

# Three-body collisions for alkali atoms near Feshbach resonances

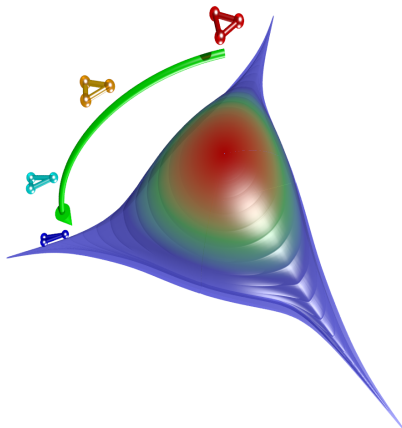
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March 2013



# Outline

- 1 Introduction
  - The Efimov effect
- 2 Universal three-body parameter near broad Feshbach resonances
- 3 Three-body problem with multichannel interactions
  - Hyperfine physics in alkali atoms
  - Efimov resonances in Cs-Li admixture
- 4 Summary

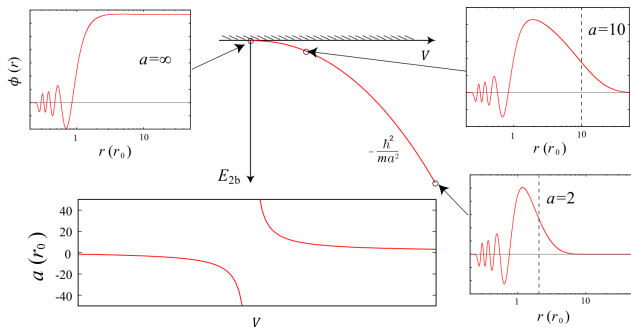
# In this section

- 1 Introduction
  - The Efimov effect
- 2 Universal three-body parameter near broad Feshbach resonances
- 3 Three-body problem with multichannel interactions
  - Hyperfine physics in alkali atoms
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# Few-body problem near unitarity

A quantum criticality  $\rightarrow$  unitarity limit ( $a \rightarrow \infty$ )

scattering length  $a = \lim_{k \rightarrow 0} -\tan \delta(k)/k$



- Halo state:  ${}^6\text{He}$ ,  ${}^{11}\text{Li}$  nuclei,  $\text{He}_3$  molecule
- **Three-body Efimov effect**

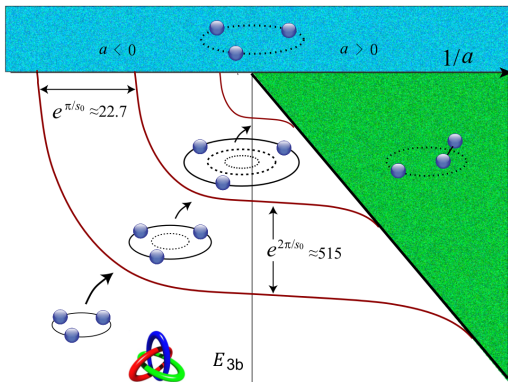
# The Efimov effect: if two do not bind, three would



Vitaly Efimov, 1970

Three particles with *isotropic, short-range* interactions form an *infinite* series of three-body bound states with  $E_n = E_0 e^{-2n\pi/s_0}$  when  $a \rightarrow \infty$  ( $s_0 \approx 1.00624$ ) [1].

## Three-body energy spectrum

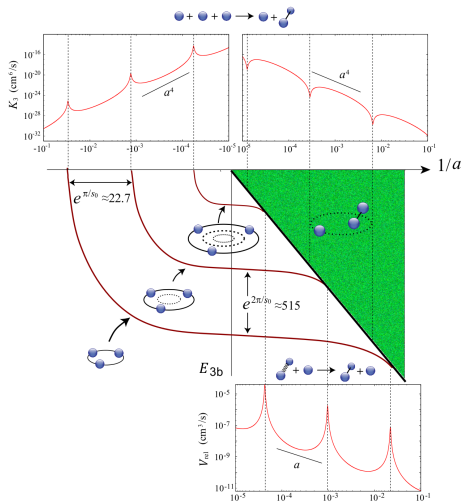
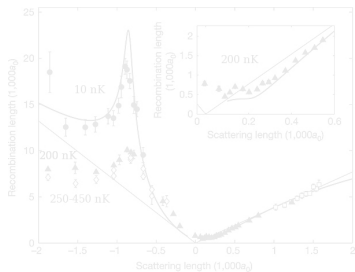


- Borromean: one leaves, all fall apart ( $a < 0$ )
- $\infty$  large state with short-range interactions
- Less states with stronger interactions ( $a > 0$ )

[1] Efimov, Phys. Lett. B 33, 563 (1970).

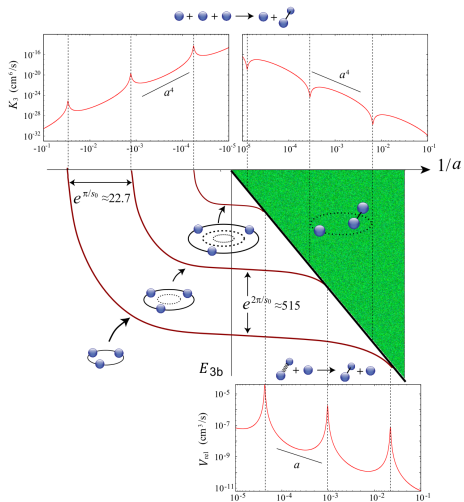
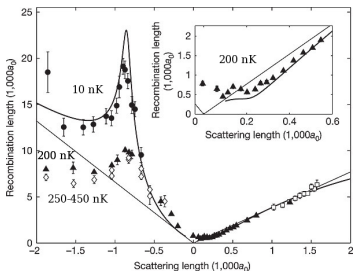
## Efimov states: manifestations

Ultracold atoms ✓

First experimental observation  
in Cs atomsKraemer, *et al.* Nature (2006)

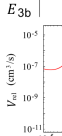
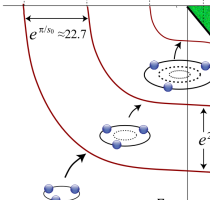
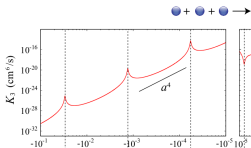
## Efimov states: manifestations

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# Efimov states: manifestations

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nature physics

### Observation of an Efimov-like trimer resonance in ultracold atom-dimer scattering

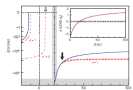
S. Knopp<sup>1</sup>, F. Feriaini<sup>1</sup>, M. Mark<sup>1</sup>, M. Berninger<sup>1</sup>, H. Schöberl<sup>1</sup>, H.-C. Nölger<sup>1</sup> and R. Grimm<sup>1,2</sup>

The field of few-body physics has originally been motivated by understanding nuclear matter, but in the past few years advanced gases with tunable interactions have emerged as ideal systems to experimentally explore few-body quantum phenomena. From the energy states involved in one-body optical lattices to the complex, few-body physics of an atomic universal scattering length near magnetic Feshbach resonances, so-called Efimov states represent a paradigm for universal quantum states in the three-body sector<sup>1</sup>. After decades of theoretical work, a full experimental signature of such a weakly bound trimer state was recently found under conditions where a weakly bound dimer state is absent<sup>2</sup>. Here, we report on a trimer state in the opposite regime, where such a dimer state exists. The trimer state manifests itself in a resonant enhancement of isotropic collisions in a mixture of atoms and dimers. Our observation is directly related to an atom-dimer resonance as predicted by Efimov<sup>3</sup>, but occurs in the experimentally challenging regime where the trimer component resonantly couples beyond the scattering length.

Three-body states of a few particles are few-body systems. Since, for the general understanding of a quantum state of three interacting particles is a nontrivially difficult task. For constant two-body interactions, however, the energy spectrum follows the well-known rules, as manifested in Efimov's conjecture describing a series of critical states<sup>4</sup>. Systems of nuclear physics<sup>5</sup> and molecular physics<sup>6,7</sup> were considered as candidates for Efimov states, but only recently ultracold atomic gases have opened up the possibility to probe and explore the required interaction conditions in a controlled way<sup>8,9</sup>. In view of these new developments, a particularly important question has been how the trimer component is connected to near-atom-dimer systems existing in the real world.

A universal three-body system of identical bosons can be fully characterized by two parameters, the two-body scattering length  $a$  and an atom-dimer scattering length, the latter results from short range physics<sup>10</sup>. In principle, knowledge on one Efimov trimer state, for example its binding energy for a given value of  $a$ , is sufficient to determine the three-body parameter and predict the complete spectrum. A test of universality in a real three-body system is possible, when at least two different types of information on the trimer spectrum become experimentally available. For the caesium system, information was obtained by measuring these resonances<sup>11</sup>. The observation of a trimeric resonance marked the particular value of the scattering length where a trimer state of bosonic character reaches the threshold for dissociation into three-body states. Observations at positive scattering lengths revealed a dimer resonance, but an interpretation in terms of universal

LETTERS  
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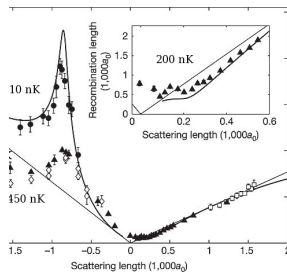
**Figure 1** Three-body spectrum of caesium. The energies of the most relevant resonances for an isotropic scattering length of the magnetic field  $B$ . The red dashed line shows the Efimov-like trimer state, with the scattering length  $a$  and the blue dashed line the scattering length of the scattering length of  $1/2$ , respectively. For the trimer state, the energy dependence is approximately linear. The green dashed line represents a bound  $(1,0,0)$  state. The purple dashed line represents an Efimov state with three atom-trimer states. The observation of an Efimov state with an atom-dimer threshold (blue arrow) leads to a resonance in atom-dimer scattering. The energy dependence of the trimer state in the lowest spin state, indicated by the red spin quantum number  $F=1$  and the scattering  $\mu=1$ . The red dashed line shows the scattering length as a function of the magnetic field  $B$ . The grey area represents the non-universal regime, where for  $|a| \ll a_0 \ll 1000 a_0$ ,  $a_0 = 3.67 \times 10^{-6}$  m.

approximation is questionable because of anisotropy concerning the sign of the scattering<sup>12</sup>.

Here, we follow a new experimental approach and show that an atom-dimer resonance provides experimental access to the situation where a weakly bound trimer state couples to the threshold for dissociation into a dimer and an atom. The observation that we observe is a resonance in atom-dimer scattering, which exhibits itself in resonantly enhanced isotropic decay. The trimeric location provides an unambiguous piece of information that complements the previous results on caesium and facilitates a comparison with universal predictions concerning the spectrum of trimer states.

For caesium atoms in the lowest internal state (hyperfine and Zeeman splitting quantum numbers  $F=1$  and  $m_F=0$ ), the wave scattering length  $a$  shows a pronounced dependence on the magnetic field  $B$  (see the blue field region below 400 G and Fig. 1a, inset). Over a wide range,  $|a|$  is very large and exceeds the

## experimental observation of atoms mer, *et al.* Nature (2006)



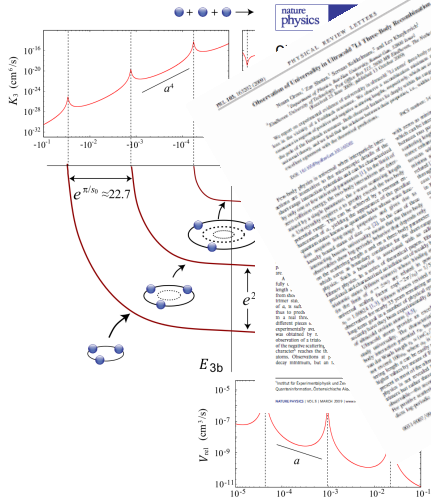
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## Efimov states: manifestations

Ultracold atoms ✓



nature physics

PRACTICAL REVIEW LETTERS

Observation of Universality in Ultracold  $^7\text{Li}$  Three-Body Recombination

Y. T. Chou, C. Chin, S. B. Papp, S. J. Kim, and L. Khaykovich

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www.sciencemag.org

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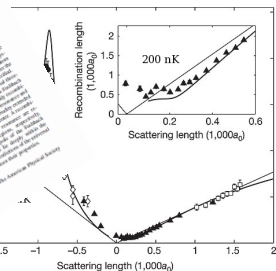
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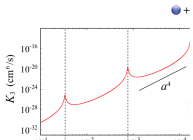
in

experimental observation  
of Efimov states  
in ultracold  
atoms  
Chou, *et al.* Nature (2006)



# Efimov states: manifestations

## Ultracold atoms ✔



PHYSICAL REVIEW LETTERS

### Three-Body Recombination in a Three-State Fermi Gas with Widely Tunable Interactions

J. H. Huckles, J. R. Williams, E. L. Haller, R. W. Sites, and K. M. O'Hara  
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 (Received 17 October 2008; revised manuscript received 17 March 2009; published 20 April 2009)

We investigate the stability of three-spin state mixtures of ultracold neutral  $^6\text{Li}$  atoms over a range of magnetic fields encompassing three Feshbach resonances. For most field values, we attribute the decay of the lowest population to three-body processes involving one atom from each spin state and find that the three-body loss coefficient varies by over 4 orders of magnitude. We observe high stability when at least two of the three scattering lengths are small, rapid loss near the Feshbach resonances, and two companion resonance loss features. At our highest fields, where all pairwise scattering lengths are approaching  $a \approx -230a_0$ , we measure a three-body loss coefficient  $L_3 = 5 \times 10^{-20} \text{ cm}^3/\text{s}$  and a weak second lower decay rate for higher fields indicating that Feshbach studies of color superfluidity and state formation in a BEC/condensate Fermi gas may be feasible.

DOI: 10.1103/PhysRevLett.102.163002

Multicomponent Fermi gases with tunable interactions are conceptually well suited to the study of few- and many-body quantum physics. Ultracold two-state Fermi gases near a Feshbach resonance have been used to characterize the contact term, Ruderman-Coppersmith superfluidity to three elements conditions of distance entanglement [1], Fermi, a normal to superfluid transition and phase separation have been observed in imbalanced two-state spin mixtures [2]. The stability of the two-state resonances against two- and three-body loss processes was critically important to the success of these experiments.

Recently, there has been considerable interest in the study of three-state Fermi gases with tunable interactions [3–6]. Whereas three-body interactions in imbalanced two-state Fermi gases are suppressed by the exclusion principle, the addition of a third spin component allows for the study of three-body phenomena such as the Efimov effect [7]. Indeed, with this additional spin state there may be competition between different pairing states and even formation [3,4,6]. For example, if pairwise interactions are all attractive and of equal magnitude, the system is expected to exhibit a novel superfluid phase analogous to color superconductivity in quarks, chromosuperconductivity (CSC) [8]. A quantum phase transition to a Fermi liquid of states analogous to the one that we observed if the ratio of the interaction energy to the kinetic energy is increased to  $g \geq 8$  by an external field [5].

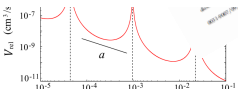
Four studies of the above phenomena critically depend on the magnitude of two- and three-body loss rates, particularly when two or more scattering lengths are resonantly enhanced. Two- and three-body loss and heating processes can impose intrinsic limits on the maximum achievable phase space density. Further, the unambiguous observation of three-body resonances requires negligible two-body loss [9].

Most investigations of Fermi gases with tunable interactions have focused on two-state mixtures. A small addition of a third spin component has been used for

fluorescence [10] and recently the rapid decay of a three-state Fermi gas near a Feshbach resonance was noted during an investigation of Cooper pair size by radio-frequency (rf) spectroscopy in a two-state Fermi gas [11]. Very recently one group reported measurements of three-body loss in thermal and degenerate three-state Fermi gases of  $^6\text{Li}$  for magnetic fields between 400 and 900 G which included three Feshbach resonances [12]. Independently, Oshinoya et al. observed loss in a three-state  $^6\text{Li}$  gas for field values between 0 and 750 G (including a single Feshbach resonance) and reported three-body loss rate coefficients for field values up to 600 G (including spin pairwise scattering resonances) [13].

In this Letter, we report the first measurements of three-body loss coefficients for a three-state Fermi gas over a range of magnetic fields where pairwise scattering resonances are resonantly enhanced. This enhancement is due to three competing Feshbach resonances and zero-energy resonances in the  $^6\text{Li}$  triplet molecular potential. Further, we demonstrate that all two-state mixtures of the three lowest hyperfine states of  $^6\text{Li}$  are stable against two-body loss processes for almost the entire range of field values between 10 and 900 G, making this three-state mixture well suited to studies of three-body physics. We observe a narrow loss feature at 277 G [14] which may be due to a three-body resonance and an additional state-dependent loss feature at 656 G. Finally, we measure the three-body loss coefficient at high fields where all three scattering lengths are approaching the right scattering lengths,  $-230a_0$ . This measurement has important implications for studying cold dense ensembles of color superfluid and baryon phases in QCD.

We study a  $^6\text{Li}$  Fermi gas with equal populations in the three lowest energy hyperfine states. In zero field, the three states correspond to  $|1\rangle = |1, 0, 0\rangle$ ,  $|2\rangle = |1, 1, -1\rangle$ , and  $|3\rangle = |1, 1, -2\rangle$  in the  $(|l, m_l, m_s\rangle)$  basis. For fields above  $\approx 200$  G, these states become increasingly electron-spin

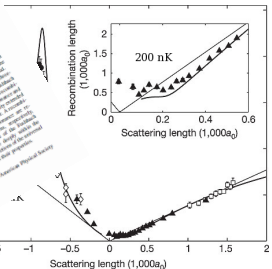


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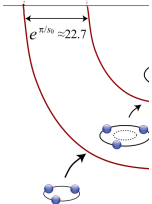
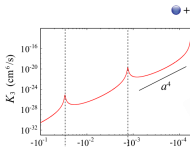
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# Efimov states: manifestations

## Ultracold atoms ✔



**PHYSICAL REVIEW LETTERS** 95, 105702 (2005)

**Three-Body Recombination in a Three-Dimensionally Tunable Interacting Atomic System**

J. H. Huckans, J. R. Williams, Department of Physics, Pennsylvania State University, University Park, Pennsylvania 16802 (Received 17 October 2004; revised manuscript received 12 December 2004)

We investigate the stability of a Bose-Einstein condensate near a Feshbach resonance for three-body recombination. We show that the recombination rate is significantly enhanced near the resonance. We also show that the recombination rate is significantly enhanced near the resonance. We also show that the recombination rate is significantly enhanced near the resonance.

**ARTICLES** **PHYSICAL REVIEW LETTERS** 95, 105702 (2005)

**Observation of an Efimov spectrum in an atomic system**

M. Zaccaro<sup>1</sup>, S. Diederichs<sup>1</sup>, C. D'Errico<sup>1</sup>, M. Fetter<sup>1</sup>, M. Jona-Lasinio<sup>1</sup>, S. Müller<sup>1</sup>, G. Roati<sup>1</sup>, M. Inguscio<sup>1</sup>, and G. Morigioli<sup>1</sup>

In this study, we provide the first experimental observation of an Efimov spectrum in an atomic system. We observe a series of discrete energy levels that are characteristic of Efimov states. The spacing between these levels is constant, as predicted by theory. We also observe a characteristic divergence of the recombination rate near the resonance.

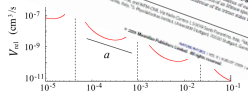
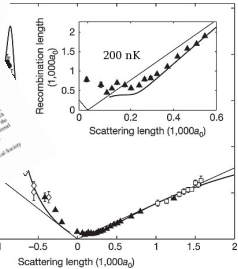
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**Observation of an Efimov spectrum in an atomic system**

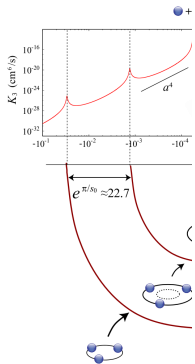
M. Zaccaro<sup>1</sup>, S. Diederichs<sup>1</sup>, C. D'Errico<sup>1</sup>, M. Fetter<sup>1</sup>, M. Jona-Lasinio<sup>1</sup>, S. Müller<sup>1</sup>, G. Roati<sup>1</sup>, M. Inguscio<sup>1</sup>, and G. Morigioli<sup>1</sup>

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experimental observation of atoms *et al.* Nature (2006)



## Efimov states: manifestations

Ultracold atoms 

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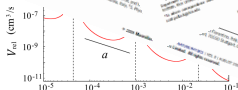
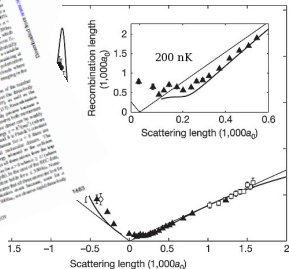
## Three-Body Recombination

Recently, the study of three-body recombination in ultracold atomic gases has attracted significant attention. In particular, the observation of Efimov states in ultracold atomic gases has provided a unique opportunity to study the universal behavior of three-body recombination. The Efimov effect, first proposed in 1971, predicts that in a system of three particles, there exists a discrete spectrum of bound states with a characteristic spacing of  $e^{\pi/s_0}$ , where  $s_0$  is the scattering length. This prediction has been experimentally verified in ultracold atomic gases, demonstrating the universality of the Efimov effect.

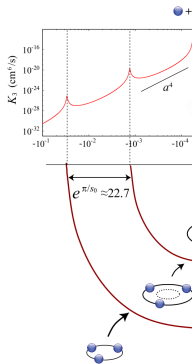

## Universality in Three- and Four-Body Bound States of Ultracold Atoms

Scott B. Papp, Daniel Soto, and Michael J. Holland

Quantum mechanics predicts that in a system of three particles, there exists a discrete spectrum of bound states with a characteristic spacing of  $e^{\pi/s_0}$ , where  $s_0$  is the scattering length. This prediction has been experimentally verified in ultracold atomic gases, demonstrating the universality of the Efimov effect.

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## Efimov states: manifestations

Ultracold atoms 

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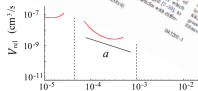
## Three-Body Recombination

Recently, the study of three-body recombination in ultracold atomic gases has attracted significant attention. In particular, the observation of Efimov states in alkali atoms has provided a unique opportunity to study the universal behavior of three-body recombination near a Feshbach resonance. The scattering length  $a$  is a key parameter in this context, and its dependence on the magnetic field is crucial for understanding the experimental results.

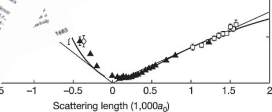
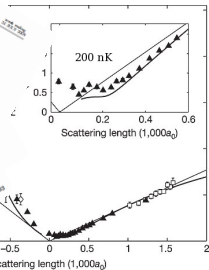
Universality Bound

Scattering length  $a$

Scattering length  $a^4$

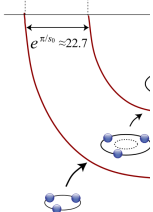
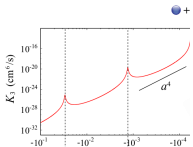


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# Efimov states: manifestations

## Ultracold atoms ✔



PHYSICAL REVIEW LETTERS 100, 120401 (2008)

### Universality of the Three-Body Parameter for Efimov States in Ultracold Cesium

M. Hennings,<sup>1</sup> A. Zurek,<sup>1</sup> B. Huang,<sup>1</sup> W. Haver,<sup>1</sup> H.-C. Nienke,<sup>1</sup> P. Fedun,<sup>1</sup> R. Grimm,<sup>1,2</sup> P. S. Julienne,<sup>3</sup> and J. M. Rost<sup>1</sup>

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<sup>2</sup>Institute for Quantum Optics and Quantum Information, Österreichische Akademie der Wissenschaften, 6020 Innsbruck, Austria  
<sup>3</sup>Johns Hopkins University, NIST and University of Maryland, College Park, Maryland 20742-2441, USA  
<sup>4</sup>Department of Chemistry, Durham University, South Road, Durham, DH1 1TA, United Kingdom  
 (Received 20 June 2011; published 14 September 2011)

We report on the observation of universal Efimov resonances in an ultracold gas of cesium atoms. Exploiting the wide tunability of ultracold scattering lengths from these bound Feshbach resonances in the same spin channel, we measure magnetic field dependent three-body recombination loss. The positions of the loss resonances yield corresponding values for the three-body parameter, which in ultracold few-body physics is required to describe three-body processes and, in particular, to fix the spectrum of Efimov states. Our observations show a robust universal behavior with a three-body parameter that is approximately constant.

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PACS numbers: 53.75.+n, 24.45.-v, 34.30.ck, 47.45.-d

The concept of universality manifests itself in the fact that different physical systems can exhibit basically the same behavior, even if the relevant energy and length scales differ by many orders of magnitude [1]. Universality thus allows us to understand in the same theoretical framework physical situations that at first glance seem completely different. In ultracold atomic collisions, the universal regime is realized when the *s*-wave scattering length *a*, characterizing the two-body interaction in the zero-energy limit, is much larger than the characteristic length of the interaction potential. Then the essential properties of the two-body system such as the binding energy of the most weakly bound dimer state and the scattering length of the two-body wave function can simply be described in terms of *a*, independent of any other system-dependent parameters. In the three-body sector, the description of universal systems requires an additional parameter, which incorporates all relevant short-range interactions not already included in *a*. In few-body physics, this important quantity is commonly referred to as the three-body parameter (3BP).

In Efimov's famous scenario [2], the infinite ladder of three-body bound states follows a discrete scaling law, which determines the relative energy spectrum of the states. The 3BP fixes the starting point of the ladder and thus the absolute energies of all states. The parameter enters the theoretical description as a short-range boundary condition for the three-body wave function in real space or as a high-frequency cutoff in momentum space. To determine the 3BP from theory would require precise knowledge of both the two-body interactions and the genuine three-body interaction at short range. In real systems, this is extremely difficult and the 3BP needs to be determined experimentally through the observation of few-body features such as Efimov resonances.

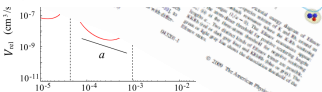
In the past few years, ultracold atomic systems have opened up the possibility to explore Efimov's scenario experimentally and to test further predictions of universal theory [3–6]. The key ingredient of such experiments is the possibility to control *a* by an external magnetic field *B* via the Feshbach resonance phenomenon [7]. This naturally raises to the important question whether the 3BP remains constant or whether it is affected by the magnetic tuning, in particular, when different Feshbach resonances are involved.

The current state of theoretical and experimental research does not provide a conclusive picture on possible variations of the 3BP. A theoretical study [8] points to strong possible variations when different two-body resonances contribute to the zero energy, and even suggests a change of the 3BP on the two sides of a zero crossing of the scattering length. Other theoretical papers point to the importance of the particular character of the Feshbach resonance [1–6]. While closed-channel dominated ("narrow") resonances involve an additional length scale that sets the 3BP [1–10], the case of entrance-channel dominated ("broad") resonances leaves the 3BP in principle open. However, predictions based on two-body scattering properties exist that apparently fix the 3BP for broad resonances as well [20–22]. The available experimental observations provide only fragmentary information. The first observation of Efimov physics in an ultracold Cs gas [3] is consistent with the assumption of a constant 3BP on both sides of a zero crossing. A later observation on <sup>71</sup>Li indicated different values of the 3BP on both sides of a Feshbach resonance. A similar observation was done for experiments on <sup>71</sup>Li, but other experiments on <sup>71</sup>Li showed universal behavior at a constant 3BP for the whole tuning range of a single resonance [1] and for another spin channel [12]. Besides these observations on

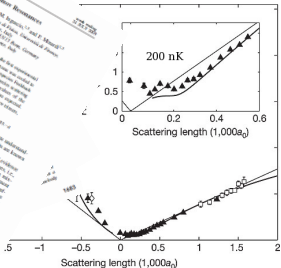
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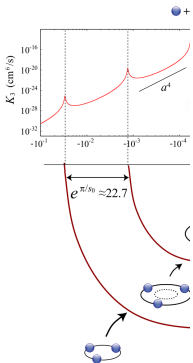


## experimental observation atoms *al. Nature (2006)*



# Efimov states: manifestations

## Ultracold atoms ✔



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**REPORTS**  
**Radio-Frequency Association of Efimov Trimers**  
 Thomas Lomte,<sup>1,2,3,4</sup> Yuxin Li,<sup>1,2,3,4</sup> ...

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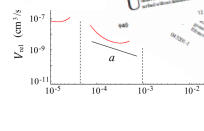
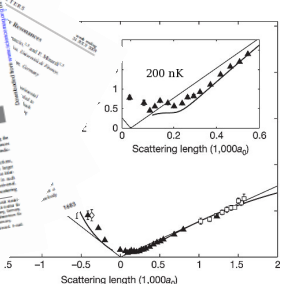
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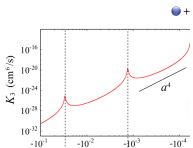
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experimental observation  
 atoms  
*al. Nature (2006)*



# Efimov states: manifestations

Ultracold atoms ✔



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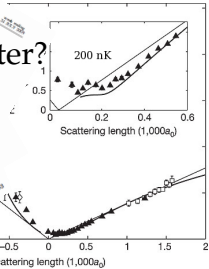
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Non-universal three-body parameter?

experimental observation  
atoms  
*al. Nature (2006)*



## REPORTS

### Radio-Frequency Association of Efimov Trimers

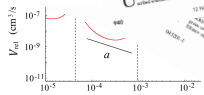
Thomas Köhl,<sup>1,2,3,4</sup> Tassilo Busch,<sup>1,2,3,4</sup> Johannes Lehmann,<sup>1,2,3,4</sup> and J. J. Braaten<sup>1,2,3,4</sup>

<sup>1</sup>Department of Physics, University of Colorado, Boulder, Colorado 80509, USA

<sup>2</sup>Department of Physics, University of Colorado, Boulder, Colorado 80509, USA

<sup>3</sup>Department of Physics, University of Colorado, Boulder, Colorado 80509, USA

<sup>4</sup>Department of Physics, University of Colorado, Boulder, Colorado 80509, USA



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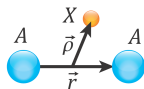


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  - The Efimov effect
- 2 Universal three-body parameter near broad Feshbach resonances
- 3 Three-body problem with multichannel interactions
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## Efimov effect in Born-Oppenheimer picture

## Hyperspherical vs Born-Oppenheimer (BO)



$$\left[ -\frac{1}{m_A} \nabla_r^2 - \frac{2m_A + m_X}{2m_A m_X} \nabla_\rho^2 + V_{AA}(r) + V_{AX} \left( \left| \rho + \frac{r}{2} \right| \right) + V_{AX} \left( \left| \rho - \frac{r}{2} \right| \right) \right] \Psi = E\Psi$$

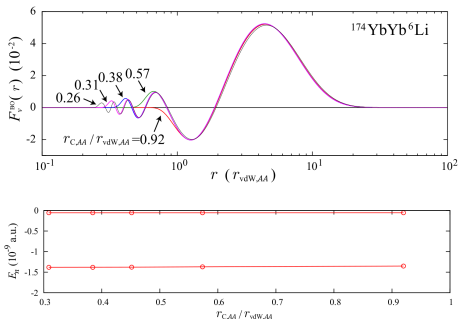
In adiabatic hyperspherical representation  $\Psi = \sum_{\nu} F_{\nu,E}(R) \Phi_{\nu}(R; \Omega)$ :

- Exact solution by coupling hyperradial equations for solving  $F_{\nu,E}$ .  
 $(\mu R^2 = \frac{1}{2} m_A r^2 + \frac{2m_A m_X}{2m_A + m_X} \rho^2)$

In BO approximation  $\Psi = F_{\nu,E}^{\text{BO}}(\mathbf{r}) \Phi_{\nu}^{\text{BO}}(\mathbf{r}; \rho)$ :

- Long-range Efimov behavior  $U_{\nu}^{\text{BO}}(r) \simeq -\chi_0^2 / 2m_X r^2$  ( $\chi_0 \approx 0.57$ ).
- Short-range van der Waals behavior  $U_{\nu}^{\text{BO}}(r) \simeq V_{AA}(r) = -C_{6,AA} / r^6$ .

## Efimov-favored AAX systems — universal three-body parameter



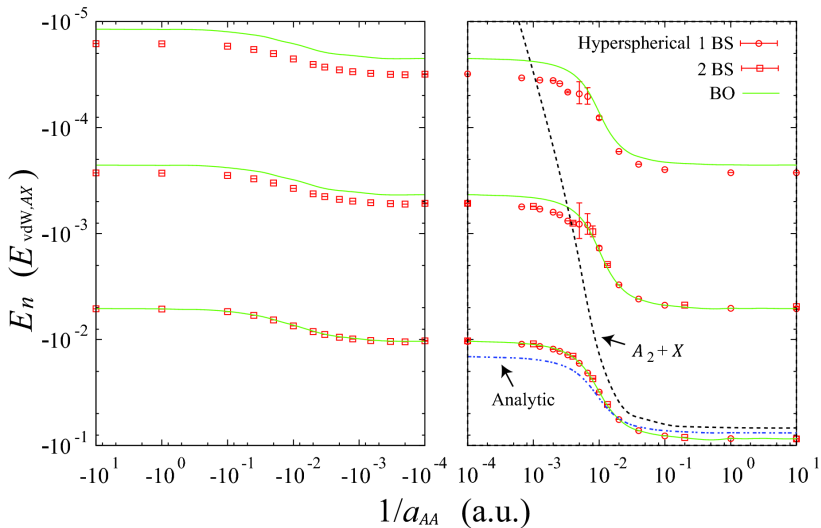
- Nodal positions in the van der Waals region are determined by  $a_{AA}$  and  $r_{\text{vdW},AA}$ .
- Universal Efimov state energies independent of  $r_c$

$r_+$  determines the ground Efimov state energy:

$$E_{0,\text{analytic}} = -\frac{4}{Mr_+^2} \exp\left(-\frac{2}{s_0} \{\text{Arg}[\Gamma(1 - is_0)] - \pi\}\right), r_+ \text{ can be found by}$$

$$J_{-\frac{i\alpha s_0}{2}}\left(2\frac{r_{\text{vdW},AA}^2}{r_+^2}\right) N_{-\frac{i\alpha s_0}{2}}\left(2\frac{r_{\text{vdW},AA}^2}{r_-^2}\right) = N_{-\frac{i\alpha s_0}{2}}\left(2\frac{r_{\text{vdW},AA}^2}{r_+^2}\right) J_{-\frac{i\alpha s_0}{2}}\left(2\frac{r_{\text{vdW},AA}^2}{r_-^2}\right),$$

$$N_{\frac{1}{4}}\left(2\frac{r_{\text{vdW},AA}^2}{r_-^2}\right) = \left[1 - \sqrt{2}\frac{a_{AA}}{r_{\text{vdW},AA}} \frac{\Gamma(5/4)}{\Gamma(3/4)}\right] J_{\frac{1}{4}}\left(2\frac{r_{\text{vdW},AA}^2}{r_-^2}\right). (\alpha \approx 2)$$

Efimov-favored  $AAX$  systems — universal three-body parameter

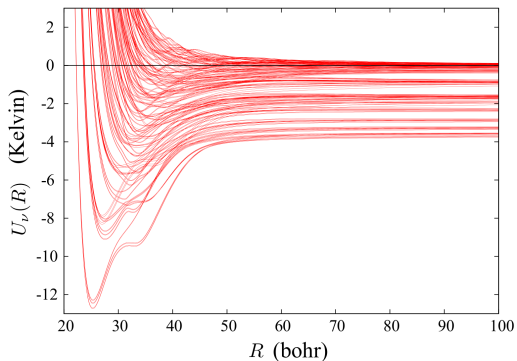
Universal Efimov spectrum for YbYbLi [1]

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## Hyperspherical potentials with nuclear spins

$$\psi = \sum_{\nu} F_{\nu}(R) \sum_{K,J} \phi_{\nu}^{J,K,\lambda}(R; \theta, \varphi) D_{MK}^J(\alpha, \beta, \gamma) \sum_{\lambda=m_{s_i}, m_{i_i}} |m_{s_1} m_{s_2} m_{s_3} m_{i_1} m_{i_2} m_{i_3}\rangle$$

Hyperspherical potentials for three  ${}^7\text{Li}$  atoms ${}^7\text{Li}$  atom:

- $s = 1/2, i = 3/2$
- 8 hyperfine levels  
 $f = 1, 2$
- $\Delta E_{\text{hf}} \approx 20\text{mK}$

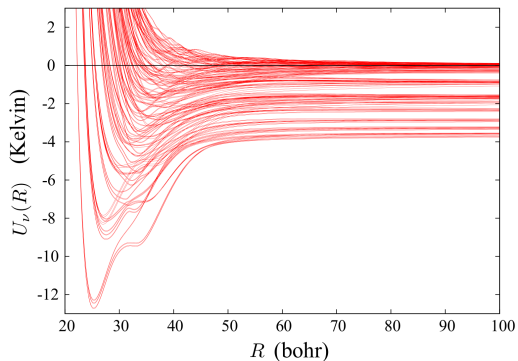
In the calculation:

- $|f = 1, m_f = 1\rangle,$   
 $|f = 2, m_f = 1\rangle$
- $J = 0, 2$
- $B = 200\text{G}$

More work needs to be done for  $> 50$  spin channels.

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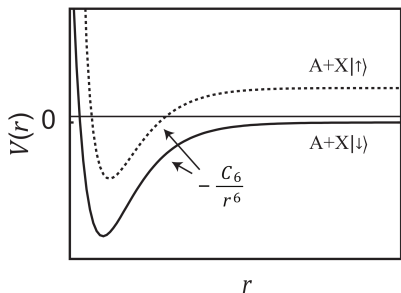
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More work needs to be done for  $> 50$  spin channels.

# A two-channel model for $A + A + X$ two-body interactions

Two-channel model for isolated Feshbach resonances:



$$s_{res} = 2\mu_2 \delta\mu \delta B a_{bg} r_{vdW} / \hbar^2$$

$$s_{res} \gg 1: \text{broad resonances}$$

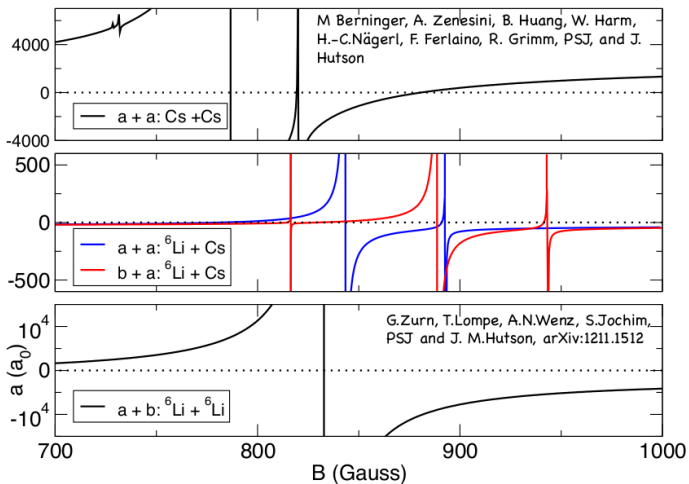
$$s_{res} \ll 1: \text{narrow resonances}$$

Three-body in two-channel representation:

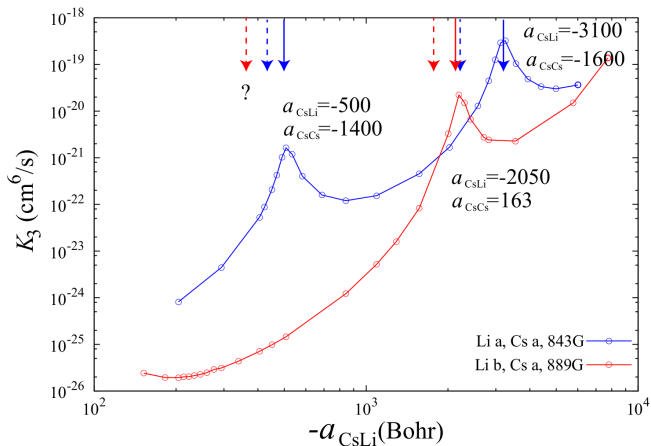
$$\left( -\frac{1}{2\mu} \frac{\partial^2}{\partial R^2} + \frac{\Lambda^2}{2\mu R^2} + V_{AA}(r_{31}) \right) \psi_\alpha + \sum_{\beta=g,e;i < j} V_{\alpha\beta}(r_{ij}) \psi_\beta = (E - \epsilon_\alpha) \psi_\alpha$$



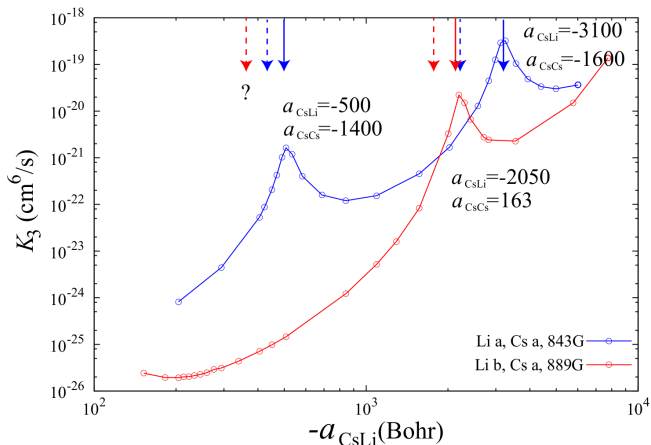
## Feshbach resonances in Cs+Li systems



Cs-Li broad resonances:  $a_{\text{bg}} \approx -29$  bohr,  $\Delta_B \approx 60G$ ,  $s_{\text{res}} \approx 0.7$

Efimov resonances in Cs+Cs+Li recombination ( $e^{\pi/s_0} \approx 4.9$ )

No Efimov resonance is found for  $|a_{\text{CsLi}}| < 10^4$  near 893G (aa) and 943G (ba) narrow Cs+Li Feshbach resonances.

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# Summary

- The three-body parameter is universal for atoms near broad Feshbach resonances.
- Three-body hyperspherical potentials are calculated by including hyperfine interactions.
- The positions of Efimov resonances are predicted in Cs-Li ultracold admixture.