α_S from e^+e^- machines (LEP & SLC)

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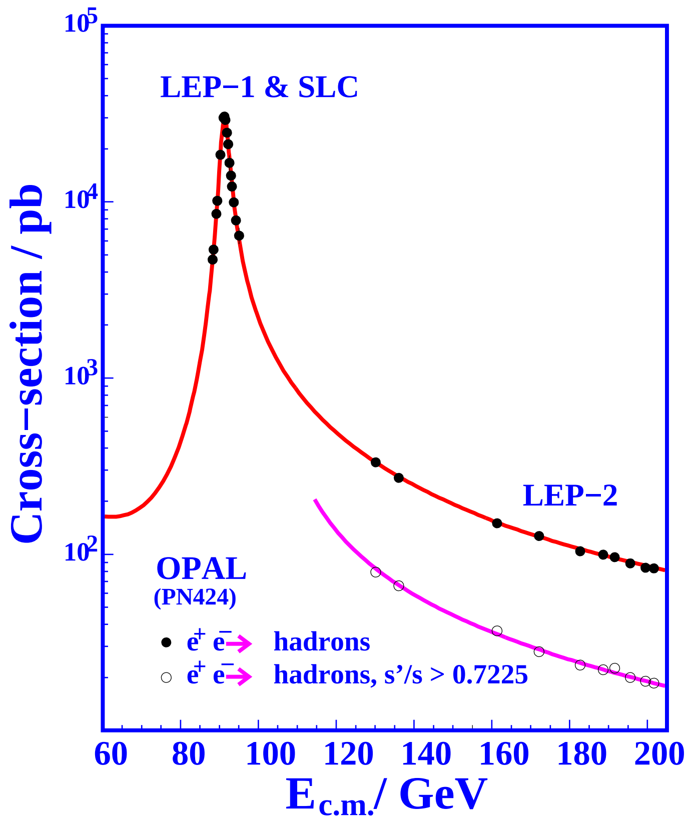
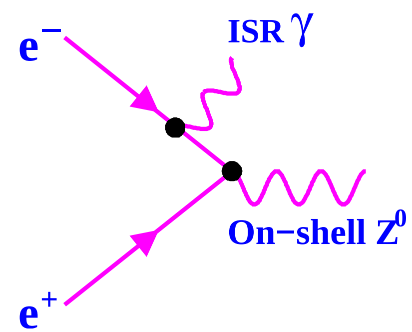
Data samples



Most LEP-2 QCD events:

Radiative returns:
$e^+e^- \to Z^0\gamma$



→ Cut on hadronic energy s'

For $\mathrm{E_{c.m.}} \gtrsim$ 160 GeV, $e^+e^- \to W^+W^-$ events contribute significant background, especially to the multi-jet (high thrust) region

⟶ Reduce WW background using cuts e.g. on QCD 4-jet matrix element, subtract residual background ($\sim 10\%$) using “4-fermion” MCs

Measurements of α_S from LEP/SLC

$\longrightarrow$ Inclusive: (LEP-1)

- $R_\ell = \dfrac{\Gamma(Z^0 \to hadrons)}{\Gamma(Z^0 \to \ell^+\ell^-)}$ $\ell = e, \mu$ or τ
- $R_\tau = \dfrac{\Gamma(\tau \to hadrons)}{\Gamma(\tau \to \ell^+\ell^-)}$

$\longrightarrow$ 3-jet dominated: (LEP-1/SLC or LEP-2)

- Event shapes: Thrust, Jet broadening, $\cdots$
- Jet rates
- Energy correlations

$\longrightarrow$ Scaling violations: (LEP-1 and LEP-2)

- "Q^2" evolution of fragmentation functions

α_S from $R_\ell = \frac{\Gamma(Z^0 \rightarrow hadrons)}{\Gamma(Z^0 \rightarrow \ell^+\ell^-)}$

$\longrightarrow$ Ratio of the total hadronic to the single species (massless) leptonic branching ratios of the Z^0: $\ell = e, \mu$ or τ

Experiment

$\longrightarrow$ Based exclusively on event counting

(experimentally, "only" need to understand the acceptance for the different types of events: Internal characteristics of the events are irrelevant)

$\longrightarrow$ No correction for the effects of hadronization

(but some hadronization uncertainty in the acceptance corrections)

Theory

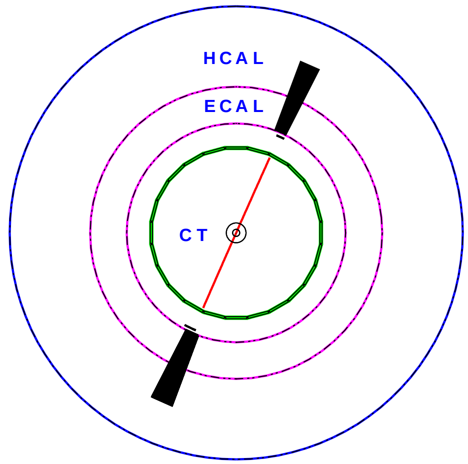
$\longrightarrow$ Complete $\mathcal{O}(\alpha_S^3)$ (3 loop) calculation available

(the only observable, along with R_τ, for which this is true)

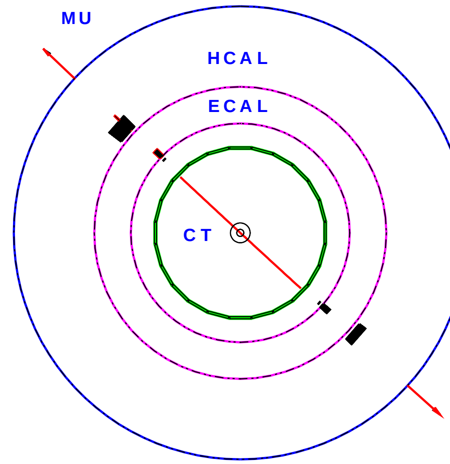
α_S from R_ℓ has intrinsically small experimental and theoretical uncertainties !

$Z^0 \to \ell^+\ell^-$ and $Z^0 \to hadrons$

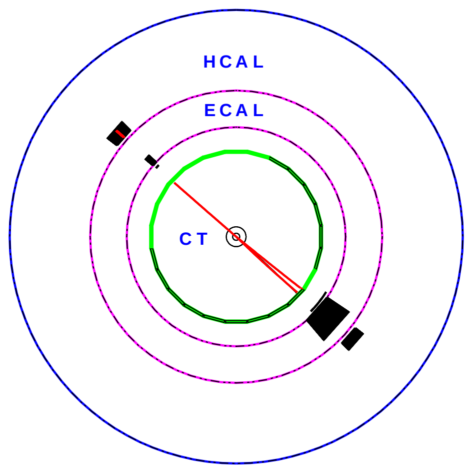
e+e- --> e+e-



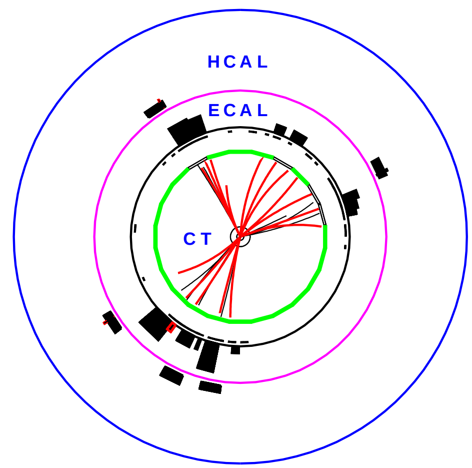
e+e- -->$\mu+\mu-$



e+e- --> $\tau+\tau-$

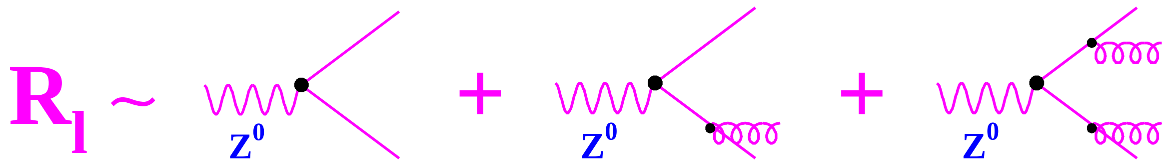


e+e- --> $q\bar{q}$(g)



However, the dependence of R_ℓ on α_S is

non-leading and therefore weak:



or $R_\ell = R_\ell^0 \, (1 + \delta_{QCD})$

with $R_\ell^0 = R_\ell(\alpha_S = 0) = 19.934$

and $\delta_{QCD} = 1.045 \left(\frac{\alpha_S(m_Z)}{\pi}\right) + 0.94 \left(\frac{\alpha_S(m_Z)}{\pi}\right)^2 - 15 \left(\frac{\alpha_S(m_Z)}{\pi}\right)^3 \approx 0.042$

$\longrightarrow$ An accurate determination of α_S from R_ℓ requires the total LEP-1 event statistics from the four experiments combined !

Total LEP-1 event statistics (combined):

$Z^0 \to hadrons$: ~ 15 million

$Z^0 \to leptons$: ~ 2 million
(all species)

$$R_\ell = 20.767 \pm 0.025 \rightarrow \alpha_S(M_Z) = 0.1224 \pm 0.0038$$

[LEP Electroweak Working Group, CERN-EP/2003-091 (Dec. 2003)]

3% precision $\longrightarrow$ Uncertainty dominated by experimental systematics (e.g. acceptance for narrow 2-jet-like events near the beam axis) and statistics.

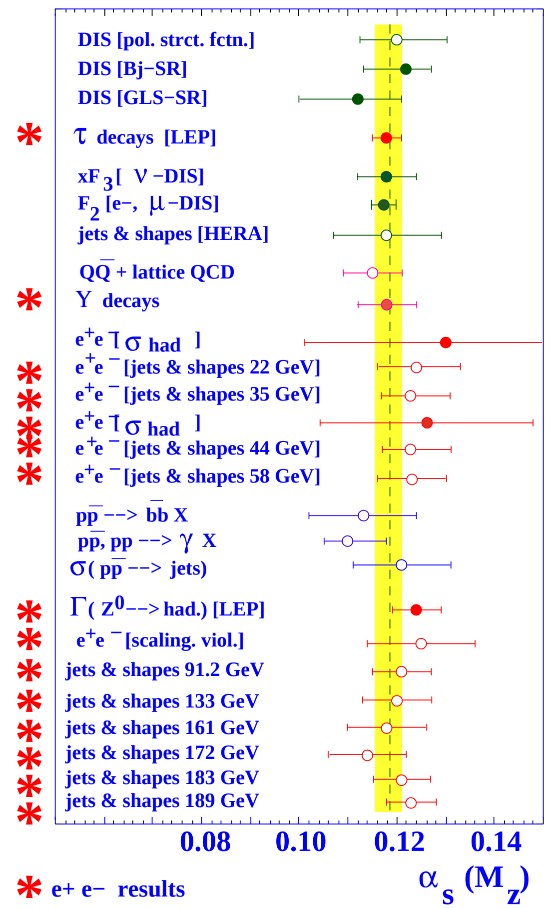
S. Bethke, J. Phys. G26 (2000) R27:
$[\alpha_S(M_Z) = 0.1184 \pm 0.0031]$

Note the smallness of the α_S result from τ decays:

$\longrightarrow$ The “shrinking error” of QCD:

$$\frac{\delta\alpha_S(M_Z)}{\alpha_S(M_Z)} \sim \frac{\alpha_S(M_Z)}{\alpha_S(Q)} < 1$$

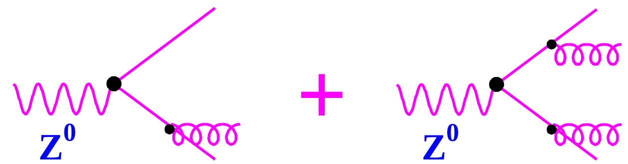
for $\mathrm{Q} < \mathrm{M_Z}$



α_S from Event Shapes

$\longrightarrow$ Internal momentum structure of an event, one entry "y" per event

$\longrightarrow$ 3-jet dominated quantities, leading terms $\sim\ \alpha_S$

y $\sim$ 

- Thrust T: $$T = \max\left(\frac{\sum_i \vec{p}_i\cdot\hat{n}}{\sum|\vec{p}_i|}\right) \qquad i = particles$$

  $$\text{resulting}\ \hat{n} = \hat{n}_T \longrightarrow \text{the thrust axis}$$

- Jet broadening variables B_T and B_W:

$\longrightarrow$ Divide event into hemispheres using plane $\perp$ to $\hat{n}_T$

$$B_k = \frac{\sum_{i\,\text{in hemis.}\,k} |\vec{p}_i\times\hat{n}_T|}{\sum|\vec{p}_i|} \qquad k = 1,2\ \text{(hemispheres)}$$

$$B_T = B_1 + B_2\ \ \underline{\text{Total}} \qquad B_W = \max(B_1, B_2)\ \ \underline{\text{Wide}}$$

- plus many others : C parameter, jet rate "flip" value y_{23}, $\cdots$

⟶ Many of these variables are equivalent to each other at LO but have different higher order corrections

⟶ Unlike R_ℓ, event shape distributions are based on the internal characteristics of events

⟶ A hadronization correction (~ 10%) is usually applied to the data before being fitted by theoretical expressions

⟶ The hadronization correction is determined by the ratio of the Monte Carlo predictions at the parton & hadron levels

⟶ Use Jetset versus Herwig, parton shower versus fixed order α_S^2, etc.

QCD predictions for event shape variables

- Exact $\mathcal{O}(\alpha_S^2)$ expressions:

$$\frac{1}{\sigma_0}\frac{d\sigma}{dy} = A(y)\frac{\alpha_S(\mu)}{2\pi} + \left[B(y) + A(y)2\pi c_0 \log\left(\frac{\mu^2}{s}\right)\right]\left(\frac{\alpha_S(\mu)}{2\pi}\right)^2$$

$$c_0 = (33 - 2n_f)/12\pi$$

→ The renormalization scale μ is an unphysical parameter

→ If the calculation were available to all orders in perturbation theory, there would be no dependence on μ

→ For finite orders, a residual dependence $\sim \left(\log \mu^2/s\right)^n$ is present

→ Need $\mu \approx \sqrt{s}$ for the effects of higher order terms to be negligible

→ Two parameter fits of $\Lambda_{\overline{\mathrm{MS}}}$ and μ to the hadronization corrected data typically yield $\mu \approx \sqrt{s}/20$, indicating the importance of the missing higher order terms

→ Theory uncertainties due to the missing higher orders (renormalization scale dependence) dominate the total uncertainty of α_S from event shapes

Importance of NLO terms (LEP-1 data)

[N. Magnoli, P. Nason, R. Rattazzi, PL B252 (1990) 271]

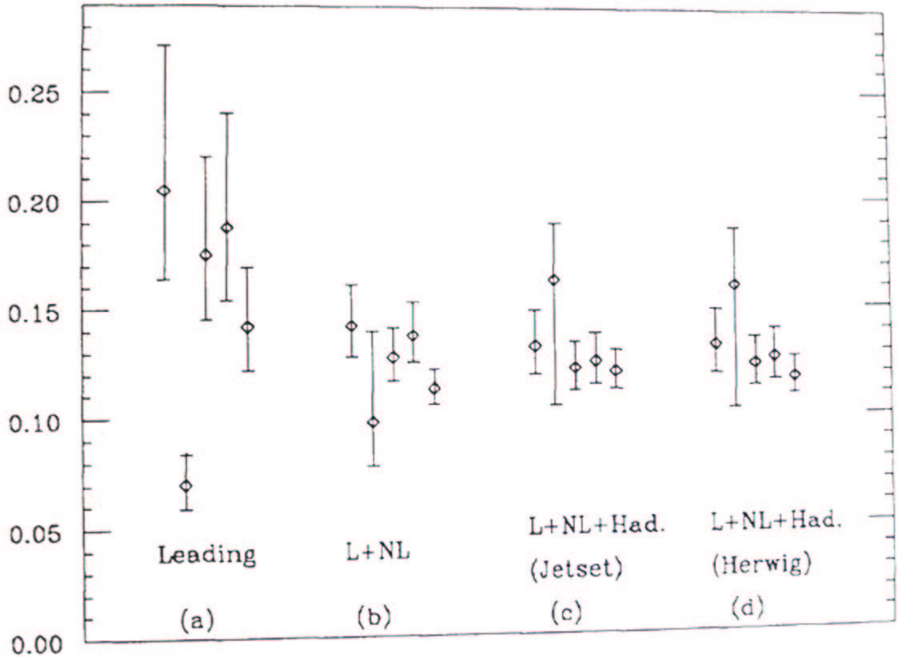
$\alpha_S(M_Z)$ from Thrust, Oblateness, Major, C & N_3 (Jade jet finder, $y_{\rm cut} = 0.08$)

(a) LO term only, no hadronization correction

⟶ Result from Oblateness strongly inconsistent with the others

(b) NLO term also, no hadronization correction

⟶ Results more consistent, smaller uncertainties because of a reduction in the scale dependence (vary μ between M_Z and $M_Z/4$)



(c) and (d) NLO theory fitted to hadronization-corrected data

⟶ At LEP, the effect of the NLO perturbative terms is more important than hadronization effects (this had not been the case at PEP & PETRA)

- $\mathcal{O}(\alpha_S^2)$ + NLLA expressions:

$\rightarrow$ Perturbative expansion for the cumulative event shape: $R(y) = \int_0^y \frac{1}{\sigma}\frac{d\sigma}{dy'}dy'$

$\rightarrow$ Expressed as a series in $L = \ln(1/y)$

$\rightarrow$ Most singular (largest) terms are in the 2-jet region, $y \rightarrow 0$

$\rightarrow$ Leading and next-to-leading logarithmic terms have been summed to all orders of α_S for a number of event shape variables: NLLA

NLLA

	Leading logs	Next-to-leading logs	Sub-leading terms
$\mathcal{O}(\alpha_S)$	$\alpha_S\, L^2$	$\alpha_S\, L$	$\alpha_S,\, \alpha_S\, \frac{1}{L}$
$\mathcal{O}(\alpha_S^2)$	$\alpha_S^2\, L^3$	$\alpha_S^2\, L^2$	$\alpha_S^2\, L,\, \alpha_S^2,\, \alpha_S^2\, \frac{1}{L}$
	$\alpha_S^3\, L^4$	$\alpha_S^3\, L^3$	$\cdots$
	$\cdots$	$\cdots$	

$\mathcal{O}(\alpha_S^2)$

Common to $\mathcal{O}(\alpha_S^2)$ and NLLA

$\longrightarrow$ Terms up to $\mathcal{O}(\alpha_S^2)$ in the NLLA expression are replaced by the exact $\mathcal{O}(\alpha_S^2)$ results $\longrightarrow$ $\boxed{\mathcal{O}(\alpha_S^2) \text{ + NLLA}}$ [ln(R) matching]

$\longrightarrow$ The most complete analytic description of event shapes currently available

$\longrightarrow$ Fits of the $\mathcal{O}(\alpha_S^2)$ + NLLA expressions to data yield results for μ much closer to the physical scale $\sqrt{s}$ than the pure $\mathcal{O}(\alpha_S^2)$ expressions

$\longrightarrow$ The NLLA terms reduce the sensitivity of the α_S result to the choice of μ

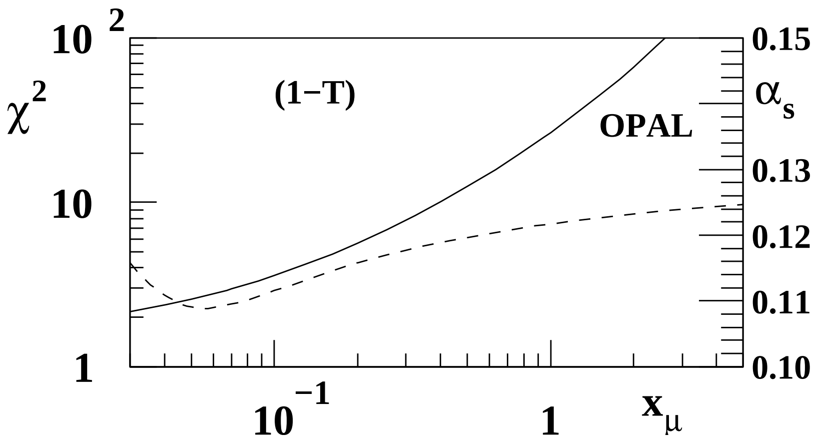
$\longrightarrow$ The perturbative description of the data is more sensible

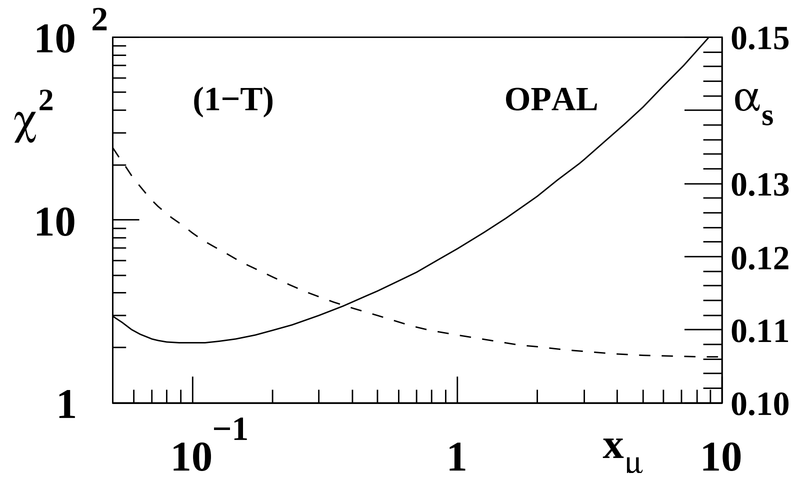
$\longrightarrow$ The description of the 2-jet region is improved
(important for LEP-2, where statistics are limited, and where the multi-jet region is contaminated by $e^+e^- \longrightarrow$ WW events)

$\longrightarrow$ The theory uncertainty due to missing higher order terms remains the dominant uncertainty, however

Example: $x_\mu \equiv \frac{\mu}{E_{c.m.}}$ for $\mathcal{O}(\alpha_S^2)$ and $\mathcal{O}(\alpha_S^2)$ + NLLA

[OPAL Collab., Z. Phys. C 59 (1993) 1]



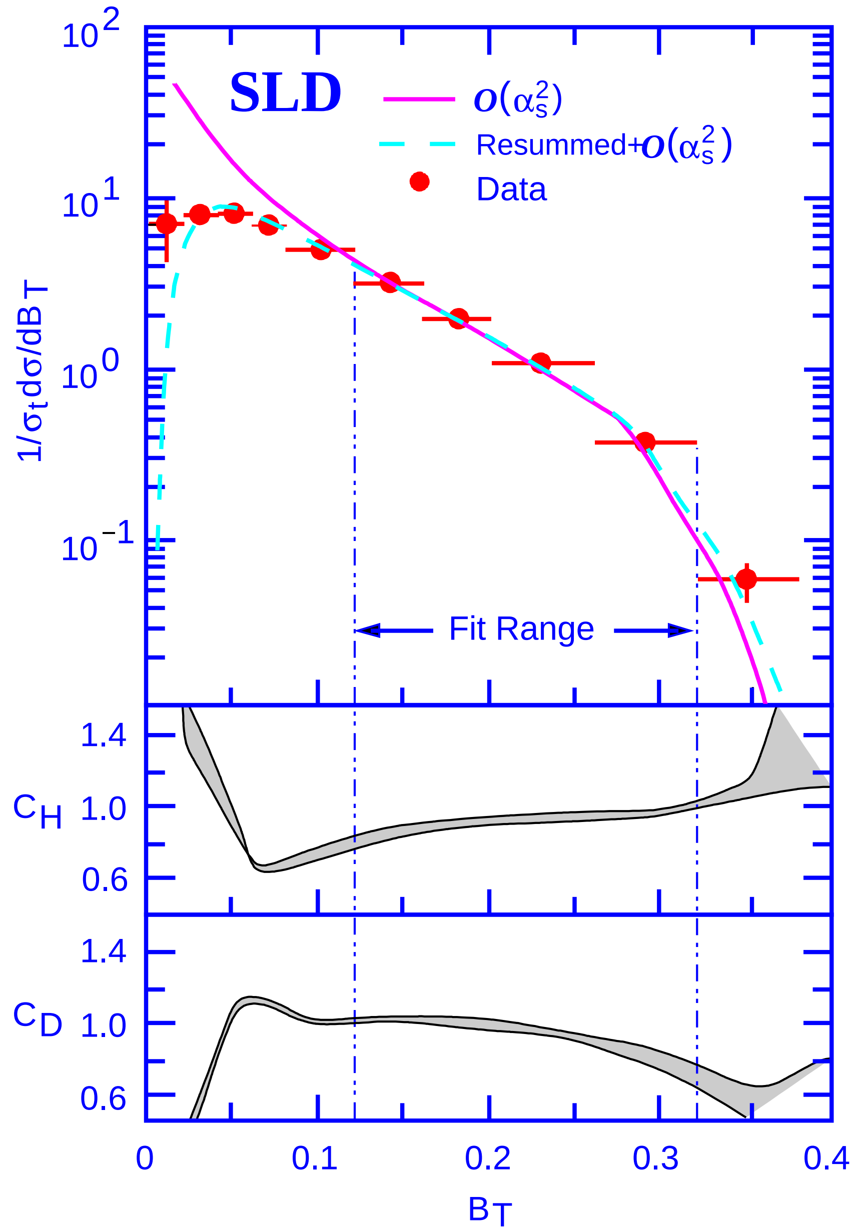


Dashed curves: χ^2 of fit of the theory curve to the data for different choices of μ

$\mathcal{O}(\alpha_S^2)$ (left plot): Minimum χ^2 occurs for $x_\mu \approx 0.06$

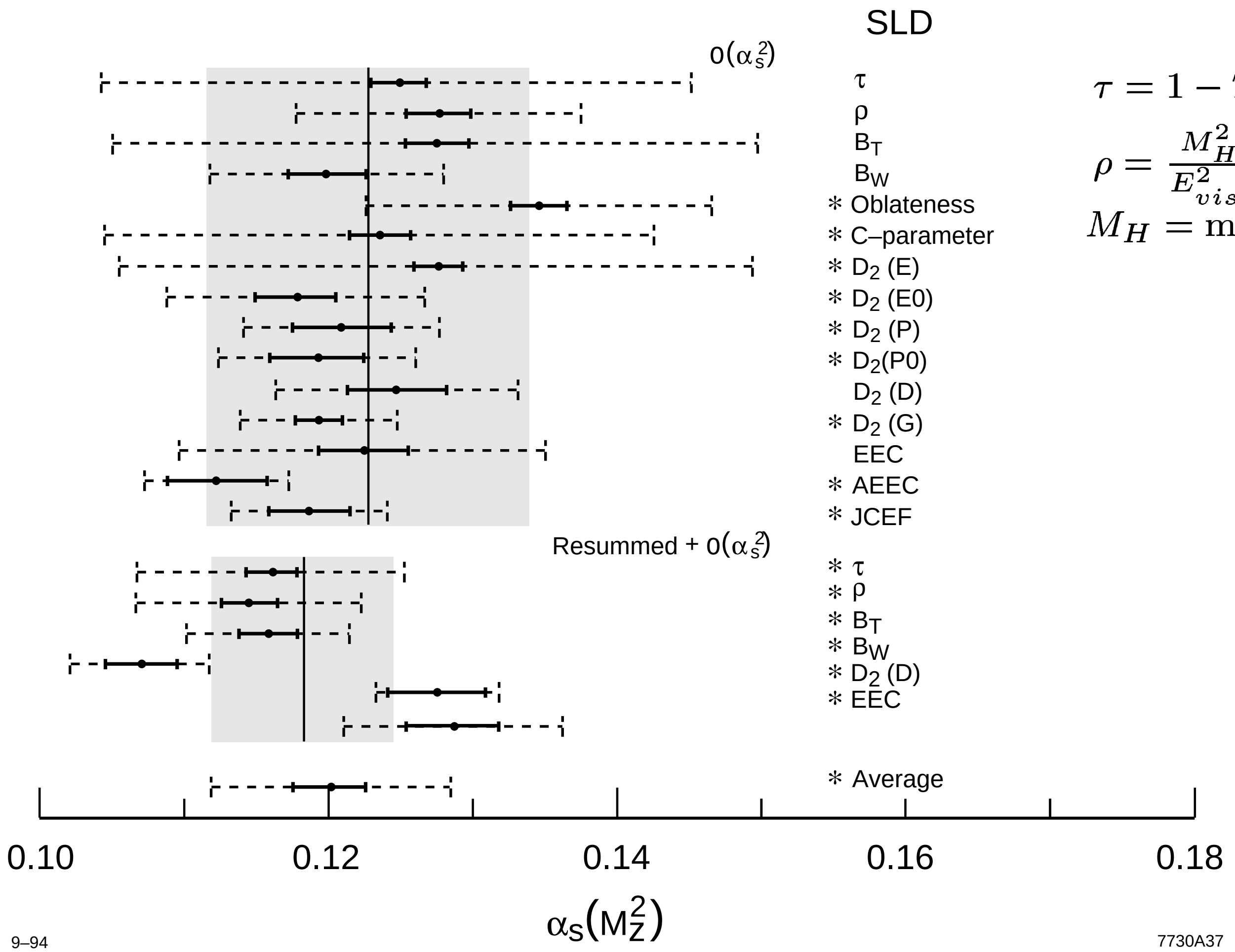
$\mathcal{O}(\alpha_S^2)$ + NLLA (right plot): Reasonable χ^2, flat χ^2 curve for $x_\mu \approx 1$

[SLD Collab., Phys. Rev. D51 (1995) 962]



C_H = Hadronization correction

C_D = Correction for detector acceptance & resolution



$\tau = 1 - T$

$\rho = \frac{M_H^2}{E_{vis.}^2}$

$M_H = \max(M_1, M_2)$

Solid bars $\longrightarrow$ Experimental uncertainties

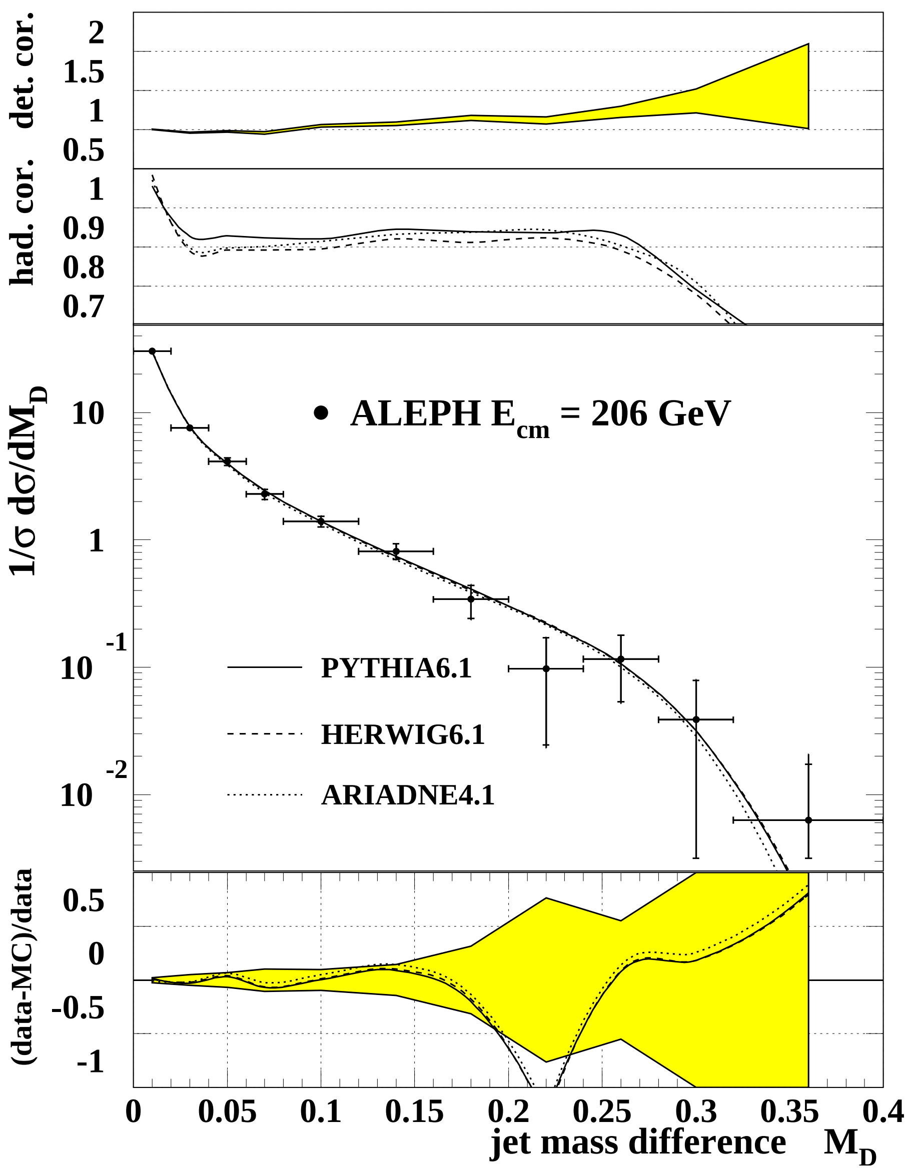
Dashed bars $\longrightarrow$ Experimental & theory uncertainties

Top section $\longrightarrow$ $\mathcal{O}(\alpha_S^2)$ results

Bottom section $\longrightarrow$ $\mathcal{O}(\alpha_S^2)$ + NLLA results (Note the reduced theoretical uncertainties compared to the $\mathcal{O}(\alpha_S^2)$ results)

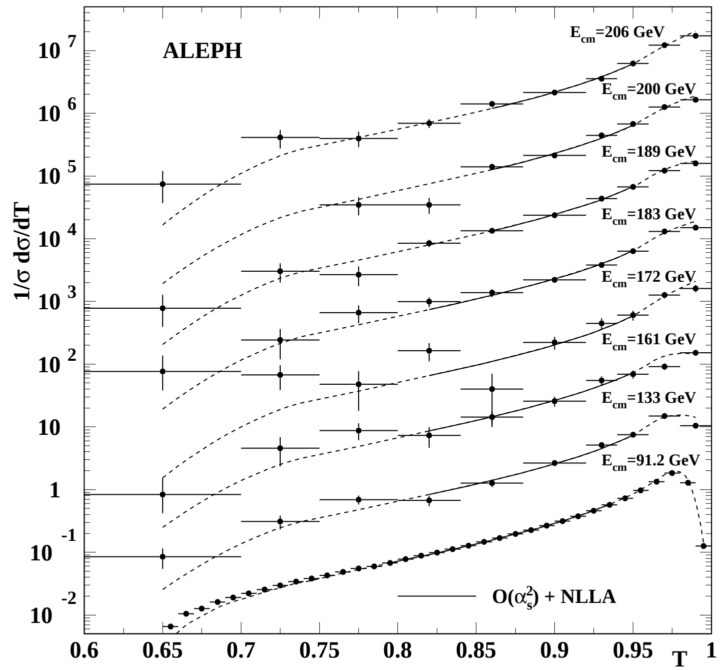
Shaded regions $\longrightarrow$ Average α_S value and total uncertainty

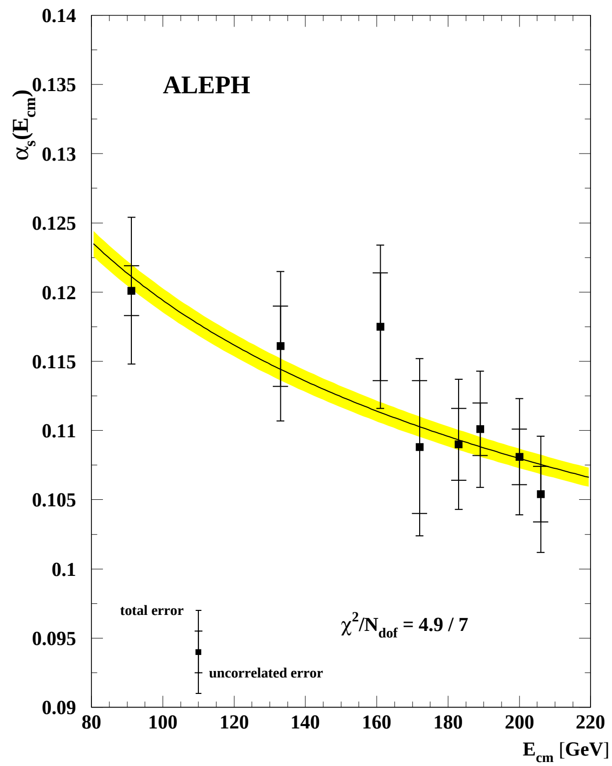
[ALEPH Collab., CERN-EP/2003-084 (Dec. 2003)]



Hadronization correction can still be $\sim 10\%$, even at the highest LEP-2 energies

Combine α_S results based on six event shapes ($T, -\ln y_{23}, \rho, B_W, B_T, C$) and $\mathcal{O}(\alpha_S^2)$ + NLLA calculations





Inner error bars in plot on right exclude the perturbative uncertainty:

⟶ Choice of μ: $0.5 \leq \mu/M_Z \leq 2$

⟶ Arbitrariness in the definition of logarithms to be summed
[e.g. whether to sum powers of $\alpha_S \ln(y_0/y)$ or $\alpha_S \ln(y_0/2y)$; $y_0 =$ constant depending on the event shape variable]

⟶ Sum powers of $\alpha_S \ln[y_0/(x_L\, y)]$, $\frac{2}{3} \leq x_L \leq \frac{3}{2}$
[see R.W.L. Jones et al., JHEP0312 (2003) 007]

[ALEPH Collab., CERN-EP/2003-084 (Dec. 2003)]

$$\alpha_S(M_Z) = 0.1214 \pm 0.0045(\text{perturbative}) \pm 0.0018$$

(combination of the results of six event shape measurements at eight energies)

→

Q [GeV]	91.2	133	161	172	183	189	200	206
$\alpha_s(M_Z)$	0.1201	0.1229	0.1285	0.1193	0.1207	0.1227	0.1212	0.1183
stat. error	0.0001	0.0028	0.0044	0.0056	0.0027	0.0018	0.0020	0.0020
exp. error	0.0008	0.0012	0.0013	0.0013	0.0013	0.0012	0.0013	0.0012
pert. error	0.0050	0.0048	0.0048	0.0047	0.0044	0.0042	0.0042	0.0042
hadr.error	0.0016	0.0012	0.0011	0.0010	0.0010	0.0009	0.0009	0.0009
total error	0.0053	0.0058	0.0067	0.0075	0.0054	0.0049	0.0049	0.0048

$\longrightarrow$ 4% precision

The perturbative uncertainty decreases with increasing energy, faster than α_S itself (16% versus 1.5%), but it remains the dominant uncertainty of the measurement

α_S using NLO corrections to 4-jet observables

[ALEPH Collab., Eur.Phys.J.C27 (2003) 1]

Fit of NLO predictions for the 4-jet rate (Durham jet finder) to the hadronization & detector corrected data

NLO + resummed 4-jet predictions from DEBRECAN MC, Z. Trócsányi

$$\alpha_S(M_Z) = 0.1170 \pm 0.0001(\text{stat.}) \pm 0.0013(\text{syst.})$$

(1% uncertainty, Bayesian method: systematic variations which result in larger χ^2 are given less weight)

$$\alpha_S(M_Z) = 0.1170 \pm 0.0001(\text{stat.}) \pm 0.0022(\text{syst.})$$

(2% uncertainty, add systematic uncertainties in quadrature)

Include color factors C_A and C_F in the fit:

$\alpha_S(M_Z) = 0.119 \pm 0.006(\text{stat.}) \pm 0.026(\text{syst.})$ (ALEPH 2003)

$\alpha_S(M_Z) = 0.120 \pm 0.011(\text{stat.}) \pm 0.020(\text{syst.})$ [OPAL Eur.Phys.J.C20 (2001) 601]

Outlook

- The basic experimental situation with respect to measuring α_S at e^+e^- colliders hasn't much changed in the past 10 years !!

  $\longrightarrow$ Availability of $\mathcal{O}(\alpha_S^2)$ + NLLA expressions

  $\longrightarrow$ Uncertainties dominated by the lack of higher orders

  Improvements in the experimental situation (higher energies of LEP-2) haven't had too much impact on the overall uncertainty attributed to α_S

- We need improvements in the theory !

  $\longrightarrow$ NNLO [order $\mathcal{O}(\alpha_S^3)$] calculations of event shapes should lead to a reduction in the dependence of the result on the choice of the renormalization scale

  $\longrightarrow$ Re-analysis of LEP/SLD data

  For the two-jet region, we need NNLLA resummed calculations ??