QCD RESUMMATIONS

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Theoretical motivations, phenomenological applications, directions

1. Why resummation?
2. When can we resum?
3. Resummation for electroweak annihilation
4. Resummation with color exchange
5. Resummed to nonperturbative
6. Directions

⋆ Why Resum?

Every final state in hard scattering carries the imprint of QCD dynamics from at all distance scales

- **Phenomenological**
  - **Logarithmic corrections: explicit**

$$\frac{d\sigma(Q)}{dQ_1} \propto \frac{1}{Q_1} \sum_n C_n \alpha_s^n \ln^{an+b}\left(\frac{Q}{Q_1}\right) \quad \Lambda \ll Q_1 \ll Q$$

⋆ **Z, H p_T, $\mathrm{e^+e^-}$ event shapes, BFKL**

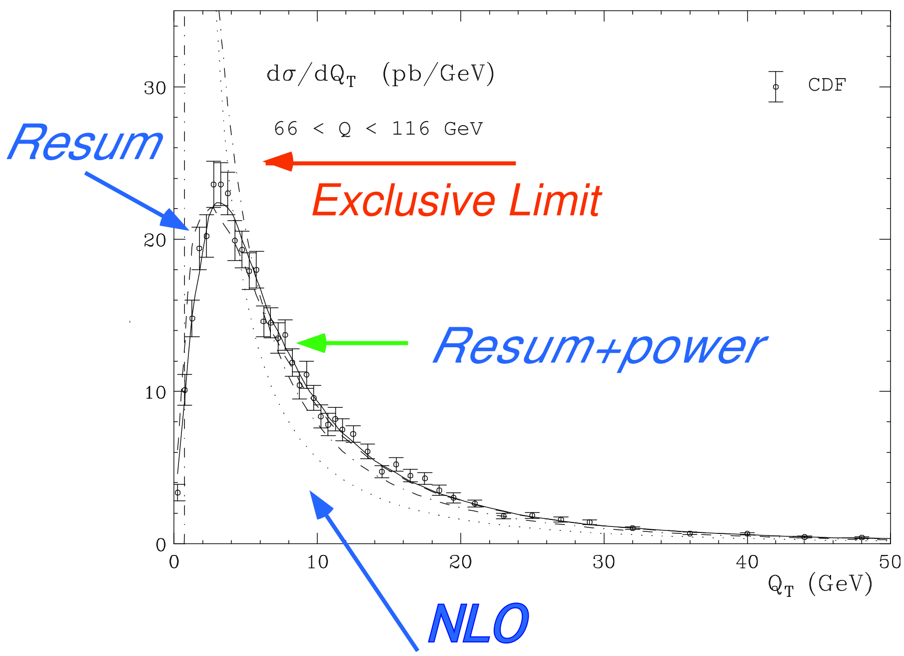
  - **Logarithmic corrections: implicit**

$$\sigma(Q) \propto \int \frac{dQ_1}{Q_1} \, F(Q_1) \, \sum_n C_n \alpha_s^n \ln^{an+b}\left(\frac{Q}{Q_1}\right) \quad F(0) = 0$$

⋆ **Threshold resummations, 1PI high-p_T**

- **Theoretical**
  - **QCD where the corrections are large!**
  - **Exploration of gauge theory**
    - ∗ **all-orders predictions; strong coupling**
    - ∗ **guide to nonperturbative dynamics**
    - ∗ **study distributions near their maxima**

• **Explicit logs: Z p_T at Run 1**



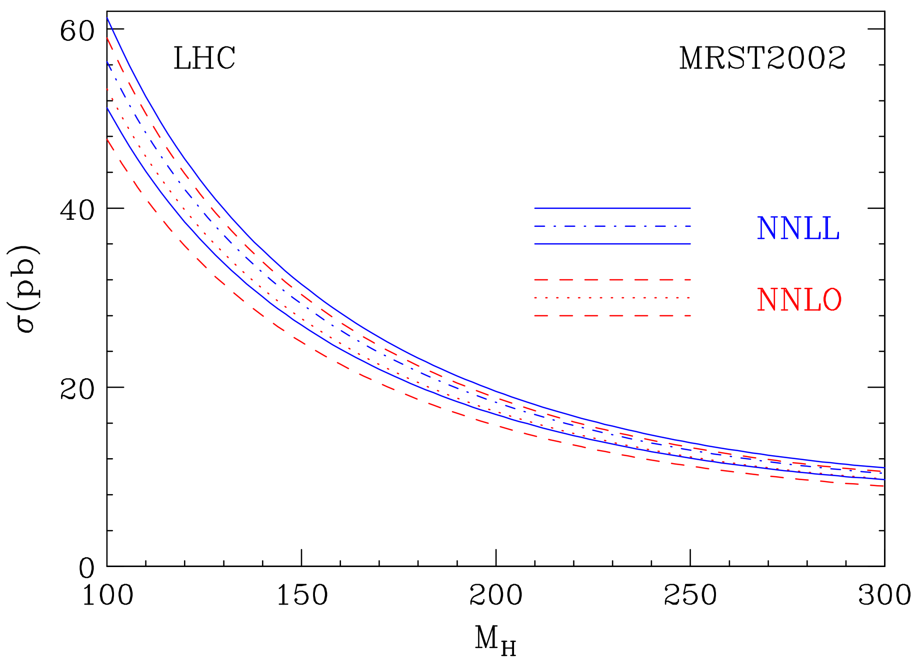
(From Kulesza, G.S., Vogelsang (2002) –

Balázs, de Florian, Kulesza (2002), E.L. Berger, Qiu (2003)

Bozzi, Catani, de Florian, Grazzini (NLO, 2003) qualitatively similar . . .)

- **maximum then decrease near "exclusive" limit (parton model kinematics) replaces divergence**
- **Soft but perturbative radiation broadens distribution**
- **Typically, NP correction necessary for quantitative description of data**
- **recover fixed order away from exclusive limit**

- **Implicit logs: threshold resummation vs. fixed order for H at LHC**



(From Catani, de Florian, Grazzini, Nason (2003))

  - **Modest change ↔ increased confidence**
  - **Modest decrease in scale dependence**
  - **Can expand to give predictions for extrapolation to higher fixed order**

    (e.g. Kidonakis, Laenen, Moch, Vogt (2003) for $c\bar{c}$)

⋆ When Can We Resum?

- **Infrared safety & asymptotic freedom:**

$$Q^2\ \hat{\sigma}_{\rm SD}(Q^2,\mu^2,\alpha_s(\mu)) \ = \ \sum_n c_n(Q^2/\mu^2)\ {\alpha_s}^n(\mu) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$
$$= \ \sum_n c_n(1)\ {\alpha_s}^n(Q) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$

  – $\mathrm{e^+e^-}$ **total; jets**

- **Generalization: factorization**

$$Q^2\sigma_{\rm phys}(Q,m) = \omega_{\rm SD}(Q/\mu,\alpha_s(\mu)) \otimes f_{\rm LD}(\mu,m) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$

μ = **factorization scale;** m= **IR scale**
(m *may* **be perturbative)**

  – **New physics in** $\omega_{\rm SD}$**;** $f_{\rm LD}$ **"universal"**
  – **Deep-inelastic,** $\mathrm{p\bar{p}} \rightarrow Q\bar{Q}$ **. . .**
  – **Exclusive decays:** $B \rightarrow \pi\pi$
  – **Exclusive limits:** $\mathrm{e^+e^-} \rightarrow \mathrm{JJ}$ **as** $m_J \rightarrow 0$

- **Whenever there is factorization, there is evolution**

$$0 = \mu \frac{d}{d\mu} \ln \sigma_{\rm phys}(Q,m)$$
$$\mu \frac{d \ln f}{d\mu} = -P(\alpha_s(\mu)) = -\mu \frac{d \ln \omega}{d\mu}$$

- **Wherever there is evolution, there is resummation**

$$\ln \sigma_{\rm phys}(Q,m) = \exp\left\{ \int_q^Q \frac{d\mu'}{\mu'} P\left(\alpha_s(\mu')\right) \right\}$$

- **Can resum when we can factorize**
- **This is the viewpoint I will mainly take here**
- **But other viewpoints fruitful: e.g. coherent branching: recursive properties of diagrams with collinear kinematics. (Angular ordering.) Each branching a mini-factorization**
- **Flexible. Many results first shown this way.**
- **Relation of factorization/branching/showering bears further study**

- **Factorization structure and proofs:**
  - **(1)** $\omega_{\rm SD}$ **incoherent with LD dynamics**
  - **(2) mutual incoherence when** $v_{\rm rel} = c$
  - **For large** $Q \sim s$**: long-distance logs from**

$$\frac{d\sigma(Q, a+b \rightarrow N_{\rm jets})}{dQ}$$
$$= \int dx_a dx_b \; H(x_a p_a, x_b p_b, Q)_{a'b' \rightarrow c_1 \dots c_{N_{\rm jets}}}$$
$$\times \mathcal{P}_{a'/a}(x_a p, X_a)\, \mathcal{P}_{b'/b}(x_b p, X_b)$$
$$\otimes_{\rm soft} \prod_{i=1}^{N_{\rm jets}} \; J_{c_i}(X_i) \;\; \otimes_{\rm soft} S_{a'b' \rightarrow c_1 \dots c_{N_{\rm jets}}}(X_{\rm soft})$$

  - **A story with only these pieces:**
    * **Evolved incoming partons** $\mathcal{P}_{a'/a}$, $\mathcal{P}_{b'/b}$ **collide at** H**, with** $X_{a,b}$ **"fragments",**
    * **to produce outgoing jets** J_{c_i}
    * **and coherent soft emission** S**,**
    * **to any fixed** α_s^n**, all** $\ln^a \mu/Q$**,**
    * **with only power corrections.**

- **Why this structure?**
- **Any diagram can be written as a sum of ordered time integrals: $\tau_i \to \infty$ give logs**

  (c.f. Forde, Signer (2003), DelDuca, Magnea, GS (1990))

$$
\begin{aligned}
\Gamma \; = \; & \sum_{\tau\,\mathrm{orders}} \int_{-\infty}^{\infty} d\tau_n \ldots \int_{-\infty}^{\tau_2} d\tau_1 \\
& \times \prod_{\mathrm{loops}\, i} \int \frac{d^3\ell_i}{(2\pi)^3} \prod_{\mathrm{lines}\, j} \frac{1}{2E_j} \\
& \times \exp\left[i \sum_{\mathrm{states}\, m} \left(\sum_{j\,\mathrm{in}\, m} E(\vec{p}_j) \right) (\tau_m - \tau_{m-1}) \right] \\
& \times \; (\text{spin factors})
\end{aligned}
$$

  - **Long times require stationary phase**

$$
\frac{\partial}{\partial \ell_{i\mu}} \left[\mathrm{phase} \right] = \sum_{\mathrm{states}\, m} \; \sum_{j\,\mathrm{in}\, m} (\pm \beta_j^\mu)(\tau_{m+1} - \tau_m) = 0
$$

  - $\beta_j = \pm \partial E_j / \partial \ell_i$ **for j in loop i is four-velocity**
  - **distance travelled around any loop is zero**
  - **Long times $\leftrightarrow$ free classical propagation**

    (Coleman-Norton interpretation of Landau equations)

    Fragments and outgoing jets can never rescatter with finite momentum transfer

- **Extension: "non-global" observables Indefinite number of jets (see below)**

  (Dasgupta & Salam (2001))

- **Cancellation of FS Interactions**
  - **The phase:**

$$\text{phase} = \sum_{\text{states}\, m} \; \sum_{j\, \text{in}\, m} E_j(\vec{p}_j)\,(\tau_{m+1} - \tau_m)$$

  - **At stationary phase**

$$\text{phase} = \sum_{\text{jets}\, i} E_i^{(\text{total})} \times (\,\text{time elapsed}\,)$$

  - **Phase unchanged by IR emission, CO rearrangement**
  - **If also weight all states within jets the same at stationary phase:**
  - **Cross sections is sum over all long time FS interactions**
  - **Inclusive cross section "forgets" large times $\leftrightarrow$ Infrared safety**
- **In applications . . .**

$\star$ Electroweak Annihilation

- **Initial State: EW boson production (Q^μ)**
- **Hadronic FS unobserved except $-Q_T$**

$$\sigma(Q) \sim \int dx_a dx_b \; H(x_a p_a, x_b p_b, Q)_{a'b'\to c_1\dots c_{N_{\rm jets}}}$$
$$\times \mathcal{P}_{a'/a}(x_a p, X_a)\, \mathcal{P}_{b'/b}(x_b p, X_b)$$
$$\otimes_{\rm soft} \prod_{i=1}^{N_{\rm jets}} \; J_{c_i}(X_i) \;\; \otimes_{\rm soft} S_{a'b'\to c_1\dots c_{N_{\rm jets}}}(X_{\rm soft})$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$\frac{d\sigma_{ab\to QX}}{dQ d^2Q_T} = \int dx_a dx_b \;\; H(x_a p_a, x_b p_b, Q, n)_{a'b'\to c_1\dots c_{N_{\rm jets}}}$$
$$\times \mathcal{P}_{a'/a}(x_a, p{\cdot}n, k_{aT})\, \mathcal{P}_{\bar{a}'/b}(x_b, p{\cdot}n, k_{bT})$$
$$\otimes_{Q_T=-k_{aT}-k_{bT}-k_{sT}} \; S_{a'\bar{a}'}(k_{sT}, n)$$

- n^μ **specifies frame for jets**
- $S_{a'\bar{a}'}$ **coupled only in $Q_T \Rightarrow$ Fourier** $e^{i\vec{Q}_T\cdot\vec{b}}$
- **LHS independent of $\mu_{\rm ren}$, n**
- $\Rightarrow$ **two equations**

$$\mu_{\rm ren}\frac{d\sigma}{d\mu_{\rm ren}} = 0 \qquad n^\alpha \frac{d\sigma}{dn^\alpha} = 0$$

(Method of Collins and Soper (1981) for Sudakov resummation)

- **Threshold Resummation: same method, fix** $E_{\rm had} \sim (1-z)Q \quad z = \frac{Q^2}{x_a x_b S} \rightarrow$ Mellin transform z^N
- **Joint Resummation: same method, fix** Q_T AND $E_{\rm had} \;\Rightarrow\;$ Fourier AND Mellin
- **The jointly resummed cross section (Higgs)**

$$\frac{d\sigma^{\rm res}_{AB}}{dQ^2\, d^2\vec{Q}_T} = H(\alpha_s(Q^2)) \int_{C_N} \frac{dN}{2\pi i}\, \tau^{-N} \int \frac{d^2b}{(2\pi)^2}\, e^{i\vec{Q}_T\cdot\vec{b}} \times \exp\left[\, E^{\rm PT}_{gg}(N,b,Q,\mu)\,\right] \times \prod_{H=A,B} \mathcal{C}_{g/H}(Q,b,N,\mu,\mu_F)$$

  - **Double inverse transform**
  - **Soft gluon exponent has familiar form**

$$E^{\rm PT}_{gg} = -\int_{\frac{Q^2}{\chi^2}}^{Q^2} \frac{dk_T^2}{k_T^2} \left[A_g(\alpha_s(k_T))\, \ln\left(\frac{Q^2}{k_T^2}\right) + B_g(\alpha_s(k_T))\right]$$

  - **But with lower limit:** Q/χ**,** $\chi \sim bQ + N$
  - **Evolved Distributions, coefficients**

$$\mathcal{C}_{g/H} = \sum_j C_{g/j}\,(N, \alpha_s(Q/\chi))\; f_{j/H}(N, Q/\chi)$$

  * $b \rightarrow 0$, $j \rightarrow g$ only**: threshold;** $N \rightarrow 0$**:** $\mathbf{Q_T}$
  * **Joint curve slightly below pure** Q_T **for H**

    (Kulesza, GS Vogelsang (2003); Catani et al. (2003))

- **Final State: Two-jet event shapes in** $\mathrm{e^+e^-}$
  - **Interpolating event shapes**

    (C.F. Berger, Kúcs, GS (2003))

    $\tau_a = \frac{1}{Q}\ \Sigma_{i\,\mathrm{in}\,N}\ E_i\ (\sin\theta_i)^a\ (1-|\cos\theta_i|)^{1-a}$

    * θ_i **angle to thrust** ($a=0$) **axis**
    * **broadening:** $a=1$**; inclusive limit** $a\to\infty$

$$\sigma(Q)\ \sim\ \int dx_a dx_b\ H(x_a p_a, x_b p_b, Q)_{a'b'\to c_1\ldots c_{N_{\mathrm{jets}}}}$$
$$\times \mathcal{P}_{a'/a}(x_a p, X_a)\, \mathcal{P}_{b'/b}(x_b p, X_b)$$
$$\otimes_{\mathrm{soft}} \prod_{i=1}^{N_{\mathrm{jets}}}\ J_{c_i}(X_i)\ \otimes_{\mathrm{soft}}\ S_{a'b'\to c_1\ldots c_{N_{\mathrm{jets}}}}(X_{\mathrm{soft}})$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$\frac{d\sigma_{\mathrm{e^+e^-}\to 2J}}{d\tau} = \ H(p_{J1}, p_{J2}, n)_{c\bar{c}}$$
$$\times \prod_{i=c,\bar{c}} J_i(\tau_{Ji}, p_{Ji}, n)\ \otimes_{\tau=\tau_{J1}+\tau_{J2}+e\tau_S}\ S_{c\bar{c}}(\tau_S, n)$$

  - **Convolution in** $\tau \Rightarrow$ **Laplace transform** $\mathrm{e}^{-\nu\,\tau_a}$
  - **Independence of** μ_{ren}, $n \Rightarrow$ **two equations**
  - **NLL analysis: jets absorb** S**; equivalent to coherent branching result**

– **NLL resummed cross section**

$$\sigma\left(\tau_a, Q, a\right) = \sigma_{\rm tot} \int_C d\nu\, {\rm e}^{\nu\,\tau_a} \left[\, J_i(\nu, p_{Ji}, n) \,\right]^2$$

– **At NLL can define $S_{c\bar{c}} = 1$: normalizes jets**

– **Indepenent jet evolution in coherent branching**

(Catani, Turnock, Trentadue, Webber (1990-92))

$$J_i(\nu, p_{Ji}, n) = \int_0 d\tau_a\, {\rm e}^{-\nu\tau_{Ji}}\, J_i(\tau_{Ji}, p_{Ji}, n) = {\rm e}^{\frac{1}{2}E(\nu,Q,a)}$$

$$E(\nu, Q, a) = 2\int_0^1 \frac{du}{u}\Big[\int_{u^2Q^2}^{uQ^2} \frac{dp_T^2}{p_T^2} A\left(\alpha_s(p_T)\right)\left({\rm e}^{-u^{1-a}\nu(p_T/Q)^a} - 1\right) + \frac{1}{2} B\left(\alpha_s(\sqrt{u}Q)\right)\left({\rm e}^{-u(\nu/2)^{2/(2-a)}} - 1\right)\Big]$$

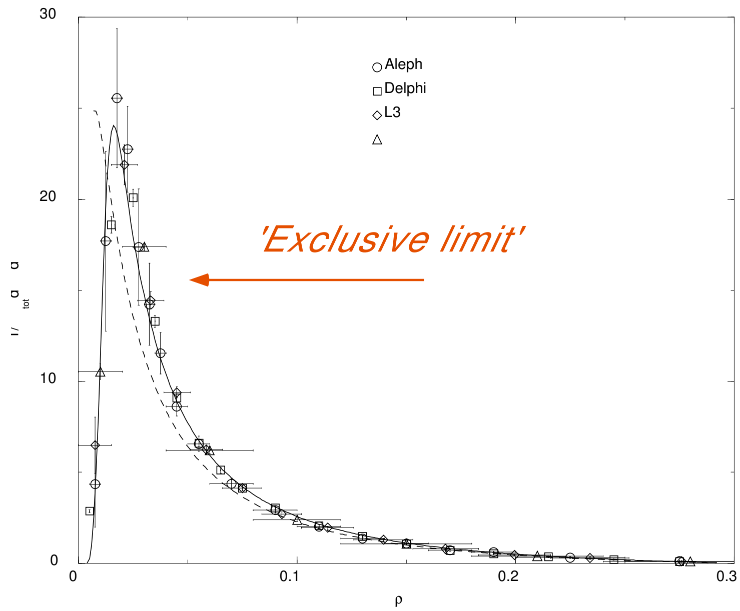
– **n-independent**

– **$a < 1$: $a \geq 1$ recoil non-negligible**

(Dokshitzer, Lucenti, Marchesini, Salam (1998))

– **Example: Heavy jet distribution at the Z pole ($\sim \tau_0$)**

(Korchemsky and Tafat (2000))



  * **Dashed line: NP "shape function" fit**

– **Jet shapes in DIS similar only if overall final state limited (global)**

(Dasgupta and Salam (2000, 2002))

– **Semi-numerical resummation (CAESER)**

(Banfi, Salam, Zanderighi (2003))

⋆ Resummation with Color Exchange

- **Resummed amplitudes in dimensional regularization**

  (Tejeda-Yeomans & GS (2002) Kosower (2003))

  – **Amplitude for partonic process**

  f : $f_A(p_A,r_A)+f_B(p_B,r_B) \longrightarrow f_1(p_1,r_1)+f_2(p_2,r_2)$

$$\mathcal{M}^{[\mathrm{f}]}_{\{r_i\}}\left(p_j,\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right)=\mathcal{M}^{[\mathrm{f}]}_{L}\left(p_j,\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right)\,(c_L)_{\{r_i\}}$$

  – **Jet/soft factorization** (Sen (1983))**:**

$$\mathcal{M}^{[\mathrm{f}]}_{L}\left(p_i,\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right)=\prod_{i=A,B,1,2}J_i^{[\mathrm{virt}]}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right)$$
$$\times\mathbf{S}^{[\mathrm{f}]}_{LI}\left(p_i,\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right)\,h_I^{[\mathrm{f}]}\left(\wp_i,\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)$$

  – **Soft function labelled by color exchange (singlet, octet . . .)**

  – **Factors require dimensional regularization**

  – **Same factorization → resummation**

  – **Poles at 2- and higher loops . . .**

– **Dimensionally-regularized jets normalized as above by $f\bar{f}$ annihilation**

(Magnea & GS (1990), Magnea (2000))

$$J_i\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right) = \exp\Big\{ \frac{1}{4}\int_0^{-Q^2}\frac{d\xi^2}{\xi^2}\Big[\mathcal{K}^{[i]}(\alpha_s(\mu^2),\epsilon) + \mathcal{G}^{[i]}\left(-1,\bar{\alpha}_s\left(\frac{\mu^2}{\xi^2},\alpha_s(\mu^2),\epsilon,\right)\epsilon\right) + \frac{1}{2}\int_{\xi^2}^{\mu^2}\frac{d\tilde{\mu}^2}{\tilde{\mu}^2}\gamma_K^{[i]}\left(\bar{\alpha}_s\left(\frac{\mu^2}{\tilde{\mu}^2},\alpha_s(\mu^2),\epsilon\right)\right)\Big]\ \Big\}.$$

– γ_K, $\mathcal{K}$ **related to** A **above,** $\mathcal{G}+\mathcal{K}$ **to** B

– **Dimensionally-regularized** S

$$\mathbf{S}^{[\mathrm{f}]}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right) = \mathrm{P}\ \exp\left[-\frac{1}{2}\int_0^{-Q^2}\frac{d\tilde{\mu}^2}{\tilde{\mu}^2}\mathbf{\Gamma}^{[\mathrm{f}]}\left(\bar{\alpha}_s\left(\frac{\mu^2}{\tilde{\mu}^2},\alpha_s(\mu^2),\epsilon\right)\right)\right]$$

– $\Gamma^{[\mathrm{f}]}$**: anomalous dimension; color mixing**

- **Color Mixing** (Date (1983) Sen (1983) . . .
  Kidonakis & GS (1996) Bonciani et al. (1998,2003))

  – **Cross sections & amplitudes: NLL exponentiation in basis that diagonalizes Γ**

$$\exp \int_{Q_1}^{Q} \frac{dm}{m} \left[\lambda^{(\mathrm{f})}(\alpha_s(m))\right]$$

  – **[f] color exchange basis, λs: eigenvalues of color exchange (anom. dim.) matrix Γ_S**

  – **Eigenvalues control (NLL) probability for no (wide-angle) radiation between scales Q and Q_1**

  – **Generalized showering?**

  – **If radiation at Q_1: new anomalous dimension matrix with more partons . . .**

  – **Example: f: $g + g \to g + g$**

$$\mathrm{Tr}\left[\,T_{a_1}T_{a_2}T_{a_3}T_{a_4}\,\right] \text{ and 5 perms}$$
$$\left[\,T_{a_1}T_{a_2}\right]\ \mathrm{Tr}\left[T_{a_3}T_{a_4}\right] \text{ and 2 perms}$$

  – **Color mixing governed by a 9×9 matrix**

– $g + g \to g + g$

(Kidonakis, Oderda, GS (1998))

$$\Gamma_{S'} = \frac{\alpha_s C_A}{\pi} \begin{pmatrix} T & 0 & 0 & 0 & 0 & 0 & -\frac{U}{N_c} & \frac{T-U}{N_c} & 0 \\ 0 & U & 0 & 0 & 0 & 0 & 0 & \frac{U-T}{N_c} & -\frac{T}{N_c} \\ 0 & 0 & T & 0 & 0 & 0 & -\frac{U}{N_c} & \frac{T-U}{N_c} & 0 \\ 0 & 0 & 0 & (T+U) & 0 & 0 & \frac{U}{N_c} & 0 & \frac{T}{N_c} \\ 0 & 0 & 0 & 0 & U & 0 & 0 & \frac{U-T}{N_c} & -\frac{T}{N_c} \\ 0 & 0 & 0 & 0 & 0 & (T+U) & \frac{U}{N_c} & 0 & \frac{T}{N_c} \\ \frac{T-U}{N_c} & 0 & \frac{T-U}{N_c} & \frac{T}{N_c} & 0 & \frac{T}{N_c} & 2T & 0 & 0 \\ -\frac{U}{N_c} & -\frac{T}{N_c} & -\frac{U}{N_c} & 0 & -\frac{T}{N_c} & 0 & 0 & 0 & 0 \\ 0 & \frac{U-T}{N_c} & 0 & \frac{U}{N_c} & \frac{U-T}{N_c} & \frac{U}{N_c} & 0 & 0 & 2U \end{pmatrix}$$

– **Same matrix generates $\epsilon = 4 - D$ poles in $\mathcal{O}(\alpha_s^2)$ $gg \to gg$ scattering:** $\exp(I^{(1)})$

(Bern, Dixon, Kosower (2000) Anastasiou, Glover, Oleari, Tejeda-Yeomans (2000) Catani (1998))

$$I^{(1)}(\epsilon)(s = \mu_R^2) \sim \left(\frac{1}{\epsilon^2} + \frac{\beta_0}{N_c \epsilon}\right) \Gamma_S$$

- **Threshold Resummation for Jet Production**
  - **Moments:** $\left(M_{JJ}^2/S\right)^N$
  - **Factorization:** **2 incoming, 2 outgoing jets**
  - **Soft function with two color indices ($M,\ M^*$)**

$$\tilde{\sigma}_\alpha(N) \propto \exp\Big\{ \sum_{i=A,B} \Big[E_{(f_i)}(N, M_{JJ}) - 2\int_\mu^{M_{JJ}} \frac{d\mu'}{\mu'} \left[\gamma_{f_i}(\alpha_s(\mu'^2)) - \gamma_{f_i f_i}(N, \alpha_s(\mu'^2))\right]\Big]\Big\}$$
$$\times \exp\Big\{ \sum_{j=1,2} E'_{(f_j)}(N, M_{JJ})\Big\}$$
$$\times \mathrm{Tr}\Big\{H^{(\alpha)}\left(\frac{M_{JJ}}{\mu}, \Delta y, \alpha_s(\mu^2)\right)$$
$$\times \bar{P}\exp\left[\int_\mu^{M_{JJ}/N} \frac{d\mu'}{\mu'}\, \Gamma_S^{(\alpha)\dagger}\left(\alpha_s(\mu'^2)\right)\right]\ \tilde{S}^{(\alpha)}(1, \Delta y, \alpha_s\left(M_{JJ}^2/N^2\right))$$
$$\times P\exp\left[\int_\mu^{M_{JJ}/N} \frac{d\mu'}{\mu'}\, \Gamma_S^{(\alpha)}\left(\alpha_s(\mu'^2)\right)\right]\Big\}$$

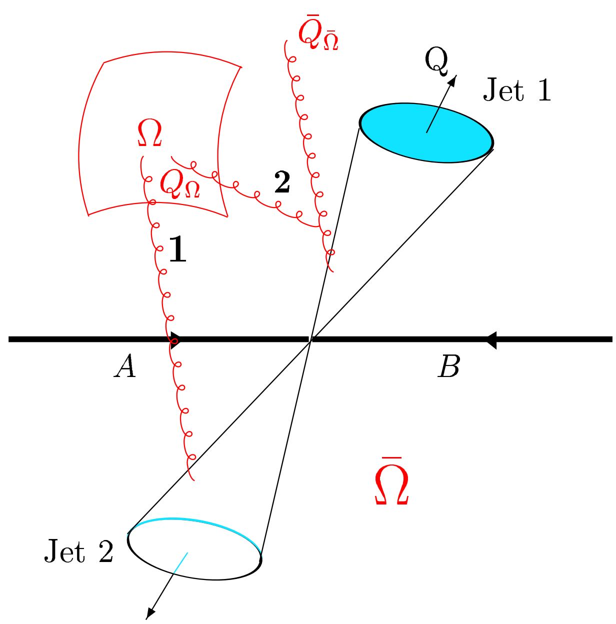
  - **S solution to:**

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\right) S_{LI}^{(\alpha)} = -(\Gamma_S^{(\alpha)\dagger})_{LB} S_{BI}^{(\alpha)} - S_{LA}^{(\alpha)}(\Gamma_S^{(\alpha)})_{AI}$$

  - **NLO can be stable at colliders** (Kidonakis and Owens (2001), FONLL: Cacciari, Frixione, Mangano, Nason, Ridolfi (2003), 2003 RHIC results for 1PI)

- **Non-global logs: color and energy flow**
  (Dasgupta & Salam (2001))

  - **Observe 2 jets**
  - **Measure distribution $\Sigma_\Omega(E)$:**
    - ∗ **a) Correlation with event shape: $\rightarrow$ factorization** (Berger, Kúcs, GS (2003))
    - ∗ **b) Inclusive for momentum transfer Q Number of jets outside Ω not fixed!**



    - ∗ **for a) Resum as above**
    - ∗ **for b) $\rightarrow$ No unique factorization: need recursive relation** (Banfi, Marchesini, Smye (2002))

– **To LL in E/Q ($\Delta \sim \ln(E)$)**

$$\partial_\Delta \Sigma_{ab}(E) = -\partial_\Delta R_{ab}\, \Sigma_{ab}(E) + \int_{k \text{ not in } \Omega} dN_{ab\to k}\, \left(\Sigma_{ak}\Sigma_{kb} - \Sigma_{ab}\right)$$

$$dN = \frac{d\Omega_k}{4\pi}\, \frac{\beta_a \cdot \beta_b}{\beta_k \cdot \beta_b\, \beta_k \cdot \beta_a}$$

$$R_{ab} = \int_E^Q \frac{dE'}{E'}\, \int_\Omega dN_{ab\to k}$$

– **A large-N result: planar color structure quark dipole radiates and produces a pair of radiating dipoles**

– **Intriguing relation with approach to small-x saturation**

(Balitsky (1995), Kovchegov (1998), Weigert (2003))

– **New perspective on energy flow analysis rapidity gaps & as diagnostic of hard scattering**

Marchesini, Webber (1987)

- **No hard scattering: BFKL & beyond from factorization**

  (Sen (1980) Balitsky (1996) Kúcs (2003))

  - $q^2 = -t \ll s$; **Regge limit in PT**

$$
\begin{aligned}
A(t,s) \;=\; & \sum_{n,m} \int \left(\prod_{i=1}^{n-1} \mathrm{d}^{\mathrm{D}-2} \mathrm{k}_{\mathrm{i}\perp} \right) \left(\prod_{j=1}^{m-1} \mathrm{d}^{\mathrm{D}-2} \mathrm{p}_{\mathrm{j}\perp} \right) \\
& \times \Gamma_A^{(n)\, a_1 \dots a_n}(p_A, q, n, k_{1\perp}, \dots, k_{n\perp}) \\
& \times S'^{(n,m)}_{a_1 \dots a_n, b_1 \dots b_m}(q, n; k_{1\perp}, \dots, k_{n\perp}; p_{1\perp}, \dots, p_{m\perp}) \\
& \times \Gamma_B^{(m)\, b_1 \dots b_m}(p_B, q, n; p_{1\perp}, \dots, p_{m\perp})
\end{aligned}
$$

  - **Factorization at fixed rapidity separation: Jets and soft exchange**
  - **Generically** m **convolutions at** $N^m LL$

$$
\begin{aligned}
& \left(p_A \cdot n \frac{\partial}{\partial\, p_A \cdot n} - 1 \right) \Gamma_A^{(n)\, a_1 \dots a_n}(p_A, q, n; k_{1\perp}, \dots, k_{n\perp}) = \\
& \sum_m \int \prod_{j=1}^{m} \mathrm{d}^{D-2} l_{j\perp}\, \mathcal{K}^{(n,m)}_{a_1 \dots a_n; b_1 \dots b_m}(k_{1\perp}, l_{1\perp}, \dots; q, n) \\
& \times \Gamma_A^{(m)\, b_1 \dots b_m}(p_A, q, n; l_{1\perp} \dots)
\end{aligned}
$$

  - **Project color exchange:**
    octet, $m = 0$ **LL reggeized gluon**
    singlet, $m = 1$**, BFKL LL pomeron**

⋆ From Resummed PT to NP QCD

- **Resummed logs →**
  resummmed power corrections
- **How to interpret expressions like**

$$E(\nu,Q,a) = 2\int_0^1 \frac{du}{u}\Big[\int_{u^2Q^2}^{uQ^2}\frac{dp_T^2}{p_T^2}A\left(\alpha_s(p_T)\right)\left(\mathrm{e}^{-u^{1-a}\nu(p_T/Q)^a}-1\right) + \frac{1}{2}B\left(\alpha_s(\sqrt{u}Q)\right)\left(\mathrm{e}^{-u(\nu/2)^{2/(2-a)}}-1\right)\Big]$$

- **Shape function approach:** $\mathrm{e^+e^-}$ **jets**
  - $p_T > \kappa$, **PT**
  - $p_T < \kappa$, **expand exponentials**
  - **Low** p_T **replaced by** f_{NP}

$$E(\nu,Q,a) = E_{\mathrm{PT}}(\nu,Q,\kappa,a) + \frac{2}{1-a}\sum_{n=1}^{\infty}\frac{1}{n\,n!}\left(-\frac{\nu}{Q}\right)^n\int_0^{\kappa^2}\frac{dp_T^2}{p_T^2}\,p_T^n\,A\left(\alpha_s(p_T)\right)\left[1-\left(\frac{p_T}{Q}\right)^{n(1-a)}\right] + \ldots$$

$$\equiv E_{\mathrm{PT}}(\nu,Q,\kappa,a) + \ln\tilde{f}_{a,\mathrm{NP}}\left(\frac{\nu}{Q},\kappa\right)$$

– **Analogous to power corrections like $\langle(1-T)\rangle \sim PT + 1/Q$**

  * **integral of universal anomalous dimension $A(\alpha_s)$ (c.f. integral of α_s in dispersive treatment of integrated event shapes: α_0)**

    (Banfi, Dokshitzer, Lucenti, Marchesini, Salam, Webber, Zanderighi)

  * **enhanced by powers of moment variables ($\nu/Q \leftrightarrow 1/\tau Q$) in exponent. Factorization**
  * **generalizes beyond NLL form shown above**

- **Shape function properties**

  – **$f_{\rm NP}$ factorizes in moments → convolution**

$$\begin{aligned}\sigma(\tau_a, Q) &= \frac{1}{2\pi i}\int_C d\nu\, \tilde{f}_{a,\rm NP}\left(\frac{\nu}{Q}\right)\,\sigma_{\rm PT}(\nu, Q, a)\\ &= \int d\xi\, f_{a,\rm NP}(\xi)\;\sigma(\tau_a - \xi, Q)\end{aligned}$$

  – **$f_{\rm NP}$ function of ν/Qonly**

  – **Linear in ν/Q: shift in PT distribution**

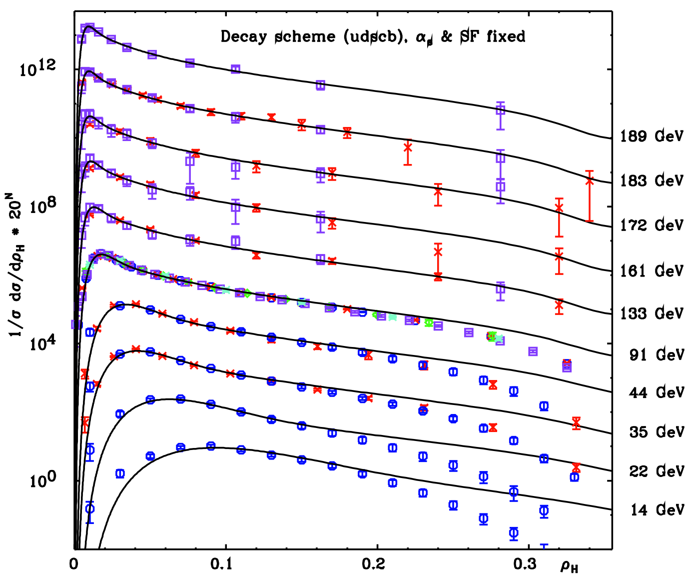
  (Korchemsky & GS (1995), Dokshitzer & Webber (1997))

$$\tilde{f}_{a,\rm NP}\left(\frac{\nu}{Q}\right) \longrightarrow {\rm e}^{\nu\left(\tau_a - \frac{1}{1-a}\frac{\lambda_1}{Q}\right)}$$

  – **Fit at Z: predictions for all Q**

– Shape function phenomenology for thrust

$(\mathrm{e}^+\mathrm{e}^- \to \mathbf{Z})$



Strategy: $f_{\mathrm{NP}}(\epsilon)$ at Z pole; predict other Q

(Korchemsky,GS, Belitsky; Gardi Rathsman,Magnea (1998 . . .))

First pass:

$$f_{0,\mathrm{NP}}(\rho) = \mathrm{const}\ \rho^{a-1}\ \mathrm{e}^{-b\rho^2}$$

a : $\langle$no. particles / unit rapidity$\rangle$

– **Scaling property for τ_a event shapes**

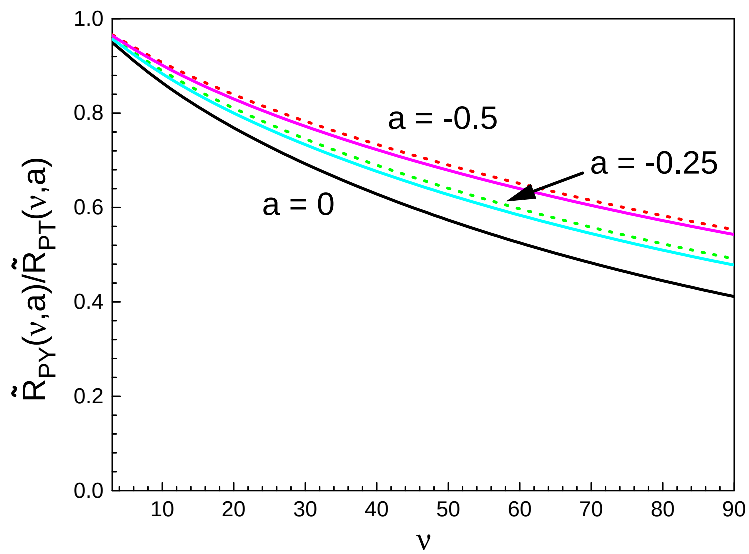
(C.F. Berger & GS (2003))

– **Test of rapidity-independence of NP dynamics**

$$\ln \tilde{f}_{a,\mathrm{NP}}\left(\frac{\nu}{Q},\kappa\right) = \frac{1}{1-a}\sum_{n=1}^{\infty}\lambda_n(\kappa)\left(-\frac{\nu}{Q}\right)^n$$

$$\tilde{f}_a\left(\frac{\nu}{Q},\kappa\right) = \left[\tilde{f}_0\left(\frac{\nu}{Q},\kappa\right)\right]^{\frac{1}{1-a}}$$

– **What PYTHIA gives**



– **Most event shapes were invented for jet physics of the late 70's**

– **Address existing data with new analysis**

– **New observables to analyze final states; aid in searches for new physics**

(Tkachov (1995), C.F. Berger et al. (Snowmass, 2001))

- **Shape function approach: 1PI cross sections**

  - **Self-consistent recoil in Joint Resummation of 1PI Cross sections**

    (Laenen, GS, Vogelsang (2001), (2004))

  - **Analyze transition: fixed target to collider energies**

  - **1PI Cross section as double inverse transform**

  - **"Instrinsic" logs of initial-state Q_T integrated over (viz. $\ln(1-z)$ in threshold resummation)**

$$p_T^3 \frac{d\sigma_{ab}}{dp_T} \sim \int_{-i\infty}^{i\infty} dN \int_{-\infty}^{\infty} d^2b \, d^2Q_T \, \mathrm{e}^{i\,\vec{Q}_T\cdot\vec{b}}$$

$$\times \; \tilde{\sigma}_{ab}^{(0)}(N)\, \mathrm{e}^{E(N,b,p_T)}$$

$$\times \quad \underbrace{\left(\frac{S}{4(\vec{p}_T-\frac{1}{2}\vec{Q}_T)^2}\right)^{N+1}}_{=\left(x_T^2\right)^{-N-1}\,\mathrm{e}^{N\vec{Q}_T\cdot\vec{p}_T/p_T^2}\left(1+\mathcal{O}(1/N,Q_T^2/p_T^2)\right)}$$

– Q_T, b **integrals (N imaginary)** $\Rightarrow$

$$p_T^3\,\frac{d\sigma_{ab}}{dp_T} \sim \int_{-i\infty}^{i\infty} dN\ \tilde{\sigma}_{ab}^{(0)}(N)\ \left(x_T^2\right)^{-N-1}$$

$$\mathrm{e}^{E\mathrm{thr}(N,p_T)}\ \mathrm{e}^{\delta\, E\mathrm{recoil}(N,p_T)}$$

$$\delta\, E_{\mathrm{recoil(N,p_T)}}\ =\ \pi\ \int_0^{Q^2} \frac{dk_T^2}{k_T^2}\ \sum_{i=a,b}\ A_i\left(\alpha_s(k_T^2)\right)$$

$$\times\left[\left(I_0\left(\frac{N\,k_T}{p_T}\right)-1\right)K_0\left(\frac{N\,k_T}{p_T}\right)\right]$$

– **Isolate perturbative recoil; NLL in N:**

$$\delta\, E_{\mathrm{recoil}}(N,p_T) = \delta E_{\mathrm{PT}} + \delta\, E_{\mathrm{np}}$$

$$\delta E_{\mathrm{PT}} \propto \frac{\alpha_s(p_T^2/N^2)}{\pi}\ \frac{\zeta(2)}{2}$$

– **isolate low scales $\leftrightarrow$ strong coupling**

$$\delta\, E_{\mathrm{recoil}}\ =\ \mathrm{PT}\ +\ \lambda_{\mathrm{ab}}\,\frac{\mathrm{N}^2}{\mathrm{p_T^2}}\ \ln\frac{\mathrm{p_T}}{\mathrm{N}}$$

$$\lambda_{ab}\ \sim\ 2g_{\mathrm{EW}}\ \propto\ \int dk_T^2\,\alpha_s(k_T^2)$$

$$N\ \leftrightarrow\ \frac{1}{\ln x_T^2}$$

– **Leading power now quadratic**

(viz. Beneke, Braun (1994) for NNLO DY; GS, Vogelsang (1999) for resummed DY)

$$\delta\, E_{\rm recoil} \;=\; {\rm PT} \;+\; \lambda_{ab}\, \frac{N^2}{p_T^2 \ln^2\left(\frac{4p_T^2}{S}\right)}\; \ln\left(p_T \ln\left(\frac{4p_T^2}{S}\right)\right)$$

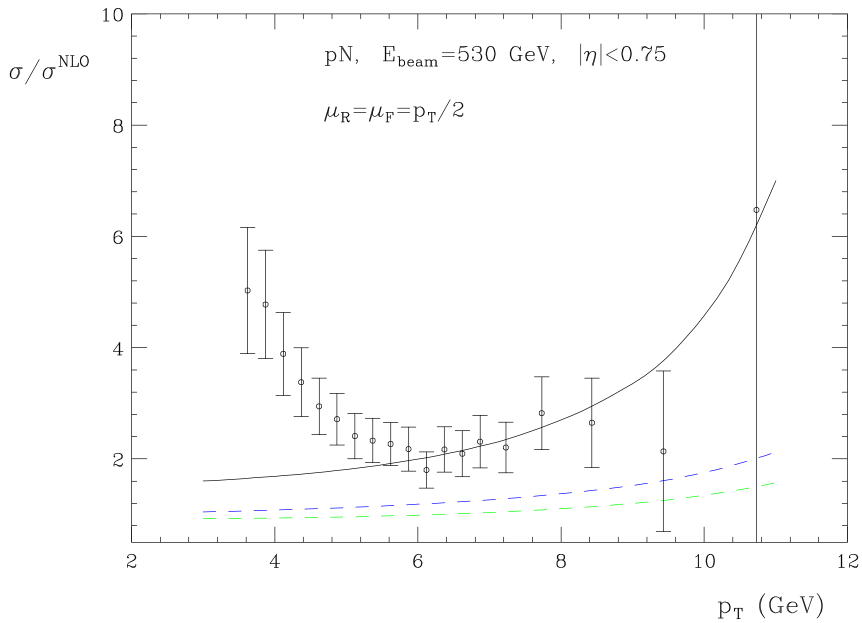
– **power suppressed in** p_T

– **decreases with** S **at fixed** p_T

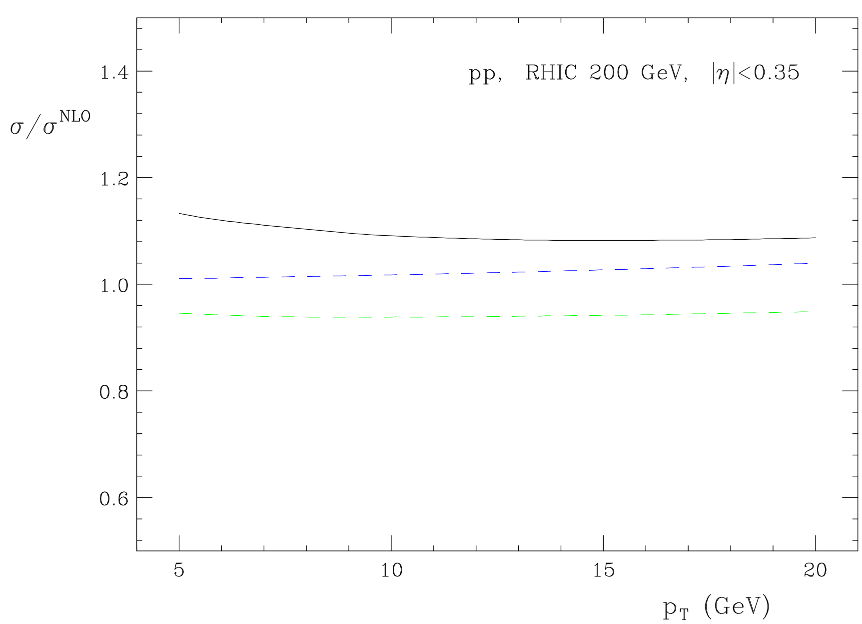
– **Tailor to large- and small-**N **behavior of Bessel functions** $\rightarrow$
sample 'shape function' of N/p_T **only:**

$$\delta E_{\rm np} = \frac{N^2}{p_T^2}\; \frac{ln\left(1+\frac{2p_T}{N}\right)}{\left(1+\frac{p_T}{N}\right)^2}$$

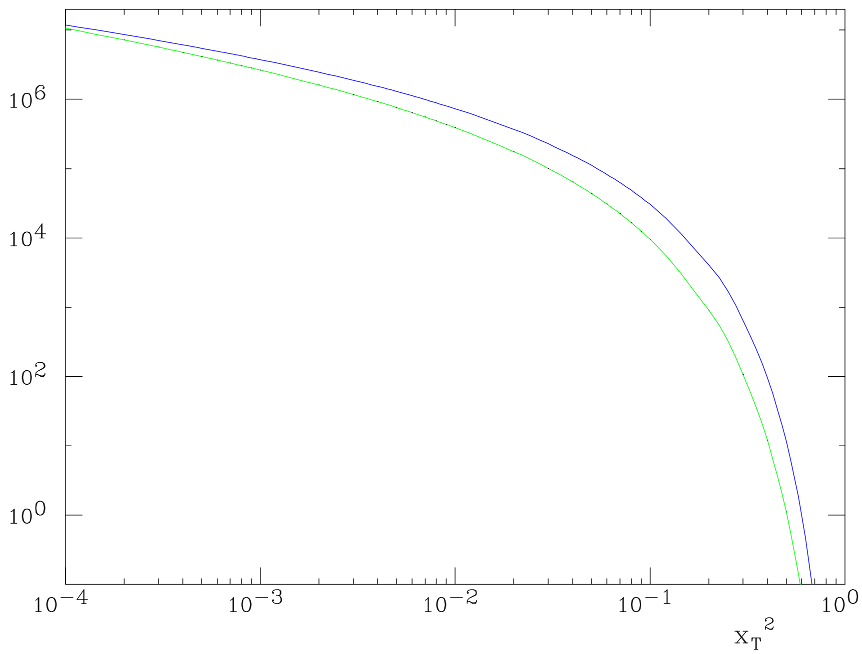
(p_T **in GeV)**



Including $\delta E_{\rm recoil}$ for direct photons at E706



Including $\delta E_{\rm recoil}$ for RHIC



– **The NLO cross section** $p_T^3 d\sigma_\gamma/dp_T$ *vs.* x_T^2**.**

– **Origin of high-**p_T **enhancement:**

$\exp CN^2/p_T^2$ **on** $p_T^3 d\sigma_\gamma/dp_T \sim (x_T^2)^{-\lambda} \Rightarrow (x_T^2)^{-\lambda} \times \mathrm{e}^{C\lambda^2/p_T^2}$

  – **Collider kinematics "on top", fixed-target on steep slope**

⋆ Directions

No particular order: all with question marks

- **Investigation of resummed coherent radiation with color exchange to showering**
- **Interjet radiation as a diagnostic for new physics**
  (Ellis, Khoze, Stirling (1997))
- **Further development of event shapes for hadronic collisions**
- **Event shape/energy flow correlations for hadronic collisions: separating jet branching from coherent radiation& underlying event**
- **Energy flow for rapidity gaps in the light of non-global logarithms**
- **Fluctuations in event structure analyzed in terms of energy flow or other infrared safe quantities**
- **The implications of multiple interactions at LHC**