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Topological transitions in mixed-geometry lattices, and dynamics of fermions in one dimension

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Aalto University

Fundamental science and applications of ultra-cold polar molecules, KITP workshop, Santa Barbara 28.2.2013

Contents

- The FFLO state in 1D-3D crossover (DMFT) (briefly)
- Pairing in ***mixed geometries*** (mean field)
- Expansion of a band insulator in a lattice (t-DMRG)
- Expansion of an FFLO state (t-DMRG)
- Dynamics of a polaron in 1D (t-DMRG)

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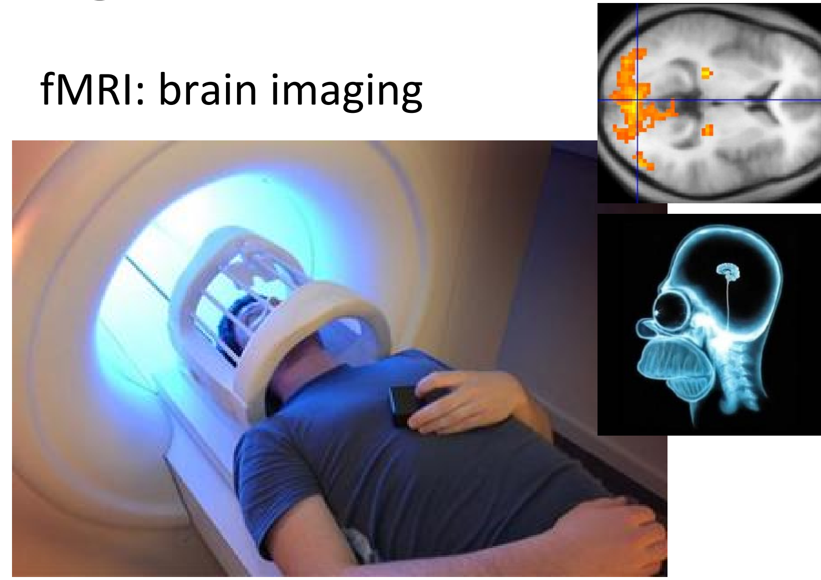
SUPERCONDUCTIVITY IN OUR LIVES

JR Maglev MLX01 – 581 km/h (Japan, 2003)

fMRI: brain imaging



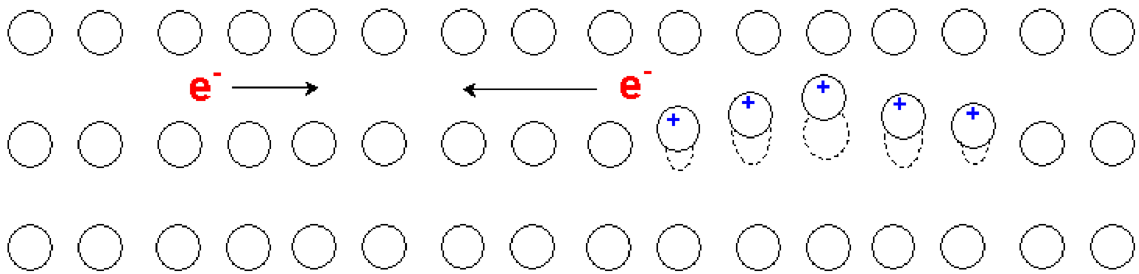
LHC: Go Higgs!

Future? Superconducting power grid

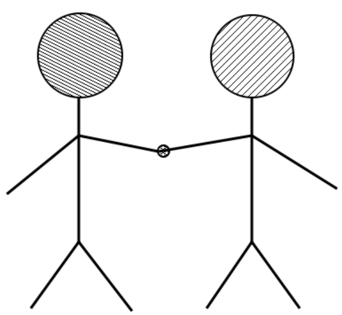
Long Island, NY

What is superconductivity?

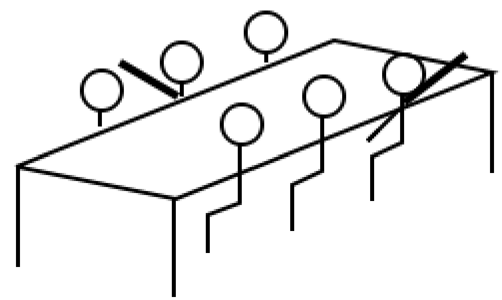
Cooper pairs of spin up and spin down electrons

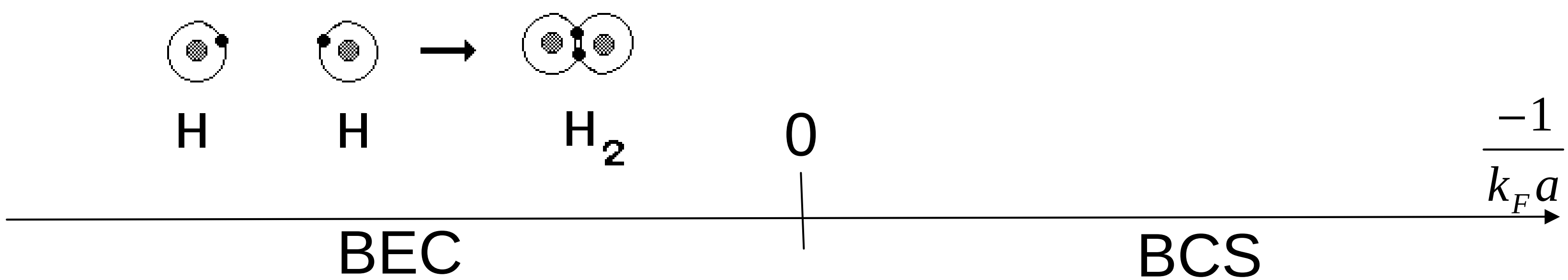


Not really:

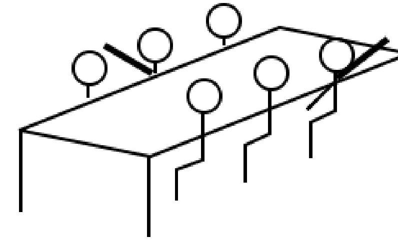


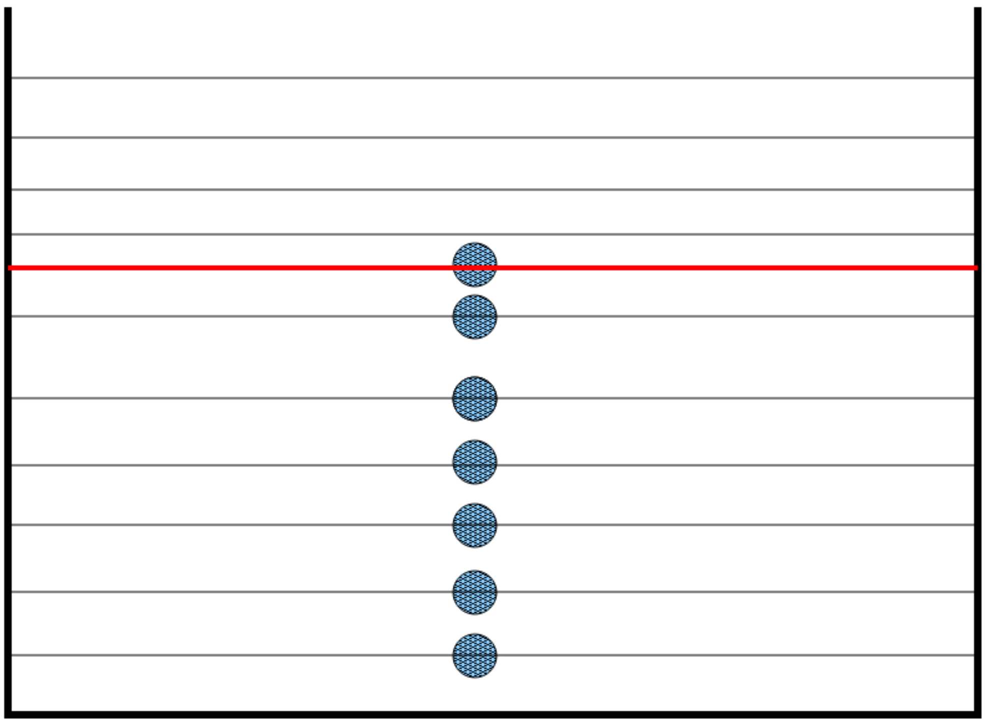
More like:





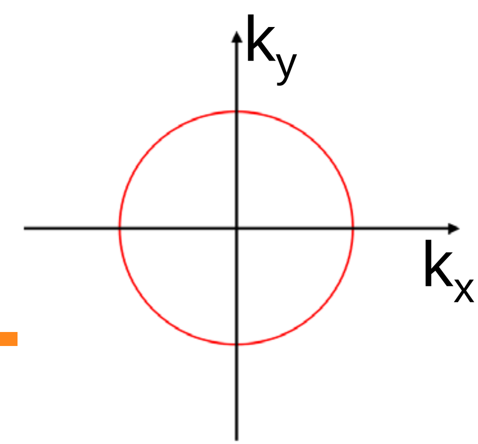
The Fermi surface





E_F

(k_F)

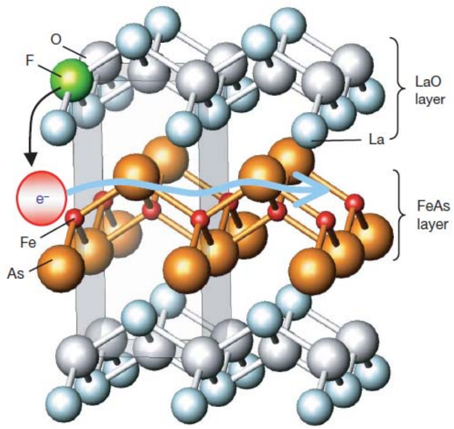


$$E_F \propto k_x^2 + k_y^2$$

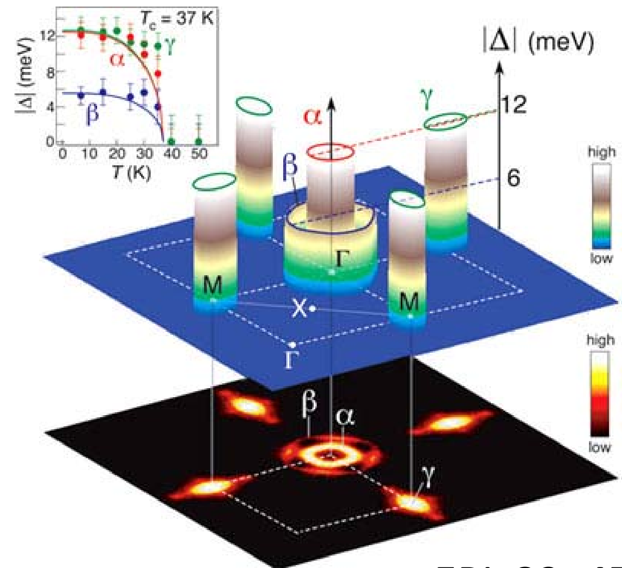
High-temperature superconductors: complicated structure, spin plays a role (?)

Fermi surfaces (BaKFeAs)

Fe-based superconductors



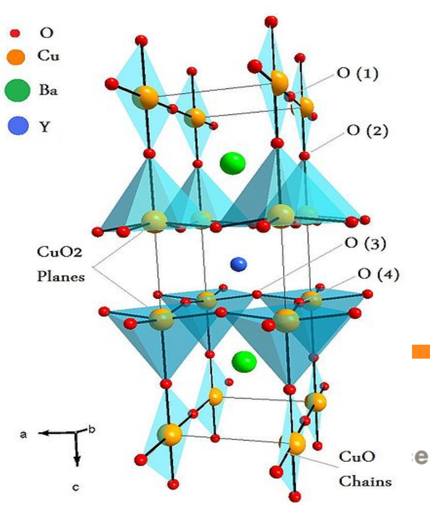
LaOFeAs



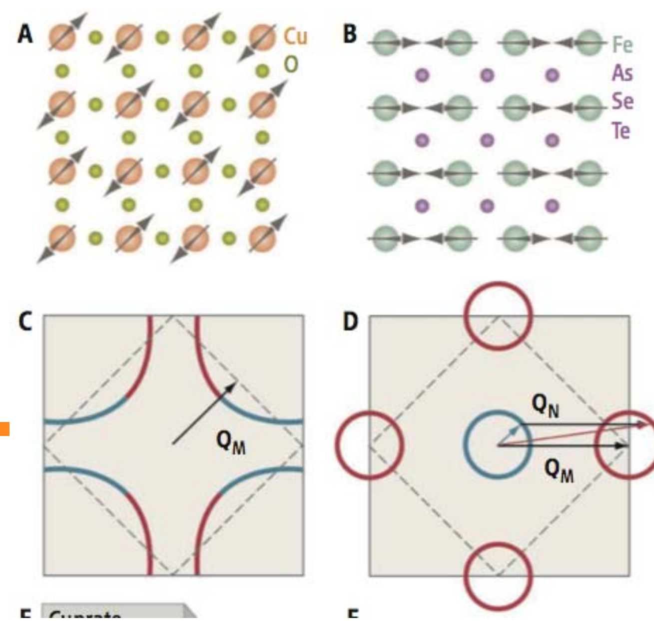
EPL 83, 47001 (2008).

Cuprates

YBCO



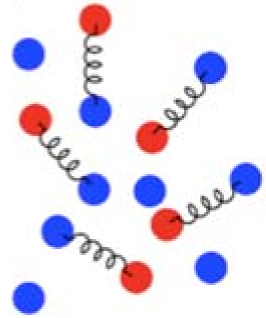
Spin fluctuations



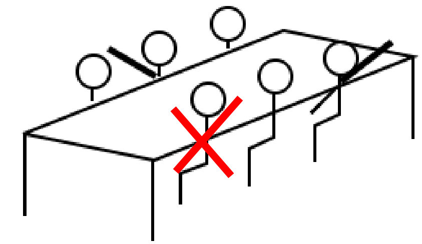
FeSc

Science 328, 441 (2010)

Spin-Population Imbalanced Fermi Gases



$N_\uparrow \neq N_\downarrow$



1. Magnetism versus Superconductivity

Chandrasekhar-Clogston limit

Chandrasekhar, APL 1962.
Clogston, PRL 1962.

critical magnetic field to break superconductivity

2. Exotic superconducting phase?

Fulde-Ferrell-Larkin-Ovchinnikov States

FF, PR 1964
LO, JETP 1965

Oscillating order parameter

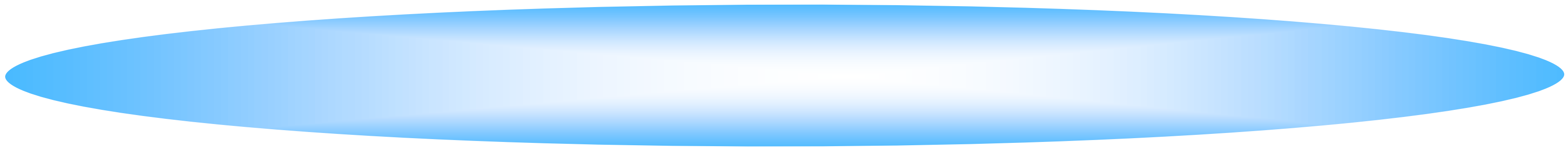
$$\Delta \equiv \Delta_0 \exp(iqx) \quad \text{(FF)}$$
$$\Delta \equiv \Delta_0 \cos(qx) \quad \text{(LO)}$$

Polarized Superfluid States

fully paired
+ excess unpaired

Sarma, J Phys Chem Solids 1963
Liu & Wilczek, PRL 2003; Sheehy & Radzihovsky, PRL 2006,
Pao, Wu, Yip PRB 2006; Parish et al., PRL 2007,
Pilati & Giorgini PRL 2008, etc.

Spin-imbalanced fermions in 3D elongated traps



Experiments:

Shin et. al., PRL 97, 030401 (2006)
Partridge et. al., Science 311, 5760 (2006)
Partridge et. al., PRL 97, 190407 (2006)
Nascimbene et. al., PRL 103, 170402 (2009)

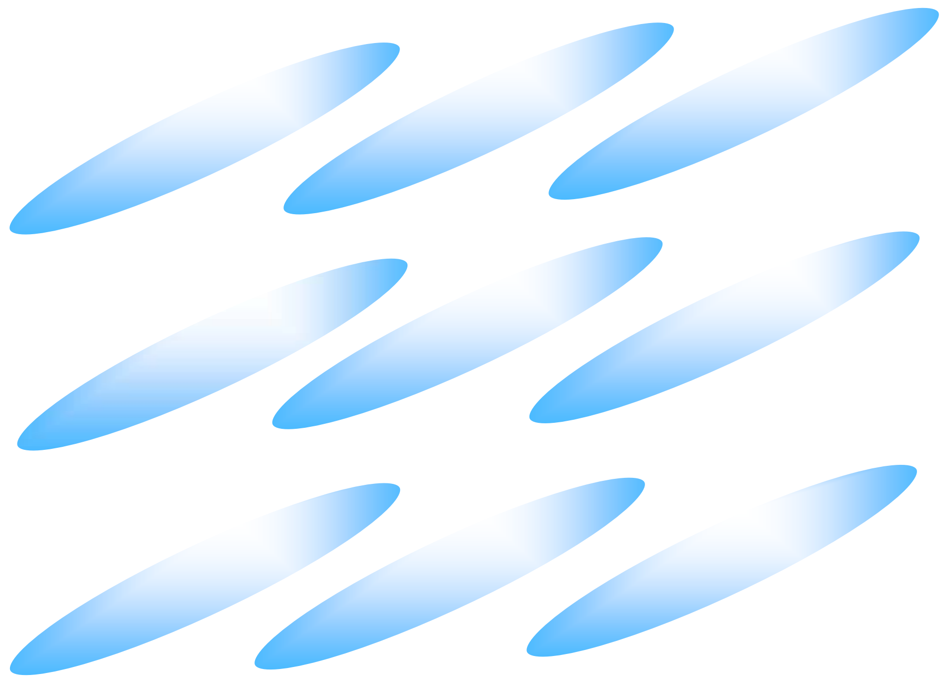
QMC:

C. Lobo, A. Recati, S. Giorgini, S. Stringari, PRL 97, 200403 (2006)

DMFT:

D.-H. Kim, J. J. Kinnunen, J.-P. Martikainen, PT, PRL 106, 095301 (2011)

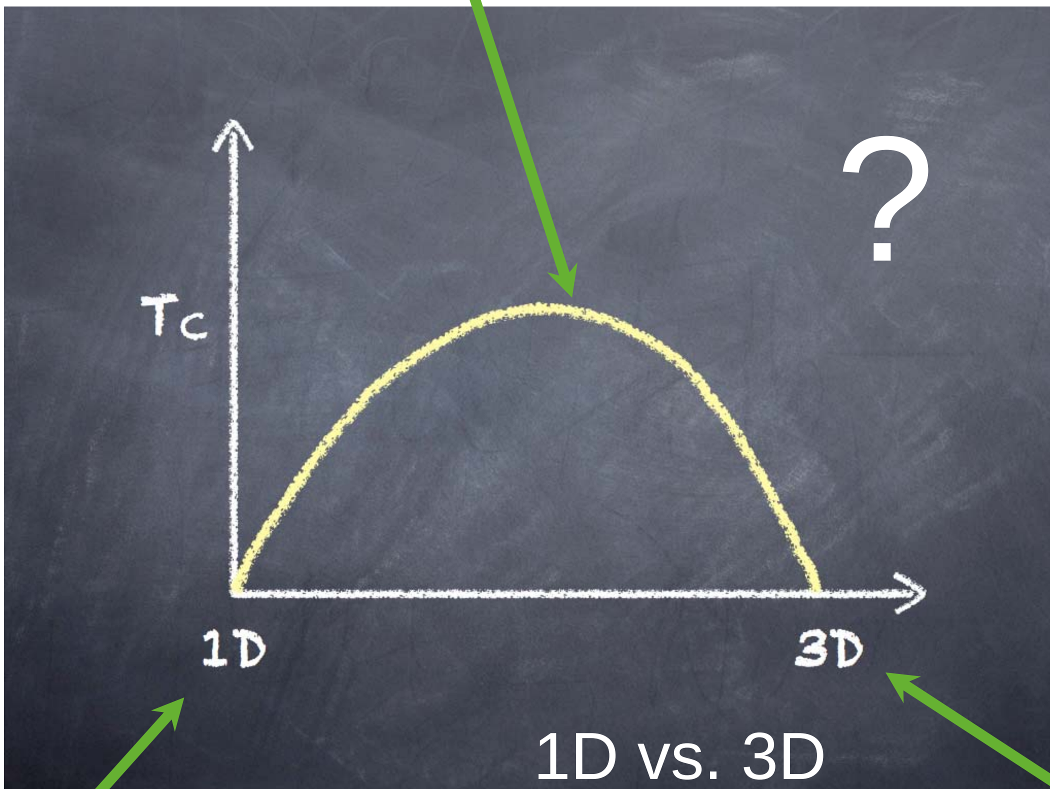
FFLO in quasi-1D lattices



D. H. Kim, PT, PRB 85, 180508(R) (2012)
M.O.J. Heikkinen, D.H. Kim, PT, in preparation (2013)

Stronger FFLO signature in quasi-1D?

Somewhere between 3D and 1D: an ideal place for FFLO?



The long-range order may stabilize FFLO.

Mean-field theory (T=0)

Parish et al., PRL 99, 250403 (2007)

Experiment in 1D (density profiles)

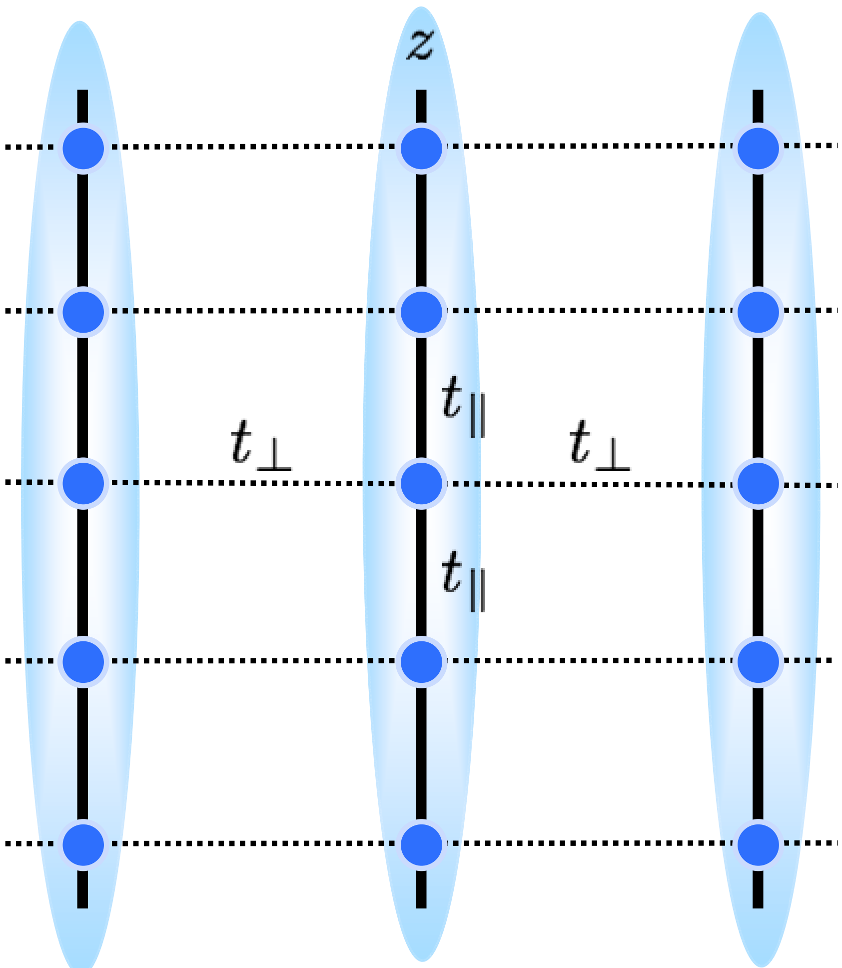
Liao et al., Nature 467, 567 (2010)

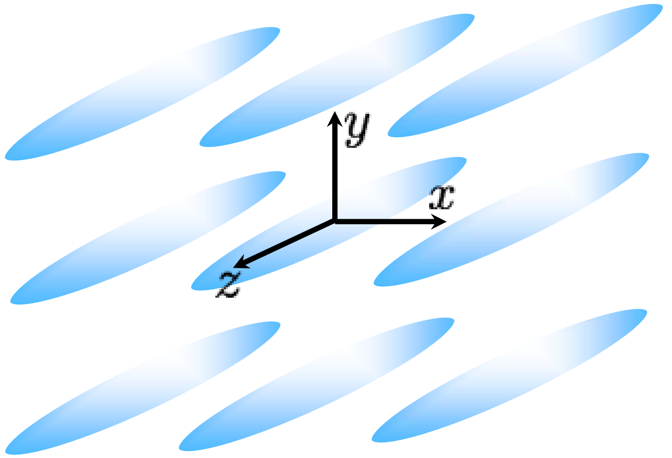
In 1D, an exact solution gives FFLO, but no long-range order is possible.

In 3D, the FFLO area may be very narrow.

Stronger FFLO signature in quasi-1D?

A bundle of chains : 3D to 1D





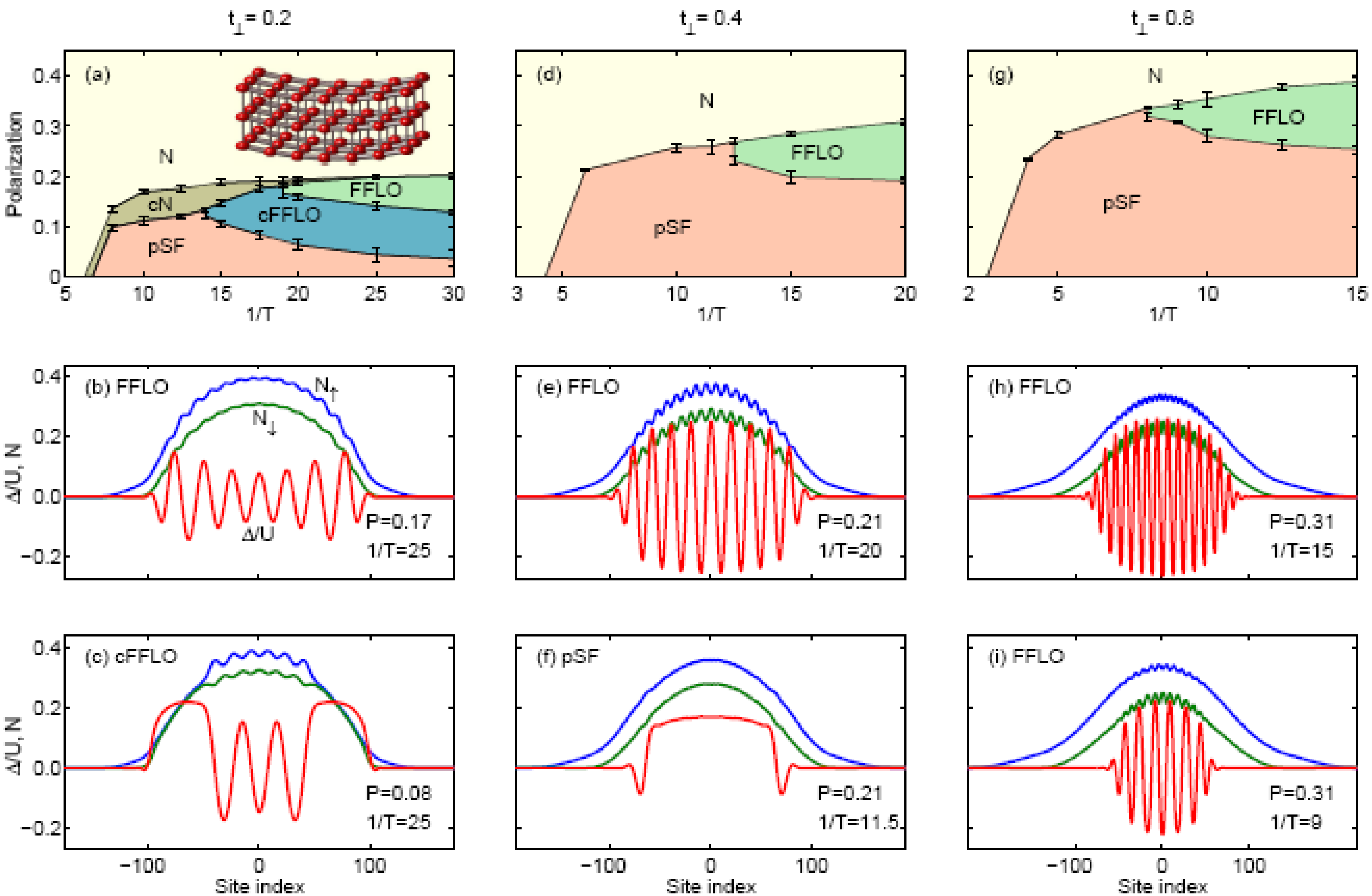
Anisotropic 3D optical lattice for DMFT calculations

(1D) $0 < t \equiv \frac{t_{\perp}}{t_{\parallel}} \leq 1$ (3D)

Homogeneous 2D lattice of trapped 1D chains

$$U \to U_{*} \ \ (a \to \infty)$$

Finite temperature phase diagram



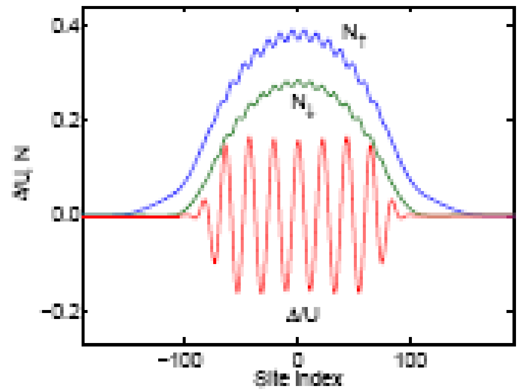
Confirms the mean-field predictions about stability of FFLO in lattices (due to nesting)

FFLO enhancement in optical lattices

- T.K. Koponen, T. Paananen, J.-P. Martikainen, and P. Törmä, Phys. Rev. Lett. 99, 120403 (2007)
- Y.L. Loh and N. Trivedi, Phys. Rev. Lett. 104, 165302 (2010)

In contrast, dimensionality does not play a large role, unlike suggested by mean-field: the FFLO is prominent throughout the crossover

However, a clear dimensionality effect: extremely uniform order parameter in intermediate dimensionality



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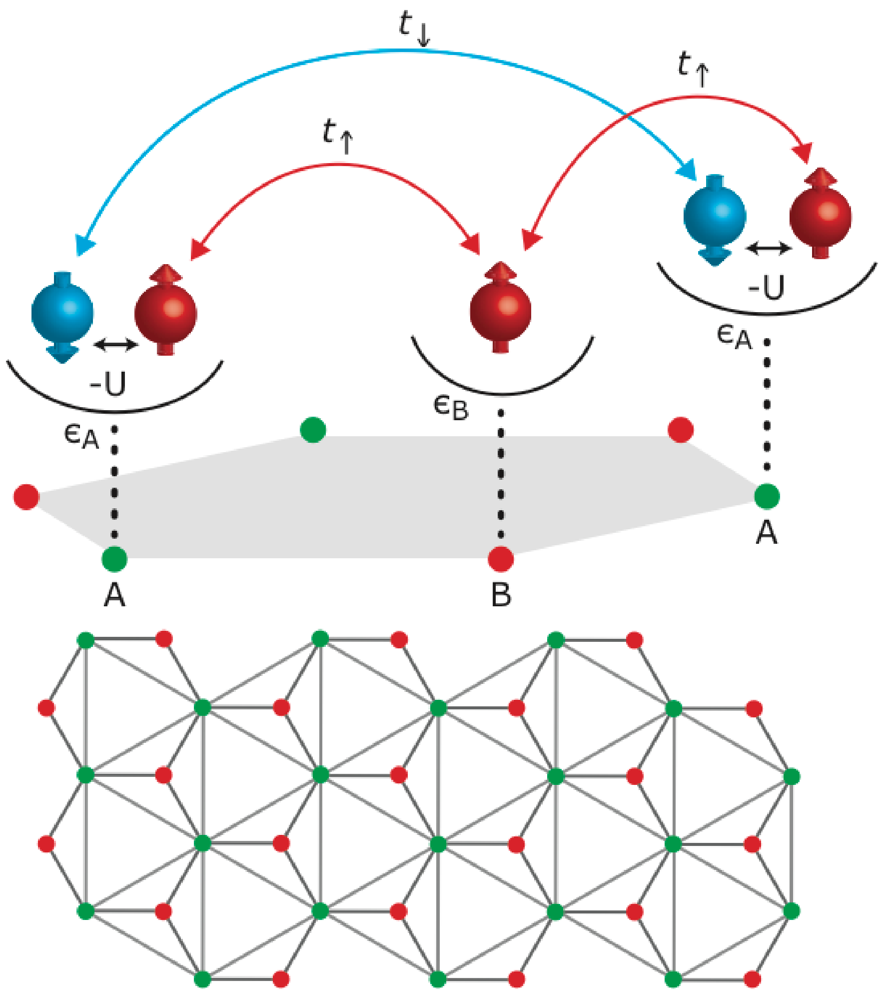
Pairing in *mixed geometries*

- Pairing with spin-imbalance (mass-imbalance)
  - Chandrasekhar-Clogston limit, FFLO, polarized superfluids, breach pair superfluids, etc.
- Pairing in mixed dimensions
  - A couple of theory papers (Tan, Iskin)
- Our question: pairing in ***mixed geometries***
  - D.-H. Kim, J.S.J. Lehikoinen, PT, PRL 110, 055301 (2013)
- Motivation
  - ***Spin-dependent confinement***
  - Spin-dependent lattices experimentally possible (Hamburg, NIST, Dusseldorf, others)
  - Two fermionic atoms (Li, K) trapped, Feshbach resonances (Innsbruck, MIT, others)
  - Novel superfluids? High T_c? Polarized superfluid?

Our choice of geometry: honeycomb lattice for the up-component, triangular for the down-component

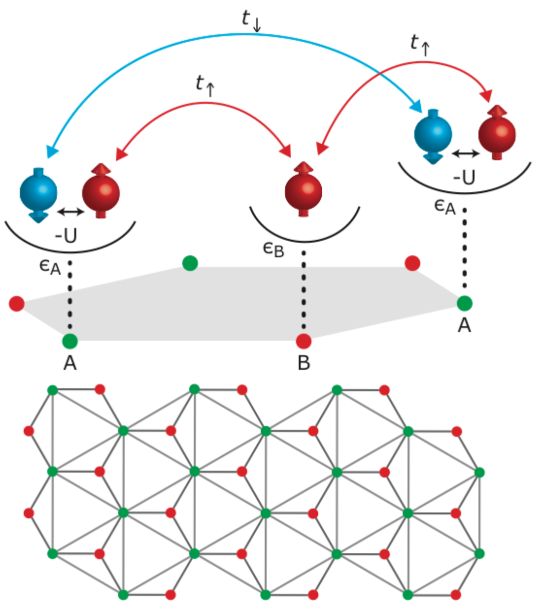


Mean-field theory for this system

$$\mathcal{H} = -\,t_\uparrow \sum_{\langle i,j\rangle \in \mathcal{L}_\uparrow} (\hat{a}^\dagger_{i\uparrow}\hat{b}_{j\uparrow} + h.c.) + \epsilon^a_\uparrow \sum_i \hat{n}^a_{i\uparrow} + \epsilon^b_\uparrow \sum_i \hat{n}^b_{i\uparrow}$$

$$-\,t_\downarrow \sum_{\langle i,j\rangle \in \mathcal{L}_\downarrow} (\hat{a}^\dagger_{i\downarrow}\hat{a}_{j\downarrow} + h.c.) + \epsilon^a_\downarrow \sum_i \hat{n}^a_{i\downarrow}$$

$$\sum_{i,j}(\hat{n}^a_{i\uparrow} + \hat{n}^b_{j\uparrow}) - \mu_\downarrow \sum_i \hat{n}^a_{i\downarrow} - U \sum_i \hat{n}^a_{i\uparrow}\hat{n}^a_{i\downarrow}$$



$$t_\uparrow = t_\downarrow = 1 \qquad U = 5$$

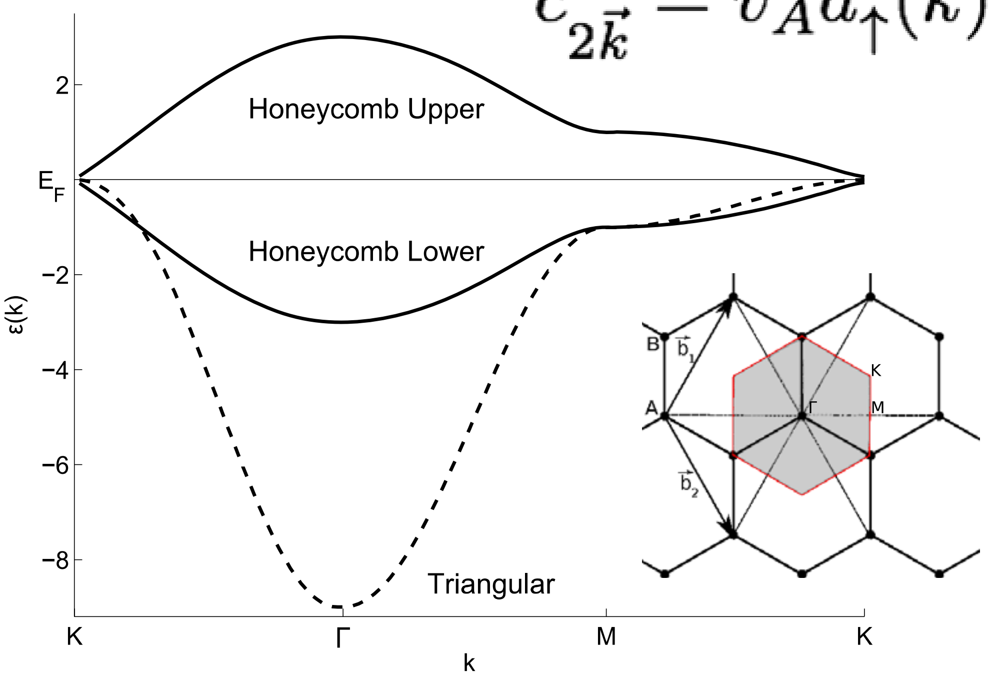
$$\epsilon^a_\uparrow = \frac{\tilde{\epsilon}}{2} \quad \epsilon^b_\uparrow = -\frac{\tilde{\epsilon}}{2} \quad \epsilon^a_\downarrow = -3$$

The non-interacting system: two braches for the honeycomb

$$\mathcal{H}_0 = \sum_{\vec{k}} \xi_1(\vec{k})\hat{c}^\dagger_{1\vec{k}}\hat{c}_{1\vec{k}} + \xi_2(\vec{k})\hat{c}^\dagger_{2\vec{k}}\hat{c}_{2\vec{k}} + \xi_3(\vec{k})\hat{c}^\dagger_{3\vec{k}}\hat{c}_{3\vec{k}}$$

$$\hat{c}^\dagger_{1\vec{k}} = u_A\tilde{a}^\dagger_\uparrow(\vec{k}) + u_B\tilde{b}^\dagger_\uparrow(\vec{k})$$

$$\hat{c}^\dagger_{2\vec{k}} = v_A\tilde{a}^\dagger_\uparrow(\vec{k}) + v_B\tilde{b}^\dagger_\uparrow(\vec{k})$$



Fermi-surface match:

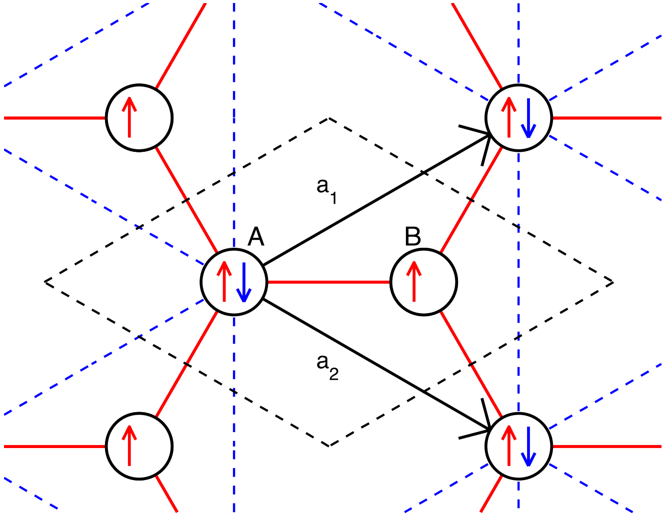
$$\mu_\downarrow = -\mu^2_\uparrow$$

Interaction at site A

$$\mathcal{H}_{\mathrm{U}} = -U \sum_{i} \hat{n}^{a}_{i\uparrow} \hat{n}^{a}_{i\downarrow}$$

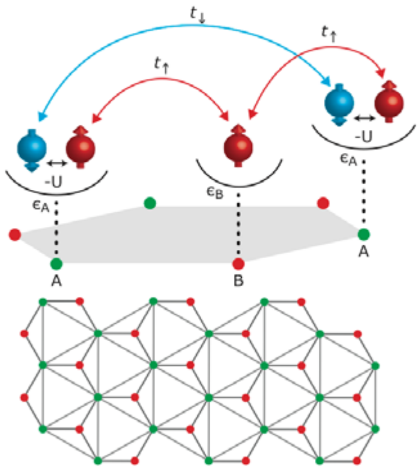
Corresponds to pairing between spin-down and the two honeycomb braches

$$\mathcal{H}_{\mathrm{U}} = \sum_{\vec{k}} [g_1(\vec{k}) \hat{c}^{\dagger}_{1\vec{k}} \hat{c}^{\dagger}_{3,-\vec{k}} + g_2(\vec{k}) \hat{c}^{\dagger}_{2\vec{k}} \hat{c}^{\dagger}_{3,-\vec{k}} + h.c.] + \frac{|\Delta|^2}{U}$$



Pairing ansatz at site A

$$\Delta = U\langle \hat{a}_{i\downarrow}\hat{a}_{i\uparrow}\rangle$$



Leads to a momentum-dependent coupling in the momentum space

$$g_{1,2}(\vec{k}) = -\frac{\Delta}{\sqrt{2}}\left[1 \pm \frac{\tilde{\epsilon}}{\sqrt{\tilde{\epsilon}^2 + |h_\uparrow(\vec{k})|^2}}\right]^{\frac{1}{2}}$$

$$h_\uparrow(\vec{k}) = -t_\uparrow\left[e^{\frac{ik_x}{\sqrt{3}}} + 2e^{\frac{-ik_x}{2\sqrt{3}}}\cos\frac{k_y}{2}\right]$$

Diagonalize by a Bogoliubov transformation (third order eigenvalue equation)

$$\mathcal{H} = \sum_{\vec{k}} E_1(\vec{k})\tilde{\gamma}^\dagger_{1\vec{k}}\tilde{\gamma}_{1\vec{k}} + E_2(\vec{k})\tilde{\gamma}^\dagger_{2\vec{k}}\tilde{\gamma}_{2\vec{k}} + E_3(\vec{k})\tilde{\gamma}^\dagger_{3\vec{k}}\tilde{\gamma}_{3\vec{k}}$$

$$+ \sum_{\vec{k}} \xi_\downarrow(-\vec{k}) + \frac{|\Delta^2|}{U}$$

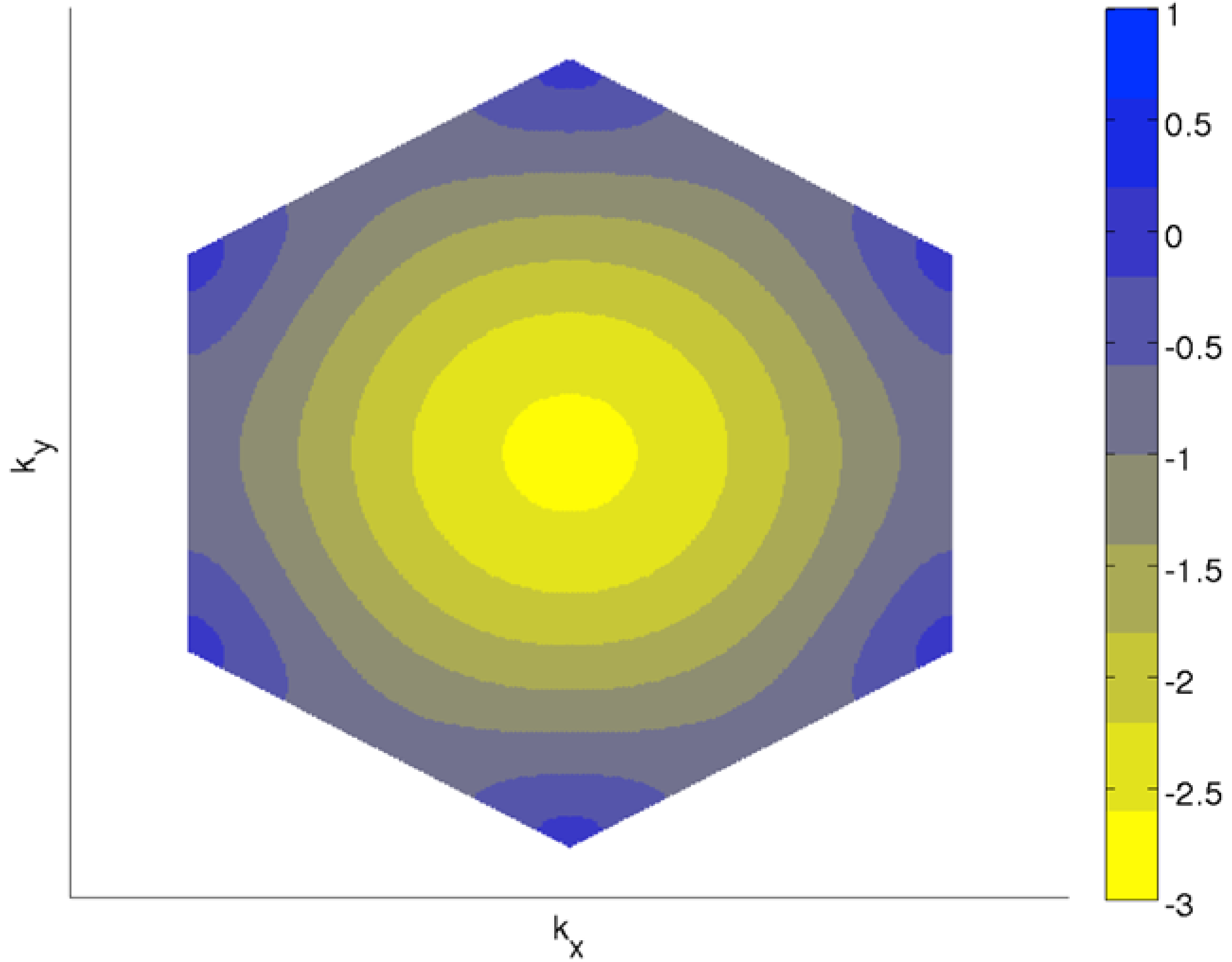
$$\begin{pmatrix} \tilde{\gamma}_{1\vec{k}} \\ \tilde{\gamma}_{2\vec{k}} \\ \tilde{\gamma}_{3\vec{k}} \end{pmatrix} = \mathbf{U}^\dagger_{\vec{k}} \begin{pmatrix} \hat{c}_{1\vec{k}} \\ \hat{c}_{2\vec{k}} \\ \hat{c}^\dagger_{3,-\vec{k}} \end{pmatrix}$$

Find the phases by minimizing the grand potential, and from the gap equation

$$\Omega = -\frac{1}{\beta}\sum_{\vec{k}}\left[\ln\left(1+e^{-\beta E_1(\vec{k})}\right)+\ln\left(1+e^{-\beta E_2(\vec{k})}\right)+\ln\left(1+e^{-\beta E_3(\vec{k})}\right)\right]$$

$$+\sum_{\vec{k}}\xi_{\downarrow}(-\vec{k})+\frac{|\Delta|^2}{U}$$

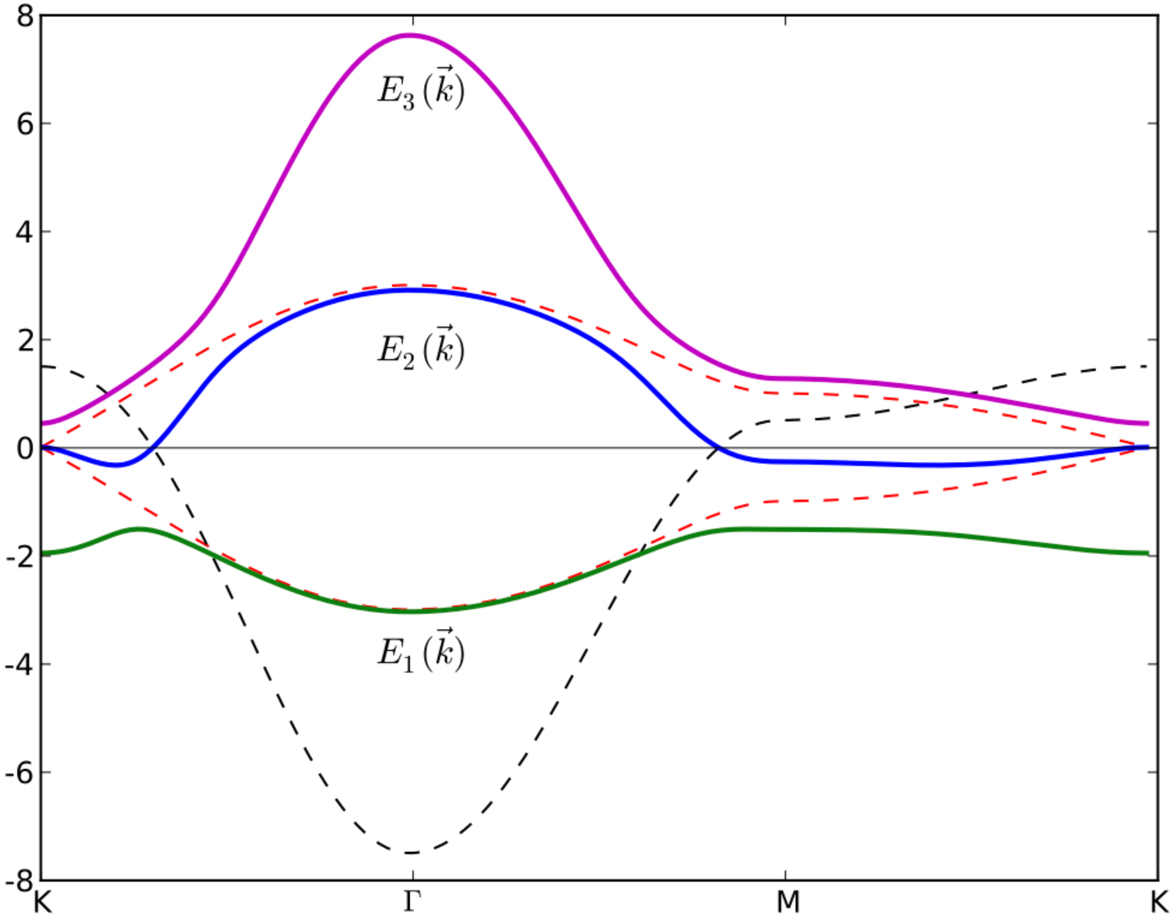
The Fermi surface

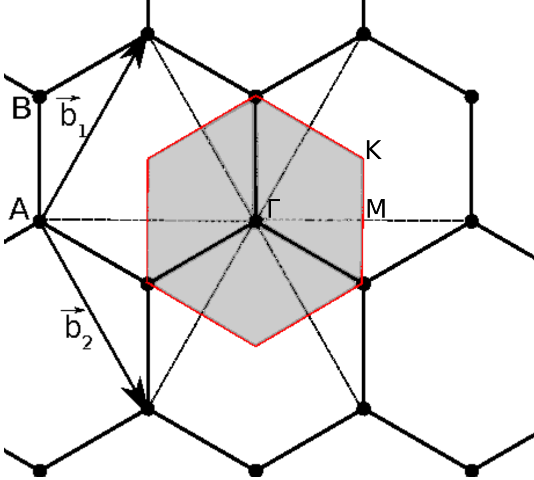


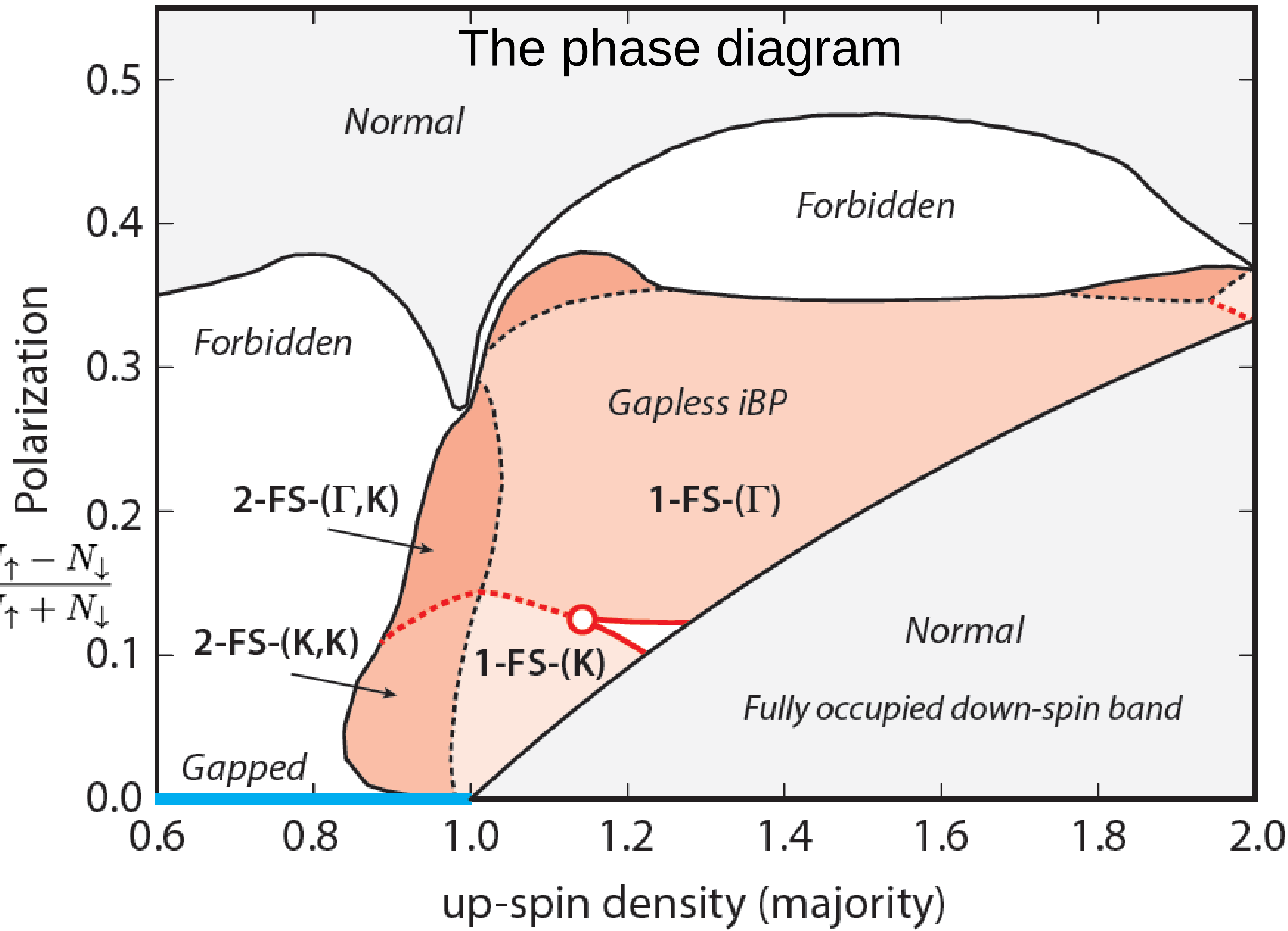
Results

Three quasiparticle branches

$$\mu_\uparrow = -0.1,\ \mu_\downarrow = -1.505,\ U = 5,\ \Delta = 0.93$$







$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

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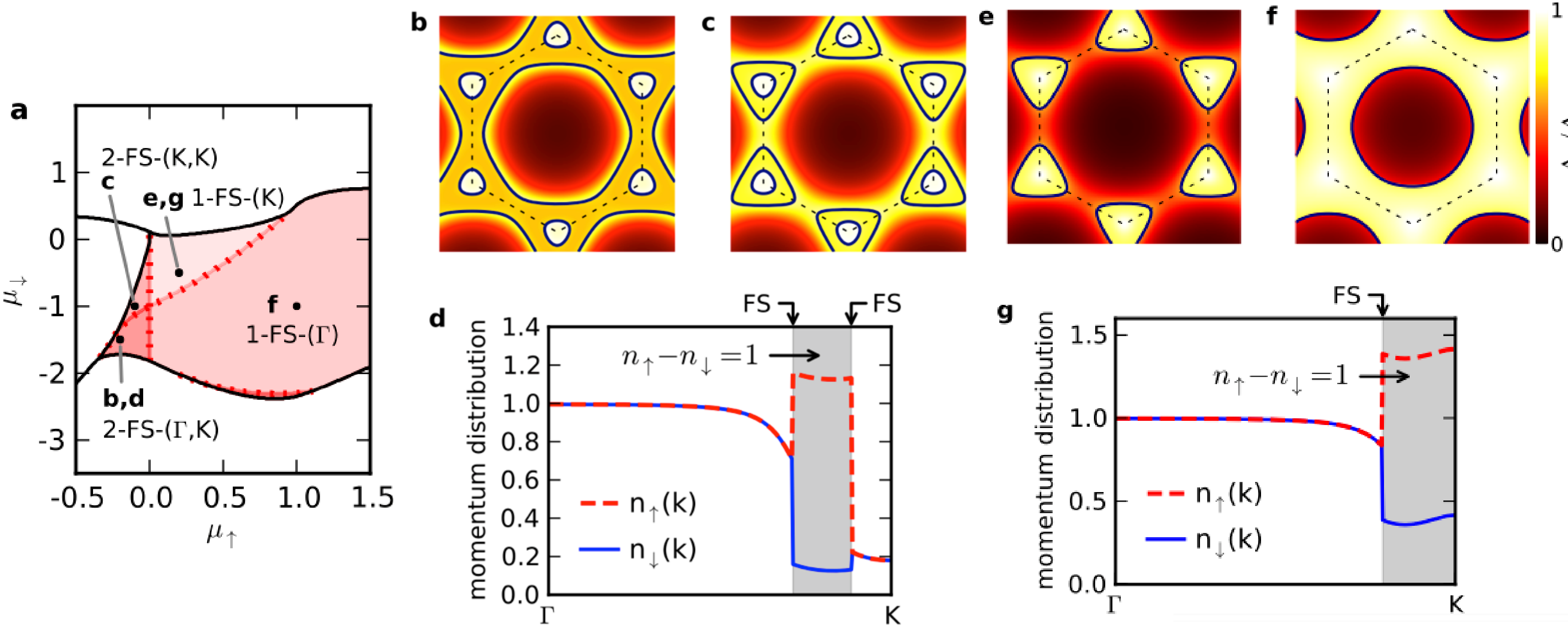
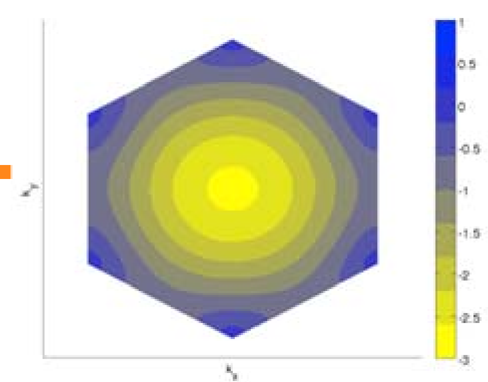
A new stable polarized superfluid phase: ***incomplete breach pair (iBP) state***



Quantum phase transitions between topologically distinct states



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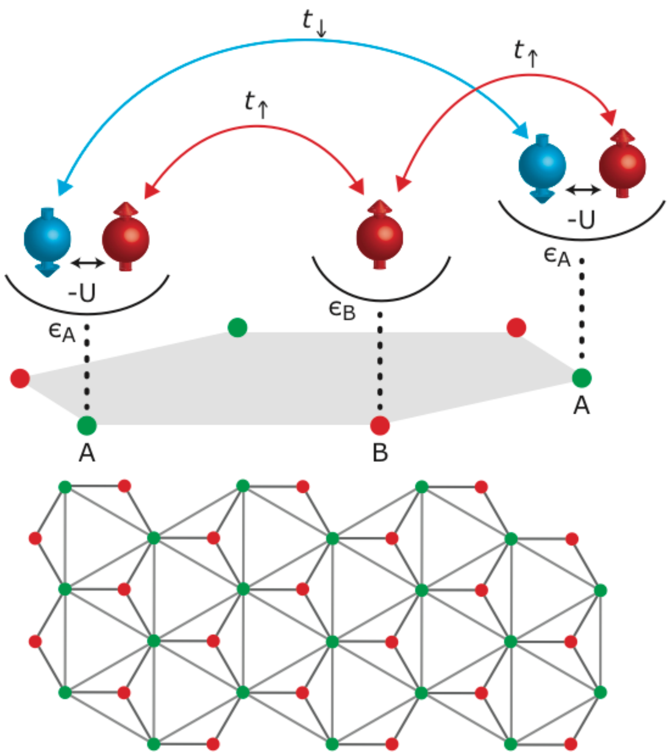
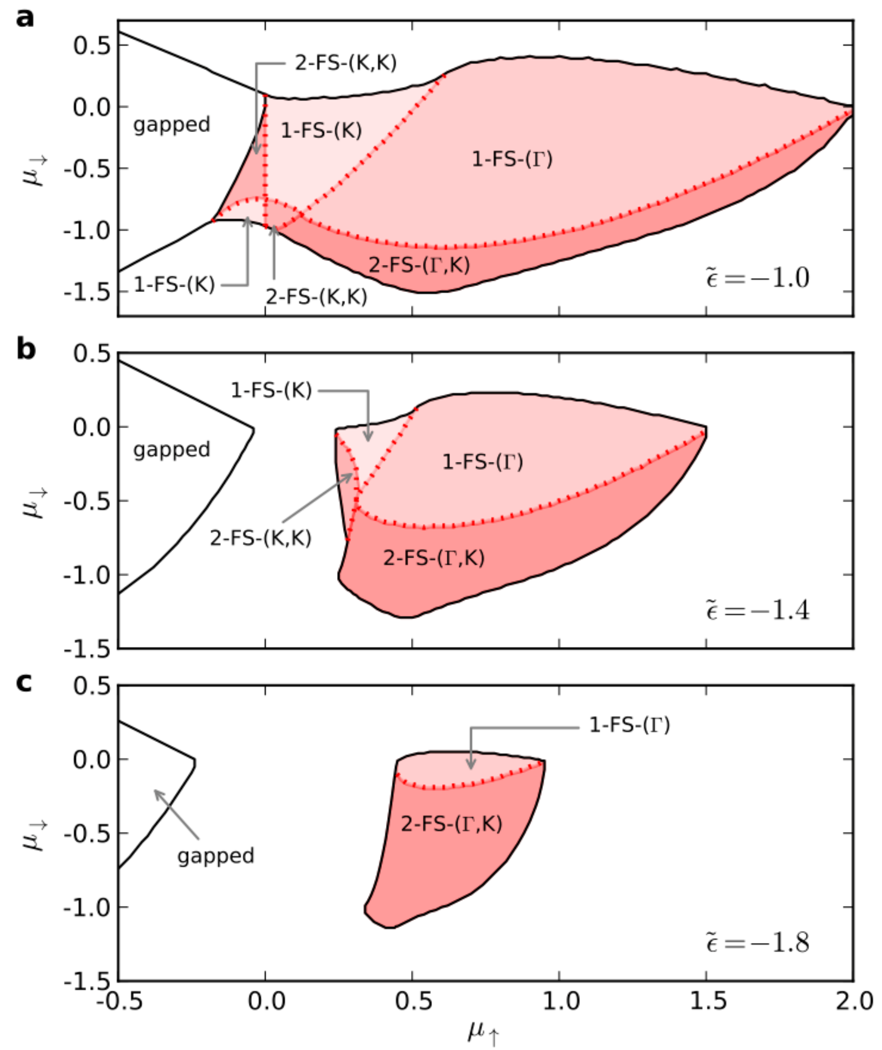
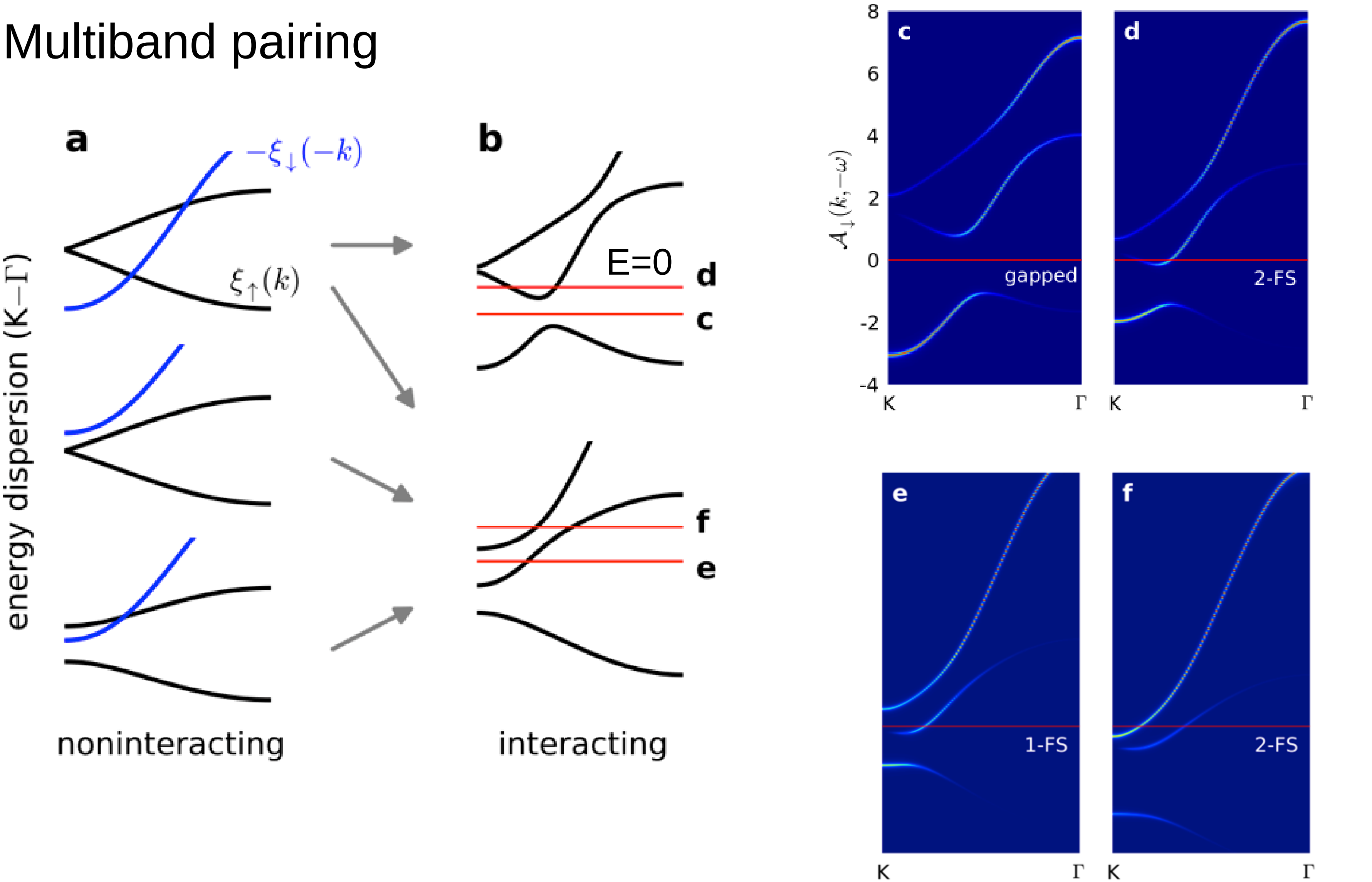
Changing the energy offset between the A and B sites





Multiband pairing



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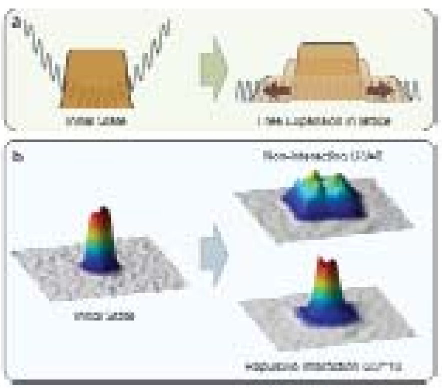
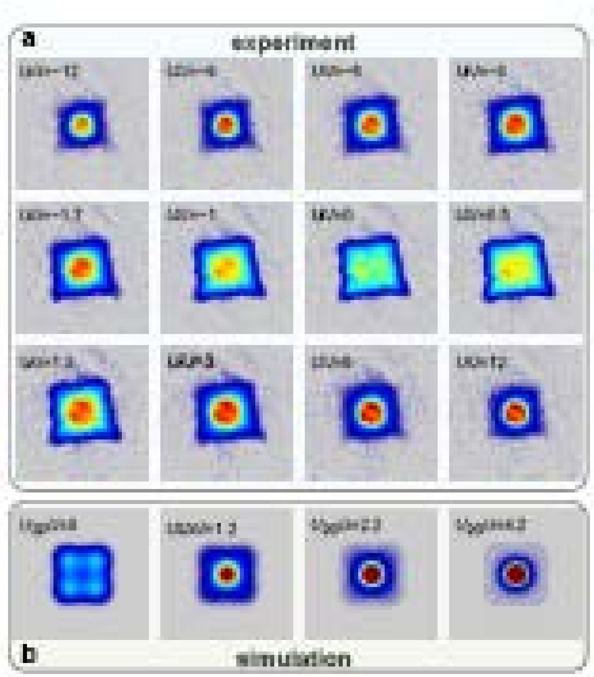
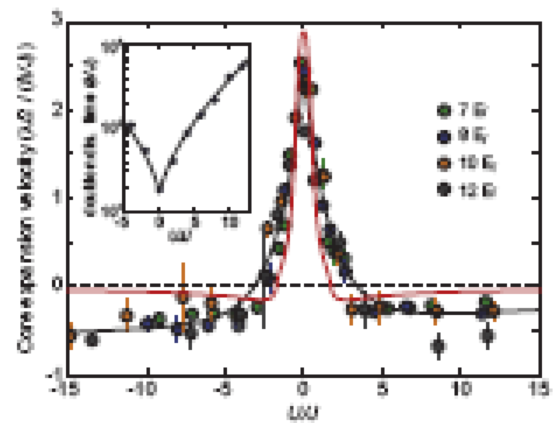
- Dynamics of a polaron in 1D (t-DMRG)

Motivation

- Dynamics of a many-body Fermion system
- U. Schneider, L. Hackermuller, J.P. Ronzheimer, S. Will, S. Braun, T. Best, I. Bloch, E. Demler, S. Mandt, D. Rasch, A. Rosch, *Breakdown of diffusion: From collisional hydrodynamics to a continuous quantum walk in a homogeneous Hubbard model*, Nat. Phys. 2012







Core expansion speed as a function of interaction

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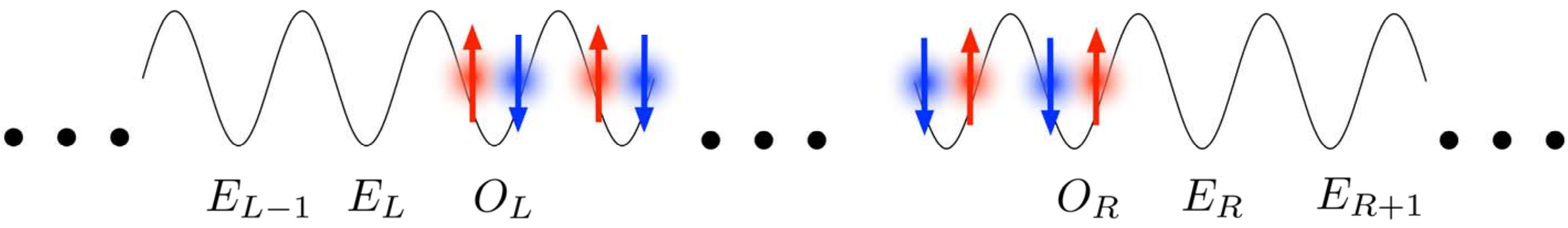
The system



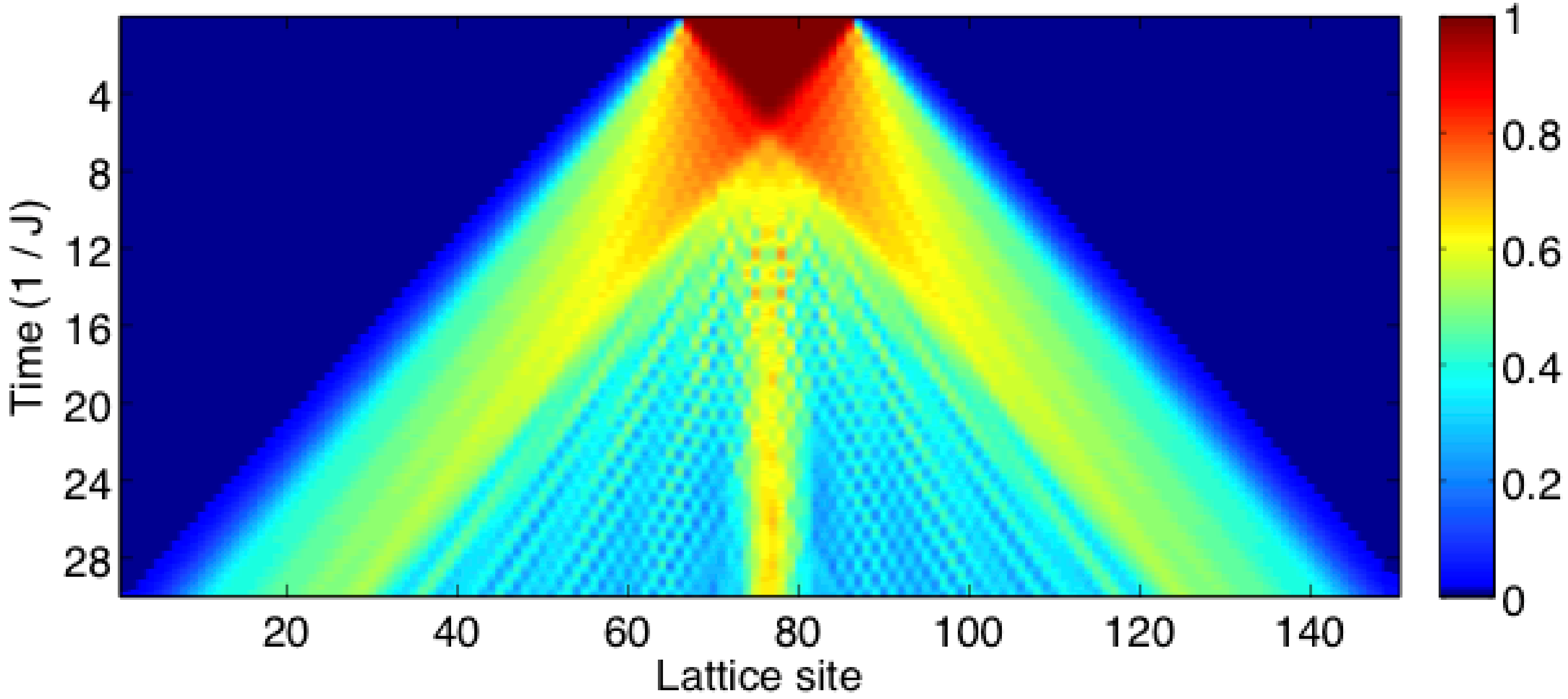
$$H = U\sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow} - J\sum_{i,\sigma=\uparrow,\downarrow} c^{\dagger}_{i\,\sigma} c_{i+1\,\sigma} + h.c.$$

J. Kajala, F. Massel, PT, PRL 106, 206401 (2011)

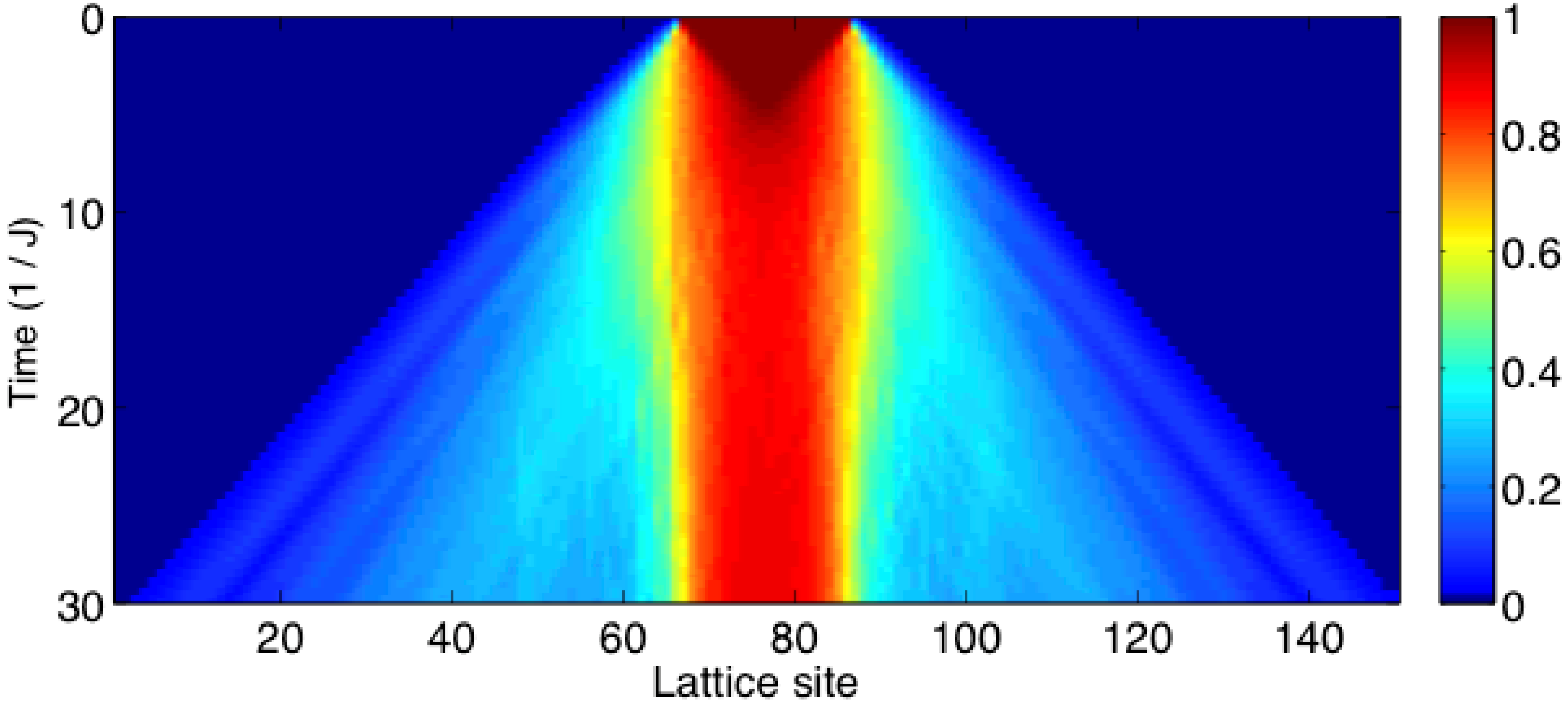
Earlier 1D dynamics: Kollath, Schollwöck, Zwerger, Heidrich-Meisner, Rigol, Cirac, Zoller, Shlyapnikov, Daley, Santos, Carr, Pupillo, Tezuka, Ueda, Diehl, our group, etc

Results (t-DMRG)

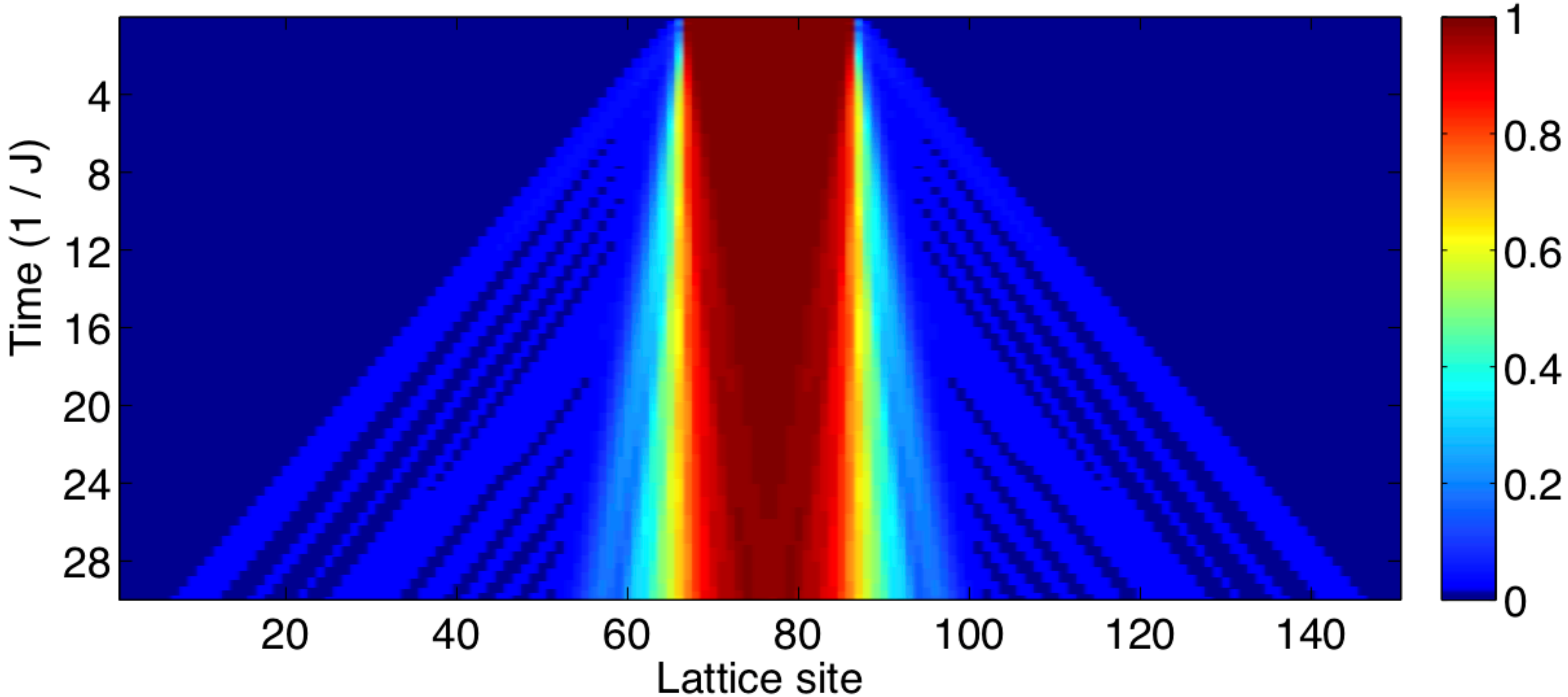
$$\sqrt{n_\uparrow} \text{ for } \frac{|U|}{J} = 0.0$$



$$\sqrt{n_\uparrow} \text{ for } \frac{|U|}{J} = 1.0$$



$$\sqrt{n_\uparrow} \text{ for } \frac{|U|}{J} = 10.0$$

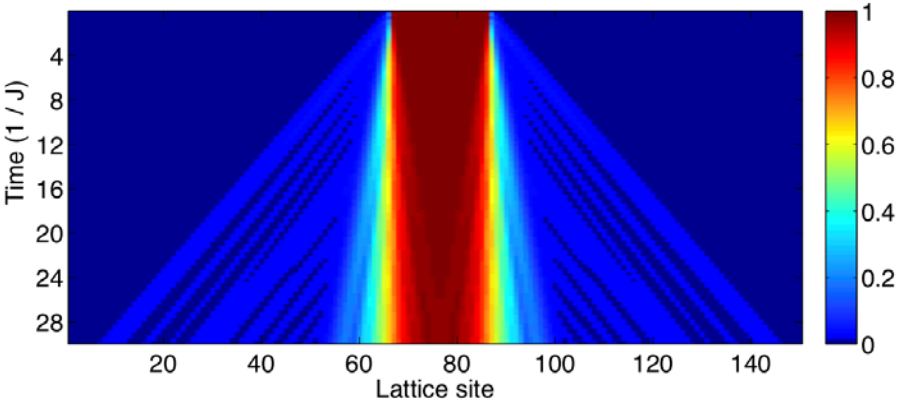


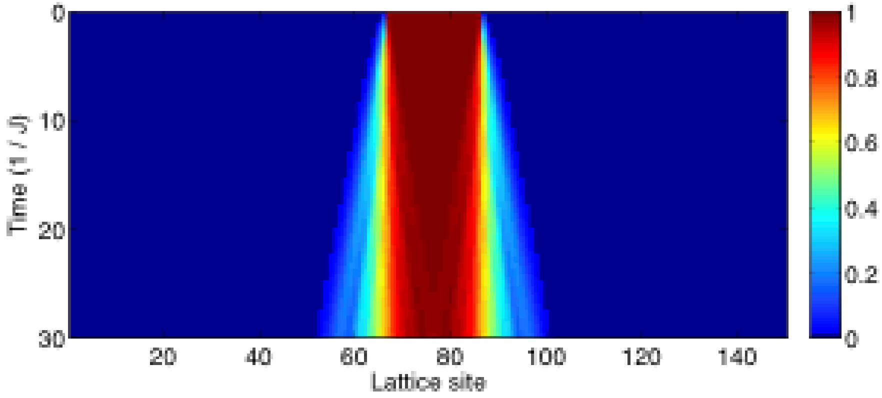
- With TEBD one can also calculate the doublon density

$$n_{i\uparrow\downarrow}(t) = <\Phi(t)|c_{i\uparrow}^\dagger c_{i\uparrow} c_{i\downarrow}^\dagger c_{i\downarrow}|\Phi(t)>$$

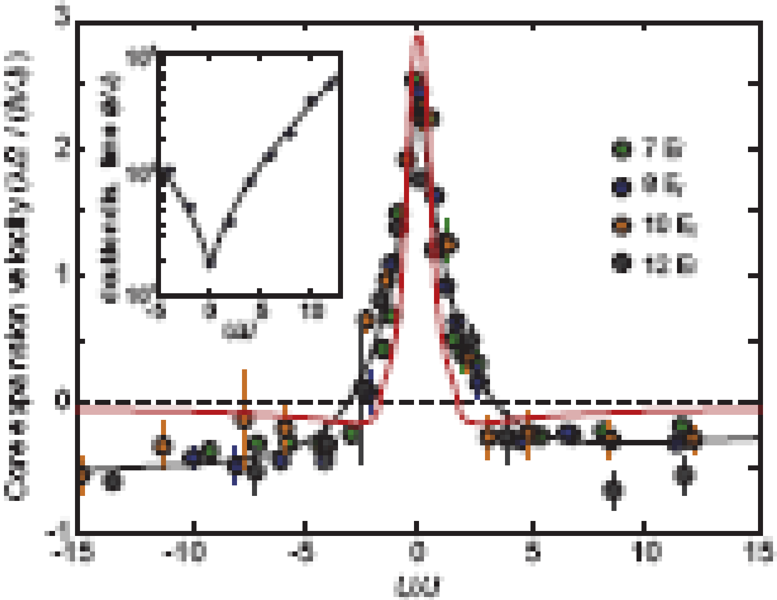
- Here we call doublons excitations of the form $c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger|\emptyset\rangle$

Inspired by two-fluid models in the imbalanced 1D ultracold gas context: E. Zhao, W. Liu, G. Orso, H. Hu, X. Liu, P. Drummond, etc.



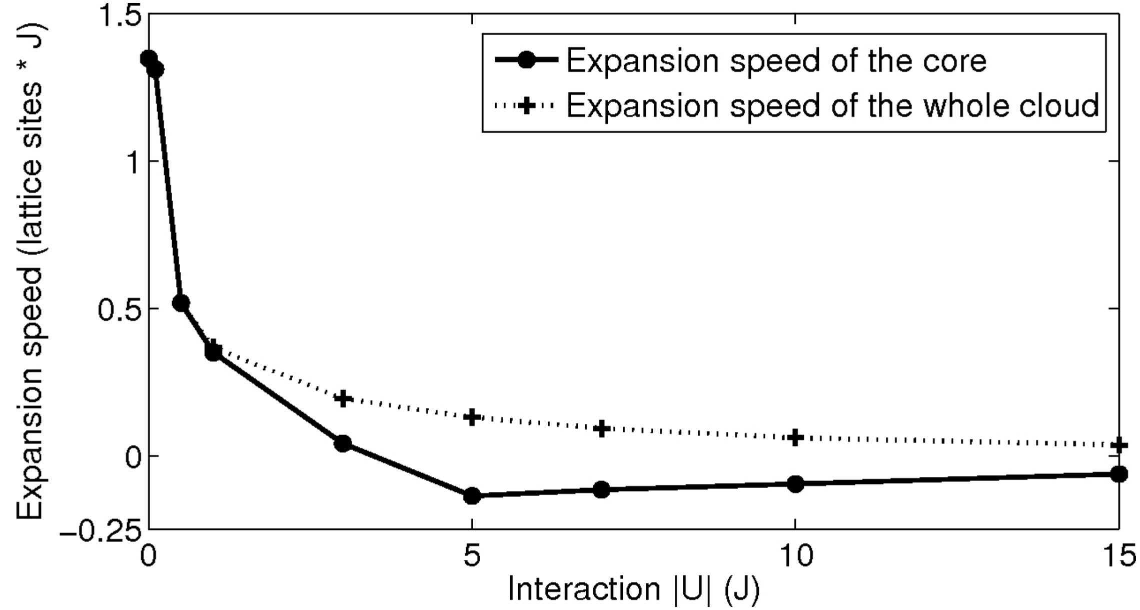


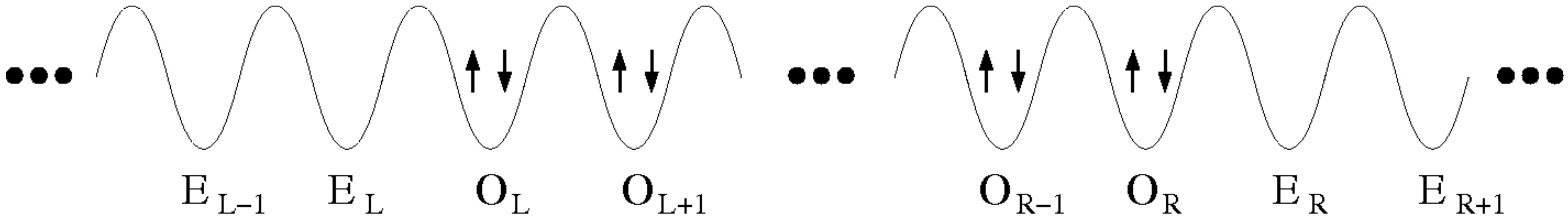
$$\text{Left: } \sqrt{n_{\uparrow}}, \text{ Right:} \sqrt{n_{\uparrow\downarrow}}, \ \frac{|U|}{J} = 10.0$$



Experiment by Schneider et al., 2D

Our simulation, 1D

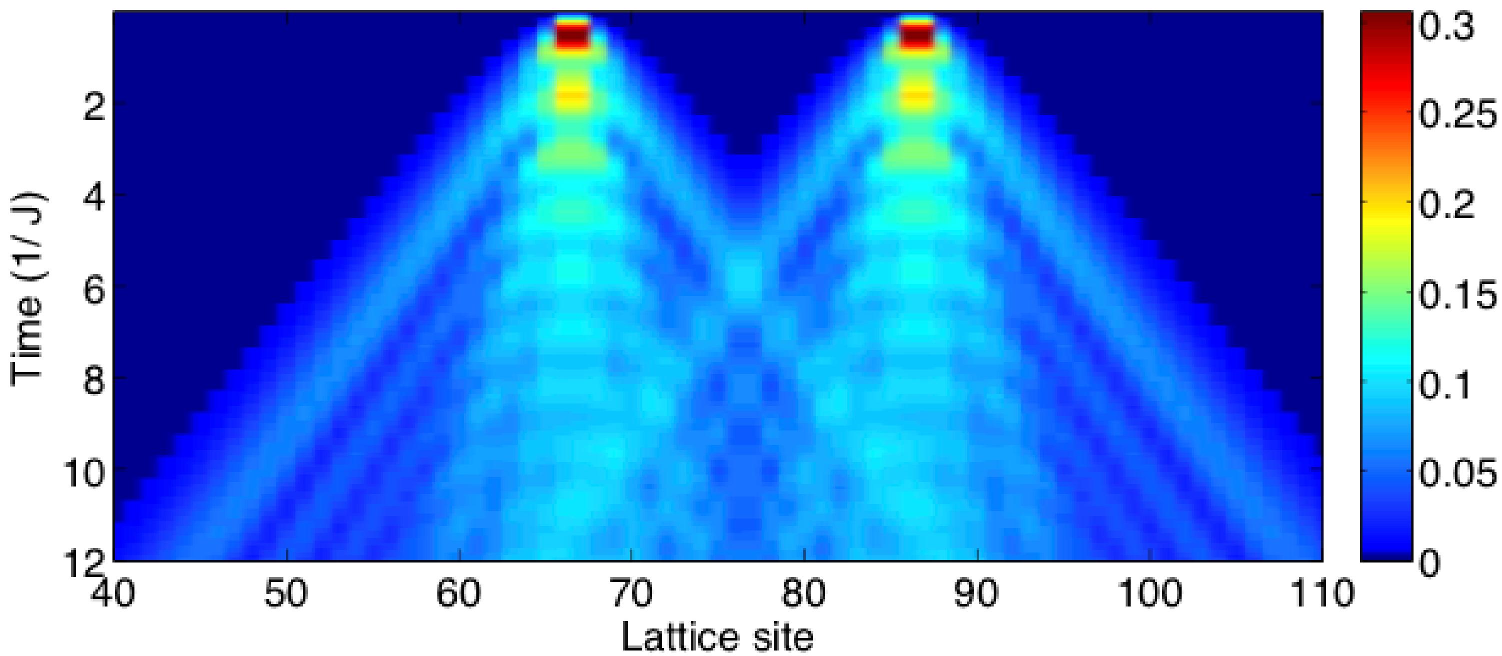




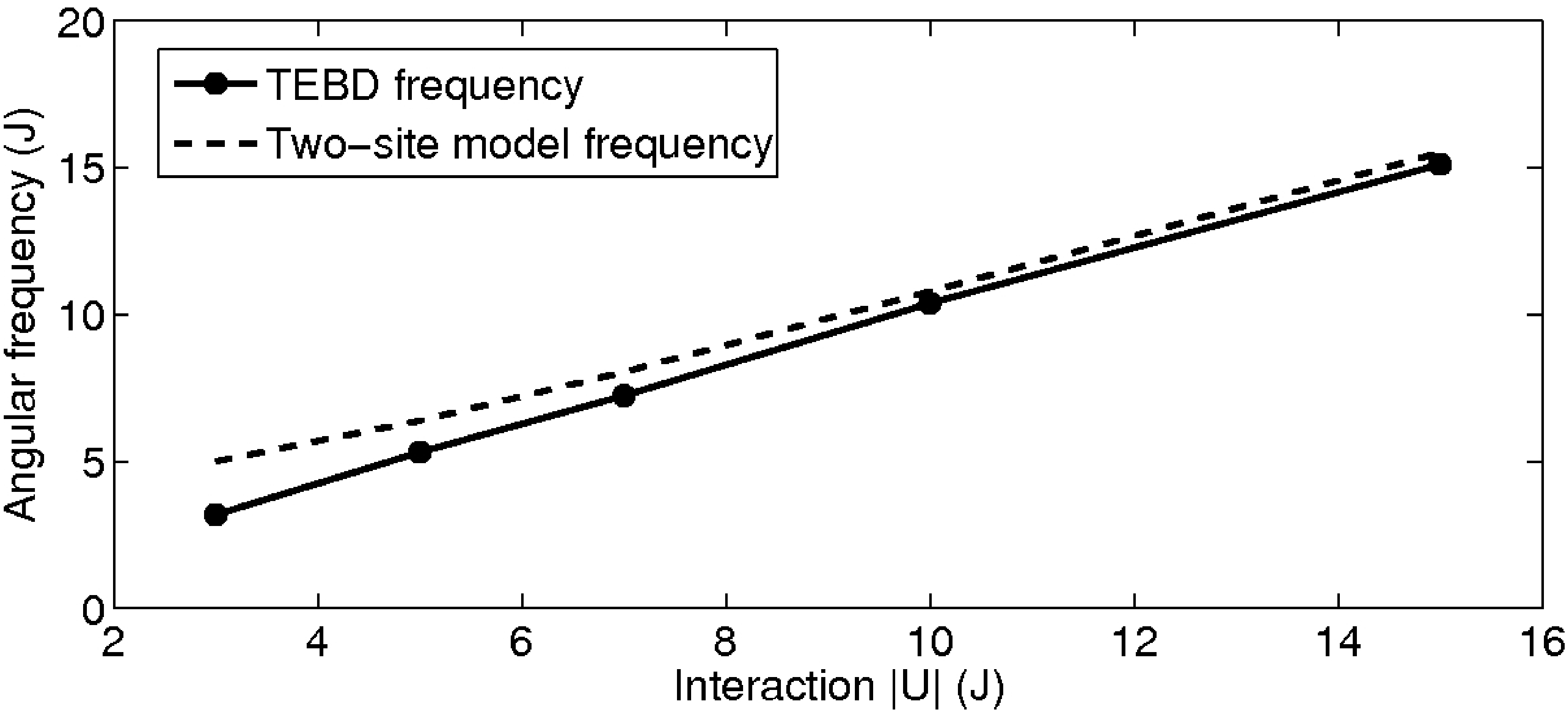
- Our explanation for the numerical findings: describe the two last sites by the Hubbard dimer

- We are interested in the paired state <-> singlet time evolution.
- The singlet state $\frac{1}{\sqrt{2}}(|\uparrow,\downarrow> -|\downarrow,\uparrow>)$
- $$n_{Singlet}(t) = \frac{8}{16+\frac{U^2}{J^2}}\left(1-\cos\left(\sqrt{U^2+16J^2}t\right)\right)$$
- Compare to numerics

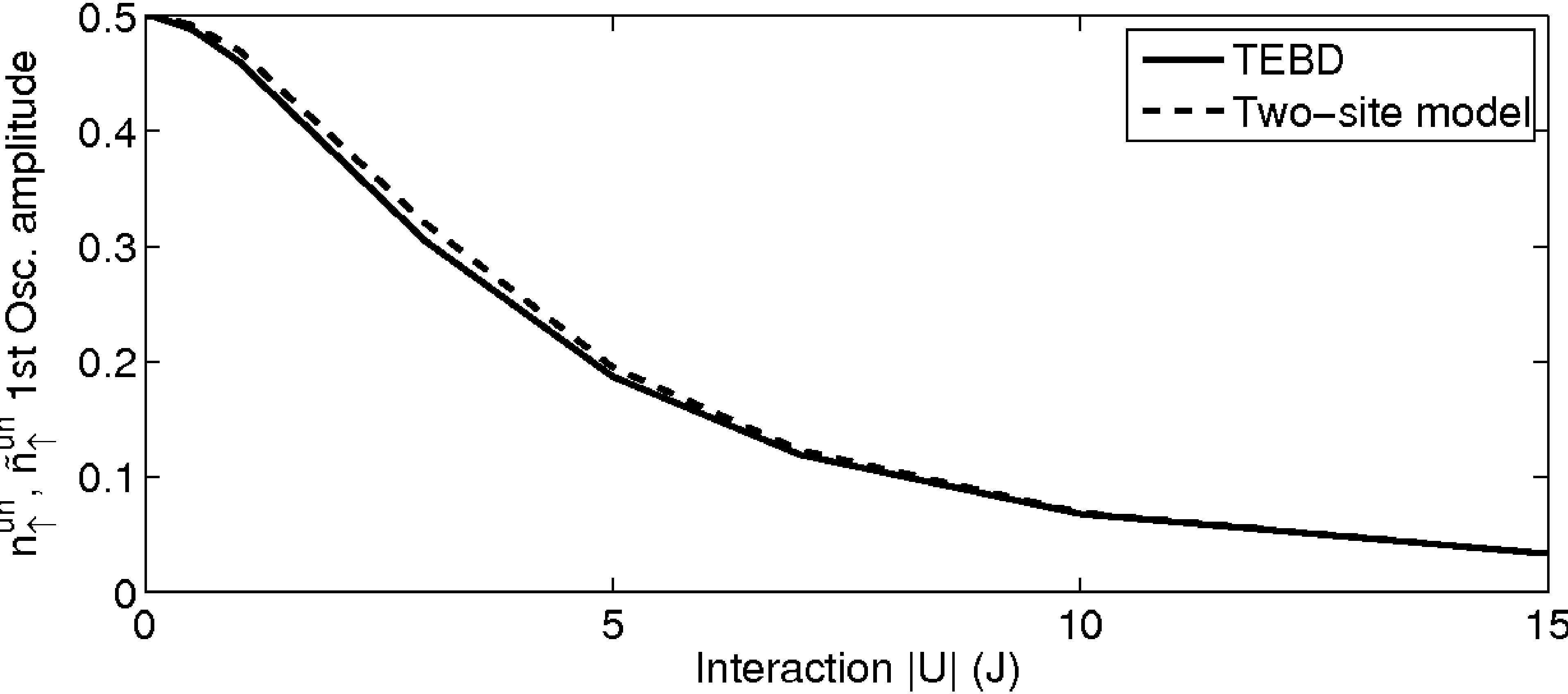
$$\sqrt{n_{Singlet}(t)} \text{ for } \frac{|U|}{J} = 5.0$$



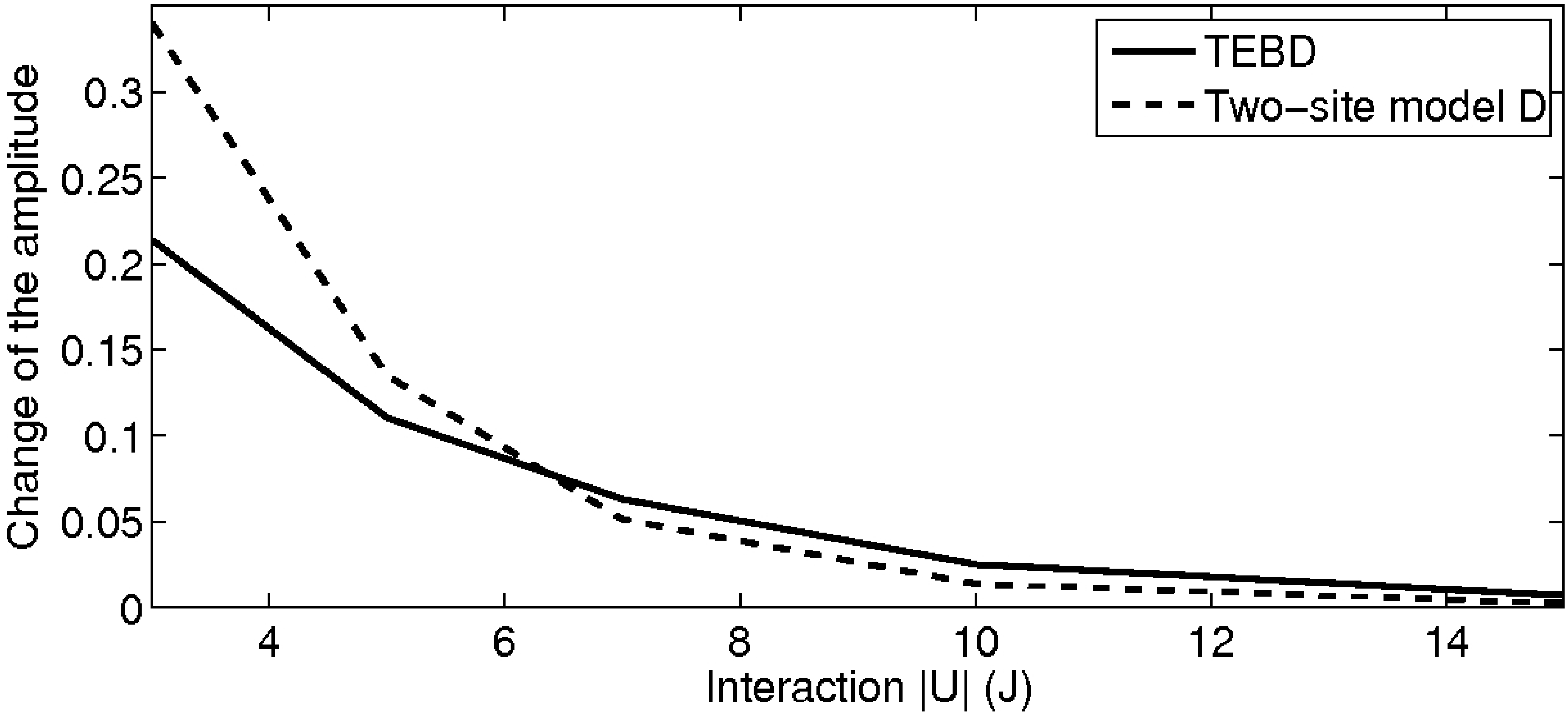
The frequency comparison.



The amplitude comparison.

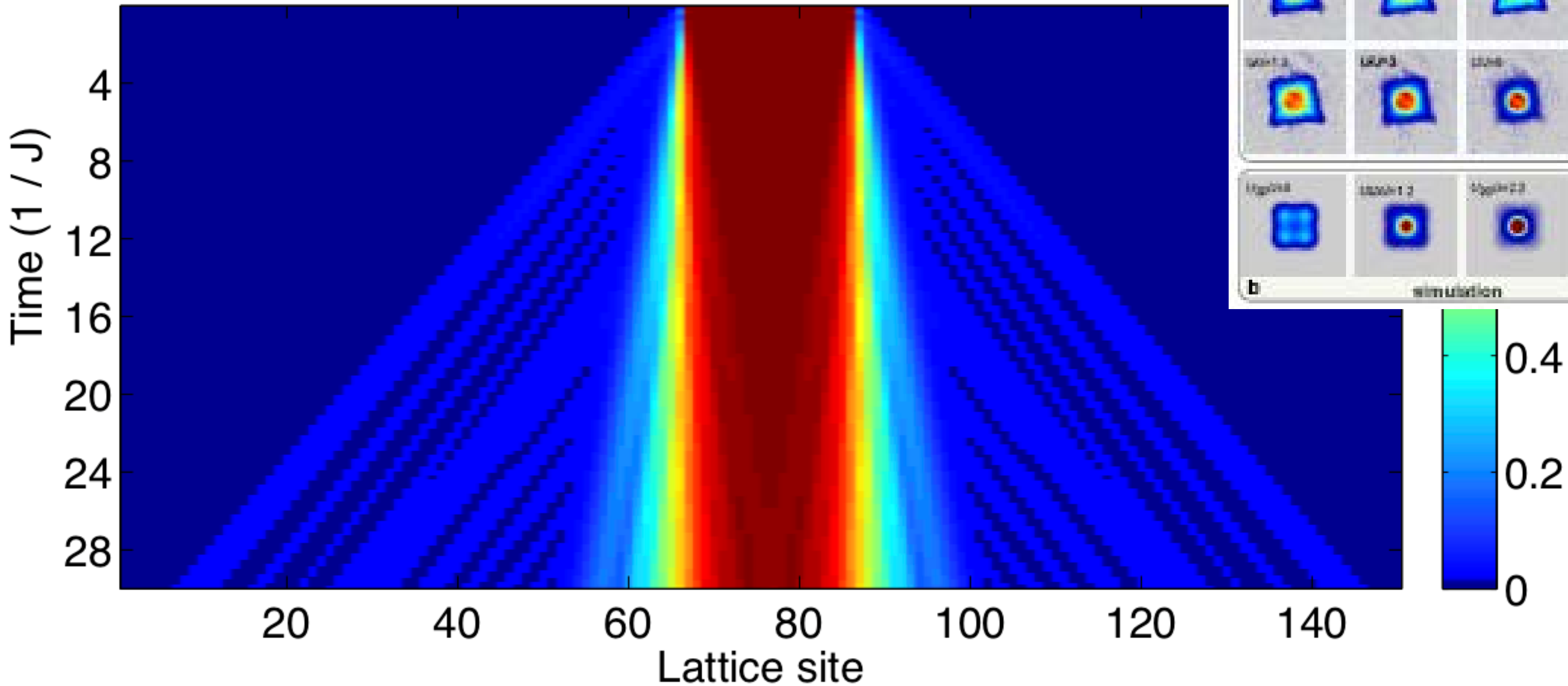


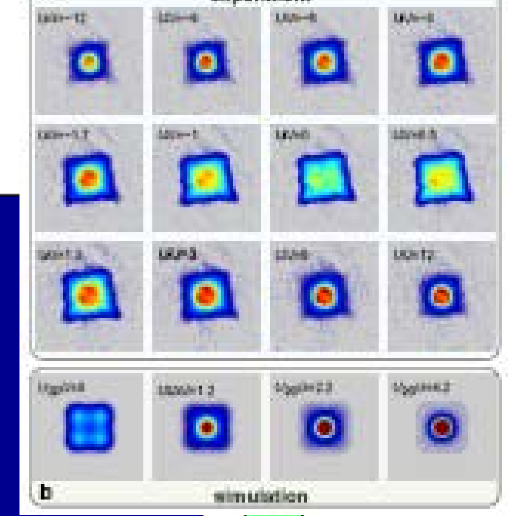
The amplitude decay comparison.



- In the light of the Dimer analysis, let us take another look at the TEBD results.

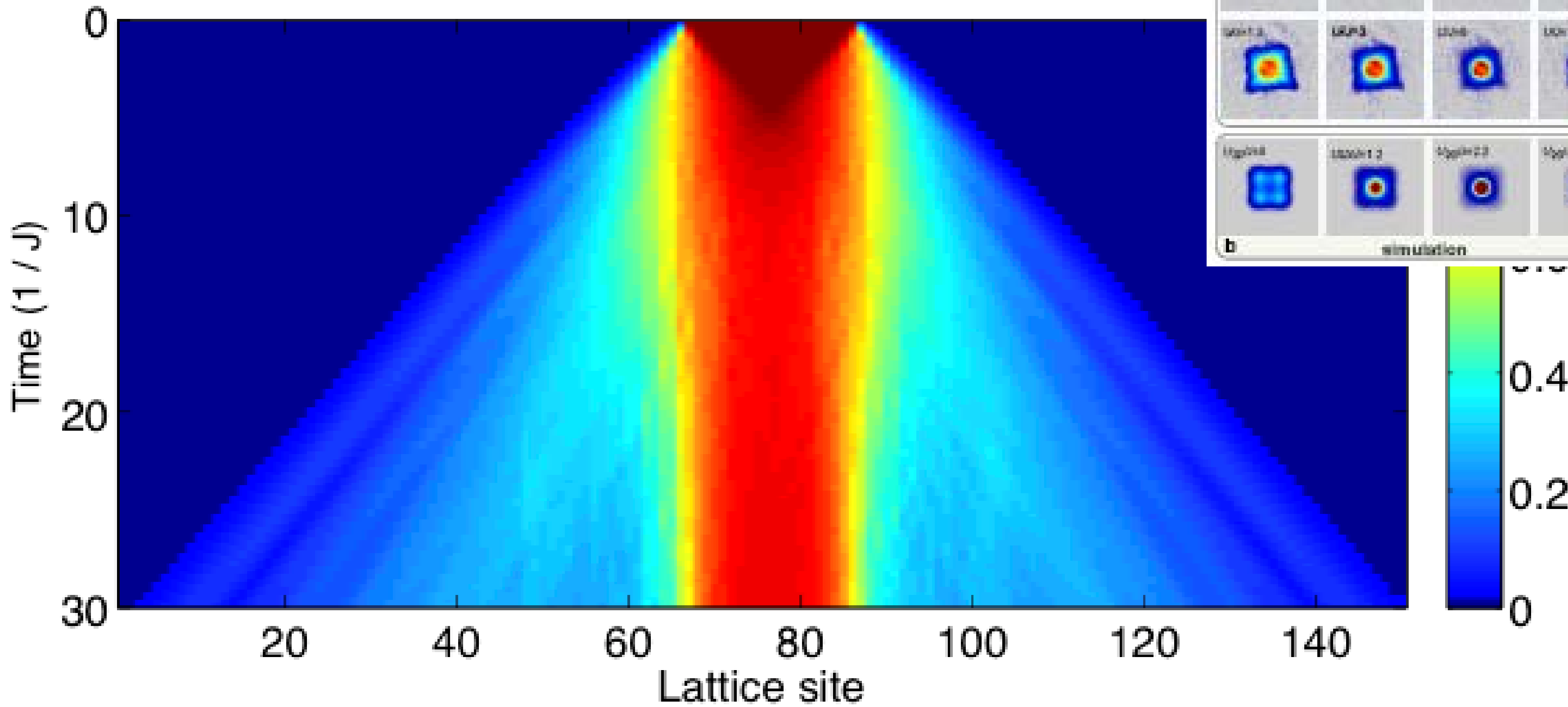
$$\sqrt{n_\uparrow} \text{ for } \frac{|U|}{J} = 10.0$$

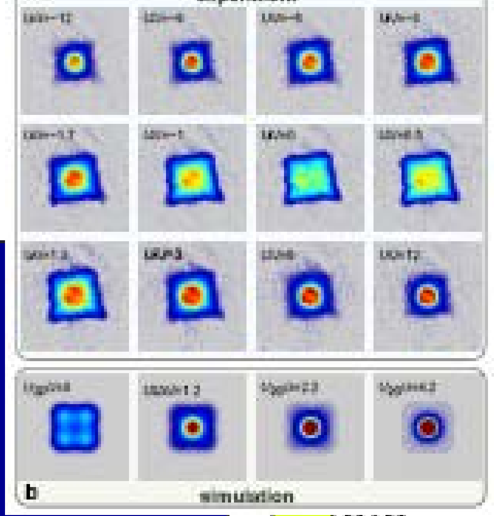




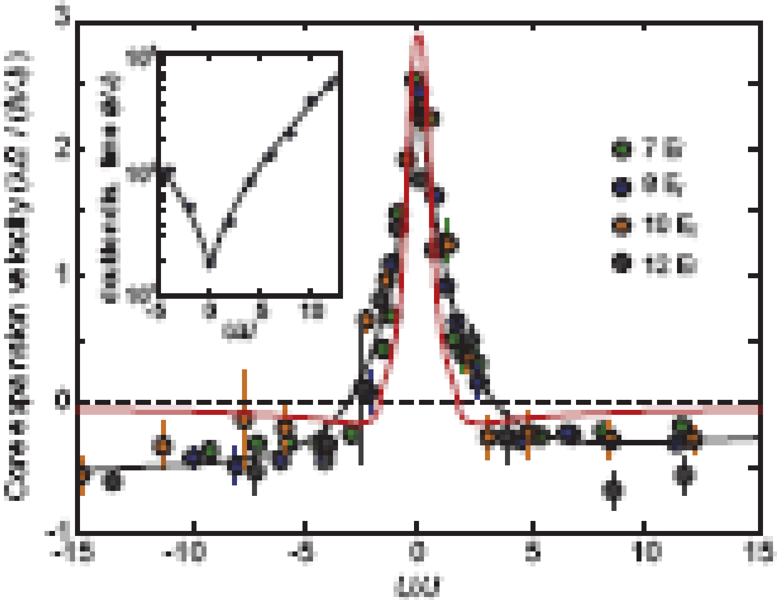
$$n_{Singlet}(t) = \frac{8}{16 + \frac{U^2}{J^2}} \left(1 - \cos\left(\sqrt{U^2 + 16J^2}t\right)\right)$$

$$\sqrt{n_{\uparrow}(1)} \text{ for } \frac{|U|}{J} = 1.0$$





$$n_{Singlet}(t) = \frac{8}{16 + \frac{U^2}{J^2}} \left(1 - \cos\left(\sqrt{U^2 + 16J^2 t}\right)\right)$$



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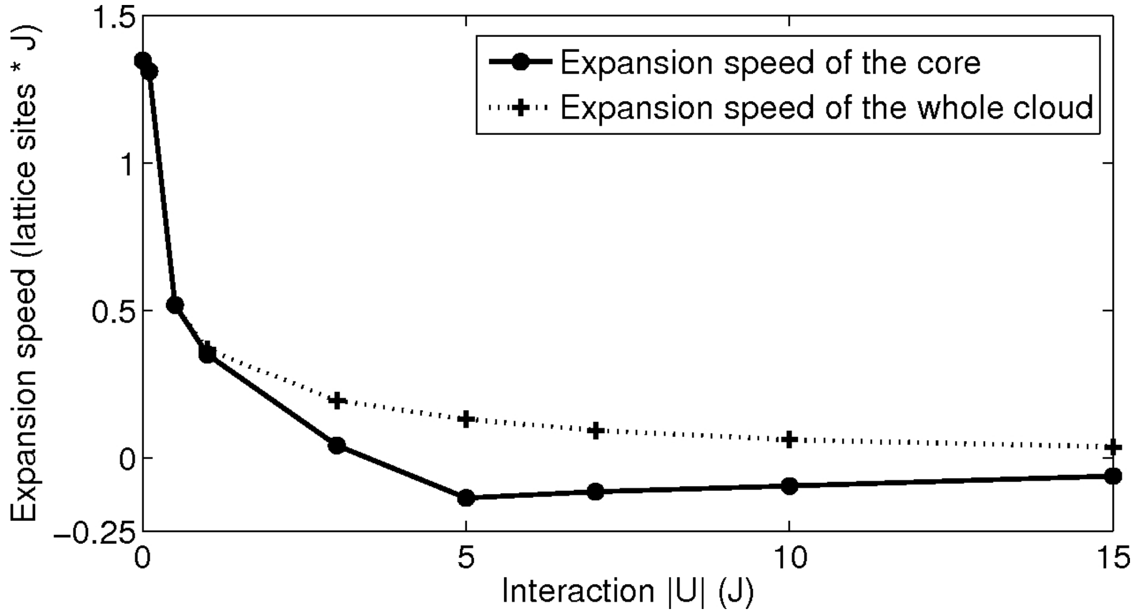
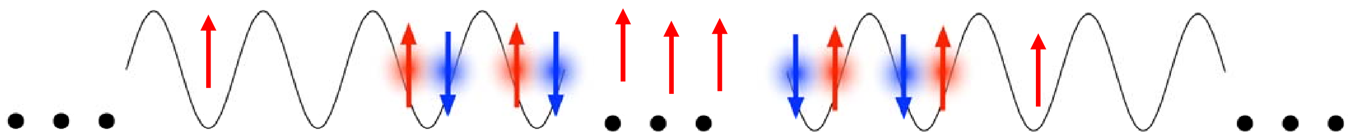
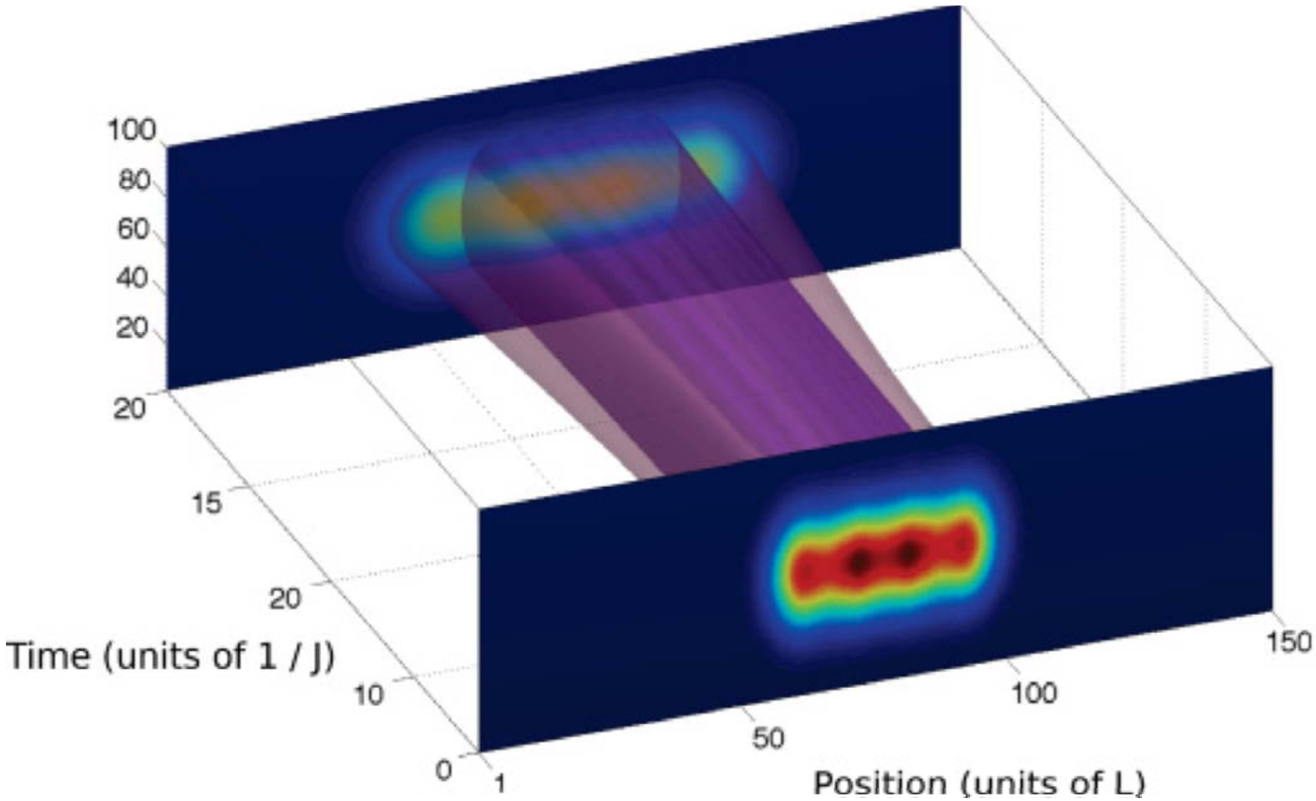


- The FFLO state in 1D-3D crossover (DMFT) (briefly)
- Pairing in ***mixed geometries*** (mean field)
- Expansion of a band insulator in a lattice (t-DMRG)
- **Expansion of an FFLO state (t-DMRG)**
- Dynamics of a polaron in 1D (t-DMRG)

Expansion of an FFLO state in a 1D lattice

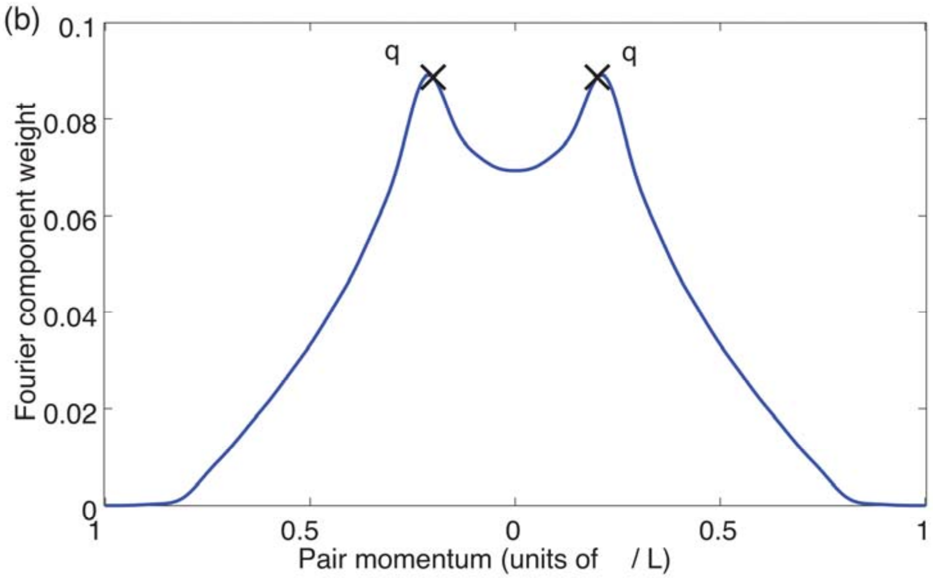


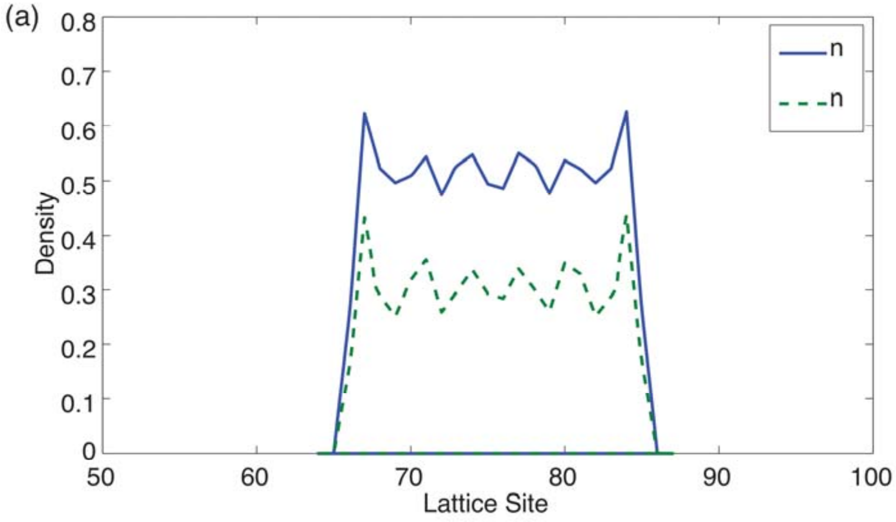
Inspired by the (continuum) 1D experiment: Liao et al., Nature 467, 567 (2010)



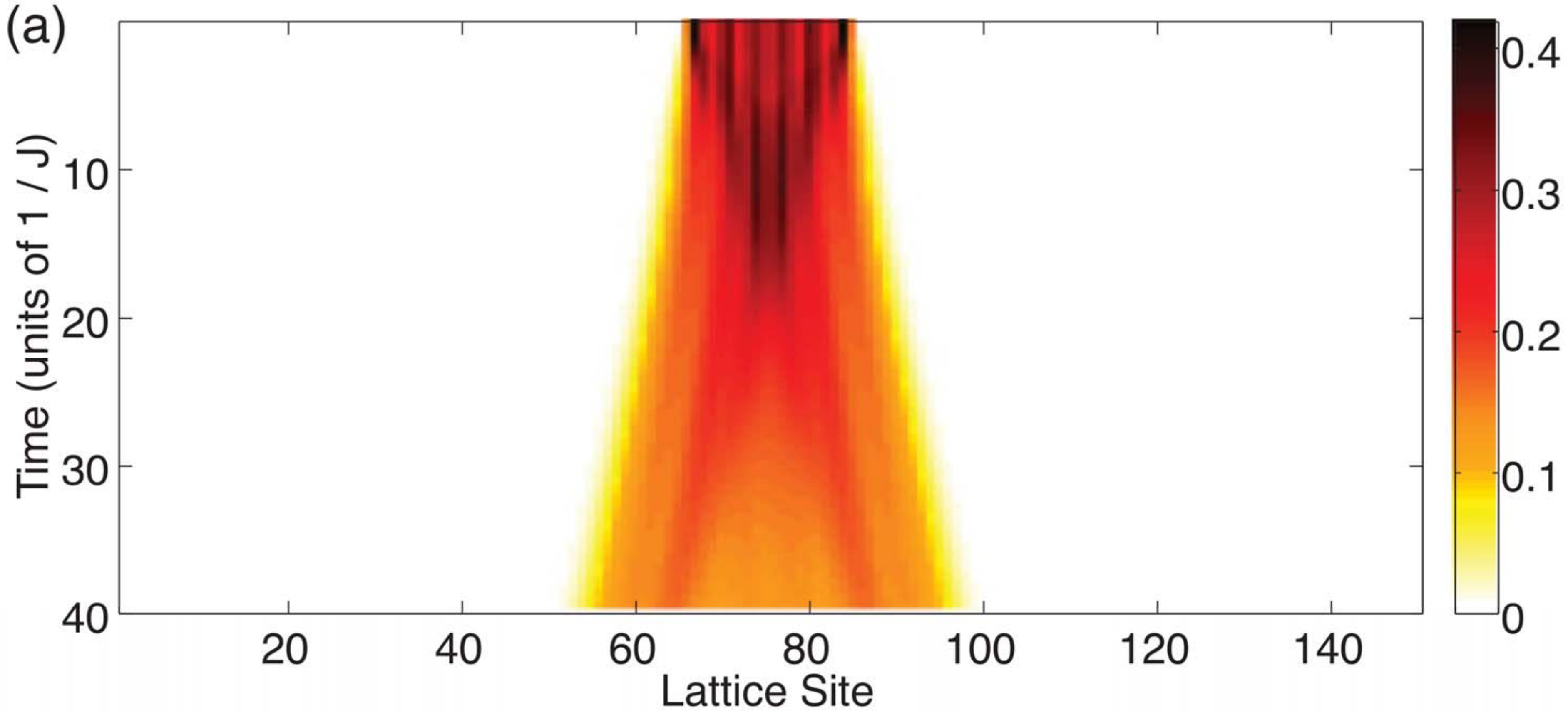
J. Kajala, F. Massel, PT, PRA 106, 206401(R) (2011)

FFLO in 1D lattice

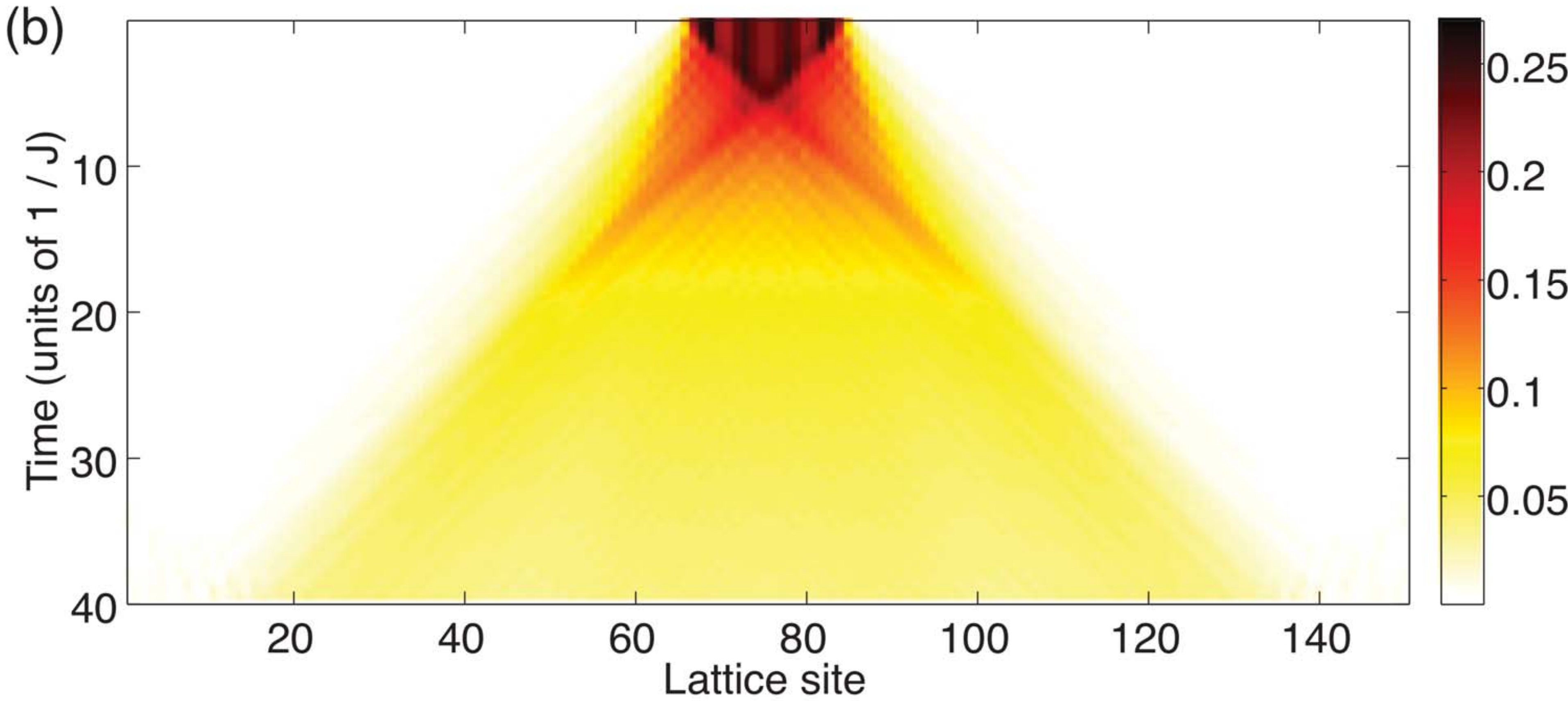




Feiguing and Heidrich-Meisner 2007, Batrouni et al. 2008, Rizzi et al. 2008



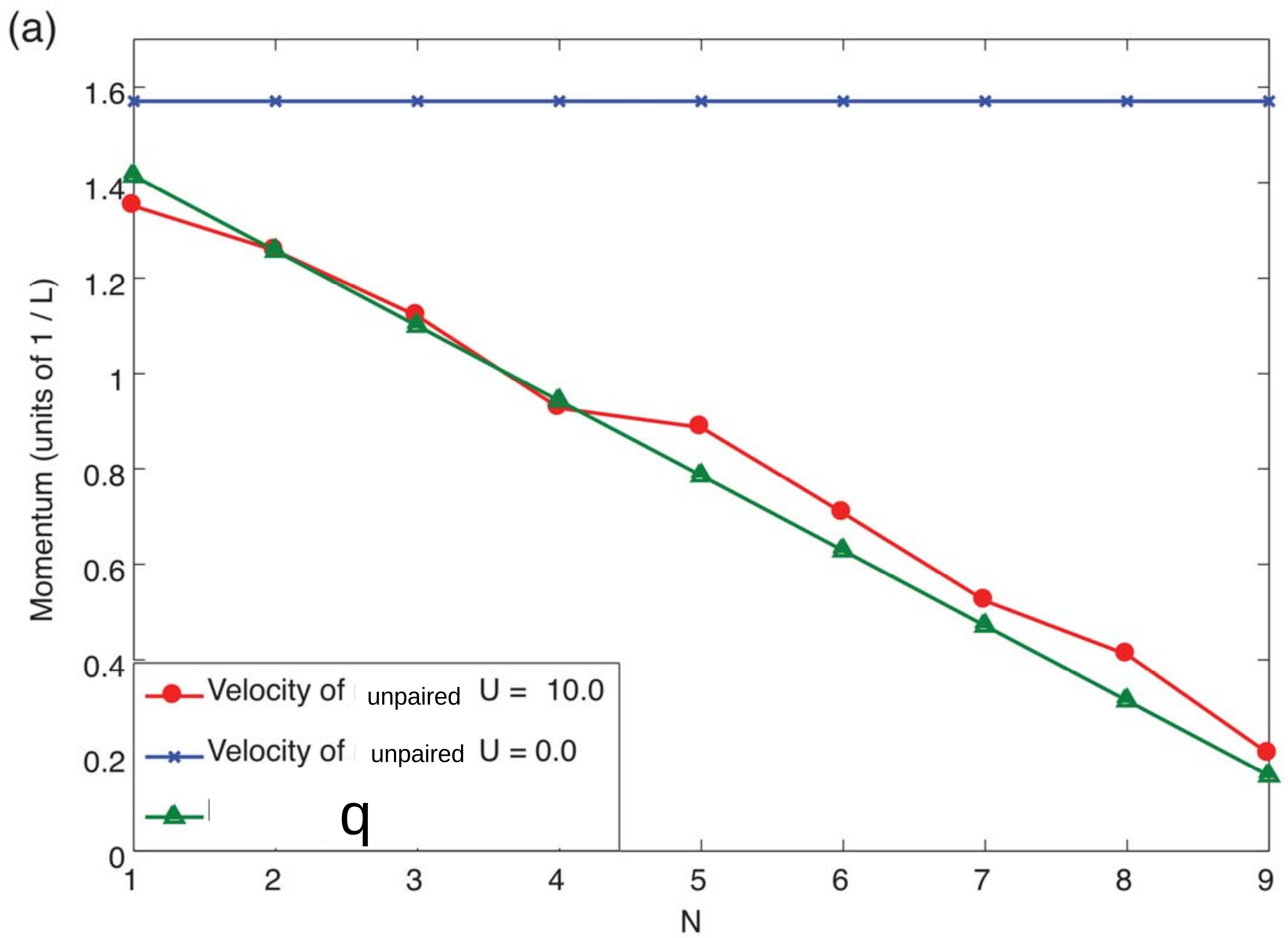
The doublon density as a function of time $n_{\uparrow\downarrow}(t)$. Interaction $\frac{|U|}{J} = 10.0$.



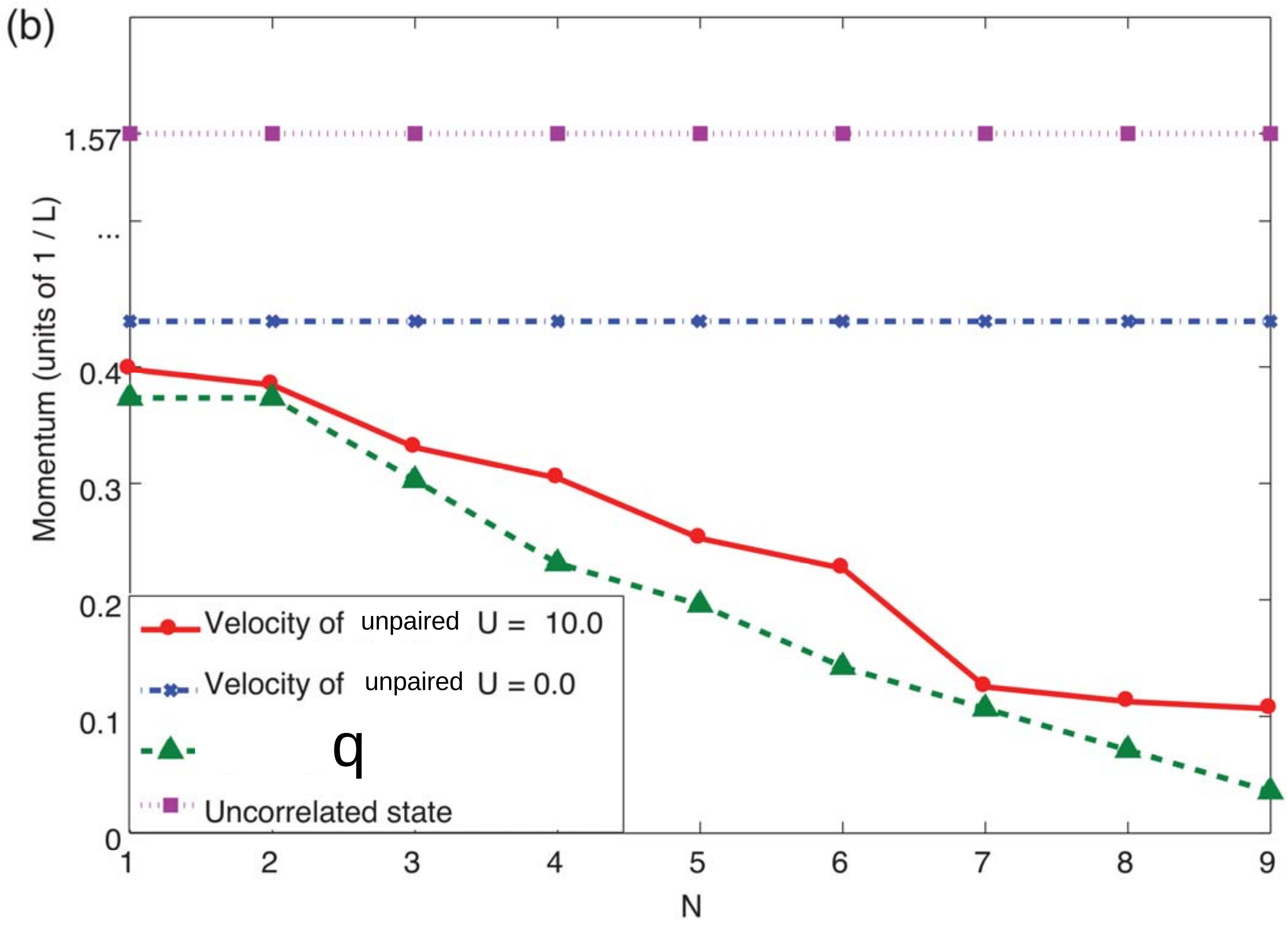
The unpaired particle density as a function of time $n_{\uparrow}(t) - n_{\uparrow\downarrow}(t)$. Interaction $\frac{|U|}{J} = 10.0$.

Consistent with Bethe ansatz in the large U limit

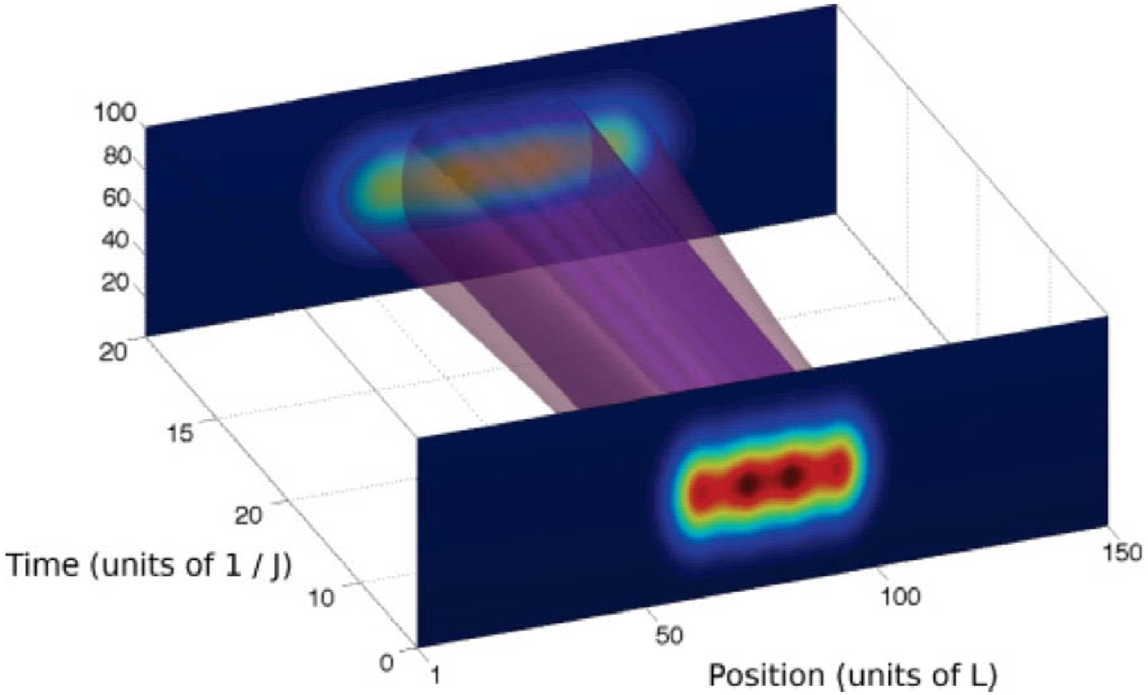
- $v_{un}^{max} = 2J\sin(k_{un}^{max})$
- $v_{\uparrow\downarrow}^{max} = 2J\sin(k_{\uparrow\downarrow}^{max})$
- We find that $k_{\uparrow\downarrow}^{max} = k_{F\downarrow}$
- and $k_{un}^{max} = q$.
- Therefore, by measuring the maximum expansion velocity of the unpaired particles, one can detect the *FFLO* momentum. The wavefront corresponding to the maximum velocity is the cloud edge.
- $q = \arcsin(\frac{v_{max}}{2J})$.


(a)
Momentum (units of 1 / L)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8
9
N
Velocity of unpaired U = 10.0
Velocity of unpaired U = 0.0
q

In trap, + comparison with an uncorrelated (non-FFLO) state



Summary: measuring the expansion velocity of the edge (majority particles) gives the FFLO q-vector!



J. Kajala, F. Massel, PT, PRA 106, 206401(R) (2011)

c.f. C.J. Bolech, F. Heidrich-Meisner, S. Langer, I.P. McCulloch, G. Orso, M. Rigol, PRL 109, 110602 (2012): FFLO correlations lost during the expansion; however, as we point out the initial FFLO q is imprinted to the fastest majority particles that travel at the edge of the cloud.

- The FFLO state in 1D-3D crossover (DMFT) (briefly)
- Pairing in ***mixed geometries*** (mean field)
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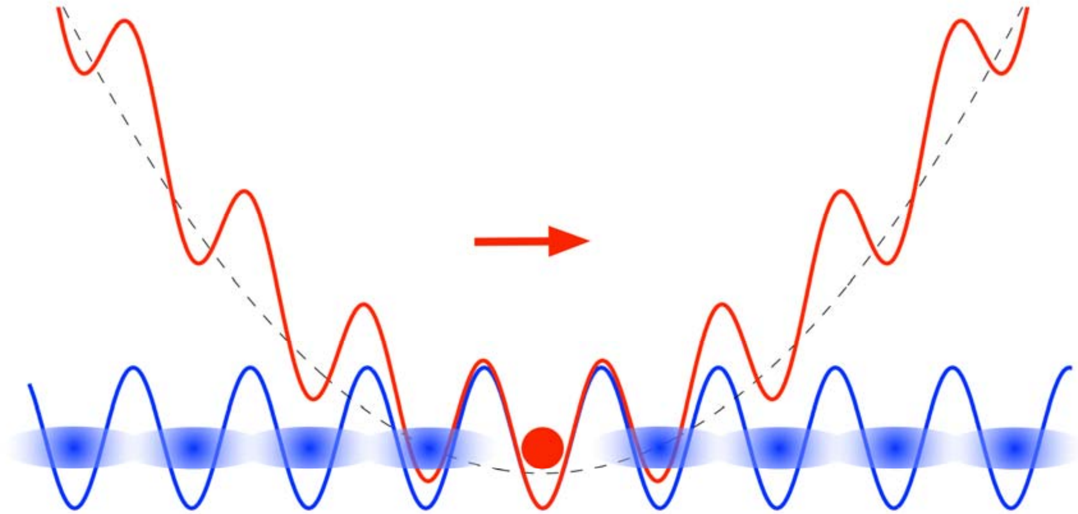
- Expansion of an FFLO state (t-DMRG)
- **Dynamics of a polaron in 1D (t-DMRG)**

Dynamics of an impurity in a one-dimensional lattice



Polarons in 2D/3D Grimm, Zwierlein, Köhl, Salomon, Zwerger, Chevy, Lobo, Bruun etc.

F. Massel, A. Kantian, A.D. Daley, T. Giamarchi, PT, submitted to New J. Phys., arXiv:1210.4270

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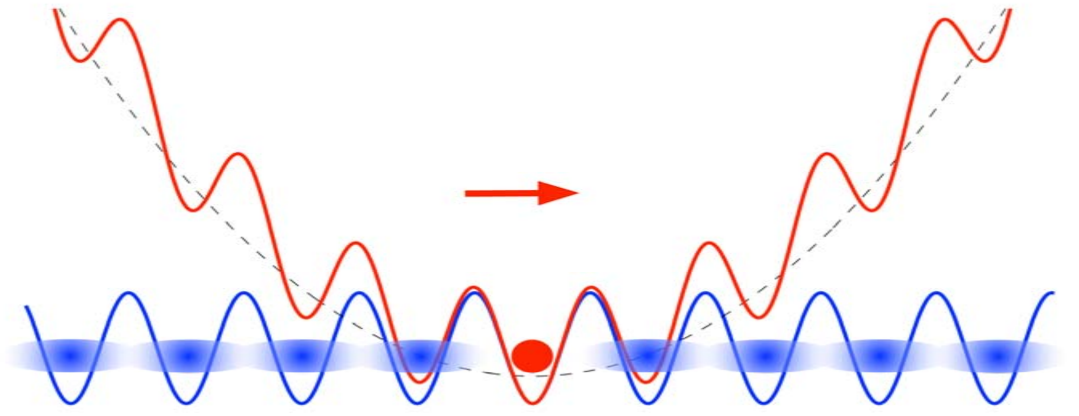
c.f. 1D impurity dynamics experiments for bosons: Inguscio group, Bloch group

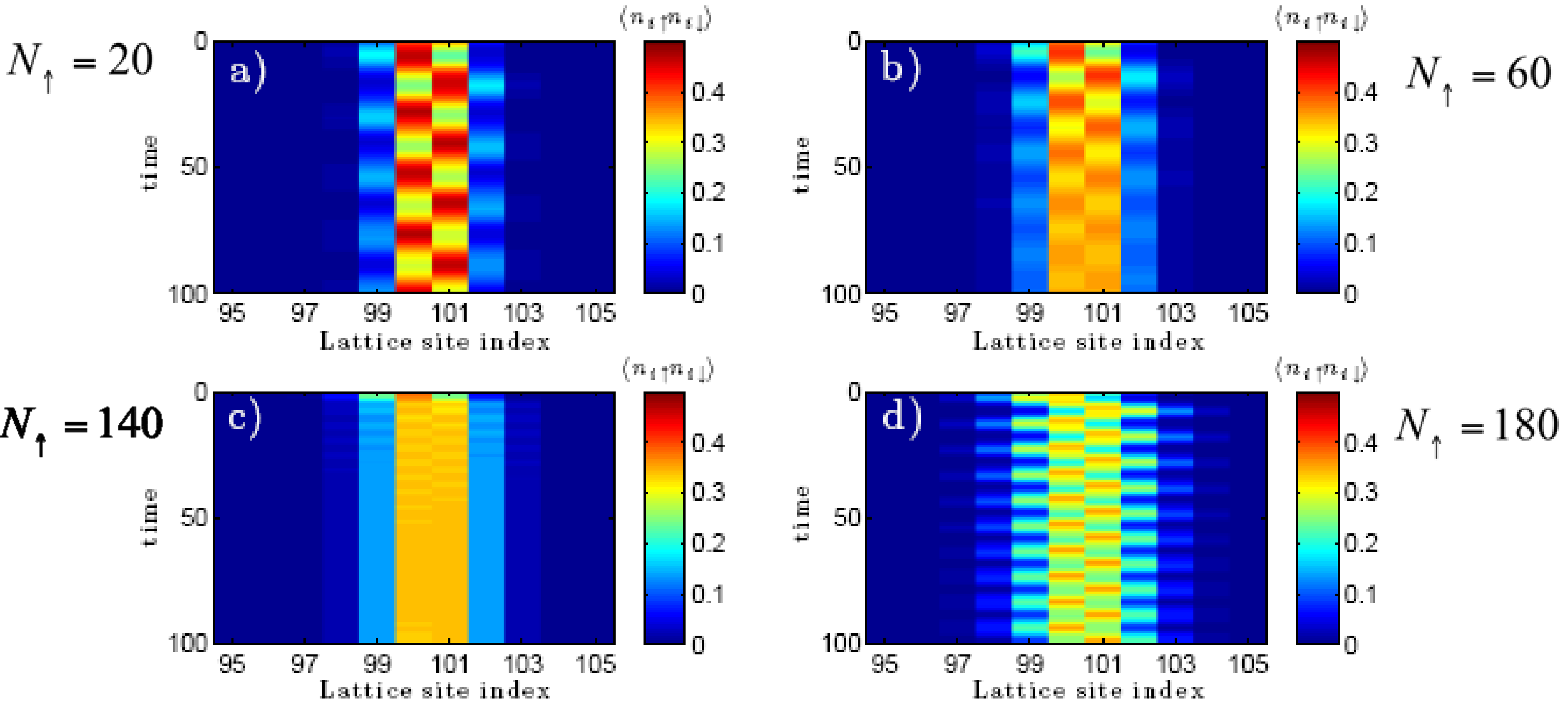
Kick to the down particle: k = 0.1 π



$$H = -J \sum_{i\,\sigma} c^{\dagger}_{i\,\sigma} c_{i+1\,\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} + V \sum_{i} n_{i\downarrow} \left(i - \frac{L-1}{2} \right)$$

Bath of up-fermions, one down-particle,
lattice for both, trap only for the impurity.
Kick of k to the impurity:
oscillations observed (t-DMRG)

Doublon dynamics: strong interactions $\langle n_{i\uparrow} n_{i\downarrow} \rangle (t)$



$N_\uparrow = 100$ half filling

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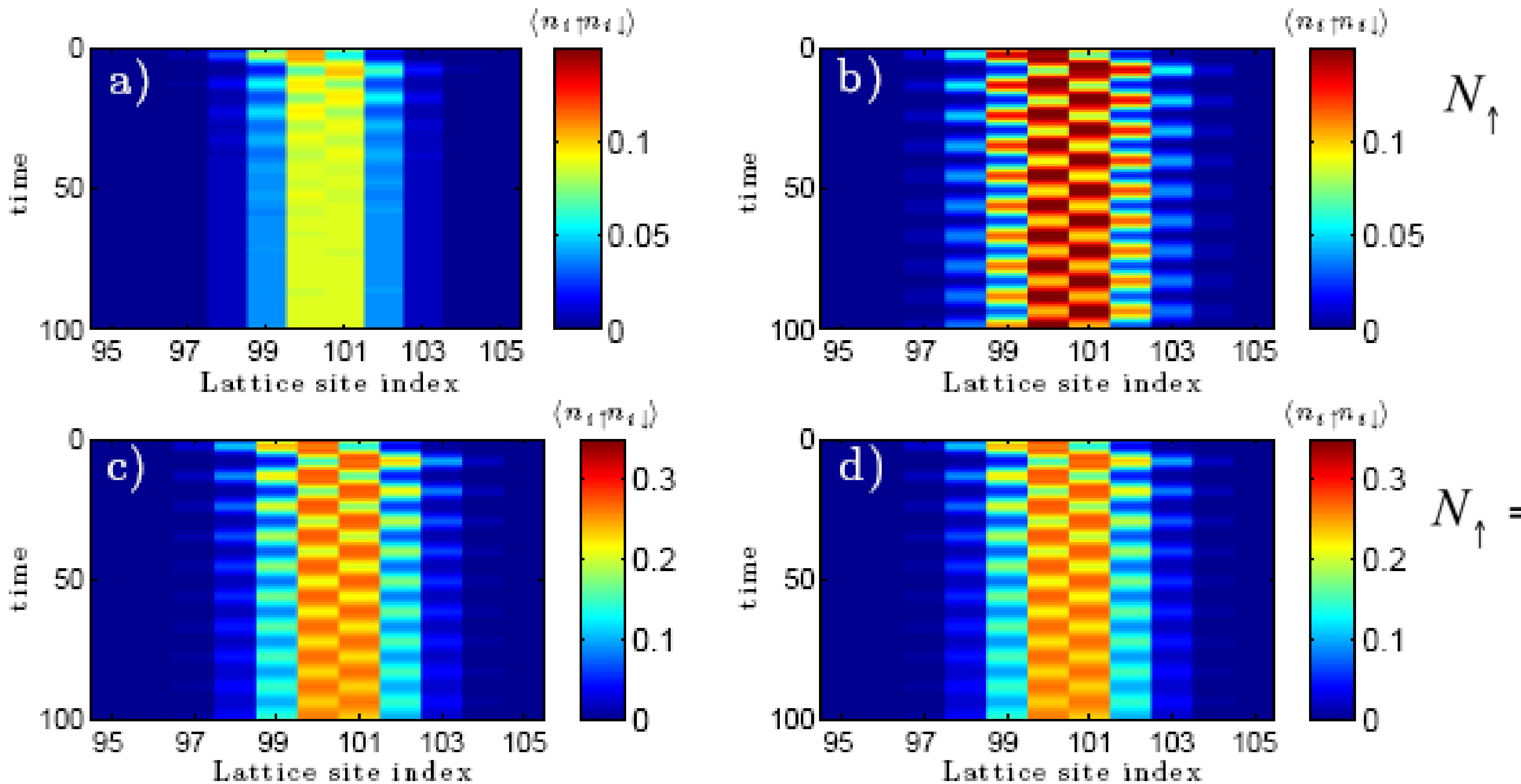
Doublon dynamics: weak interactions $\langle n_{i\uparrow} n_{i\downarrow} \rangle (t)$

$N_\uparrow = 20$

$N_\uparrow = 60$



$N_\uparrow = 140$

$N_\uparrow = 180$

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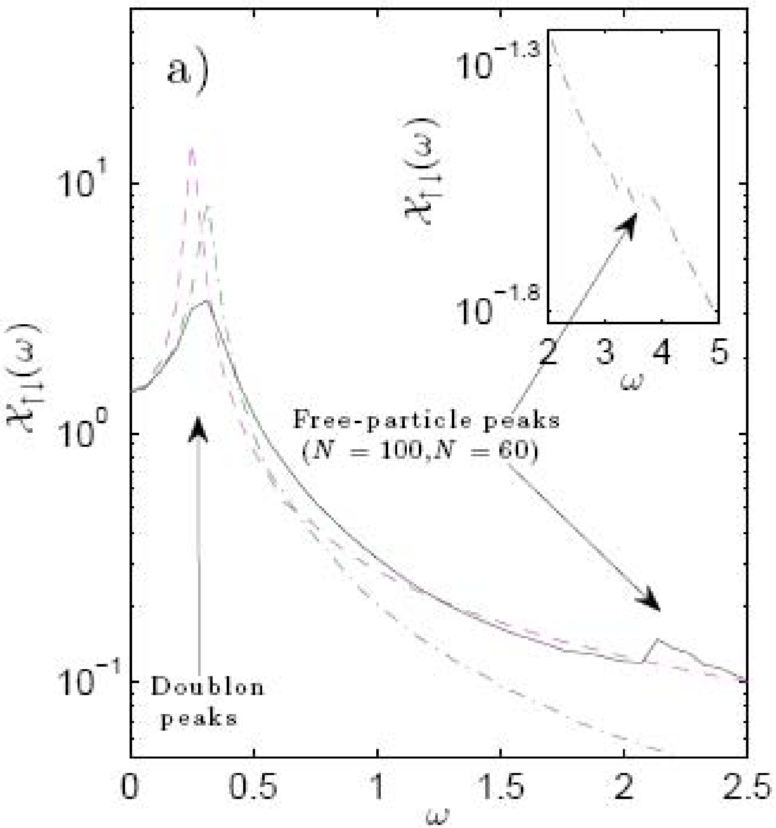
$N_\uparrow = 100$ half filling

Fourier transform (doublon center of mass motion)

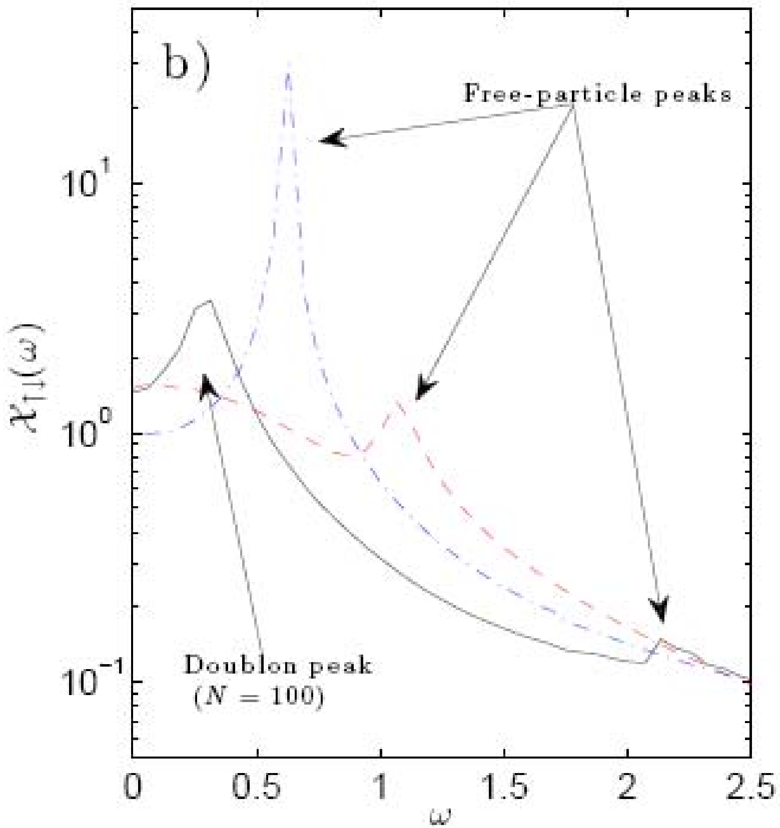
strong interactions U/J=10

Below half-filling

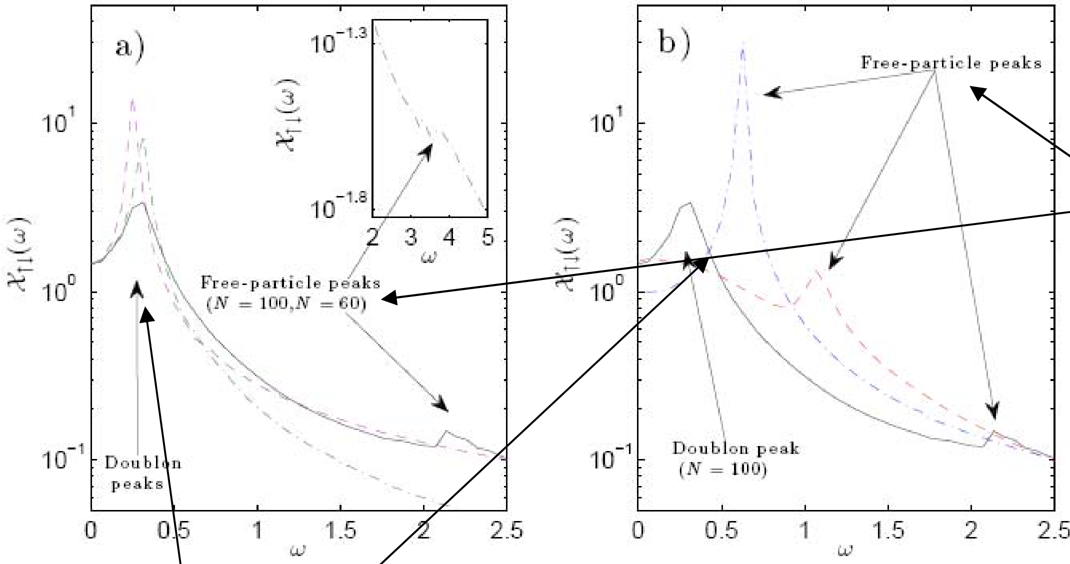
Above half-filling



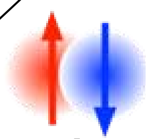
$N_{\uparrow} = 20, 60, 100$



$N_{\uparrow} = 100, 140, 180$

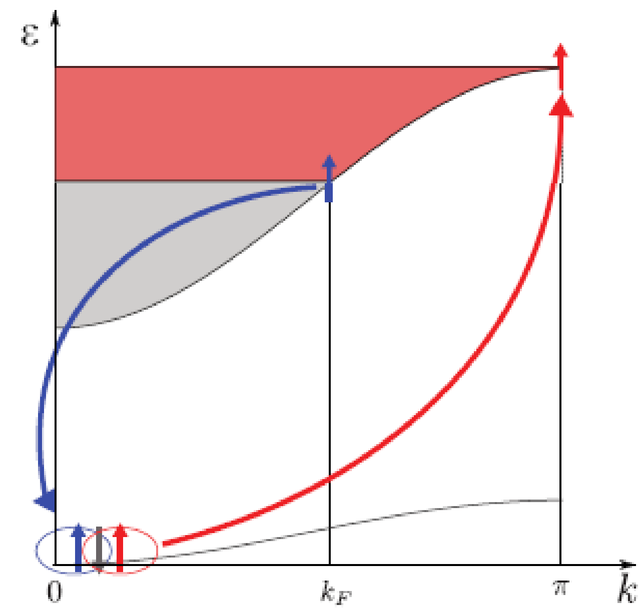


Dynamics of free impurity + impurity mediated bath scattering



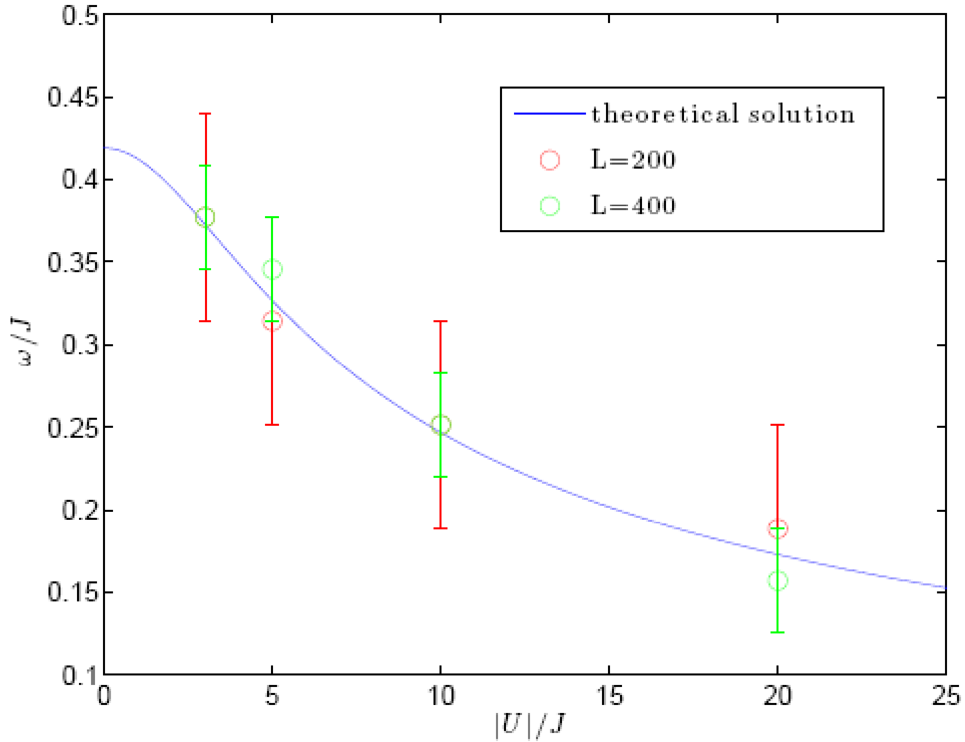
Bound on-site pair dynamics (derived from Bethe ansatz)

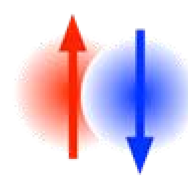
$$m_{\text{doublon}} = \left[\left(\frac{\partial^2 E}{\partial \kappa^2}\right)\right]^{-1}_{\kappa \to 0} = \frac{1}{J}\sqrt{1 + \frac{U^2}{16J^2}} \xrightarrow{|U|/J \gg 1} \frac{|U|}{4J^2}$$



$$\Delta E = \omega_{hi} + 2J\left[(1 - \cos k_p) - (1 - \cos k_{F\uparrow})\right]$$

Independent of U!





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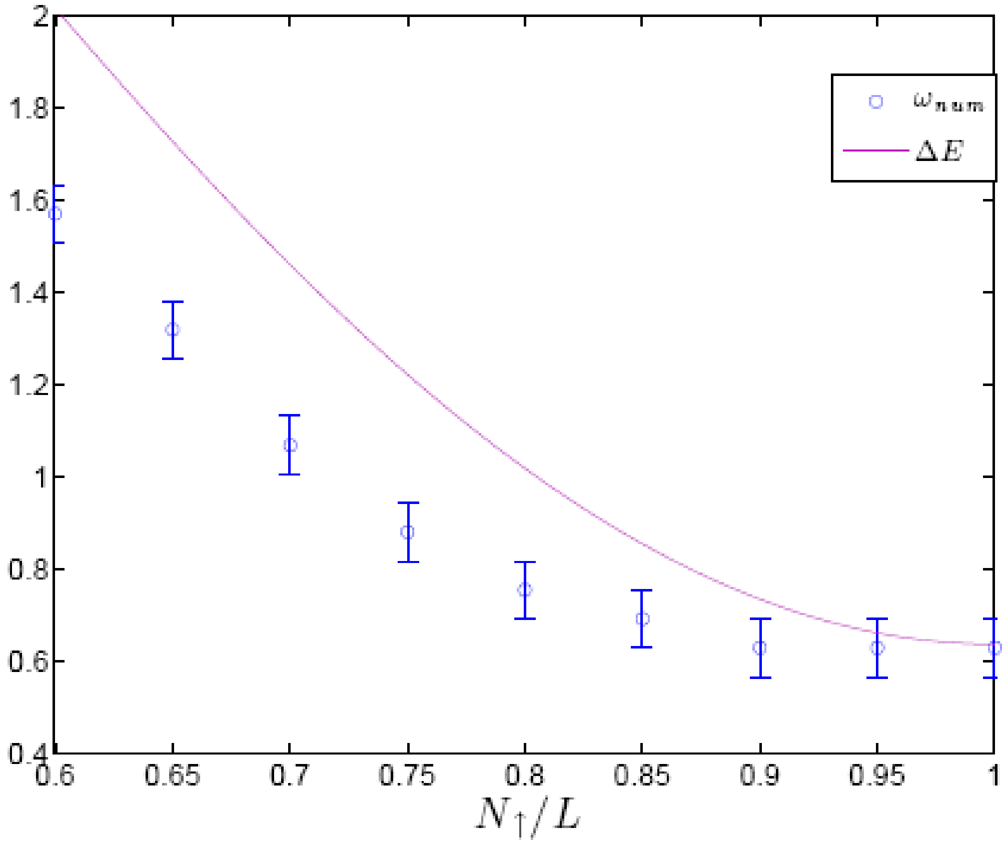
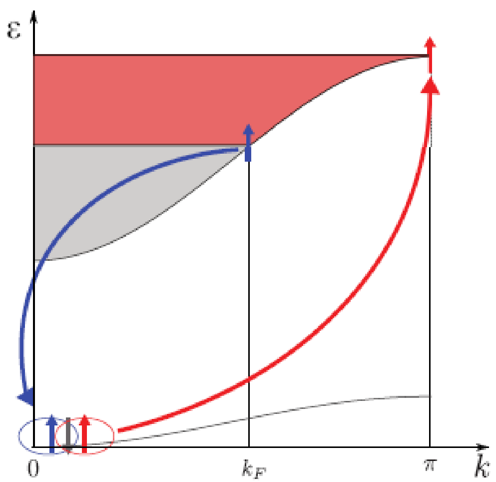
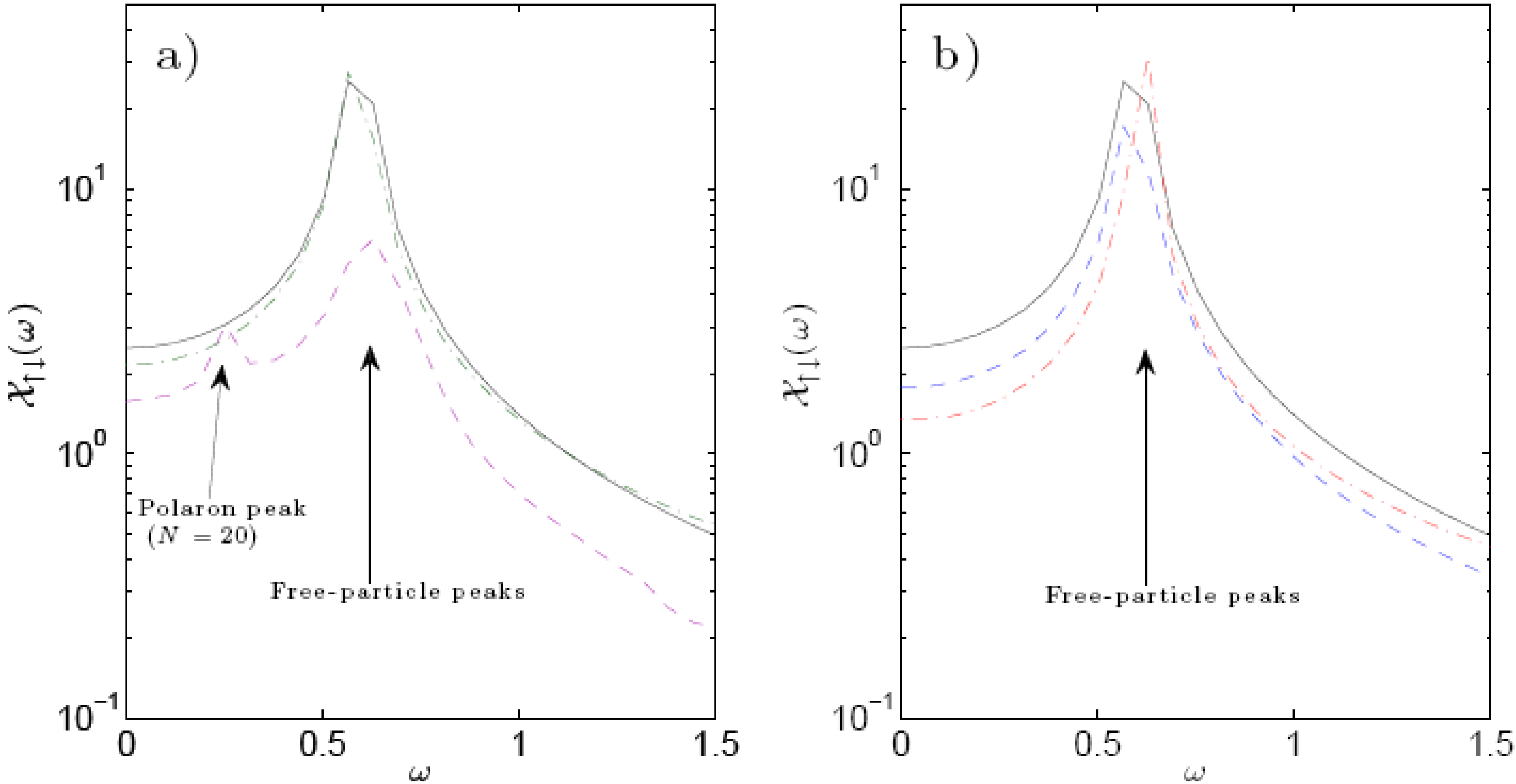




Offset: non-uniform $k_{F\uparrow}(x)$
due to Friedel oscillations

$$\Delta E = \omega_{hi} + 2J\left[(1-\cos k_p) - (1-\cos k_{F\uparrow})\right]$$

Fourier transform (doublon center of mass motion),
weak interactions U/J=1



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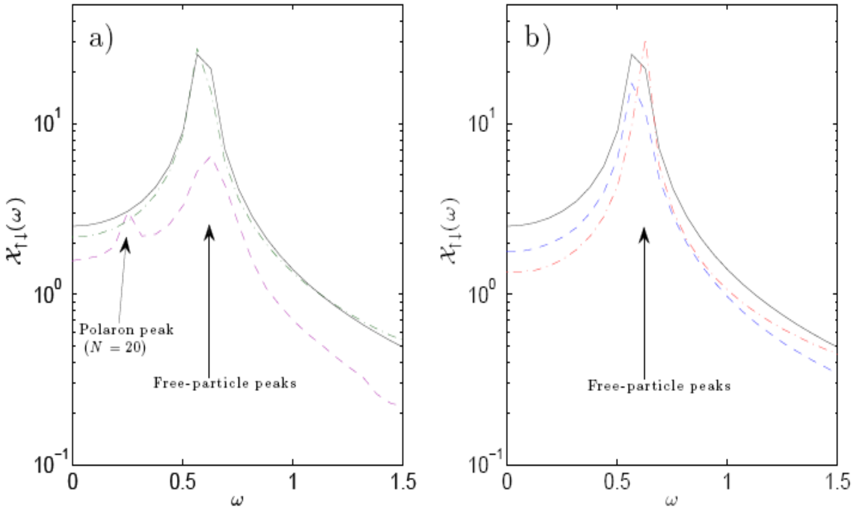
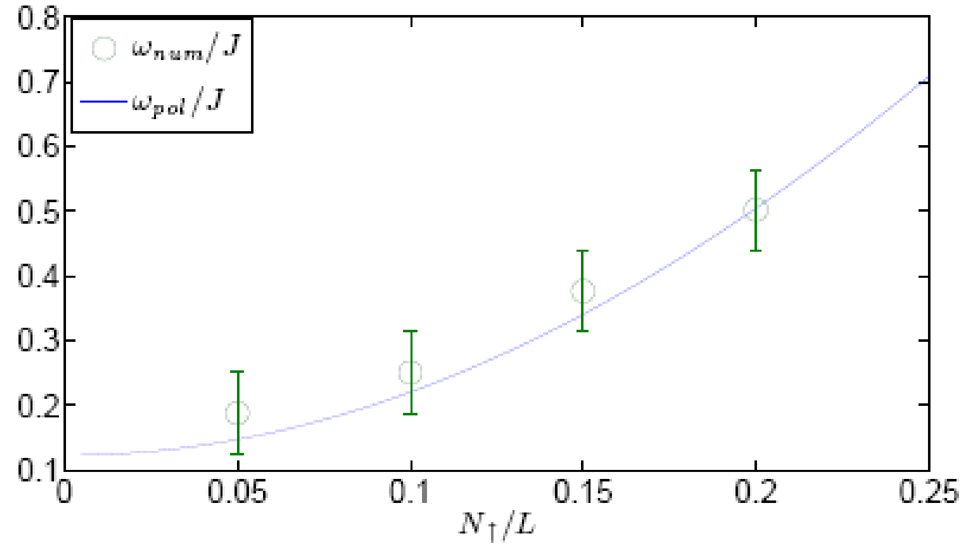




Polaron peak:

$$\omega_{pol} = E_\uparrow + E_\downarrow - E_{\uparrow\downarrow} = -2J\left[1 + \cos(k_{F\uparrow})\right] + \sqrt{U^2 + 16J^2}$$

$$|\Psi\rangle = \sqrt{Z}\, c^\dagger_{0\downarrow} |FS\rangle_\uparrow |0\rangle_\downarrow + \sum_{k > k_F^\uparrow,\, q < k_F^\uparrow} \phi_{k,q}\, c^\dagger_{k\uparrow} c_{q\uparrow} c^\dagger_{q-k\downarrow} |FS\rangle_\uparrow |0\rangle_\downarrow$$

Summary

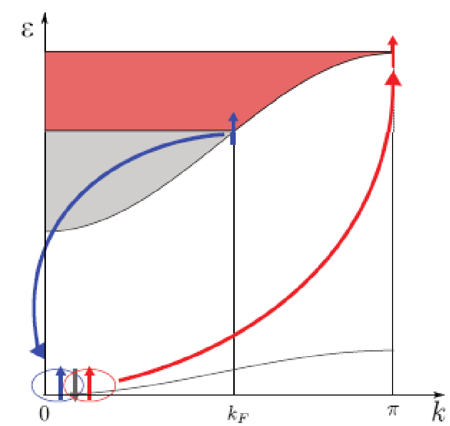
Same phenomena seen also for strongly repulsive bosonic bath, in the weak interaction case

$\lvert U \rvert$ range	Bath population	Dynamics regime
Strong interaction	Large $N_\uparrow$	Free particle
Strong interaction	Intermediate $N_\uparrow$	Bound pair + polaron internal dynamics
Strong interaction	Small $N_\uparrow$	Bound pair
Weak interaction	Large $N_\uparrow$	Free particle
Weak interaction	Intermediate & small $N_\uparrow$	Free particle + polaron

Pair –free particle scattering;
Pair breaking

$$\omega_{pol} = E_\uparrow + E_\downarrow - E_{\uparrow\downarrow} = -2J\left[1+\cos(k_{F\uparrow})\right] + \sqrt{U^2+16J^2}$$

$$\lvert\Psi\rangle = \sqrt{Z}\, c^+_{0\downarrow}\lvert FS\rangle_\uparrow \lvert 0\rangle_\downarrow + \phi_{k_{F\uparrow},0}\, c^+_{k_{F\uparrow}\uparrow} c_{0\uparrow} c^+_{0\downarrow}\lvert FS\rangle_\uparrow \lvert 0\rangle_\downarrow$$



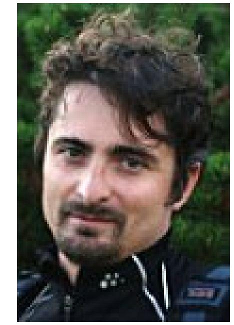
Virtual pair breaking;
Polaron internal dynamics

$$\lvert\Psi\rangle = \sqrt{Z}\lvert FS-1\rangle_\uparrow \lvert Pair\rangle_{\uparrow\downarrow} + \phi_{\pi,k_{F\uparrow}}\, c^+_{\pi\uparrow} c_{k_{F\uparrow}\uparrow}\lvert FS-1\rangle_\uparrow \lvert Pair\rangle_{\uparrow\downarrow}$$

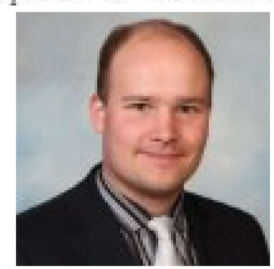
Conclusions

- The FFLO state in 1D-3D crossover (DMFT) (briefly)
  - Intermediate dimension and polarization stabilizes FFLO
- Pairing in ***mixed geometries*** (mean field)
  - New iBP state, topological transitions, stability of exotic superfluidity connected with multiband pairing
- Expansion of a band insulator in a lattice (t-DMRG)
  - Two-site Hubbard physics describes the dynamics well
- Expansion of an FFLO state (t-DMRG)
  - FFLO directly visible in the free particle expansion
- Dynamics of a polaron in 1D (t-DMRG)
  - Bound pair and new types of polaron dynamics

Francesco Massel
(now at University
of Helsinki)



Jussi Kajala
now at
SPINVERSE
Capital & Consulting



Dong-Hee Kim (1.3.2013 assistant professor at GIST, South Korea)

Joel Lehikoinen
(now at his own start-up)

Miikka Heikkinen

Thanks: Academy of Finland

Aalto University
School of Science