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Soft gluons
Non-global logs and Monte Carlo

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- Multi-soft-gluon distribution
- Generating functional for observables
- Global and non-global observables (and logs)
- Improving Monte Carlo branching

Multi-soft-gluon distribution

Bassetto,Ciafaloni&GM, Phys.Rep.100(1983)201

Example: $\gamma^* \to q\bar{q}g_1 \cdots g_n$

$$\mathcal{M}_n(p\bar{p}q_1 \cdots q_n) = \sum_{\text{perm.}} \{\lambda^{a_{i_1}} \cdots \lambda^{a_{i_n}}\}_{\beta\bar{\beta}} M_n(i_1 \cdots i_n)$$

Leading order in soft limit (including large angles):

- strong energy ordering: $\omega_n \ll \cdots \omega_1 \ll Q$
- softest gluon emitted off external legs *without* modification of internal legs
- Factorization $\Rightarrow$ recurrent relation

$$M_n(\cdots \ell\, n\, \ell' \cdots) = g_s\, M_{n-1}(\cdots \ell\ell' \cdots) \cdot J^{\mu_n}_{\ell\ell'}(q_n)$$

$$J^{\mu}_{\ell\ell'}(q) = \frac{q^{\mu}_{\ell}}{(q_{\ell}q)} - \frac{q^{\mu}_{\ell'}}{(q_{\ell'}q)}$$

- Distribution (colour and polarization sum)

$$|\mathcal{M}_n|^2 = \sum_{\text{perm.}} N_c^n \alpha_s^n\, W_{p\bar{p}}(1_1 \cdots i_n) + \cdots$$

$$W_{ab}(1 \cdots n) = \frac{(ab)}{(aq_1) \cdots (q_n b)}$$

- In pure Yang-Mills $\Rightarrow$ Parke-Taylor MHV amplitude

Distribution and generating functional (soft and planar limit)

Banfi,Smye&GM,JHEP0208:006,2002

- Factorize phase space (soft and planar limit)
- Factorize constraints (observable) ⇒ source $u(q)$

$$\Sigma_{ab}^{\mathrm{real}}(Q,u) = \prod_{i=1}^{n} \int^{Q} \left\{ \frac{dq_{it}}{q_{ti}} \frac{d\Omega_{q_i}}{4\pi} \, u(q_i) \, \bar{\alpha}_s \right\} \cdot W_{ab}(1 \dots n)$$

- Obtain the generating functional $(\bar{\alpha}_s = N_c \alpha_s / \pi)$

$$Q\partial_Q \Sigma_{ab} = \bar{\alpha}_s \int \frac{d\Omega_q}{4\pi} \frac{\xi_{ab}}{\xi_{aq}\,\xi_{qb}} \, [u(q)\, \Sigma_{aq} \cdot \Sigma_{qb} - \Sigma_{ab}]$$

$$\xi_{ij} = 1 - \cos\theta_{ij}$$

- Use factorization structure of multi-soft gluon distribution

$$W_{ab} = \omega_\ell^{-2} \frac{\xi_{ab}}{\xi_{a\ell}\,\xi_{\ell b}} \, W_{a\ell} \cdot W_{\ell b}$$

- Include virtual corrections (by Cauchy integration)
- Sudakov form factor

$$Q\partial_Q S_{ab} = -\bar{\alpha}_s \int \frac{d\Omega_q}{4\pi} \frac{\xi_{ab}}{\xi_{aq}\,\xi_{qb}} \, S_{ab}$$

- IR cancellation: $\Sigma(Q, u\!=\!1)\!=\!1$

New discovery: non-global logs

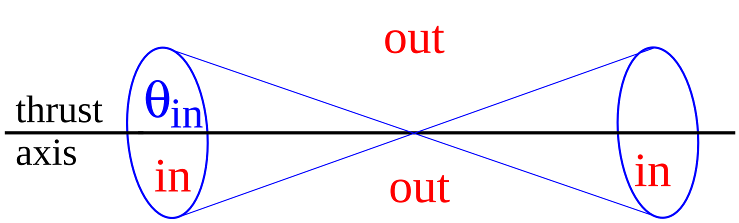
M. Dasgupta and G.Salam, JHEP 08(02) 032; JHEP 03(02) 3311

Jet-shape with only part of phase space involved. Examples:

- Sterman-Weinberg distribution (energy in a cone)
- photon isolation
- away from jet radiation
- rapidity cuts in hadron-hadron (e.g. pedestal dist.)
- DIS jet in current hemisphere

Relevant configuration: large angle soft emission
General features: M.Dasgupta,G.Salam; C.Berger,T.Kúcs,G.Sterman; A.Banfi,G.Smye&GM; Yuri Dokshitzer &GM.

Simplest case e^+e^-: Soft emission off $q\bar{q}$-dipole in out-region



$$\Sigma_{e^+e^-}(E_{\rm out}) = \sum_n \int \frac{d\sigma_n}{\sigma_T}\, \Theta\left(E_{\rm out} - \sum_{\rm out} q_{ti}\right)$$

Basis for the analysis: multi-soft gluon emission (large N_c)

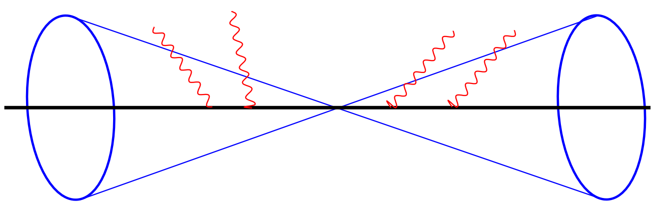
$$\Theta\left(E_{\rm out} - \sum_{\rm out} q_{ti}\right) \simeq \prod_{\rm out} \Theta(E_{\rm out} - q_{ti}) \;\Rightarrow\; u(q) = \Theta_{\rm out}\, \Theta(E_{\rm out} - q_t\,)$$

Generating functional with $u(q) = \Theta_{\rm out}\, \Theta(E_{\rm out} - q_t)$

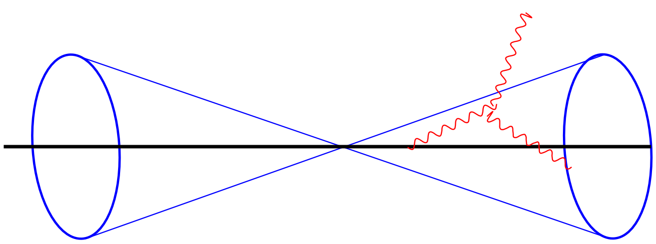
$$\partial_\Delta \Sigma_{ab} = -(\partial_\Delta R_{ab})\, \Sigma_{ab} + \int_{\rm in} \frac{d\Omega_q}{4\pi} \frac{\xi_{ab}}{\xi_{aq}\, \xi_{qb}} \left[\Sigma_{aq} \cdot \Sigma_{qb} - \Sigma_{ab}\right]$$

$$R_{ab} = \Delta \int_{\rm out} \frac{d\Omega_q}{4\pi} \frac{\xi_{ab}}{\xi_{aq}\, \xi_{qb}}\,, \qquad \Delta = \int_{E_{\rm out}}^{Q} \frac{dq_t}{q_t} \bar{\alpha}_{\rm s}(q_t)$$

Two QCD components:



Bremsstrahlung component:
SL Sudakov factor: $S_{ab} = e^{-R_{ab}}$
Linear evolution (DGLAP type)



Soft branching inside Jet region
correlation function C_{ab}
SL only for non-global obs.
beyond SL for global obs.

Result: $\Sigma_{ab} = S_{ab} \cdot C_{ab}$

$$C_{ab} \simeq e^{-\frac{c}{2}\Delta^2}, c = 4.8834\ldots \qquad \text{for large} \quad \Delta$$

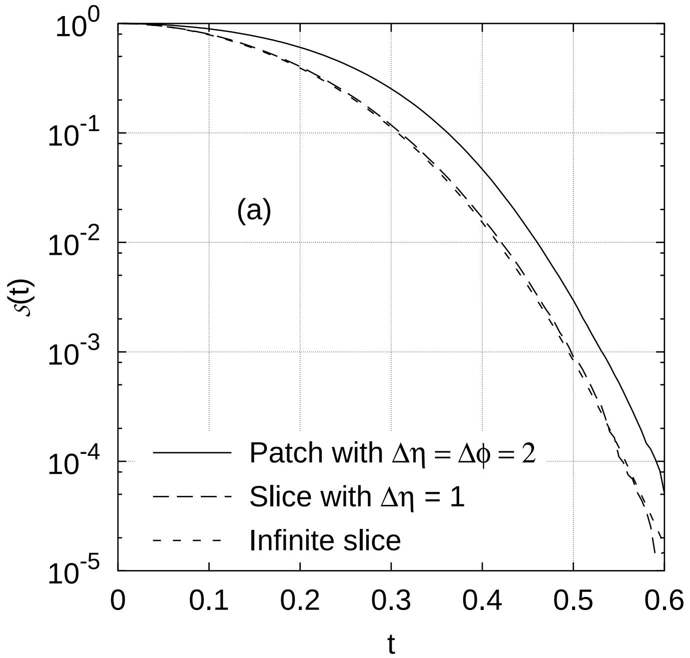
- Large buffer (Dasgupta&Salam):
  branching in region in around p (or $\bar{p}$)

$$\theta_{pq} < \theta_{\rm crit}\,, \qquad \theta_{\rm crit} \simeq \theta_0\, e^{-\frac{c}{2}\Delta}$$

- Puzzle: connection to small x-physics (BFKL)!
  Mueller&GM,PLB575(2003)37

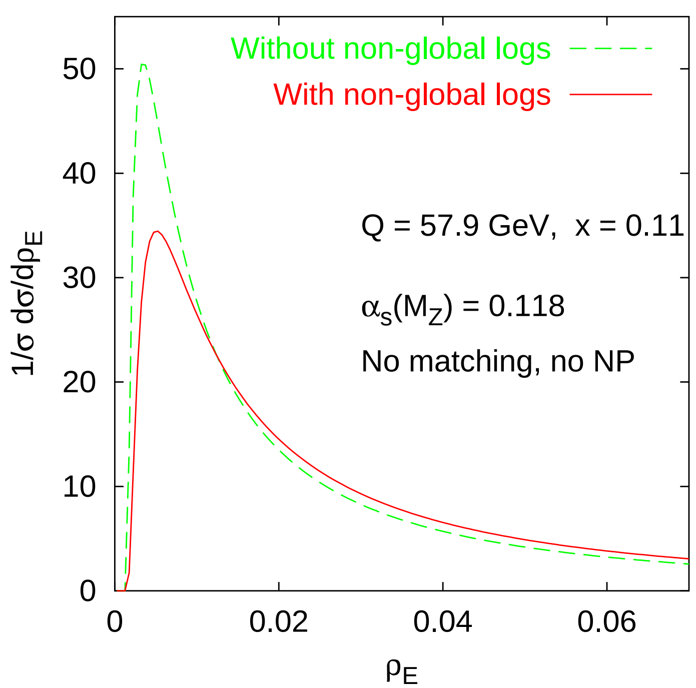
Impact of non-global logs

M.Dasgupta,G.Salam,PLB512(2001)323, JHEP0203:017:2002



Correlation function

$C_{e^+e^-}(\Delta)$ with $\Delta = N_c\, t$



Current-hemisphere

jet mass in DIS

Improve Monte Carlo

Include large angle soft branching (large N_c) (with S.Gieseke)
se also ARIADNE by L.Lonnblad

Monte Carlo: numerical solution of the generating functional

HERWIG: collinear singularities with coherence (angular ordering)

Branching with only soft gluons

$$g_1\, g_2 \to g_1\, g_2 + g_3\,, \qquad \omega_3 \ll \omega_1, \omega_2$$

$$\frac{\xi_{12}}{\xi_{13}\,\xi_{32}} = \frac{A_{13}}{\xi_{13}} + \frac{A_{23}}{\xi_{23}}\,, \qquad \xi_{ij} = 1 - \cos\theta_{ij}$$

$$\int \frac{d\phi_a}{2\pi}\, A_{a3} = \Theta(\xi_{12} - \xi_{a3}) \equiv \Theta_a$$

Herwig coherent branching (very schematic):

$$dP_{12\to 3} = dP_{1\to 3}\cdot dP_{2\to 3} \qquad dP_{a\to 3} = dn_a \cdot S_a$$

$$dn_a = dz_3 \frac{\bar{\alpha}_s}{z_3}\, \frac{d\xi_{a3}}{\xi_{a3}} \cdot \Theta_a$$

Improved branching (exact large angle soft emission)

$$\frac{\xi_{12}}{\xi_{13}\,\xi_{32}} = \frac{\rho_{13}}{\xi_{13}}\Theta_1 + \frac{\rho_{23}}{\xi_{23}}\Theta_2 + \sigma\, \not{\Theta}_1 \not{\Theta}_2\,, \qquad \rho_{a3}, \sigma > 0$$

then $dP_{12\to 3} = dP_{1\to 3}\cdot dP_{2\to 3}\cdot dP_{\mathrm{soft}}$

$$dn_a = dz_3 \frac{\bar{\alpha}_s}{z_3}\, \frac{d\xi_{a3}}{\xi_{a3}}\, \rho_{a3}\, \Theta_a\,, \qquad dP_a = dn_a \cdot S_a$$

$$dn_{\mathrm{soft}} = dz_3 \frac{\bar{\alpha}_s}{z_3}\, \frac{d\Omega_3}{4\pi} \sigma\, \not{\Theta}_1 \not{\Theta}_2\,, \qquad dP_{\mathrm{soft}} = dn_{\mathrm{soft}} \cdot S_{\mathrm{soft}}$$

Working program

- Soft amplitude (leading order) and MHV Parke-Taylor amplitude

$$J_{ab}(q) \sim \sqrt{w^q_{ab}}$$

  NL order factorization: S.Catani,PLB427(1998)161

- Exploit large angle soft emission distribution:
  - Global observables probe global jet structure beyond SL
  - Non-global obs. gives information at SL level
  - Is large angle soft emission in MC "phenomenological relevant"? Was not coherence in MC phenom. relevant? (Stefan Gieseke)
  - explore BFKL connection (Al Mueller)
- Beyond planar approximation, at least next-to-planar $(1/N_c^2)$ Beyond parton picture. See Appendix
- Multi-soft emission in multi-jet environment (Yuri Dokshitzer)

Appendix on $1/N_c^2$

Take $\gamma \to p\bar{p}q_1 \cdots q_n$ with $\omega_n \ll \omega_1 \ll Q$
Starting point: Fiorani,Reina&GM,Nucl.Phys.309(1988)439

$$\mathcal{M}(q_1 \cdots q_n) = \sum_{\text{perm}} (\alpha_{i_1} \cdots \alpha_{i_n})_{\beta\beta'} \, M_n(i_1 \cdots i_n)$$

Insert softest gluon

$$M_n(\cdots \ell \, n \, \ell' \cdots) = J_{\ell\ell'}(n) \cdot M_{n-1}(\cdots \ell\ell' \cdots)$$

$$J_{\ell\ell'}(n) = \frac{q_\ell}{q_n q_\ell} - \frac{q_{\ell'}}{q_n q_{m\ell'}}$$

Then (up constant factor)

$$|\mathcal{M}_n|^2 = \sum_{\text{perm}} \text{Tr}([1 \cdots n] \cdot [i_n \cdots i_1]) \, M_n(1 \cdots n) \, M^*(i_1 \cdots i_n)$$

NB: opposite order in left square bracket

Planar contribution: $W_n(1 \cdots n) = |M_n(1 \cdots n)|^2$

$$\text{Tr}(1 \cdots n \, n \cdots 1) = N C_F^n$$

$$W_n(\cdots \ell \, n \, \ell' \cdots) = J_{\ell\ell'} J_{\ell'\ell} \cdot W_{n-1}(\cdots \ell\ell' \cdots)$$

$$J_{\ell\ell'} J_{\ell'\ell} = \frac{2(q_\ell q_{\ell'})}{(q_\ell q_n)(q_n q_{\ell'})} = 2 w_{\ell\ell'}^n$$

$$W_n^{\text{planar}}(12 \cdots n) = \frac{(p\bar{p})}{(p1)(12)\cdots(n\bar{p})}$$

Next-to-planar (down by $1/N^2$)

Case 1: Insert n on a planar distribution in a non planar way

$$\mathrm{Tr}([\cdots m n m' \cdots][\ldots \ell' n \ell \cdots]) \,, \quad m \neq \ell$$

$$W_n^{\mathrm{NP}}([\cdots m n m' \cdots][\ldots \ell' n \ell \cdots]) = J_{mm'} J_{\ell' \ell} \cdot W_{n-1}^{\mathrm{planar}}(\cdots)$$

$$J_{mm'} J_{\ell' \ell} = w_{m\ell} + w_{m'\ell'} - w_{m\ell'} w_{m'\ell}$$

Dots with same order.

Case 2: Insert n on a next-to-planar distribution in a planar way
Most general NP trace ($B = D = 0$ previuos case)

$$\mathrm{Tr}\left([\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}] \cdot [\bar{\mathcal{E}}, \bar{\mathcal{B}}, \bar{\mathcal{C}}, \bar{\mathcal{D}}, \bar{\mathcal{A}}]\right)$$

$$\mathcal{A} = (a \cdots a') \,, \quad \bar{\mathcal{A}} = (a' \cdots a) \,, \quad \ldots$$

Associated NP distribution

$$W_{n-1}^{\mathrm{NP}} [\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}]$$

Insert in a planar way the softest gluon n.

There are various cases:

- inside $\mathcal{A} = (a \cdots \ell \ell' \cdots a') \to \mathcal{A}_n = (a \cdots \ell \, n \, \ell' \cdots a')$

$$W_n^{\mathrm{NP}}[\mathcal{A}_n, \ldots] = 2 w_{\ell\ell'} \, W_{n-1}^{\mathrm{NP}}[\mathcal{A}, \ldots]$$

  Same for insertion on $\mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}$

- Insert n at the end of $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$

$$W_n^{\rm NP}[\mathcal{A}n, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}] = J_{a'b} J_{da'} \cdot W_{n-1}^{\rm NP}[\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}]$$
$$W_n^{\rm NP}[\cdots \mathcal{B}n \cdots] = J_{b'c} J_{eb'} \cdot W_{n-1}^{\rm NP}[\cdots \mathcal{B} \cdots]$$
$$W_n^{\rm NP}[\cdots \mathcal{C}n \cdots] = J_{c'd} J_{bc'} \cdot W_{n-1}^{\rm NP}[\cdots \mathcal{C} \cdots]$$
$$W_n^{\rm NP}[\cdots \mathcal{D}n \cdots] = J_{d'e} J_{cd'} \cdot W_{n-1}^{\rm NP}[\cdots \mathcal{D} \cdots]$$

- Insert n at the beginning of $\mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}$

$$W_n^{\rm NP}[\mathcal{A}, n\mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}] = J_{a'b} J_{bc'} \cdot W_{n-1}^{\rm NP}[\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}]$$
$$W_n^{\rm NP}[\cdots n\mathcal{C} \cdots] = J_{b'c} J_{cd'} \cdot W_{n-1}^{\rm NP}[\cdots \mathcal{C} \cdots]$$
$$W_n^{\rm NP}[\cdots n\mathcal{D} \cdots] = J_{c'd} J_{da'} \cdot W_{n-1}^{\rm NP}[\cdots \mathcal{D} \cdots]$$
$$W_n^{\rm NP}[\cdots n\mathcal{E}] = J_{d'e} J_{eb'} \cdot W_{n-1}^{\rm NP}[\cdots \mathcal{E} \cdots]$$

where $J_{12}\, J_{34} = w_{14} + w_{23} - w_{13} - w_{24}$

Next-to-planar distribution $W_{n-1}^{\rm NP}[\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}]$ associated to trace

$$\mathrm{Tr}\left([\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}] \cdot [\bar{\mathcal{E}}, \bar{\mathcal{B}}, \bar{\mathcal{C}}, \bar{\mathcal{D}}, \bar{\mathcal{A}}]\right)$$
$$\mathcal{A} = (\cdots a')\,, \quad \mathcal{B} = (b \cdots b')$$
$$\mathcal{C} = (c \cdots c')\,, \quad \mathcal{D} = (d \cdots d')\,, \quad \mathcal{E} = (e \cdots)$$