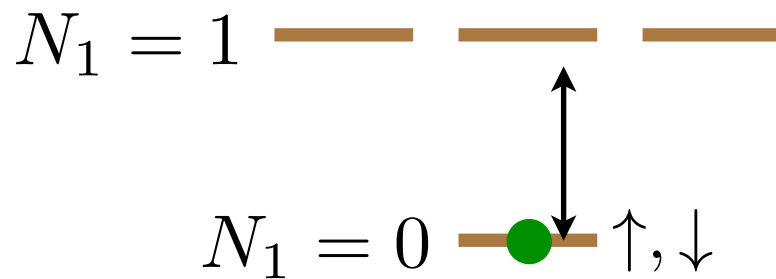
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