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Automated resummation of QCD final state observables

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Fermilab

☞ In collaboration with

A. Banfi (Amsterdam) and G. Salam (Paris)

Jet observables

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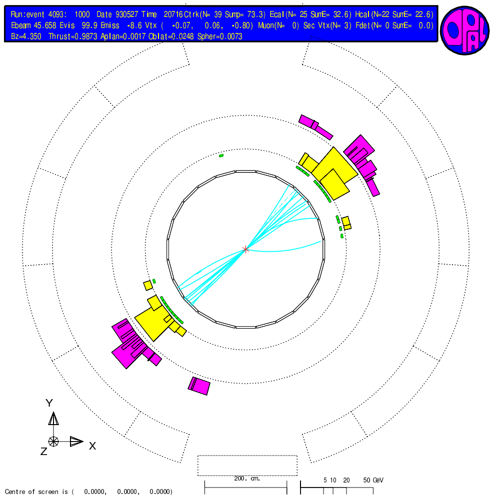
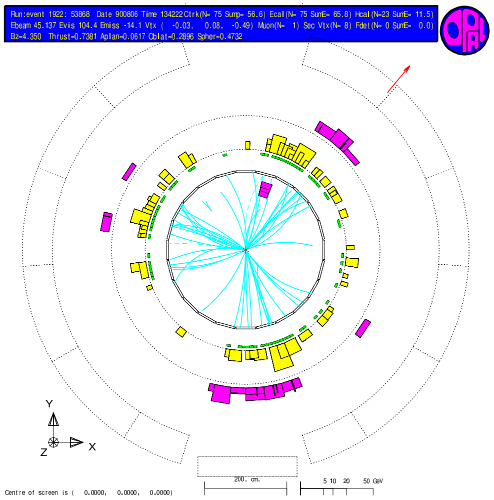
Pencil-like event: $\tau \equiv 1 - T \ll 1$

Planar event: $T \simeq 2/3$





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Event shapes are a good compromise between

- simplicity (it is feasible to make theoretical predictions about them)
- sensitivity to properties of QCD radiation

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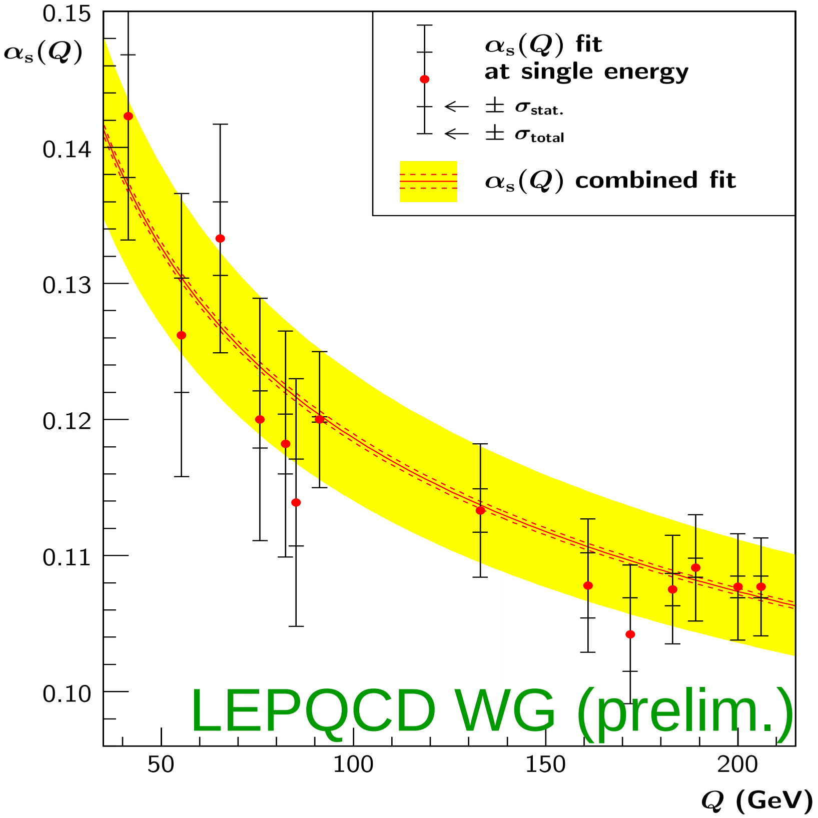
Provide a wealth of information, e.g.:

- Measurements of the coupling α_s and its renormalization group running



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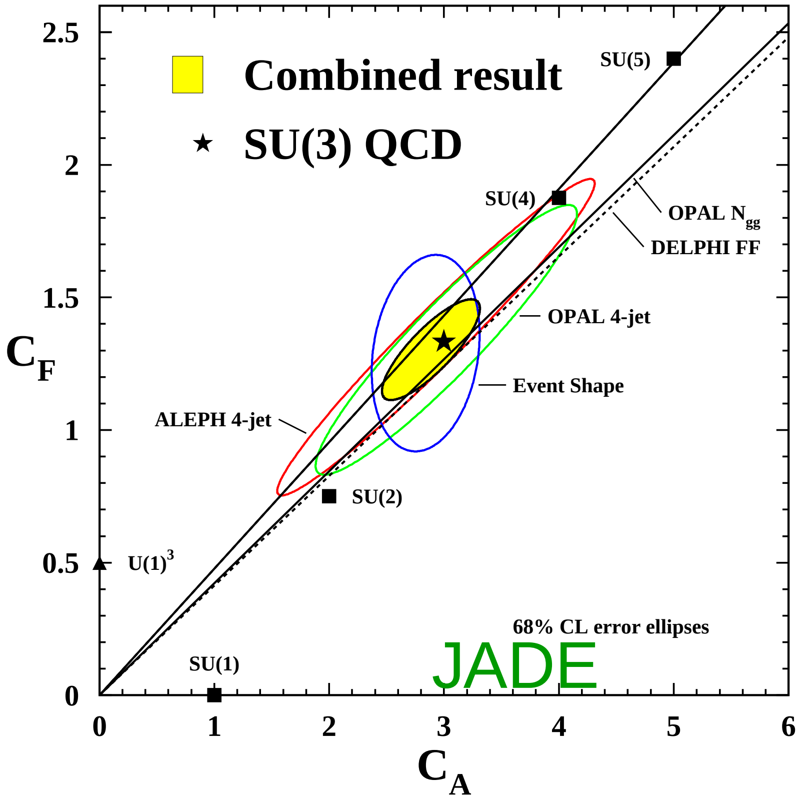

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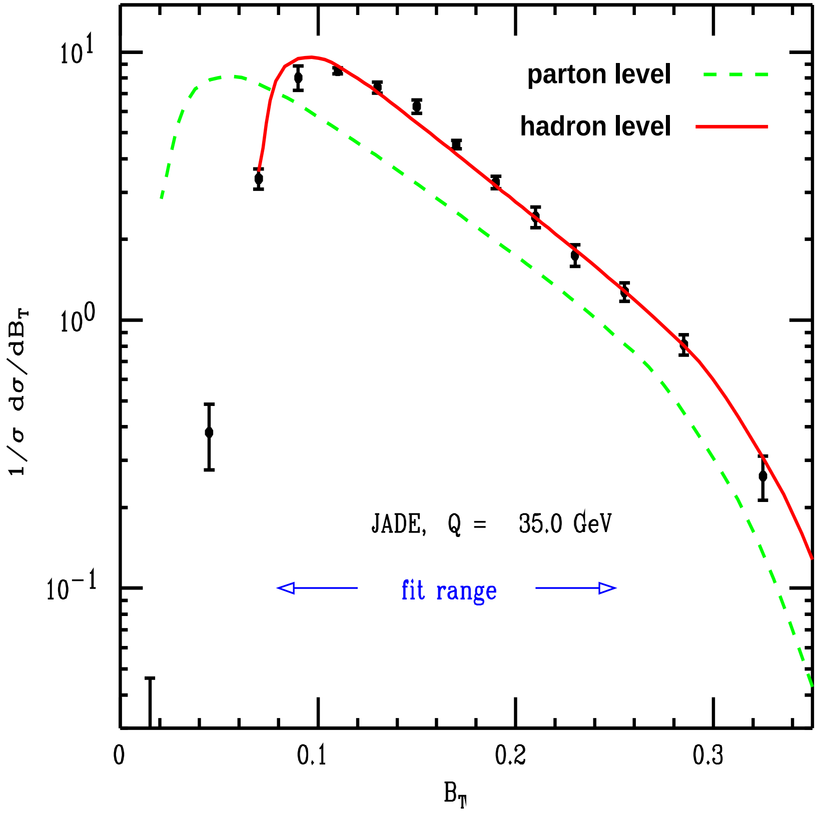
- Measurements/cross checks of the values of the colour factors of QCD
- Studies of connection between parton-level (perturbative description of quarks and gluons) and hadron-level (the real)



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Given an event shape (or a jet-rate) V then

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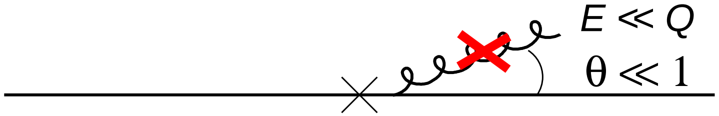
This has an *unequal effect* on real and virtual contributions

- *hard, large angle* emissions *forbidden* (no additional jet)
- *soft and collinear* real emissions are *constrained*

$$\xcancel{\frac{dE}{E}\frac{d\theta}{\theta}\alpha_s(\theta E)}$$



- *virtual* corrections are *unaffected*

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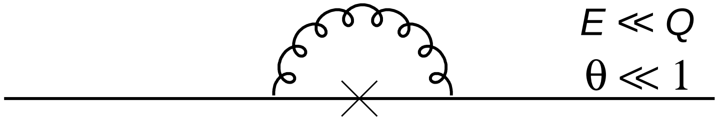

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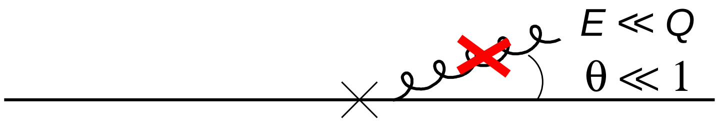

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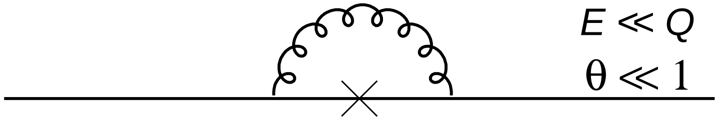


Imbalance leads to *large logarithms* in distributions

$$\mathrm{Prob}(V < v) \simeq 1 - \frac{\#\alpha_s C_F}{2\pi}\ln^2 v + \dots \quad \left[v \ll 1 \quad \Rightarrow \quad \frac{\alpha_s C_F}{2\pi}\ln^2 v = \mathcal{O}(1)\right]$$

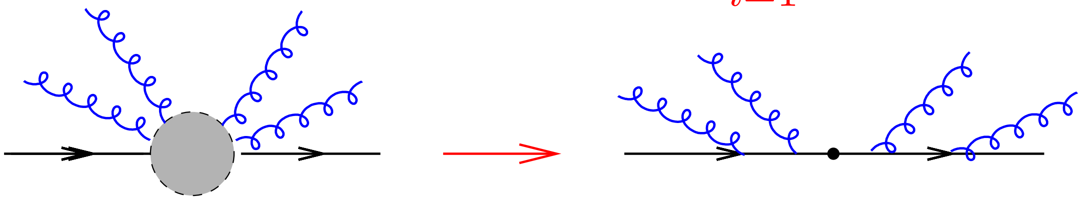
which need to be *resummed to all orders*

Basics of resummation: factorization

First half of the history: Matrix elements and phase space

exploit *angular ordering* ⇒ soft *independent emissions* (⇒ QED)

e.g. $e^+e^- \to 2\,\text{jets} \Rightarrow w_{p\bar{p}}(k_1,\ldots,k_n) = \frac{1}{n!}\prod_{i=1}^{n} w_{p\bar{p}}(k_i) \sim \frac{1}{n!}\prod_{i=1}^{n}\frac{dE}{E}\frac{d\theta}{\theta}$



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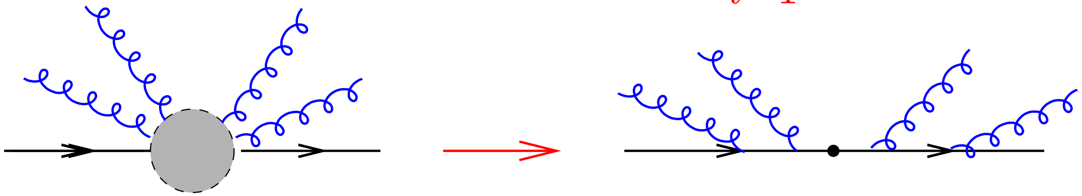

Second half of the history: The observable definition

analyse the observable & use Mellin transforms

$$1 - T \simeq \frac{1}{Q}\sum_{i=1}^{n}\frac{E_i\theta_i^2}{2} \qquad \longrightarrow \qquad \Theta(1 - T < \tau) = \int \frac{d\nu}{2\pi i\nu} e^{\nu\tau}\prod_{i=1}^{n} e^{-\nu\frac{E_i\theta_i^2}{2Q}}$$

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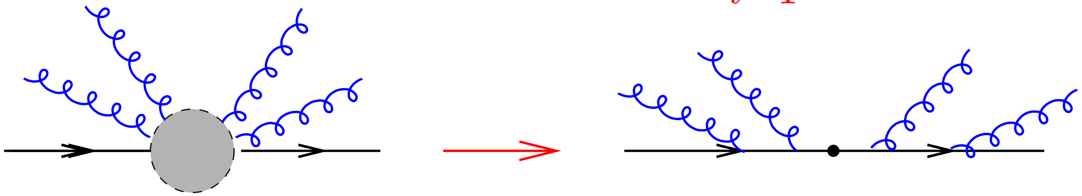

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THE ANSWER

$$\boxed{\Sigma(\tau)\int\frac{d\nu}{2\pi i\nu}e^{\nu\tau}\exp\left[\int\frac{d\theta}{\theta}\frac{dE}{E}\alpha_s(E\theta)\left(e^{-\nu\frac{E_i\theta_i^2}{2Q}} - 1\right)\right]}$$

A selection of analytical NLL predictions

$e^+e^- \to$ 2 jets

- S. Catani, G. Turnock, B. R. Webber and L. Trentadue, *Thrust distribution in e^+e^- annihilation,* Phys. Lett. B **263** (1991) 491.
- S. Catani, G. Turnock and B. R. Webber, *Heavy jet mass distribution in e^+e^- annihilation,* Phys. Lett. B **272** (1991) 368.
- S. Catani, Yu. L. Dokshitzer, M. Olsson, G. Turnock and B. R. Webber, *New clustering algorithm for multi-jet cross-sections in e^+e^- annihilation,* Phys. Lett. B **269** (1991) 432.
- S. Catani, L. Trentadue, G. Turnock and B. R. Webber, *Resummation of large logarithms in e^+e^- event shape distributions,* Nucl. Phys. B **407** (1993) 3.
- S. Catani, G. Turnock and B. R. Webber, *Jet broadening measures in e^+e^- annihilation,* Phys. Lett. B **295** (1992) 269.
- G. Dissertori and M. Schmelling, *An Improved theoretical prediction for the two jet rate in e^+e^- annihilation,* Phys. Lett. B **361** (1995) 167.
- Y. L. Dokshitzer, A. Lucenti, G. Marchesini and G. Salam, *On the QCD analysis of jet broadening,* JHEP **9801** (1998) 011
- S. Catani and B. R. Webber, *Resummed C-parameter distribution in e^+e^- annihilation,* Phys. Lett. B **427** (1998) 377
- S. J. Burby and E. W. Glover, *Resumming the light hemisphere mass and narrow jet broadening distributions in e^+e^- annihilation,* JHEP **0104** (2001) 029
- M. Dasgupta and G. Salam, *Resummation of non-global QCD observables,* Phys. Lett. B **512** (2001) 323
- C. F. Berger, T. Kucs and G. Sterman, *Event shape / energy flow correlations,* Phys. Rev. D **68** (2003) 014012

DIS 1+1 jet

- V. Antonelli, M. Dasgupta and G. Salam, *Resummation of thrust distributions in DIS,* JHEP **0002** (2000) 001
- M. Dasgupta and G. Salam, *Resummation of the jet broadening in DIS,* Eur. Phys. J. C **24** (2002) 213
- M. Dasgupta and G. Salam, *Resummed event-shape variables in DIS,* JHEP **0208** (2002) 032

e^+e^-, DY, DIS 3 jets

- A. Banfi , G. Marchesini, Y. L. Dokshitzer and GZ, *QCD analysis of near-to-planar 3-jet events,* JHEP **0007** (2000) 002
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$\sim$ 1 observable per article

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Error prone business [✗ $\sim 30\%$]!

Automated resummed predictions

Our goal: develop a computer code which resums final state observables at NLL accuracy in an automated way – as for fixed order calculations.

The user just

- ✗ fixes the Born process and the number of hard jets (legs)
- ✗ provides the definition of the observable in the form of a computer routine

☛ To achieve this one needs to understand the origin of all NLL terms in observable distributions in a general way.

The master formula

$$\boxed{\Sigma(v) =_{NLL} \sum_{\text{sub.}} \int [d\Phi]_{\text{hard}} \Sigma_s(v) \cdot \mathcal{F}(R')}$$

Banfi , Salam, GZ hep-ph/0304148

✓ Analytical resummation for the “easy” Σ_s: *pure LL and NLL terms*

$$\Sigma_s(v) = \prod_{\ell=1}^{n_{inc}} \underbrace{f_\ell(v^{\frac{2}{a+b_\ell}} \mu_F^2)}_{\text{pdfs}} \otimes \prod_{\ell=1}^{N} \underbrace{J_\ell(L)}_{\text{jet function}} \cdot \underbrace{S\left(T(L/a)\right)}_{\text{soft}}$$

- soft and collinear emission ⇒ jet function $J_\ell(L)$
  (all LL Sudakov suppression and some NLL terms)
- hard collinear splitting ⇒ evolution of the pdfs
- soft large angle ⇒ QCD coherence and geometry dependence in S

✓ the “difficult” $\mathcal{F}$ is computed numerically but is by construction a pure NLL function

Single emission properties

IDEA: Define a simpler observable with the same double logs but factorizes trivially

$$V(k_1, \ldots k_n) \Rightarrow V_s \equiv \max[V(k_1), \ldots, V(k_n)]$$

- Simple factorization $\Theta(V_s - v) = \prod_i \Theta(V_i - v) \Rightarrow$ analytical resummation straightforward!

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Fix a Born event and emit a soft gluon k collinear to a given hard leg ℓ.

We parametrize

$$\boxed{V(k) \simeq d_\ell \left(\frac{k_t}{Q}\right)^{a_\ell} e^{-b_\ell\,\eta} g_\ell(\phi)}$$

$k_t \Rightarrow$ transverse momentum

$\eta \Rightarrow$ rapidity

$\phi \Rightarrow$ azimuth

✓ Σ_s known given the (automatically determined) quantities $a_\ell, b_\ell, d_\ell, g_\ell(\phi)$

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To resum $V(k_1 \dots k_n)$ one needs to account for the observable specific mismatch between $V(k_1, \dots k_n)$ and $V_s \Rightarrow$ multiple emission effects

Multiple emission effects

The function $\mathcal{F}$ which encodes the information on how precisely the observable depends on multiple emissions, e. g.

- if $V(k_1,\dots k_n) = \max\{V(k_1,\dots V(k_n)\} \quad \Longrightarrow \quad \mathcal{F} = 1 \qquad [y_3^{\mathrm{Cam.}}]$
- if $V(k_1,\dots k_n) \;=\; V(k_1) + \cdots + V(k_n) \quad \Longrightarrow \quad \mathcal{F} = \dfrac{e^{-\gamma_E R'}}{\Gamma(1+R')} \qquad [\tau]$
- in general, compute $\mathcal{F}$ via Monte Carlo event samples targeted to be observable

$$\mathcal{F} = \langle \exp\left\{ -R' \ln \frac{V(k_1,\dots k_n)}{\max\{V(k_1),\dots V(k_n)\}} \right\} \rangle$$

☛ Notation: $R' \equiv -dR/dL$ with $R(v)$ the LL Sudakov exponent $\Sigma_s(v) = e^{-R(v)}$

➥R' and so $\mathcal{F}$ are pure NLL functions!

Requirements on the observable

For the observable to be resummed automatically it should

- ✗ vanish in the Born limit and be positive defined
- ✗ behave as $V(k) \simeq d_\ell \left(\frac{k_t}{Q}\right)^{a_\ell} e^{-b_\ell \eta} g_\ell(\phi)$ for 1 SC gluon along leg ℓ
- ✗ be infrared and collinear safe
- ✗ be continuously global ($a_\ell = a \;\; \forall$ hard legs ℓ)
- ✗ exponentiate (no JADE)

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  (all other conditions are satisfied by all observables resummed so far)

- ☞ the essential feature of the program is the ability to perform all checks automatically and to resum the observable only when correctness of the result is guaranteed at NLL

Exponentiation

Some observables have exponentiating double (and single) logs

$$\mathrm{P}(v) = 1 - X\frac{\alpha_s C_F}{\pi}\ln^2 v + \frac{1}{2}X^2\left(\frac{\alpha_s C_F}{\pi}\right)^2 \ln^4 v + \ldots$$

Exponentiation

Some observables have exponentiating double logs, others do not, e.g. Jade-algorithm jet rates:

$$\mathrm{P}_{\mathrm{Jade2-jet}}(y_{\mathrm{cut}}) = 1 - \frac{\alpha_s C_F}{\pi} \ln^2 y_{\mathrm{cut}} + \frac{1}{2} \cdot \frac{5}{6} \left(\frac{\alpha_s C_F}{\pi}\right)^2 \ln^4 y_{\mathrm{cut}} + \dots$$

Brown and Striling, Phys.Lett.B 252 (1990)

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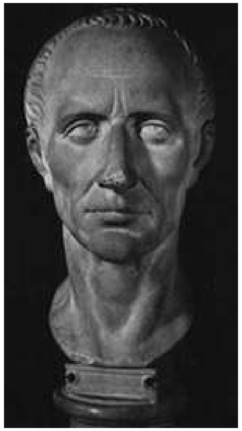
Brown and Striling, Phys.Lett.B 252 (1990)

- ☛ No one jet knows how to resum Double Logs, let alone what matrix-element ingredients are needed to achieve NLL accuracy!

> Any automated approach to NLL resummation has better be able to establish whether an observables exponentiates

CAESAR: conquering resummations



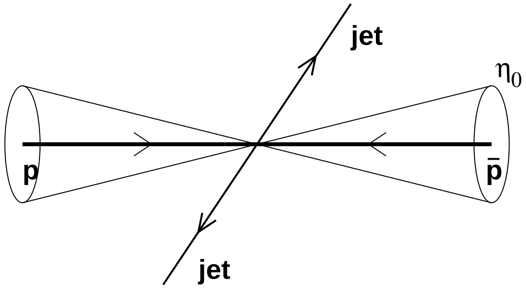
Computer Automated Expert
Semi-Analytical Resummation

- currently limited to global observables
- tested against all known global, exponentiable event shapes
- results from an early version used by the LEP-QCD-WG for fits of α_s
- can be applied to
  - 2 & 3 jets in e^+e^-
  - [1+1] & [1+2] jets in DIS
  - Drell-Yan + 1 jet
  - hadron-hadron dijet events [⇐ first resummations]

Observables in hadronic dijet production

Cut around the beam $|\eta| < \eta_0$

➥ Problems with globalness ⚠



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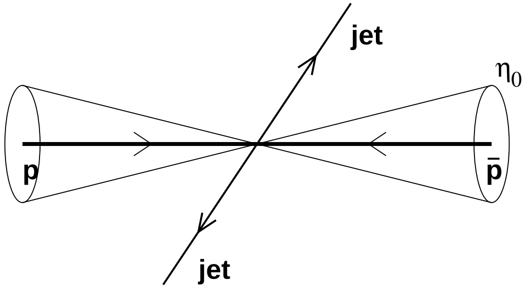


Directly global observables: $\eta_0 > 1$

- ✗ Transverse thrust

$$T_T = \frac{1}{E_T} \max_{\vec{n}_T} \sum_i |\vec{p}_{ti} \cdot \vec{n}_T|$$

- ✗ Thrust minor

$$T_m = \frac{1}{E_T} \sum_i |p_i^{out}|$$

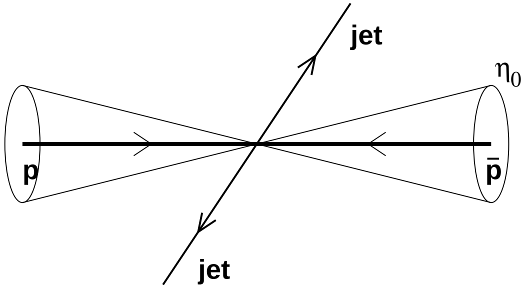
Predictions valid as long as

$$|\log v| < (a + b_\ell)|\eta_0|$$

Observables in hadronic dijet production

Cut around the beam $|\eta| < \eta_0$

➥ Problems with globalness ⚠



Directly global observables: $\eta_0 > 1$

✗ Transverse thrust

$$T_T = \frac{1}{E_T} \max_{\vec{n}_T} \sum_i |\vec{p}_{ti} \cdot \vec{n}_T|$$

✗ Thrust minor

$$T_m = \frac{1}{E_T} \sum_i |p_i^{out}|$$

Predictions valid as long as

$$|\log v| < (a + b_\ell)|\eta_0|$$

Indirectly global observables: $\eta_0 = \mathcal{O}(1)$

✗ Transverse thrust

$$T_T = \frac{1}{E_{T,\eta_0}} \left(\max_{\vec{n}_T} \sum_{|\eta_i|<\eta_0} |\vec{p}_{ti} \cdot \vec{n}_T| - \left| \sum_{|\eta_i|<\eta_0} \vec{p}_{ti} \right| \right)$$

✗ Thrust minor

$$T_m = \frac{1}{E_{T,\eta_0}} \left(\sum_{|\eta_i|<\eta_0} |p_i^{out}| + \left| \sum_{|\eta_i|<\eta_0} \vec{p}_{ti} \right| \right)$$

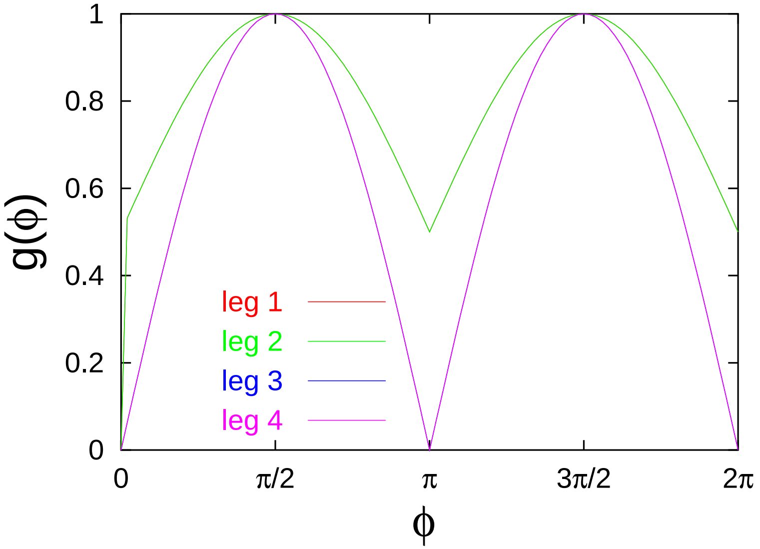
Predictions valid as usual,

but $\mathcal{F}$ diverges at $R' = R'_c$

Sample output: the indirectly global thrust minor

- ✗ Tests on the observable

Test	result
check number of jets	T
all legs positive	T
global	T
continuously global	T
additive	F
exponentiate	T
eliminate subleading effects	T
opt. probe region exists	T



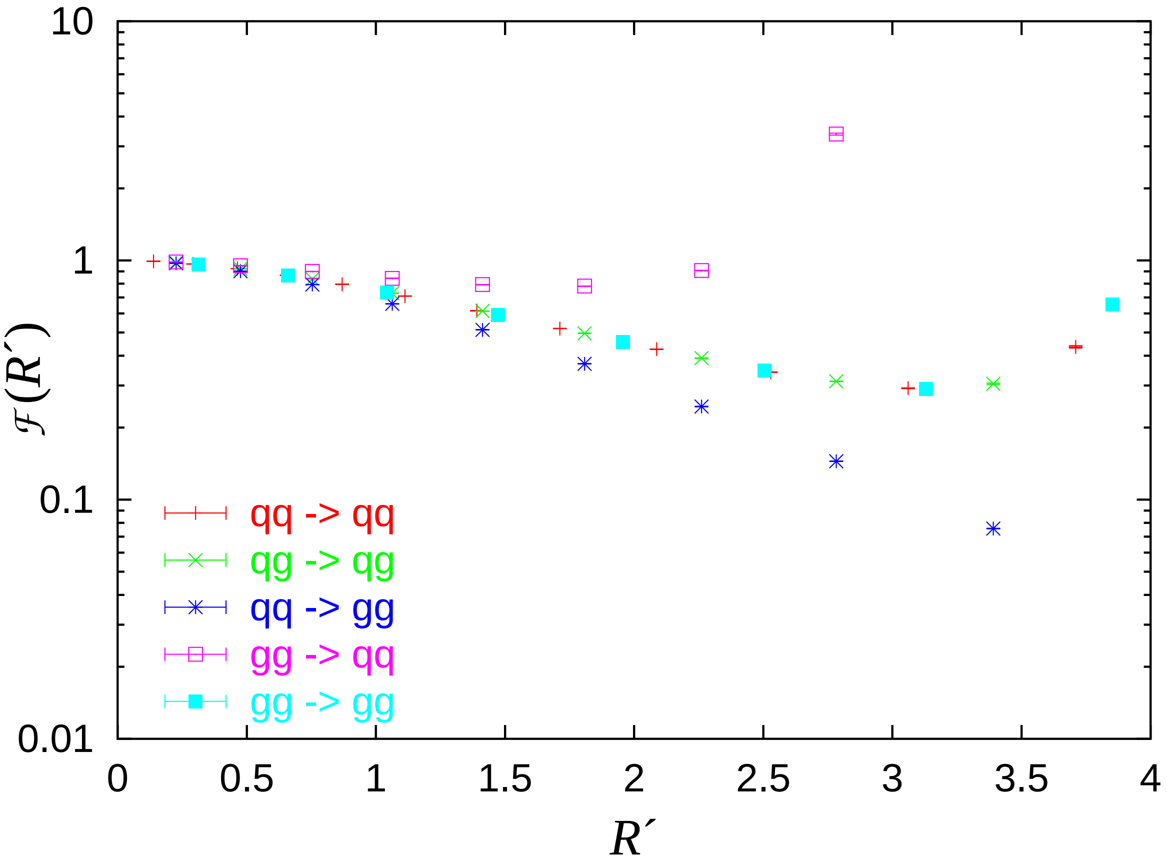
- ✗ Single emission properties

leg ℓ	a_ℓ	b_ℓ	$g_\ell(\phi)$	d_ℓ	$\langle \ln g_\ell(\phi) \rangle$
1	1	0	tabulated	2	-0.2201
2	1	0	tabulated	2	-0.2201
3	1	0	$\sin(\phi)$	2	-Ln(2)
4	1	0	$\sin(\phi)$	2	-Ln(2)

☞ Tables and plots generated automatically by CAESAR

$\mathcal{F}(R')$ *for the indirectly global thrust minor*

The multiple emission function $\mathcal{F}(R')$

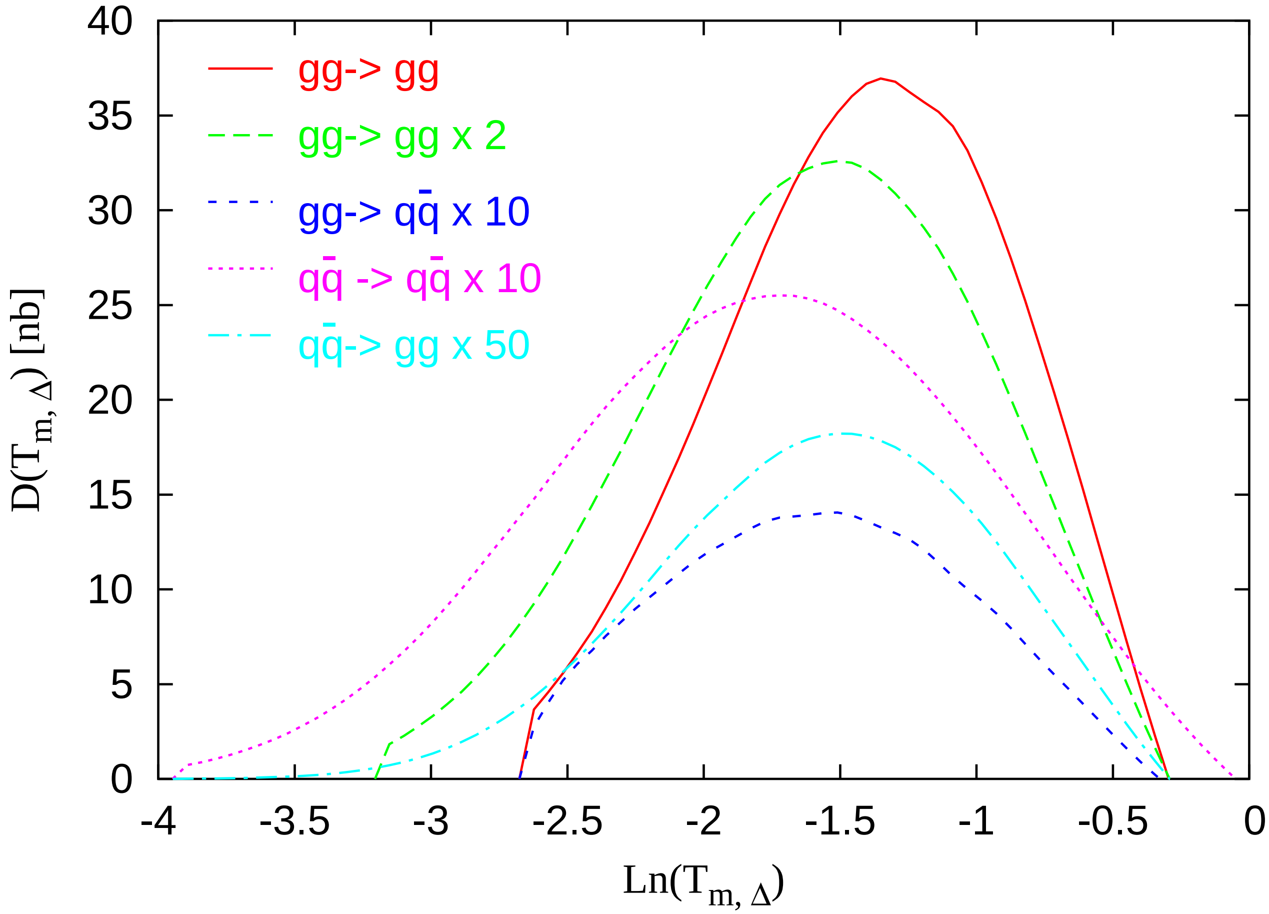


☞ Different result for different colour configurations

The indirectly global thrust minor

Dijets events at Tevatron run II regime

- run II regime $\sqrt{s} = 1.96$ TeV
- cut on jet transverse energy $E_T > 50$GeV and on rapidity $|\eta| < 1$



$\alpha_s(M_Z) = 0.118$

$\mu_F = \mu_R = P_T$

$X_c = 1$

PDFS: CTEQ6M

Physical/mathematical/technical content of CAESAR

- ✓ Born processes currently implemented

✌ e^+e^--collisions:	$e^+e^- \longrightarrow$ 2 jets	$e^+e^- \longrightarrow$ 3 jets
✌ DIS collision:	$p\ \ e \quad \longrightarrow$ 2 jets	$p\ \ e \quad \longrightarrow$ 3 jets
✌ Drell Yan collision:	$p_1p_2 \longrightarrow Z_0 +$ jet	
✌ Hadronic collisions:	$p_1p_2 \longrightarrow$ 2jets	

  $(p_i = q, \bar{q}, g)$

- ✓ Implementation of exact analytical formulas whenever possible
- ✓ Recoil in dipole method

  Catani & Seymour, Nucl. Phys. B **485** (1997) 291
- ✓ Evolution of colour charge (soft radiation at large angle)

  Kidonakis, Oderda & Sterman, Nucl. Phys. B **531** (1998) 365
- ✓ PDF evolution code

  Dasgupta & Salam, Eur. Phys. J. C **24**, 213 (2002)
- ✓ Extended arbitrary precision arithmetic package

  Bailey, RNR Technical Report RNR-94-013

Conclusions & outlook

- ☛ In less inclusive regions fixed order calculations insufficient
  $\Rightarrow$ resummation of logarithmic enhanced terms mandatory
- ☛ the use of resummations limited by availability of analytical results

Main result: rigorous procedure to perform resummation semi-analytically

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- EX: first NLL predictions in hh collisions (indirectely globalness)
- TH: necessary and sufficient condition for exponentiation

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- ● release CAESAR v1.0

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Wish-list

- extension non-global observables and inclusion of mass effects