Antiferrosmectic ground state of two component dipolar Fermi gases

- an analog of meson condensation in nuclear matter

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Fundamental Science and Applications of Ultra-cold Polar Molecules

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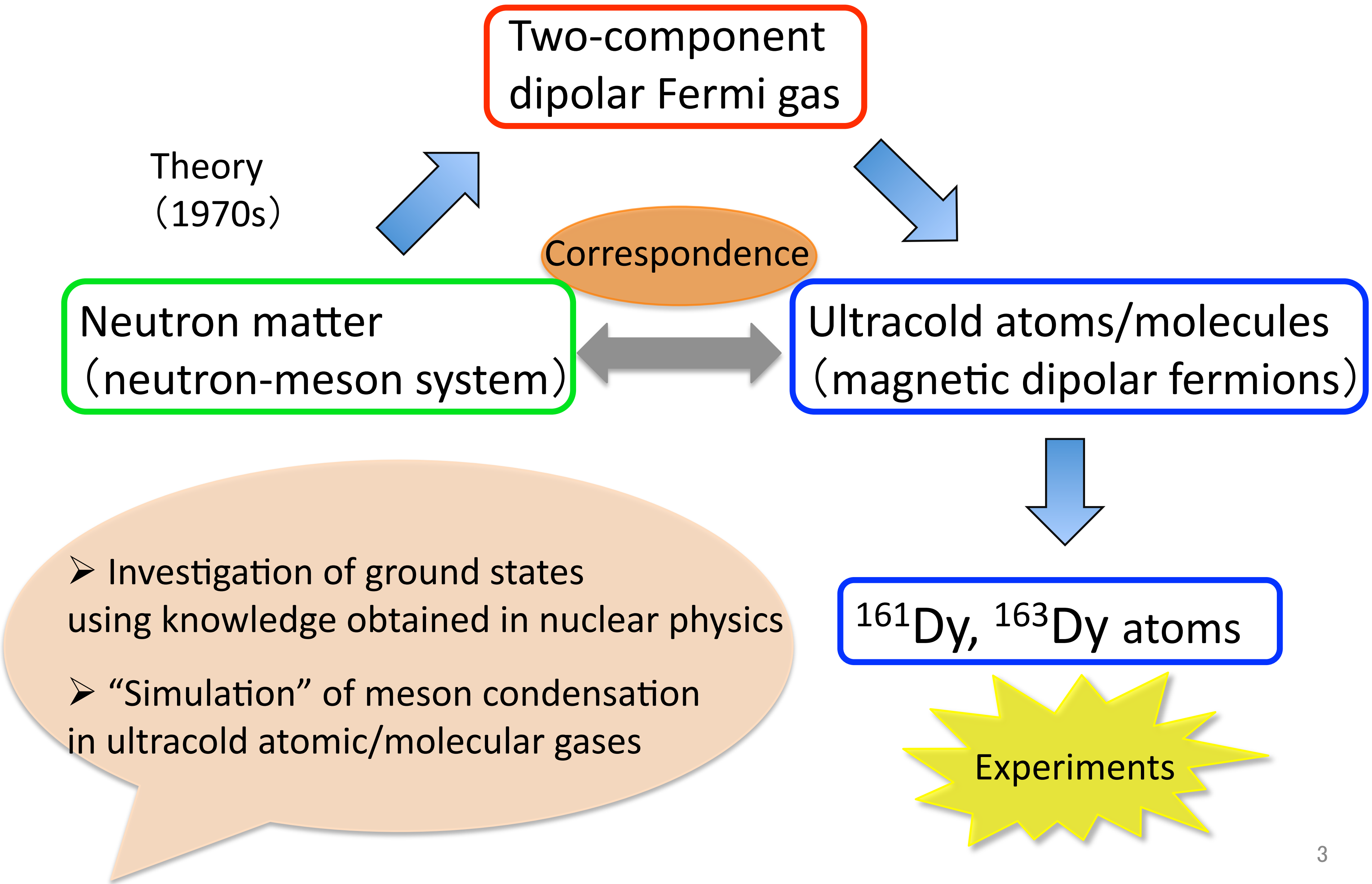
Antiferrosmectic ground state of two component dipolar Fermi gases

I. Antiferro Smectic-C State

II. Phase Diagram

Motivation

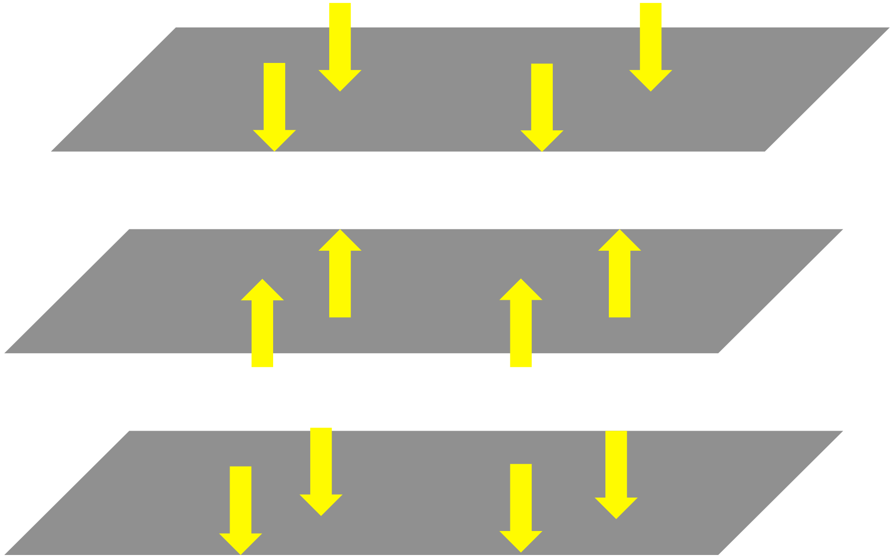


Two-component dipolar Fermi gas
Theory (1970s)
Correspondence
Neutron matter (neutron-meson system)
Ultracold atoms/molecules (magnetic dipolar fermions)
➢ Investigation of ground states using knowledge obtained in nuclear physics
➢ "Simulation" of meson condensation in ultracold atomic/molecular gases
^{161}Dy, ^{163}Dy atoms
Experiments

Meson Condensation in Neutron Matter

π^0 condensation

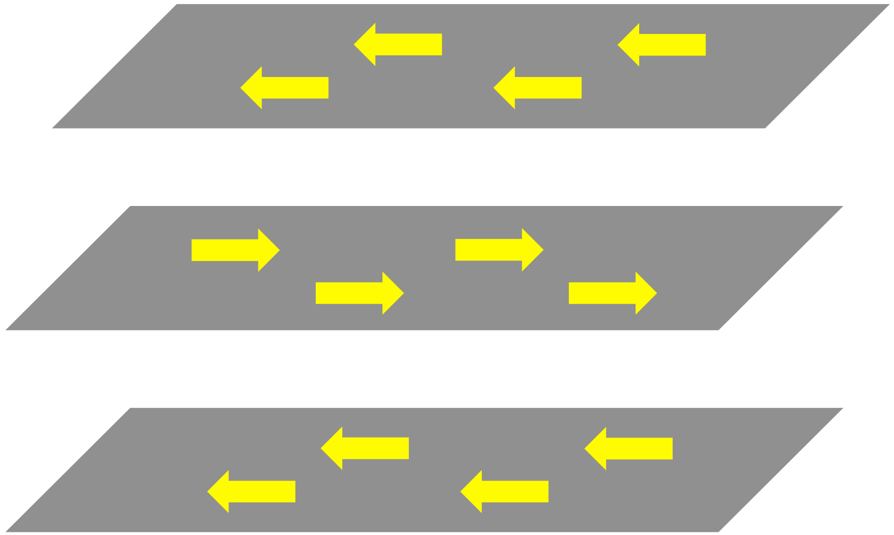
$$(-\nabla^2 + m_\pi^2)\varphi_c(\boldsymbol{r}) = (f/m_\pi)\boldsymbol{\nabla}\cdot\langle\psi^\dagger\boldsymbol{\sigma}\psi\rangle$$



A.B. Migdal, Zh.Eksp.Teor.Fiz. **61**, 2209 (1971)
T. Takatsuka, et al., PTP **59**, 1933 (1978)
T. Matsui, et al., PTP **60**, 1442 (1978)

ρ^0 condensation

$$(-\nabla^2 + m_\rho^2)\boldsymbol{\rho}_c(\boldsymbol{r}) = (f_\rho/m_\rho)\boldsymbol{\nabla}\times\langle\psi^\dagger\boldsymbol{\sigma}\psi\rangle$$



T. Kunihiro, PTP **60**, 1229 (1978)

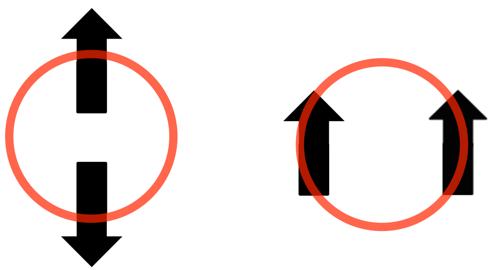
Pion condensation (1) --- tensor force potential

$$
\begin{aligned}
&V_{\mathrm{OPEP}}(r) \\
&= \frac{f_{\pi N}^2}{4\pi}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1 \cdot \nabla_1)(\boldsymbol{\sigma}_2 \cdot \nabla_2)\frac{e^{-m_\pi r}}{r} \\
&= \frac{g_{\pi N}^2}{4\pi}\left(\frac{m_\pi}{2M_N}\right)^2 \frac{(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)}{3}\left[(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + S_{12}\left(1 + \frac{3}{m_\pi r} + \frac{3}{m_\pi^2 r^2}\right)\right]\frac{e^{-m_\pi r}}{r} \\
&\xrightarrow[\text{chiral limit}]{} \frac{g_A^2}{16\pi F_\pi^2}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)\frac{S_{12}}{r^3}
\end{aligned}
$$

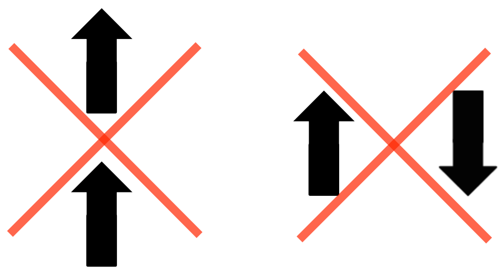
$$
S_{12} = 3(\boldsymbol{\sigma}_1 \cdot \hat{\boldsymbol{r}})(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{r}}) - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2
$$

"anti-magnets"

Favored



Disfavored

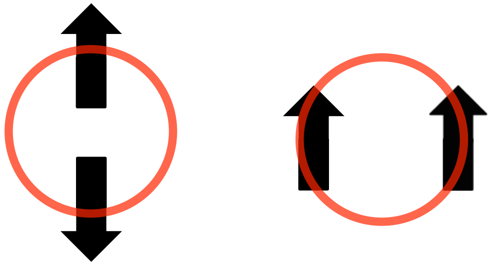


Pion condensation (2) --- ALS structure

T. Takatsuka, et al., Prog. Theor. Phys. **59**, 1933 (1978)

"anti-magnets"

Favored



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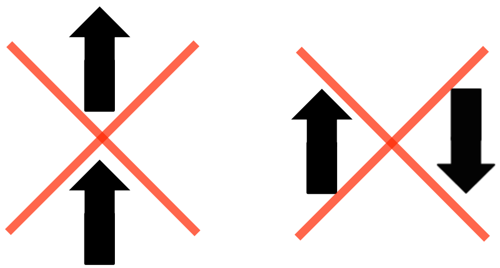
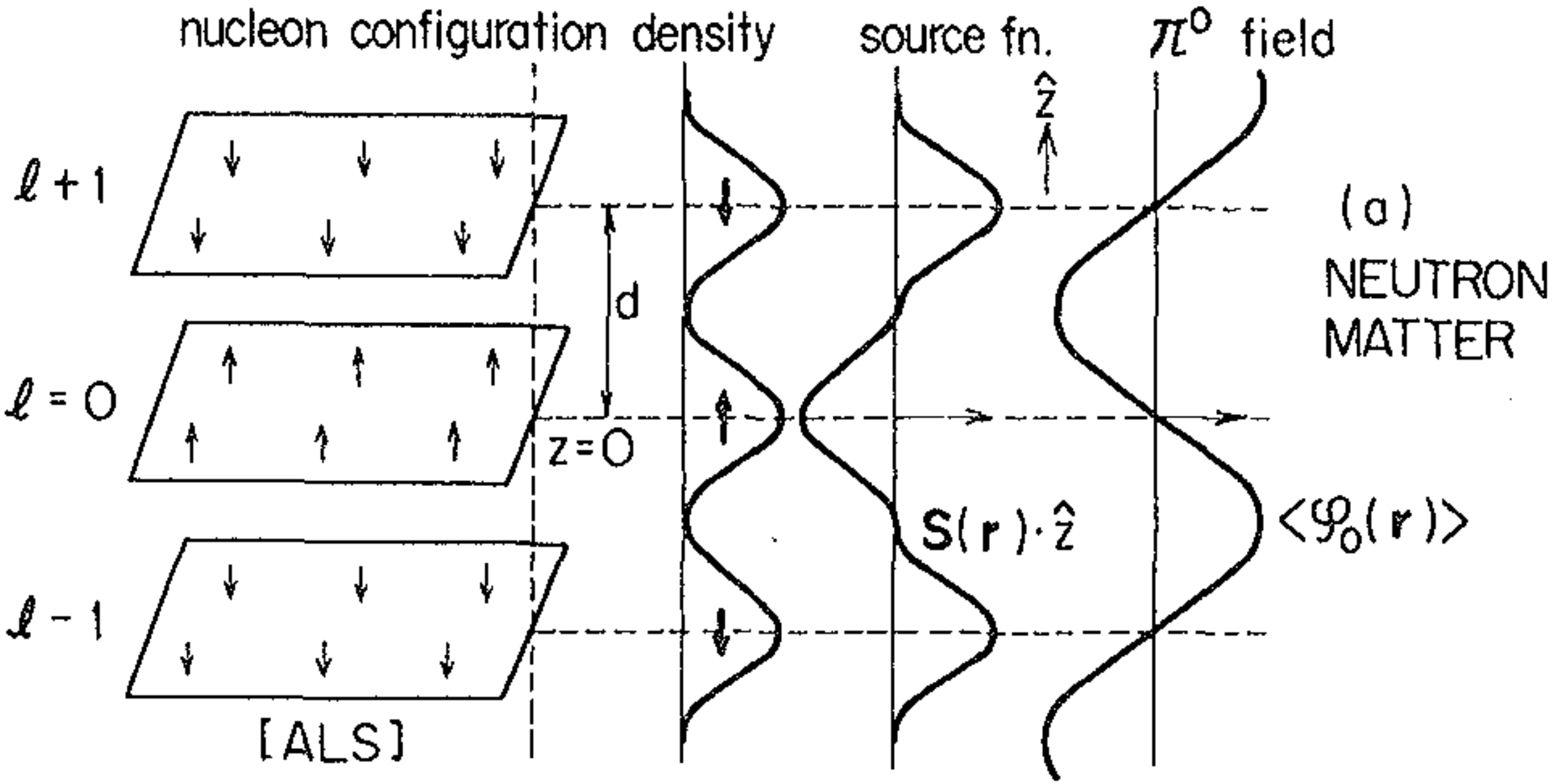


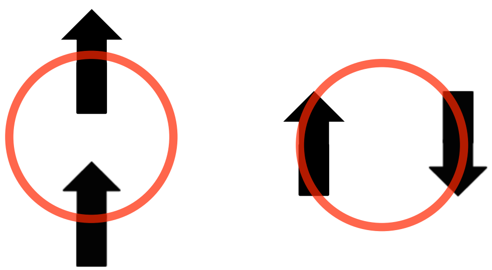


Rho-meson condensation (1) --- tensor force potential

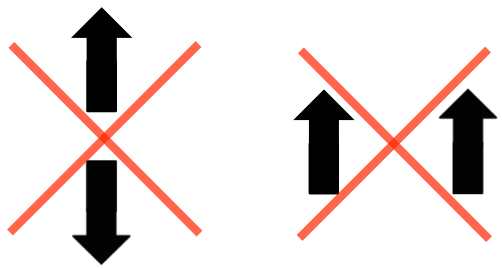
$$
\begin{aligned}
&V_{\mathrm{OREP}}(r) \\
&= \frac{f_{\rho N}}{4\pi}(\tau_1 \cdot \tau_2)(\sigma_1 \times \nabla_1)(\sigma_2 \times \nabla_2)\frac{e^{-m_\rho r}}{r} \\
&= \frac{g_{\rho N}^2}{4\pi}\left(\frac{m_\rho}{2M_N}\right)^2 \frac{\tau_1 \cdot \tau_2}{3}\left[2(\sigma_1 \cdot \sigma_2) - S_{12}\left(1 + \frac{3}{m_\rho r} + \frac{3}{m_\rho^2 r^2}\right)\right]\frac{e^{-m_\rho r}}{r} \\
&\to -\frac{g_{\rho N}^2}{16\pi M_N^2}(\tau_1 \cdot \tau_2)\frac{S_{12}}{r^3}
\end{aligned}
$$

"magnets"

Favored

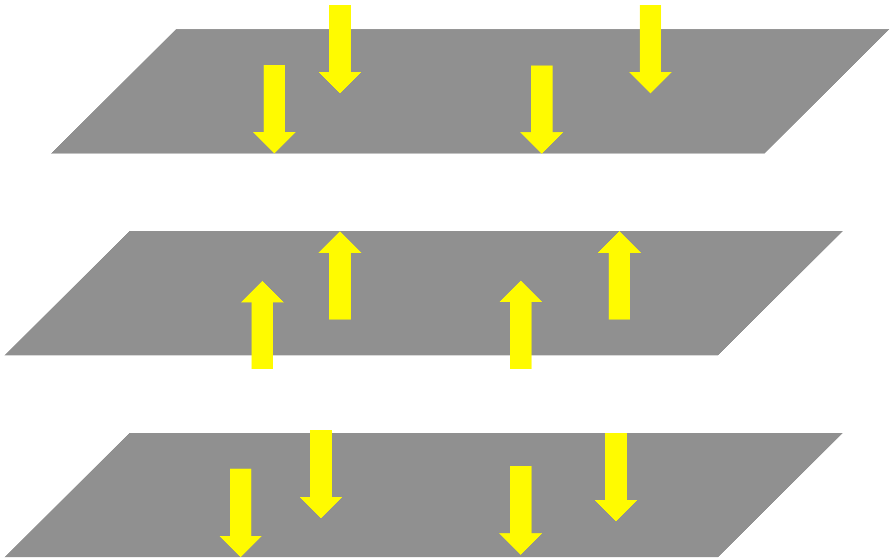


Disfavored



Meson Condensation in Neutron Matter (revisit)

π^0 condensation



$$(-\nabla^2 + m_\pi^2)\varphi_c(\boldsymbol{r}) = (f/m_\pi)\boldsymbol{\nabla}\cdot\langle\psi^\dagger\boldsymbol{\sigma}\psi\rangle$$

A.B. Migdal, Zh.Eksp.Teor.Fiz. **61**, 2209 (1971)
T. Takatsuka, et al., PTP **59**, 1933 (1978)
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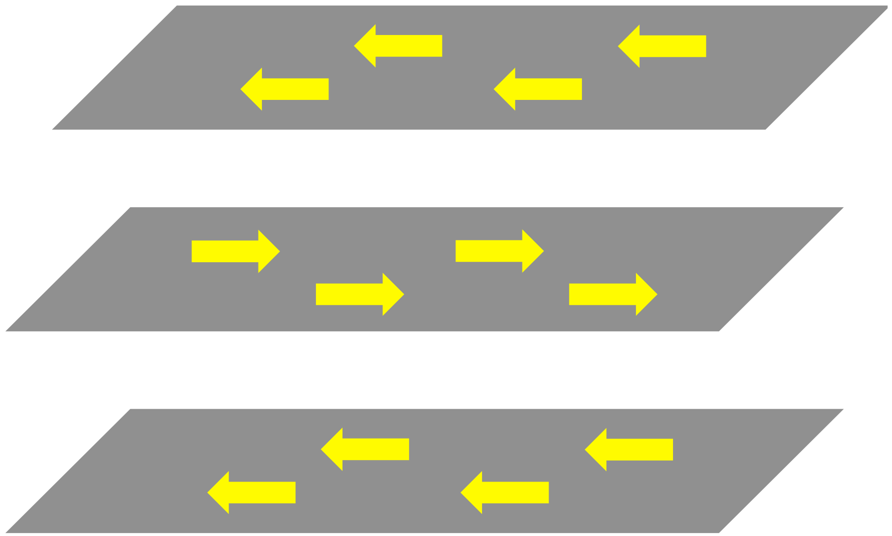
ρ^0 condensation



$$(-\nabla^2 + m_\rho^2)\boldsymbol{\rho}_c(\boldsymbol{r}) = (f_\rho/m_\rho)\boldsymbol{\nabla}\times\langle\psi^\dagger\boldsymbol{\sigma}\psi\rangle$$

T. Kunihiro, PTP **60**, 1229 (1978)

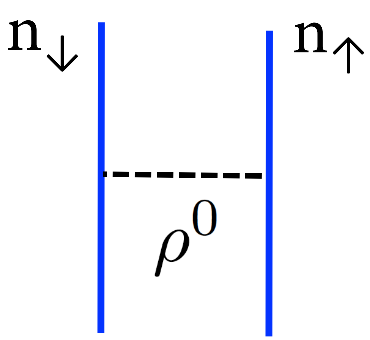
Correspondence (1) --- Maxwell Eq. with magnetic source

(a) Neutron system

$$(-\nabla^2 + m_\rho{}^2)\,\boldsymbol{\rho}_c(\boldsymbol{r}) = (f_\rho/m_\rho)\,\boldsymbol{\nabla}\times\langle\psi^\dagger\boldsymbol{\sigma}\psi\rangle$$

rho-meson
(neutral vector boson)

neutron
(spin-up & -down)

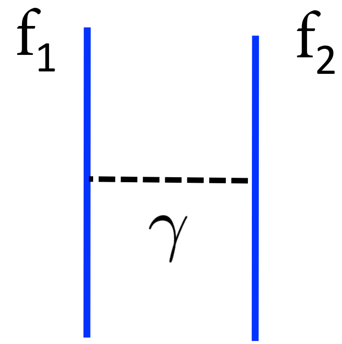


(b) Fermionic dipolar system

$$-\nabla^2\langle\vec{A}(\vec{r})\rangle = 4\pi\mu\nabla\times\langle\Psi^\dagger\vec{\sigma}\Psi\rangle$$

photon
(neutral vector boson)

fermionic dipolar
(two of pseudospin states)



Correspondence (2)

KM, T. Hatsuda, G. Baym, arXiv:1205.1086

neutron matter	atomic dipolars
neutron stars	atomic gases (^{163}Dy)
neutrons	fermionic diplolars
spin	pseudo-spin (hyperfine states)
neutral vector meson (ρ^0)	photon
tensor-force potential	dipolar interaction potential
meson cond.	cond. of gauge filed

Let us consider ***two-component fermionic atoms with a magnetic dipole interaction***.

Model --- the potential description

$$H = \int d\vec{r}_1 \frac{\nabla\Psi^\dagger(\vec{r}_1) \cdot \nabla\Psi(\vec{r}_1)}{2m}$$

$$+\frac{1}{2}\sum_{i,j=1}^{3}\sum_{\alpha,\alpha',\beta,\beta'}^{\uparrow,\downarrow}\int d\vec{r}_1 d\vec{r}_2\, \psi_\alpha^\dagger(\vec{r}_1)\psi_\beta^\dagger(\vec{r}_2)V(\vec{r}_1,\vec{r}_2)^{ij}_{\alpha\alpha',\,\beta\beta'}\psi_{\beta'}(\vec{r}_2)\psi_{\alpha'}(\vec{r}_1)$$

with

$$V(\vec{r}_1,\vec{r}_2)^{ij}_{\alpha\alpha',\,\beta\beta'} = \frac{\mu^2}{r^3}\{\sigma^i_{\alpha\alpha'}(\delta_{ij} - 3\hat{r}_i\hat{r}_j)\sigma^j_{\beta\beta'}\} + g\delta_{\alpha\alpha'}\frac{\delta_{ij}}{3}\delta(\vec{r}_1 - \vec{r}_2)\delta_{\beta\beta'}$$

Physical parameters (dimensionless)

dipole-dipole interaction strength : $\lambda_d = n\mu^2/\epsilon_F$

contact interaction strength : $\lambda_s = gn/\epsilon_F$

Investigation of phase structure in $\lambda_s - \lambda_d$ plane

Variational state : EALS state

$$|\Phi_{\rm EALS}\rangle = \prod_{l,\vec{q}_\perp;\alpha}^{(\rm occ.)} c^\dagger_{\ell,\vec{q}_\perp;\alpha}|0\rangle$$

$$\chi_l(\cdot) = \begin{cases} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} & l: odd \\ \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} & l: even \end{cases}$$

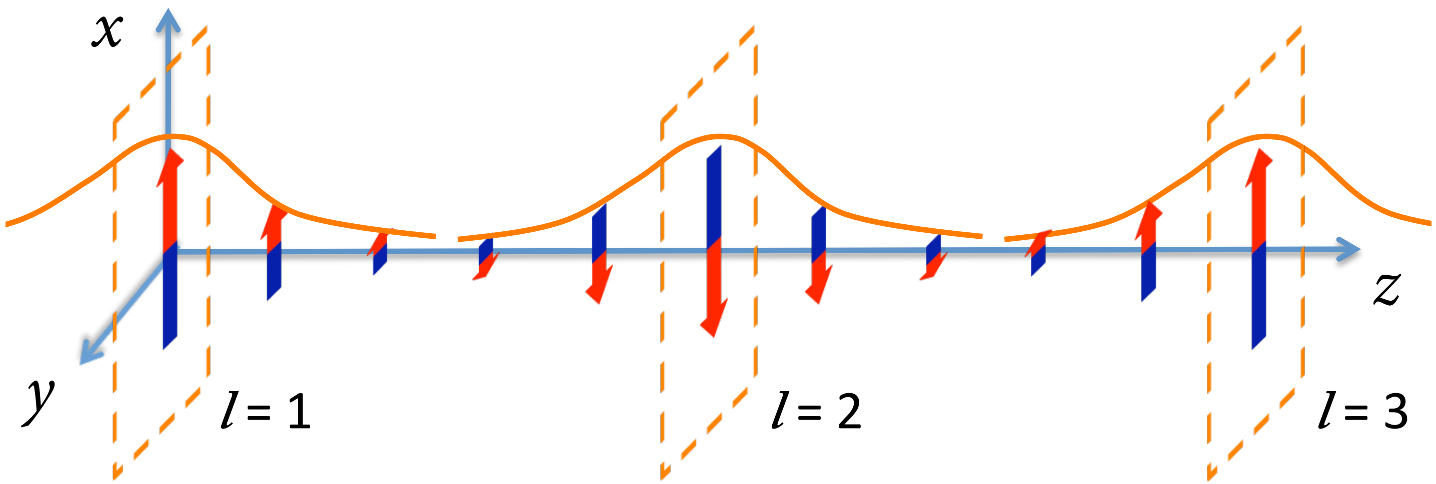
with the single particle basis

staggered in z direction

$$\phi_{\ell,\vec{q}_\perp}(x,y,z) = \frac{e^{-(z-d\ell)^2/2b^2}}{(\pi b^2)^{1/4}} \frac{e^{i\vec{q}_\perp\cdot\vec{r}_\perp}}{\sqrt{V_\perp}}\chi_\ell$$

localized in z direction

uniform in x-y plane



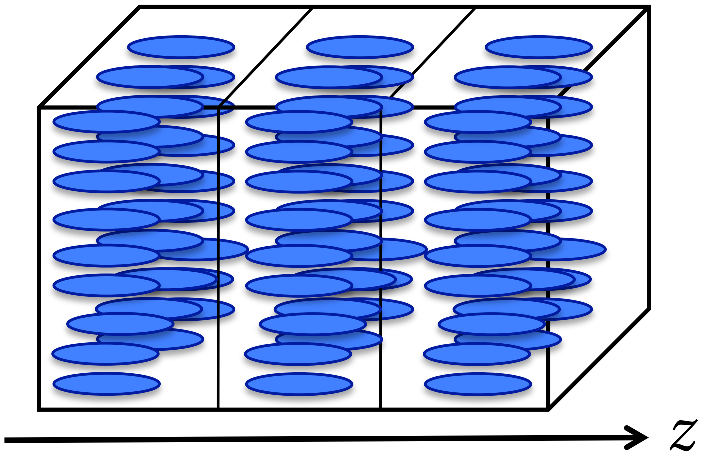
Extended
Alternating
Layered
Spin

T. Kunihiro (1978)

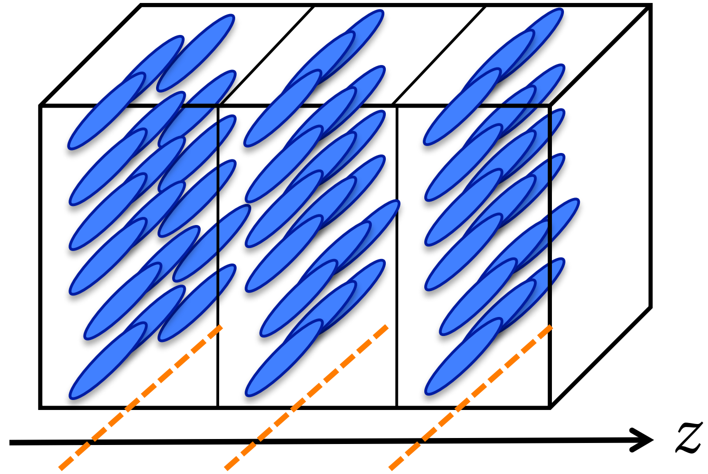
AFSC state --- analog of liquid-crystal

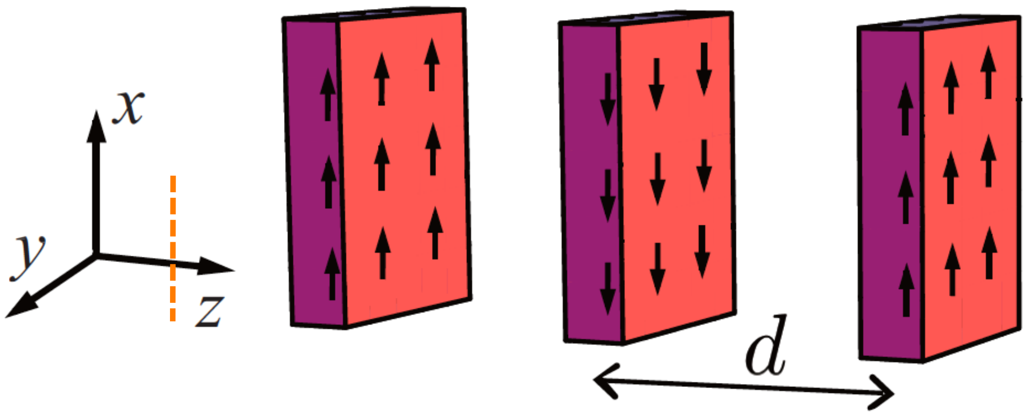
Liquid Crystal

Smectic-*A* phase



Smectic-*C* phase





Anti-**F**erro
Smectic-**C** **(AFSC)** state

KM, T. Hatsuda, G. Baym, arXiv:1205.1086

Observables in the AFSC (EALS) state

Local number density

$$n(\vec{r}) = \frac{nd}{b\sqrt{\pi}} \sum_{\ell=-\infty}^{\infty} e^{-(z-d\ell)^2/b^2}$$

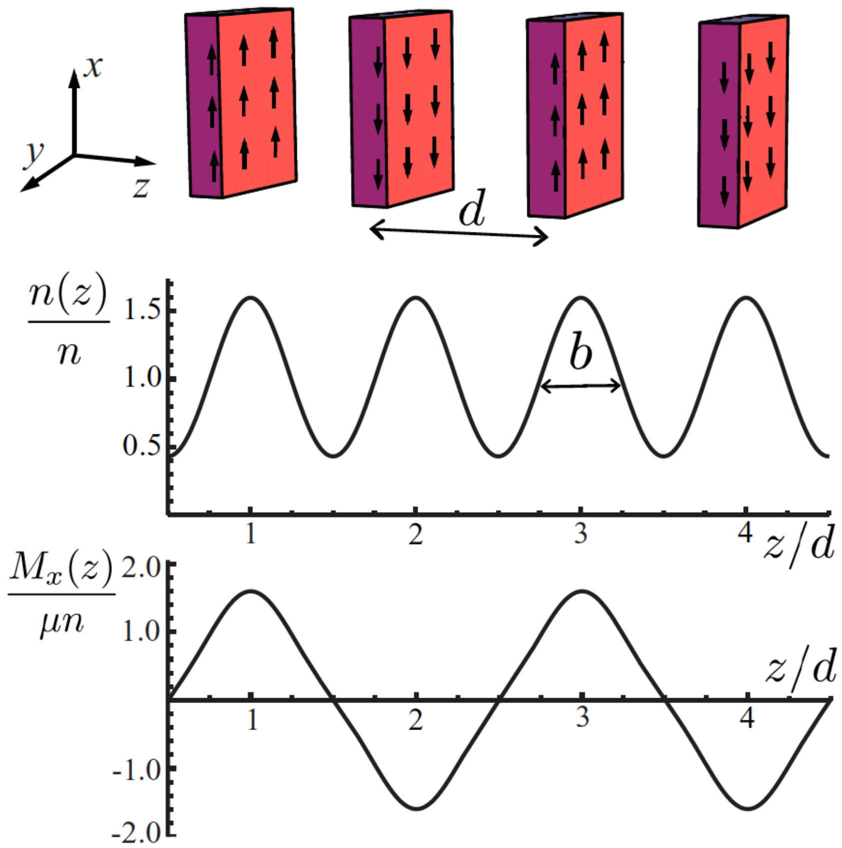
$$= n + 2n \sum_{j=1}^{\infty} e^{-j^2\pi^2/\Gamma} \cos(2j\pi z/d)$$

Local magnetization

$$\langle M_x(\vec{r})\rangle = -\frac{\mu nd}{b\sqrt{\pi}} \sum_{\ell=-\infty}^{\infty} (-1)^\ell e^{-(z-d\ell)^2/b^2}$$

$$= -2\mu n \sum_{j=1}^{\infty} e^{-(2j-1)^2\pi^2/4\Gamma} \cos\{(2j-1)\pi z/d\}$$

$$\langle M_y(\vec{r})\rangle = \langle M_z(\vec{r})\rangle = 0$$



$$d = \sqrt{8}b$$

Dimensionless parameter;

$$\Gamma = (d/b)^2 = 8$$

I. Antiferro Smectic-C State

II. Phase Diagram

Candidates for the ground state

1. Fermi Gas State

$$|\Phi_{\rm FG}\rangle := \prod_{\alpha=\uparrow,\downarrow}\left(\prod_{p\le p_{\rm F}} a^\dagger_{\alpha,\vec{p}}\right)|0\rangle$$

2. Polarized Ferronematic State

$$|\Phi_{\rm FN}\rangle = \prod_{\gamma^{-1}(p_x^2+p_y^2)+\gamma^2 p_z^2 \le p_{\rm F,\uparrow}^2} a^\dagger_{\uparrow,\vec{p}}\,|0\rangle$$

B.M. Fregoso, E. Fradkin, PRL **103**, 205301 (2009)

3. AFSC State

$$|\Phi_{\rm AFSC}\rangle = \prod_{\ell,\vec{q}_\perp}^{\rm (occ.)} c^\dagger_{(\ell,\vec{q}_\perp)}|0\rangle$$

KM, T. Hatsuda, G. Baym, arXiv:1205.1086

Kinetic Energy	Contact Repulsion	Dipole-dipole interaction
Low	Repulsive	None
High	None	Repulsive or Attractive (in total)
Very High	Little	Attractive (in total)

Energy density (1)--- Fermi gas state

In the following, we will represent energy densities in units of the energy density of the two-component free Fermi gas.

1. Fermi gas state

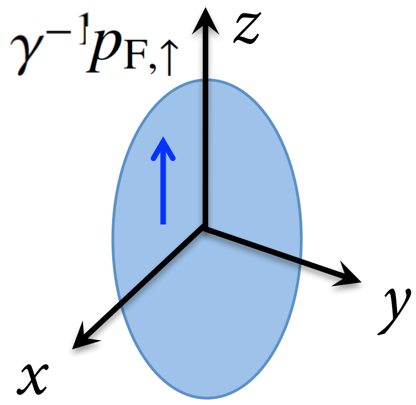
$$\tilde{\mathcal{E}}_{\mathrm{FG}} = 1 + \frac{5}{12}\lambda_{\mathrm{s}}$$

2-component Fermi gas energy + Contact Repulsion

No contribution from dipole-dipole interaction.

Energy density (2) --- FN state

$$p_{F,\uparrow} = (6\pi^2 n)^{1/3}$$



$$\tilde{\mathcal{E}}_{\rm FN} = \frac{2^{2/3}}{3}\left(2\gamma + \frac{1}{\gamma^2}\right) - \frac{5\pi}{9}\lambda_d I(\gamma)$$

2-comp. Fermi gas energy with spheroidal deformation

Dipole-dipole interaction

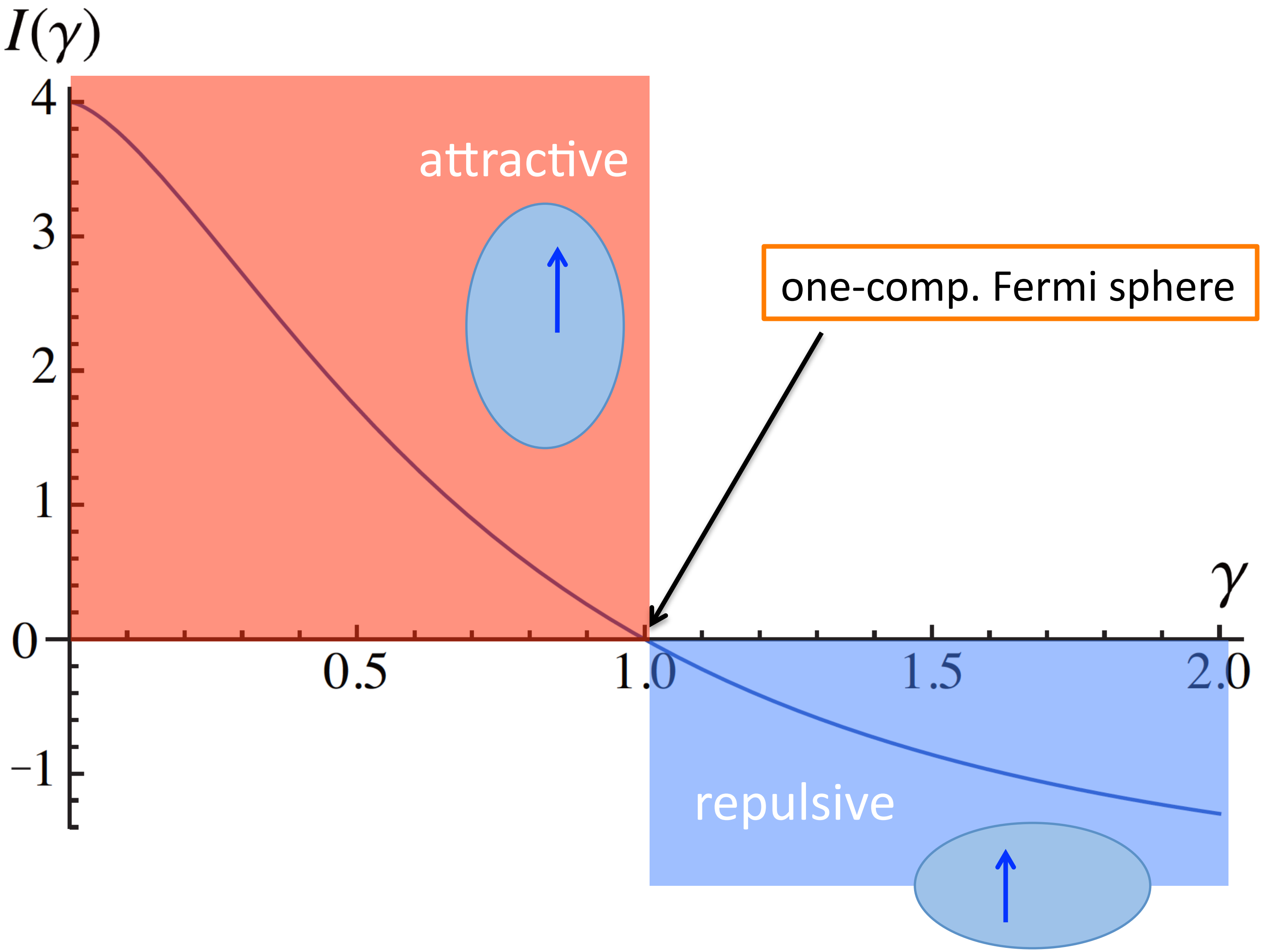
No contribution from contact repulsion.

with *Deformation function*

$$I(\gamma) = -2 - \frac{6}{\gamma^3 - 1} - \frac{6 \arccos \gamma^{3/2}}{(\gamma^{-1} - \gamma^2)^{3/2}}$$

Deformation function

$$\tilde{\mathcal{E}}_{\mathrm{FN}} = \frac{2^{2/3}}{3}\left(2\gamma + \frac{1}{\gamma^2}\right) - \frac{5\pi}{9}\lambda_d\, I(\gamma)$$



Energy density (3) --- AFSC state

$$\tilde{\mathcal{E}}_{\mathrm{AFSC}}$$

$$= \frac{10}{3(3\pi)^{2/3}}\Gamma^{1/3}\alpha^{2/3} + \frac{5}{3(3\pi)^{2/3}}\Gamma^{1/3}\alpha^{-1/3}$$

$$-\frac{20\pi}{3}\lambda_d \sum_{j=1}^{\infty} e^{-(2j-1)^2\pi^2/2\Gamma}\left\{\frac{1}{3} - F(\alpha)\right\}$$

$$+\frac{5}{6}\lambda_s\left\{\frac{1}{2} - \sum_{j=1}^{\infty}\left[e^{-(2j-1)^2\pi^2/2\Gamma} - e^{-2j^2\pi^2/\Gamma}\right]\right\}$$

Kinetic Energy

Zero-point motion in z-axis

Fermi gas energy in xy plane.

Compete

Dipole-dipole pot.

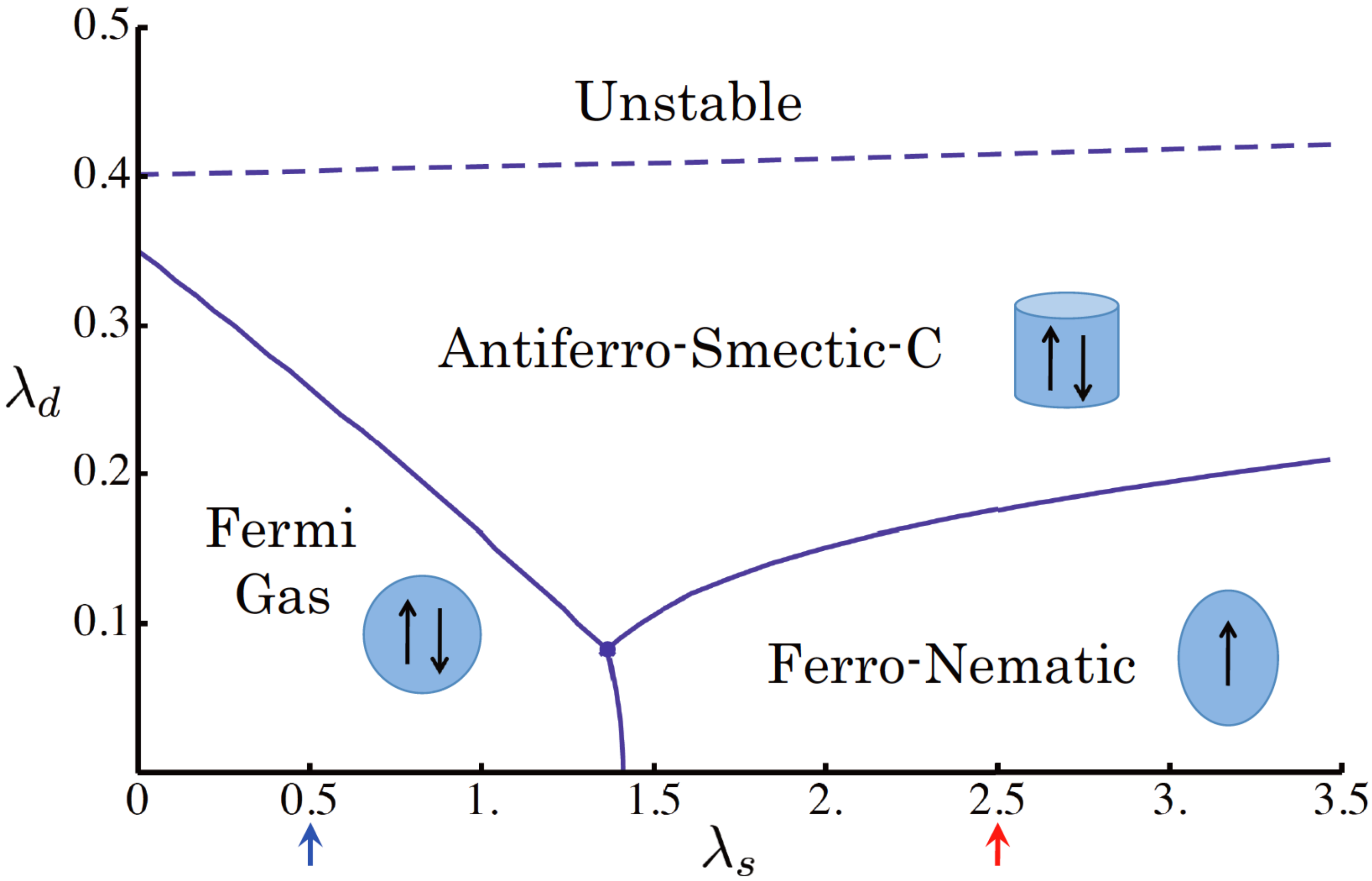
Contact repulsion

little contribution (Pauli exclusion principle)

Variation parameters;

$$\Gamma = (d/b)^2 \qquad \alpha = 1/(2q_{\mathrm{F}}^2 b^2)$$

Phase diagram (1)



Recent Progress

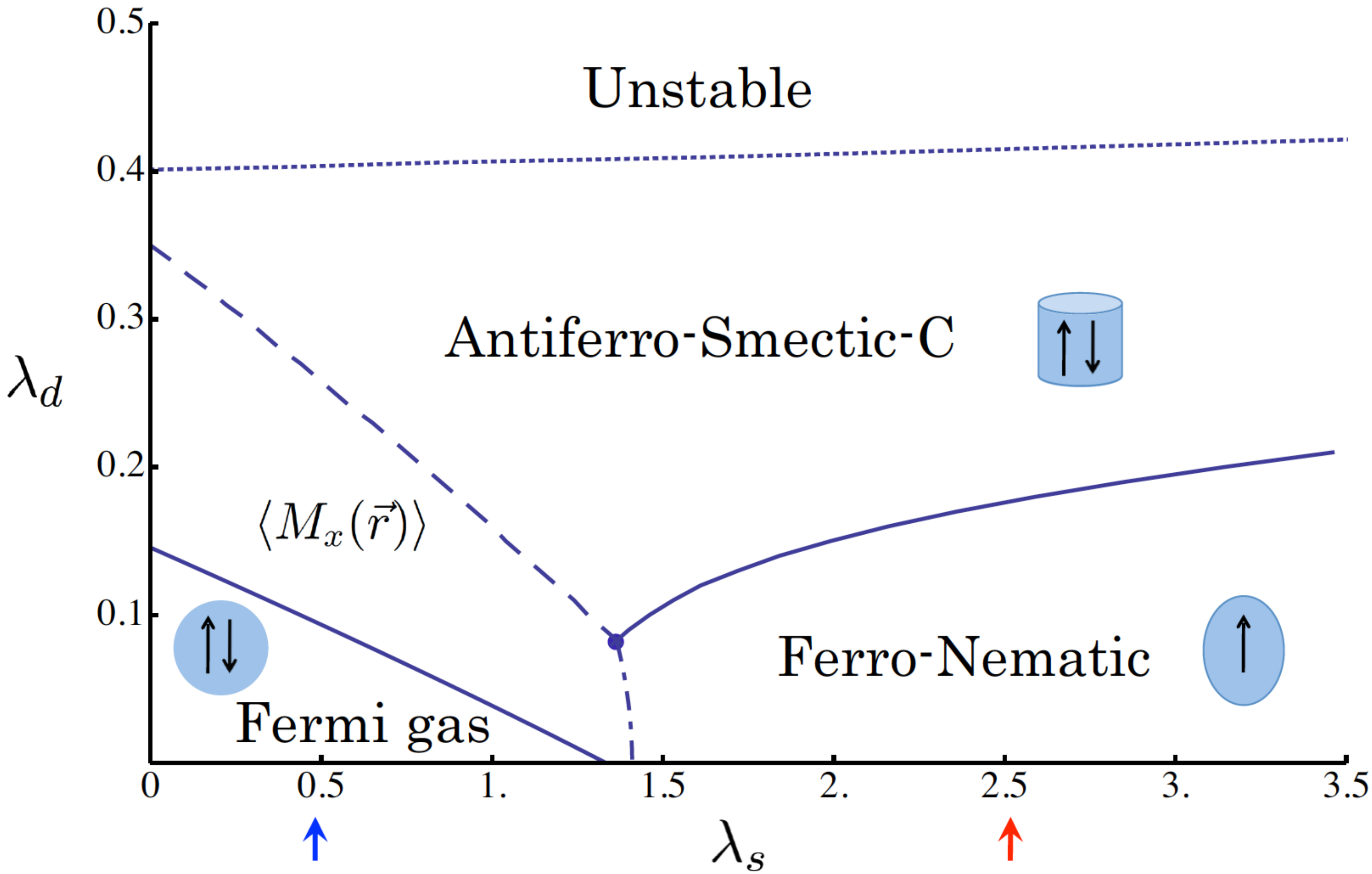
Coupled channel RPA analysis of the spin-triplet (S = 1) excitation
with the angular momentum L = 0 and L = 2 states

Fermi gas state becomes unstable
against spatially varying magnetization along the line :

$$\left(1-\frac{3}{4}\lambda_s-2\pi\lambda_d\right)\left(1+\frac{\pi}{2}\right)-\frac{\pi^2}{2}\lambda_d^2 \;=\; 0$$

T. Sogo, M. Urban, P. Shuck, T. Miyakawa , PRA **85**, 031601(R) (2012)
Yi Li and Congjun Wu, PRB **85**, 205126 (2012)

Phase diagram (2)



KM, T. Hatsuda, G. Baym, arXiv:1205.1086

Summary

✓ rho-meson in neutron matter corresponds

to photon in fermionic dipolars.

✓ AFSC state (localized, staggered state in one-dimension)

is favored at strong coupling or high density regime.

✓ Phase diagram of two-comp. dipolar Fermi gas is updated.

Concept of the AFSC state in nuclear physics
may be tested by table-top experiments in ultra-cold dipolars.