Theoretical and experimental aspects in the search for gravitational waves

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Content

- How gravitational waves interact: evidence and astrophysical signatures
- Sources of gravitational waves: which information we can extract
- Compact binaries made of black holes and/or neutron stars
- Laser interferometer gravitational-wave detectors: very high experimental challenge
- What limits the interferometer sensitivity and how to improve the performances for the years to come

Einstein's theory of gravitation and gravitational waves

In weak field limit Einstein's equations are described by linear equation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + \mathcal{O}(h^2_{\mu\nu})$$

Using transverse traceless gauge $(h_{\mu 0} = 0, h_{kj,j} = 0, h_{kk} = 0) \Rightarrow$

familiar wave equation: $\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) h_{\mu\nu} = 0$

The strain $h_{\mu\nu}$ takes the form of a plane wave propagating at light's speed

Since graviton is spin-2 boson, the waves have two components

$$h = h_+(t - R/c) + h_\times(t - R/c)$$

How gravitational waves interact with free test particles

A freely moving particle travels through spacetime along a geodesic

The curvature of spacetime pushes neighboring spacetime geodesics together or apart

GW detector: body of mass m at distance L from fiducial laboratory point

$$m\,\ddot{\xi} = m\,\frac{L}{2}\,\left[F_{\times}(\theta,\phi,\psi)\,\ddot{h}_{\times} + F_{+}(\theta,\phi,\psi)\,\ddot{h}_{+}\right]$$

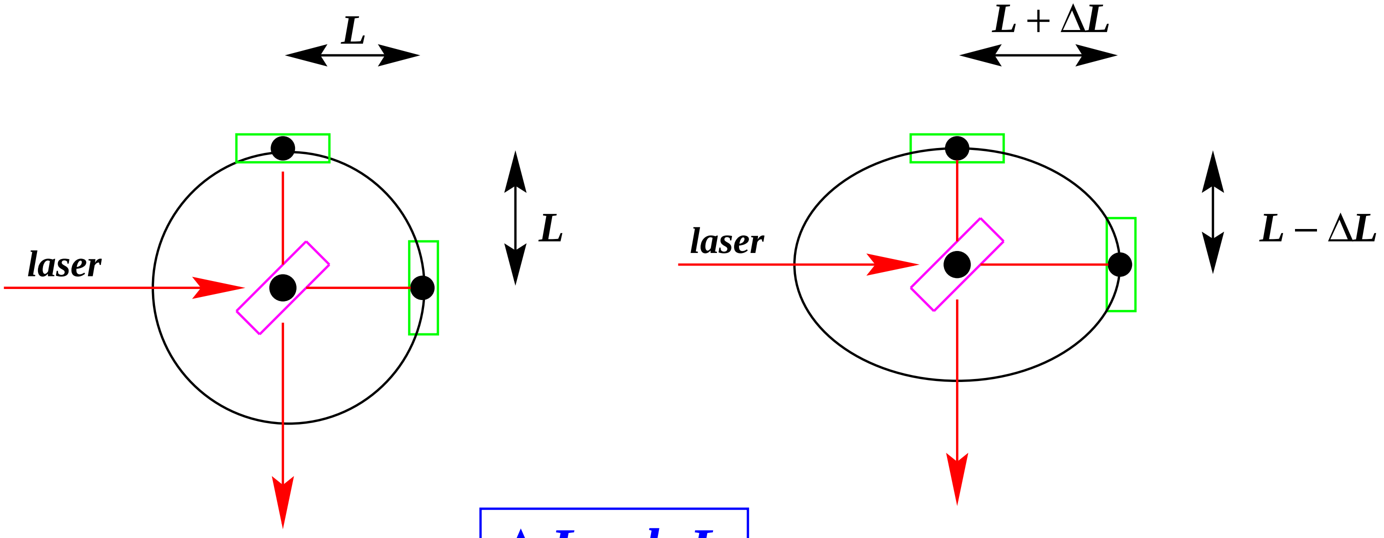
$\xi \to$ infinitesimal displacement of the mass m

$$\Rightarrow \Delta L \equiv \xi \sim L\,h$$

$F_{+,\times}(\theta,\phi,\psi) \to$ beam-pattern factors

Gravitational wave can induce a phase-shift on the light

Assumption: $L \ll \lambda_{\mathrm{GW}}/(2\pi)$



$\Phi = 0$

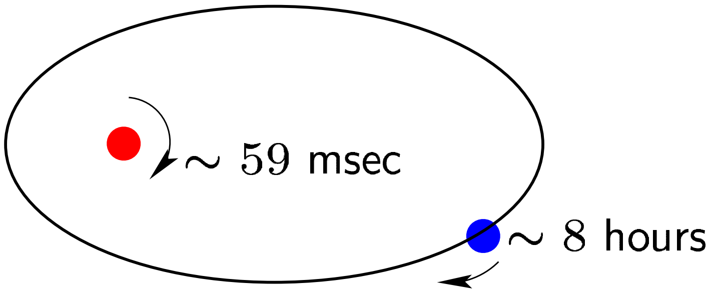
$$\boxed{\Delta L = h\, L}$$

$\Phi = \Delta L$

h → gravitational-wave strain

Indirect observation of gravitational wav

Neutron Binary System: PSR 1913 +16 - Timing Pulsars



Hulse & Taylor discovery (1974)

Separated by $\sim 10^6$ Km, $m_1 = 1.4 M_\odot$, $m_2 = 1.36 M_\odot$, $\epsilon = 0.617$

- Prediction from GR: rate of change of orbital period
- Emission of gravitational waves:
  - due to loss of orbital energy
  - orbital decay in agreement with GR at the level of 0.5%

Astrophysical sources and signatures

- **Compact binary inspiral: "chirp"** [duration $\sim$ minutes to seconds]
  - NS/NS, NS/BH and BH/BH

- **Supernovae and Gamma-ray bursts: "bursts"** [duration $\sim$ mseconds]
  - non axisymmetric collapse
  - "coincidence" with signals in EMR
  - neutrino detection
  - Gamma-ray bursts triggered by merger of NS/NS or NS/BH

- **Pulsars in our galaxy: "periodic"**
  - search for observed spinning neutron stars: Vela, Crab
  - all sky search

- **Cosmological signals: "stochastic"**

How to detect a gravitational-wave signal from inspiraling binaries

Track the signal phase and build up the signal-to-noise ratio

by integrating the signal for the time it stays in the detector band

Filter the detector output, $o = h + n$, with a *template* t which is an

(approximate) copy of the exact, observed signal h

Signal-to-noise ratio:

$$\frac{S}{N} = 2\int_0^{+\infty} df\ \left[o(f)\, t^*(f) + o^*(f)\, t(f)\right] \frac{1}{S_n(f)}$$

h → gravitational waveform, t → template, o → detector's output, n → detector's noise

Coalescing binary systems

Approximating compact bodies to point particles

Two-body equation of motions:

[Damour & Deruelle 81, 82; Damour, Blanchet & Iyer 95; Will & Wiseman 95; Blanchet 96]

[Jaranowski & Schäfer 98,99; Damour, Jaranowski & Schäfer 00, 01; Blanchet & Faye 00,01]

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{GM}{r^2}\left[-\widehat{n} + A_{\rm 1PN} + A_{\rm 2PN} + A_{\rm 2.5PN} + A_{\rm 3PN} + A_{\rm 3.5PN} + \cdots\right]$$

$$M = m_1 + m_2, \qquad r = |\vec{x}_1 - \vec{x}_2|, \qquad \vec{v} = \vec{v}_1 - \vec{v}_2, \qquad \widehat{n} = (\vec{x}_1 - \vec{x}_2)/r$$

$A_{\rm nPN} \to \mathcal{O}(\epsilon^n)$ relative to Newtonian term

$\epsilon = \frac{v^2}{c^2} \sim \frac{GM}{c^2 r} \to$ post-Newtonian parameter

Gravitational waveforms

Gravitational field far from source:

[Wagoner & Will '76; Wiseman '92; Blanchet, Damour, Iyer, Will & Wiseman '96]

[Blanchet '96, '98; Blanchet, Joguet & Iyer, '01]

$$h^{ij} = \frac{2GM}{c^4 R} \left[Q^{ij} + Q^{ij}_{0.5\mathrm{PN}} + Q^{ij}_{1\mathrm{PN}} + Q^{ij}_{1.5\mathrm{PN}} + Q^{ij}_{2\mathrm{PN}} + Q^{ij}_{2.5\mathrm{PN}} + \cdots \right]$$

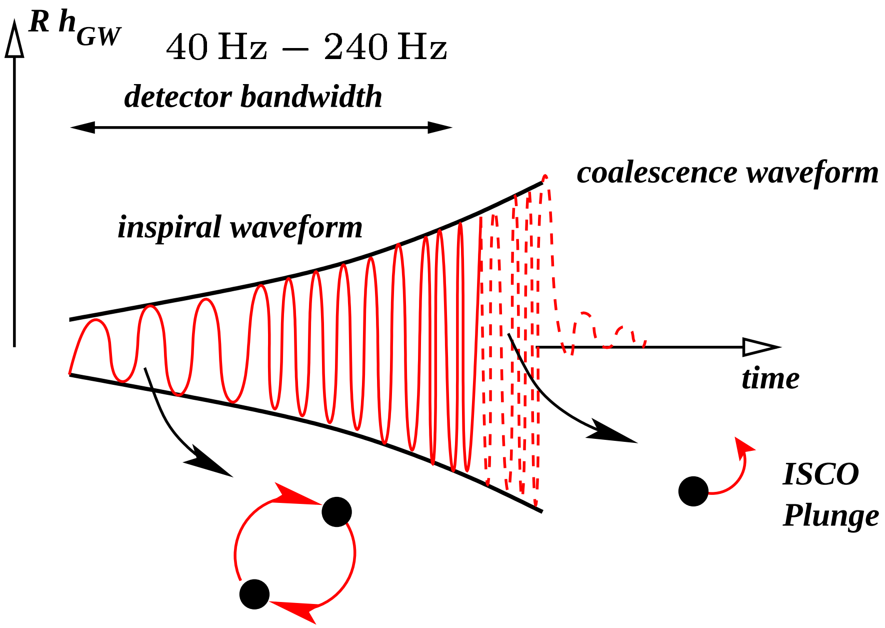
$R \rightarrow$ distance from the source

Quadrupole formula: $h^{ij} = \frac{2G}{Rc^4} \ddot{I}^{ij}(t - R/c)$ $I^{ij} \rightarrow$ source's quadrupole moment

$h \sim \frac{GMr^2}{Rc^4} f_{\mathrm{GW}}^2$ for $r \simeq 20$ Km, $M \simeq 10^{30}$ Kg, $f_{\mathrm{GW}} \sim 100$ Hz

R at 20 Mpc (Virgo cluster) $\Rightarrow h \sim 10^{-21}$! $h = \Delta L / L$

Science from observed inspiral, merger of NS/NS and tidal disruption of NS/BH



$f^{\rm GW}_{\rm ISCO} \simeq 0.022\, c^3/GM$

BH/NS: $M = (10 + 1.4)\, M_\odot \to f^{\rm GW}_{\rm ISCO} \simeq 380$ Hz

NS/NS: $M = (1.4 + 1.4)\, M_\odot \to f^{\rm GW}_{\rm ISCO} \simeq 1600$ Hz

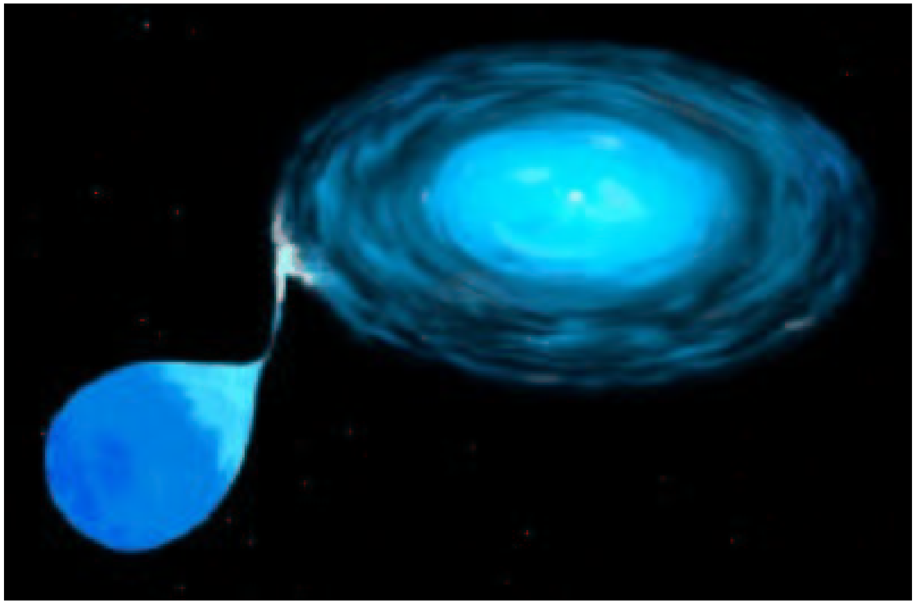
- **Information carried in NS/NS inspiral** [Abramovici et al. ‘92; Cutler et al. ‘93; Schutz ‘86]

  Masses (a few %), Spins (few %), Distance ($\sim 10\%$), Location on sky (~ 1 degree)

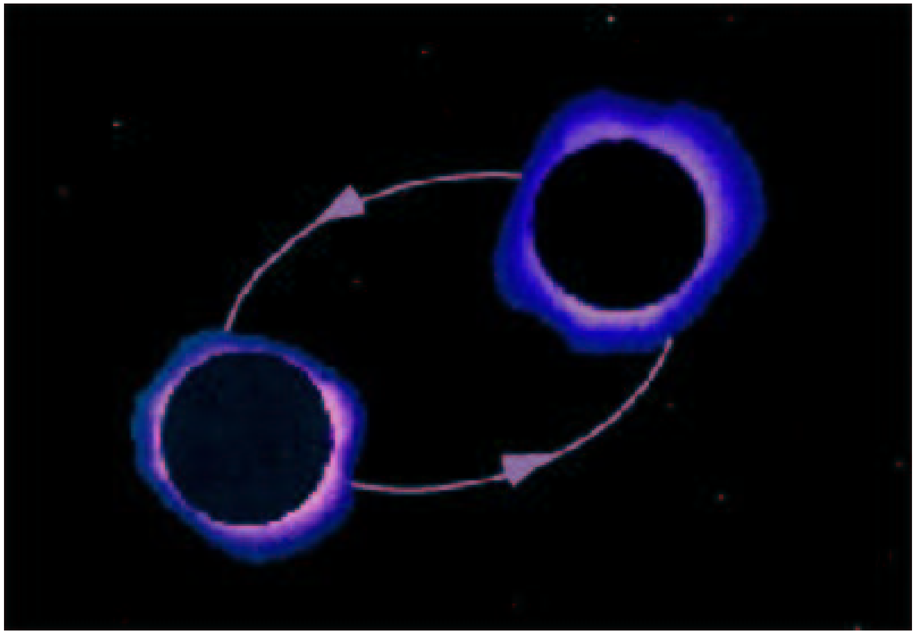
- **Information carried in NS/BH inspiral and NS tidal disruption** [Thorne ‘87; Vallisneri ‘00]

  NS radius to 15%, equation of state, nuclear physics

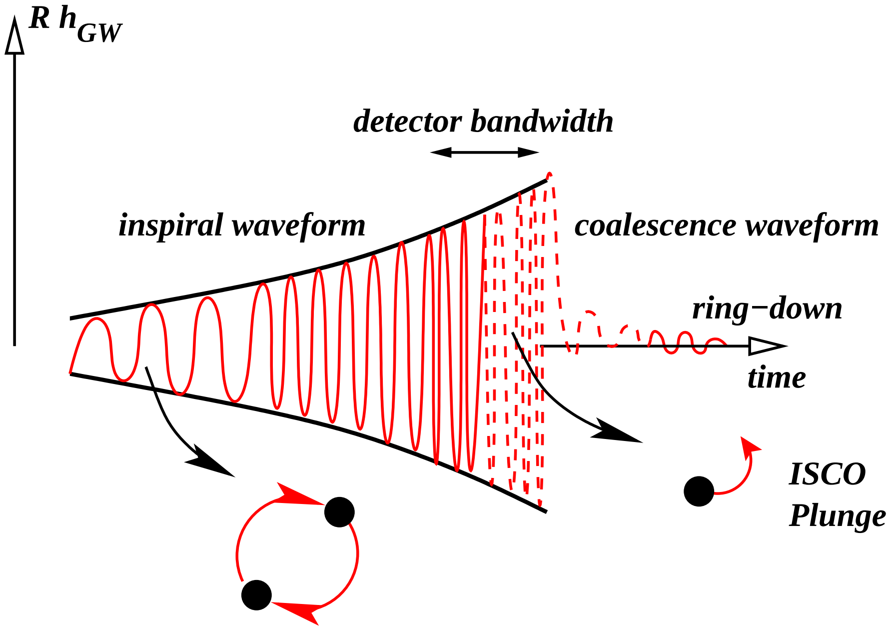
Black hole - Neutron star binary



Black hole - Black hole binary



Main issues for most promising sources: comparable-mass BH/BH



BH/BH: $M = (15 + 15)\, M_\odot \rightarrow f^{\rm GW}_{\rm ISCO} \simeq 150$ Hz

BH/BH: $M = (10 + 10)\, M_\odot \rightarrow f^{\rm GW}_{\rm ISCO} \simeq 220$ Hz

Issues:

- **GWs emitted where PN expansion fails ($v/c \sim 0.3$)**
- **Spin effects: precession orbital plane, modulations, etc.**

Solutions:

- **Numerical relativity** [Potsdam, Cornell, ...]
- **Analytical approaches: PN resummation techniques**

[Damour, Iyer & Sathyaprakash ‘97; A.B. & Damour ‘00,‘01; Damour, Jaranowski & Schaefer ‘01]

Effective-one-body resummation (up to 3 post-Newtonian order)

[A.B. & Damour '99, '00]

$\mu = m_1\, m_2/M$

$\nu = m_1\, m_2/M^2$

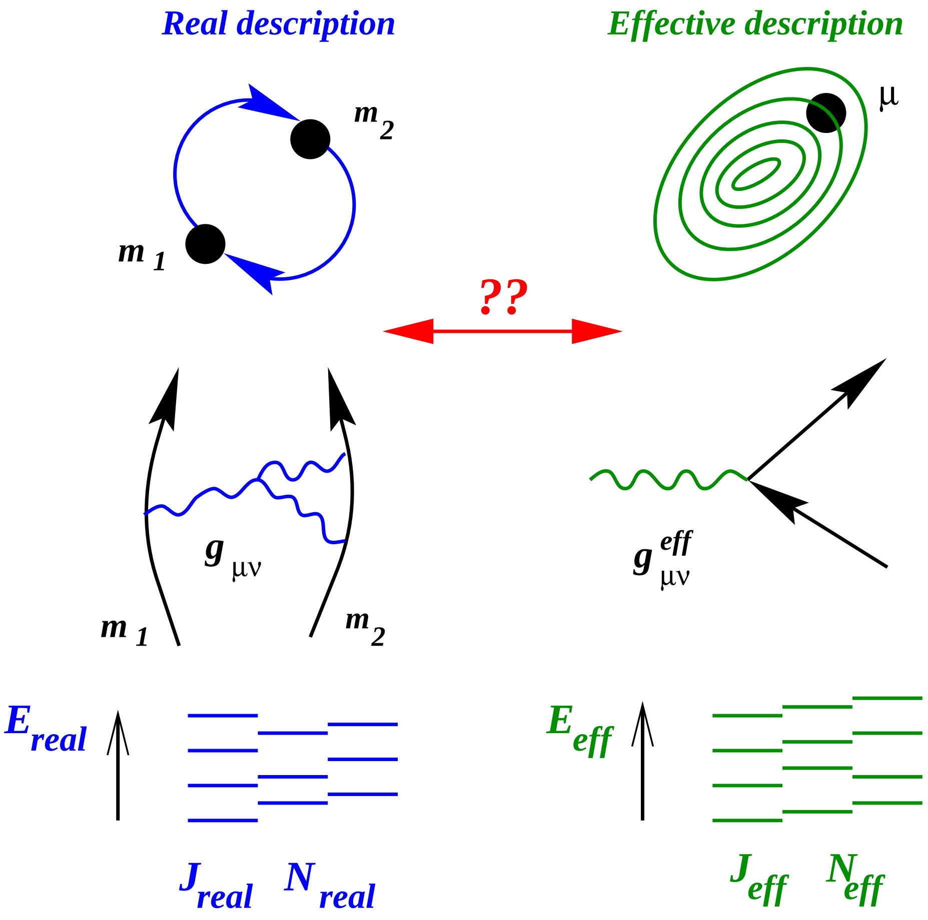
$0 \leq \nu \leq 1/4$

matching rules:

$E_{\rm real} = f(E_{\rm eff})$

$J_{\rm real} = J_{\rm eff}$

$N_{\rm real} = N_{\rm eff}$



Relativistic energy for two-body system of comparable masses

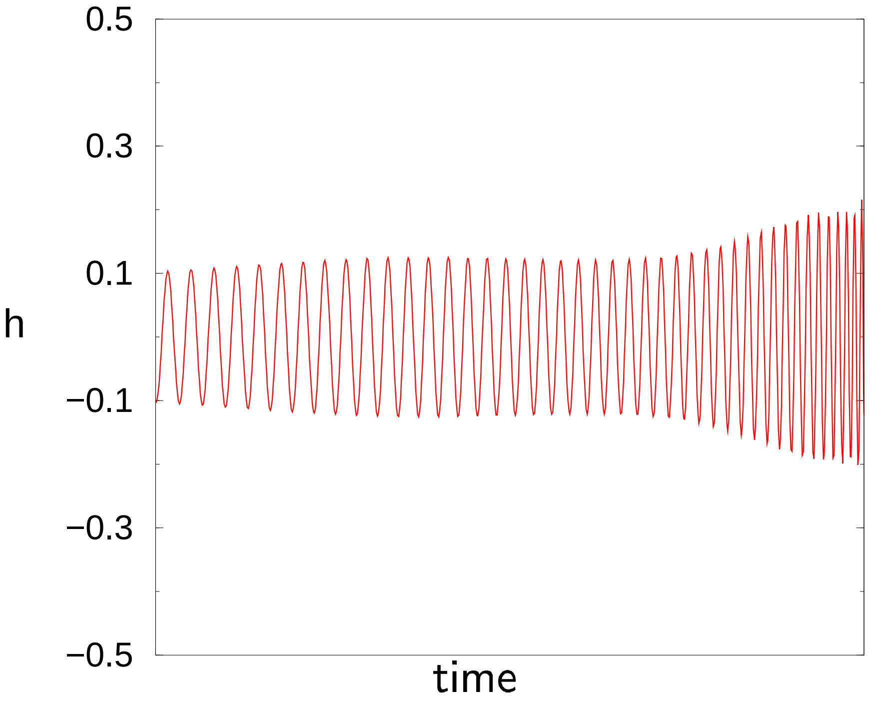
Classical gravity (up to 3PN)

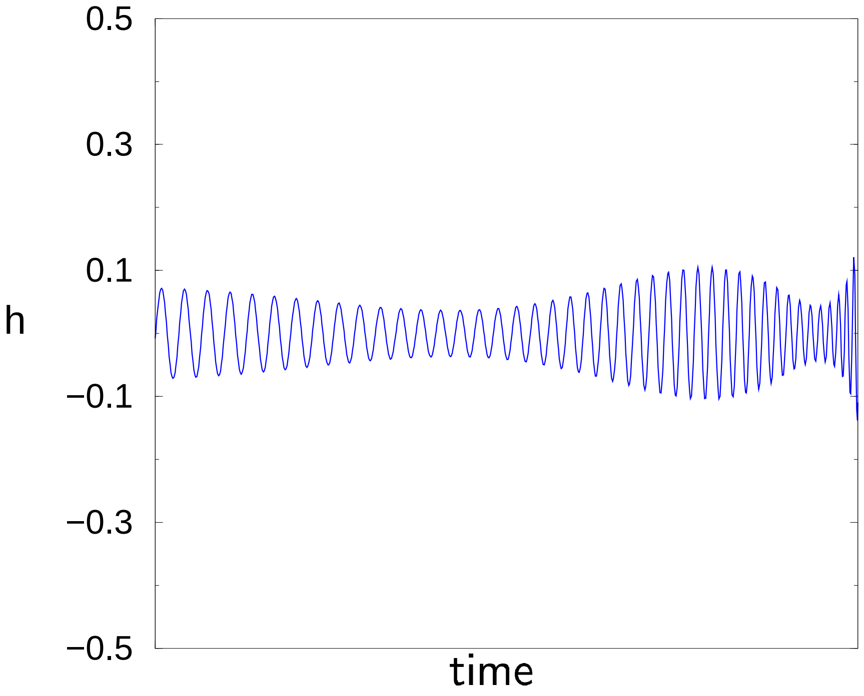
$$E_{\rm real}^2 = m_1^2 + m_2^2 + 2m_1\, m_2 \left(\frac{E_{\rm eff}}{\mu}\right)$$

Quantum electrodynamics [Brézin, Itzykson & Zinn-Justin 70]

$$E_{\rm real}^2 = m_1^2 + m_2^2 + 2m_1\, m_2\, \frac{1}{\sqrt{1 + Z^2\alpha^2/(n-\epsilon_j)^2}}$$

Including spin effects





[A.B., Chen, Chernoff & Vallisneri, work in progress]

International network of earth-based GW interferometers

Livingston Observatory ⇒

Frequency band: $10 - 10^4$ Hz

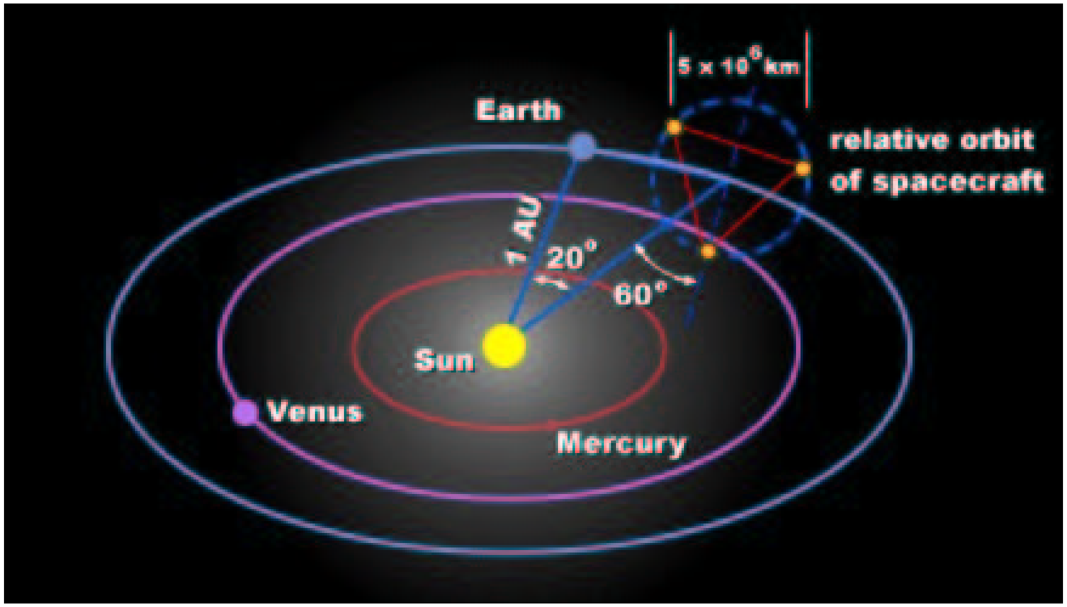
⇐ Hanford Observatory

Virgo Observatory (France-Italy) ⇒

GEO Observatory (UK-Germany)

TAMA Observatory (Japan)

Laser Interferometer Space Antenna (LISA)



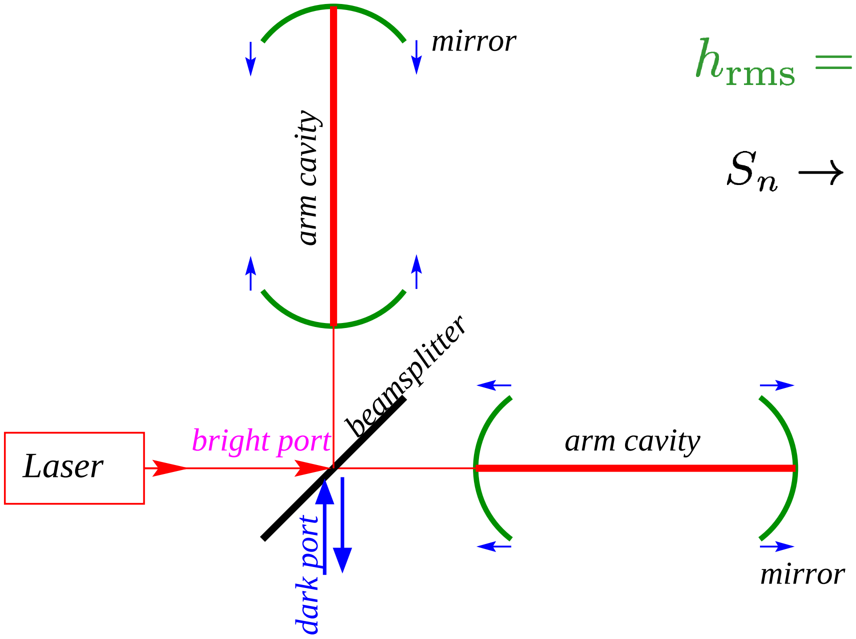
Frequency band: $10^{-4} - 0.1$ Hz

- Giant stars, main sequence stars, white dwarfs, neutron stars, black holes spiraling around supermassive black hole of $\sim 10^5 - 10^8 M_\odot$

- Binaries made of white dwarfs, neutron stars or black holes

Earth-based GW interferometers

Frequency band: $10 - 10^4$ Hz



$$h_{\rm rms} = \sqrt{S_n(f)\,\Delta f} = \frac{\Delta L}{L}, \qquad h_n \equiv \sqrt{S_n(f)}$$

$S_n \to$ noise power per unit frequency, $\Delta f \to$ bandwidth

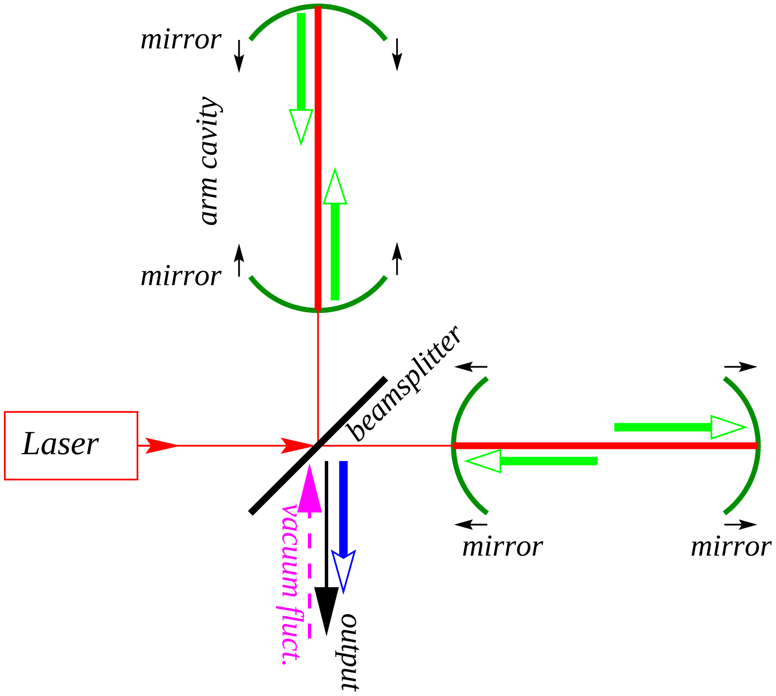
$L \to$ arm-cavity length (4 Km)

LIGO-I/VIRGO at $f \sim 100$ Hz: $\Delta L \sim 10^{-16}\,{\rm cm}$

LIGO-II at $f \sim 100$ Hz: $\Delta L \sim 10^{-17}\,{\rm cm} \simeq$ $1/10^8\times$ **radius of hydrogen atom!**

Quantum mechanical formalism to describe optical noise and internal dynamics

First generation GW interferometers



The detector output contains the GW signal, noisy terms scaling such as $\sqrt{I_0}$ (radiation pressure) and $1/\sqrt{I_0}$ (shot noise), and fluctuations associated to initial quantum displacement of mirrors

I_0 → laser light at beamsplitter

Quantum optical-mechanical noise

$$\textit{Output:} \qquad \widehat{\mathcal{O}}(\Omega) = \overbrace{[\widehat{\mathcal{Z}}(\Omega) - \frac{1}{\mu\Omega^2}\widehat{\mathcal{F}}(\Omega)]}^{\text{optical field}} + \overbrace{L\,h(\Omega)}^{\text{GW signal}} + \overbrace{\widehat{x}^{(0)}(\Omega)}^{\text{free test mass}}$$

$\widehat{\mathcal{Z}} \to$ shot noise $\qquad \widehat{\mathcal{F}} \to$ radiation-pressure force

$\Omega \to$ GW sideband frequency, $\quad \mu = m_{\mathrm{mirror}}/4, \quad h(\Omega) \to$GW strain, $\quad L \to$ arm-cavity's length

- GW interferometers: The output noise is not influenced by the test-mass initial quantum state [Braginsky, Gorodetsky, Khalili, Matsko, Thorne & Vyatchanin '01]

⇒ Shot noise and radiation-pressure noise are the only sources of quantum noise

First-generation interferometer

First science-run planned for 2002

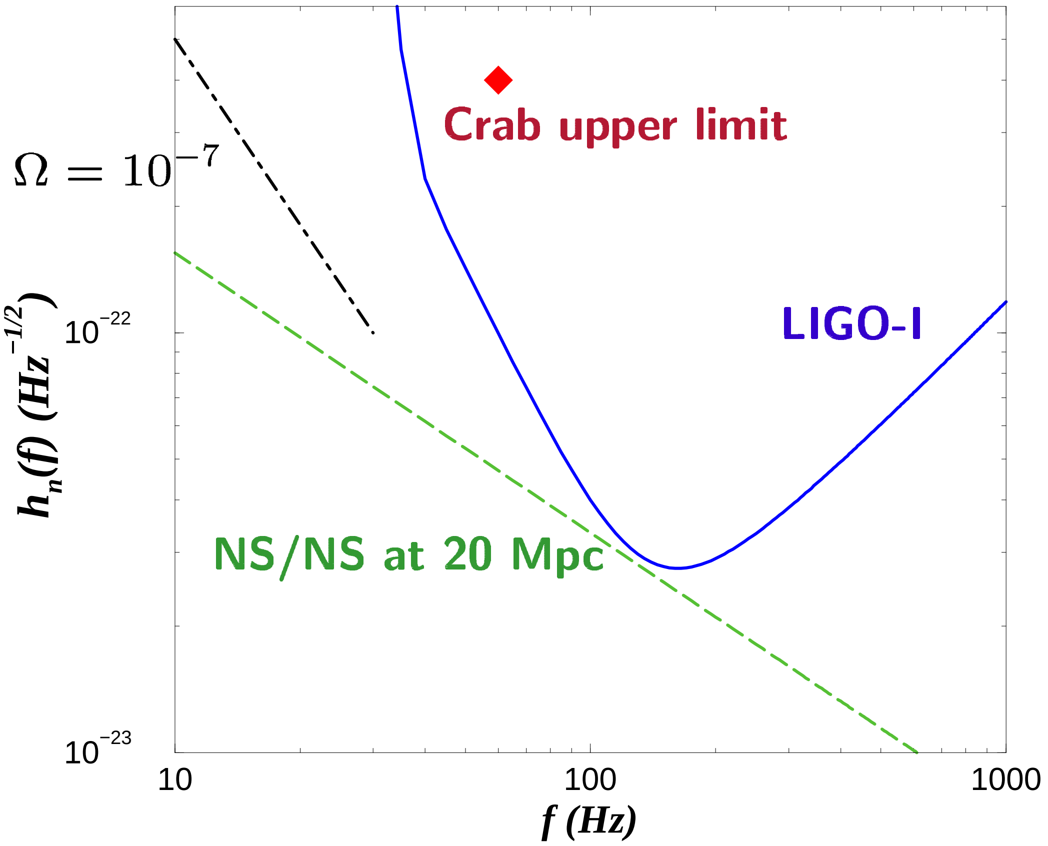
[Thorne '01]

Rate NS/NS at 20 Mpc:

1/3000 yrs to 1/3 yrs

Rate BH/BH at 100 Mpc:

1/300 yrs to 1/ yrs



Standard quantum limit for GW interferometer **[Braginsky '68, '70; Caves '81]**

Naive derivation of SQL: independent measurements of free test-mass displacements

If positions measured with high precision then test-mass momenta perturbed ("Heisenberg microscope")

As time passes momentum perturbations produce position uncertainties

If momentum perturbations and measurement errors are not correlated we have minimum possible spectral density

$\Rightarrow \quad S_n^{\mathrm{SQL}}(\Omega) = \frac{2\hbar}{\mu\,\Omega^2\,L^2}$ for GW signal $h = \frac{\Delta L}{L}$

Free mass SQL for GW interferometers

Noise spectral density (noise power per unit frequency)

$$\boxed{S_h(\Omega) = S_n^{\rm shot} + S_n^{\rm rad\ press} + 2\,S_n^{\rm corr}}$$

$$S_n^{\rm shot} \propto S_{\mathcal{Z}\mathcal{Z}} \qquad S_n^{\rm rad\ press} \propto S_{\mathcal{F}\mathcal{F}} \qquad S_n^{\rm corr} \propto S_{\mathcal{Z}\mathcal{F}}$$

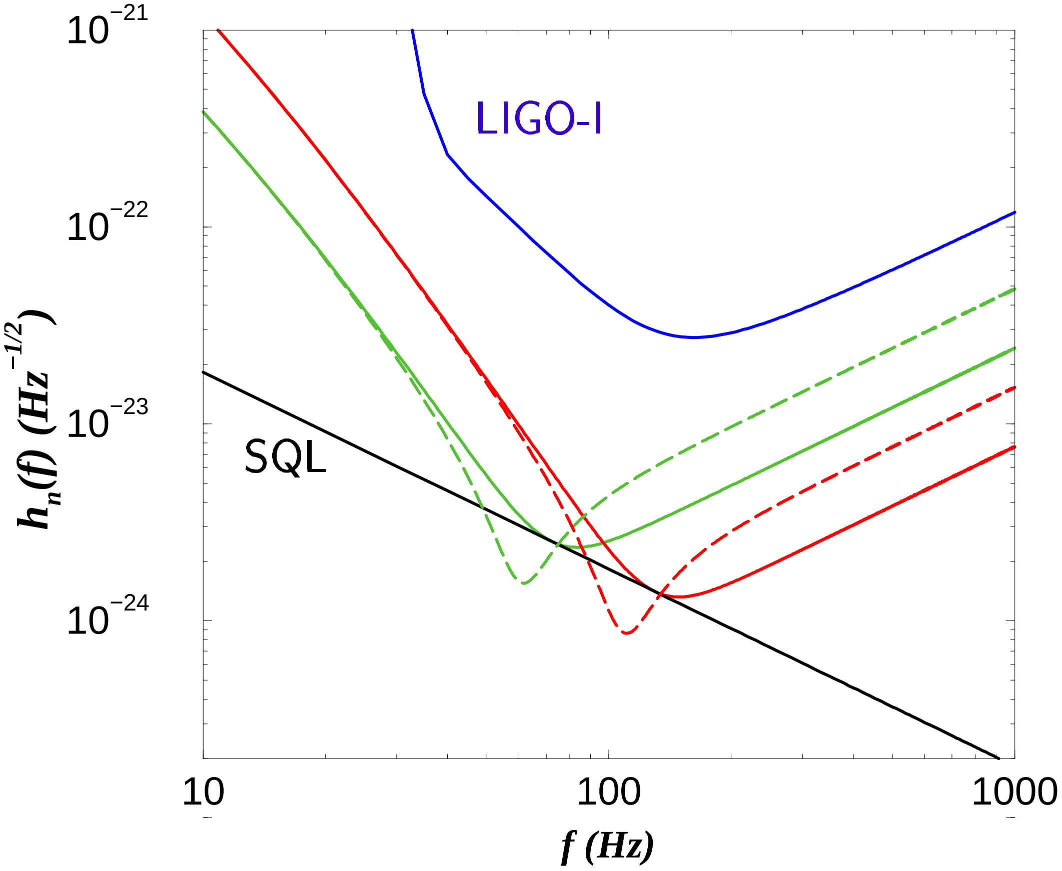
$$\boxed{S_n^{\rm shot}\,S_n^{\rm rad\ press} - |S_n^{\rm corr}|^2 \geq S_n^{\rm SQL}/4} \qquad S_n^{\rm SQL}(\Omega) \equiv h^2_{\rm SQL}(\Omega) = \frac{2\hbar}{\mu\,\Omega^2\,L^2}$$

Standard configuration of first generation interferometer (LIGO-I, TAMA, VIRGO)

$$S_n^{\rm corr} = 0 \quad \Rightarrow \quad \boxed{S_n(\Omega) \equiv h_n^2(\Omega) \geq S_n^{\rm SQL}(\Omega)}$$

Toward quantum-limited GW interferometers

$h_{\rm SQL} = 2 \times 10^{-24}/{\rm Hz}^{1/2}$ at $f = 100$ Hz



Building up correlations by changing the internal dynamics

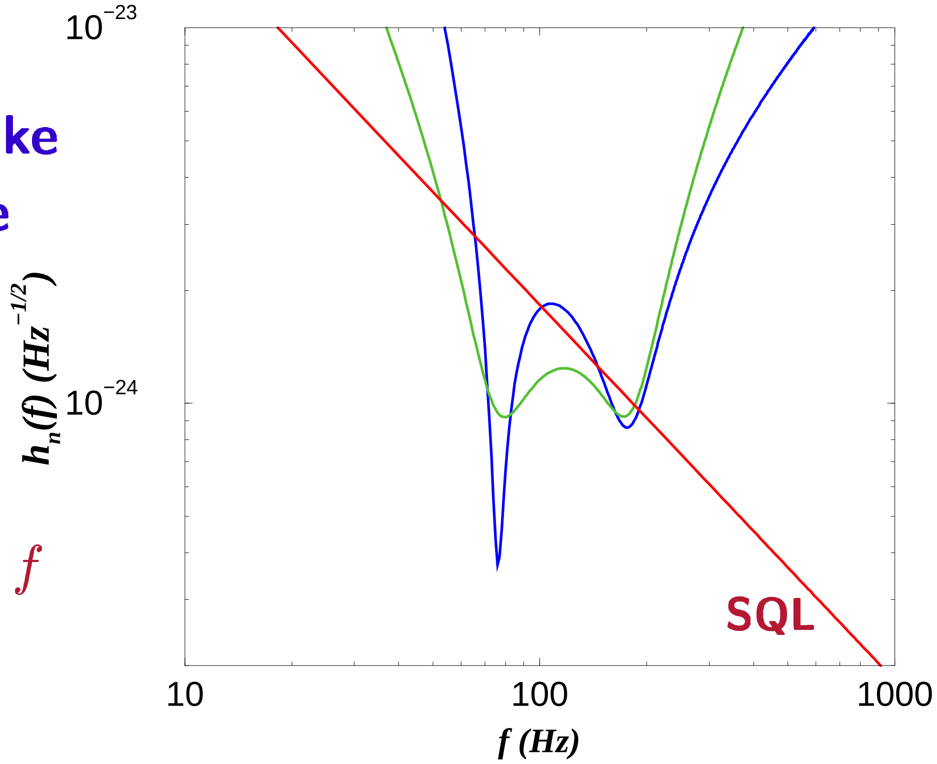
- **Signal-recycling interferometers: LIGO-II** (2007)

  [Drever ‘82; Vinet et al ‘88; Meers ‘88; Mizuno et al. ‘93]

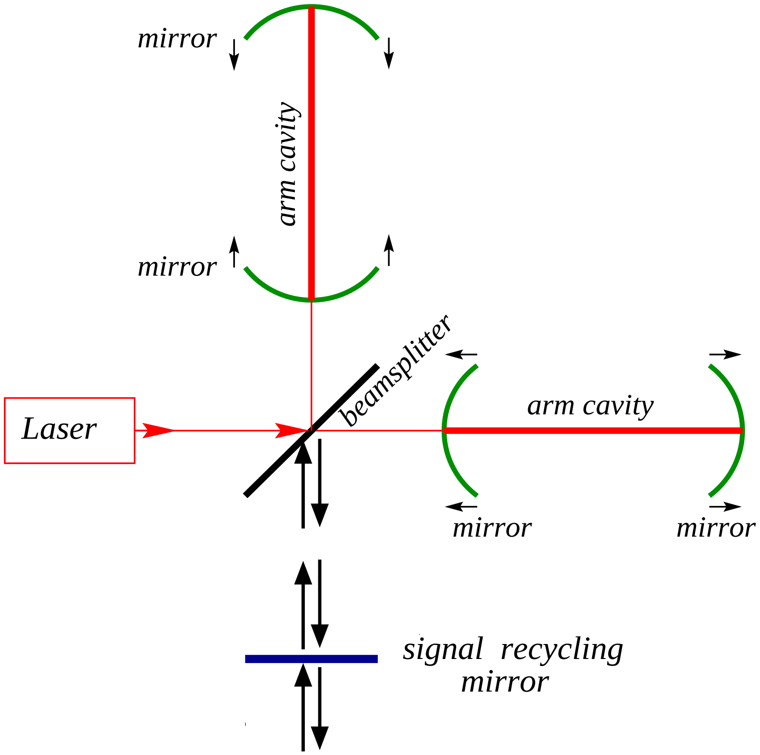
- **With high light power is crucial to take into account radiation-pressure force**

  [A.B. & Chen ‘00, ‘01]

$h_n^{\rm LIGOII}/h_{\rm SQL} \simeq 0.5$ over band of $\Delta f \sim f$



Signal recycled interferometer: LIGO-II (2007)



LIGO-II as an optical spring [A.B. & Chen '00, '01]

Equation of motion for $\widehat{x}(\Omega)$:

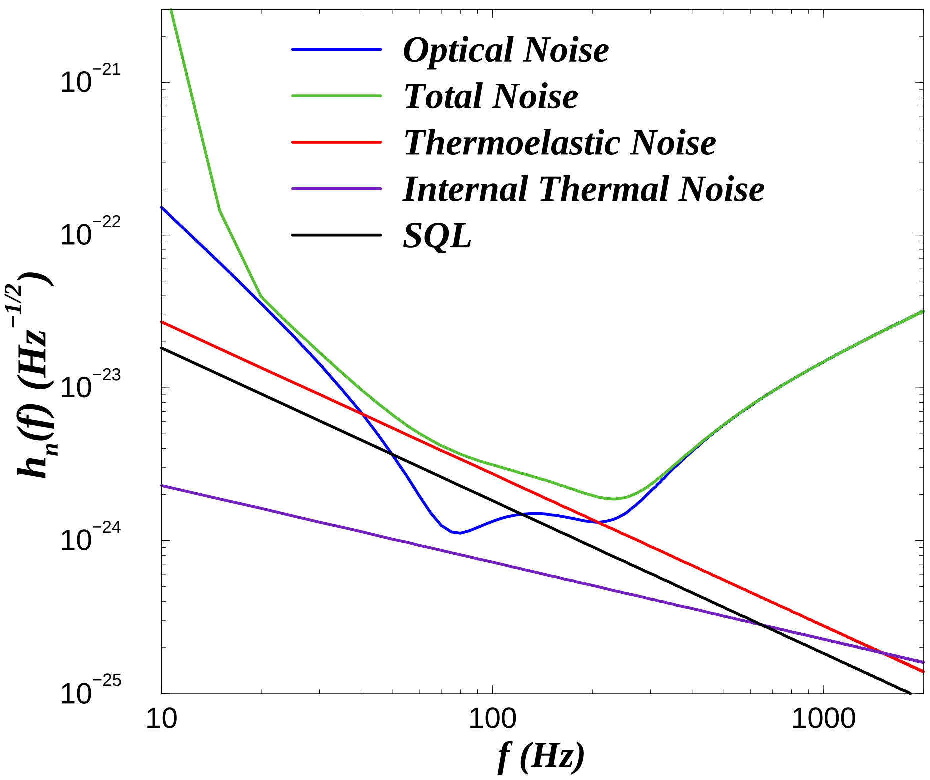
$$-\mu\,\Omega^2\,\widehat{x}(\Omega) = \mathrm{GW\ Force} + \overbrace{[\widehat{F}_0(\Omega) - K_{\mathrm{spring}}(\Omega)\,\widehat{x}(\Omega)]}^{\mathrm{radiat.-press.\,force}}$$

Test-mass mirrors buffeted by radiation pressure $\widehat{F}_0$, but also subject to harmonic restoring force with frequency-dependent spring constant

Optical-mechanical resonances: no longer free test-mass!

Quantum-optical noise augmented by other sources of noise

Current estimate of internal thermal, thermoelastic and seismic noises



Science in LIGOII (2007)

[Thorne '01]

Rate NS/NS at 300 Mpc:

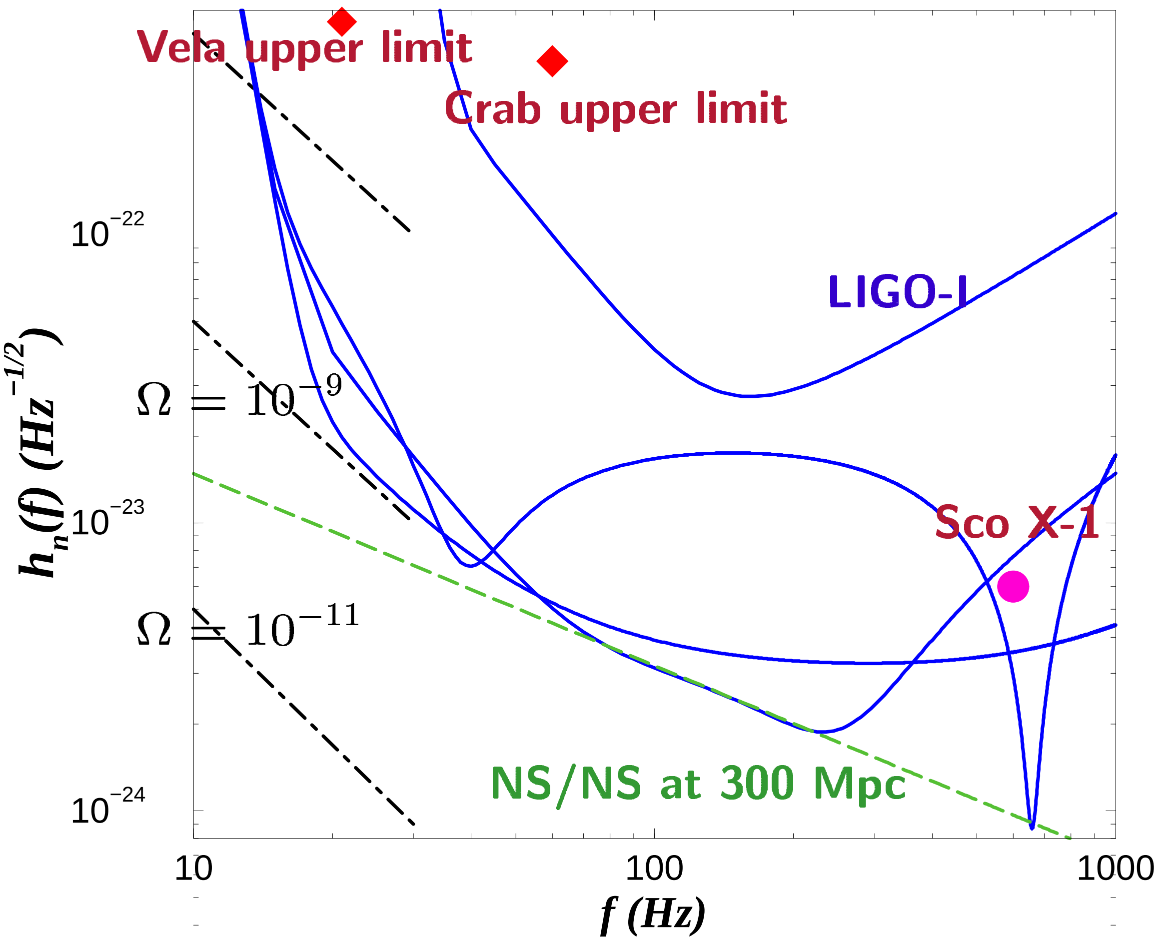
$1/$ yr to $2/$ day

Rate BH/BH at $z = 0.4$:

$2/$ month to $10/$ day

Rate BH/NS at 650 Mpc:

$1/$ yrs to $4/$ day



How to improve at low frequency ($\sim 10 - 10^2$ Hz)

- **Thermal noise**
  - **Cryogenic techniques**

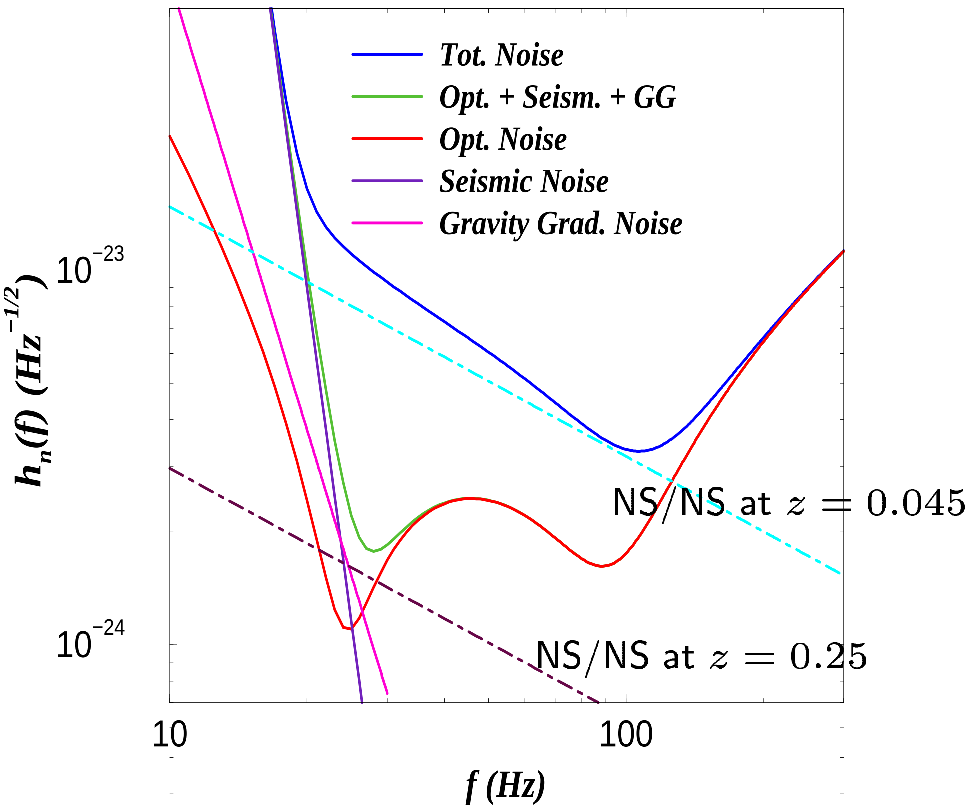
  (TAMA, Glasgow, ...)

- **Radiation-pressure noise**
  - **Larger mirror masses** $\sim 100 - 200$ **Kg**
  - **Low laser power**

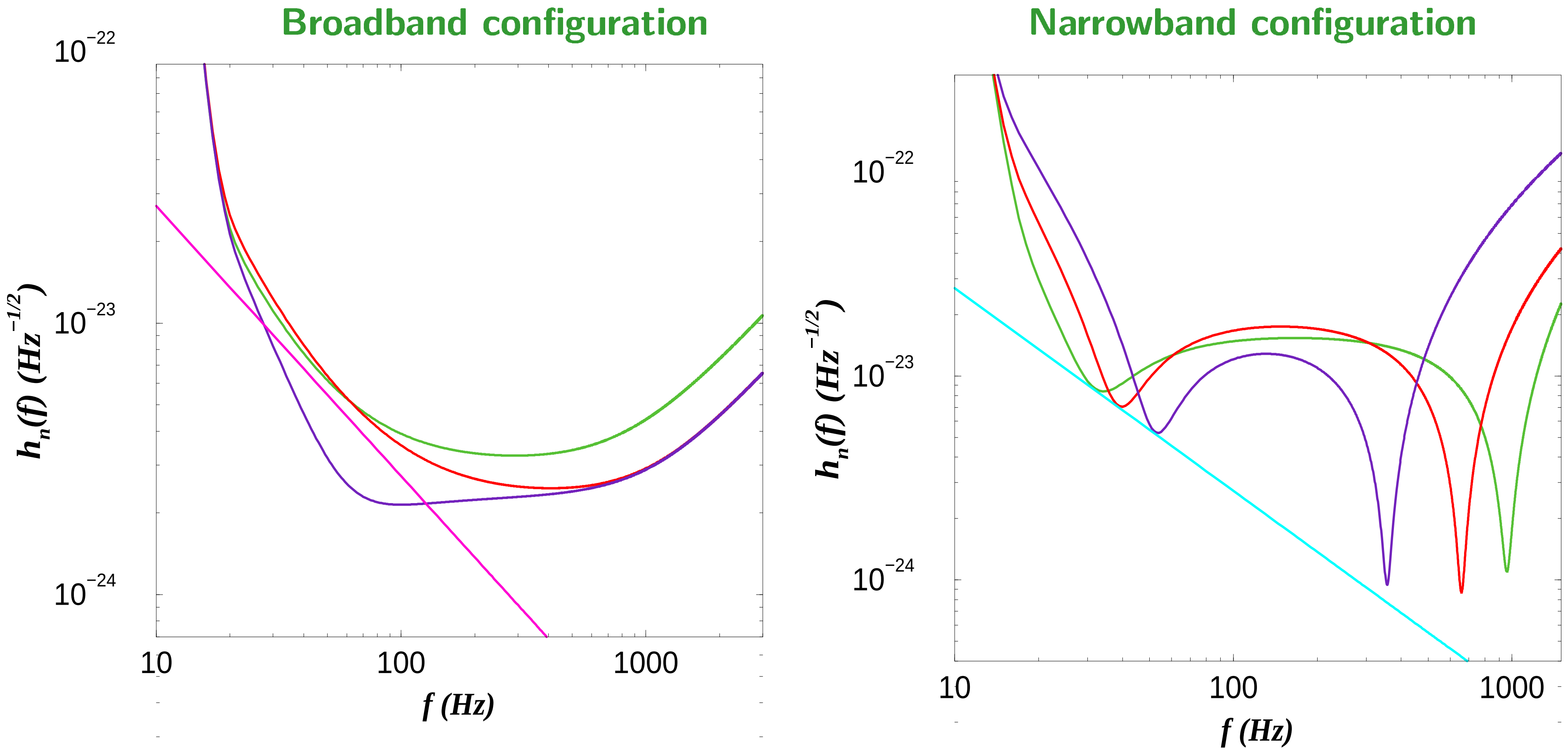
- **Seismic noise**

- **Seismic gravity-gradient noise**

[Hughes & Thorne '98, Cella & Cuoco '98]



How to improve at high frequency ($\sim 10^2 - 3 \times 10^3$ Hz)



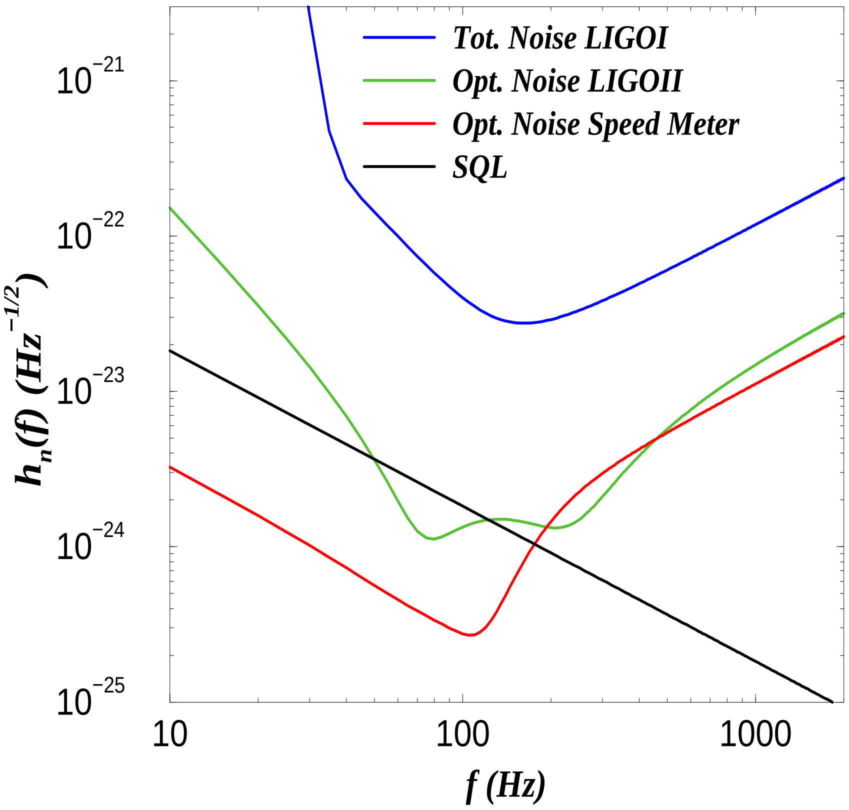
Speed meter

Output signal proportional to the relative speeds of test masses rather than relative positions

[Braginsky, Gorodetsky, Khalili & Thorne '99]

New optical topologies

[Chen & Purdue, work in progress]



[Purdue '01]

Summary

- Gravitational-wave research: joint effort of high-energy physicists, astrophysicists, relativists and experimentalists
- Interesting astrophysics from direct detection of gravitational waves
- Binary black holes: delicate issue of late dynamical evolution
- Advanced GW detectors: quantum mechanical formalism to describe optical-mechanical noise and build up correlations
- For the years to come: reduce thermal noise, use low power circulating in arm cavities and ... new designs!