A Tour of Supersymmetry Parameter Space

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We still do not know what causes
electroweak symmetry breaking.

In the past few years, we have seen a plethora of new models for physics at the TeV scale:

extra dimensions, little Higgs, fat Higgs, Higgsless, ...

yet supersymmetry remains the best candidate in contention

- SUSY solves the whole list of problems for a theory of TeV physics

- SUSY is intrinsically a weak-coupling theory, allowing deep theoretical analysis

In this lecture, I will take SUSY extremely seriously and discuss issues for its experimental program.

If you are not a fan of SUSY,

remember that any reasonable alternative
(except for "G-D made it so")
will be at least as complicated
in SUSY we can at least work out the details explicitly

In this lecture, I will work mainly with the
"Minimal" Supersymmetric Standard Model (MSSM),
with 2 Higgs doublets.

Extra electroweak singlets may be needed; I'll give a few hints of this as we go along.

I remind you that the MSSM solves* the following problems:

- hierarchy $\langle H \rangle \ll m_{Pl}$
- origin of EWSB   (associated with large m_t)
- consistency of grand unification
- smallness of precision electroweak corrections
- smallness of flavor-changing neutral currents
- origin of cosmic dark matter

and also gives

- direct connection to string theory
- conceptual zero of energy
  (prerequisite to a theory of "dark energy")

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*It is more accurate to say that solutions to all of these problems are accomodated by (hopefully, natural) restrictions of the parameter space.

Grand unification:

using 1-loop RGE's and no threshold effects, SU(5)/SO(10)/E_6 normalization of U(1):

$$\alpha_1 = \frac{5}{3}\alpha'$$

$$B = \frac{\alpha_3^{-1} - \alpha_2^{-1}}{\alpha_2^{-1} - \alpha_1^{-1}} = \frac{b_3 - b_2}{b_2 - b_1} = \begin{cases} \frac{5}{7} = 0.714 & \text{MSSM} \\ 0.53 & \text{MSM} \end{cases}$$

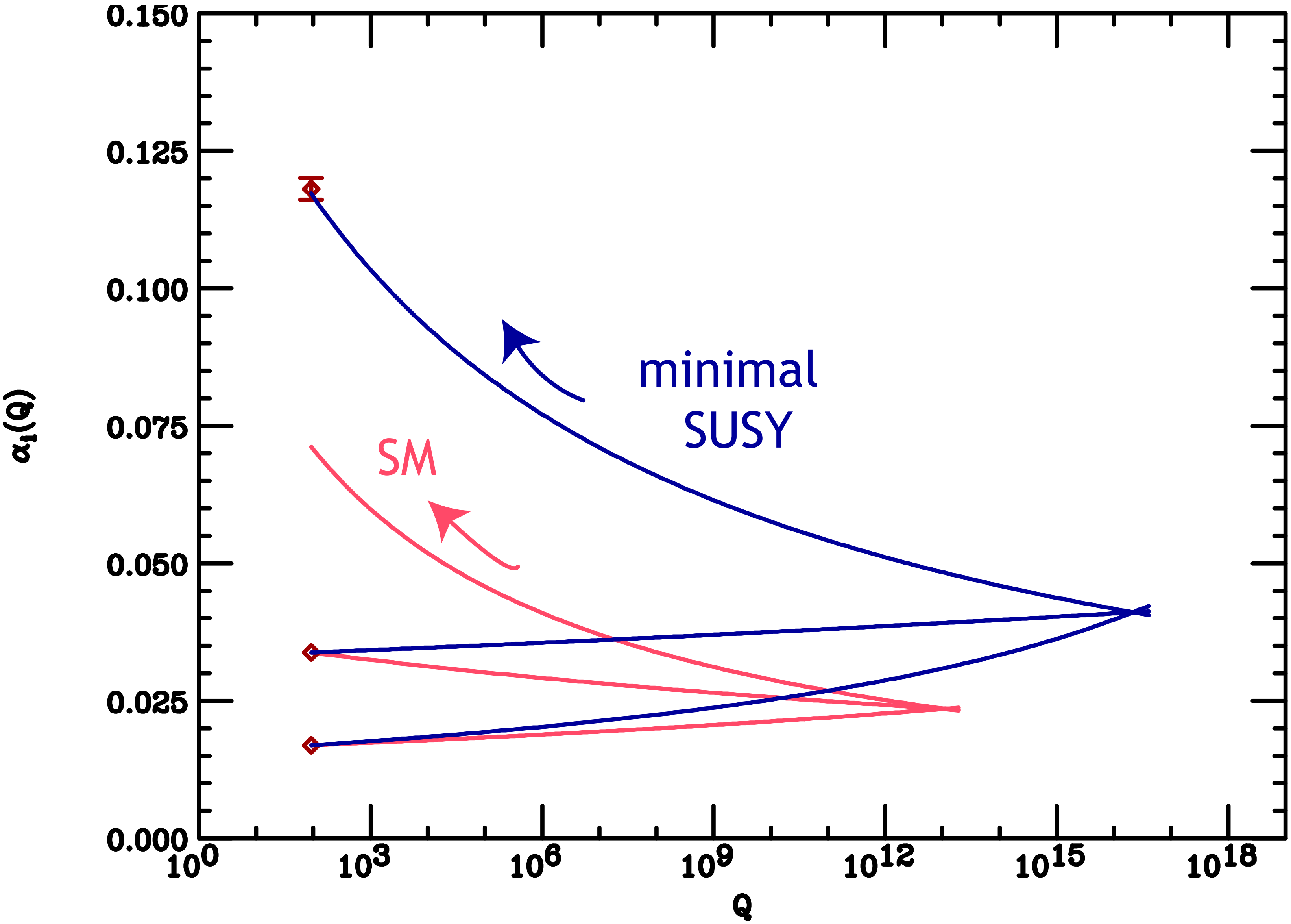
experimentally: $B = 0.717 \pm 0.008$

Unfortunately, TeV-scale threshold effects and 2-loop RGE's weaken the agreement

$$\begin{array}{ll} \text{SUSY:} & \alpha_s > 0.13 \\ \text{expt.:} & \alpha_s = 0.1172 \pm 0.002 \end{array}$$

Langacker, Polonsky
Pierce

To restore agreement, we need relatively large GUT-scale threshold corrections. Their value might be a clue to properties of the true GUT.



0.150
0.125
0.100
0.075
0.050
0.025
0.000
$\alpha_i(Q)$
minimal
SUSY
SM
10^0
10^3
10^6
10^9
10^{12}
10^{15}
10^{18}
Q

Precision electroweak:

Altarelli discussed this on Monday; I have little to add

- specific pieces of data (SLAC A_{LR}, Tevatron m_W) strongly favor a light Higgs boson (fitting in MSM)
- most likely explanation: the Higgs is actually light (and seen at LEP)
- if SUSY thresholds are just above the LEP range, W/Z/Higgs superpartners can contribute a small negative ΔS, which improves agreement

Altarelli, Barbieri, Caravaglios

Parameters of the MSSM:

supersymmetric Lagrangian:

gauge interactions: SM gauge couplings α_1 , α_2 , α_3

superpotential: SM Yukawa couplings
+ 1 extra parameter μ

2 Higgs doublets:

$$\langle H_d \rangle = \frac{1}{\sqrt{2}} v \cos\beta \qquad \langle H_u \rangle = \frac{1}{\sqrt{2}} v \sin\beta$$

Parametrize effects of spontaneous supersymmetry breaking by soft operators

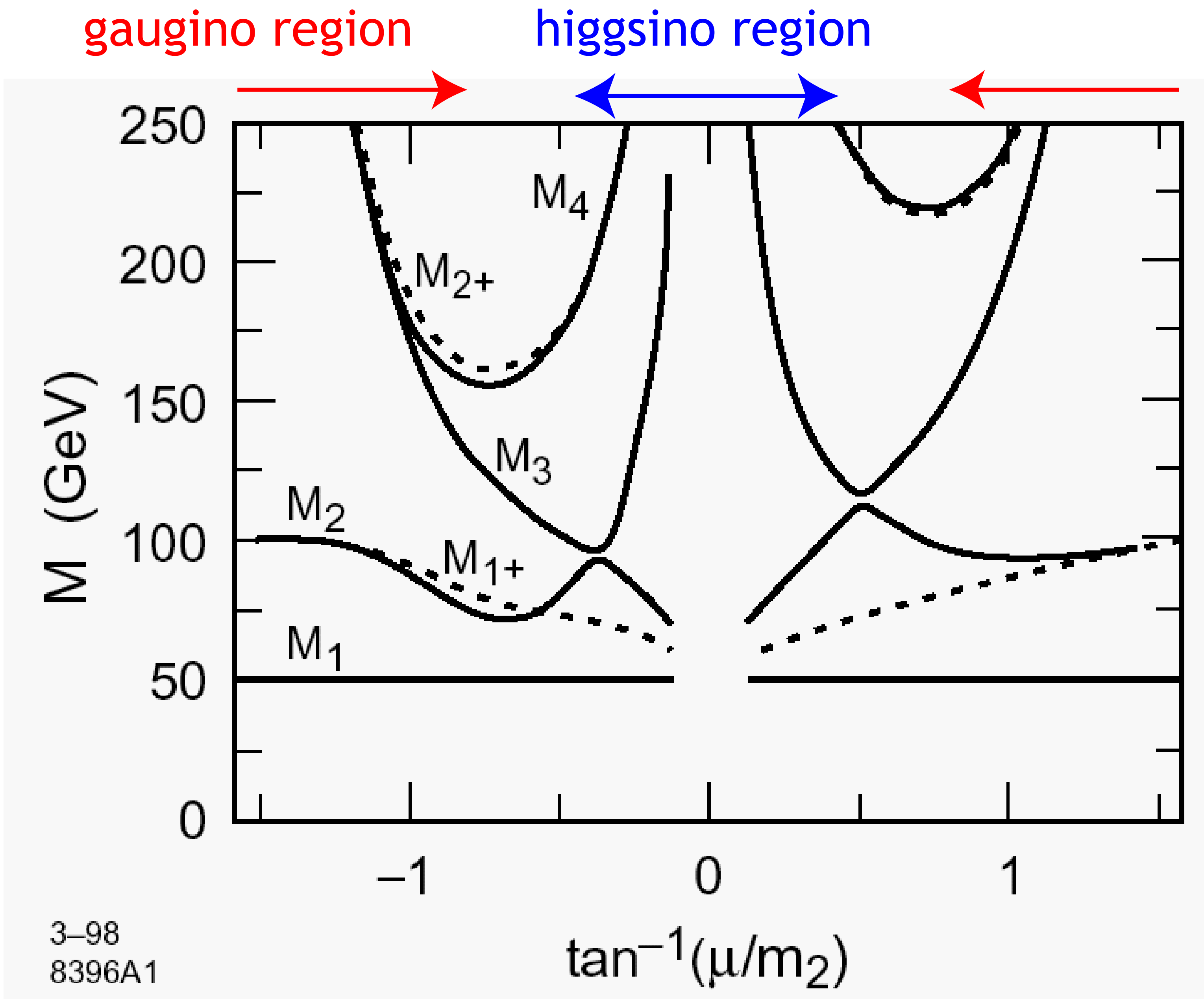
$$
\begin{aligned}
L_{soft} = -M_f^2|\phi_f|^2 - \frac{1}{2}m_a\lambda^a\lambda^a + h.c. \\
-A_e y_e \overline{e} H_1 L - A_d y_d \overline{d} H_1 Q + A_u y_u \overline{u} H_2 Q \\
+B\mu H_1 H_2 + h.c.
\end{aligned}
$$

Each coefficient here could be a matrix in flavor with CP-violating phases $\rightarrow$ 108 new parameters

Superpartner mixing can affect the final spectrum:

gaugino-Higgsino mixing $\rightarrow$ neutralinos ($\tilde{N}$), charginos ($\tilde{C}$)

$\tilde{f}_L$ - $\tilde{f}_R$ mixing: $X_\tau = (A_\tau - \mu \tan\beta)$ $\qquad X_t = (A_t - \mu \cot\beta)$



gaugino region
higgsino region
250
200
150
100
50
0
M (GeV)
M4
M2+
M3
M2
M1+
M1
–1
0
1
tan−1(μ/m2)
3–98
8396A1

Cartoon of models of SUSY breaking:

$$M_f^2 = c_f \frac{\langle F \rangle^2}{\mathcal{M}^2} \qquad m_a = d_a \frac{\langle F \rangle}{\mathcal{M}}$$

$\mathcal{M}$ = "messenger scale" -- somewhere between 30 TeV and mPl, depending on the physics of spontaneous SUSY breaking

Convert the parameters at $\mathcal{M}$ to parameters at TeV using RG evolution.

The ultimate goal of experimentation on SUSY particles is to determine $\mathcal{M}$ and the pattern of the c_f, d_a;

these are clues to a deeper level of physical theory.

Squarks and sleptons with the same quantum numbers must be given soft masses that are either highly degenerate or highly aligned with Yukawa matrices, to avoid FCNC effects.

for example: Gabbiani and Masiero

$$\frac{(V_R M_{dR}^2 V_R^\dagger)_{12}}{M_d^2} < 10^{-2}\left(\frac{M_d}{300\text{ GeV}}\right)^2$$

There are now several schemes for obtaining this degeneracy naturally:

gauge mediation	requires low $\mathcal{M}$	Dine-Nelson
anomaly mediation	problem w. sleptons	Giudice et al Randall-Sundrum
gaugino mediation	requires RG running above MGUT	Schmaltz-Skiba

Most phenomenological studies are done in the context of

messenger scale at M_{GUT}

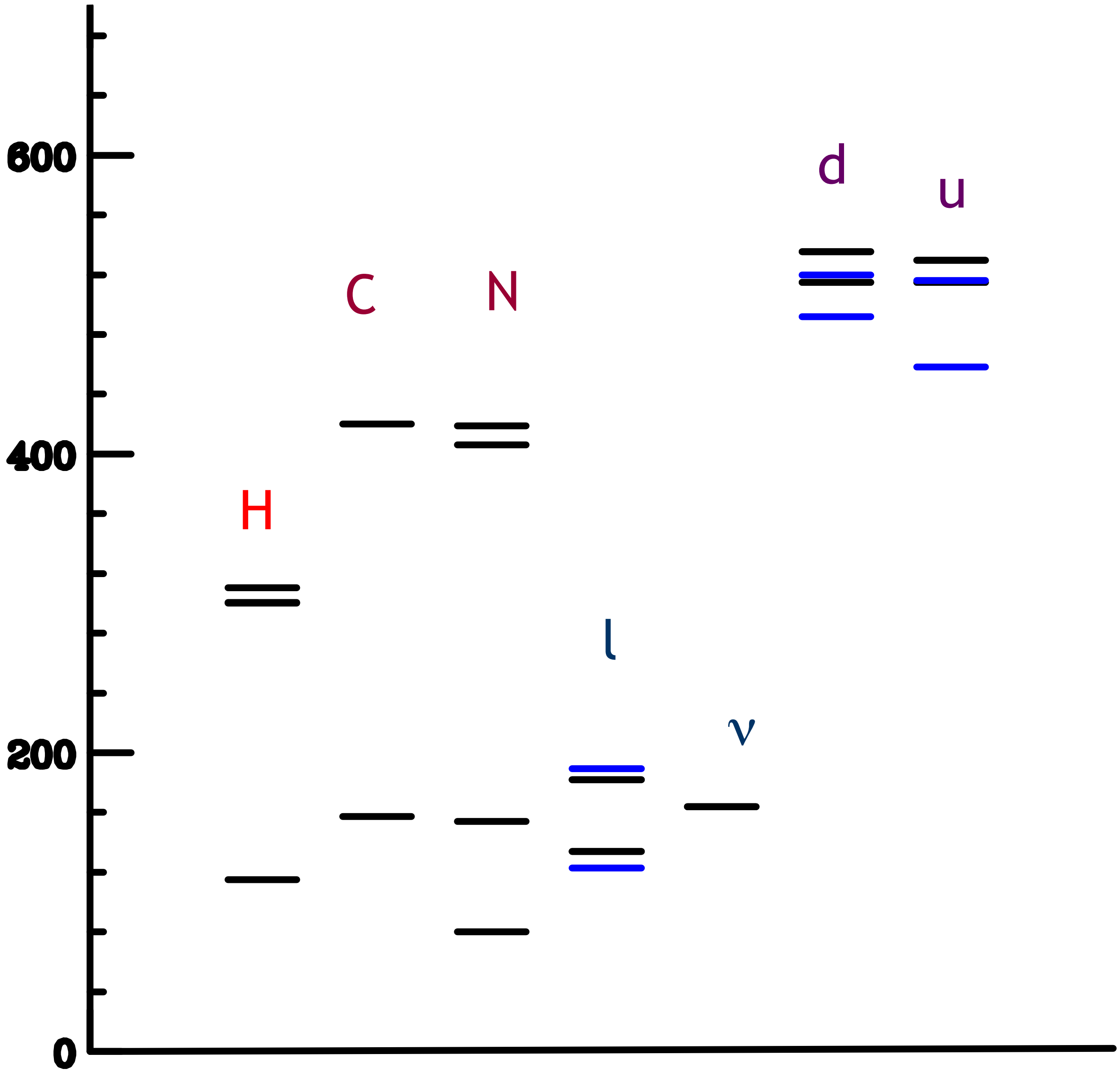
universal c_f, d_a (parametrized by m_0, $m_{1/2}$)

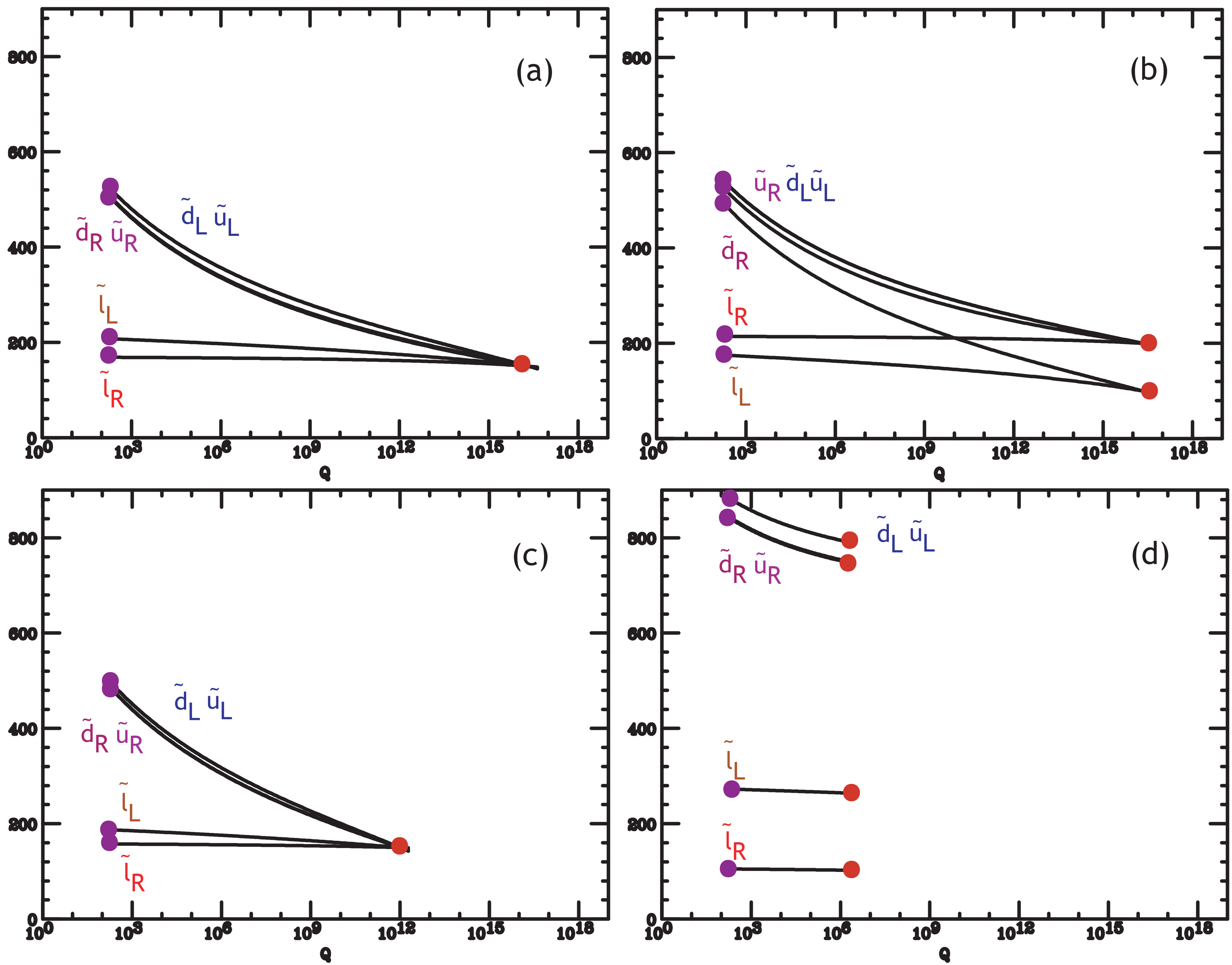
"minimal SUGRA" or "cMSSM"

But there is no good reason why the c_f cannot depend on the SU(2)XU(1) quantum numbers

In time, we will investigate this experimentally.

Here is a spectrum generated from universal initial conditions at the Planck scale:





(a)
(b)
(c)
(d)
$\tilde{d}_L$ $\tilde{u}_L$
$\tilde{d}_R$ $\tilde{u}_R$
$\tilde{l}_L$
$\tilde{l}_R$
$\tilde{u}_R$ $\tilde{d}_L\tilde{u}_L$
$\tilde{d}_R$
0
200
400
600
800
10^0
10^3
10^6
10^9
10^{12}
10^{15}
10^{18}
Q

For gauginos, induction of masses above the GUT scale:

$$\Delta\mathcal{L} = \int d^2\theta \, \frac{\Phi}{\mathcal{M}} \mathrm{tr}(W^\alpha W_\alpha)$$

leads at low energy to "gaugino unification":

$$m_1 \;:\; m_2 \;:\; m_3 \;=\; \alpha_1 \;:\; \alpha_2 \;:\; \alpha_3 \;=\; 0.5 \;:\; 1.0 \;:\; 3.5$$

the simplest models of gauge mediation lead to the same result.

Other models can give a qualitatively different spectrum

e.g. anomaly mediation:

$$m_1 \;:\; m_2 \;:\; m_3 \;=\; 3.3 \;:\; 1.0 \;:\; 10.5$$

The lightest gauginos are an almost degenerate W^0, W^+, W^-

It is very important to measure m_1, m_2, m_3 independently.

the μ parameter is paradoxical; it is a SUSY-invariant parameter, but its scale is that of SUSY breaking

We cannot have $\mu = 0$ in the MSSM; this leaves one $\tilde{N}$ massless.

μ can have its origin in SUSY breaking:

Nilles-Kim $$\Delta W = S H_d H_u \qquad \langle S \rangle = \frac{\langle F_S \rangle^2}{\mathcal{M}}$$

Giudice-Masiero $$\Delta K = \frac{1}{\mathcal{M}} \overline{S} H_d H_u \qquad \langle \overline{S} \rangle = \theta^2 \langle F_S \rangle$$

By adding 2 singlets to the MSSM, one can construct a model with $\mu = 0$ satisfying all phenomenological constraints

Nelson, Ruis, Sanz, Unsal

In phenomenological studies, μ is usually fixed by the physics of EWSB:

$$\mu^2 = \frac{M^2(H_d) - M^2(H_u)\tan^2\beta}{\tan^2\beta - 1} - \frac{1}{2}m_Z^2$$

typically in minimal SUGRA $|M^2(H_u)| \sim M_{\tilde{t}}^2 \sim m_{\tilde{g}}^2$

leading to $\mu \gg m_1 \ , \ m_2$

However, there is a parameter region ("focus point") with μ << M(q), M(l) even if we assume universality at $\mathcal{M}$

Feng, Moroi, Matchev

The physics of EWSB also gives qualitative upper limits on the mass scale of SUSY:

$$m_W^2 = -1.3\mu^2 + 0.3m_{\tilde{g}}^2 + \cdots \qquad (\tan\beta = 10)$$

Now I will review various constraints on the parameter space:

Higgs mass

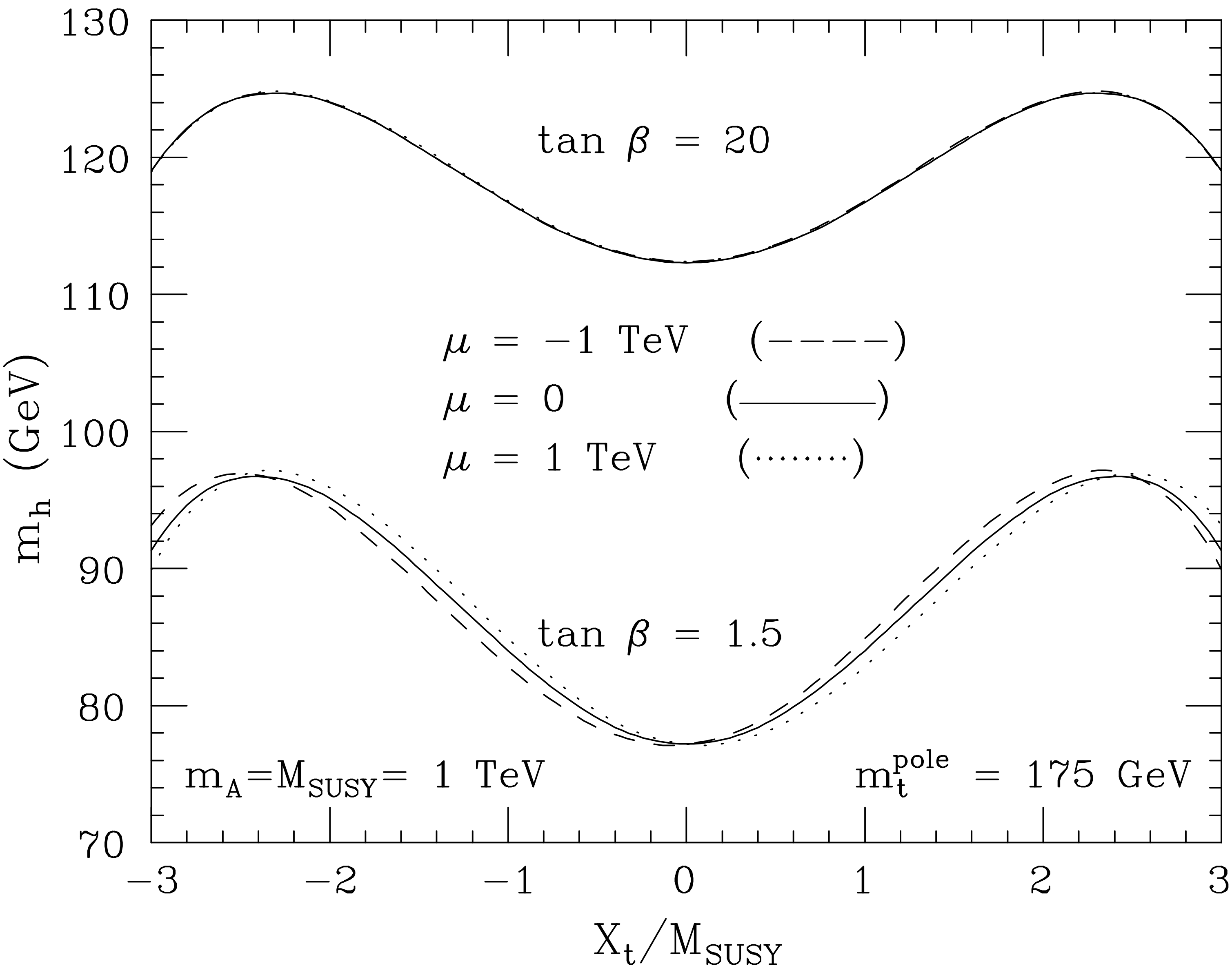
$$m_h^2 = m_Z^2 \cos^2\beta + \frac{3\alpha_w m_t^4}{2\pi m_W^2 \sin^2\beta} \log \frac{M^2}{m_t^2}$$

It appears that m_h is larger than m_Z: $m_h > 114$ GeV

Enhance the correction with

- large SUSY masses
- large mass specifically for $\tilde{t}$, $\tilde{b}$
- large $\tilde{t}$ mixing

If LEP did not see the Higgs, relatively little phase space is left. This is the best argument for adding singlets to the MSSM; then one can easily obtain $m_h \sim 150$ GeV and above.



Haber, Hempfling, Hoang

$b \to s\gamma$

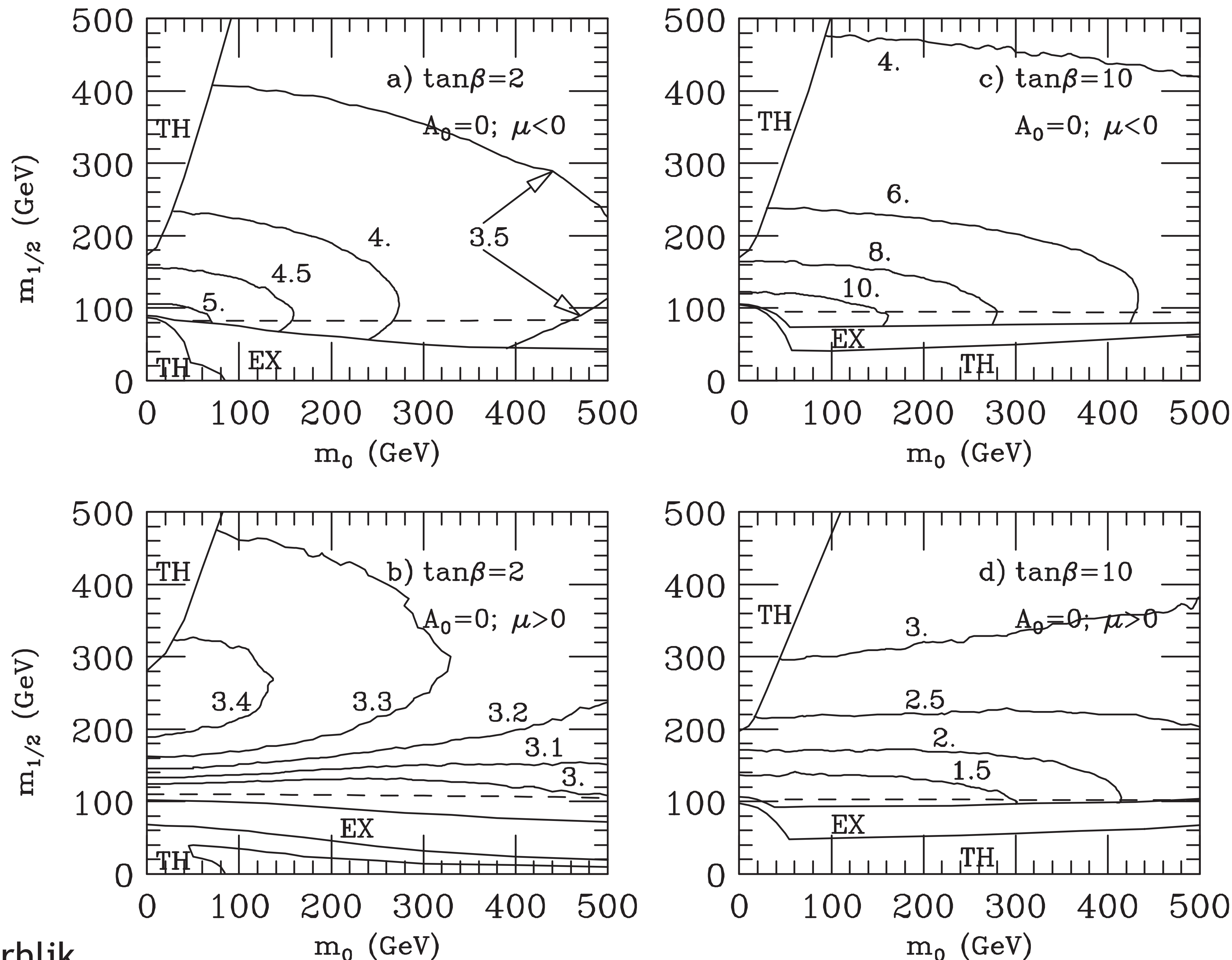
$$BR(b \to s\gamma) = (3.3 \pm 0.4) \times 10^{-4}$$

This is in good agreement with the MSM.

$H^+ + t\ , \quad \widetilde{C}^+ + \tilde{t}$ diagrams can give contributions of the same order for $m(\widetilde{C}^+) \sim m_W$

Fortunately, for $\mu > 0$, these two new contributions have destructive interference.

(Please note that there is currently a 3 σ discrepancy with the SM in the CP phase in $B^0 \to \phi K_S$.)


a) tanβ=2
A0=0; μ<0
TH
EX
4.
4.5
5.
3.5
m1/2 (GeV)
m0 (GeV)
b) tanβ=2
A0=0; μ>0
3.4
3.3
3.2
3.1
3.
c) tanβ=10
A0=0; μ<0
6.
8.
10.
d) tanβ=10
A0=0; μ>0
2.5
2.
1.5

Brhlik

muon (g-2)

new BNL measurement: average of μ^+ , μ^-

$$a_\mu = 11\,659\,208\,(6) \;\; \times 10^{-10}$$

or

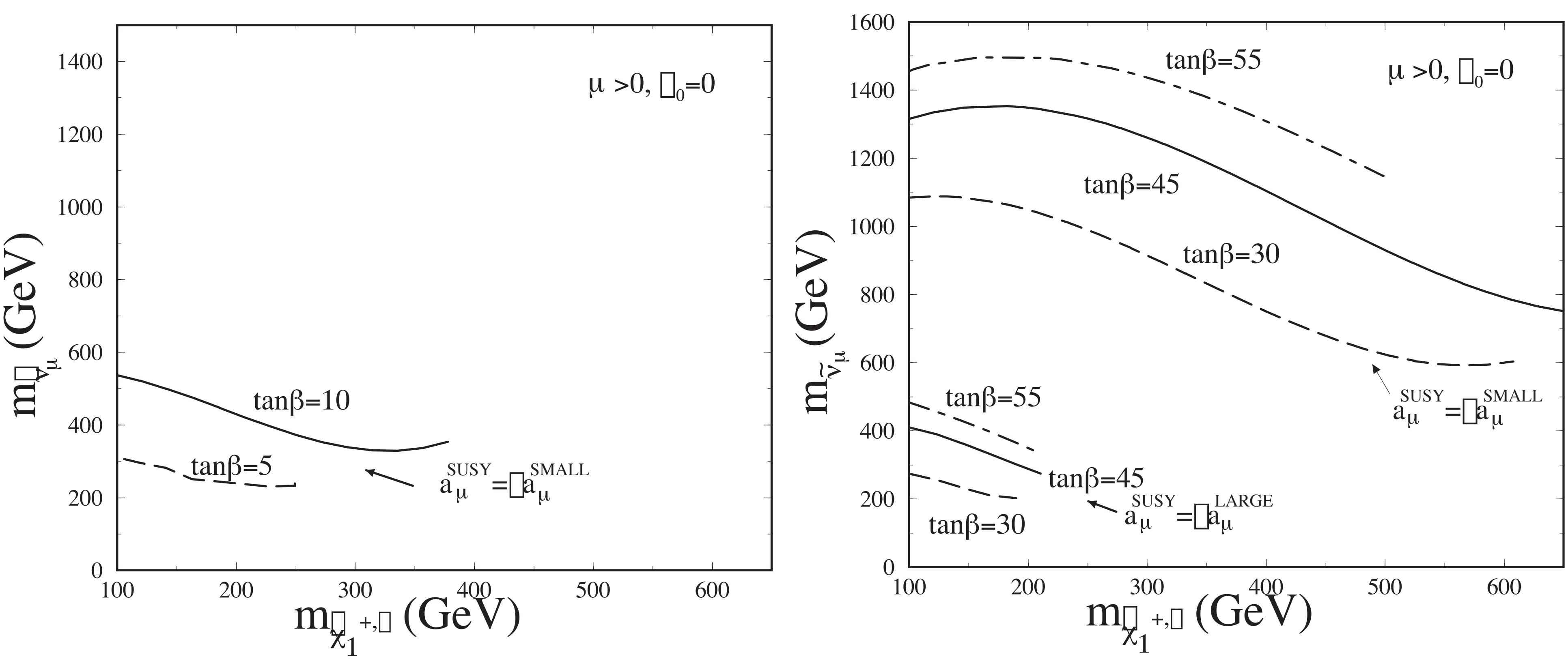
$$\Delta a_\mu = \begin{cases} +17 \;\; \times 10^{-10} \\ +32 \times 10^{-10} \end{cases}$$

in SUSY,

$$\Delta a_\mu \sim \frac{\alpha_w}{4\pi}\frac{m_\mu^2 m_2 \mu}{M^4}\tan\beta$$

The BNL value strongly excludes negative μ
with light $\tilde{\mu}$, $\tilde{C}^+$ Martin, Wells

if the effect is real, it favors light sleptons or large $\tan\beta$



Chattopadhyay-Nath

baryogenesis

different options are consistent with SUSY;
the choice is coupled to the choice of the scale of inflation

leptogenesis requires $T_R \sim M_{GUT}$

CP violation in the ν sector
slepton flavor violation Nojiri, Nomura

weak scale baryogenesis needed if $T_R \sim$ TeV

challenging for the MSSM

$$m_h < 115 \text{ GeV} \qquad 105 < m_{\tilde{t}} < 165 \text{ GeV} \qquad \varphi(\mu) > 0.04$$

but wonderful if correct !

Dvali-Kachru "new old inflation" : imprinting of density fluctuations by singlet-Higgs mixing dark matter

This is a general feature of theories of EWSB that should receive more popular attention.

general hypothesis: DM is a "thermal relic"

stable, in thermal equilibrium at early times, isolated by the expansion of the universe

$$\Omega_{DM} h^2 \approx \frac{s_0}{\rho_c/h^2} \left(\frac{45}{\pi g_*}\right)^{1/2} \frac{1}{m_{\rm Pl}} \left(\frac{1}{\langle \sigma_{ann} v \rangle}\right)$$

WMAP has measured the dark matter abundance accurately:

$$\Omega_{DM} h^2 = 0.113 \pm 0.009$$

putting in the numbers:

$$\langle \sigma_{ann} v \rangle = 1 \ \ \mathrm{pb} \\ = \frac{\pi \alpha^2}{8 m^2}$$

for m = 100 GeV

This implies that LHC will see

multi-jets + missing energy ($\not{E}_T$ > 300 GeV)

at a rate ~100 times that predicted in the SM !

The assumption of supersymmetry is not needed for this conclusion.

DM poses a small problem for SUSY

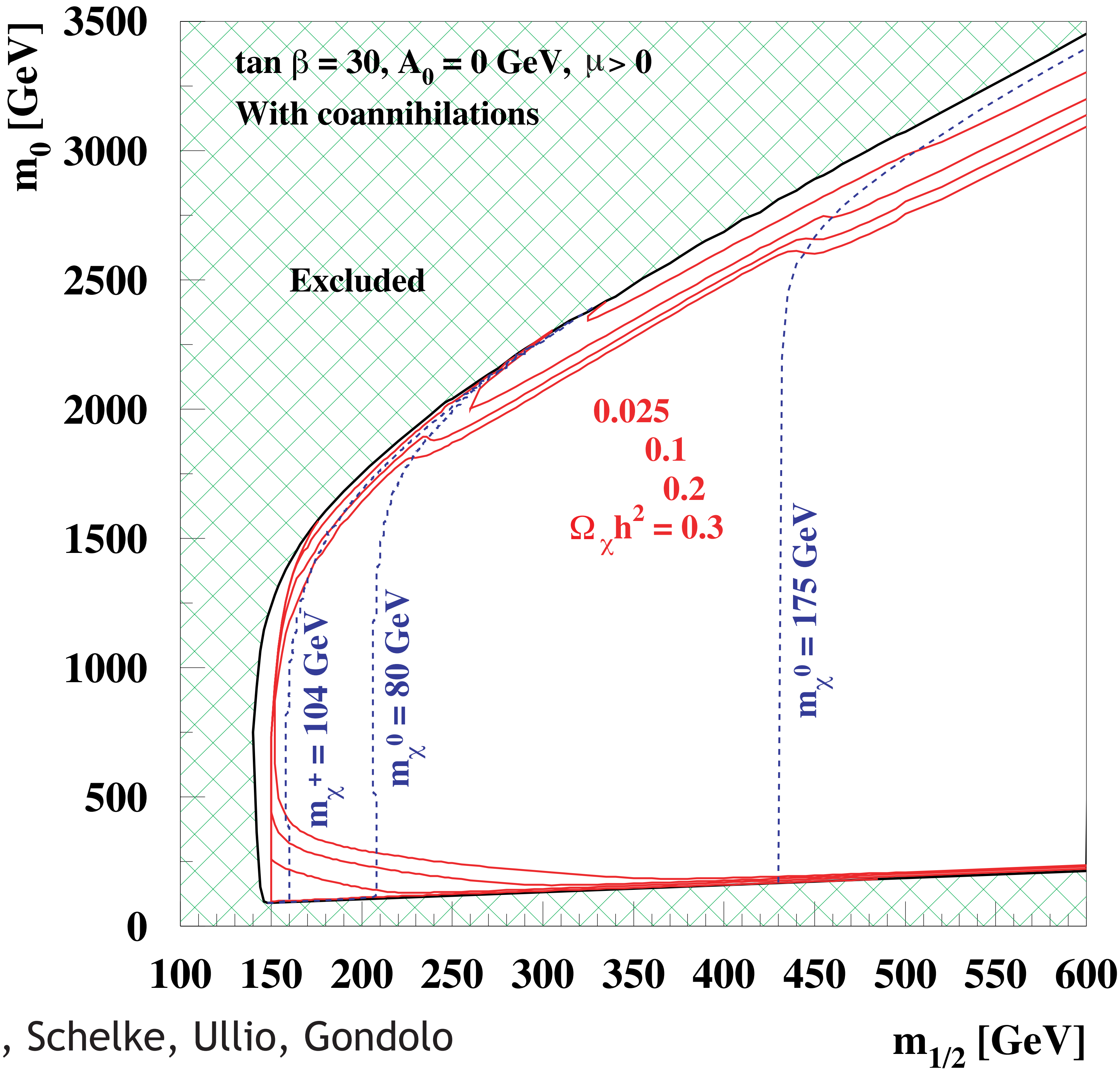
(at least in "minimal SUGRA")

$\langle \sigma_{ann} v \rangle = 1 \ \ \mathrm{pb}$ is a large cross section

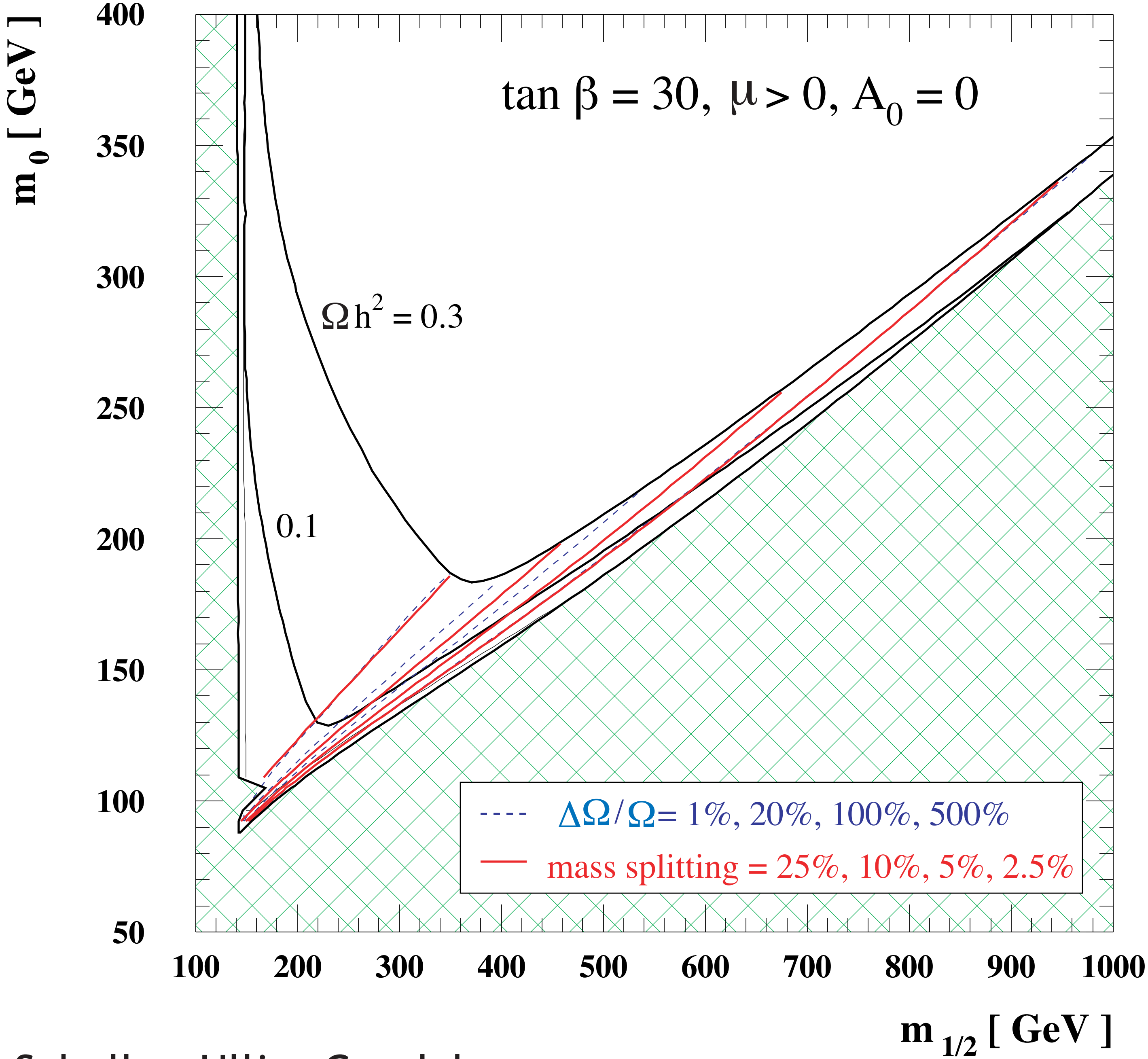
$\tilde{N} \ \tilde{N} \to f \ \bar{f}$ is helicity suppressed in the S-wave Goldberg

so $\Omega_{DM} h^2 = 0.1$ occurs only in special regions of the parameter space

$$
\begin{array}{rl}
m(\widetilde{C}^+) \approx m(\widetilde{N}) & \widetilde{N}\widetilde{N} \to W^+W^- \\
m(\tilde{\ell}) \approx m(\widetilde{N}) & \widetilde{N}\tilde{\ell} \to \gamma\ell, \tilde{\ell}\tilde{\ell} \to \ell\ell \\
m(A^0) \approx 2\, m(\widetilde{N}) & \widetilde{N}\widetilde{N} \to A^0 \\
m(\tilde{t}) \approx m(\widetilde{N}) & \widetilde{N}\tilde{t} \to \cdots
\end{array}
$$



Edsjo, Schelke, Ullio, Gondolo



Edsjo, Schelke, Ullio, Gondolo

I have raised many issues about the values of the SUSY parameters.

Eventually, all of these questions should be answered by determining these parameters experimentally.

Can we actually expect to do this ?

LHC

supersymmetric particle production is dominated by

$$gg \longrightarrow \tilde{g}\tilde{g} \qquad gq \longrightarrow \tilde{g}\tilde{q}$$

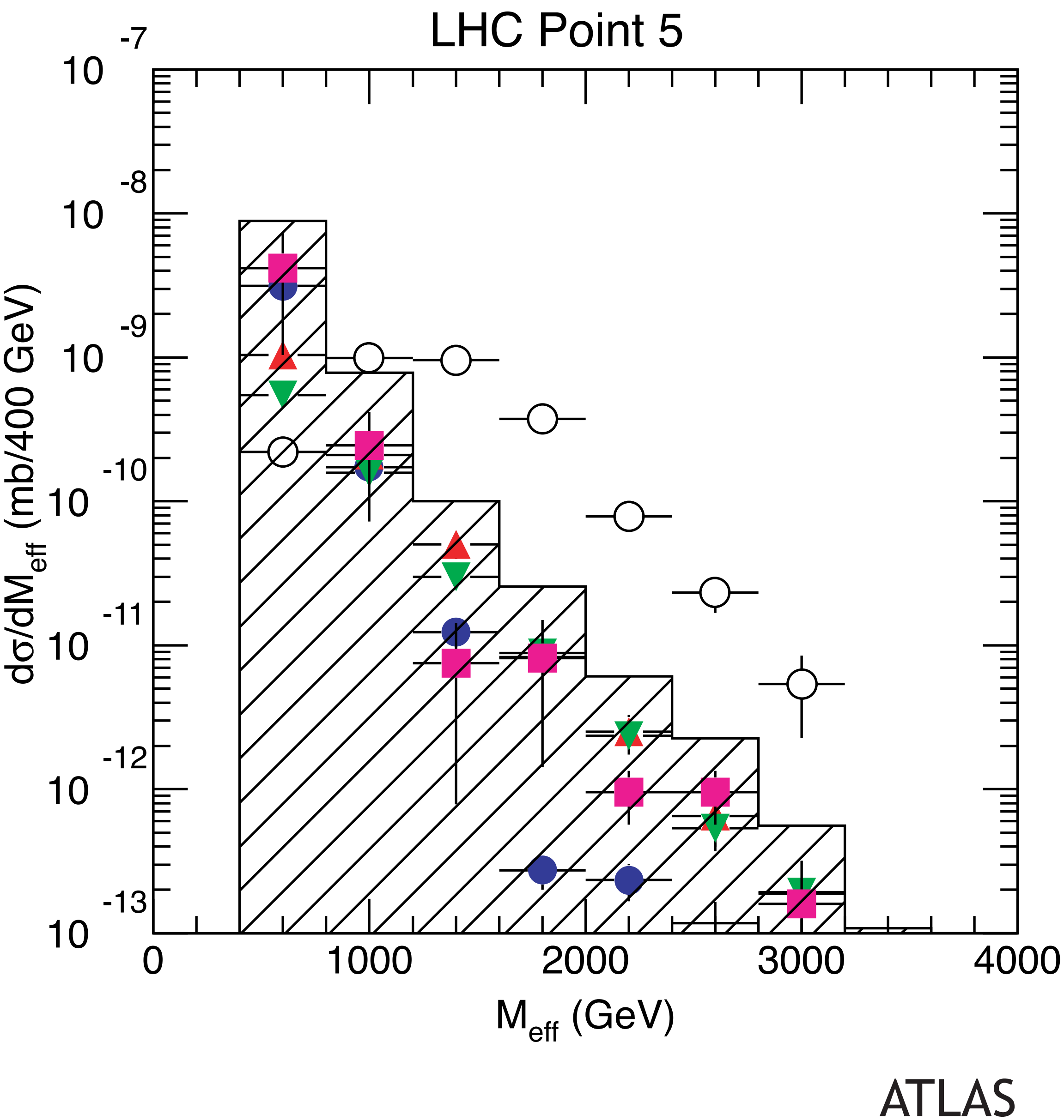
$\tilde{g}$, $\tilde{q}$ are relatively heavy ; lighter SUSY particles appear in their decay chains

The generic SUSY signal is robust: Hinchliffe, Paige

$$M_{\text{eff}} = \not{E}_T + \sum_1^4 E_{Ti}$$

Precision measurements require opportunistic use of special features of the SUSY spectrum

(a gifted experimenter will always find them)

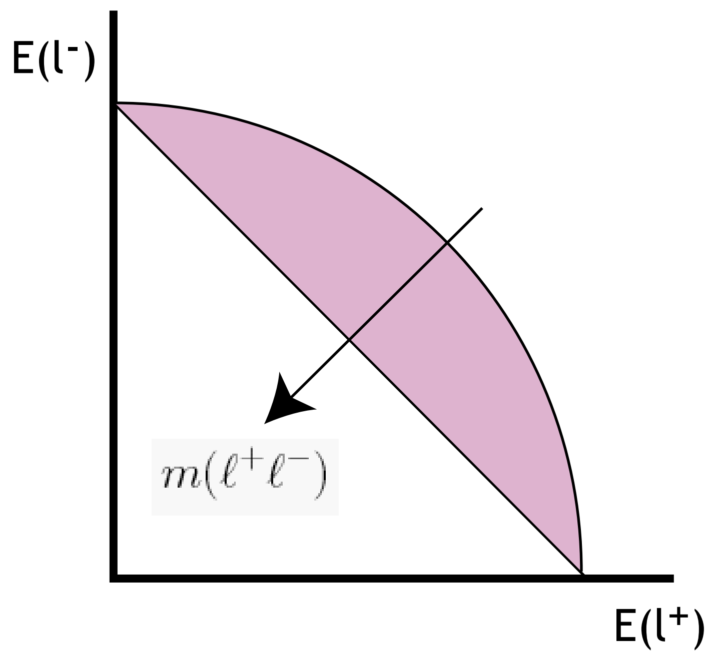


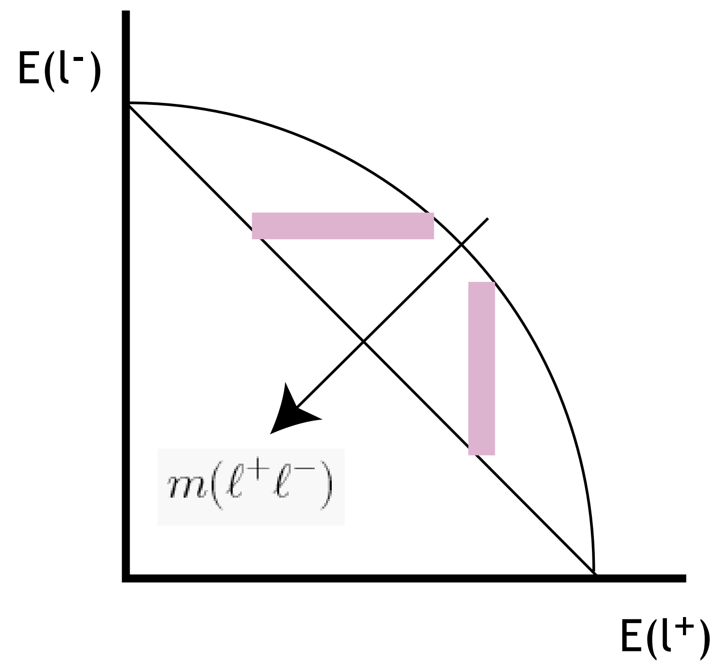
LHC Point 5
10^-7
10^-8
10^-9
10^-10
10^-11
10^-12
10^-13
dσ/dM_eff (mb/400 GeV)
0
1000
2000
3000
4000
M_eff (GeV)
ATLAS

example: (many more in the ATLAS Physics TDR)

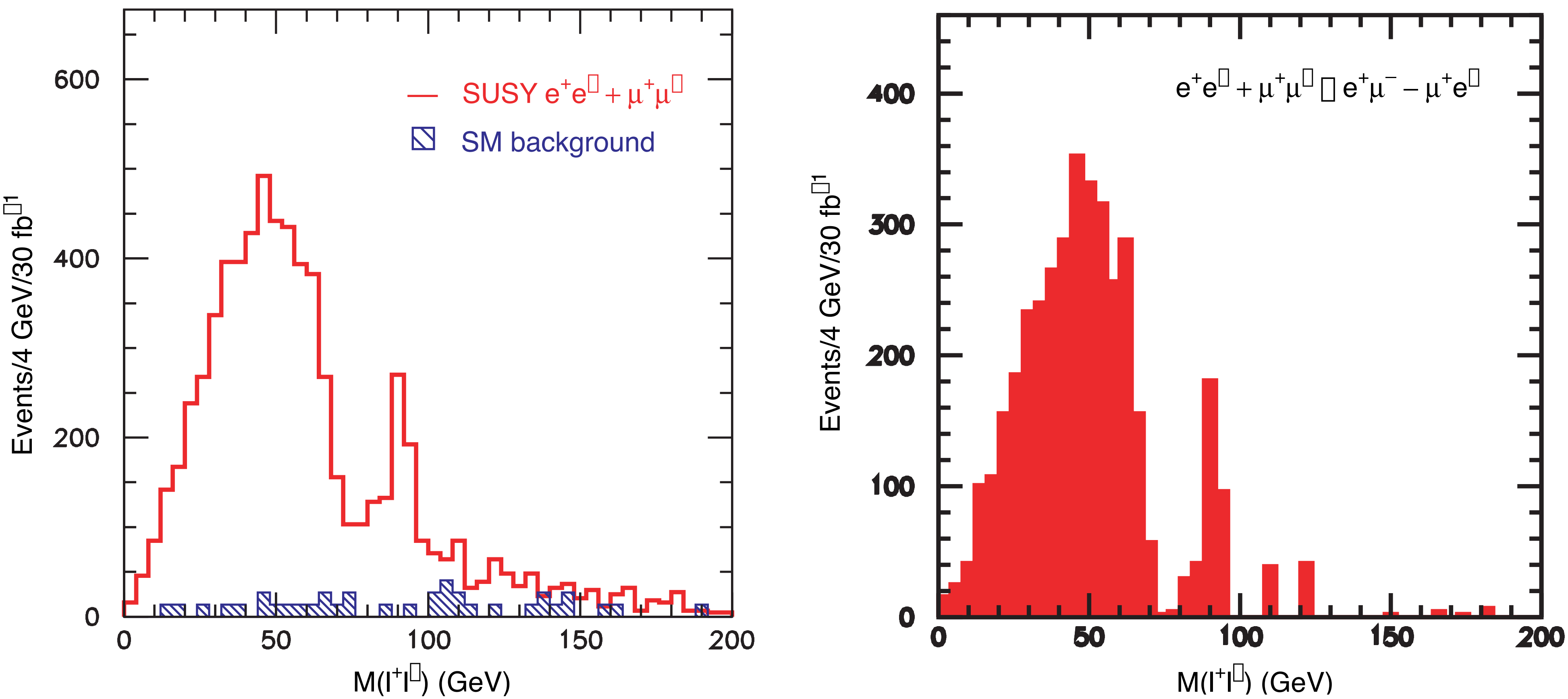
$\tilde{q}_L \to q\ \widetilde{N}_2^0 \to \ell^+\ell^-\ N_1^0$ or $\tilde{q}_L \to q\ \widetilde{N}_2^0 \to \ell^+\ \tilde{\ell}^- \to \ell^+\ell^-\ N_1^0$

in either case, the spectrum of m(ll) has a sharp kinematic endpoint



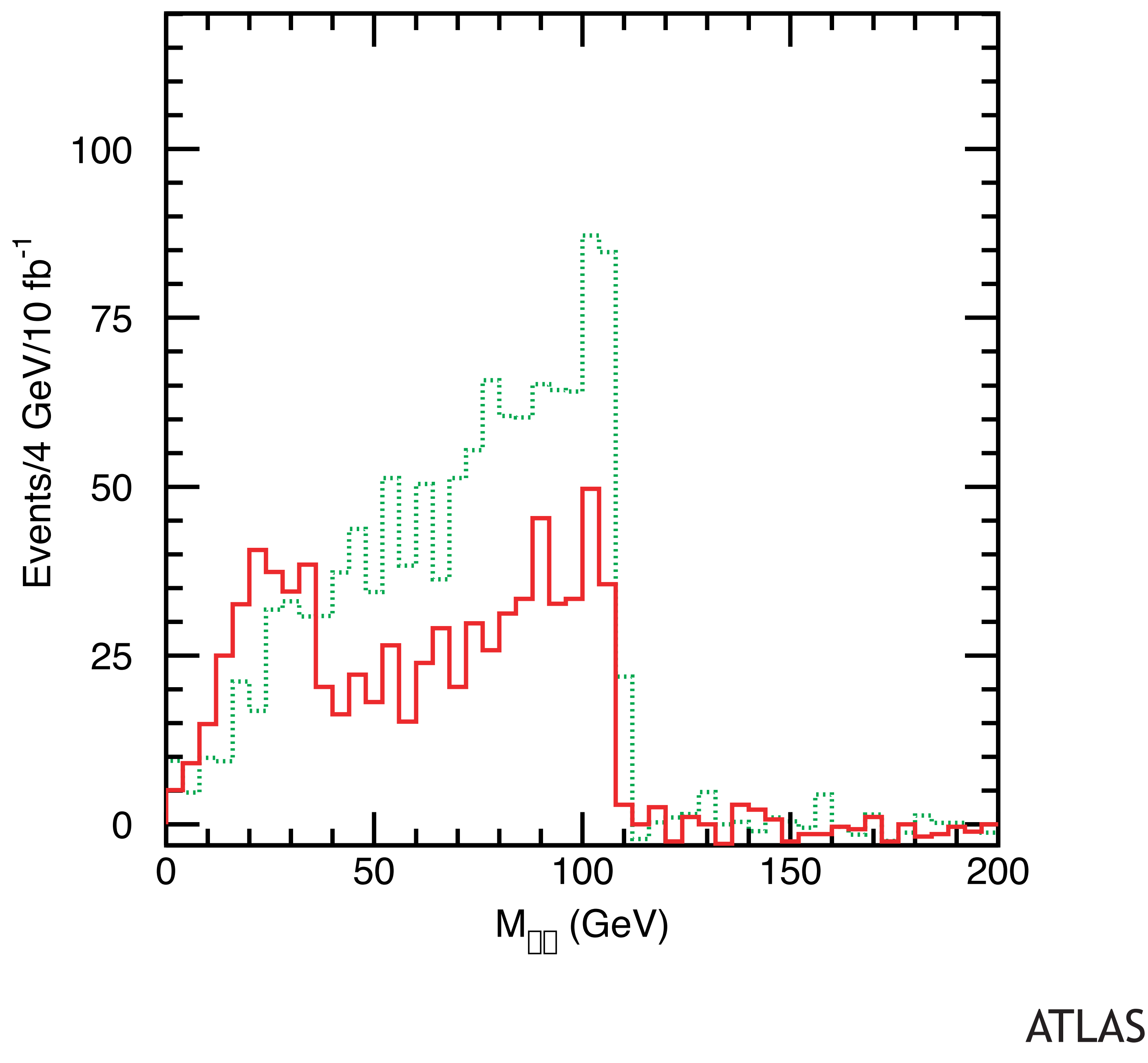


In the case with on-shell sleptons, there are multiple visible upper and lower endpoints -- more constraints than unknown masses.



SUSY $e^+e^- + \mu^+\mu^-$
SM background
Events/4 GeV/30 fb^{-1}
M(l^+l^-) (GeV)
$e^+e^- + \mu^+\mu^- - e^+\mu^- - \mu^+e^-$
Events/4 GeV/30 fb^{-1}
M(l^+l^-) (GeV)

ATLAS



Events/4 GeV/10 fb^{-1}
100
75
50
25
0
0
50
100
150
200
M (GeV)
ATLAS

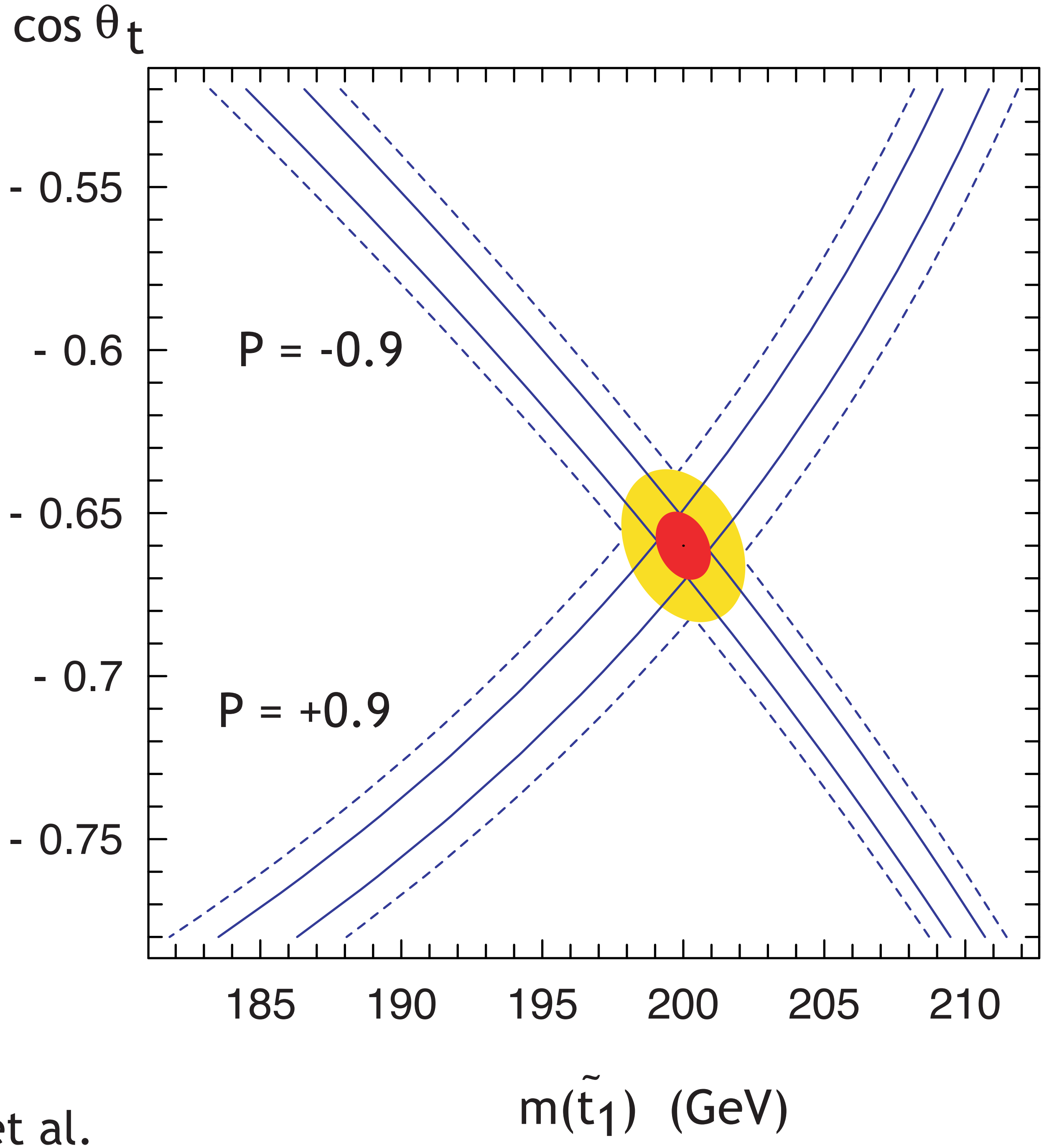
Linear Collider :

For $e^+e^- \to \gamma^*, Z^* \to \tilde{f}\tilde{\bar{f}}$

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\left|-Q + \frac{Q_{Z_e}Q_{Z_f}}{s_w^2 c_w^2}\frac{s}{s-m_Z^2}\right|^2 \sin^2\theta$$

so we can make an unambiguous determination of spin and electroweak quantum numbers

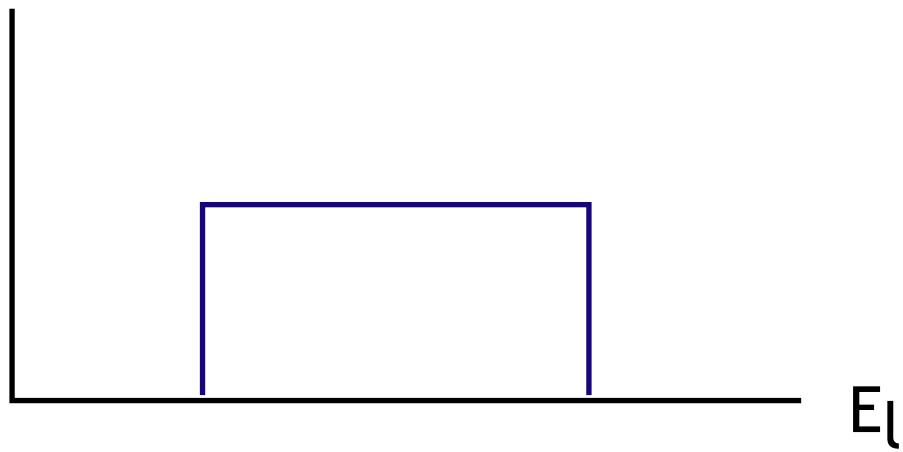
measuring $\sigma(e_R^-)/\sigma(e_L^-)$ determines the mixing angles for third-generation sfermions $\tilde{\tau}$, $\tilde{b}$, $\tilde{t}$.



Eberl et al.

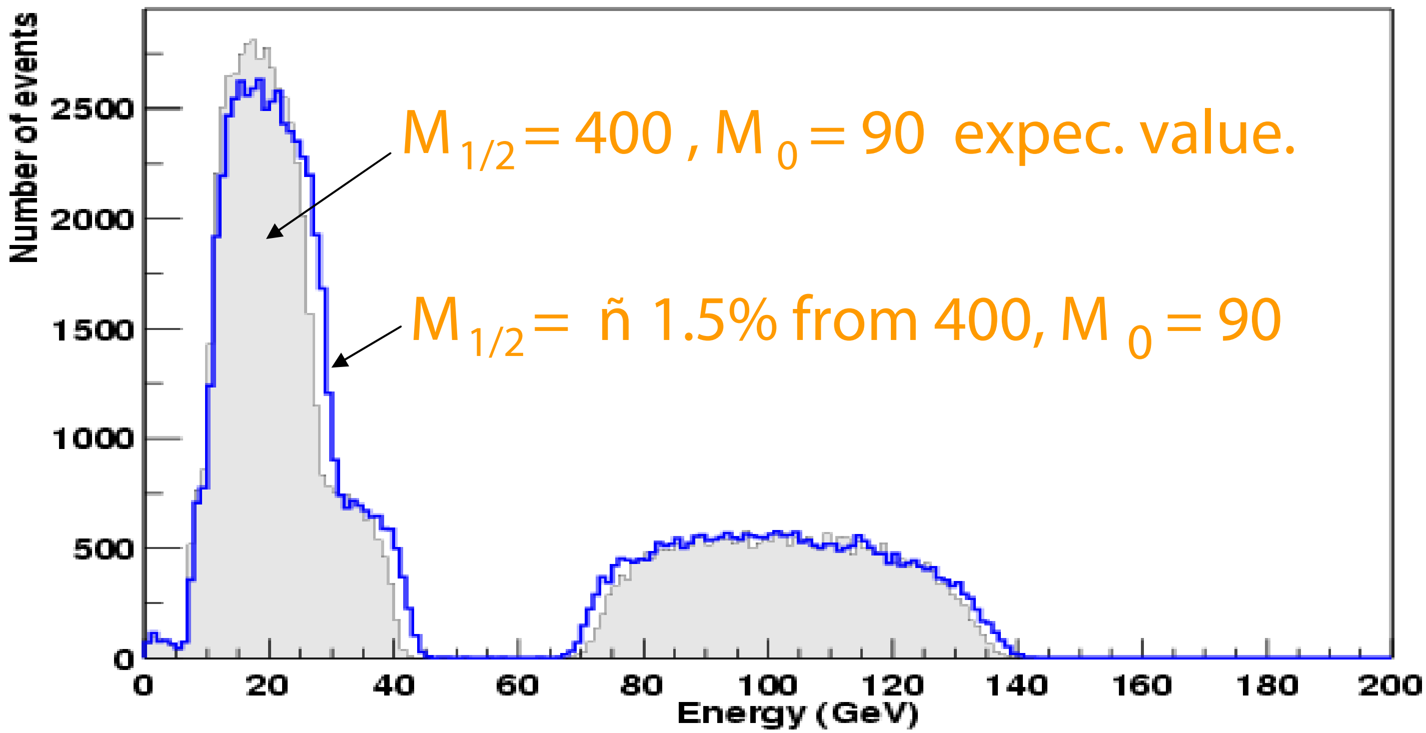
The decay of a scalar is isotropic in its rest frame.

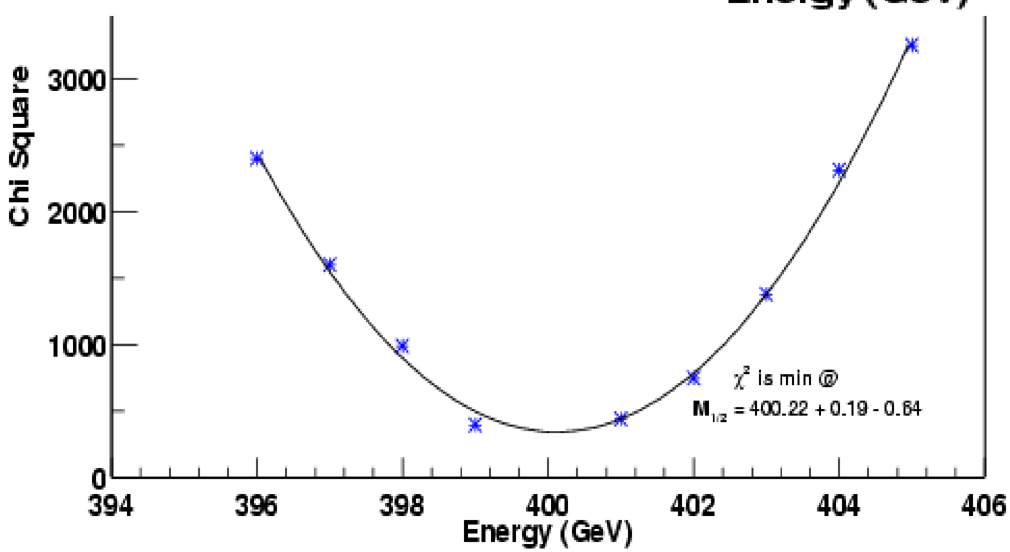
Boosting this gives an idealized flat energy distribution (to the extent that E_{CM} is fixed)



The kinematic endpoints can be used to determine SUSY masses to part-per-mil accuracy.

500 GeV



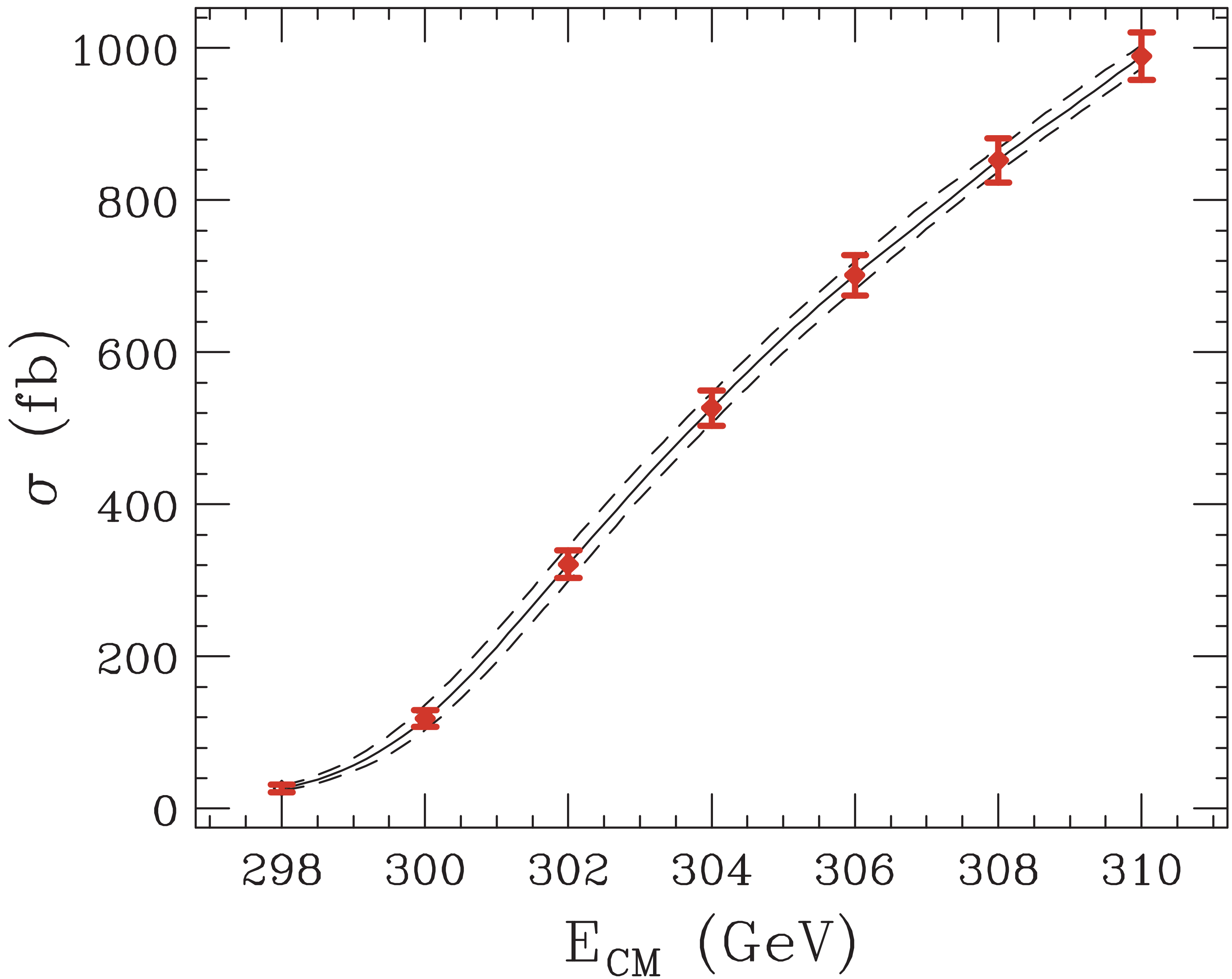


Mass $\tilde{e}_R$ = 178.3 + 0.06 - 0.2

Mass $\tilde{e}_L$ = 287.1 + 0.1 - 0.6

Mass $\tilde{\chi}_1^0$ = 160.5 + 0.09 - 0.3

Nauenberg



Feng et al.

$$e^+e^- \to \tilde{e}^+\tilde{e}^- \qquad e^-e^- \to \tilde{e}^-\tilde{e}^-$$

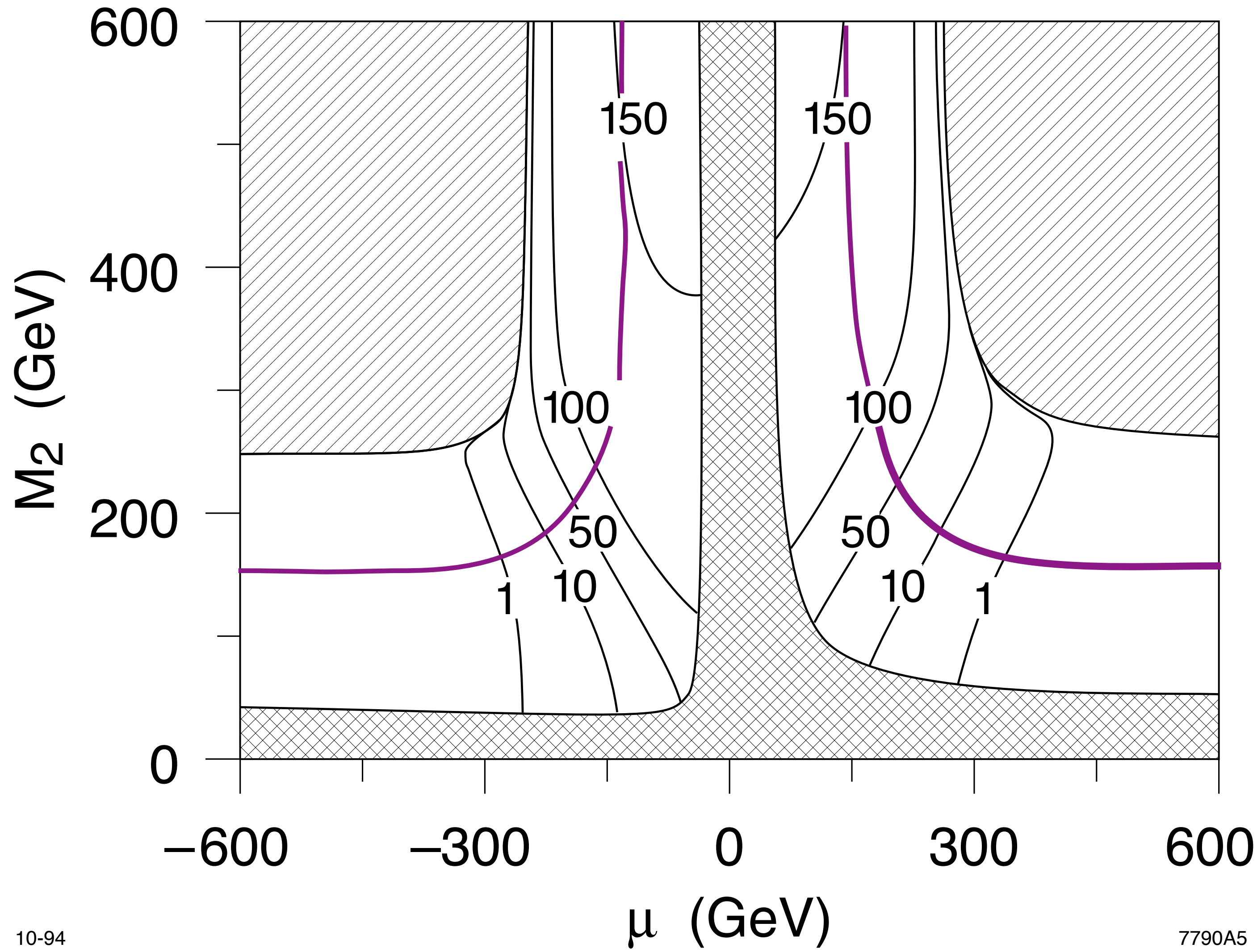
are dominated by t-channel $\widetilde{N}$ exchange.

$$e^+e^- \to \widetilde{C}^+\widetilde{C}^-$$

is strongly affected by beam polarization.

Both effects can be used to measure
gaugino/Higgsino mixing.

$\sigma(e^-_R e^+ \to \tilde{C}^+_1 \tilde{C}^-_1)$ (fb)



line of fixed $\tilde{C}_1$ mass

Feng et al.

Given that we have good reason to take SUSY seriously as the theory of TeV-scale physics, we should think through the experimental program that SUSY will require.

The questions to be answered are complex. But the answers are within reach.

The experiments will be rich and exiciting, well worth foresight and preparation.

Issues for Collider Theory (Linear Collider)

Radiative corrections to SUSY signal processes

this is straightforward, and many results are in hand

Radiative corrections to background processes

$$e^+e^- \to W^+W^-$$
$$\gamma e \to Ze \ , \ W\nu$$
$$\gamma\gamma \to \ell^+\ell^- \ , \ W^+W^-$$

tails of distributions must be undeterstood well

Precision (NNLO) calculation of processes needed for absolute and differential luminosity measurements

$$e^+e^- \to e^+e^-$$
$$e^+e^- \to Z\gamma$$