The Ongoing Saga Surrounding the Velocity Fluctuations in Sedimentation

Michael P. Brenner
Division of Engineering and Applied Sciences
Harvard University

Collaborators:

Theory.
Peter Mucha (Georgia Tech)

+ help from Boris Shraiman…

Experiments
Dave Weitz Harvard
Shang Tee
Suliana Manley
Luca Cipelletti

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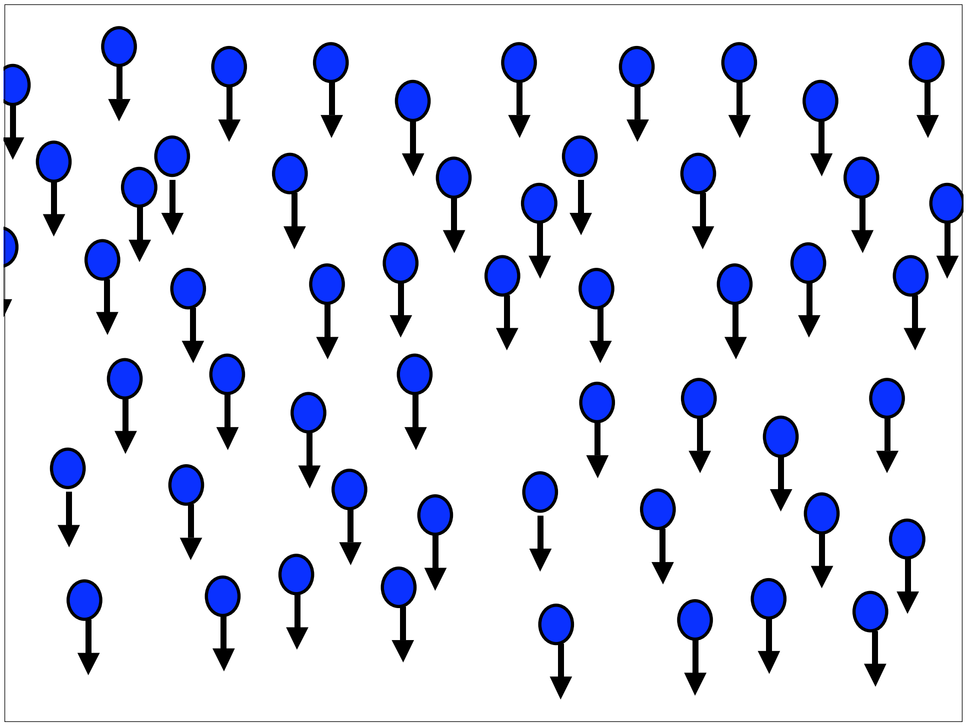

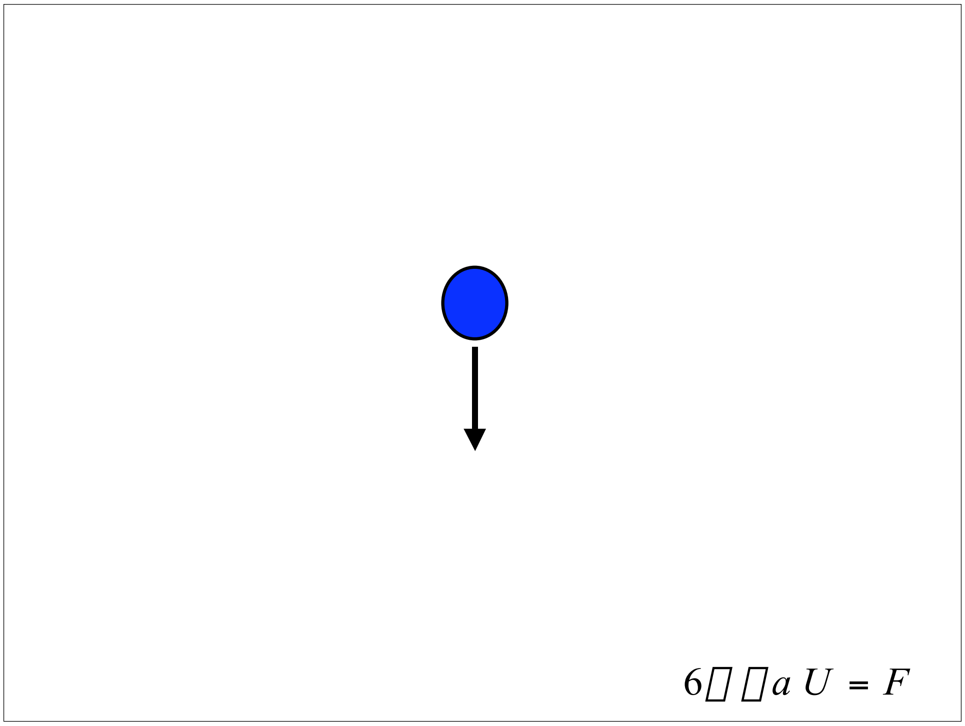

$$6\square\ \square a\, U = F$$

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$$\langle U\rangle = U_{stokes}(1 - 6.55\phi)$$

Batchelor (1972)

Kermack (1929)

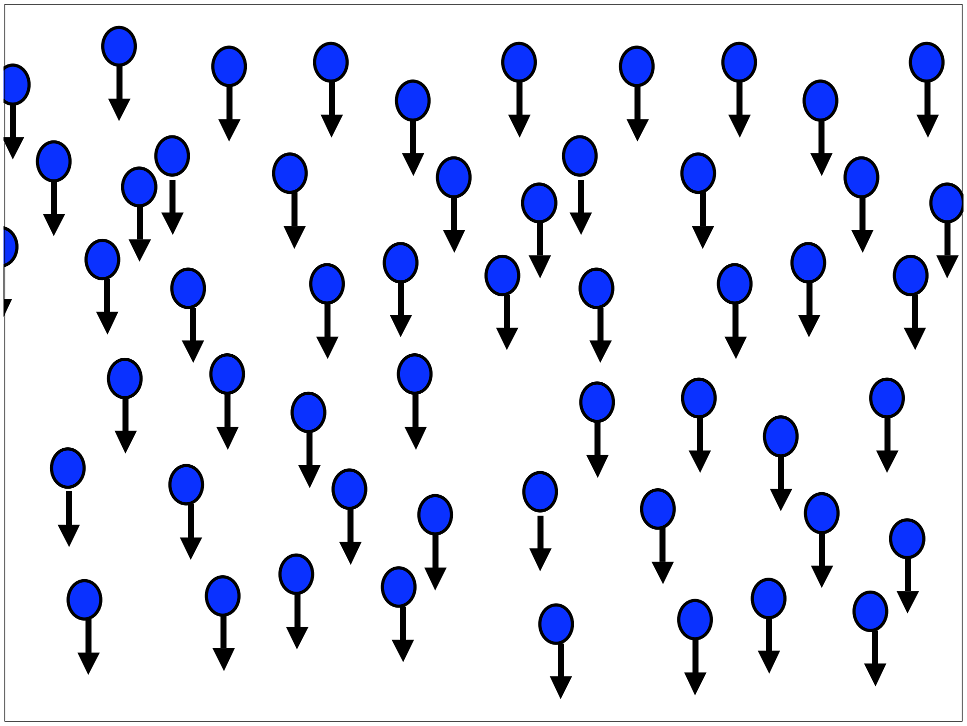
Assumptions

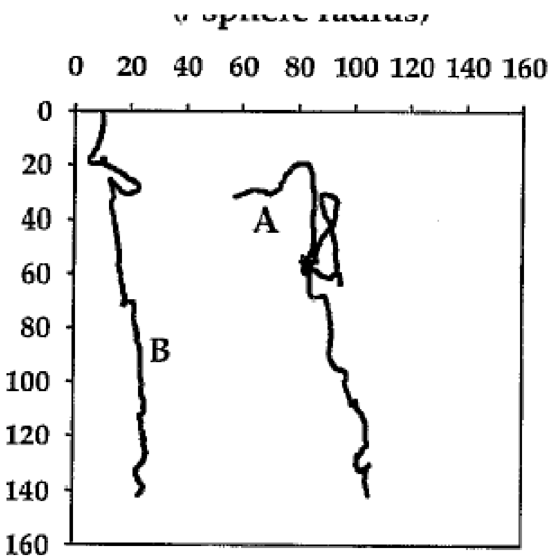
Vanishing Reynolds Number

Monodisperse

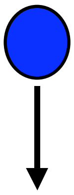
Dilute

No Brownian Motion



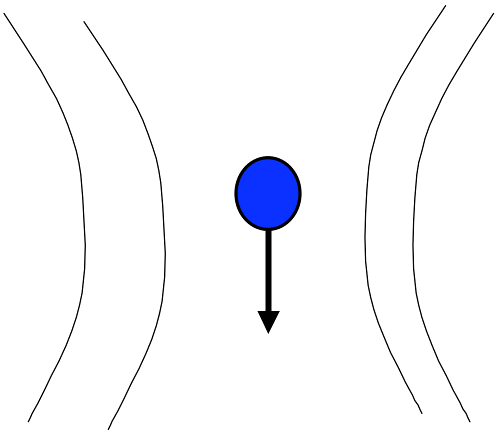


E. Guazzelli et.al,
Phys Fluids, 1995



$6\pi\mu a U = F$

$$\mu \nabla^2 u - \nabla p = F\delta(x)\hat{z}, \qquad \nabla \cdot u = 0$$



Corrections to satisfy B.C.

$$u(x) = \underbrace{\frac{F}{8\pi\mu}\left[\frac{I}{|x-x'|} + \frac{(x-x')(x-x')}{|x-x'|^3}\right]}_{S(x-x')} + O\left(\frac{a}{|x-x'|}\right)^2$$

$$\frac{dx_i}{dt} = U_{stokes} + \sum_{j \neq i} S(x_j - x_i)$$

$$\frac{dx_i}{dt} = U_{stokes} + \underbrace{\sum_{j \neq i} S(x_j - x_i)}$$

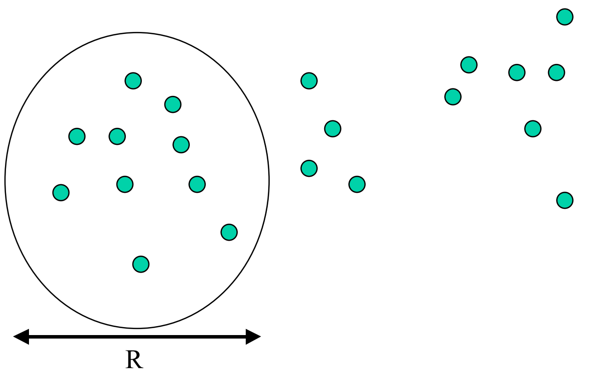
$$\sim U_{stokes}^2 \phi \int \frac{d^3r}{a^3} S(r)^2$$

Caflisch + Luke, 1985.

$$\sim U_{stokes}^2 \phi \int \frac{d^3r}{a^3} \left(\frac{a}{r}\right)^2 \sim U_{stokes}^2 \phi \frac{r}{a}$$

Physical Mechanism:

E. J. Hinch



$$\delta N \sim \pm\sqrt{N} \sim \sqrt{\phi R^3 / a^3}$$

Number Fluctuation

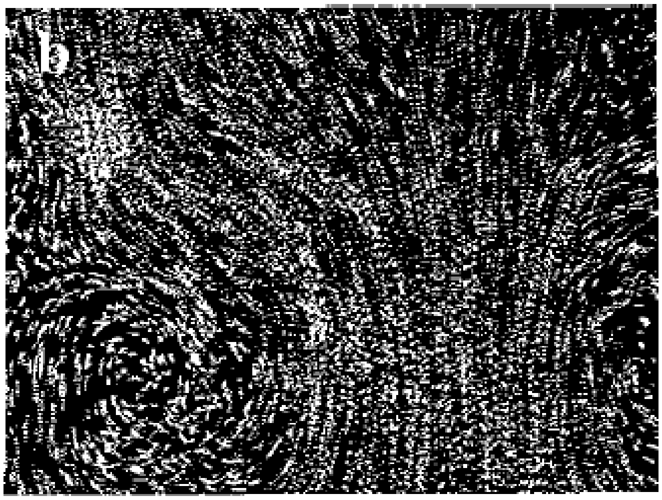
$$\delta U \sim U_{stokes} \frac{\sqrt{N}}{R} \sim U_{stokes} \sqrt{\phi R / a}$$

Two Views:

(a) **The argument is wrong.** No way the velocity fluctuations depend on system size. Some type of “screening mechanism” exists.

(b) **The arguments are correct.** Diffusion in a sediment qualitatively different than normal diffusion.

Experiments



Lei, Tong & Ackerson PRL 2001

The Ongoing Saga Surrounding the Velocity Fluctuations in Sedimentation

:**Experiments:**

Velocity Fluctuations independent of system size

(1) Ham and Homsy (1988);

(2) Nicolai and Guazzelli (1994)

Particle Tracking

(3) **Segre', Herboltzheimer and Chaikin**, 1997, PIV:

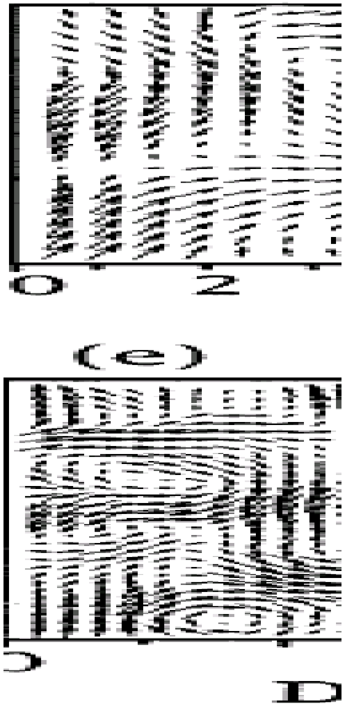
(4) Guazzelli (Phys. Fluids, 2001) (larger cells)

P.I.V.

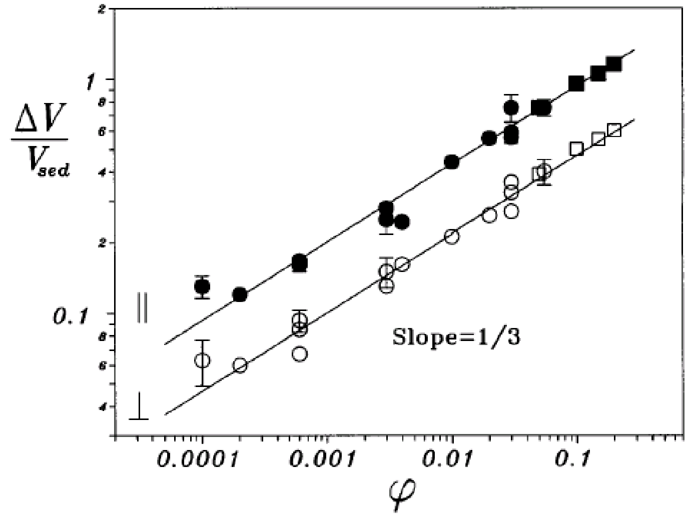
(5) Lei, Tong and Ackerson (Phys. Rev. Lett. 2001)

Density Fluct.

Segre', Herboltzheimer and Chaikin, *Phys. Rev. Lett.*,1997



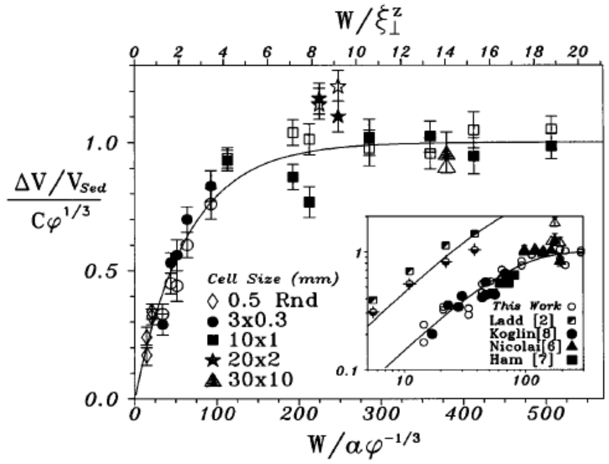
The Ongoing Saga Surrounding the Velocity Fluctuations in Sedimentation



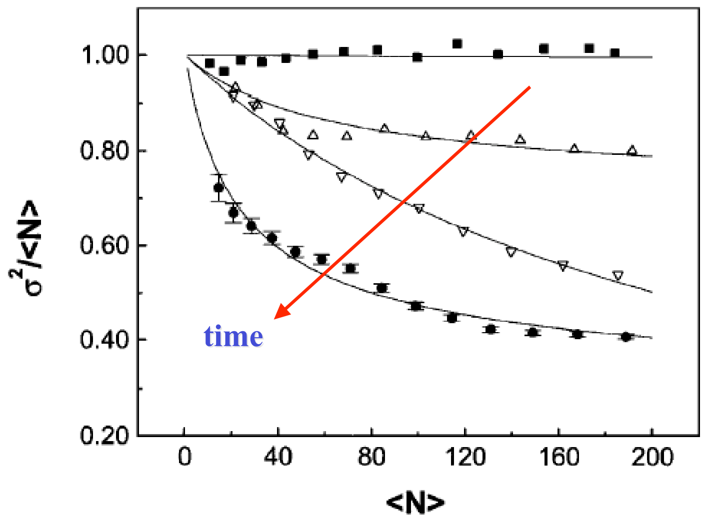
$$\Delta V \sim U_{stokes}\varphi^{1/3}$$

$$\ell \sim a\varphi^{-1/3}$$

The Fluctuations are universal, independent of cell size.



Change in Character of Number Fluctuations



X. Lei, B. Ackerson and P. Tong , Phys Rev Lett, 2001

Theories and Simulations

(1) Koch and Shaqfeh, 1991
special particle distribution

(2) Bruinsma, Frey, Levine, PRL 1999.
Renormalization arguments => screening

SCREENING (Theory)

(1) Ladd, 1994: Lattice Boltzman simulations.

(2) Koch, 1994: point particle simulations
No evidence of screening in periodic box

particle number? ~ 30000

NO SCREENING (Simulation)

Outline:

(1) The Dilemma

---

(2) Finite Cells: Expectations and Simulations

(3) Wide Cell experiments (Weitz)

(4) A mystery.

(5) A resolution.

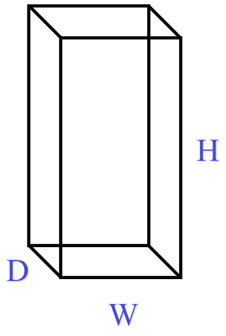
If there is time: A short story
Elastic Instability of a Growing Tissue

All theories and simulations of fluctuations assumed that system is:

(a) infinite
(b) homogeneous.

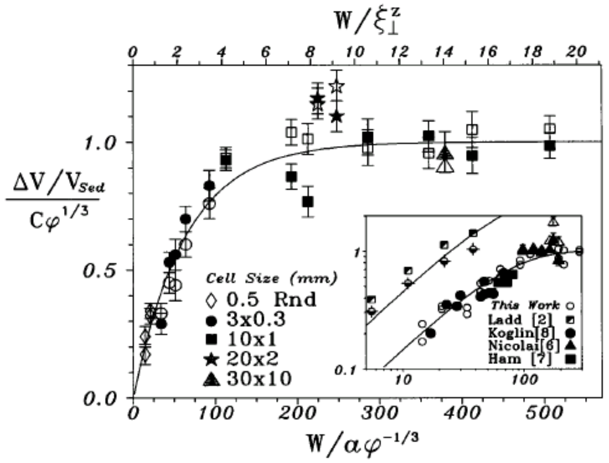
Experiments are definitely not infinite.

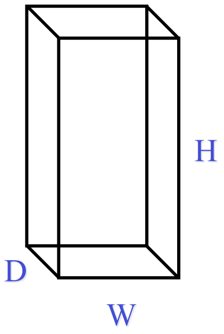
$D/a < 100$



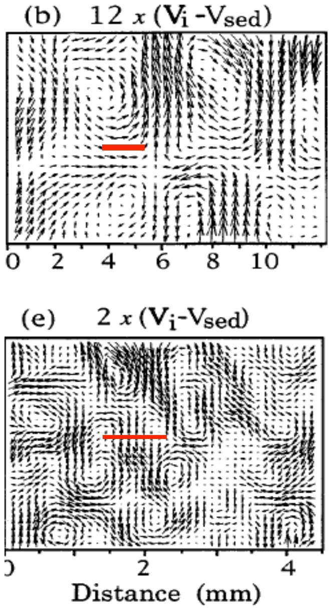
The Ongoing Saga Surrounding the Velocity Fluctuations in Sedimentation

“Universal” Fluctuations, independent of cell size.



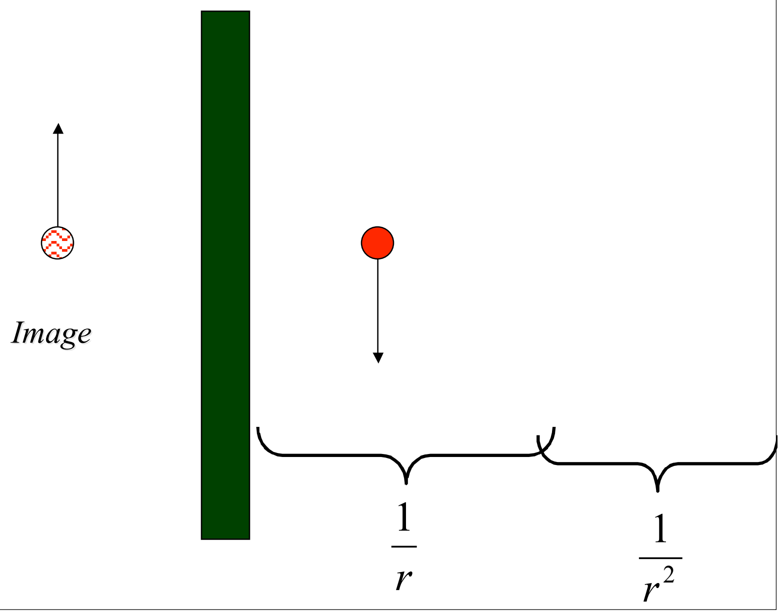


D/a ~ 100 (fixed)



Segre et.al.

Wall Effects.

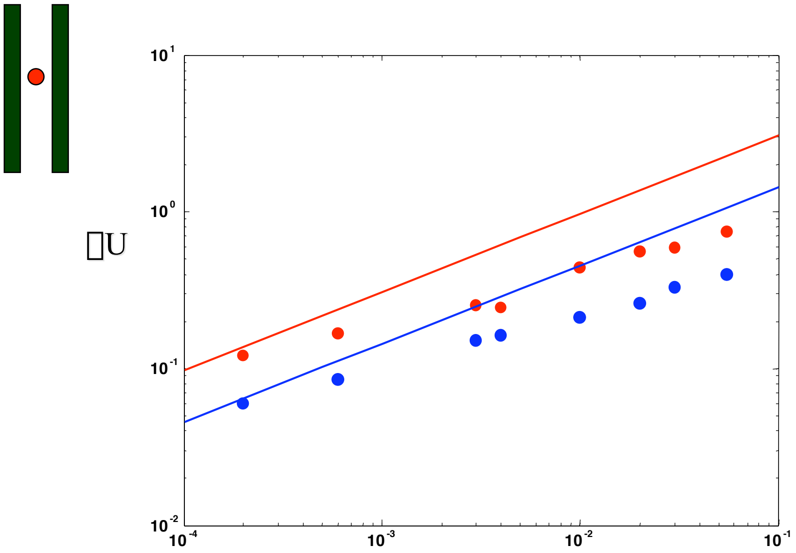


$$\square U_{\|}^{2}(x) = \square\square\frac{d^{3}x'}{a^{3}}\,\mathrm{u}_{\|}(x \square x')^{2}$$

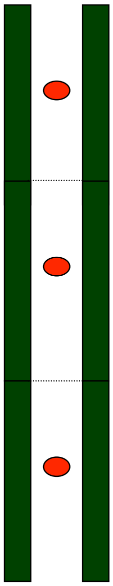
Integral is convergent when wall effects are taken into account!

$$\frac{\square U_{\|}^{2}}{U_{stokes}^{2}} = \mathrm{c(geometry)}\ \frac{\mathrm{d}}{\mathrm{a}}\square \qquad \ell \sim \frac{d}{2}$$

How does it compare to experiments?



$$\square V_{\|} = V_{stokes}\, 0.864\sqrt{\frac{d}{a}\square}^{\,\square}$$



Perhaps the uniform distribution is destabilized by boundaries?

Developed method for solving

$$\frac{dx_i}{dt} = \square_{j \neq i} S(x_j \square x_i)$$

With bounding walls in O(N log(N)) operations.

N $< 4\ 10^6$ particles

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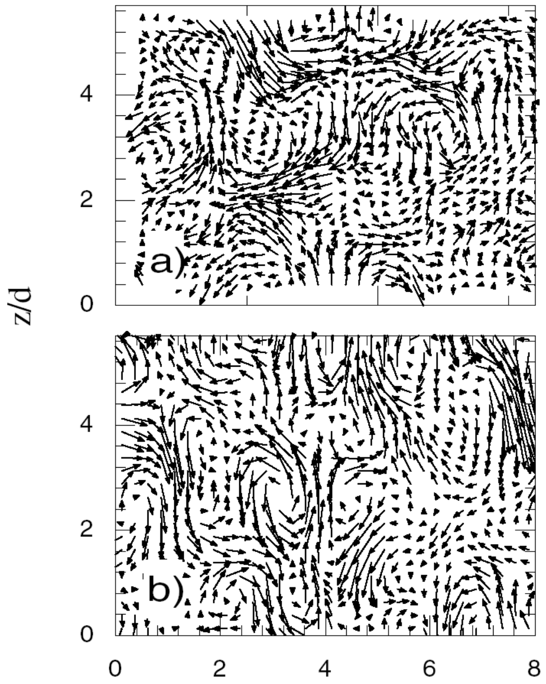


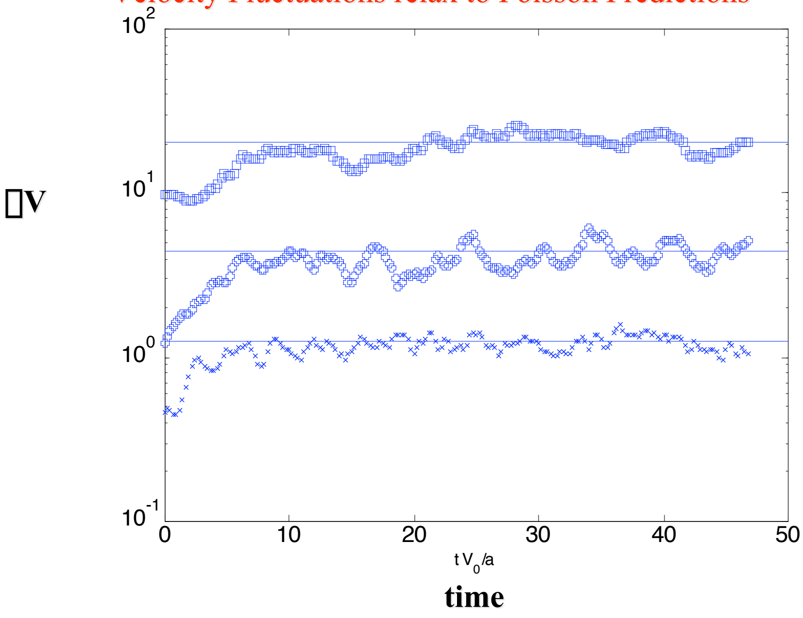
Experiments

(S. Tee)

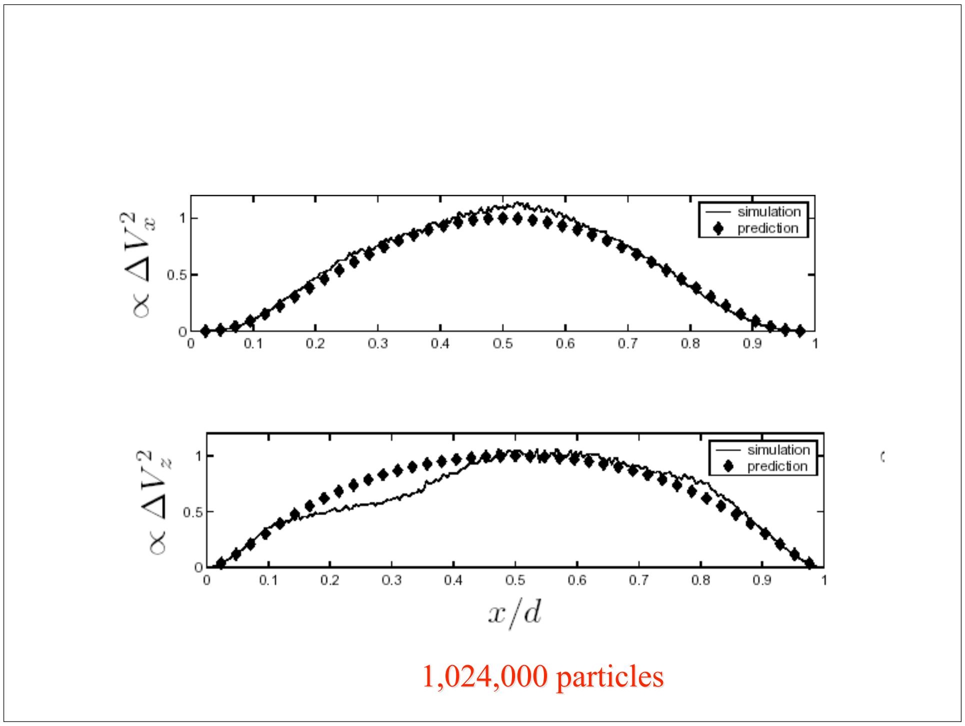
Simulations

(P. Mucha)

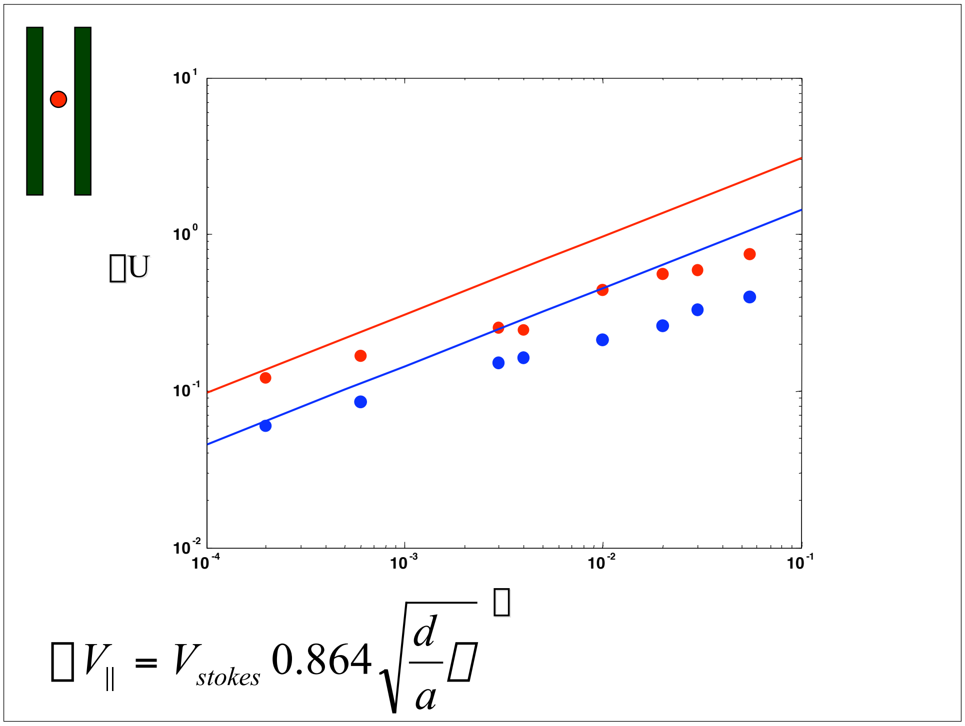
Velocity Fluctuations relax to Poisson Predictions



64,000 particles



1,024,000 particles



$$\square V_{\parallel} = V_{stokes}\, 0.864 \sqrt{\frac{d}{a}\square}^{\,\square}$$

Experiments

Shang Tee, Suliana Manley, Luca Cipelletti
Dave Weitz

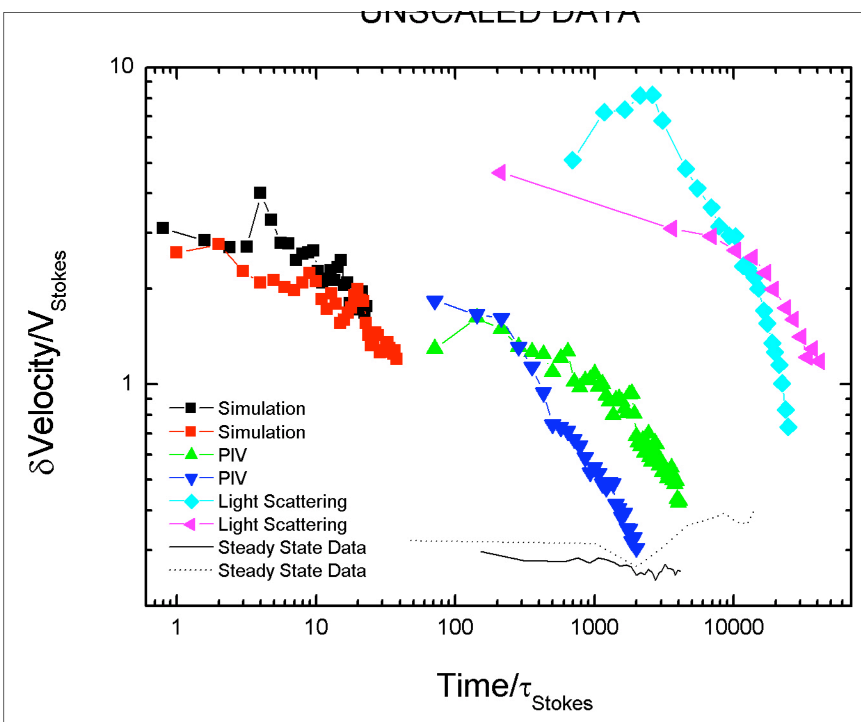
Increase the cell depth.

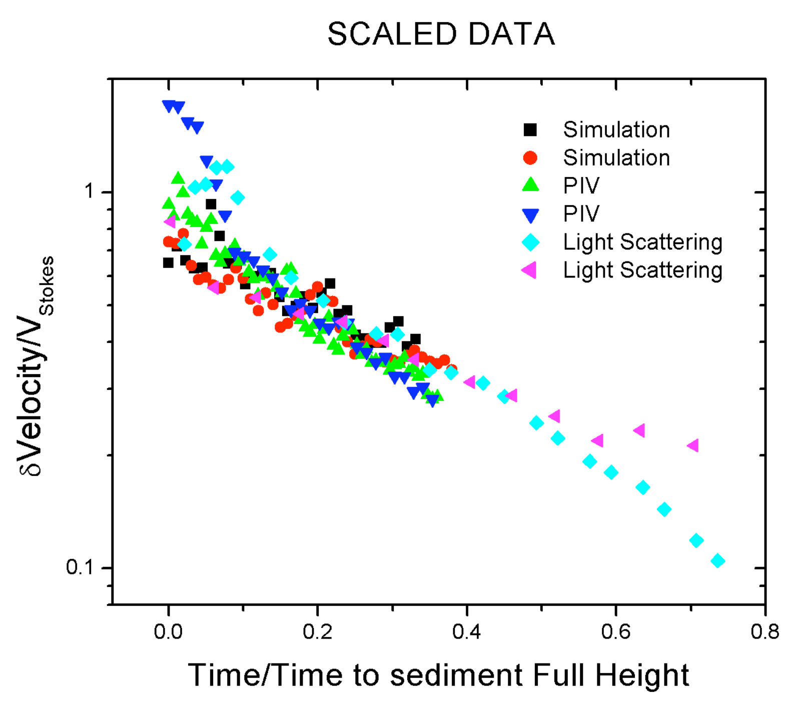
Do the velocity fluctuations increase or not?

Two techniques:

Particle Imaging Velocimetry	(~25 micron particles)
Light Scattering	(~2.5 micron particles)

Cell Sizes: (D/a, W/a, H/a) ~ (1000,5000,15000)





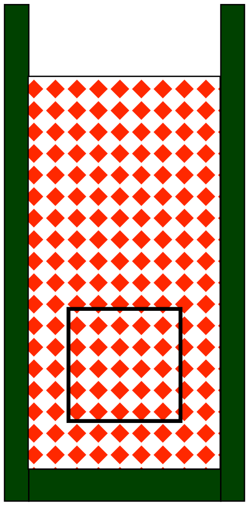
Simulations do not produce decay
There must be an additional physical effect.

Possible Effects:

(1) Polydispersity ⊘
(2) Inertia ⊘
(3) Boycott Effect (Cell is tilted?) ⊘
(4) Thermal Convection? ⊘

Radius	phi	Viscosity	Cell D	Cell W	Cell H	MIS	D MIS	W MIS	H MIS	Vsed	Vstokes	Tstokes
1.5e-4	0.087	0.16	0.5	1.8	3	5.46E-04	9.16E+02	3.30E+03	5.50E+03	1.28E-05	2.67782E-05	5.601563
1.5e-4	0.048	0.02	0.2	1.8	3	6.65E-04	3.01E+02	2.71E+03	4.51E+03	1.72E-04	0.000241933	0.620007
1.5e-4	0.038	0.02	0.2	2.5	6.8	7.19E-04	2.78E+02	3.48E+03	9.45E+03	2.16E-04	0.000279674	0.53634
7.8e-4	0.001	0.28	0.08	1.5	15	1.26E-02	6.36E+00	1.19E+02	1.19E+03	4.18E-04	0.000420523	1.854833
2.5e-4	0.0223	0.43	0.165	1.8	7.05	1.43E-03	1.15E+02	1.26E+03	4.92E+03	2.44E-05	2.8169E-05	8.875
2.5e-4	0.02	0.30	0.085	1.8	7	1.48E-03	5.73E+01	1.21E+03	4.71E+03	3.60E-05	4.09091E-05	6.111111
2.5e-4	0.0247	0.37	0.1	1.8	7.1	1.38E-03	7.23E+01	1.30E+03	5.13E+03	2.82E-05	3.31064E-05	7.551418
2.5e-4	0.0266	0.43	0.63	1.8	7.2	1.35E-03	4.67E+02	1.33E+03	5.33E+03	2.42E-05	2.87958E-05	8.681818
2.5e-4	0.0235	0.41	0.32	1.8	6.6	1.41E-03	2.27E+02	1.28E+03	4.69E+03	2.56E-05	2.98021E-05	8.388672
2.5e-4	0.0213	0.42	0.32	1.8	6.6	1.45E-03	2.20E+02	1.24E+03	4.54E+03	2.55E-05	2.92364E-05	8.55098
2.5e-4	0.0315	0.32	0.165	1.8	3	1.28E-03	1.29E+02	1.41E+03	2.35E+03	3.10E-05	3.82244E-05	6.540323
2.5e-4	0.028	0.40	0.165	1.8	3.9	1.33E-03	1.24E+02	1.36E+03	2.94E+03	2.52E-05	3.02885E-05	8.253968
2.5e-4	0.018	0.38	0.165	1.8	7	1.54E-03	1.07E+02	1.17E+03	4.55E+03	2.84E-05	3.18386E-05	7.852113
2.65E-3	.01	0.10	0.6	6	28	1.98E-02	3.03E+01	3.03E+02	1.41E+03	1.95E-02	0.020744681	0.127744
2.65E-3	.01	0.10	0.6	6	14	1.98E-02	3.03E+01	3.03E+02	7.06E+02	1.95E-02	0.020744681	0.127744
2.65E-3	.01	0.10	2	5	28	1.98E-02	1.01E+02	2.52E+02	1.41E+03	1.95E-02	0.020744681	0.127744
2.65E-3	.01	0.10	0.3	5	28	1.98E-02	1.51E+01	2.52E+02	1.41E+03	1.95E-02	0.020744681	0.127744
2.65E-3	.01	4.57	0.6	6	28	1.98E-02	3.03E+01	3.03E+02	1.41E+03	4.40E-04	0.000468085	5.661364
0.02	0.001	10.00	20	20	40	3.22E-01	6.20E+01	6.20E+01	1.24E+02	1.00E-02	0.010060362	1.988

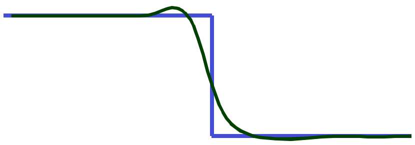
Mucha:



When

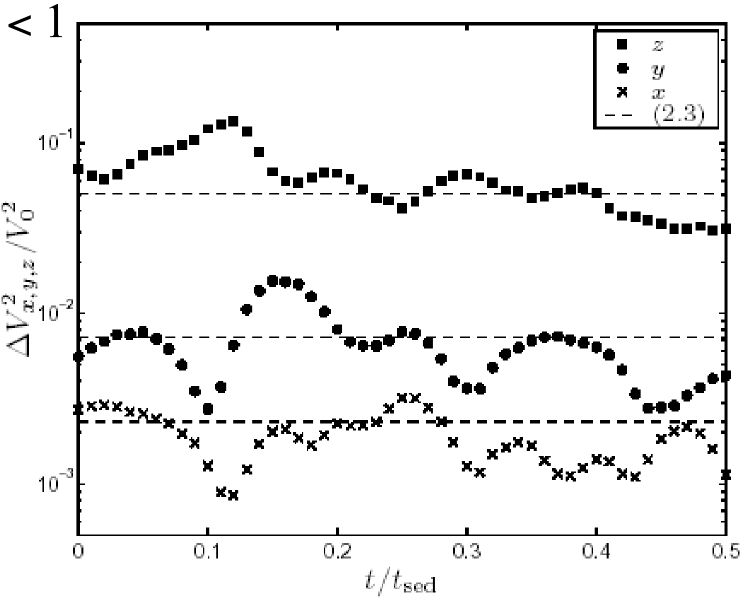
$$\frac{\Box U}{U_{stokes}} > 1$$

Sediment front disperses

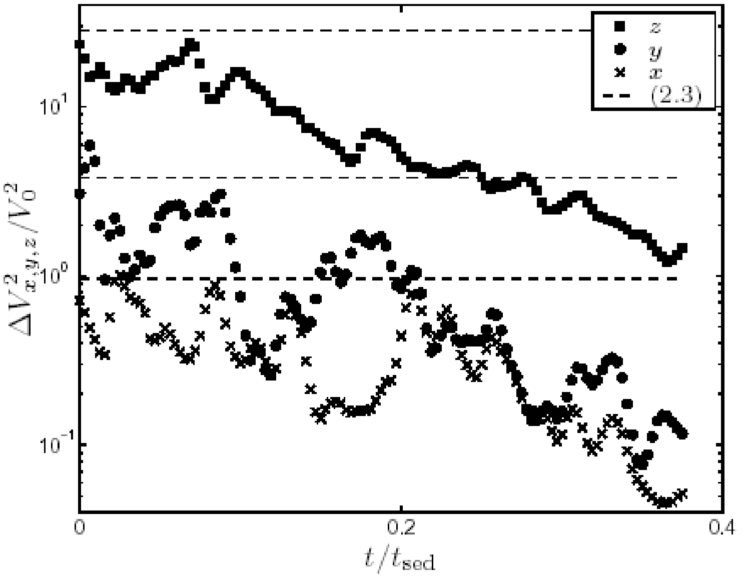


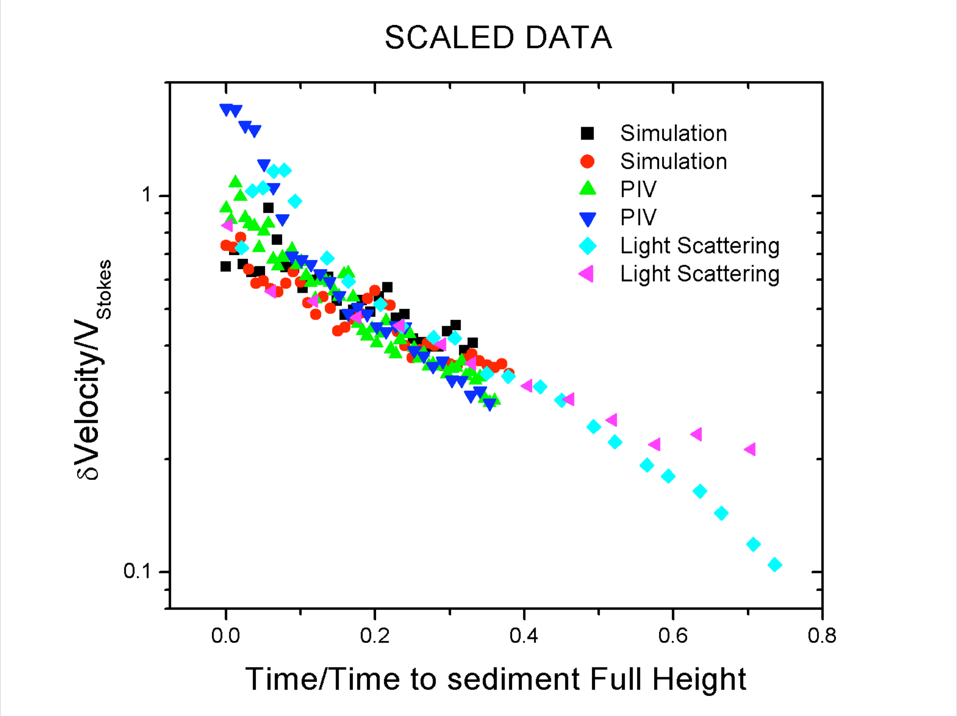
See a (time dependent) stratification in the imaging window.

$$\frac{\square U}{U_{stokes}} < 1$$

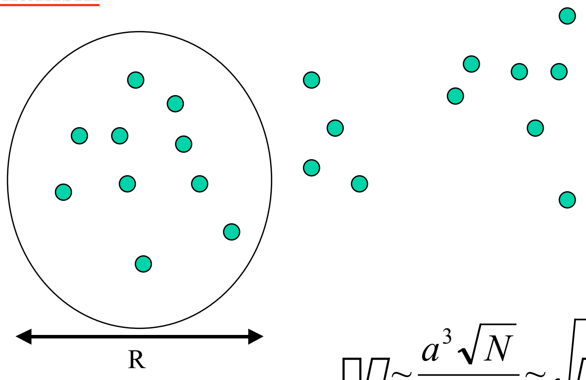


$$\frac{\square U}{U_{stokes}} > 1$$



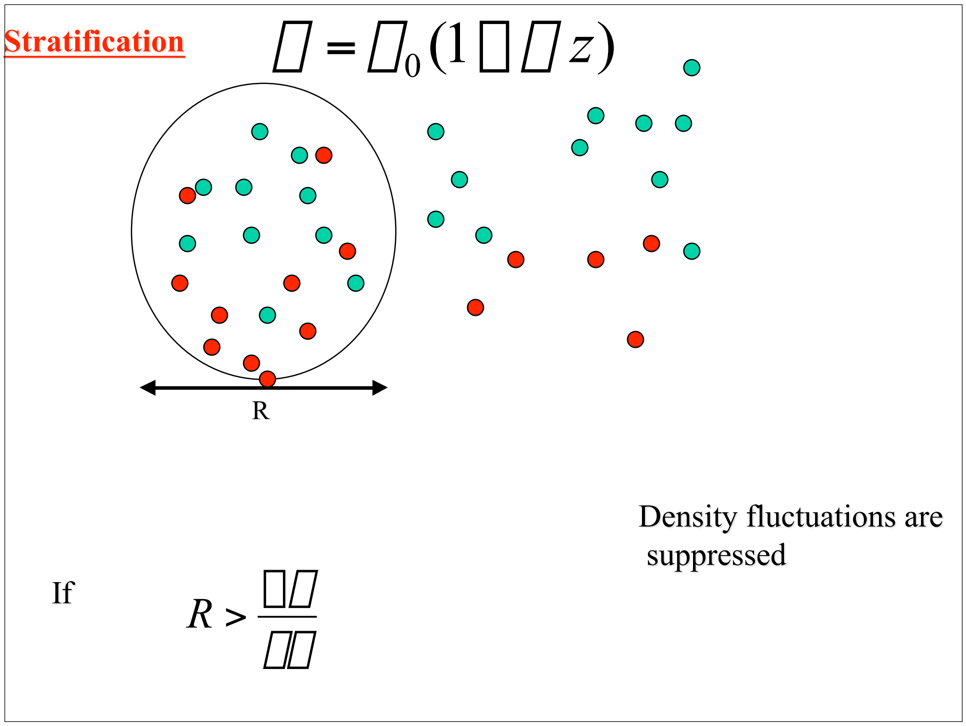


The Mechanism



$$\Box\Box \sim \frac{a^3\sqrt{N}}{R^3} \sim \sqrt{\Box\frac{a^3}{R^3}}$$

$$\Box U \sim U_{stokes}\frac{\sqrt{N}}{R} \sim U_{stokes}\sqrt{\Box\frac{R}{a}}$$

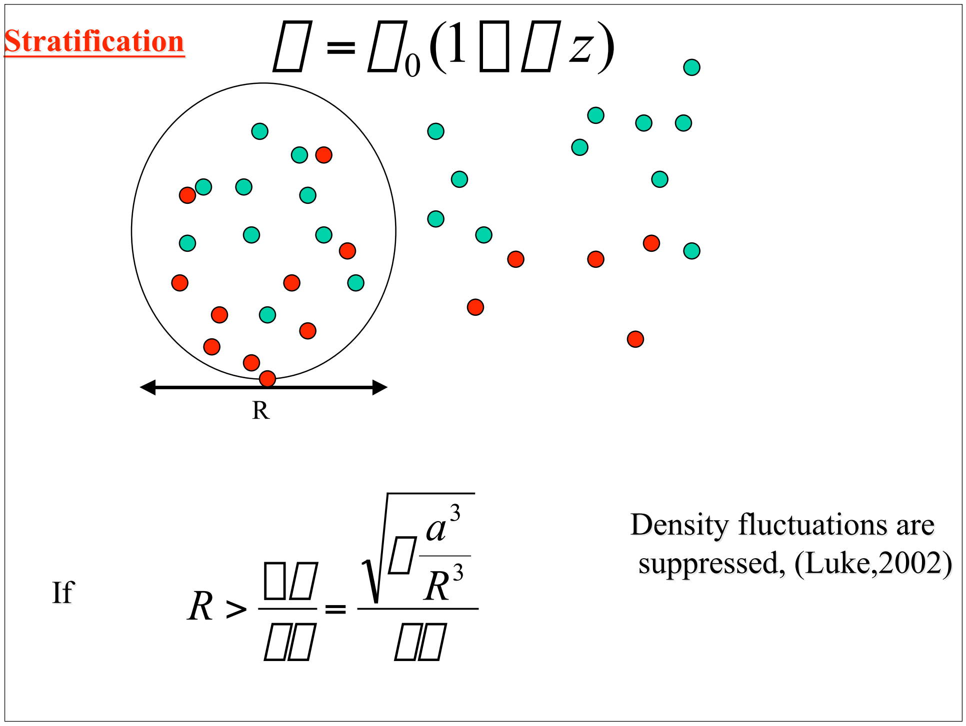


Stratification

$$\Box = \Box_0(1 \Box \Box z)$$

Density fluctuations are suppressed

If $R > \frac{\Box\Box}{\Box\Box}$



Stratification

$$\Box = \Box_0(1 \Box \Box z)$$

Density fluctuations are suppressed, (Luke,2002)

If $R > \frac{\Box\Box}{\Box\Box} = \frac{\sqrt{\Box \frac{a^3}{R^3}}}{\Box\Box}$

$$R^* \sim a \square^{\square 1/5} (\square a)^{\square 2/5}$$

$$\square V \sim V_{sed} \square^{2/5} (\square a)^{\square 1/5}$$

Critical Stratification! $R^* \sim d$

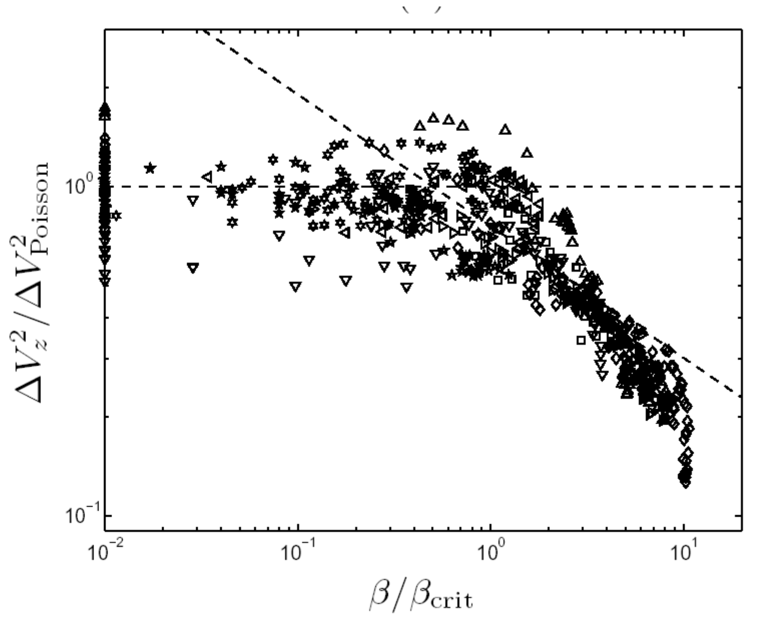
$$\square_{crit}\, d \sim \frac{1}{\sqrt{N_d}} = \frac{1}{\sqrt{\square (d/a)^3}}$$

$$(\square d) \sim (10^{\square 3})^{3/2} 10^{\square 2 \square 1/2}$$

$$\sim \quad 3 \square 10^{\square 4}$$

!

Simulations Show Critical Stratification



A theoretical Issue

A theoretical Issue

Particle Dynamics

$$\frac{dx_i}{dt} = U_{stokes} + \sum_{j\neq i} S(x_j - x_i)$$

A theoretical Issue

Particle Dynamics

$$\frac{dx_i}{dt} = U_{stokes} + \sum_{j\neq i} S(x_j - x_i)$$

Continuum Model:

$$\partial_t \phi + \boldsymbol{\nabla}\cdot\left[\underbrace{\mathbf{u}(\boldsymbol{r},t)}_{\text{average vel.}}\phi - \underbrace{\mathbf{D}\cdot\nabla\phi}_{\substack{\text{large scale}\\ \text{fluctuations}}} + \underbrace{\boldsymbol{\xi}(\boldsymbol{r},t)}_{\substack{\text{short wavelength}\\ \text{noise}}}\right] = 0$$

A theoretical Issue

Particle Dynamics

$$\frac{dx_i}{dt} = U_{stokes} + \sum_{j \neq i}^{\square} S(x_j \,\square\, x_i)$$

Continuum Model:

$$\partial_t \phi + \nabla \cdot [\underbrace{\mathbf{u}(\boldsymbol{r},t)\phi}_{\text{average vel.}} - \underbrace{\mathbf{D} \cdot \nabla \phi}_{\text{large scale fluctuations}} + \underbrace{\boldsymbol{\xi}(\boldsymbol{r},t)}_{\text{short wavelength noise}}] = 0$$

$$u = \square S(x \,\square\, x')\square(x')\,dx' \qquad D = \square V \ell$$

$$\square ?$$

Including □ (or not including it) is important:

Without □, noise is only from initial condition.

A constant stratification would decay cause fluctuations to decay continuously in time.

But, no fluctuation dissipation theorem (just self consistency argument).

Experimental Tests

(0) Experiments with Constant Stratification

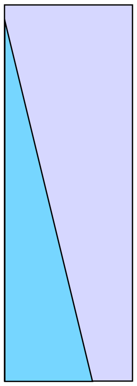
(1) Time dependent stratification

(2) Fluctuation decay rate is cell-height dependent.

(3) Suppression of number density fluctuations (Tong and Ackerson)

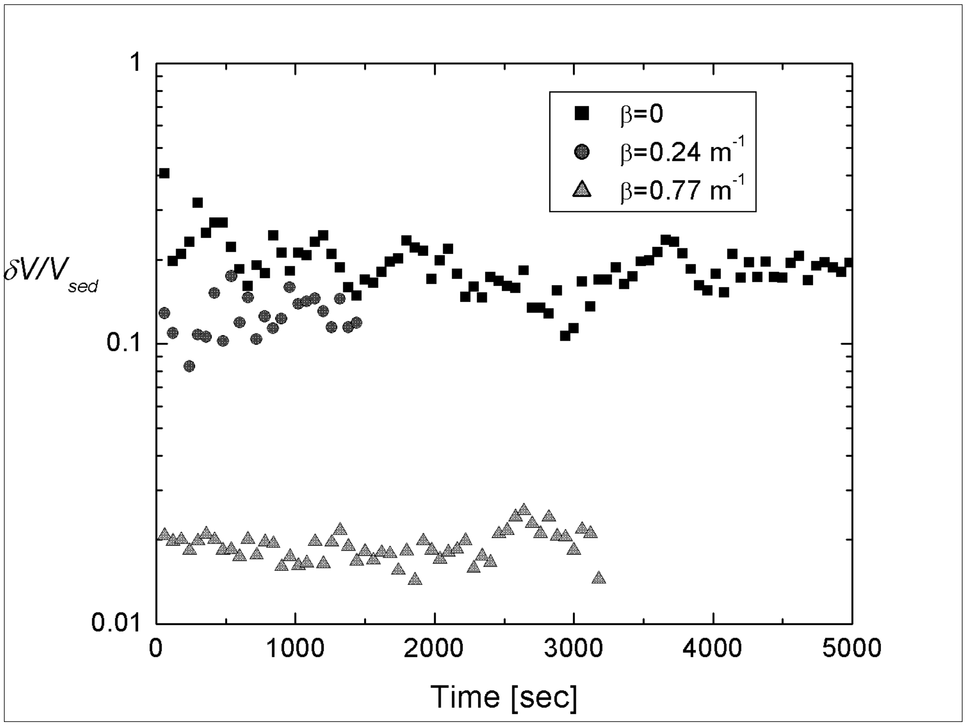
(4) $\square^{1/3}$ law

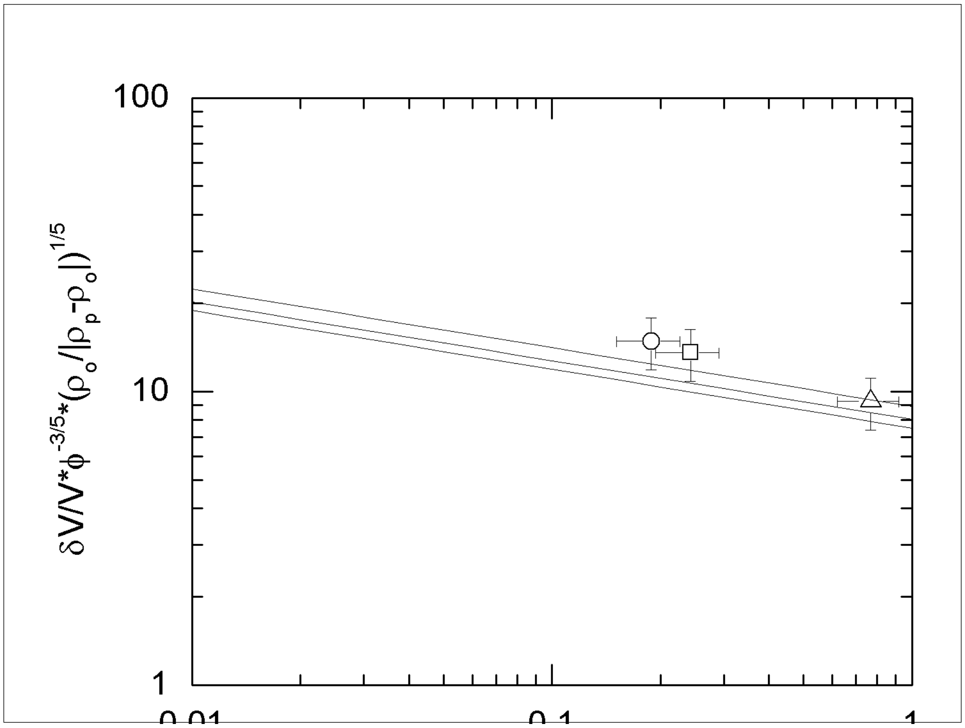
(5) Calculation of structure factor



Impose a salt gradient

The Ongoing Saga Surrounding the Velocity Fluctuations in Sedimentation





Experimental Tests

(0) Experiments with Constant Stratification

(1) Time dependent stratification

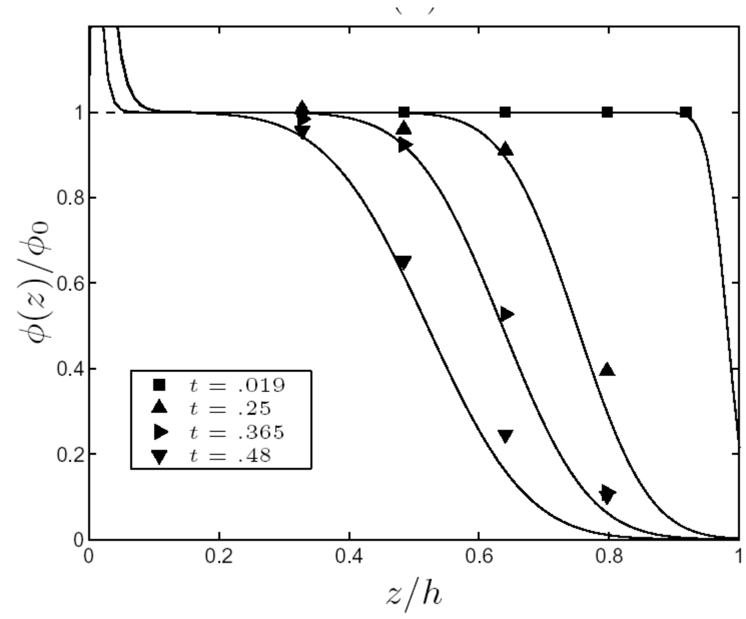
(2) Fluctuation decay rate is cell-height dependent.

(3) Suppression of number density fluctuations (Tong and Ackerson)

(4) □$^{1/3}$ law

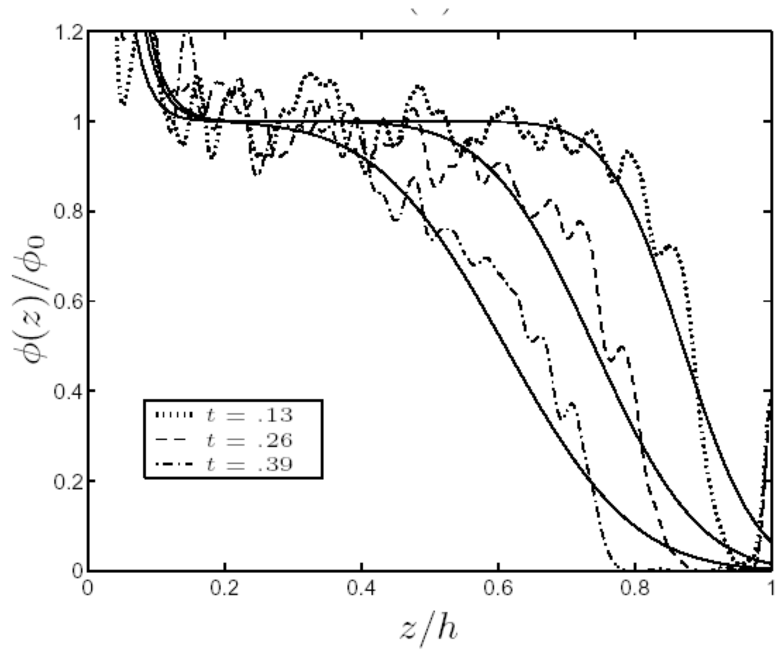
(5) Calculation of structure factor

Experiments



□=0.01

Simulations



Predict the gradient:
Diffusion Model

$$\partial_t n + V_{sed} \partial_z n = D \partial_z^2 n$$

D taken from simulations (cell size dependent!)

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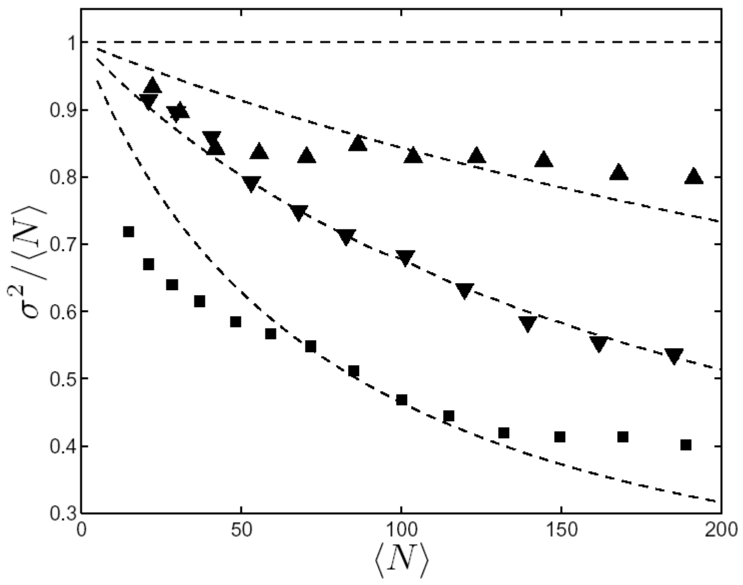
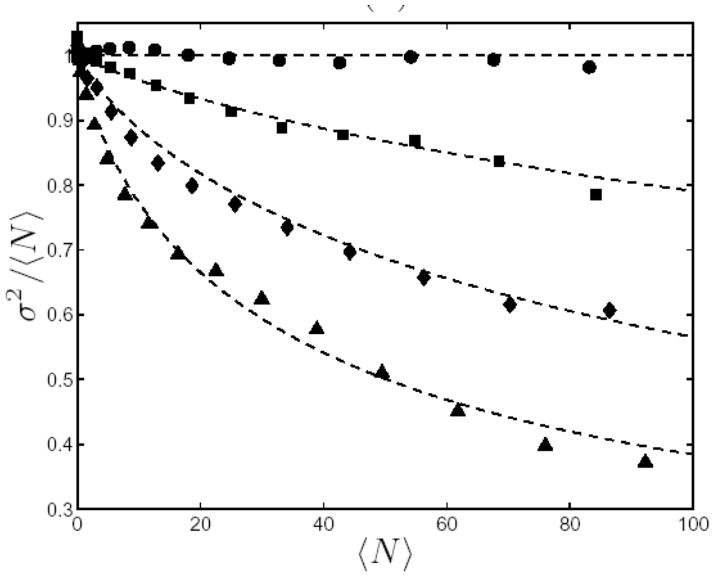
Number Fluctuations



X. Lei, B. Ackerson and P. Tong , Phys Rev Lett, 2001

Simulations



(Dashed lines: Theoretical Model with no fitting parameters)

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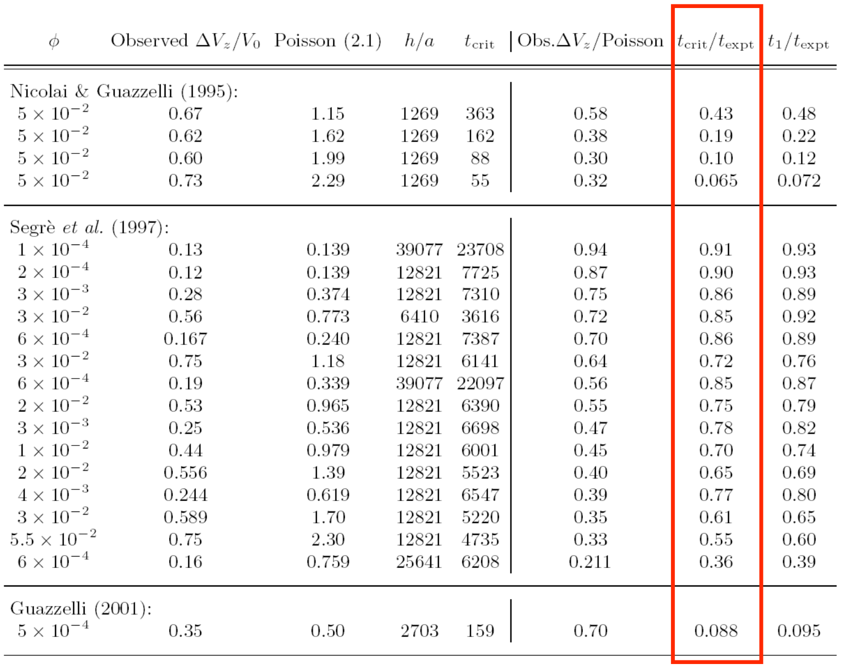

ϕ	Observed $\Delta V_z/V_0$	Poisson (2.1)	h/a	t_{crit}	Obs.ΔV_z/Poisson	$t_{\mathrm{crit}}/t_{\mathrm{expt}}$	t_1/t_{expt}
Nicolai & Guazzelli (1995):							
5×10^{-2}	0.67	1.15	1269	363	0.58	0.43	0.48
5×10^{-2}	0.62	1.62	1269	162	0.38	0.19	0.22
5×10^{-2}	0.60	1.99	1269	88	0.30	0.10	0.12
5×10^{-2}	0.73	2.29	1269	55	0.32	0.065	0.072
Segrè *et al.* (1997):							
1×10^{-4}	0.13	0.139	39077	23708	0.94	0.91	0.93
2×10^{-4}	0.12	0.139	12821	7725	0.87	0.90	0.93
3×10^{-3}	0.28	0.374	12821	7310	0.75	0.86	0.89
3×10^{-2}	0.56	0.773	6410	3616	0.72	0.85	0.92
6×10^{-4}	0.167	0.240	12821	7387	0.70	0.86	0.89
3×10^{-2}	0.75	1.18	12821	6141	0.64	0.72	0.76
6×10^{-4}	0.19	0.339	39077	22097	0.56	0.85	0.87
2×10^{-2}	0.53	0.965	12821	6390	0.55	0.75	0.79
3×10^{-3}	0.25	0.536	12821	6698	0.47	0.78	0.82
1×10^{-2}	0.44	0.979	12821	6001	0.45	0.70	0.74
2×10^{-2}	0.556	1.39	12821	5523	0.40	0.65	0.69
4×10^{-3}	0.244	0.619	12821	6547	0.39	0.77	0.80
3×10^{-2}	0.589	1.70	12821	5220	0.35	0.61	0.65
5.5×10^{-2}	0.75	2.30	12821	4735	0.33	0.55	0.60
6×10^{-4}	0.16	0.759	25641	6208	0.211	0.36	0.39
Guazzelli (2001):							
5×10^{-4}	0.35	0.50	2703	159	0.70	0.088	0.095

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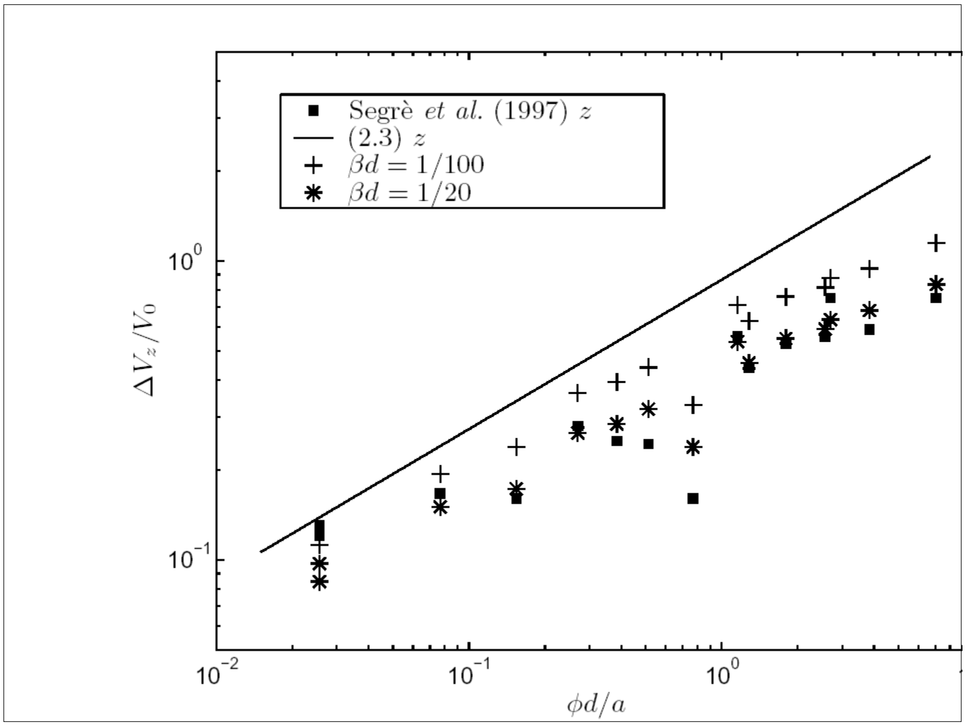


Experimental Tests

(1) Time dependent stratification

(2) Fluctuation decay rate is cell-height dependent.

(3) Suppression of number density fluctuations (Tong and Ackerson)

(4) □$^{1/3}$ law

(5) Measure structure factor

(a) Tony Ladd, Phys. Rev. Lett. 2002 (Lattice Boltzman, Re ~1)
(b) Sarah Dance and Martin Maxey, APS FE2

Summary

(1) Velocity fluctuations in sedimentation highly nonuniversal.
Depend on container dimension + small inhomogeneities

(2) Fluctuations are sensitive to very small physical effects.

$$\square U \sim \sqrt{\text{Volume}}$$

Most physical effects ~ volume

Summary

(1) Velocity fluctuations in sedimentation highly nonuniversal.
Depend on container dimension + small inhomogeneities

(2) Fluctuations are sensitive to very small physical effects.

(3) Diffusivity of a sediment:

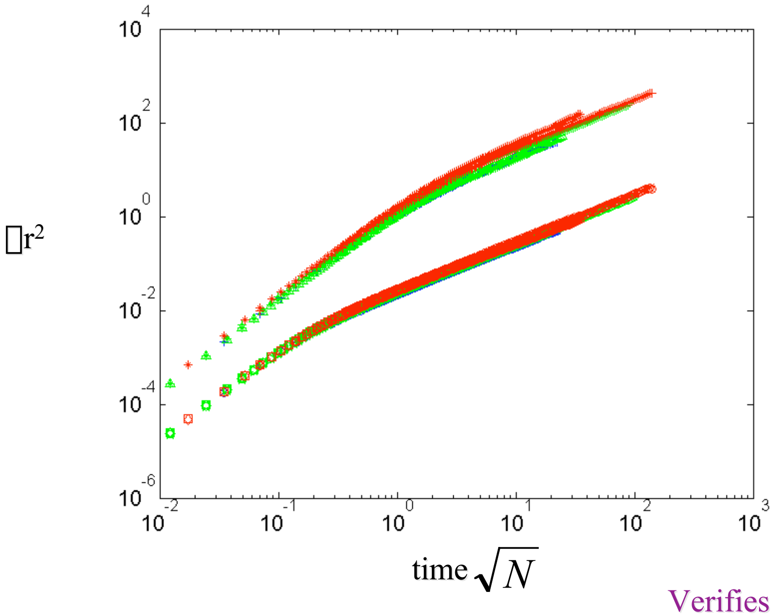
$$D\left(\phi, \frac{\partial \phi}{\partial z}\right) = \begin{cases} C d V_0 \sqrt{\phi d / a} & \text{for } \beta \leq \beta_{\text{crit}} \\ C B^{3/5} a V_0 \phi^{4/5} \left| a \frac{\partial \phi}{\partial z} \right|^{-3/5} & \text{for } \beta \geq \beta_{\text{crit}} \end{cases}$$

Higher Volume Fractions

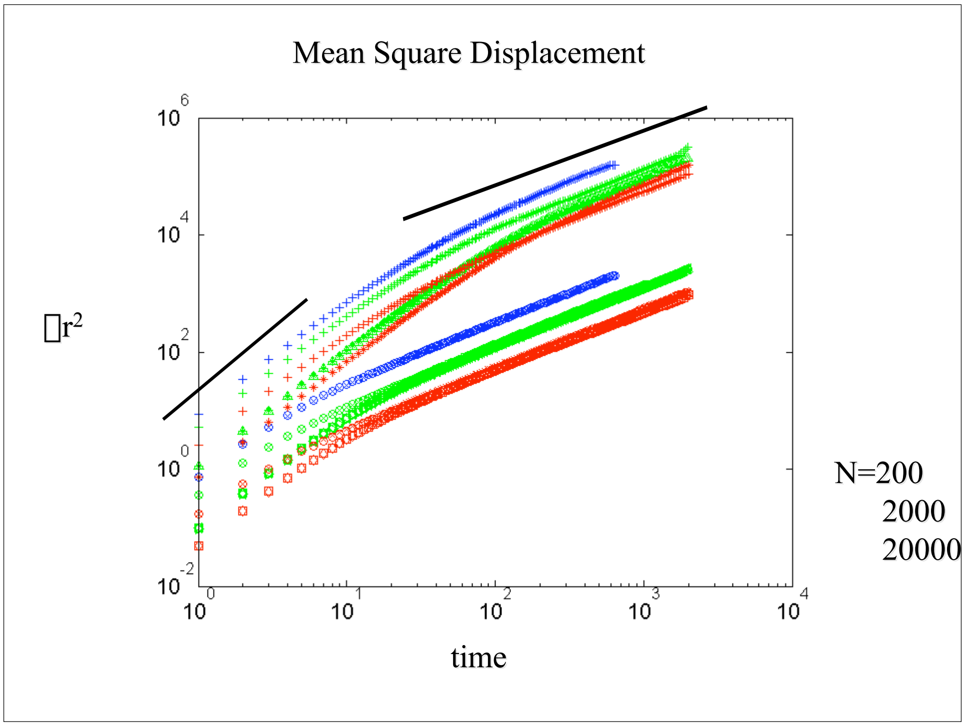
Nonzero Reynolds Numbers

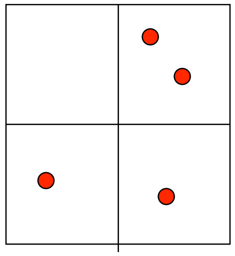
Polydispersity

Etc.



Verifies scalings…





Issues

- □ Fourier Transform || to side walls on course mesh
- □ Explicitly compute flows perpendicular sums
- □ FFT for summing || O(N log N)
- □ Clever organizational tricks for perpendicular sums
- □ Near field corrections…
- □ Back flow

➡ □Can do 10^6 particles; *c.f. previous 50000 periodic*